Complex Transformation IFS for Tiling Levy Curve

The Iterated Function System (IFS) for the Levy curve is defined by the following set of complex transformations:

$$\begin{cases} f_1(z) = (0.5 - 0.5i)(z - (-0.5 + 0.5i)) + (-0.5 + 0.5i) \\ f_2(z) = (0.5 - 0.5i)e^{i\pi/2}(z - (-0.5 + 0.5i)) + (0.5 - 0.5i) + (-0.5 + 0.5i) \end{cases} \tag{1}$$

The completed curve is rotated by multiples of 90° to construct the unit tile. Tiles are then shifted by increments of one unit to tile the plane.

The rotations and shifts are applied according to:

$$\begin{cases} R_{k,a,b}(z) = ze^{i\pi k/2} + (a + bi) \\ \text{where } k \in \{0, 1, 2, 3\}, \text{ and } a, b \in \mathbb{Z} \end{cases} \tag{2}$$

The complete Levy Mosaic is generated by applying f_1 and f_2 to the initial point, followed by all possible combinations of rotations and shifts $R_{k,a,b}$.

Explanation of Tiling

The tiling system works in two distinct steps:

1. **Initial Transformations:** First, we apply f_1 and f_2 iteratively to create the Levy curve.

2. **Symmetry Rotations:** After generating all points, we rotate the curve by multiples of 90°:

   - $k = 0$ gives $e^0 = 1$ (no rotation)
   - $k = 1$ gives $e^{i\pi/2} = i$ (90° rotation)
   - $k = 2$ gives $e^{i\pi} = -1$ (180° rotation)
   - $k = 3$ gives $e^{3i\pi/2} = -i$ (270° rotation)

   These four rotations create the square symmetry of the basic tile.

Finally, we create the tiling pattern by shifting each rotated point by integer combinations (a, b) where $a, b \in \mathbb{Z}$, allowing the pattern to extend infinitely in all directions.