REPORT

PhD in Mechanical Engineering

Validation of UMMDp

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The UMMDp library is prepared to use several yield criteria, isotropic and kinematic hardening laws. However, its implementation is not complete and few validations of the library exist. In this report a comparison between different user subroutines is carried out, with its focus on a non-linear kinematic hardening law and Yld2004-18p anisotropic yield criterion and a. A few modifications are implemented in the user subroutines to improve their performance, in addition to the implementation of a new kinematic hardening law into UMMDp. The performed numerical simulations to validate the Yld2004-18p yield criterion and the new kinematic hardening law of UMMDp demonstrate a good agreement in the results comparatively to the other subroutines.

KEYWORDS

UMMDp, Abaqus Standard, UMAT, Yld2004-18p, kinematic hardening

1 | INTRODUCTION

Unified Material Model Driver for Plasticity (UMMDp) was developed by JANCAE[1]. UMMDp is an open-source user subroutine library for plasticity models, that supports various isotropic and kinematic hardening laws, as well as several finite element software – Abaqus, ANSYS, ADINA, LS-DYNA and MSC.Marc (see Figure 1). Additionally, it is very flexible and easy for any user to implement their own models into the library [1, 2, 3, 4]. The library presents large potential for its use, but it few validations of its models are reported and none outside the original authors are found. Moreover, the authors state that kinematic hardening laws are implemented in UMMDp, but the equations they present do not agree with that statement and the available version on the website is not prepared to be used with kinematic hardening laws, even though its formulations are implemented. However, with small modifications it seems possible to prepare UMMDp for mixed hardening laws.

[1]The Japan Association for Nonlinear Computer Aided Engineering (JANCAE), a non-profit organization, has held lectures twice a year for four days each since 2001 to provide training on theories and technology related to nonlinear simulation for researchers and engineers engaged in various CAE businesses. Since 2009, as one of the subcommittee activities, it has developed and verified the user subroutine library the Unified Material Model Driver for Plasticity (UMMDp).

In order to validate some of the models in UMMDp, a developed material user subroutine for Abaqus Standard is used. This material user subroutine was initially developed by Grilo [5] and afterwards by Souto [6]. This material user subroutine has implemented the Yld2004-18p yield criterion, various isotropic hardening laws and a kinematic hardening law composed of three back-stress terms, hereon designated by UMAT_Yld2004_Mixed. The material user subroutine has been used in some works for material identification (e.g. [7, 8]), therefore providing as a reliable comparison in the validation of UMMDp. As a second reference it is used the built-in constitutive equations provided by Abaqus Standard, hereon designated by Std_Aba.

The report starts by discussing the validation of the Yld2004-18p yield criterion implemented both in UMMDp and UMAT_Yld2004_Mixed, with some focus on the formulation of both material user subroutines. Afterwards, the implementation of a new kinematic hardening law into UMMDp is presented, as well as discussing implemented modifications to the original code of UMAT_Yld2004_Mixed.

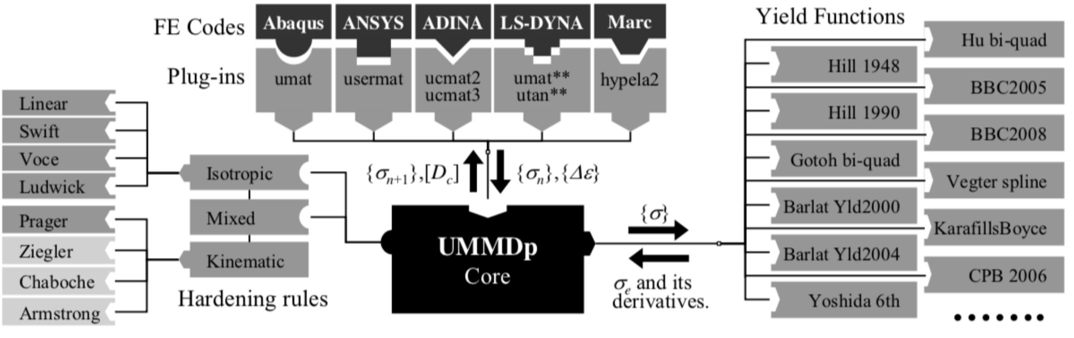


FIGURE 1 Framework of the Unified Material Model Driver for Plasticity (UMMDp) [1].

2 | FORMULATION OF Yld2004-18p

The yield criterion proposed by Barlat *et al.* [9] is given by,

$$\phi = \sum_{i=1}^{3} \sum_{j=1}^{3} \left| \tilde{\mathbf{S}}_i^{(1)} - \tilde{\mathbf{S}}_j^{(2)} \right|^a = 4\bar{\sigma}^a, \tag{1}$$

where ϕ is the yield function, a is a material parameter, $\bar{\sigma}$ the equivalent stress, and $\tilde{\mathbf{S}}_i^{(1)}$ and $\tilde{\mathbf{S}}_j^{(2)}$ are the principal values of the tensors $\tilde{\mathrm{s}}^{(1)}$ and $\tilde{\mathrm{s}}^{(2)}$ defined by two linear transformations on the stress deviator $\mathbf{s}$,

$$\begin{aligned} \tilde{\mathbf{s}}^{(1)} &= \mathbf{C}^{(1)}\mathbf{s} = \mathbf{C}^{(1)}\mathbf{T}\boldsymbol{\sigma} = \mathbf{L}^{(1)}\boldsymbol{\sigma} \\ \tilde{\mathbf{s}}^{(2)} &= \mathbf{C}^{(2)}\mathbf{s} = \mathbf{C}^{(2)}\mathbf{T}\boldsymbol{\sigma} = \mathbf{L}^{(2)}\boldsymbol{\sigma}, \end{aligned} \tag{2}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\mathbf{C}^{(1)}$ and $\mathbf{C}^{(2)}$ are the matrices containing the anisotropy coefficients, have the form of C below, and $\mathbf{T}$ the transformation matrix of the stress tensor to its deviator $\mathbf{s}$, according to:

$$\mathbf{C} = \begin{bmatrix} 0 & -c_{12} & -c_{13} & 0 & 0 & 0 \\ -c_{21} & 0 & -c_{23} & 0 & 0 & 0 \\ -c_{31} & -c_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}, \tag{3}$$

$$\mathbf{T} = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}. \tag{4}$$

In respect to the formulation of Yld2004-18p implemented in UMMDp and UMAT_Yld2004_Mixed there are two important details to discuss: (i) the order of the anisotropy coefficients and (ii) the formulation of the yield function. Firstly, it is important to note that in the formulation of Yld2004-18p in UMMDp the order of the anisotropy coefficients differ from the one in the original paper (see Equation 3). This difference is related to the Voigt notation of the stress tensor, which is given by $(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{yz}, \sigma_{zx}, \sigma_{xy})$ in the original paper and by $(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{zx})$ in UMMDp, while the formulation in UMAT_Yld2004_Mixed is known to be the same as in the original paper. This difference in the Voigt notation changes the order of the anisotropy coefficients c_{44}, c_{55} and c_{66} as follows:

$$\begin{bmatrix} c_{44} \\ c_{55} \\ c_{66} \end{bmatrix}_{\text{UMMDp}} \equiv \begin{bmatrix} c_{66} \\ c_{44} \\ c_{55} \end{bmatrix}_{\text{Barlat } et\ al.} \equiv \begin{bmatrix} c_{66} \\ c_{44} \\ c_{55} \end{bmatrix}_{\text{UMAT\_Yld2004\_Mixed}}. \tag{5}$$

Secondly, in the original formulation of the yield function (see Equation 1) the value 4 is constant, but in the formulation of UMMDp this value takes the form of a coefficient here defined as R as follows:

$$\phi = \sum_{i=1}^{3} \sum_{j=1}^{3} \left| \tilde{\mathbf{S}}_i^{(1)} - \tilde{\mathbf{S}}_j^{(2)} \right|^a = R\bar{\sigma}^a. \tag{6}$$

In the formulation of UMMDp this coefficient depends on the values of the material parameters $c_{12}^{(k)}$, $c_{13}^{(k)}$, $c_{21}^{(k)}$, $c_{23}^{(k)}$, $c_{31}^{(k)}$ and $c_{32}^{(k)}$, $k = 1, 2$, and is computed according to the equations presented in Appendix A, deducted from UMMDp source code. The reason for this formulation is not documented, but it seems related to the fact that under uniaxial loading the following condition can be imposed:

$$\frac{\sigma_{11}}{\bar{\sigma}} = \left(\frac{3^a R}{COEF} \right) = 1, \tag{7}$$

where σ_{11} is the uniaxial component of the stress tensor and $COEF$ is given by:

$$
\begin{aligned}
COEF = & \left|c_{12}^{(1)} + c_{13}^{(1)} - c_{12}^{(2)} - c_{13}^{(2)}\right|^a + \left|c_{12}^{(1)} + c_{13}^{(1)} + 2c_{21}^{(2)} - c_{23}^{(2)}\right|^a + \left|c_{12}^{(1)} + c_{13}^{(1)} - 2c_{31}^{(2)} - c_{32}^{(2)}\right|^a \\
& + \left|c_{23}^{(1)} - 2c_{21}^{(1)} - c_{12}^{(2)} - c_{13}^{(2)}\right|^a + \left|c_{23}^{(1)} - 2c_{21}^{(1)} + 2c_{21}^{(2)} - c_{23}^{(2)}\right|^a + \left|c_{23}^{(1)} - 2c_{21}^{(1)} + 2c_{31}^{(2)} - c_{32}^{(2)}\right|^a \\
& + \left|c_{32}^{(1)} - 2c_{31}^{(1)} - c_{12}^{(2)} - c_{13}^{(2)}\right|^a + \left|c_{32}^{(1)} - 2c_{31}^{(1)} + 2c_{21}^{(2)} - c_{23}^{(2)}\right|^a + \left|c_{32}^{(1)} - 2c_{31}^{(1)} + 2c_{31}^{(2)} - c_{32}^{(2)}\right|^a .
\end{aligned} \tag{8}
$$

If the condition given in Equation 7 is imposed to a given set of parameters, the coefficient R in UMMDp takes the original value of 4, otherwise the value of R is such that the condition is verified. This approach seems to be more practical in terms of an optimization procedure, as it reduces the number of externally imposed constraints. In order to validate this assumption a comparison between both implementations is performed for the case of uniaxial loading, with σ_{11} = 100 MPa, and using Yld2004-18p material parameters reported by Souto *et al.* [7] (see Table 1), with the exponent a equal to 6.

TABLE 1 Set of Yld2004-18p material parameters used in the verification of coefficient R [7].

$c_{12}^{(1)}$	$c_{13}^{(1)}$	$c_{21}^{(1)}$	$c_{23}^{(1)}$	$c_{31}^{(1)}$	$c_{32}^{(1)}$	$c_{44}^{(1)}$	$c_{55}^{(1)}$	$c_{66}^{(1)}$
1.264	0.974	1.242	1.049	0.579	0.708	1.000	1.000	1.365
$c_{12}^{(2)}$	$c_{13}^{(2)}$	$c_{21}^{(2)}$	$c_{23}^{(2)}$	$c_{31}^{(2)}$	$c_{32}^{(2)}$	$c_{44}^{(2)}$	$c_{55}^{(2)}$	$c_{66}^{(2)}$
0.792	0.672	0.838	0.929	0.996	0.768	1.000	1.000	0.678

The obtained results are presented in Table 2, where it is observed that in original formulation of the yield function the Equation 7 is not satisfied whereas in the implementation of UMMDp the condition is satisfied, with R taking a value higher than 4. Although the implementation of the yield function in UMMDp computes the R value, it is possible to assign its value to 4. This way, in order to fairly compare the results between UMAT_Yld2004_Mixed and UMMDp, the constant value of 4 is adopted in all of the following results.

TABLE 2 Comparison of the Yld2004-18p yield function formulation between the original paper and UMMDp, under uniaxial loading.

	Original	UMMDp
$COEF$	3072.416	
R	4.000	4.215
$\sigma_{11}/\bar{\sigma}$	0.991	1.000
$\bar{\sigma}$ [MPa]	100.875	100.000

Additionally, in order to verify the correct order of the anisotropic coefficients a validation test is performed, where it is given as input to the yield criterion a prescribed stress tensor and as output the yield criterion gives the equivalent stress, first and second derivative. By manually giving a stress tensor as input to the yield criterion it is possible to successfully compare its formulation. However, when running a numerical simulation with a finite element software, the input cannot be controlled and it is equal to both UMAT_Yld2004_Mixed and UMMDp. The set of material parameters used in this verification is given in Table 3, according to the values reported for the material 6111-T4 by Barlat *et al.* [9], with the exponent a equal to 8.

In Figures 2 and 3 it is presented the results for the comparisons obtained for UMAT_Yld2004_Mixed and UMMDp. In both comparisons the order of the anisotropy coefficients is maintained according to Equation 5, but for the first comparison the order of the stress tensor out-of-plane components is changed in order to obtain similar values of equivalent stress. Analyzing the results of the first comparison (see Figure 2) it is observed that the equivalent

TABLE 3 Set of Yld2004-18p material parameters used in the verification of the coefficients order [9].

$c_{12}^{(1)}$	$c_{13}^{(1)}$	$c_{21}^{(1)}$	$c_{23}^{(1)}$	$c_{31}^{(1)}$	$c_{32}^{(1)}$	$c_{44}^{(1)}$	$c_{55}^{(1)}$	$c_{66}^{(1)}$
1.241024	1.078271	1.21643	1.223867	1.093105	0.889161	0.501909	0.557173	1.349094
$c_{12}^{(2)}$	$c_{13}^{(2)}$	$c_{21}^{(2)}$	$c_{23}^{(2)}$	$c_{31}^{(2)}$	$c_{32}^{(2)}$	$c_{44}^{(2)}$	$c_{55}^{(2)}$	$c_{66}^{(2)}$
0.775366	0.922743	0.765487	0.793356	0.918689	1.027625	1.115833	1.112273	0.589787

stress is identical for both material user subroutines, but the first and second derivatives present some differences in the out-of-plane components. These differences are not in terms of values, rather in terms of the position at which they appear, which is nonetheless coherent to the order of the stress tensor given as input. Although this situation is not documented in UMMDp documentation, it is understood that the formulation used in the yield criterion cannot be the same for each software which UMMDp is prepared for, as it is shown in Table 4 that there are two different orders in the Voigt notation used by the software. As the software used is Abaqus Standard, the formulation of the yield criterion in UMMDp is modified (see Appendix B) according to the Voigt notation of the stress tensor used in this finite element software $(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{xz}, \sigma_{yz})$, resulting in a new relation of the anisotropy coefficients c_{44}, c_{55} and c_{66} as follows:

$$\begin{bmatrix} c_{44} \\ c_{55} \\ c_{66} \end{bmatrix}_{\text{UMMDp}} = \begin{bmatrix} c_{44} \\ c_{55} \\ c_{66} \end{bmatrix}_{\text{Barlat } et\ al.} = \begin{bmatrix} c_{44} \\ c_{55} \\ c_{66} \end{bmatrix}_{\text{UMAT\_Yld2004\_Mixed}} . \tag{9}$$

TABLE 4 Convention of Voigt notation for each finite element software prepared for UMMDp.

	Voigt Notation
Abaqus Standard	(σ_{xx} σ_{yy} σ_{zz} σ_{xy} σ_{xz} σ_{yz})
ADINA	(σ_{xx} σ_{yy} σ_{zz} σ_{xy} σ_{xz} σ_{yz})
LS-DYNA	(σ_{xx} σ_{yy} σ_{zz} σ_{xy} σ_{yz} σ_{xz})
ANSYS	(σ_{xx} σ_{yy} σ_{zz} σ_{xy} σ_{yz} σ_{xz})
MSC.Marc	(σ_{xx} σ_{yy} σ_{zz} σ_{xy} σ_{yz} σ_{xz})

The second comparison (see Figure 3) is performed in order to validate the modification to the yield criterion in UMMDp. The stress tensor given as input is now equal for both material user subroutines as well as the order of the anisotropy coefficients according to Equation 9. The results show that all of the output variables are exactly the same in UMAT_Yld2004_Mixed and UMMDp, therefore validating the modification which will be maintained used in all of the following results.

3 | IMPLEMENTATION OF KINEMATIC HARDENING LAW IN UMMDp

As stated above, the authors state that UMMDp is prepared to be used with kinematic hardening laws. However, the source code available online is not prepared for the use of mixed hardening laws. In order to be able to use them, it is necessary to complete some parts of the source code, in particular the subroutine where the dimension of properties are defined for each law (yield criterion, isotropic hardening and kinematic hardening). In Listing 1 a snippet is shown of this subroutine, where in Listing 1a the original subroutine is presented and in Listing 1b the applied modifications.

Moreover, in order to be able to compare the results with UMAT_Yld2004_Mixed, it is necessary to implement a

	INPUT	OUTPUT							
	$\sigma =$	$\bar{\sigma} =$	$\partial\phi/\partial\sigma =$	$\partial^2\phi/(\partial\sigma)^2 =$					
UMAT _Yld2004 _Mixed	-149	355.769	-0.632	0.004	-0.002	-0.001	0.000	0.000	0.003
	-5		0.266	-0.002	0.003	-0.001	0.001	0.000	-0.001
	35		0.366	-0.001	-0.001	0.002	0.000	0.000	-0.001
	-12		-0.168	0.000	0.001	0.000	0.017	-0.002	0.001
	-19		-0.120	0.000	0.000	0.000	-0.002	0.008	0.000
	204		1.205	0.003	-0.001	-0.001	0.001	0.000	0.002
UMMDp	-149	355.769	-0.632	0.004	-0.002	-0.001	0.000	0.003	0.000
	-5		0.266	-0.002	0.003	-0.001	0.001	-0.001	0.000
	35		0.366	-0.001	-0.001	0.002	0.000	-0.001	0.000
	-12		-0.168	0.000	0.001	0.000	0.017	0.001	-0.002
	204		1.205	0.003	-0.001	-0.001	0.001	0.002	0.000
	-19		-0.120	0.000	0.000	0.000	-0.002	0.000	0.008

FIGURE 2 Comparison of the yield criterion formulation for UMAT_Yld2004_Mixed and UMMDp without modification.

	INPUT	OUTPUT							
	$\sigma =$	$\bar{\sigma} =$	$\partial\phi/\partial\sigma =$	$\partial^2\phi/(\partial\sigma)^2 =$					
UMAT _Yld2004 _Mixed	-149	355.769	-0.632	0.004	-0.002	-0.001	0.000	0.000	0.003
	-5		0.266	-0.002	0.003	-0.001	0.001	0.000	-0.001
	35		0.366	-0.001	-0.001	0.002	0.000	0.000	-0.001
	-12		-0.168	0.000	0.001	0.000	0.017	-0.002	0.001
	-19		-0.120	0.000	0.000	0.000	-0.002	0.008	0.000
	204		1.205	0.003	-0.001	-0.001	0.001	0.000	0.002
UMMDp	-149	355.769	-0.632	0.004	-0.002	-0.001	0.000	0.000	0.003
	-5		0.266	-0.002	0.003	-0.001	0.001	0.000	-0.001
	35		0.366	-0.001	-0.001	0.002	0.000	0.000	-0.001
	-12		-0.168	0.000	0.001	0.000	0.017	-0.002	0.001
	-19		-0.120	0.000	0.000	0.000	-0.002	0.008	0.000
	204		1.205	0.003	-0.001	-0.001	0.001	0.000	0.002

FIGURE 3 Comparison of the yield criterion formulation for UMAT_Yld2004_Mixed and UMMDp with modification.

```
c------------------------------------------------------------------------------
c      set dimensions of material properties
c
       subroutine jancae_prop_dim ( prop,mxprop,ndela,ndyld,ndihd,ndkin,npbs )
c------------------------------------------------------------------------------
       (...)
       n=ndela+ndyld+ndihd
       nkin=nint(prop(n+1))
       select case (nkin)
         case ( 0) ; nd= 0                          ! No Kinematic Hardening
         case default
           write (6,*) 'error␣kinematic␣hardening␣id␣:',nkin
           call jancae_exit ( 9000 )
       end select
       ndkin=nd+1
       npbs=0
c
       return
       end
```

(a)

```
c------------------------------------------------------------------------------
c      set dimensions of material properties
c
       subroutine jancae_prop_dim ( prop,mxprop,propdim,ndela,ndyld,ndihd,ndkin,npbs )
c------------------------------------------------------------------------------
       (...)
       n=ndela+ndyld+ndihd
       nkin=nint(prop(n+1))
       select case (nkin)
         case ( 0) ; nd= 0 ; npbs= 0                ! No Kinematic Hardening
         case ( 1) ; nd= 1 ; npbs= 1                ! Prager
         case ( 2) ; nd= 1 ; npbs= 1                ! Ziegler
         case ( 3) ; nd= 2 ; npbs= 1                ! Armstrong & Frederic
         case ( 4) ; nd= propdim-(n+1) ; npbs= nd/2 ! Chaboche (1979)
         case ( 5) ; nd= 5 ; npbs= 2                ! Yoshida-Uemori
         case default
           write (6,*) 'error␣kinematic␣hardening␣id␣:',nkin
           call jancae_exit ( 9000 )
       end select
       ndkin=nd+1
c
       return
       end
```

(b)

LISTING 1 Snippet of UMMDp subroutine for the definition of properties dimension (a) original and (b) modified.

new kinematic hardening law into UMMDp. In UMMDp, the implemented kinematic hardening law is based on the additive contribution of several terms proposed based on the strain tensor as proposed by Chaboche [10] following a modification of the kinematic hardening law proposed by Armstrong and Frederick (1966) [11]. The evolution of the backstress tensor $\boldsymbol{\alpha}$ from the additive contribution of each backstress term $\boldsymbol{\alpha}_i$ expressed in terms of the strain tensor is given by

$$\dot{\boldsymbol{\alpha}} = \sum_{i=1}^{n} \dot{\boldsymbol{\alpha}}_i = \sum_{i=1}^{n} \left(\frac{2}{3} C_i \dot{\boldsymbol{\epsilon}}^{\mathrm{p}} - \gamma_i \boldsymbol{\alpha}_i \dot{p} \right), \tag{10}$$

where C_i and $\gamma_i, i = 1, ..., n$, are material dependent parameters, $\boldsymbol{\epsilon}^{\mathrm{p}}$ is the plastic strain tensor and p is the equivalent plastic strain. In opposition, the kinematic hardening law implemented in UMAT_Yld2004_Mixed, is derived from the formulation of Chaboche, but based on the stress tensor. Carbonnière *et al.* [12] present an explanation for this difference in both types of formulation, naming the one based on the stress tensor Ziegler's model, because it leads to an uncoupling between the initial anisotropy and the hardening behavior, which is not the case when using the Armstrong–Frederick representation. The evolution of the backstress tensor expressed in terms of the Ziegler model is given by:

$$\dot{\boldsymbol{\alpha}} = \sum_{i=1}^{n} \dot{\boldsymbol{\alpha}}_i = \sum_{i=1}^{n} \left(\frac{C_i}{\bar{\eta}} \boldsymbol{\eta} - \gamma_i \boldsymbol{\alpha}_i \right) \dot{p}, \tag{11}$$

where $\bar{\eta}$ is the equivalent stress which expresses the equivalent value of the tensor $\boldsymbol{\eta} = \boldsymbol{\sigma} - \boldsymbol{\alpha}$. The implementation in UMMDp for this kinematic hardening law is shown in Listing C.1. Additionally, it is required to add information about this new law into the subroutine provided to define the dimension of material properties, as well as other subroutines used to debug and print information.

4 | MODIFICATIONS IN UMAT

This section is related to modifications introduced to UMAT_Yld2004_Mixed, which were identified as necessary after running some tests with the different material user subroutines. Two observations are pointed out regarding UMAT_Yld2004_Mixed subroutine. The first one is concerned with the elastic/plastic transition, which in the original version of the material user subroutine is verified if the yield condition is smaller than a prescribed tolerance *Tol*. Even though the original implementation is not wrong, as a residual tolerance can be used, the computed value for this tolerance was always equal to 1, in case the unit used for stress is MPa, resulting in an error of 1 MPa for the yield stress. In Listing 2a it is presented a snippet of the code in UMAT_Yld2004_Mixed corresponding to this condition and in Listing 2b the modifications applied to the code in order to avoid this error in the yield stress.

To validate this modification, a tensile test is simulated using a single 3D element of 1 mm^3 with boundary conditions according to Figure 6, where it is used the von Mises yield criterion with isotropic hardening. In the numerical simulation, small increments are used in order to better understand the impact of the applied modification. In Figure 4 it is shown the stress-strain curve for the tensile test, where no visible differences are observed and both converge to the same values of stress. However, looking closer to the yield stress point it is visible that with the original formulation it presents difficulties of convergence after the yield stress point, while with the modified formulation it presents a smooth evolution in stress. Therefore, it is possible to assume that the implemented modification corrects the observed problems in the original subroutine.

The second observation was identified when trying to perform a numerical simulation of a cup deep drawing. Ini-

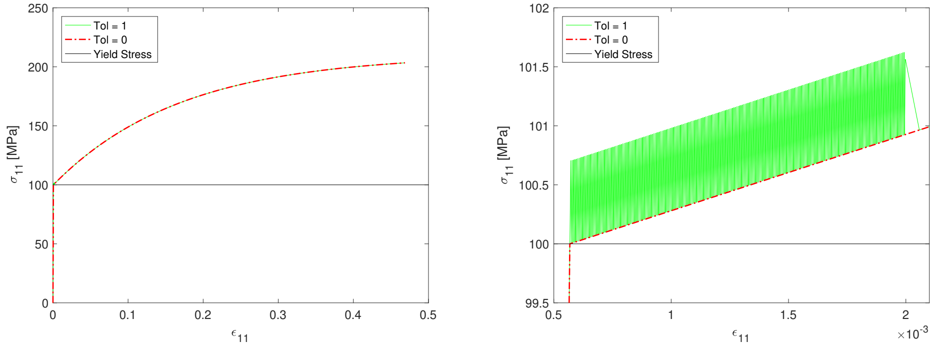
```
Tol=aint((1.0D-7)*STRSS_LIM)+1
RES_ACT=STRESSEF_LIN-STRSS_LIM
if (RES_ACT < Tol) then
  write (*,*) 'State = Elastic'
  STRESSEFk = STRESSEF_LIN
  goto 20
else
  write (*,*) 'State = Plastic'
  goto 10
end if
```

(a)

```
RES_ACT=STRESSEF_LIN-STRSS_LIM
if (RES_ACT < 0) then
  write (*,*) 'State = Elastic'
  STRESSEFk = STRESSEF_LIN
  goto 20
else
  write (*,*) 'State = Plastic'
  goto 10
end if
```

(b)

LISTING 2 Snippet of UMAT_Yld2004_Mixed source code for the elastic/plastic transition: (a) original and (b) modified.



(a) (b)

FIGURE 4 Numerical stress-strain curve of the tensile test with UMAT_Yld2004_Mixed: (a) whole range of strain and (b) range in the yield stress point.

tially, the numerical simulation was successfully performed with UMMDp and Std_Aba, but using UMAT_Yld2004_Mixed it is not able to converge, stopping at the very early beginning. After analyzing the material user subroutine, it was identified a difference between the theoretical multi-stage return mapping algorithm between the one implemented in UMAT_Yld2004_Mixed and the theoretical algorithm presented by Grilo [5] (see Listing 3). Even though there were no assurances that the problem was concerned with this difference, the theoretical algorithm is implemented in UMAT_Yld2004_Mixed to verify its impact in the numerical simulation. It is important to mention that the theoretical algorithm presented in Listing 3 is formulated for the same kinematic hardening law, but with only one backstress term, whereas in UMAT_Yld2004_Mixed the kinematic hardening law has three backstress terms, requiring the additional auxiliary residuum $r^0_{4_k}$ and $r^0_{5_k}$. The observed difference is concerned with the stopping condition of the multi-stage return mapping algorithm, whereas in UMAT_Yld2004_Mixed the only auxiliary residuum being verified is $r^0_{1_k}$ (yield condition), while ignoring the other auxiliary residua $r^0_{2_k}$, $r^0_{3_k}$, $r^0_{4_k}$ and $r^0_{5_k}$ that should as well converge to zero, within a prescribed tolerance (see Listing 4a). According to Grilo [5], the stopping condition takes into account all the residua in the shape of a total residuum $\mathbf{r}^0$, which is implemented in UMAT_Yld2004_Mixed to investigate its influence in the results (see Listing 4b). Using the modified UMAT_Yld2004_Mixed to perform the numerical simulation of a cup deep drawing it was possible to finish the analysis, and the results were similar to the analysis performed with UMMDp and Std_Aba (see Section 5.2).

Initiate variables
  $\boldsymbol{\sigma}^{i=0}_{k=0} = \boldsymbol{\sigma}_n$, $\boldsymbol{\alpha}^{i=0}_{k=0} = \boldsymbol{\alpha}_n$, and $\bar{\varepsilon}^{i=0}_{\mathrm{p}_{k=0}} = \bar{\varepsilon}_{\mathrm{p}_n}$

Multi-stage procedure:

DO $k = 1 : N$

1. Define the residuum Φ_k

$$\Phi_k = \left(1 - \frac{k}{N}\right)\Phi^{\mathrm{trial}}$$

2. Compute the auxiliary residua, $r^0_{1_k}$, $\mathbf{r}^0_{2_k}$, and $\mathbf{r}^0_{3_k}$ (Equations 4.81-4.83)
3. Evaluate the root-mean-square value of the total residuum

$$\| \mathbf{r}^0 \| = \sqrt{(r^0_{1_k})^2 + \| \mathbf{r}^0_{2_k} \|^2 + \| \mathbf{r}^0_{3_k} \|^2}$$

4. Iterative procedure ($TOL = 10^{-8}$):

   DO WHILE $\| \mathbf{r} \| > TOL$

   a) Compute the auxiliary variables $\mathbb{E}^i_k$, $\mathbb{A}^i_{1_k}$, $\mathbf{a}^i_{2_k}$, and $\mathbf{a}^i_{3_k}$ (Equations 4.73-4.76)
   b) Compute the increment i of the increment of the plastic multiplier, $\Delta\Delta\lambda^i_k$ (Equation 4.86)
   c) Update the state variables $\Delta\lambda^i_k$, $\boldsymbol{\sigma}^i_k$, and $\boldsymbol{\alpha}^i_k$ (Equations 4.77-4.79)
   d) Re-evaluate the auxiliary residua, $r^i_{1_k}$, $\mathbf{r}^i_{2_k}$, and $\mathbf{r}^i_{3_k}$ and the root-mean-square value of the total residuum, $\| \mathbf{r}^i \|$

   ENDDO

ENDDO

LISTING 3 Scheme of the backward-Euler algorithm with the multi-stage return mapping procedure [5].

To further investigate the impact of the implemented modifications, in particular the evolution of the kinematic hardening, a tensile test is simulated using a single 3D element of 1 mm^3 with boundary conditions according to Figure 6, where it is used the von Mises yield criterion with mixed hardening and very small increments of strain. The isotropic hardening, controlling the size of the yield surface is defined by the Voce law written as

$$\sigma_{\mathrm{Y}}(p) = \sigma_0 + Q\left(1 - \exp(-bp)\right), \tag{12}$$

where σ_{Y} is the yield stress related to the isotropic hardening, σ_0 the initial yield stress, Q and b material parameters. The of parameters used for the isotropic and kinematic hardening laws are given in Table 5. The simulation if performed

```
Tol=aint((1.0D-7)*STRSS_LIM)+1
(...)
G1=STRESSEFk-STRSS_LIMk-Fk
(...)
if (G1<Tol) then
  CONDICAO = .TRUE.
elseif ((G1>=Tol) .and. (G1>=G1temp)) then
  ank=2.d0*ank
  if (ank>300) then
    PNEWDT=0.5d0
    goto 999
  endif
  (...)
endif
```

(a)

```
Tol=1.0D-8
(...)
call EUCLIDEANORM(G2,NTENS,R2)
call EUCLIDEANORM(G3,NTENS,R3)
call EUCLIDEANORM(G4,NTENS,R4)
call EUCLIDEANORM(G5,NTENS,R5)
R0 = (G1**2.0+R2**2.0+R3**2.0+R4**2.0+R5**2.0)**(1.d0/2.d0)
(...)
if (R0<Tol) then
  CONDICAO = .TRUE.
elseif ((R0>=Tol) .and. (R0>=R0temp)) then
  ank=2.d0*ank
  if (ank>300) then
    PNEWDT=0.5d0
    goto 999
  endif
  (...)
endif
(...)
subroutine EUCLIDEANORM(G,NTENS,R)
dimension G(NTENS)
R = 0.d0
do i=1,NTENS
  R = R + G(i)**2
end do
R = R**(1.d0/2.d0)
return
end
```

(b)

LISTING 4 Snippet of UMAT_Yld2004_Mixed source code for the multistage return mapping stopping condition (a) original and (b) modified.

for Std_Aba, UMMDp and UMAT_Yld2004_Mixed, with and without the implemented modifications. The results are presented in Figure 5c, where it is observed that the level of stress is similar for each simulation (see Figure 5a). However, analyzing the results for the evolution of the components of the total backstress (α_{11} and $\alpha_{22} = \alpha_{33}$) visible differences compared to other simulations are observed for the UMAT_Yld2004_Mixed in its original formulation, whereas in the modified UMAT_Yld2004_Mixed the difference in α_{11} is not visible and α_{22} is closer to zero as it would be expected (see Figure 5b). Even though these differences are observed, they do not affect the stress level as it is calculated using the backstress deviator $\boldsymbol{\alpha}^{'}$ and from Figure 5c it is possible to observe that there are no visible differences between the simulations. These results in UMAT_Yld2004_Mixed are directly associated with the residuum of the back stress terms, which are used in the calculation of the backstress increments, so that when the residua are not able to converge to zero, its values affect the values of the backstress terms [6]. It is a fact that the implemented modification in the residuum does not alter the stress levels in the case of simple tests (e.g. tensile, shear and biaxial test with a single element), but as it is previously mentioned without the modification the cup drawing test is not able to converge. As the implemented modification proved to be successful, it is adopted in all of the following results, as well as the modification of the elastic/plastic transition.

TABLE 5 Parameters of Voce law and kinematic hardening law.

Voce law					
σ_0 [MPa]		Q [MPa]		b	
100		110.3		5.92	
Kinematic hardening law					
C_1 / γ_1 [MPa]	γ_1	C_2 / γ_2 [MPa]	γ_2	C_3 / γ_3 [MPa]	γ_3
44.57	22.85	106.2	258.38	5629.7	0.0258

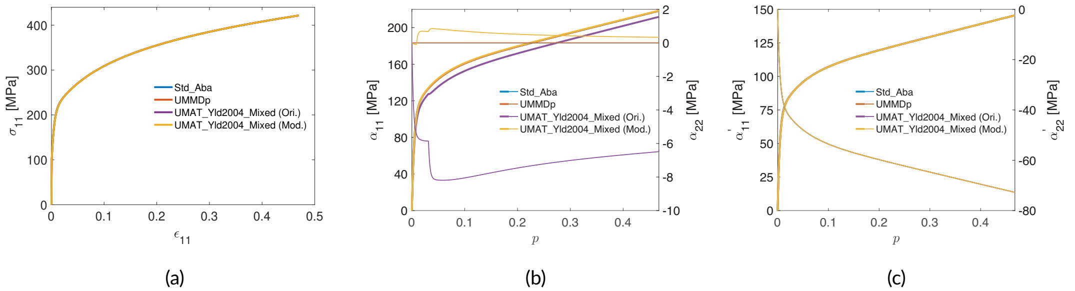


FIGURE 5 Results of tensile test with von Mises yield criterion and mixed hardening for Std_Aba, UMMDp and UMAT_Yld2004_Mixed with and without residua modifications: (a) ϵ_{11} - σ_{11}, (b) p - $\alpha_{11,22}$ and (c) p - $\alpha^{'}_{11,22}$.

5 | NUMERICAL SIMULATIONS

After having modified UMAT_Yld2004_Mixed and UMMDp, and implemented the new kinematic hardening law into UMMDp, it is possible to directly compare the results of Abaqus, UMAT and UMMDp with numerical simulations of homogeneous tests and a cup deep drawing. The results are compared for von Mises isotropic yield criterion and Yld2004-18p anisotropic yield criterion, using the parameters given in Table 3. The Voce law is used for isotropic hardening (see Equation 12) and the kinematic hardening law of Equation 11, using the parameters given in Table 5.

5.1 | Homogeneous Tests

The numerical models of the homogeneous tests (tensile, shear and biaxial) are modeled with a single 3D element of 1 mm^3 with boundary conditions according to Figure 6. In order to validate UMMDp several simulations are performed, using different combinations of yield criterion, isotropic and kinematic hardening laws, as well as different material orientations from rolling direction (RD), according to:

- Tensile, shear and biaxial test with von Mises isotropic yield criterion and mixed hardening;
- Tensile, shear and biaxial test with Yld2004-18p anisotropic yield criterion and isotropic hardening with material orientation at 0º from RD;
- Tensile and shear test with Yld2004-18p anisotropic yield criterion and isotropic hardening with material orientation at 45º from RD;
- Tensile, shear and biaxial test with Yld2004-18p anisotropic yield criterion and mixed hardening with material orientation at 0º from RD;
- Tensile and shear test with Yld2004-18p anisotropic yield criterion and mixed hardening with material orientation at 45º from RD;
- Tensile test with Yld2004-18p anisotropic yield criterion and mixed hardening with material orientation at 0º, 15º, 30º, 45º, 60º, 75º and 90º from RD.

The results of the simulations are presented in Figures 7 to 12, where it is observed that for every simulation the results are identical between UMMDp, UMAT_Yld2004_Mixed and Std_Aba.

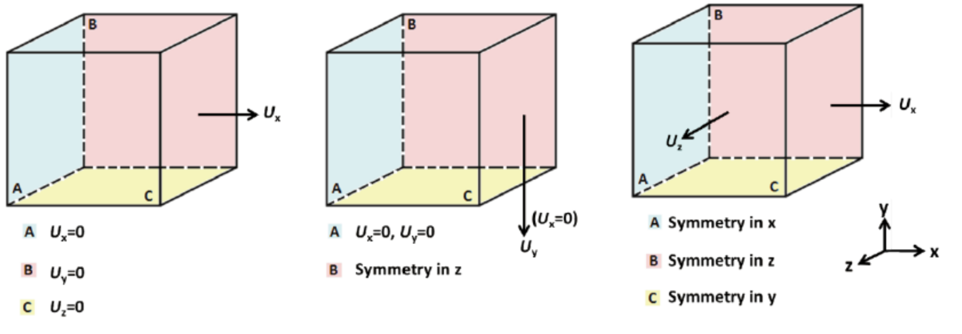
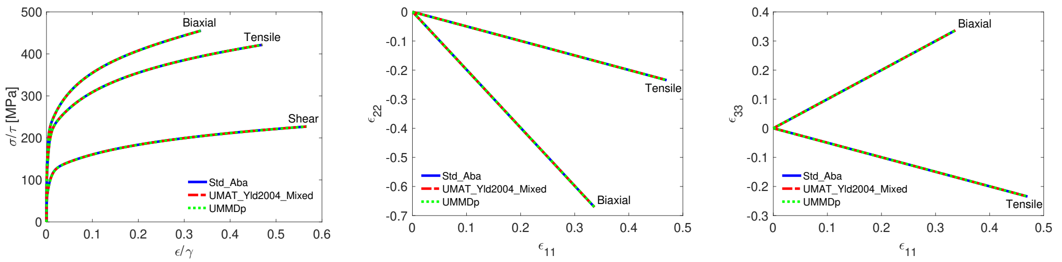
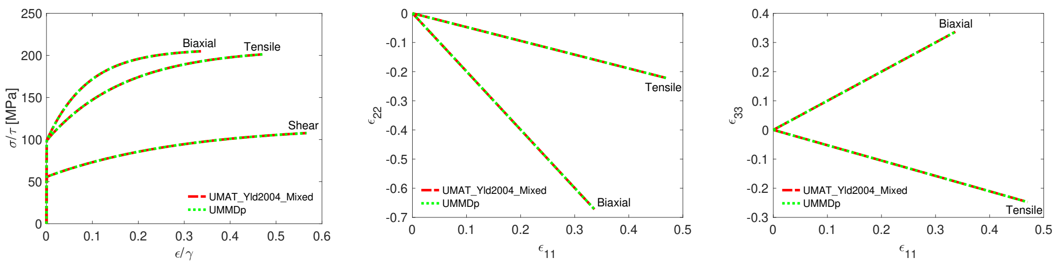


FIGURE 6 Boundary conditions applied on the numerical models of tensile, shear and biaxial test [6].



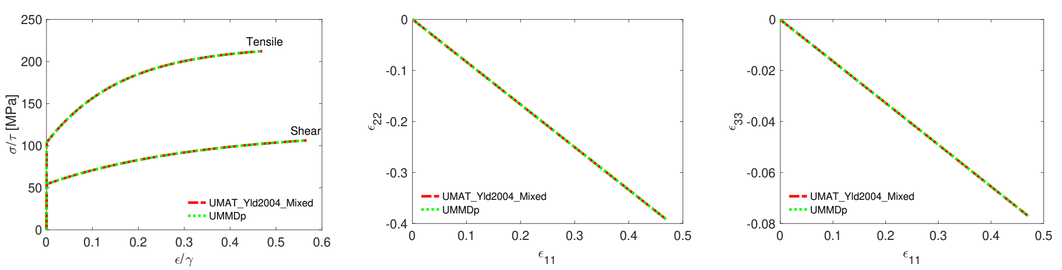
(a) (b) (c)

FIGURE 7 Results of tensile, shear and biaxial test with von Mises isotropic yield criterion and mixed hardening for Std_Abq, UMAT_Yld2004_Mixed and UMMDp: (a) ϵ_{11} - σ_{11}, (b) ϵ_{11} - ϵ_{22} and (c) ϵ_{11} - ϵ_{33}.



(a) (b) (c)

FIGURE 8 Results of tensile, shear and biaxial test with Yld2004-18p anisotropic yield criterion and isotropic hardening with material orientation at 0° from rolling direction (RD) for UMAT_Yld2004_Mixed and UMMDp: (a) ϵ_{11} - σ_{11}, (b) ϵ_{11} - ϵ_{22} and (c) ϵ_{11} - ϵ_{33}.



(a) (b) (c)

FIGURE 9 Results of tensile and shear test with Yld2004-18p anisotropic yield criterion and isotropic hardening with material orientation at 45° from rolling direction (RD) for UMAT_Yld2004_Mixed and UMMDp: (a) ϵ_{11} - σ_{11}, (b) ϵ_{11} - ϵ_{22} and (c) ϵ_{11} - ϵ_{33}.

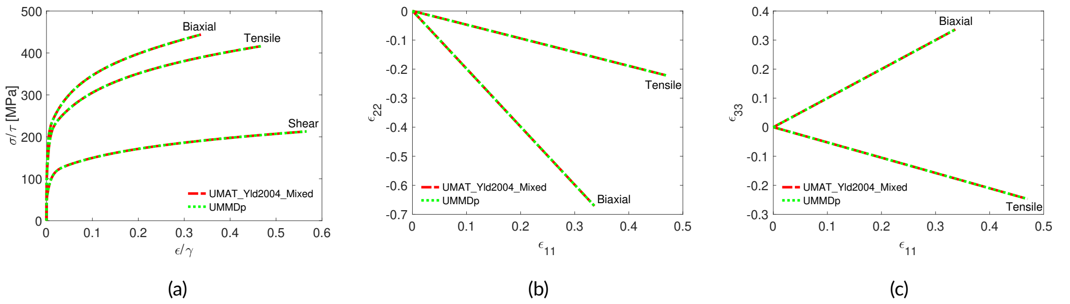


FIGURE 10 Results of tensile, shear and biaxial test with Yld2004-18p anisotropic yield criterion and mixed hardening with material orientation at 0º from rolling direction (RD) for UMAT_Yld2004_Mixed and UMMDp: (a) ϵ_{11} - σ_{11}, (b) ϵ_{11} - ϵ_{22} and (c) ϵ_{11} - ϵ_{33}.

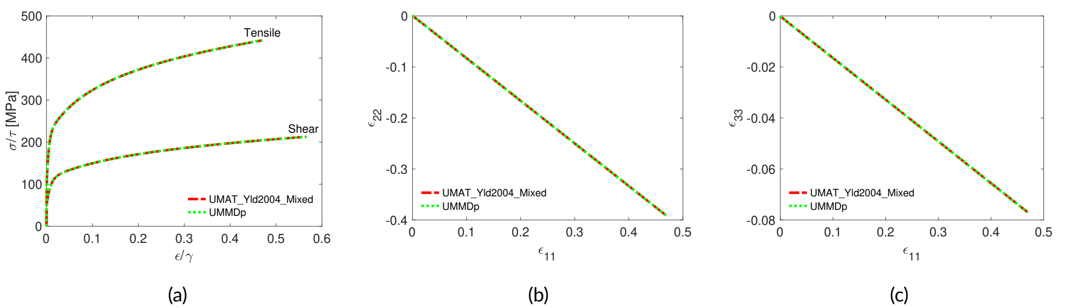


FIGURE 11 Results of tensile and shear test with Yld2004-18p anisotropic yield criterion and mixed hardening with material orientation at 45º from rolling direction (RD) for UMAT_Yld2004_Mixed and UMMDp: (a) ϵ_{11} - σ_{11}, (b) ϵ_{11} - ϵ_{22} and (c) ϵ_{11} - ϵ_{33}.

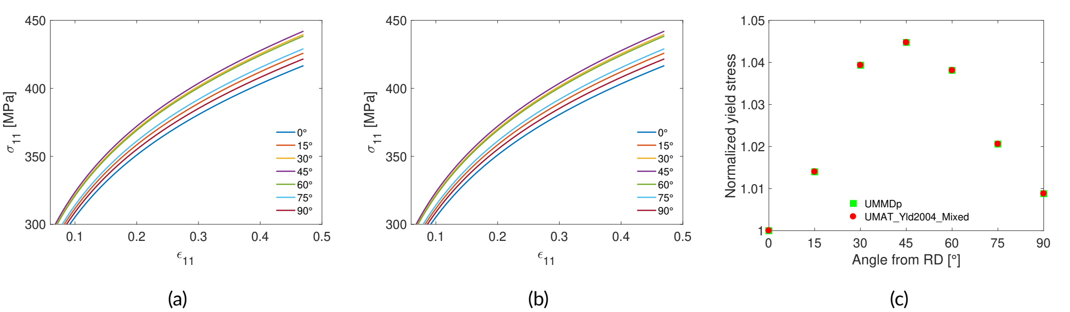


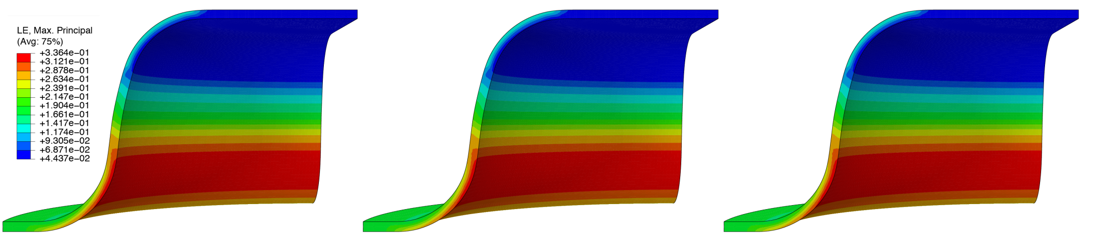
FIGURE 12 Results of tensile test with Yld2004-18p anisotropic yield criterion and mixed hardening with material orientation at different angles from rolling direction (RD): (a) UMAT_Yld2004_Mixed ϵ_{11} - σ_{11}, (b) UMMDp ϵ_{11} - σ_{11} and (c) rolling direction vs. normalized yield stress.

5.2 | Cup Deep Drawing

Additionally to the homogeneous tests, it is recommended to compare the results of UMMDp, UMAT_Yld2004_Mixed and Std_Aba.for a more complex model and to compare the convergence of each material user subroutine. A model of a cup deep drawing is used to perform the numerical simulations according to the dimensions used by Souto *et al.* [7]. The simulations are performed combining von Mises isotropic or Yld2004-18p anisotropic yield criteria, with isotropic or mixed hardening laws. The material parameters are similar to the ones used in the homogeneous tests numerical simulations and are given in Tables 3 and 5. The combination of constitutive equations used in the simulations are given according to:

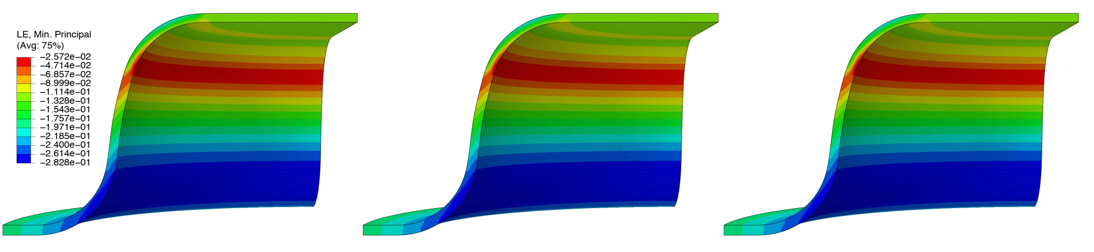
- von Mises isotropic yield criterion and mixed hardening (Test 1);
- Yld2004-18p anisotropic yield criterion and isotropic hardening (Test 2);
- Yld2004-18p anisotropic yield criterion and mixed hardening (Test 3).

Similarly to the results of the homogeneous tests, it is observed that the results for the cup deep drawing simulations are identical between UMMDp, UMAT_Yld2004_Mixed and Std_Aba (see Figures 13 to 20.



(a) (b) (c)

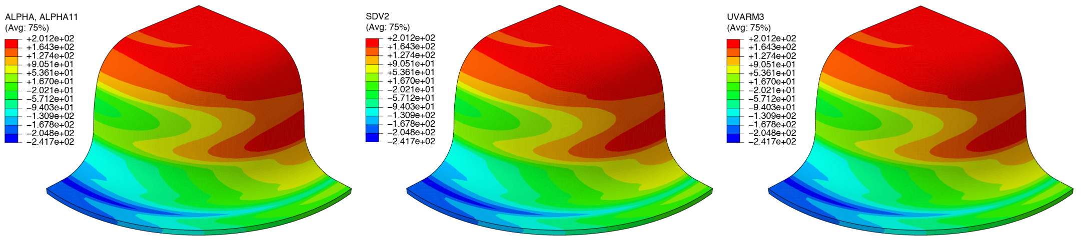
FIGURE 13 Results of the major strain for the cup deep drawing with von Mises isotropic yield criterion and mixed hardening for (a) Std_Abq, (b) UMAT_Yld2004_Mixed and (c) UMMDp.



(a) (b) (c)

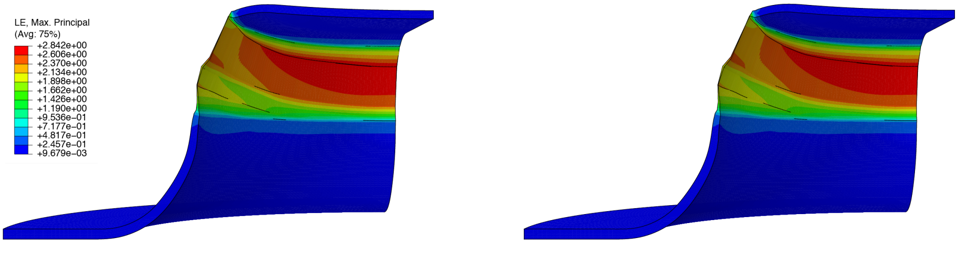
FIGURE 14 Results of the minor strain for the cup deep drawing with von Mises isotropic yield criterion and mixed hardening for (a) Std_Abq, (b) UMAT_Yld2004_Mixed and (c) UMMDp.

Analyzing the convergence and computational time of UMAT_Yld2004_Mixed and UMMDp, it is important to refer that both have the both are implemented with a backward Euler scheme with a sub-incremental method. However, it is not known if the algorithm is implemented with the same conditions and the computation of the consistent tangent matrix is not identical. In terms of the number of increments required by each simulation for the step corresponding to the displacement of the punch, the results are very similar between each simulation, with only one difference observed for Test 2, where UMMDp required five more increments than UMAT_Yld2004_Mixed (see Table



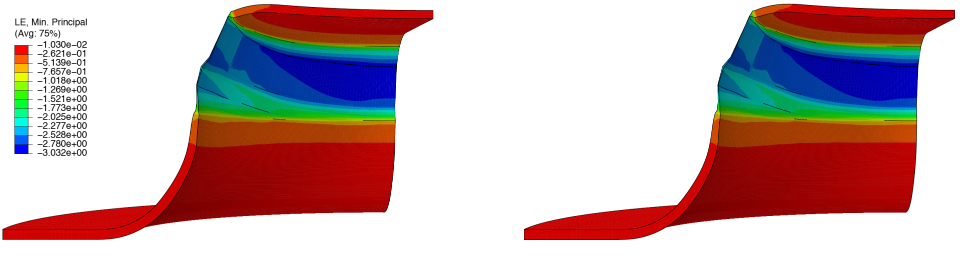
(a) (b) (c)

FIGURE 15 Results of the first component of total back stress for the cup deep drawing with von Mises isotropic yield criterion and mixed hardening for (a) Std_Abq, (b) UMAT_Yld2004_Mixed and (c) UMMDp.



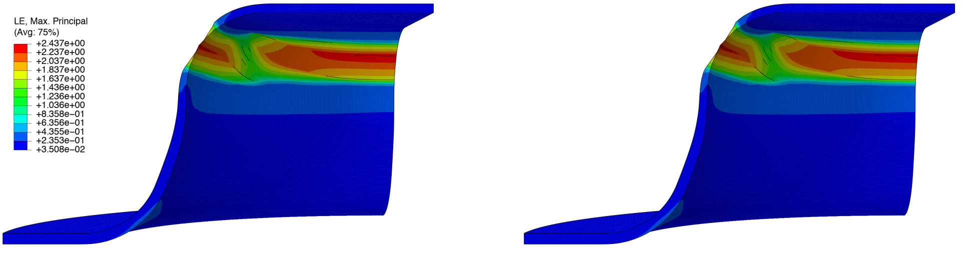
(a) (b)

FIGURE 16 Results of the major strain for the cup deep drawing with Yld2004-18p anisotropic yield criterion and isotropic hardening for (a) UMAT_Yld2004_Mixed and (b) UMMDp.



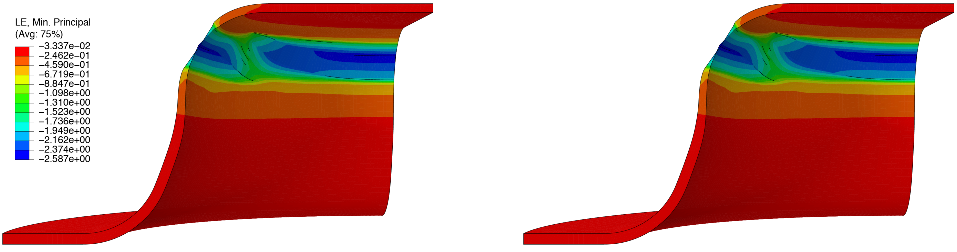
(a) (b)

FIGURE 17 Results of the minor strain for the cup deep drawing with Yld2004-18p anisotropic yield criterion and isotropic hardening for (a) UMAT_Yld2004_Mixed and (b) UMMDp.



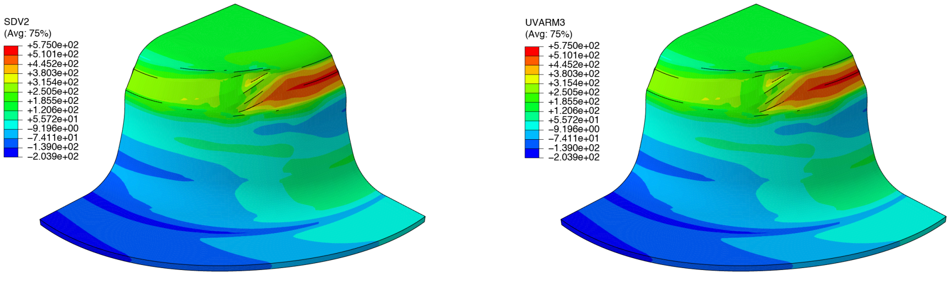
(a) (b)

FIGURE 18 Results of the major strain for the cup deep drawing with Yld2004-18p anisotropic yield criterion and mixed hardening for (a) UMAT_Yld2004_Mixed and (b) UMMDp.



(a) (b)

FIGURE 19 Results of the minor strain for the cup deep drawing with Yld2004-18p anisotropic yield criterion and mixed hardening for (a) UMAT_Yld2004_Mixed and (b) UMMDp.



(a) (b)

FIGURE 20 Results of the first component of the total back stress for the cup deep drawing with Yld2004-18p anisotropic yield criterion and mixed hardening for (a) UMAT_Yld2004_Mixed and (b) UMMDp.

6). Additionally, in Test 1 and Test 3 the number of increments is the same, as well as the number of cutbacks in each increment.

TABLE 6 Number of increments of the punch step in the cup deep drawing simulations.

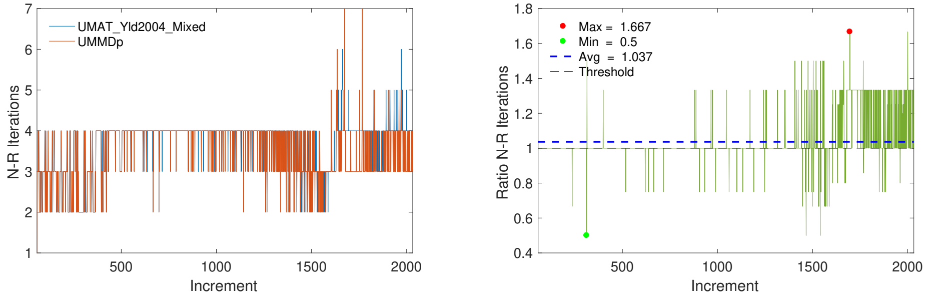
	Test 1	Test 2	Test 3
Std_Aba	2032	-	-
UMAT_Yld2004_Mixed	2032	2037	2032
UMMDp	2032	2042	2032

The computational time is analyzed for Test 3 in terms of total CPU time, and in this case UMMDp is approximately 14% slower than UMAT_Yld2004_Mixed (see Table 7). This difference is very significant and for that reason it is relevant to investigate if it is related to the stress integration algorithm or programming efficiency. For that purpose, it is monitored for a single element of the model, beginning in the increment of yielding, the number of Newton-Raphson (N-R) iterations in the global equilibrium equation per increment, the number of sub-increments in the stress integration scheme and the total number of N-R iterations in the stress integration scheme, per increment. Moreover, the CPU Time required per increment is also monitored.

TABLE 7 Total CPU time of cup deep drawing test 3.

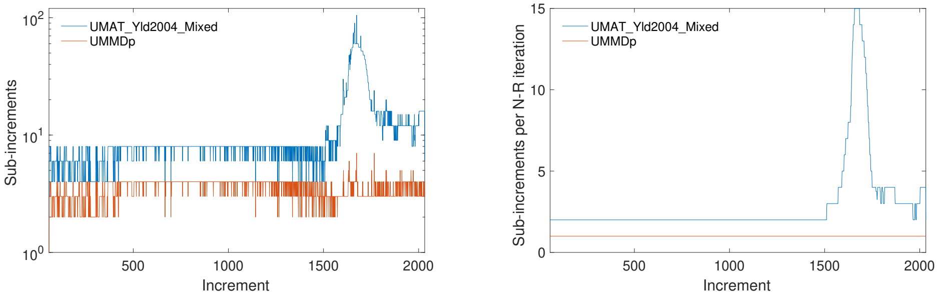
	CPU Time [h]
UMAT_Yld2004_Mixed	47.544
UMMDp	53.946

Analyzing the results of the number of N-R iterations in the global equilibrium equation per increment presented in Figure 21a, differences are not easily identified. However, analyzing the ratio of N-R iterations in Figure 21b, where above the threshold are the points where UMAT_Yld2004_Mixed takes more iterations than UMMDp and *vice versa*, a slight difference is observed. The average ratio of 1.037 indicates that UMAT_Yld2004_Mixed requires on average more iterations than UMMDp. Further observing the results of the sub-increments required in each N-R iteration (see Figure 23), it is interesting to observe that UMMDp only requires one sub-increment per N-R iteration in each increment, while UMAT_Yld2004_Mixed requires at least two sub-increments and a maximum of fifteen. Regarding the results of the number of N-R iterations in the stress integration scheme a large difference between the two material user subroutines is observed (see Figure 23), with the average ratio of 2.624 indicating the ability of UMMDp to converge faster than UMAT_Yld2004_Mixed. It seems evident from these results that UMMDp presents higher levels of efficiency of its algorithm, even though the total CPU time was largely higher than UMAT_Yld2004_Mixed. The results of the CPU time per increment presented in Figure 24 show that the average ratio is similar to the one obtained in total CPU time, therefore supporting its results. However, it is interesting to observe that in some increments UMMDp is faster than UMAT_Yld2004_Mixed. Overall, UMMDp requires less iterations than UMAT_Yld2004_Mixed to converge to the solution, but the difference in CPU time is significantly in favor of UMAT_Yld2004_Mixed. Thereby, it is possible to assume that this difference is related to the programming efficiency of the material user subroutines and not to the algorithm efficiency. Nonetheless, the programming efficiency of UMMDp is related to the fact that it is prepared to use several yield criteria, isotropic and hardening laws, in opposition to UMAT_Yld2004_Mixed that is coded specifically to one yield criterion and one kinematic hardening law, allowing for a more efficient and straightforward implementation.



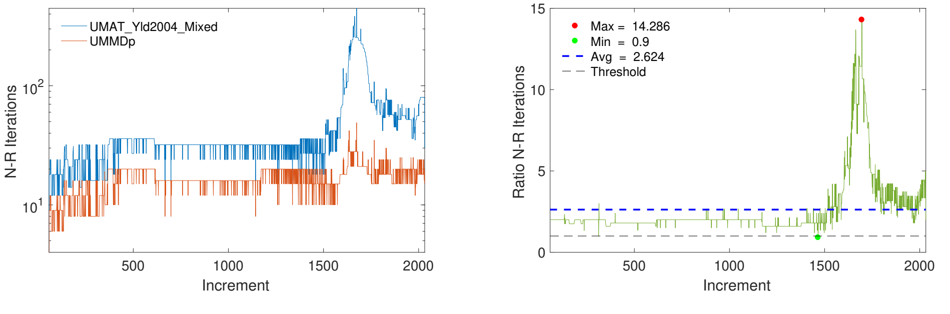
(a) (b)

FIGURE 21 Results of the number of Newton-Raphson (N-R) iterations in the global equilibrium equation per increment for the cup deep drawing with Yld2004-18p anisotropic yield criterion and mixed hardening: (a) N-R iterations and (b) ratio of N-R iterations.



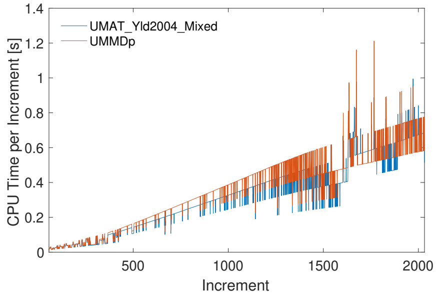
(a) (b)

FIGURE 22 Results of the number of sub-increments in the stress integration scheme per increment for the cup deep drawing with Yld2004-18p anisotropic yield criterion and mixed hardening: (a) sub-increments and (b) sub-increments per N-R iteration.

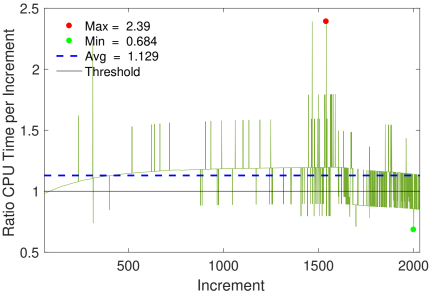


(a) (b)

FIGURE 23 Results of the number of N-R iterations in the stress integration scheme per increment for the cup deep drawing with Yld2004-18p anisotropic yield criterion and mixed hardening: (a) N-R iterations and (b) ratio of N-R iterations.



(a)



(b)

FIGURE 24 Results of the CPU time number of N-R iterations in the stress integration scheme per increment for the cup deep drawing with Yld2004-18p anisotropic yield criterion and mixed hardening: (a) CPU time and (b) ratio of CPU time.

6 | CONCLUSIONS

The Unified Material Model Driver for Plasticity (UMMDp) presents as a valuable library to be used in numerical simulations thanks to its wide range of constitutive models. Even thought it presents a great potential, it still lacks in the number of numerical validations of its models. The Yld2004-18p anisotropic yield criterion in UMMDp presented an interesting formulation of the yield function, which might be of great use in material parameter identifications. However, the observed incoherence between the Voigt notation of the stress tensor used in its formulation and the finite element software conventions is of great importance and it is recommended that notation is checked for the other yield criteria implemented in UMMDp and considering which finite element software is used. It is recommended that UMMDp is modified in order to take into account which finite element software is being used and change the Voigt notation accordingly. When accounting for the implemented modifications to Yld2004-18p the results are similar to the material user subroutine used as reference, therefore validating its formulation. The new kinematic hardening law implemented in UMMDp also presents similar results to the material user subroutine used as reference, validating its formulation and enlarging the library, proving that any user can easily implement new models into UMMDp. At last, the numerical simulations performed definitely validate the use of Yld2004-18p yield criterion, isotropic hardening laws and new kinematic hardening law in UMMDp. Even though UMMDp proved to be significantly slower in the cup deep drawing simulation than the material user subroutine used as reference, it is nevertheless advantageous as it enables the use of complex yield criteria, without the need to implemented them from zero.

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A | FORMULATION OF R COEFFICIENT IN YLD2004-18p OF UMMDp

$$
\begin{aligned}
H_1 &= \left(c_{12} + c_{13} + c_{23} - 2c_{21} + c_{32} - 2c_{31}\right)/9 \\
H_2 &= \left[\left(c_{23} - 2c_{21}\right)\left(c_{32} - 2c_{31}\right) + \left(c_{32} - 2c_{31}\right)\left(c_{12} + c_{13}\right) + \left(c_{12} + c_{13}\right)\left(c_{23} - 2c_{21}\right)\right]/27 \\
H_3 &= \left(c_{12} + c_{13}\right)\left(c_{23} - 2c_{21}\right)\left(c_{32} - 2c_{31}\right)/54
\end{aligned}
\tag{A.1}
$$

$$
\begin{aligned}
p &= H_1^2 + H_2 \\
q &= \left(2H_1^3 + 2H_1 H_2 + 2H_3\right)/2 \\
\theta &= \arccos\left(\frac{q}{p^{3/2}}\right)
\end{aligned}
\tag{A.2}
$$

$$
\begin{aligned}
B_1 &= 2\sqrt{p}\cos\left(\frac{\theta}{3}\right) + H_1 \\
B_2 &= 2\sqrt{p}\cos\left(\frac{\theta + 4\pi}{3}\right) + H_1 \\
B_3 &= 2\sqrt{p}\cos\left(\frac{\theta + 2\pi}{3}\right) + H_1
\end{aligned}
\tag{A.3}
$$

$$
R = \sum_{i=1}^{3}\sum_{j=1}^{3}\left|B_i^{(1)} - B_j^{(2)}\right|^a
\tag{A.4}
$$

B | SUBROUTINE OF YLD2004-18p IMPLEMENTATION IN UMMDp

```
      subroutine jancae_yld2004_18p ( s,se,
     1   dseds,d2seds2,nreq,pryld,ndyld
c
        (...)
        cp1(4,4) = pryld(1+7)
        cp1(5,5) = pryld(1+8)
        cp1(6,6) = pryld(1+9)
        (...)
        cp2(4,4) = pryld(1+16)
        cp2(5,5) = pryld(1+17)
        cp2(6,6) = pryld(1+18)
        (...)
c                         ---- d(hp)/d(sp)
        (...)
        do i = 1,3
          (...)


          (...)
        end do
        do i=4,6
          (...)


          (...)
        end do
         (...)
c                         ---- d2(hp)/d(sp)2
          (...)
          do i=4,6
          (...)


          (...)
          end do
          do i=1,3
          (...)


          (...)
          end do
          (...)
      return
      end
```

(a)

```
      subroutine jancae_yld2004_18p ( s,se,
     1   dseds,d2seds2,nreq,pryld,ndyld
c
        (...)
        cp1(4,4) = pryld(1+9)
        cp1(5,5) = pryld(1+8)
        cp1(6,6) = pryld(1+7)
        (...)
        cp2(4,4) = pryld(1+18)
        cp2(5,5) = pryld(1+17)
        cp2(6,6) = pryld(1+16)
        (...)
c                         ---- d(hp)/d(sp)
        (...)
        do i = 1,3
          (...)
          if ( i == 1 ) l=6
          if ( i == 2 ) l=5
          (...)
        end do
        do i=4,6
          (...)
          if ( i == 5 ) k=2
          if ( i == 6 ) k=1
          (...)
        end do
         (...)
c                         ---- d2(hp)/d(sp)2
          (...)
          do i=4,6
            (...)
            if ( i == 5 ) k = 2
            if ( i == 6 ) k = 1
            (...)
          end do
          do i=1,3
            (...)
            if ( i == 1 ) j = 6
            if ( i == 2 ) j = 5
            (...)
          end do
          (...)
      return
      end
```

(b)

LISTING B.5 Snippet of UMMDp source code of the Yld2004-18p anisotropic yield criterion main subroutine: (a) original and (b) modified.

```fortran
      subroutine jancae_yld2004_18p_yfsub( s,
     1   pi, sp, psp, hp, cetpq)
      (...)
      hp(3)=(2.0d0*sp(5)*sp(6)*sp(4)+
     1   sp(1)*sp(2)*sp(3)-sp(1)*sp(5)**2-
     2   sp(2)*sp(6)**2-sp(3)*sp(4)**2)/2.0d0
      (...)
      return
      end
```

(a)

```fortran
      subroutine jancae_yld2004_18p_yfsub( s,
     1   pi, sp, psp, hp, cetpq)
      (...)
      hp(3)=(2.0d0*sp(5)*sp(6)*sp(4)+
     1   sp(1)*sp(2)*sp(3)-sp(1)*sp(6)**2-
     2   sp(2)*sp(5)**2-sp(3)*sp(4)**2)/2.0d0
      (...)
      return
      end
```

(b)

LISTING B.6 Snippet of UMMDp source code of the Yld2004-18p anisotropic yield criterion principal values subroutine: (a) original and (b) modified.

C | SUBROUTINE OF NEW KINEMATIC HARDENING LAW IN UMMDp

```
c---------------------------------------------------------------------
c     Chaboche (1979) - Ziegler Type
c
      subroutine jancae_kin_chaboche1979_ziegler ( vk,dvkdp,dvkds,dvkdx,dvkdxt,p,s,x,
     &                                             xt,nttl,nnrm,nshr,mxpbs,npbs,prkin,
     &                                             ndkin,pryld,ndyld )
c---------------------------------------------------------------------
      implicit real*8 (a-h,o-z)
      dimension vk(npbs,nttl),dvkdp( npbs,nttl),dvkdx( npbs,npbs,nttl,nttl),
     &          dvkdxt(npbs,nttl,nttl),s(nttl),x(mxpbs,nttl),xt(nttl),prkin(ndkin),
     &          pryld(ndyld)
c
      dimension eta(nttl),dseds(nttl),d2seds2(nttl,nttl),am(nttl,nttl)
c
      do i=1,nttl
        eta(i)=s(i)-xt(i)
      enddo
c
      call jancae_dseds_kin ( eta,seta,dseds,d2seds2,nttl,nnrm,nshr,pryld,ndyld )
c
      call jancae_setunitm ( am,nttl )
      do n=1,npbs
        n0=(n-1)*2
        c=prkin(1+n0+1)
        g=prkin(1+n0+2)
        do i=1,nttl
          vk(n,i)=(c/seta)*eta(i)-g*x(n,i)
        enddo
        dcdp=0.0d0
        dgdp=0.0d0
        do i=1,nttl
          dvkdp(n,i)=(dcdp/seta)*eta(i)-dgdp*x(n,i)
        enddo
        do i=1,nttl
          do j=1,nttl
            dvkds( n,  i,j)= (c/seta)*am(i,j)
            dvkdx( n,n,i,j)=-g*am(i,j)
            dvkdxt(n,  i,j)=-(c/seta)*am(i,j)
          enddo
        enddo
      enddo
c
      return
      end
```

LISTING C.1 Snippet of UMMDp source code for the new kinematic hardening law implementation.