**Proposition 1.** Let $v_{ij}\colon (0,1) \to [0,1]$, $i,j \in \{0,1\}$ be non-decreasing, left-continuous functions with $v_{11} + v_{00} = v_{10} + v_{01}$. Let additionally $Y_0$ and $X_0$ be random variables supported on the unit interval, and let $U_1, U_2, \dots$ and $V_1, V_2, \dots$ be *iid* sequences of random variables in the unit interval, with both sequences mutually independent and also independent from $Y_0$ and $X_0$. Define moreover, for $t \geqslant 1$,

$$\begin{aligned} Y_t &\coloneqq \alpha_0(U_t) + \alpha_1(U_t)Y_{t-1} + \theta_1(U_t)X_{t-1} \\ X_t &\coloneqq Q_t(V_t, X_{t-1}, \dots, X_0, Y_t, Y_{t-1}, \dots, Y_0) \end{aligned} \tag{1}$$

where, for all $t \geqslant 1$, the measurable mapping $Q_t\colon (0,1) \times [0,1]^{2t+1} \to [0,1]$ is a quantile function of its first argument, and where $\alpha_0 \coloneqq v_{00}$, $\alpha_1 \coloneqq v_{10} - v_{00}$ and $\theta_1 \coloneqq v_{01} - v_{00}$. Then the process $(X_t, Y_t)_{t\in\mathbb{Z}}$ has state space contained in the unit square $[0,1]^2$ and satisfies

$$Q_{Y_t}(\tau|\mathscr{F}_{t-1}) = \alpha_0(\tau) + \alpha_1(\tau)Y_{t-1} + \theta_1(\tau)X_{t-1} \tag{2}$$

for all $0 < \tau < 1$ and all $t \geqslant 1$, where $\mathscr{F}_t \coloneqq \sigma\{(X_s, Y_s)\colon\ s \leqslant t\}$.

**Proposition 2.** Let $v_{10}, v_{01}\colon (0,1) \to \mathbb{R}$ be any two non-decreasing, continuously differentiable functions, with $v_{10}(0+) = v_{01}(0+) = 0$ and $v_{10}(1-) = v_{01}(1-) = 1$. Also, let $\lambda\colon (0,1) \to \mathbb{R}$ be any non-negative, continuous function satisfying the requirement that $0 < \int_0^1 (v_{10}'(u) + v_{01}'(u))\lambda(u)\,\mathrm{d}u < \infty$. Then the following holds:

1. The function $v_{00}\colon (0,1) \to \mathbb{R}$ defined through

   $$v_{00}(\tau) \coloneqq \frac{\int_0^\tau (v_{10}'(u) + v_{01}'(u))\lambda(u)\,\mathrm{d}u}{\int_0^1 (v_{10}'(u) + v_{01}'(u))\lambda(u)\,\mathrm{d}u}, \quad 0 < \tau < 1, \tag{3}$$

   is continuously differentiable and non-decreasing, with $v_{00}(0+) = 0$ and $v_{00}(1-) = 1$.
2. The function $v_{11} \coloneqq v_{10} + v_{01} - v_{00}$ is continuously differentiable, satisfies $v_{11}(0+) = 0$, $v_{11}(1-) = 1$ and, for $0 < \tau < 1$,

   $$v_{11}'(\tau) = (v_{10}'(\tau) + v_{01}'(\tau))\left(1 - \frac{\lambda(\tau)}{\int_0^1 (v_{10}'(u) + v_{01}'(u))\lambda(u)\,\mathrm{d}u}\right). \tag{4}$$

   In particular, in order that $v_{11}$ be non-decreasing, it is necessary and sufficient that the bound

   $$\lambda(\tau) \leqslant \int_0^1 (v_{10}'(u) + v_{01}'(u))\lambda(u)\mathrm{d}u$$

   holds for each $\tau$ such that $v_{10}'(\tau) + v_{01}'(\tau) \neq 0$.

sqrt – n=1000

Table 1: *Location-shift model 1: bias and MSE of estimators*

| | *Bias* | | | | | | | | *MSE* | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | | *smoothed* | | | | *canonical* | | | | *smoothed* | | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ |
| 0.01 | 0.2615 | -0.0144 | -0.0028 | 0.0060 | -0.0910 | -0.0146 | -0.0012 | 0.0074 | 0.0684 | 0.0002 | 0 | 0 | 0.0083 | 0.0002 | 0 | 0.0001 |
| 0.03 | 0.0585 | -0.0148 | -0.0010 | 0.0088 | -0.1168 | -0.0156 | -0.0001 | 0.0085 | 0.0034 | 0.0002 | 0 | 0.0001 | 0.0137 | 0.0002 | 0 | 0.0001 |
| 0.05 | 0.0325 | -0.0144 | -0.0003 | 0.0095 | -0.1082 | -0.0163 | 0.0006 | 0.0088 | 0.0011 | 0.0002 | 0 | 0.0001 | 0.0117 | 0.0003 | 0 | 0.0001 |
| 0.07 | 0.0245 | -0.0156 | 0.0018 | 0.0092 | -0.0985 | -0.0167 | 0.0007 | 0.0089 | 0.0006 | 0.0002 | 0 | 0.0001 | 0.0097 | 0.0003 | 0 | 0.0001 |
| 0.09 | 0.0207 | -0.0173 | 0.0019 | 0.0098 | -0.0899 | -0.0168 | 0.0007 | 0.0088 | 0.0004 | 0.0003 | 0 | 0.0001 | 0.0081 | 0.0003 | 0 | 0.0001 |
| 0.11 | 0.0181 | -0.0179 | 0.0008 | 0.0092 | -0.0822 | -0.0170 | 0.0006 | 0.0086 | 0.0003 | 0.0003 | 0 | 0.0001 | 0.0068 | 0.0003 | 0 | 0.0001 |
| 0.13 | 0.0153 | -0.0179 | 0.0013 | 0.0081 | -0.0755 | -0.0171 | 0.0007 | 0.0085 | 0.0002 | 0.0003 | 0 | 0.0001 | 0.0057 | 0.0003 | 0 | 0.0001 |
| 0.15 | 0.0132 | -0.0178 | 0.0003 | 0.0087 | -0.0694 | -0.0172 | 0.0006 | 0.0085 | 0.0002 | 0.0003 | 0 | 0.0001 | 0.0048 | 0.0003 | 0 | 0.0001 |
| 0.17 | 0.0120 | -0.0174 | 0.0003 | 0.0078 | -0.0639 | -0.0172 | 0.0007 | 0.0085 | 0.0001 | 0.0003 | 0 | 0.0001 | 0.0041 | 0.0003 | 0 | 0.0001 |
| 0.19 | 0.0107 | -0.0175 | 0.0005 | 0.0082 | -0.0586 | -0.0172 | 0.0008 | 0.0085 | 0.0001 | 0.0003 | 0 | 0.0001 | 0.0034 | 0.0003 | 0 | 0.0001 |
| 0.21 | 0.0093 | -0.0174 | 0.0004 | 0.0086 | -0.0537 | -0.0172 | 0.0009 | 0.0085 | 0.0001 | 0.0003 | 0 | 0.0001 | 0.0029 | 0.0003 | 0 | 0.0001 |
| 0.23 | 0.0086 | -0.0174 | 0.0005 | 0.0083 | -0.0490 | -0.0172 | 0.0011 | 0.0086 | 0.0001 | 0.0003 | 0 | 0.0001 | 0.0024 | 0.0003 | 0 | 0.0001 |
| 0.25 | 0.0073 | -0.0171 | 0.0006 | 0.0083 | -0.0429 | -0.0172 | 0.0011 | 0.0086 | 0.0001 | 0.0003 | 0 | 0.0001 | 0.0018 | 0.0003 | 0 | 0.0001 |
| 0.27 | 0.0078 | -0.0172 | 0.0011 | 0.0081 | -0.0364 | -0.0172 | 0.0012 | 0.0087 | 0.0001 | 0.0003 | 0 | 0.0001 | 0.0013 | 0.0003 | 0 | 0.0001 |
| 0.29 | 0.0066 | -0.0169 | 0.0015 | 0.0084 | -0.0328 | -0.0171 | 0.0013 | 0.0087 | 0 | 0.0003 | 0 | 0.0001 | 0.0011 | 0.0003 | 0 | 0.0001 |
| 0.31 | 0.0060 | -0.0173 | 0.0015 | 0.0082 | -0.0294 | -0.0171 | 0.0014 | 0.0087 | 0 | 0.0003 | 0 | 0.0001 | 0.0009 | 0.0003 | 0 | 0.0001 |
| 0.33 | 0.0053 | -0.0169 | 0.0014 | 0.0081 | -0.0261 | -0.0171 | 0.0015 | 0.0087 | 0 | 0.0003 | 0 | 0.0001 | 0.0007 | 0.0003 | 0 | 0.0001 |
| 0.35 | 0.0047 | -0.0167 | 0.0016 | 0.0086 | -0.0229 | -0.0171 | 0.0016 | 0.0088 | 0 | 0.0003 | 0 | 0.0001 | 0.0005 | 0.0003 | 0 | 0.0001 |
| 0.37 | 0.0039 | -0.0171 | 0.0017 | 0.0093 | -0.0197 | -0.0170 | 0.0016 | 0.0088 | 0 | 0.0003 | 0 | 0.0001 | 0.0004 | 0.0003 | 0 | 0.0001 |
| 0.39 | 0.0030 | -0.0172 | 0.0022 | 0.0097 | -0.0166 | -0.0170 | 0.0018 | 0.0089 | 0 | 0.0003 | 0 | 0.0001 | 0.0003 | 0.0003 | 0 | 0.0001 |
| 0.41 | 0.0023 | -0.0170 | 0.0028 | 0.0096 | -0.0135 | -0.0170 | 0.0019 | 0.0089 | 0 | 0.0003 | 0 | 0.0001 | 0.0002 | 0.0003 | 0 | 0.0001 |
| 0.43 | 0.0017 | -0.0169 | 0.0028 | 0.0098 | -0.0104 | -0.0169 | 0.0020 | 0.0089 | 0 | 0.0003 | 0 | 0.0001 | 0.0001 | 0.0003 | 0 | 0.0001 |
| 0.45 | 0.0019 | -0.0168 | 0.0026 | 0.0098 | -0.0073 | -0.0169 | 0.0021 | 0.0089 | 0 | 0.0003 | 0 | 0.0001 | 0.0001 | 0.0003 | 0 | 0.0001 |
| 0.47 | 0.0010 | -0.0166 | 0.0025 | 0.0097 | -0.0043 | -0.0169 | 0.0021 | 0.0088 | 0 | 0.0003 | 0 | 0.0001 | 0 | 0.0003 | 0 | 0.0001 |
| 0.49 | 0.0008 | -0.0164 | 0.0028 | 0.0087 | -0.0012 | -0.0169 | 0.0022 | 0.0088 | 0 | 0.0003 | 0 | 0.0001 | 0 | 0.0003 | 0 | 0.0001 |
| 0.51 | 0 | -0.0163 | 0.0030 | 0.0086 | 0.0018 | -0.0168 | 0.0023 | 0.0088 | 0 | 0.0003 | 0 | 0.0001 | 0 | 0.0003 | 0 | 0.0001 |
| 0.53 | -0.0008 | -0.0165 | 0.0031 | 0.0094 | 0.0049 | -0.0168 | 0.0023 | 0.0087 | 0 | 0.0003 | 0 | 0.0001 | 0 | 0.0003 | 0 | 0.0001 |
| 0.55 | -0.0010 | -0.0168 | 0.0029 | 0.0093 | 0.0079 | -0.0167 | 0.0023 | 0.0087 | 0 | 0.0003 | 0 | 0.0001 | 0.0001 | 0.0003 | 0 | 0.0001 |
| 0.57 | -0.0015 | -0.0170 | 0.0027 | 0.0091 | 0.0109 | -0.0167 | 0.0023 | 0.0086 | 0 | 0.0003 | 0 | 0.0001 | 0.0001 | 0.0003 | 0 | 0.0001 |
| 0.59 | -0.0014 | -0.0169 | 0.0030 | 0.0090 | 0.0139 | -0.0166 | 0.0023 | 0.0085 | 0 | 0.0003 | 0 | 0.0001 | 0.0002 | 0.0003 | 0 | 0.0001 |
| 0.61 | -0.0019 | -0.0171 | 0.0030 | 0.0091 | 0.0169 | -0.0166 | 0.0023 | 0.0084 | 0 | 0.0003 | 0 | 0.0001 | 0.0003 | 0.0003 | 0 | 0.0001 |
| 0.63 | -0.0026 | -0.0171 | 0.0025 | 0.0088 | 0.0200 | -0.0165 | 0.0022 | 0.0083 | 0 | 0.0003 | 0 | 0.0001 | 0.0004 | 0.0003 | 0 | 0.0001 |
| 0.65 | -0.0037 | -0.0168 | 0.0028 | 0.0082 | 0.0231 | -0.0164 | 0.0022 | 0.0082 | 0 | 0.0003 | 0 | 0.0001 | 0.0005 | 0.0003 | 0 | 0.0001 |
| 0.67 | -0.0046 | -0.0164 | 0.0025 | 0.0076 | 0.0262 | -0.0163 | 0.0021 | 0.0081 | 0 | 0.0003 | 0 | 0.0001 | 0.0007 | 0.0003 | 0 | 0.0001 |
| 0.69 | -0.0064 | -0.0160 | 0.0018 | 0.0074 | 0.0294 | -0.0162 | 0.0021 | 0.0080 | 0 | 0.0003 | 0 | 0.0001 | 0.0009 | 0.0003 | 0 | 0.0001 |
| 0.71 | -0.0069 | -0.0157 | 0.0013 | 0.0073 | 0.0327 | -0.0161 | 0.0019 | 0.0078 | 0 | 0.0002 | 0 | 0.0001 | 0.0011 | 0.0003 | 0 | 0.0001 |
| 0.73 | -0.0074 | -0.0151 | 0.0011 | 0.0070 | 0.0363 | -0.0160 | 0.0018 | 0.0077 | 0.0001 | 0.0002 | 0 | 0 | 0.0013 | 0.0003 | 0 | 0.0001 |
| 0.75 | -0.0083 | -0.0154 | 0.0010 | 0.0068 | 0.0425 | -0.0159 | 0.0016 | 0.0076 | 0.0001 | 0.0002 | 0 | 0 | 0.0018 | 0.0003 | 0 | 0.0001 |
| 0.77 | -0.0097 | -0.0154 | 0.0011 | 0.0071 | 0.0485 | -0.0158 | 0.0014 | 0.0074 | 0.0001 | 0.0002 | 0 | 0.0001 | 0.0024 | 0.0002 | 0 | 0.0001 |
| 0.79 | -0.0110 | -0.0151 | 0.0009 | 0.0063 | 0.0531 | -0.0156 | 0.0012 | 0.0073 | 0.0001 | 0.0002 | 0 | 0 | 0.0028 | 0.0002 | 0 | 0.0001 |
| 0.81 | -0.0118 | -0.0149 | 0.0008 | 0.0065 | 0.0578 | -0.0155 | 0.0010 | 0.0071 | 0.0001 | 0.0002 | 0 | 0 | 0.0033 | 0.0002 | 0 | 0.0001 |
| 0.83 | -0.0131 | -0.0147 | -0.0001 | 0.0070 | 0.0629 | -0.0153 | 0.0008 | 0.0069 | 0.0002 | 0.0002 | 0 | 0 | 0.0040 | 0.0002 | 0 | 0 |
| 0.85 | -0.0148 | -0.0145 | -0.0008 | 0.0062 | 0.0683 | -0.0152 | 0.0006 | 0.0067 | 0.0002 | 0.0002 | 0 | 0 | 0.0047 | 0.0002 | 0 | 0 |
| 0.87 | -0.0171 | -0.0141 | -0.0002 | 0.0054 | 0.0742 | -0.0150 | 0.0004 | 0.0066 | 0.0003 | 0.0002 | 0 | 0 | 0.0055 | 0.0002 | 0 | 0 |
| 0.89 | -0.0188 | -0.0148 | 0.0002 | 0.0052 | 0.0808 | -0.0148 | 0.0002 | 0.0065 | 0.0004 | 0.0002 | 0 | 0 | 0.0065 | 0.0002 | 0 | 0 |
| 0.91 | -0.0218 | -0.0140 | -0.0009 | 0.0056 | 0.0880 | -0.0147 | -0.0001 | 0.0066 | 0.0005 | 0.0002 | 0 | 0 | 0.0077 | 0.0002 | 0 | 0 |
| 0.93 | -0.0272 | -0.0143 | -0.0015 | 0.0065 | 0.0961 | -0.0145 | -0.0001 | 0.0069 | 0.0007 | 0.0002 | 0 | 0 | 0.0092 | 0.0002 | 0 | 0 |
| 0.95 | -0.0380 | -0.0144 | -0.0007 | 0.0077 | 0.1053 | -0.0143 | 0.0002 | 0.0074 | 0.0014 | 0.0002 | 0 | 0.0001 | 0.0111 | 0.0002 | 0 | 0.0001 |
| 0.97 | -0.0596 | -0.0132 | 0.0020 | 0.0068 | 0.1140 | -0.0141 | 0.0008 | 0.0075 | 0.0036 | 0.0002 | 0 | 0 | 0.0130 | 0.0002 | 0 | 0.0001 |
| 0.99 | -0.2651 | -0.0108 | -0.0001 | 0.0069 | 0.0869 | -0.0132 | 0.0015 | 0.0075 | 0.0703 | 0.0001 | 0 | 0 | 0.0075 | 0.0002 | 0 | 0.0001 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{\zeta^*/2}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $\zeta^*/2$.

Table 2: *Location-shift model 2: bias and MSE of estimators*

| | *Bias* | | | | | | | | *MSE* | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | | *smoothed* | | | | *canonical* | | | | *smoothed* | | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ |
| 0.01 | -0.1268 | 0.0033 | 0.0113 | -0.0101 | -0.2165 | -0.0018 | 0.0080 | -0.0043 | 7.6610 | 0.2630 | 0.7827 | 0.7171 | 7.2056 | 0.1815 | 0.5577 | 0.5048 |
| 0.03 | -0.0695 | -0.0202 | 0.0046 | 0.0077 | -0.1221 | -0.0213 | 0.0042 | 0.0074 | 0.7263 | 0.0708 | 0.1734 | 0.1734 | 0.6582 | 0.0555 | 0.1364 | 0.1363 |
| 0.05 | -0.0559 | -0.0215 | 0.0023 | 0.0077 | -0.1004 | -0.0221 | 0.0023 | 0.0087 | 0.2981 | 0.0377 | 0.0875 | 0.0911 | 0.2729 | 0.0307 | 0.0720 | 0.0742 |
| 0.07 | -0.0465 | -0.0200 | 0.0007 | 0.0037 | -0.0852 | -0.0194 | -0.0001 | 0.0047 | 0.1800 | 0.0256 | 0.0580 | 0.0609 | 0.1624 | 0.0208 | 0.0477 | 0.0491 |
| 0.09 | -0.0354 | -0.0180 | -0.0010 | 0.0028 | -0.0729 | -0.0171 | -0.0008 | 0.0034 | 0.1232 | 0.0186 | 0.0415 | 0.0440 | 0.1116 | 0.0154 | 0.0346 | 0.0364 |
| 0.11 | -0.0276 | -0.0155 | -0.0008 | 0.0033 | -0.0640 | -0.0159 | -0.0006 | 0.0040 | 0.0911 | 0.0145 | 0.0325 | 0.0348 | 0.0836 | 0.0122 | 0.0273 | 0.0288 |
| 0.13 | -0.0216 | -0.0151 | -0.0006 | 0.0032 | -0.0566 | -0.0151 | -0.0001 | 0.0040 | 0.0724 | 0.0120 | 0.0269 | 0.0283 | 0.0663 | 0.0101 | 0.0225 | 0.0237 |
| 0.15 | -0.0159 | -0.0147 | 0.0009 | 0.0033 | -0.0505 | -0.0143 | -0.0001 | 0.0041 | 0.0597 | 0.0103 | 0.0228 | 0.0240 | 0.0549 | 0.0087 | 0.0190 | 0.0202 |
| 0.17 | -0.0139 | -0.0139 | 0.0005 | 0.0046 | -0.0455 | -0.0136 | -0.0001 | 0.0039 | 0.0516 | 0.0092 | 0.0193 | 0.0212 | 0.0471 | 0.0077 | 0.0163 | 0.0176 |
| 0.19 | -0.0123 | -0.0131 | -0.0001 | 0.0043 | -0.0411 | -0.0127 | -0.0001 | 0.0036 | 0.0442 | 0.0082 | 0.0171 | 0.0188 | 0.0411 | 0.0069 | 0.0144 | 0.0157 |
| 0.21 | -0.0112 | -0.0121 | -0.0010 | 0.0044 | -0.0371 | -0.0118 | -0.0002 | 0.0032 | 0.0396 | 0.0074 | 0.0152 | 0.0169 | 0.0365 | 0.0062 | 0.0130 | 0.0142 |
| 0.23 | -0.0097 | -0.0118 | -0.0004 | 0.0027 | -0.0335 | -0.0111 | -0.0002 | 0.0029 | 0.0360 | 0.0068 | 0.0137 | 0.0153 | 0.0329 | 0.0057 | 0.0119 | 0.0129 |
| 0.25 | -0.0088 | -0.0112 | 0 | 0.0022 | -0.0288 | -0.0105 | -0.0002 | 0.0027 | 0.0332 | 0.0062 | 0.0129 | 0.0141 | 0.0301 | 0.0053 | 0.0110 | 0.0119 |
| 0.27 | -0.0060 | -0.0101 | -0.0001 | 0.0014 | -0.0244 | -0.0100 | -0.0002 | 0.0026 | 0.0304 | 0.0057 | 0.0121 | 0.0129 | 0.0278 | 0.0050 | 0.0104 | 0.0111 |
| 0.29 | -0.0048 | -0.0091 | -0.0006 | 0.0018 | -0.0219 | -0.0096 | -0.0002 | 0.0026 | 0.0284 | 0.0053 | 0.0113 | 0.0120 | 0.0259 | 0.0047 | 0.0098 | 0.0105 |
| 0.31 | -0.0047 | -0.0088 | -0.0007 | 0.0021 | -0.0197 | -0.0092 | -0.0002 | 0.0026 | 0.0273 | 0.0051 | 0.0108 | 0.0115 | 0.0243 | 0.0044 | 0.0093 | 0.0099 |
| 0.33 | -0.0043 | -0.0087 | -0.0004 | 0.0024 | -0.0176 | -0.0089 | -0.0003 | 0.0027 | 0.0257 | 0.0048 | 0.0102 | 0.0110 | 0.0229 | 0.0042 | 0.0089 | 0.0095 |
| 0.35 | -0.0026 | -0.0087 | -0.0013 | 0.0032 | -0.0156 | -0.0087 | -0.0003 | 0.0028 | 0.0245 | 0.0046 | 0.0098 | 0.0107 | 0.0217 | 0.0041 | 0.0086 | 0.0091 |
| 0.37 | -0.0020 | -0.0083 | -0.0006 | 0.0035 | -0.0135 | -0.0085 | -0.0003 | 0.0029 | 0.0232 | 0.0045 | 0.0096 | 0.0102 | 0.0207 | 0.0039 | 0.0083 | 0.0088 |
| 0.39 | -0.0011 | -0.0079 | -0.0005 | 0.0029 | -0.0114 | -0.0084 | -0.0003 | 0.0031 | 0.0225 | 0.0043 | 0.0093 | 0.0099 | 0.0199 | 0.0038 | 0.0081 | 0.0086 |
| 0.41 | -0.0012 | -0.0085 | 0.0004 | 0.0043 | -0.0093 | -0.0083 | -0.0002 | 0.0033 | 0.0217 | 0.0043 | 0.0090 | 0.0097 | 0.0192 | 0.0037 | 0.0079 | 0.0084 |
| 0.43 | -0.0013 | -0.0081 | -0.0002 | 0.0037 | -0.0072 | -0.0082 | -0.0001 | 0.0034 | 0.0213 | 0.0042 | 0.0089 | 0.0095 | 0.0187 | 0.0036 | 0.0078 | 0.0082 |
| 0.45 | -0.0008 | -0.0081 | 0 | 0.0038 | -0.0052 | -0.0081 | 0 | 0.0035 | 0.0208 | 0.0041 | 0.0089 | 0.0094 | 0.0183 | 0.0036 | 0.0077 | 0.0081 |
| 0.47 | -0.0004 | -0.0081 | 0.0003 | 0.0040 | -0.0030 | -0.0081 | 0.0001 | 0.0036 | 0.0205 | 0.0041 | 0.0089 | 0.0092 | 0.0180 | 0.0035 | 0.0076 | 0.0080 |
| 0.49 | -0.0002 | -0.0078 | 0.0002 | 0.0036 | -0.0008 | -0.0080 | 0.0002 | 0.0037 | 0.0205 | 0.0040 | 0.0087 | 0.0091 | 0.0179 | 0.0035 | 0.0076 | 0.0080 |
| 0.51 | 0.0011 | -0.0080 | 0.0003 | 0.0039 | 0.0015 | -0.0079 | 0.0003 | 0.0037 | 0.0207 | 0.0041 | 0.0086 | 0.0091 | 0.0178 | 0.0035 | 0.0076 | 0.0080 |
| 0.53 | 0.0014 | -0.0078 | 0.0002 | 0.0035 | 0.0038 | -0.0078 | 0.0004 | 0.0037 | 0.0206 | 0.0041 | 0.0087 | 0.0093 | 0.0179 | 0.0035 | 0.0076 | 0.0080 |
| 0.55 | 0.0007 | -0.0079 | 0.0007 | 0.0038 | 0.0061 | -0.0078 | 0.0004 | 0.0038 | 0.0207 | 0.0041 | 0.0089 | 0.0093 | 0.0181 | 0.0036 | 0.0077 | 0.0081 |
| 0.57 | 0.0015 | -0.0078 | 0.0007 | 0.0037 | 0.0085 | -0.0079 | 0.0004 | 0.0038 | 0.0210 | 0.0042 | 0.0090 | 0.0094 | 0.0183 | 0.0036 | 0.0078 | 0.0082 |
| 0.59 | 0.0031 | -0.0081 | 0.0007 | 0.0040 | 0.0108 | -0.0079 | 0.0004 | 0.0038 | 0.0214 | 0.0043 | 0.0092 | 0.0096 | 0.0187 | 0.0037 | 0.0080 | 0.0084 |
| 0.61 | 0.0032 | -0.0082 | 0.0006 | 0.0046 | 0.0131 | -0.0079 | 0.0004 | 0.0038 | 0.0218 | 0.0044 | 0.0094 | 0.0097 | 0.0193 | 0.0038 | 0.0082 | 0.0085 |
| 0.63 | 0.0039 | -0.0079 | 0.0004 | 0.0042 | 0.0155 | -0.0080 | 0.0004 | 0.0038 | 0.0227 | 0.0045 | 0.0097 | 0.0101 | 0.0200 | 0.0039 | 0.0084 | 0.0088 |
| 0.65 | 0.0062 | -0.0073 | 0.0003 | 0.0032 | 0.0177 | -0.0080 | 0.0004 | 0.0037 | 0.0231 | 0.0045 | 0.0100 | 0.0104 | 0.0208 | 0.0040 | 0.0087 | 0.0091 |
| 0.67 | 0.0068 | -0.0075 | 0.0006 | 0.0034 | 0.0198 | -0.0081 | 0.0004 | 0.0036 | 0.0243 | 0.0047 | 0.0105 | 0.0110 | 0.0219 | 0.0041 | 0.0090 | 0.0094 |
| 0.69 | 0.0068 | -0.0080 | 0.0003 | 0.0033 | 0.0220 | -0.0083 | 0.0004 | 0.0036 | 0.0257 | 0.0049 | 0.0108 | 0.0115 | 0.0232 | 0.0043 | 0.0094 | 0.0098 |
| 0.71 | 0.0069 | -0.0083 | 0.0004 | 0.0037 | 0.0242 | -0.0085 | 0.0004 | 0.0035 | 0.0274 | 0.0052 | 0.0113 | 0.0120 | 0.0248 | 0.0045 | 0.0098 | 0.0104 |
| 0.73 | 0.0083 | -0.0085 | 0.0001 | 0.0036 | 0.0267 | -0.0088 | 0.0005 | 0.0035 | 0.0295 | 0.0055 | 0.0120 | 0.0129 | 0.0267 | 0.0048 | 0.0104 | 0.0110 |
| 0.75 | 0.0098 | -0.0089 | 0.0001 | 0.0026 | 0.0313 | -0.0093 | 0.0006 | 0.0034 | 0.0318 | 0.0059 | 0.0127 | 0.0138 | 0.0290 | 0.0051 | 0.0110 | 0.0117 |
| 0.77 | 0.0123 | -0.0096 | 0.0005 | 0.0021 | 0.0366 | -0.0097 | 0.0008 | 0.0034 | 0.0346 | 0.0065 | 0.0138 | 0.0148 | 0.0321 | 0.0055 | 0.0119 | 0.0125 |
| 0.79 | 0.0154 | -0.0107 | 0.0011 | 0.0036 | 0.0410 | -0.0102 | 0.0009 | 0.0035 | 0.0388 | 0.0072 | 0.0153 | 0.0162 | 0.0359 | 0.0061 | 0.0130 | 0.0137 |
| 0.81 | 0.0181 | -0.0108 | 0.0005 | 0.0031 | 0.0455 | -0.0107 | 0.0011 | 0.0035 | 0.0441 | 0.0080 | 0.0170 | 0.0177 | 0.0407 | 0.0067 | 0.0145 | 0.0152 |
| 0.83 | 0.0192 | -0.0108 | 0.0007 | 0.0027 | 0.0505 | -0.0113 | 0.0012 | 0.0039 | 0.0504 | 0.0090 | 0.0191 | 0.0203 | 0.0469 | 0.0076 | 0.0164 | 0.0172 |
| 0.85 | 0.0228 | -0.0114 | 0.0011 | 0.0042 | 0.0559 | -0.0120 | 0.0017 | 0.0044 | 0.0592 | 0.0103 | 0.0227 | 0.0236 | 0.0554 | 0.0087 | 0.0189 | 0.0199 |
| 0.87 | 0.0256 | -0.0127 | 0.0018 | 0.0047 | 0.0616 | -0.0133 | 0.0021 | 0.0055 | 0.0727 | 0.0122 | 0.0266 | 0.0281 | 0.0677 | 0.0103 | 0.0224 | 0.0235 |
| 0.89 | 0.0345 | -0.0159 | 0.0013 | 0.0073 | 0.0693 | -0.0152 | 0.0021 | 0.0074 | 0.0944 | 0.0152 | 0.0323 | 0.0347 | 0.0863 | 0.0125 | 0.0273 | 0.0286 |
| 0.91 | 0.0419 | -0.0175 | 0.0020 | 0.0102 | 0.0798 | -0.0172 | 0.0023 | 0.0099 | 0.1280 | 0.0195 | 0.0425 | 0.0444 | 0.1161 | 0.0158 | 0.0345 | 0.0366 |
| 0.93 | 0.0521 | -0.0216 | 0.0008 | 0.0131 | 0.0925 | -0.0196 | 0.0022 | 0.0121 | 0.1870 | 0.0261 | 0.0565 | 0.0605 | 0.1707 | 0.0213 | 0.0465 | 0.0501 |
| 0.95 | 0.0638 | -0.0209 | 0.0012 | 0.0159 | 0.1114 | -0.0210 | 0.0004 | 0.0144 | 0.3184 | 0.0391 | 0.0886 | 0.0928 | 0.2923 | 0.0316 | 0.0705 | 0.0758 |
| 0.97 | 0.0976 | -0.0236 | -0.0045 | 0.0225 | 0.1472 | -0.0234 | -0.0017 | 0.0206 | 0.7632 | 0.0722 | 0.1654 | 0.1762 | 0.7028 | 0.0567 | 0.1307 | 0.1388 |
| 0.99 | 0.1356 | -0.0165 | -0.0083 | 0.0133 | 0.2290 | -0.0215 | -0.0052 | 0.0172 | 7.9011 | 0.2421 | 0.6875 | 0.7477 | 7.5178 | 0.1673 | 0.5041 | 0.5491 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{\zeta^*/2}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $\zeta^*/2$.

Table 3: *Location-shift-scale model 1: bias and MSE of estimators*

| | *Bias* | | | | | | | | *MSE* | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | | *smoothed* | | | | *canonical* | | | | *smoothed* | | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ |
| 0.01 | -0.2014 | -0.0020 | 0.1110 | 0.0024 | -0.4127 | -0.0022 | 0.1353 | 0.0015 | 0.2473 | 0.0046 | 0.0272 | 0.0054 | 3.7711 | 0.0028 | 0.0283 | 0.0033 |
| 0.03 | -0.1006 | -0.0025 | 0.0445 | 0.0003 | -0.2537 | -0.0024 | 0.0686 | 0.0010 | 0.0986 | 0.0024 | 0.0115 | 0.0027 | 2.7073 | 0.0018 | 0.0127 | 0.0020 |
| 0.05 | -0.0627 | -0.0029 | 0.0292 | 0.0005 | -0.1851 | -0.0023 | 0.0490 | 0.0009 | 0.0608 | 0.0016 | 0.0076 | 0.0017 | 2.1744 | 0.0013 | 0.0081 | 0.0014 |
| 0.07 | -0.0441 | -0.0025 | 0.0210 | 0.0005 | -0.1465 | -0.0020 | 0.0384 | 0.0007 | 0.0446 | 0.0012 | 0.0058 | 0.0013 | 1.8029 | 0.0010 | 0.0062 | 0.0011 |
| 0.09 | -0.0369 | -0.0017 | 0.0169 | 0.0001 | -0.1201 | -0.0017 | 0.0316 | 0.0004 | 0.0365 | 0.0010 | 0.0048 | 0.0011 | 1.5208 | 0.0009 | 0.0051 | 0.0010 |
| 0.11 | -0.0291 | -0.0014 | 0.0142 | -0.0003 | -0.1005 | -0.0015 | 0.0269 | 0.0003 | 0.0304 | 0.0008 | 0.0042 | 0.0009 | 1.2952 | 0.0008 | 0.0044 | 0.0008 |
| 0.13 | -0.0235 | -0.0015 | 0.0120 | -0.0001 | -0.0850 | -0.0014 | 0.0230 | 0.0004 | 0.0263 | 0.0007 | 0.0037 | 0.0008 | 1.1089 | 0.0007 | 0.0039 | 0.0007 |
| 0.15 | -0.0195 | -0.0014 | 0.0104 | 0 | -0.0724 | -0.0014 | 0.0200 | 0.0005 | 0.0234 | 0.0007 | 0.0034 | 0.0007 | 0.9518 | 0.0006 | 0.0036 | 0.0007 |
| 0.17 | -0.0163 | -0.0014 | 0.0090 | 0.0001 | -0.0616 | -0.0014 | 0.0173 | 0.0005 | 0.0213 | 0.0006 | 0.0032 | 0.0007 | 0.8177 | 0.0006 | 0.0033 | 0.0006 |
| 0.19 | -0.0141 | -0.0014 | 0.0078 | 0.0002 | -0.0525 | -0.0014 | 0.0151 | 0.0005 | 0.0201 | 0.0006 | 0.0030 | 0.0006 | 0.7012 | 0.0005 | 0.0031 | 0.0006 |
| 0.21 | -0.0131 | -0.0012 | 0.0069 | 0.0004 | -0.0447 | -0.0014 | 0.0131 | 0.0005 | 0.0186 | 0.0005 | 0.0028 | 0.0006 | 0.5992 | 0.0005 | 0.0029 | 0.0006 |
| 0.23 | -0.0103 | -0.0011 | 0.0058 | 0.0003 | -0.0378 | -0.0014 | 0.0113 | 0.0005 | 0.0178 | 0.0005 | 0.0027 | 0.0005 | 0.5095 | 0.0005 | 0.0028 | 0.0005 |
| 0.25 | -0.0076 | -0.0011 | 0.0047 | 0.0003 | -0.0311 | -0.0014 | 0.0097 | 0.0004 | 0.0169 | 0.0005 | 0.0026 | 0.0005 | 0.4312 | 0.0005 | 0.0027 | 0.0005 |
| 0.27 | -0.0052 | -0.0013 | 0.0040 | 0.0003 | -0.0252 | -0.0014 | 0.0084 | 0.0004 | 0.0161 | 0.0004 | 0.0025 | 0.0005 | 0.3620 | 0.0004 | 0.0026 | 0.0005 |
| 0.29 | -0.0037 | -0.0013 | 0.0038 | 0.0002 | -0.0208 | -0.0014 | 0.0075 | 0.0003 | 0.0156 | 0.0004 | 0.0025 | 0.0005 | 0.2998 | 0.0004 | 0.0025 | 0.0005 |
| 0.31 | -0.0023 | -0.0013 | 0.0036 | 0.0002 | -0.0168 | -0.0014 | 0.0066 | 0.0003 | 0.0151 | 0.0004 | 0.0024 | 0.0005 | 0.2452 | 0.0004 | 0.0024 | 0.0005 |
| 0.33 | -0.0009 | -0.0014 | 0.0033 | 0.0002 | -0.0133 | -0.0014 | 0.0058 | 0.0003 | 0.0147 | 0.0004 | 0.0023 | 0.0005 | 0.1974 | 0.0004 | 0.0024 | 0.0005 |
| 0.35 | 0.0001 | -0.0013 | 0.0028 | 0.0003 | -0.0102 | -0.0015 | 0.0049 | 0.0003 | 0.0146 | 0.0004 | 0.0023 | 0.0005 | 0.1559 | 0.0004 | 0.0024 | 0.0005 |
| 0.37 | 0.0018 | -0.0015 | 0.0023 | 0.0004 | -0.0071 | -0.0015 | 0.0041 | 0.0004 | 0.0144 | 0.0004 | 0.0023 | 0.0005 | 0.1204 | 0.0004 | 0.0023 | 0.0005 |
| 0.39 | 0.0038 | -0.0016 | 0.0016 | 0.0004 | -0.0042 | -0.0015 | 0.0032 | 0.0004 | 0.0141 | 0.0004 | 0.0023 | 0.0005 | 0.0905 | 0.0004 | 0.0023 | 0.0005 |
| 0.41 | 0.0043 | -0.0015 | 0.0011 | 0.0004 | -0.0015 | -0.0016 | 0.0023 | 0.0005 | 0.0141 | 0.0004 | 0.0022 | 0.0005 | 0.0658 | 0.0004 | 0.0023 | 0.0004 |
| 0.43 | 0.0051 | -0.0016 | 0.0007 | 0.0005 | 0.0011 | -0.0016 | 0.0016 | 0.0005 | 0.0140 | 0.0004 | 0.0022 | 0.0005 | 0.0461 | 0.0004 | 0.0022 | 0.0004 |
| 0.45 | 0.0063 | -0.0016 | 0.0004 | 0.0006 | 0.0037 | -0.0016 | 0.0009 | 0.0005 | 0.0139 | 0.0004 | 0.0022 | 0.0004 | 0.0312 | 0.0004 | 0.0022 | 0.0004 |
| 0.47 | 0.0069 | -0.0015 | -0.0002 | 0.0005 | 0.0061 | -0.0017 | 0.0001 | 0.0005 | 0.0137 | 0.0004 | 0.0022 | 0.0004 | 0.0210 | 0.0004 | 0.0022 | 0.0004 |
| 0.49 | 0.0075 | -0.0015 | -0.0006 | 0.0005 | 0.0084 | -0.0016 | -0.0006 | 0.0005 | 0.0136 | 0.0004 | 0.0022 | 0.0004 | 0.0154 | 0.0004 | 0.0022 | 0.0004 |
| 0.51 | 0.0089 | -0.0015 | -0.0012 | 0.0006 | 0.0109 | -0.0017 | -0.0014 | 0.0005 | 0.0136 | 0.0004 | 0.0021 | 0.0004 | 0.0145 | 0.0004 | 0.0022 | 0.0004 |
| 0.53 | 0.0103 | -0.0015 | -0.0018 | 0.0006 | 0.0134 | -0.0016 | -0.0022 | 0.0005 | 0.0138 | 0.0004 | 0.0022 | 0.0004 | 0.0181 | 0.0004 | 0.0022 | 0.0004 |
| 0.55 | 0.0115 | -0.0016 | -0.0022 | 0.0006 | 0.0157 | -0.0016 | -0.0029 | 0.0005 | 0.0139 | 0.0004 | 0.0022 | 0.0004 | 0.0264 | 0.0004 | 0.0022 | 0.0004 |
| 0.57 | 0.0124 | -0.0016 | -0.0023 | 0.0007 | 0.0182 | -0.0016 | -0.0036 | 0.0005 | 0.0142 | 0.0004 | 0.0022 | 0.0004 | 0.0395 | 0.0004 | 0.0023 | 0.0004 |
| 0.59 | 0.0136 | -0.0016 | -0.0029 | 0.0006 | 0.0206 | -0.0016 | -0.0043 | 0.0005 | 0.0144 | 0.0004 | 0.0022 | 0.0004 | 0.0575 | 0.0004 | 0.0023 | 0.0004 |
| 0.61 | 0.0141 | -0.0014 | -0.0036 | 0.0005 | 0.0229 | -0.0016 | -0.0049 | 0.0005 | 0.0143 | 0.0004 | 0.0023 | 0.0004 | 0.0805 | 0.0004 | 0.0023 | 0.0004 |
| 0.63 | 0.0147 | -0.0014 | -0.0036 | 0.0004 | 0.0253 | -0.0016 | -0.0055 | 0.0005 | 0.0143 | 0.0004 | 0.0023 | 0.0004 | 0.1090 | 0.0004 | 0.0023 | 0.0004 |
| 0.65 | 0.0156 | -0.0014 | -0.0039 | 0.0004 | 0.0277 | -0.0016 | -0.0062 | 0.0004 | 0.0147 | 0.0004 | 0.0023 | 0.0005 | 0.1431 | 0.0004 | 0.0024 | 0.0004 |
| 0.67 | 0.0158 | -0.0012 | -0.0040 | 0.0004 | 0.0304 | -0.0016 | -0.0069 | 0.0004 | 0.0150 | 0.0004 | 0.0024 | 0.0005 | 0.1831 | 0.0004 | 0.0024 | 0.0005 |
| 0.69 | 0.0168 | -0.0012 | -0.0045 | 0.0004 | 0.0334 | -0.0015 | -0.0076 | 0.0005 | 0.0154 | 0.0004 | 0.0025 | 0.0005 | 0.2296 | 0.0004 | 0.0025 | 0.0005 |
| 0.71 | 0.0182 | -0.0013 | -0.0050 | 0.0006 | 0.0370 | -0.0015 | -0.0085 | 0.0005 | 0.0162 | 0.0004 | 0.0025 | 0.0005 | 0.2829 | 0.0004 | 0.0026 | 0.0005 |
| 0.73 | 0.0196 | -0.0012 | -0.0056 | 0.0006 | 0.0412 | -0.0016 | -0.0097 | 0.0006 | 0.0169 | 0.0004 | 0.0026 | 0.0005 | 0.3433 | 0.0004 | 0.0026 | 0.0005 |
| 0.75 | 0.0213 | -0.0012 | -0.0064 | 0.0007 | 0.0471 | -0.0016 | -0.0112 | 0.0006 | 0.0176 | 0.0005 | 0.0026 | 0.0005 | 0.4108 | 0.0004 | 0.0027 | 0.0005 |
| 0.77 | 0.0222 | -0.0012 | -0.0073 | 0.0008 | 0.0536 | -0.0016 | -0.0129 | 0.0007 | 0.0185 | 0.0005 | 0.0028 | 0.0005 | 0.4875 | 0.0005 | 0.0029 | 0.0005 |
| 0.79 | 0.0239 | -0.0011 | -0.0080 | 0.0008 | 0.0603 | -0.0016 | -0.0145 | 0.0007 | 0.0194 | 0.0005 | 0.0029 | 0.0006 | 0.5755 | 0.0005 | 0.0030 | 0.0006 |
| 0.81 | 0.0253 | -0.0009 | -0.0090 | 0.0009 | 0.0676 | -0.0016 | -0.0164 | 0.0008 | 0.0212 | 0.0005 | 0.0031 | 0.0006 | 0.6763 | 0.0005 | 0.0032 | 0.0006 |
| 0.83 | 0.0284 | -0.0010 | -0.0102 | 0.0010 | 0.0763 | -0.0015 | -0.0186 | 0.0008 | 0.0226 | 0.0006 | 0.0033 | 0.0007 | 0.7914 | 0.0005 | 0.0034 | 0.0006 |
| 0.85 | 0.0316 | -0.0010 | -0.0119 | 0.0011 | 0.0863 | -0.0014 | -0.0212 | 0.0009 | 0.0245 | 0.0006 | 0.0036 | 0.0007 | 0.9248 | 0.0006 | 0.0037 | 0.0007 |
| 0.87 | 0.0346 | -0.0008 | -0.0132 | 0.0011 | 0.0988 | -0.0014 | -0.0243 | 0.0009 | 0.0272 | 0.0007 | 0.0039 | 0.0008 | 1.0795 | 0.0006 | 0.0041 | 0.0007 |
| 0.89 | 0.0387 | -0.0004 | -0.0152 | 0.0008 | 0.1145 | -0.0013 | -0.0283 | 0.0009 | 0.0303 | 0.0008 | 0.0044 | 0.0009 | 1.2623 | 0.0007 | 0.0046 | 0.0008 |
| 0.91 | 0.0464 | -0.0005 | -0.0186 | 0.0009 | 0.1349 | -0.0012 | -0.0335 | 0.0009 | 0.0360 | 0.0009 | 0.0050 | 0.0010 | 1.4832 | 0.0008 | 0.0053 | 0.0009 |
| 0.93 | 0.0596 | -0.0007 | -0.0236 | 0.0015 | 0.1627 | -0.0013 | -0.0405 | 0.0009 | 0.0455 | 0.0011 | 0.0060 | 0.0013 | 1.7583 | 0.0009 | 0.0065 | 0.0011 |
| 0.95 | 0.0810 | -0.0008 | -0.0318 | 0.0013 | 0.2041 | -0.0014 | -0.0515 | 0.0010 | 0.0642 | 0.0014 | 0.0078 | 0.0017 | 2.1182 | 0.0012 | 0.0084 | 0.0014 |
| 0.97 | 0.1203 | -0.0004 | -0.0484 | 0.0013 | 0.2765 | -0.0017 | -0.0719 | 0.0012 | 0.1119 | 0.0024 | 0.0121 | 0.0027 | 2.6344 | 0.0017 | 0.0128 | 0.0020 |
| 0.99 | 0.2264 | -0.0025 | -0.1145 | 0.0013 | 0.4403 | -0.0028 | -0.1379 | 0.0010 | 0.2751 | 0.0048 | 0.0283 | 0.0057 | 3.6724 | 0.0028 | 0.0293 | 0.0033 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{\zeta^*/2}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $\zeta^*/2$.

Table 4: *Location-shift-scale model 2: bias and MSE of estimators*

| | *Bias* | | | | | | | | *MSE* | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | | *smoothed* | | | | *canonical* | | | | *smoothed* | | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ |
| 0.01 | -0.7681 | -0.0125 | 0.2905 | -0.0105 | -1.1328 | -0.0129 | 0.3420 | -0.0076 | 6.6734 | 0.0482 | 0.4797 | 0.1216 | 6.0112 | 0.0346 | 0.4022 | 0.0844 |
| 0.03 | -0.2569 | -0.0157 | 0.0885 | -0.0102 | -0.4655 | -0.0147 | 0.1190 | -0.0076 | 0.6452 | 0.0147 | 0.1179 | 0.0270 | 0.6305 | 0.0117 | 0.1063 | 0.0223 |
| 0.05 | -0.1350 | -0.0117 | 0.0534 | -0.0043 | -0.2893 | -0.0117 | 0.0778 | -0.0028 | 0.2675 | 0.0068 | 0.0410 | 0.0116 | 0.2723 | 0.0058 | 0.0385 | 0.0099 |
| 0.07 | -0.0814 | -0.0097 | 0.0358 | -0.0015 | -0.2096 | -0.0092 | 0.0566 | -0.0012 | 0.1487 | 0.0039 | 0.0236 | 0.0053 | 0.1600 | 0.0035 | 0.0228 | 0.0047 |
| 0.09 | -0.0580 | -0.0076 | 0.0271 | -0.0015 | -0.1632 | -0.0076 | 0.0447 | -0.0007 | 0.1006 | 0.0026 | 0.0162 | 0.0035 | 0.1100 | 0.0024 | 0.0157 | 0.0031 |
| 0.11 | -0.0404 | -0.0065 | 0.0221 | -0.0012 | -0.1318 | -0.0064 | 0.0367 | -0.0003 | 0.0736 | 0.0020 | 0.0117 | 0.0025 | 0.0821 | 0.0018 | 0.0117 | 0.0023 |
| 0.13 | -0.0313 | -0.0057 | 0.0182 | -0.0004 | -0.1087 | -0.0057 | 0.0307 | 0 | 0.0583 | 0.0016 | 0.0092 | 0.0020 | 0.0645 | 0.0015 | 0.0093 | 0.0018 |
| 0.15 | -0.0248 | -0.0050 | 0.0152 | 0 | -0.0907 | -0.0051 | 0.0259 | 0.0003 | 0.0475 | 0.0013 | 0.0076 | 0.0016 | 0.0529 | 0.0012 | 0.0078 | 0.0015 |
| 0.17 | -0.0196 | -0.0044 | 0.0124 | 0 | -0.0769 | -0.0047 | 0.0222 | 0.0005 | 0.0404 | 0.0011 | 0.0065 | 0.0014 | 0.0448 | 0.0010 | 0.0066 | 0.0013 |
| 0.19 | -0.0146 | -0.0044 | 0.0104 | 0.0004 | -0.0657 | -0.0044 | 0.0193 | 0.0008 | 0.0353 | 0.0010 | 0.0057 | 0.0012 | 0.0390 | 0.0009 | 0.0058 | 0.0011 |
| 0.21 | -0.0116 | -0.0040 | 0.0091 | 0.0004 | -0.0561 | -0.0041 | 0.0167 | 0.0010 | 0.0316 | 0.0008 | 0.0051 | 0.0010 | 0.0349 | 0.0008 | 0.0052 | 0.0010 |
| 0.23 | -0.0093 | -0.0037 | 0.0078 | 0.0007 | -0.0479 | -0.0039 | 0.0145 | 0.0011 | 0.0289 | 0.0008 | 0.0046 | 0.0009 | 0.0316 | 0.0008 | 0.0047 | 0.0009 |
| 0.25 | -0.0080 | -0.0034 | 0.0066 | 0.0007 | -0.0402 | -0.0036 | 0.0127 | 0.0012 | 0.0263 | 0.0007 | 0.0041 | 0.0009 | 0.0288 | 0.0007 | 0.0042 | 0.0008 |
| 0.27 | -0.0068 | -0.0031 | 0.0062 | 0.0008 | -0.0334 | -0.0034 | 0.0112 | 0.0012 | 0.0244 | 0.0006 | 0.0038 | 0.0008 | 0.0264 | 0.0006 | 0.0039 | 0.0008 |
| 0.29 | -0.0053 | -0.0030 | 0.0055 | 0.0009 | -0.0285 | -0.0032 | 0.0101 | 0.0013 | 0.0228 | 0.0006 | 0.0036 | 0.0008 | 0.0245 | 0.0006 | 0.0037 | 0.0007 |
| 0.31 | -0.0037 | -0.0029 | 0.0052 | 0.0011 | -0.0240 | -0.0031 | 0.0091 | 0.0014 | 0.0214 | 0.0006 | 0.0034 | 0.0007 | 0.0230 | 0.0006 | 0.0035 | 0.0007 |
| 0.33 | -0.0026 | -0.0028 | 0.0048 | 0.0012 | -0.0200 | -0.0029 | 0.0081 | 0.0014 | 0.0200 | 0.0005 | 0.0033 | 0.0007 | 0.0218 | 0.0005 | 0.0034 | 0.0007 |
| 0.35 | -0.0017 | -0.0025 | 0.0045 | 0.0011 | -0.0163 | -0.0028 | 0.0071 | 0.0014 | 0.0190 | 0.0005 | 0.0031 | 0.0006 | 0.0207 | 0.0005 | 0.0032 | 0.0006 |
| 0.37 | -0.0005 | -0.0023 | 0.0035 | 0.0009 | -0.0127 | -0.0026 | 0.0061 | 0.0013 | 0.0184 | 0.0005 | 0.0030 | 0.0006 | 0.0199 | 0.0005 | 0.0031 | 0.0006 |
| 0.39 | 0.0012 | -0.0022 | 0.0031 | 0.0009 | -0.0091 | -0.0025 | 0.0052 | 0.0013 | 0.0176 | 0.0005 | 0.0029 | 0.0006 | 0.0193 | 0.0005 | 0.0030 | 0.0006 |
| 0.41 | 0.0022 | -0.0022 | 0.0028 | 0.0011 | -0.0060 | -0.0024 | 0.0044 | 0.0013 | 0.0174 | 0.0005 | 0.0028 | 0.0006 | 0.0188 | 0.0005 | 0.0029 | 0.0006 |
| 0.43 | 0.0033 | -0.0021 | 0.0023 | 0.0010 | -0.0029 | -0.0024 | 0.0036 | 0.0013 | 0.0170 | 0.0004 | 0.0027 | 0.0006 | 0.0184 | 0.0004 | 0.0028 | 0.0006 |
| 0.45 | 0.0050 | -0.0021 | 0.0018 | 0.0011 | 0.0002 | -0.0023 | 0.0028 | 0.0014 | 0.0169 | 0.0004 | 0.0027 | 0.0005 | 0.0182 | 0.0004 | 0.0028 | 0.0005 |
| 0.47 | 0.0063 | -0.0021 | 0.0013 | 0.0012 | 0.0033 | -0.0023 | 0.0019 | 0.0014 | 0.0168 | 0.0004 | 0.0027 | 0.0005 | 0.0180 | 0.0004 | 0.0028 | 0.0005 |
| 0.49 | 0.0074 | -0.0021 | 0.0008 | 0.0013 | 0.0063 | -0.0023 | 0.0011 | 0.0015 | 0.0167 | 0.0004 | 0.0027 | 0.0005 | 0.0180 | 0.0004 | 0.0028 | 0.0005 |
| 0.51 | 0.0085 | -0.0021 | 0.0003 | 0.0014 | 0.0094 | -0.0023 | 0.0003 | 0.0015 | 0.0170 | 0.0004 | 0.0027 | 0.0005 | 0.0180 | 0.0004 | 0.0028 | 0.0005 |
| 0.53 | 0.0097 | -0.0020 | -0.0003 | 0.0014 | 0.0126 | -0.0022 | -0.0005 | 0.0015 | 0.0172 | 0.0004 | 0.0027 | 0.0005 | 0.0182 | 0.0004 | 0.0028 | 0.0005 |
| 0.55 | 0.0110 | -0.0020 | -0.0007 | 0.0015 | 0.0158 | -0.0022 | -0.0014 | 0.0015 | 0.0171 | 0.0004 | 0.0028 | 0.0005 | 0.0184 | 0.0004 | 0.0028 | 0.0006 |
| 0.57 | 0.0116 | -0.0019 | -0.0012 | 0.0015 | 0.0191 | -0.0022 | -0.0022 | 0.0016 | 0.0173 | 0.0004 | 0.0028 | 0.0006 | 0.0187 | 0.0004 | 0.0028 | 0.0006 |
| 0.59 | 0.0137 | -0.0020 | -0.0017 | 0.0017 | 0.0224 | -0.0022 | -0.0030 | 0.0017 | 0.0178 | 0.0004 | 0.0028 | 0.0006 | 0.0191 | 0.0004 | 0.0029 | 0.0006 |
| 0.61 | 0.0154 | -0.0021 | -0.0023 | 0.0018 | 0.0259 | -0.0022 | -0.0039 | 0.0018 | 0.0182 | 0.0004 | 0.0028 | 0.0006 | 0.0197 | 0.0004 | 0.0029 | 0.0006 |
| 0.63 | 0.0157 | -0.0019 | -0.0021 | 0.0017 | 0.0296 | -0.0022 | -0.0047 | 0.0018 | 0.0188 | 0.0005 | 0.0029 | 0.0006 | 0.0204 | 0.0005 | 0.0030 | 0.0006 |
| 0.65 | 0.0167 | -0.0017 | -0.0028 | 0.0016 | 0.0336 | -0.0022 | -0.0057 | 0.0018 | 0.0197 | 0.0005 | 0.0031 | 0.0006 | 0.0213 | 0.0005 | 0.0031 | 0.0006 |
| 0.67 | 0.0187 | -0.0017 | -0.0037 | 0.0018 | 0.0381 | -0.0021 | -0.0068 | 0.0018 | 0.0202 | 0.0005 | 0.0032 | 0.0007 | 0.0223 | 0.0005 | 0.0033 | 0.0007 |
| 0.69 | 0.0207 | -0.0016 | -0.0043 | 0.0016 | 0.0428 | -0.0021 | -0.0079 | 0.0018 | 0.0215 | 0.0005 | 0.0034 | 0.0007 | 0.0235 | 0.0005 | 0.0035 | 0.0007 |
| 0.71 | 0.0232 | -0.0016 | -0.0050 | 0.0018 | 0.0480 | -0.0021 | -0.0091 | 0.0019 | 0.0229 | 0.0005 | 0.0036 | 0.0007 | 0.0250 | 0.0005 | 0.0037 | 0.0007 |
| 0.73 | 0.0253 | -0.0017 | -0.0056 | 0.0020 | 0.0538 | -0.0020 | -0.0104 | 0.0020 | 0.0243 | 0.0006 | 0.0039 | 0.0008 | 0.0269 | 0.0006 | 0.0040 | 0.0008 |
| 0.75 | 0.0273 | -0.0016 | -0.0064 | 0.0022 | 0.0618 | -0.0020 | -0.0123 | 0.0022 | 0.0263 | 0.0006 | 0.0042 | 0.0009 | 0.0295 | 0.0006 | 0.0043 | 0.0009 |
| 0.77 | 0.0292 | -0.0014 | -0.0076 | 0.0025 | 0.0704 | -0.0019 | -0.0142 | 0.0024 | 0.0289 | 0.0007 | 0.0046 | 0.0010 | 0.0328 | 0.0007 | 0.0047 | 0.0009 |
| 0.79 | 0.0306 | -0.0010 | -0.0085 | 0.0026 | 0.0791 | -0.0017 | -0.0163 | 0.0025 | 0.0317 | 0.0008 | 0.0051 | 0.0011 | 0.0364 | 0.0007 | 0.0052 | 0.0010 |
| 0.81 | 0.0361 | -0.0011 | -0.0102 | 0.0030 | 0.0895 | -0.0016 | -0.0188 | 0.0027 | 0.0359 | 0.0008 | 0.0057 | 0.0012 | 0.0411 | 0.0008 | 0.0058 | 0.0012 |
| 0.83 | 0.0397 | -0.0008 | -0.0118 | 0.0034 | 0.1020 | -0.0015 | -0.0217 | 0.0031 | 0.0404 | 0.0010 | 0.0066 | 0.0014 | 0.0474 | 0.0009 | 0.0067 | 0.0013 |
| 0.85 | 0.0462 | -0.0006 | -0.0141 | 0.0036 | 0.1171 | -0.0014 | -0.0252 | 0.0035 | 0.0491 | 0.0011 | 0.0078 | 0.0017 | 0.0568 | 0.0011 | 0.0079 | 0.0015 |
| 0.87 | 0.0556 | -0.0007 | -0.0172 | 0.0045 | 0.1369 | -0.0013 | -0.0297 | 0.0040 | 0.0600 | 0.0013 | 0.0095 | 0.0020 | 0.0700 | 0.0013 | 0.0095 | 0.0019 |
| 0.89 | 0.0671 | -0.0005 | -0.0202 | 0.0054 | 0.1626 | -0.0013 | -0.0353 | 0.0049 | 0.0786 | 0.0017 | 0.0119 | 0.0027 | 0.0902 | 0.0016 | 0.0118 | 0.0024 |
| 0.91 | 0.0885 | -0.0005 | -0.0264 | 0.0068 | 0.1979 | -0.0013 | -0.0434 | 0.0060 | 0.1084 | 0.0024 | 0.0160 | 0.0037 | 0.1213 | 0.0021 | 0.0155 | 0.0032 |
| 0.93 | 0.1197 | -0.0009 | -0.0339 | 0.0094 | 0.2492 | -0.0012 | -0.0547 | 0.0083 | 0.1642 | 0.0035 | 0.0239 | 0.0058 | 0.1778 | 0.0030 | 0.0229 | 0.0050 |
| 0.95 | 0.1767 | 0.0001 | -0.0498 | 0.0125 | 0.3350 | -0.0008 | -0.0737 | 0.0117 | 0.2996 | 0.0062 | 0.0421 | 0.0102 | 0.3144 | 0.0051 | 0.0403 | 0.0091 |
| 0.97 | 0.3089 | 0.0006 | -0.0871 | 0.0203 | 0.5229 | 0.0001 | -0.1188 | 0.0173 | 0.7262 | 0.0137 | 0.1098 | 0.0271 | 0.7215 | 0.0111 | 0.0970 | 0.0222 |
| 0.99 | 0.8397 | -0.0048 | -0.3015 | 0.0367 | 1.2038 | -0.0036 | -0.3501 | 0.0300 | 4.7482 | 0.0552 | 0.4263 | 0.5343 | 4.5949 | 0.0396 | 0.3672 | 0.4058 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{\zeta^*/2}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $\zeta^*/2$.

Table 5: *Location-shift model 1: bias and MSE of estimators*

| | *Bias* | | | | | | | | *MSE* | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | | *smoothed* | | | | *canonical* | | | | *smoothed* | | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ |
| 0.01 | 2.2266 | -0.0006 | -0.0015 | 0.0019 | 2.2063 | -0.0006 | -0.0006 | 0.0020 | 0.0143 | 0.0066 | 0.0138 | 0.0158 | 0.0117 | 0.0052 | 0.0108 | 0.0124 |
| 0.03 | 1.0763 | -0.0016 | 0.0001 | 0.0027 | 1.0597 | -0.0015 | -0.0002 | 0.0024 | 0.0065 | 0.0030 | 0.0061 | 0.0073 | 0.0056 | 0.0025 | 0.0051 | 0.0060 |
| 0.05 | 0.7123 | -0.0016 | -0.0001 | 0.0021 | 0.6985 | -0.0014 | -0.0002 | 0.0017 | 0.0046 | 0.0022 | 0.0044 | 0.0051 | 0.0041 | 0.0018 | 0.0038 | 0.0043 |
| 0.07 | 0.5220 | -0.0015 | -0.0006 | 0.0011 | 0.5096 | -0.0013 | -0.0001 | 0.0013 | 0.0037 | 0.0017 | 0.0036 | 0.0042 | 0.0034 | 0.0015 | 0.0031 | 0.0036 |
| 0.09 | 0.4034 | -0.0013 | 0.0004 | 0.0009 | 0.3915 | -0.0013 | 0.0001 | 0.0011 | 0.0032 | 0.0015 | 0.0031 | 0.0036 | 0.0029 | 0.0013 | 0.0027 | 0.0031 |
| 0.11 | 0.3207 | -0.0015 | 0.0003 | 0.0014 | 0.3103 | -0.0014 | 0.0001 | 0.0011 | 0.0028 | 0.0013 | 0.0027 | 0.0031 | 0.0026 | 0.0011 | 0.0024 | 0.0027 |
| 0.13 | 0.2609 | -0.0014 | 0.0003 | 0.0011 | 0.2511 | -0.0013 | 0.0002 | 0.0010 | 0.0026 | 0.0012 | 0.0025 | 0.0028 | 0.0024 | 0.0010 | 0.0022 | 0.0025 |
| 0.15 | 0.2153 | -0.0011 | 0.0002 | 0.0005 | 0.2061 | -0.0013 | 0.0002 | 0.0010 | 0.0024 | 0.0011 | 0.0023 | 0.0026 | 0.0022 | 0.0010 | 0.0020 | 0.0023 |
| 0.17 | 0.1794 | -0.0014 | 0.0002 | 0.0011 | 0.1709 | -0.0013 | 0.0001 | 0.0011 | 0.0022 | 0.0010 | 0.0022 | 0.0025 | 0.0021 | 0.0009 | 0.0019 | 0.0022 |
| 0.19 | 0.1499 | -0.0011 | -0.0003 | 0.0012 | 0.1426 | -0.0013 | -0.0001 | 0.0012 | 0.0022 | 0.0010 | 0.0020 | 0.0023 | 0.0020 | 0.0009 | 0.0018 | 0.0021 |
| 0.21 | 0.1265 | -0.0012 | -0.0003 | 0.0014 | 0.1196 | -0.0014 | -0.0002 | 0.0013 | 0.0021 | 0.0009 | 0.0020 | 0.0023 | 0.0019 | 0.0008 | 0.0017 | 0.0020 |
| 0.23 | 0.1070 | -0.0015 | -0.0005 | 0.0011 | 0.1007 | -0.0014 | -0.0003 | 0.0014 | 0.0020 | 0.0009 | 0.0019 | 0.0022 | 0.0018 | 0.0008 | 0.0017 | 0.0019 |
| 0.25 | 0.0906 | -0.0015 | -0.0005 | 0.0015 | 0.0849 | -0.0015 | -0.0003 | 0.0015 | 0.0019 | 0.0009 | 0.0018 | 0.0021 | 0.0018 | 0.0008 | 0.0016 | 0.0019 |
| 0.27 | 0.0766 | -0.0016 | -0.0004 | 0.0016 | 0.0717 | -0.0016 | -0.0004 | 0.0016 | 0.0019 | 0.0008 | 0.0018 | 0.0021 | 0.0017 | 0.0008 | 0.0016 | 0.0018 |
| 0.29 | 0.0650 | -0.0016 | -0.0004 | 0.0016 | 0.0604 | -0.0017 | -0.0004 | 0.0016 | 0.0018 | 0.0008 | 0.0017 | 0.0020 | 0.0017 | 0.0007 | 0.0015 | 0.0018 |
| 0.31 | 0.0551 | -0.0018 | -0.0004 | 0.0016 | 0.0508 | -0.0017 | -0.0004 | 0.0016 | 0.0018 | 0.0008 | 0.0017 | 0.0020 | 0.0016 | 0.0007 | 0.0015 | 0.0018 |
| 0.33 | 0.0461 | -0.0018 | -0.0004 | 0.0018 | 0.0425 | -0.0018 | -0.0004 | 0.0017 | 0.0017 | 0.0008 | 0.0016 | 0.0019 | 0.0016 | 0.0007 | 0.0015 | 0.0017 |
| 0.35 | 0.0385 | -0.0018 | -0.0002 | 0.0017 | 0.0353 | -0.0018 | -0.0004 | 0.0016 | 0.0017 | 0.0008 | 0.0016 | 0.0019 | 0.0015 | 0.0007 | 0.0014 | 0.0017 |
| 0.37 | 0.0319 | -0.0018 | -0.0003 | 0.0015 | 0.0289 | -0.0018 | -0.0004 | 0.0016 | 0.0017 | 0.0008 | 0.0016 | 0.0019 | 0.0015 | 0.0007 | 0.0014 | 0.0017 |
| 0.39 | 0.0256 | -0.0019 | -0.0005 | 0.0015 | 0.0233 | -0.0018 | -0.0004 | 0.0016 | 0.0017 | 0.0008 | 0.0016 | 0.0018 | 0.0015 | 0.0007 | 0.0014 | 0.0016 |
| 0.41 | 0.0202 | -0.0018 | -0.0004 | 0.0015 | 0.0182 | -0.0019 | -0.0004 | 0.0017 | 0.0016 | 0.0007 | 0.0016 | 0.0018 | 0.0015 | 0.0007 | 0.0014 | 0.0016 |
| 0.43 | 0.0152 | -0.0018 | -0.0005 | 0.0016 | 0.0136 | -0.0019 | -0.0004 | 0.0017 | 0.0016 | 0.0007 | 0.0016 | 0.0018 | 0.0014 | 0.0007 | 0.0014 | 0.0016 |
| 0.45 | 0.0105 | -0.0018 | -0.0004 | 0.0017 | 0.0093 | -0.0019 | -0.0004 | 0.0017 | 0.0016 | 0.0007 | 0.0016 | 0.0018 | 0.0014 | 0.0007 | 0.0014 | 0.0016 |
| 0.47 | 0.0059 | -0.0018 | -0.0003 | 0.0018 | 0.0053 | -0.0019 | -0.0005 | 0.0018 | 0.0016 | 0.0007 | 0.0016 | 0.0018 | 0.0014 | 0.0007 | 0.0014 | 0.0016 |
| 0.49 | 0.0016 | -0.0018 | -0.0004 | 0.0018 | 0.0013 | -0.0019 | -0.0005 | 0.0018 | 0.0016 | 0.0007 | 0.0016 | 0.0018 | 0.0014 | 0.0007 | 0.0014 | 0.0016 |
| 0.51 | -0.0029 | -0.0020 | -0.0004 | 0.0020 | -0.0026 | -0.0020 | -0.0005 | 0.0019 | 0.0016 | 0.0007 | 0.0016 | 0.0018 | 0.0014 | 0.0007 | 0.0014 | 0.0016 |
| 0.53 | -0.0071 | -0.0021 | -0.0006 | 0.0019 | -0.0065 | -0.0020 | -0.0005 | 0.0019 | 0.0016 | 0.0008 | 0.0016 | 0.0018 | 0.0014 | 0.0007 | 0.0014 | 0.0016 |
| 0.55 | -0.0117 | -0.0022 | -0.0006 | 0.0021 | -0.0106 | -0.0020 | -0.0005 | 0.0019 | 0.0016 | 0.0008 | 0.0016 | 0.0018 | 0.0014 | 0.0007 | 0.0014 | 0.0016 |
| 0.57 | -0.0166 | -0.0023 | -0.0006 | 0.0019 | -0.0149 | -0.0021 | -0.0005 | 0.0019 | 0.0016 | 0.0008 | 0.0016 | 0.0018 | 0.0014 | 0.0007 | 0.0014 | 0.0016 |
| 0.59 | -0.0218 | -0.0020 | -0.0005 | 0.0018 | -0.0195 | -0.0021 | -0.0005 | 0.0019 | 0.0016 | 0.0008 | 0.0016 | 0.0018 | 0.0014 | 0.0007 | 0.0014 | 0.0016 |
| 0.61 | -0.0270 | -0.0020 | -0.0005 | 0.0018 | -0.0246 | -0.0020 | -0.0005 | 0.0018 | 0.0016 | 0.0008 | 0.0016 | 0.0018 | 0.0014 | 0.0007 | 0.0014 | 0.0017 |
| 0.63 | -0.0329 | -0.0020 | -0.0005 | 0.0018 | -0.0301 | -0.0020 | -0.0005 | 0.0018 | 0.0016 | 0.0008 | 0.0016 | 0.0019 | 0.0014 | 0.0007 | 0.0015 | 0.0017 |
| 0.65 | -0.0395 | -0.0020 | -0.0004 | 0.0018 | -0.0364 | -0.0019 | -0.0005 | 0.0018 | 0.0016 | 0.0008 | 0.0017 | 0.0019 | 0.0015 | 0.0007 | 0.0015 | 0.0017 |
| 0.67 | -0.0472 | -0.0018 | -0.0005 | 0.0019 | -0.0435 | -0.0018 | -0.0005 | 0.0017 | 0.0017 | 0.0008 | 0.0017 | 0.0019 | 0.0015 | 0.0007 | 0.0015 | 0.0017 |
| 0.69 | -0.0561 | -0.0016 | -0.0004 | 0.0017 | -0.0517 | -0.0017 | -0.0005 | 0.0017 | 0.0017 | 0.0008 | 0.0017 | 0.0020 | 0.0015 | 0.0007 | 0.0015 | 0.0018 |
| 0.71 | -0.0657 | -0.0015 | -0.0004 | 0.0017 | -0.0613 | -0.0016 | -0.0005 | 0.0016 | 0.0017 | 0.0008 | 0.0018 | 0.0020 | 0.0016 | 0.0007 | 0.0016 | 0.0018 |
| 0.73 | -0.0778 | -0.0016 | -0.0005 | 0.0014 | -0.0724 | -0.0016 | -0.0005 | 0.0016 | 0.0018 | 0.0009 | 0.0018 | 0.0021 | 0.0016 | 0.0008 | 0.0016 | 0.0018 |
| 0.75 | -0.0914 | -0.0015 | -0.0006 | 0.0012 | -0.0855 | -0.0015 | -0.0005 | 0.0015 | 0.0018 | 0.0009 | 0.0018 | 0.0021 | 0.0017 | 0.0008 | 0.0017 | 0.0019 |
| 0.77 | -0.1073 | -0.0013 | -0.0007 | 0.0013 | -0.1011 | -0.0014 | -0.0005 | 0.0015 | 0.0019 | 0.0009 | 0.0019 | 0.0022 | 0.0017 | 0.0008 | 0.0017 | 0.0020 |
| 0.79 | -0.1267 | -0.0013 | -0.0003 | 0.0014 | -0.1198 | -0.0013 | -0.0005 | 0.0015 | 0.0020 | 0.0010 | 0.0020 | 0.0023 | 0.0018 | 0.0008 | 0.0018 | 0.0020 |
| 0.81 | -0.1500 | -0.0013 | -0.0002 | 0.0015 | -0.1426 | -0.0013 | -0.0004 | 0.0014 | 0.0021 | 0.0010 | 0.0021 | 0.0024 | 0.0019 | 0.0009 | 0.0018 | 0.0021 |
| 0.83 | -0.1787 | -0.0013 | -0.0002 | 0.0013 | -0.1707 | -0.0013 | -0.0003 | 0.0013 | 0.0022 | 0.0010 | 0.0022 | 0.0025 | 0.0020 | 0.0009 | 0.0019 | 0.0022 |
| 0.85 | -0.2146 | -0.0010 | -0.0001 | 0.0013 | -0.2060 | -0.0012 | -0.0001 | 0.0013 | 0.0024 | 0.0011 | 0.0023 | 0.0027 | 0.0022 | 0.0010 | 0.0020 | 0.0023 |
| 0.87 | -0.2610 | -0.0013 | 0.0003 | 0.0013 | -0.2511 | -0.0013 | 0.0001 | 0.0013 | 0.0026 | 0.0012 | 0.0025 | 0.0028 | 0.0023 | 0.0011 | 0.0021 | 0.0025 |
| 0.89 | -0.3214 | -0.0016 | 0.0005 | 0.0014 | -0.3104 | -0.0013 | 0.0003 | 0.0013 | 0.0028 | 0.0014 | 0.0027 | 0.0031 | 0.0025 | 0.0012 | 0.0023 | 0.0027 |
| 0.91 | -0.4026 | -0.0014 | 0.0006 | 0.0011 | -0.3914 | -0.0013 | 0.0003 | 0.0011 | 0.0032 | 0.0015 | 0.0030 | 0.0035 | 0.0028 | 0.0013 | 0.0026 | 0.0030 |
| 0.93 | -0.5215 | -0.0011 | 0.0002 | 0.0008 | -0.5091 | -0.0012 | 0.0002 | 0.0009 | 0.0036 | 0.0018 | 0.0036 | 0.0041 | 0.0032 | 0.0015 | 0.0031 | 0.0035 |
| 0.95 | -0.7121 | -0.0011 | 0.0002 | 0.0007 | -0.6978 | -0.0011 | 0.0002 | 0.0008 | 0.0044 | 0.0022 | 0.0045 | 0.0051 | 0.0039 | 0.0019 | 0.0038 | 0.0043 |
| 0.97 | -1.0750 | -0.0012 | 0.0006 | 0.0009 | -1.0587 | -0.0011 | 0.0008 | 0.0004 | 0.0063 | 0.0031 | 0.0063 | 0.0070 | 0.0054 | 0.0026 | 0.0053 | 0.0058 |
| 0.99 | -2.2276 | -0.0004 | 0.0029 | -0.0019 | -2.2067 | -0.0008 | 0.0024 | -0.0012 | 0.0141 | 0.0070 | 0.0137 | 0.0155 | 0.0116 | 0.0055 | 0.0107 | 0.0121 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{\zeta^*/2}$. Sample size of $n = 1\,000$. The number of replications is $5,000$. The optimal bandwidth is $\zeta^*/2$.

Table 6: *Location-shift model 2: bias and MSE of estimators*

| | *Bias* | | | | | | | | *MSE* | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | | *smoothed* | | | | *canonical* | | | | *smoothed* | | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ |
| 0.01 | -0.1268 | 0.0033 | 0.0113 | -0.0101 | -0.2165 | -0.0018 | 0.0080 | -0.0043 | 7.6610 | 0.2630 | 0.7827 | 0.7171 | 7.2056 | 0.1815 | 0.5577 | 0.5048 |
| 0.03 | -0.0695 | -0.0202 | 0.0046 | 0.0077 | -0.1221 | -0.0213 | 0.0042 | 0.0074 | 0.7263 | 0.0708 | 0.1734 | 0.1734 | 0.6582 | 0.0555 | 0.1364 | 0.1363 |
| 0.05 | -0.0559 | -0.0215 | 0.0023 | 0.0077 | -0.1004 | -0.0221 | 0.0023 | 0.0087 | 0.2981 | 0.0377 | 0.0875 | 0.0911 | 0.2729 | 0.0307 | 0.0720 | 0.0742 |
| 0.07 | -0.0465 | -0.0200 | 0.0007 | 0.0037 | -0.0852 | -0.0194 | -0.0001 | 0.0047 | 0.1800 | 0.0256 | 0.0580 | 0.0609 | 0.1624 | 0.0208 | 0.0477 | 0.0491 |
| 0.09 | -0.0354 | -0.0180 | -0.0010 | 0.0028 | -0.0729 | -0.0171 | -0.0008 | 0.0034 | 0.1232 | 0.0186 | 0.0415 | 0.0440 | 0.1116 | 0.0154 | 0.0346 | 0.0364 |
| 0.11 | -0.0276 | -0.0155 | -0.0008 | 0.0033 | -0.0640 | -0.0159 | -0.0006 | 0.0040 | 0.0911 | 0.0145 | 0.0325 | 0.0348 | 0.0836 | 0.0122 | 0.0273 | 0.0288 |
| 0.13 | -0.0216 | -0.0151 | -0.0006 | 0.0032 | -0.0566 | -0.0151 | -0.0001 | 0.0040 | 0.0724 | 0.0120 | 0.0269 | 0.0283 | 0.0663 | 0.0101 | 0.0225 | 0.0237 |
| 0.15 | -0.0159 | -0.0147 | 0.0009 | 0.0033 | -0.0505 | -0.0143 | -0.0001 | 0.0041 | 0.0597 | 0.0103 | 0.0228 | 0.0240 | 0.0549 | 0.0087 | 0.0190 | 0.0202 |
| 0.17 | -0.0139 | -0.0139 | 0.0005 | 0.0046 | -0.0455 | -0.0136 | -0.0001 | 0.0039 | 0.0516 | 0.0092 | 0.0193 | 0.0212 | 0.0471 | 0.0077 | 0.0163 | 0.0176 |
| 0.19 | -0.0123 | -0.0131 | -0.0001 | 0.0043 | -0.0411 | -0.0127 | -0.0001 | 0.0036 | 0.0442 | 0.0082 | 0.0171 | 0.0188 | 0.0411 | 0.0069 | 0.0144 | 0.0157 |
| 0.21 | -0.0112 | -0.0121 | -0.0010 | 0.0044 | -0.0371 | -0.0118 | -0.0002 | 0.0032 | 0.0396 | 0.0074 | 0.0152 | 0.0169 | 0.0365 | 0.0062 | 0.0130 | 0.0142 |
| 0.23 | -0.0097 | -0.0118 | -0.0004 | 0.0027 | -0.0335 | -0.0111 | -0.0002 | 0.0029 | 0.0360 | 0.0068 | 0.0137 | 0.0153 | 0.0329 | 0.0057 | 0.0119 | 0.0129 |
| 0.25 | -0.0088 | -0.0112 | 0 | 0.0022 | -0.0288 | -0.0105 | -0.0002 | 0.0027 | 0.0332 | 0.0062 | 0.0129 | 0.0141 | 0.0301 | 0.0053 | 0.0110 | 0.0119 |
| 0.27 | -0.0060 | -0.0101 | -0.0001 | 0.0014 | -0.0244 | -0.0100 | -0.0002 | 0.0026 | 0.0304 | 0.0057 | 0.0121 | 0.0129 | 0.0278 | 0.0050 | 0.0104 | 0.0111 |
| 0.29 | -0.0048 | -0.0091 | -0.0006 | 0.0018 | -0.0219 | -0.0096 | -0.0002 | 0.0026 | 0.0284 | 0.0053 | 0.0113 | 0.0120 | 0.0259 | 0.0047 | 0.0098 | 0.0105 |
| 0.31 | -0.0047 | -0.0088 | -0.0007 | 0.0021 | -0.0197 | -0.0092 | -0.0002 | 0.0026 | 0.0273 | 0.0051 | 0.0108 | 0.0115 | 0.0243 | 0.0044 | 0.0093 | 0.0099 |
| 0.33 | -0.0043 | -0.0087 | -0.0004 | 0.0024 | -0.0176 | -0.0089 | -0.0003 | 0.0027 | 0.0257 | 0.0048 | 0.0102 | 0.0110 | 0.0229 | 0.0042 | 0.0089 | 0.0095 |
| 0.35 | -0.0026 | -0.0087 | -0.0013 | 0.0032 | -0.0156 | -0.0087 | -0.0003 | 0.0028 | 0.0245 | 0.0046 | 0.0098 | 0.0107 | 0.0217 | 0.0041 | 0.0086 | 0.0091 |
| 0.37 | -0.0020 | -0.0083 | -0.0006 | 0.0035 | -0.0135 | -0.0085 | -0.0003 | 0.0029 | 0.0232 | 0.0045 | 0.0096 | 0.0102 | 0.0207 | 0.0039 | 0.0083 | 0.0088 |
| 0.39 | -0.0011 | -0.0079 | -0.0005 | 0.0029 | -0.0114 | -0.0084 | -0.0003 | 0.0031 | 0.0225 | 0.0043 | 0.0093 | 0.0099 | 0.0199 | 0.0038 | 0.0081 | 0.0086 |
| 0.41 | -0.0012 | -0.0085 | 0.0004 | 0.0043 | -0.0093 | -0.0083 | -0.0002 | 0.0033 | 0.0217 | 0.0043 | 0.0090 | 0.0097 | 0.0192 | 0.0037 | 0.0079 | 0.0084 |
| 0.43 | -0.0013 | -0.0081 | -0.0002 | 0.0037 | -0.0072 | -0.0082 | -0.0001 | 0.0034 | 0.0213 | 0.0042 | 0.0089 | 0.0095 | 0.0187 | 0.0036 | 0.0078 | 0.0082 |
| 0.45 | -0.0008 | -0.0081 | 0 | 0.0038 | -0.0052 | -0.0081 | 0 | 0.0035 | 0.0208 | 0.0041 | 0.0089 | 0.0094 | 0.0183 | 0.0036 | 0.0077 | 0.0081 |
| 0.47 | -0.0004 | -0.0081 | 0.0003 | 0.0040 | -0.0030 | -0.0081 | 0.0001 | 0.0036 | 0.0205 | 0.0041 | 0.0089 | 0.0092 | 0.0180 | 0.0035 | 0.0076 | 0.0080 |
| 0.49 | -0.0002 | -0.0078 | 0.0002 | 0.0036 | -0.0008 | -0.0080 | 0.0002 | 0.0037 | 0.0205 | 0.0040 | 0.0087 | 0.0091 | 0.0179 | 0.0035 | 0.0076 | 0.0080 |
| 0.51 | 0.0011 | -0.0080 | 0.0003 | 0.0039 | 0.0015 | -0.0079 | 0.0003 | 0.0037 | 0.0207 | 0.0041 | 0.0086 | 0.0091 | 0.0178 | 0.0035 | 0.0076 | 0.0080 |
| 0.53 | 0.0014 | -0.0078 | 0.0002 | 0.0035 | 0.0038 | -0.0078 | 0.0004 | 0.0037 | 0.0206 | 0.0041 | 0.0087 | 0.0093 | 0.0179 | 0.0035 | 0.0076 | 0.0080 |
| 0.55 | 0.0007 | -0.0079 | 0.0007 | 0.0038 | 0.0061 | -0.0078 | 0.0004 | 0.0038 | 0.0207 | 0.0041 | 0.0089 | 0.0093 | 0.0181 | 0.0036 | 0.0077 | 0.0081 |
| 0.57 | 0.0015 | -0.0078 | 0.0007 | 0.0037 | 0.0085 | -0.0079 | 0.0004 | 0.0038 | 0.0210 | 0.0042 | 0.0090 | 0.0094 | 0.0183 | 0.0036 | 0.0078 | 0.0082 |
| 0.59 | 0.0031 | -0.0081 | 0.0007 | 0.0040 | 0.0108 | -0.0079 | 0.0004 | 0.0038 | 0.0214 | 0.0043 | 0.0092 | 0.0096 | 0.0187 | 0.0037 | 0.0080 | 0.0084 |
| 0.61 | 0.0032 | -0.0082 | 0.0006 | 0.0046 | 0.0131 | -0.0079 | 0.0004 | 0.0038 | 0.0218 | 0.0044 | 0.0094 | 0.0097 | 0.0193 | 0.0038 | 0.0082 | 0.0085 |
| 0.63 | 0.0039 | -0.0079 | 0.0004 | 0.0042 | 0.0155 | -0.0080 | 0.0004 | 0.0038 | 0.0227 | 0.0045 | 0.0097 | 0.0101 | 0.0200 | 0.0039 | 0.0084 | 0.0088 |
| 0.65 | 0.0062 | -0.0073 | 0.0003 | 0.0032 | 0.0177 | -0.0080 | 0.0004 | 0.0037 | 0.0231 | 0.0045 | 0.0100 | 0.0104 | 0.0208 | 0.0040 | 0.0087 | 0.0091 |
| 0.67 | 0.0068 | -0.0075 | 0.0006 | 0.0034 | 0.0198 | -0.0081 | 0.0004 | 0.0036 | 0.0243 | 0.0047 | 0.0105 | 0.0110 | 0.0219 | 0.0041 | 0.0090 | 0.0094 |
| 0.69 | 0.0068 | -0.0080 | 0.0003 | 0.0033 | 0.0220 | -0.0083 | 0.0004 | 0.0036 | 0.0257 | 0.0049 | 0.0108 | 0.0115 | 0.0232 | 0.0043 | 0.0094 | 0.0098 |
| 0.71 | 0.0069 | -0.0083 | 0.0004 | 0.0037 | 0.0242 | -0.0085 | 0.0004 | 0.0035 | 0.0274 | 0.0052 | 0.0113 | 0.0120 | 0.0248 | 0.0045 | 0.0098 | 0.0104 |
| 0.73 | 0.0083 | -0.0085 | 0.0001 | 0.0036 | 0.0267 | -0.0088 | 0.0005 | 0.0035 | 0.0295 | 0.0055 | 0.0120 | 0.0129 | 0.0267 | 0.0048 | 0.0104 | 0.0110 |
| 0.75 | 0.0098 | -0.0089 | 0.0001 | 0.0026 | 0.0313 | -0.0093 | 0.0006 | 0.0034 | 0.0318 | 0.0059 | 0.0127 | 0.0138 | 0.0290 | 0.0051 | 0.0110 | 0.0117 |
| 0.77 | 0.0123 | -0.0096 | 0.0005 | 0.0021 | 0.0366 | -0.0097 | 0.0008 | 0.0034 | 0.0346 | 0.0065 | 0.0138 | 0.0148 | 0.0321 | 0.0055 | 0.0119 | 0.0125 |
| 0.79 | 0.0154 | -0.0107 | 0.0011 | 0.0036 | 0.0410 | -0.0102 | 0.0009 | 0.0035 | 0.0388 | 0.0072 | 0.0153 | 0.0162 | 0.0359 | 0.0061 | 0.0130 | 0.0137 |
| 0.81 | 0.0181 | -0.0108 | 0.0005 | 0.0031 | 0.0455 | -0.0107 | 0.0011 | 0.0035 | 0.0441 | 0.0080 | 0.0170 | 0.0177 | 0.0407 | 0.0067 | 0.0145 | 0.0152 |
| 0.83 | 0.0192 | -0.0108 | 0.0007 | 0.0027 | 0.0505 | -0.0113 | 0.0012 | 0.0039 | 0.0504 | 0.0090 | 0.0191 | 0.0203 | 0.0469 | 0.0076 | 0.0164 | 0.0172 |
| 0.85 | 0.0228 | -0.0114 | 0.0011 | 0.0042 | 0.0559 | -0.0120 | 0.0017 | 0.0044 | 0.0592 | 0.0103 | 0.0227 | 0.0236 | 0.0554 | 0.0087 | 0.0189 | 0.0199 |
| 0.87 | 0.0256 | -0.0127 | 0.0018 | 0.0047 | 0.0616 | -0.0133 | 0.0021 | 0.0055 | 0.0727 | 0.0122 | 0.0266 | 0.0281 | 0.0677 | 0.0103 | 0.0224 | 0.0235 |
| 0.89 | 0.0345 | -0.0159 | 0.0013 | 0.0073 | 0.0693 | -0.0152 | 0.0021 | 0.0074 | 0.0944 | 0.0152 | 0.0323 | 0.0347 | 0.0863 | 0.0125 | 0.0273 | 0.0286 |
| 0.91 | 0.0419 | -0.0175 | 0.0020 | 0.0102 | 0.0798 | -0.0172 | 0.0023 | 0.0099 | 0.1280 | 0.0195 | 0.0425 | 0.0444 | 0.1161 | 0.0158 | 0.0345 | 0.0366 |
| 0.93 | 0.0521 | -0.0216 | 0.0008 | 0.0131 | 0.0925 | -0.0196 | 0.0022 | 0.0121 | 0.1870 | 0.0261 | 0.0565 | 0.0605 | 0.1707 | 0.0213 | 0.0465 | 0.0501 |
| 0.95 | 0.0638 | -0.0209 | 0.0012 | 0.0159 | 0.1114 | -0.0210 | 0.0004 | 0.0144 | 0.3184 | 0.0391 | 0.0886 | 0.0928 | 0.2923 | 0.0316 | 0.0705 | 0.0758 |
| 0.97 | 0.0976 | -0.0236 | -0.0045 | 0.0225 | 0.1472 | -0.0234 | -0.0017 | 0.0206 | 0.7632 | 0.0722 | 0.1654 | 0.1762 | 0.7028 | 0.0567 | 0.1307 | 0.1388 |
| 0.99 | 0.1356 | -0.0165 | -0.0083 | 0.0133 | 0.2290 | -0.0215 | -0.0052 | 0.0172 | 7.9011 | 0.2421 | 0.6875 | 0.7477 | 7.5178 | 0.1673 | 0.5041 | 0.5491 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{\zeta^*/2}$. Sample size of $n = 1\,000$. The number of replications is $5,000$. The optimal bandwidth is $\zeta^*/2$.

Table 7: *Location-shift-scale model 1: bias and $MSE$ of estimators*

| | *Bias* | | | | | | | | *MSE* | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | | *smoothed* | | | | *canonical* | | | | *smoothed* | | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ |
| 0.01 | -0.0290 | -0.0014 | 0.0156 | -0.0016 | -0.1912 | -0.0010 | 0.0440 | -0.0009 | 0.0098 | 0.0002 | 0.0020 | 0.0003 | 4.5652 | 0.0002 | 0.0034 | 0.0003 |
| 0.03 | -0.0115 | -0.0008 | 0.0061 | -0.0007 | -0.1177 | -0.0004 | 0.0266 | -0.0004 | 0.0036 | 0.0001 | 0.0008 | 0.0001 | 3.1117 | 0.0001 | 0.0014 | 0.0001 |
| 0.05 | -0.0069 | -0.0006 | 0.0039 | -0.0004 | -0.0879 | -0.0004 | 0.0200 | -0.0002 | 0.0023 | 0.0001 | 0.0006 | 0.0001 | 2.4267 | 0.0001 | 0.0009 | 0.0001 |
| 0.07 | -0.0049 | -0.0005 | 0.0030 | -0.0003 | -0.0698 | -0.0004 | 0.0161 | -0.0001 | 0.0018 | 0 | 0.0004 | 0.0001 | 1.9788 | 0.0001 | 0.0007 | 0.0001 |
| 0.09 | -0.0035 | -0.0005 | 0.0021 | -0.0002 | -0.0574 | -0.0003 | 0.0133 | -0.0001 | 0.0015 | 0 | 0.0004 | 0 | 1.6489 | 0 | 0.0005 | 0.0001 |
| 0.11 | -0.0028 | -0.0004 | 0.0018 | -0.0002 | -0.0483 | -0.0003 | 0.0113 | -0.0001 | 0.0014 | 0 | 0.0003 | 0 | 1.3899 | 0 | 0.0005 | 0 |
| 0.13 | -0.0028 | -0.0003 | 0.0016 | -0.0001 | -0.0412 | -0.0002 | 0.0097 | -0.0001 | 0.0012 | 0 | 0.0003 | 0 | 1.1792 | 0 | 0.0004 | 0 |
| 0.15 | -0.0026 | -0.0002 | 0.0014 | -0.0001 | -0.0353 | -0.0002 | 0.0084 | -0.0001 | 0.0012 | 0 | 0.0003 | 0 | 1.0036 | 0 | 0.0004 | 0 |
| 0.17 | -0.0022 | -0.0002 | 0.0014 | -0.0002 | -0.0304 | -0.0001 | 0.0073 | -0.0001 | 0.0011 | 0 | 0.0003 | 0 | 0.8547 | 0 | 0.0003 | 0 |
| 0.19 | -0.0020 | -0.0002 | 0.0011 | -0.0001 | -0.0261 | -0.0001 | 0.0064 | 0 | 0.0010 | 0 | 0.0003 | 0 | 0.7267 | 0 | 0.0003 | 0 |
| 0.21 | -0.0019 | -0.0001 | 0.0009 | -0.0001 | -0.0225 | -0.0001 | 0.0055 | 0 | 0.0010 | 0 | 0.0002 | 0 | 0.6157 | 0 | 0.0003 | 0 |
| 0.23 | -0.0015 | -0.0001 | 0.0008 | -0.0001 | -0.0194 | -0.0001 | 0.0048 | 0 | 0.0009 | 0 | 0.0002 | 0 | 0.5188 | 0 | 0.0003 | 0 |
| 0.25 | -0.0013 | -0.0001 | 0.0007 | -0.0001 | -0.0165 | -0.0001 | 0.0041 | 0 | 0.0009 | 0 | 0.0002 | 0 | 0.4340 | 0 | 0.0003 | 0 |
| 0.27 | -0.0009 | -0.0002 | 0.0006 | 0 | -0.0141 | -0.0001 | 0.0035 | 0 | 0.0009 | 0 | 0.0002 | 0 | 0.3595 | 0 | 0.0002 | 0 |
| 0.29 | -0.0008 | -0.0001 | 0.0004 | 0 | -0.0121 | -0.0001 | 0.0030 | 0 | 0.0008 | 0 | 0.0002 | 0 | 0.2941 | 0 | 0.0002 | 0 |
| 0.31 | -0.0005 | -0.0002 | 0.0004 | 0 | -0.0103 | -0.0001 | 0.0026 | 0 | 0.0008 | 0 | 0.0002 | 0 | 0.2368 | 0 | 0.0002 | 0 |
| 0.33 | -0.0004 | -0.0002 | 0.0003 | 0 | -0.0089 | -0.0001 | 0.0022 | 0 | 0.0008 | 0 | 0.0002 | 0 | 0.1868 | 0 | 0.0002 | 0 |
| 0.35 | -0.0004 | -0.0001 | 0.0002 | 0 | -0.0078 | -0.0001 | 0.0019 | 0 | 0.0008 | 0 | 0.0002 | 0 | 0.1435 | 0 | 0.0002 | 0 |
| 0.37 | -0.0004 | -0.0001 | 0.0001 | 0 | -0.0068 | -0.0001 | 0.0016 | 0 | 0.0008 | 0 | 0.0002 | 0 | 0.1066 | 0 | 0.0002 | 0 |
| 0.39 | -0.0004 | -0.0001 | 0.0002 | 0 | -0.0060 | -0.0001 | 0.0014 | 0 | 0.0007 | 0 | 0.0002 | 0 | 0.0757 | 0 | 0.0002 | 0 |
| 0.41 | -0.0002 | -0.0001 | 0 | 0 | -0.0052 | -0.0001 | 0.0012 | 0 | 0.0007 | 0 | 0.0002 | 0 | 0.0504 | 0 | 0.0002 | 0 |
| 0.43 | -0.0001 | -0.0001 | 0.0001 | 0 | -0.0042 | -0.0001 | 0.0010 | 0 | 0.0007 | 0 | 0.0002 | 0 | 0.0306 | 0 | 0.0002 | 0 |
| 0.45 | 0 | -0.0001 | 0.0001 | 0 | -0.0031 | -0.0001 | 0.0007 | 0 | 0.0007 | 0 | 0.0002 | 0 | 0.0160 | 0 | 0.0002 | 0 |
| 0.47 | 0.0002 | -0.0001 | 0 | 0 | -0.0018 | -0.0001 | 0.0004 | 0 | 0.0007 | 0 | 0.0002 | 0 | 0.0064 | 0 | 0.0002 | 0 |
| 0.49 | 0.0003 | -0.0001 | -0.0001 | 0.0001 | -0.0003 | -0.0001 | 0.0001 | 0 | 0.0007 | 0 | 0.0002 | 0 | 0.0016 | 0 | 0.0002 | 0 |
| 0.51 | 0.0004 | -0.0001 | 0 | 0 | 0.0012 | -0.0001 | -0.0002 | 0 | 0.0007 | 0 | 0.0002 | 0 | 0.0015 | 0 | 0.0002 | 0 |
| 0.53 | 0.0005 | -0.0001 | -0.0001 | 0 | 0.0027 | -0.0001 | -0.0005 | 0 | 0.0007 | 0 | 0.0002 | 0 | 0.0062 | 0 | 0.0002 | 0 |
| 0.55 | 0.0008 | -0.0001 | -0.0003 | 0 | 0.0040 | -0.0001 | -0.0009 | 0 | 0.0007 | 0 | 0.0002 | 0 | 0.0158 | 0 | 0.0002 | 0 |
| 0.57 | 0.0009 | -0.0001 | -0.0003 | 0 | 0.0050 | -0.0001 | -0.0011 | 0 | 0.0007 | 0 | 0.0002 | 0 | 0.0303 | 0 | 0.0002 | 0 |
| 0.59 | 0.0010 | -0.0001 | -0.0003 | 0.0001 | 0.0060 | -0.0001 | -0.0014 | 0 | 0.0007 | 0 | 0.0002 | 0 | 0.0501 | 0 | 0.0002 | 0 |
| 0.61 | 0.0012 | -0.0001 | -0.0004 | 0.0001 | 0.0069 | -0.0001 | -0.0016 | 0 | 0.0007 | 0 | 0.0002 | 0 | 0.0752 | 0 | 0.0002 | 0 |
| 0.63 | 0.0014 | -0.0001 | -0.0003 | 0 | 0.0077 | -0.0001 | -0.0017 | 0 | 0.0008 | 0 | 0.0002 | 0 | 0.1061 | 0 | 0.0002 | 0 |
| 0.65 | 0.0012 | 0 | -0.0002 | 0 | 0.0086 | -0.0001 | -0.0019 | 0 | 0.0008 | 0 | 0.0002 | 0 | 0.1429 | 0 | 0.0002 | 0 |
| 0.67 | 0.0012 | 0 | -0.0002 | 0.0001 | 0.0096 | -0.0001 | -0.0021 | 0.0001 | 0.0008 | 0 | 0.0002 | 0 | 0.1862 | 0 | 0.0002 | 0 |
| 0.69 | 0.0011 | 0 | -0.0002 | 0.0001 | 0.0110 | -0.0001 | -0.0024 | 0.0001 | 0.0008 | 0 | 0.0002 | 0 | 0.2361 | 0 | 0.0002 | 0 |
| 0.71 | 0.0014 | 0 | -0.0003 | 0.0001 | 0.0127 | -0.0001 | -0.0029 | 0.0001 | 0.0008 | 0 | 0.0002 | 0 | 0.2934 | 0 | 0.0002 | 0 |
| 0.73 | 0.0013 | 0 | -0.0004 | 0.0002 | 0.0146 | 0 | -0.0033 | 0.0001 | 0.0008 | 0 | 0.0002 | 0 | 0.3589 | 0 | 0.0002 | 0 |
| 0.75 | 0.0012 | 0.0001 | -0.0004 | 0.0002 | 0.0170 | 0 | -0.0039 | 0.0001 | 0.0009 | 0 | 0.0002 | 0 | 0.4334 | 0 | 0.0002 | 0 |
| 0.77 | 0.0016 | 0.0001 | -0.0007 | 0.0001 | 0.0197 | 0 | -0.0046 | 0.0001 | 0.0009 | 0 | 0.0002 | 0 | 0.5183 | 0 | 0.0003 | 0 |
| 0.79 | 0.0016 | 0.0001 | -0.0008 | 0.0002 | 0.0228 | 0 | -0.0054 | 0.0001 | 0.0009 | 0 | 0.0002 | 0 | 0.6153 | 0 | 0.0003 | 0 |
| 0.81 | 0.0020 | 0.0001 | -0.0009 | 0.0002 | 0.0263 | 0 | -0.0062 | 0.0001 | 0.0010 | 0 | 0.0002 | 0 | 0.7265 | 0 | 0.0003 | 0 |
| 0.83 | 0.0023 | 0.0001 | -0.0012 | 0.0002 | 0.0305 | 0.0001 | -0.0072 | 0.0001 | 0.0011 | 0 | 0.0003 | 0 | 0.8545 | 0 | 0.0003 | 0 |
| 0.85 | 0.0024 | 0.0002 | -0.0015 | 0.0003 | 0.0355 | 0.0001 | -0.0085 | 0.0001 | 0.0011 | 0 | 0.0003 | 0 | 1.0032 | 0 | 0.0004 | 0 |
| 0.87 | 0.0026 | 0.0003 | -0.0017 | 0.0002 | 0.0414 | 0.0001 | -0.0098 | 0.0001 | 0.0012 | 0 | 0.0003 | 0 | 1.1786 | 0 | 0.0004 | 0 |
| 0.89 | 0.0034 | 0.0003 | -0.0020 | 0.0002 | 0.0487 | 0.0002 | -0.0114 | 0.0001 | 0.0014 | 0 | 0.0003 | 0 | 1.3888 | 0 | 0.0005 | 0 |
| 0.91 | 0.0042 | 0.0003 | -0.0024 | 0.0003 | 0.0581 | 0.0002 | -0.0134 | 0.0001 | 0.0015 | 0 | 0.0004 | 0 | 1.6471 | 0 | 0.0006 | 0.0001 |
| 0.93 | 0.0053 | 0.0004 | -0.0029 | 0.0003 | 0.0704 | 0.0002 | -0.0161 | 0.0001 | 0.0018 | 0.0001 | 0.0004 | 0.0001 | 1.9772 | 0.0001 | 0.0007 | 0.0001 |
| 0.95 | 0.0072 | 0.0005 | -0.0037 | 0.0004 | 0.0885 | 0.0003 | -0.0200 | 0.0002 | 0.0023 | 0.0001 | 0.0006 | 0.0001 | 2.4249 | 0.0001 | 0.0009 | 0.0001 |
| 0.97 | 0.0122 | 0.0006 | -0.0061 | 0.0006 | 0.1184 | 0.0004 | -0.0268 | 0.0003 | 0.0036 | 0.0001 | 0.0008 | 0.0001 | 3.1093 | 0.0001 | 0.0014 | 0.0001 |
| 0.99 | 0.0320 | 0.0012 | -0.0161 | 0.0014 | 0.1933 | 0.0008 | -0.0447 | 0.0007 | 0.0104 | 0.0002 | 0.0021 | 0.0003 | 4.5564 | 0.0002 | 0.0034 | 0.0002 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 1\,000$. The number of replications is $5,000$. The optimal bandwidth is $\zeta^*/2$.

Table 8: *Location-shift-scale model 2: bias and MSE of estimators*

| | *Bias* | | | | | | | | *MSE* | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | | *smoothed* | | | | *canonical* | | | | *smoothed* | | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\theta_2$ |
| 0.01 | -0.1517 | -0.0060 | 0.0587 | -0.0127 | -0.4102 | -0.0055 | 0.1107 | -0.0108 | 0.1734 | 0.0030 | 0.0406 | 0.0058 | 0.2719 | 0.0027 | 0.0407 | 0.0051 |
| 0.03 | -0.0422 | -0.0020 | 0.0167 | -0.0032 | -0.1954 | -0.0016 | 0.0474 | -0.0022 | 0.0237 | 0.0005 | 0.0064 | 0.0008 | 0.0559 | 0.0005 | 0.0074 | 0.0007 |
| 0.05 | -0.0220 | -0.0011 | 0.0094 | -0.0020 | -0.1353 | -0.0009 | 0.0323 | -0.0013 | 0.0103 | 0.0002 | 0.0029 | 0.0003 | 0.0272 | 0.0002 | 0.0036 | 0.0003 |
| 0.07 | -0.0140 | -0.0009 | 0.0069 | -0.0013 | -0.1038 | -0.0006 | 0.0249 | -0.0009 | 0.0062 | 0.0001 | 0.0018 | 0.0002 | 0.0166 | 0.0002 | 0.0022 | 0.0002 |
| 0.09 | -0.0094 | -0.0007 | 0.0050 | -0.0010 | -0.0834 | -0.0005 | 0.0197 | -0.0006 | 0.0043 | 0.0001 | 0.0012 | 0.0001 | 0.0113 | 0.0001 | 0.0015 | 0.0002 |
| 0.11 | -0.0078 | -0.0005 | 0.0037 | -0.0007 | -0.0691 | -0.0004 | 0.0161 | -0.0004 | 0.0032 | 0.0001 | 0.0009 | 0.0001 | 0.0083 | 0.0001 | 0.0012 | 0.0001 |
| 0.13 | -0.0057 | -0.0005 | 0.0028 | -0.0005 | -0.0581 | -0.0003 | 0.0134 | -0.0003 | 0.0025 | 0.0001 | 0.0007 | 0.0001 | 0.0063 | 0.0001 | 0.0009 | 0.0001 |
| 0.15 | -0.0046 | -0.0004 | 0.0022 | -0.0004 | -0.0492 | -0.0003 | 0.0114 | -0.0002 | 0.0022 | 0.0001 | 0.0006 | 0.0001 | 0.0049 | 0.0001 | 0.0007 | 0.0001 |
| 0.17 | -0.0037 | -0.0004 | 0.0021 | -0.0003 | -0.0420 | -0.0003 | 0.0098 | -0.0001 | 0.0019 | 0 | 0.0005 | 0.0001 | 0.0040 | 0.0001 | 0.0006 | 0.0001 |
| 0.19 | -0.0029 | -0.0004 | 0.0018 | -0.0002 | -0.0361 | -0.0003 | 0.0085 | -0.0001 | 0.0016 | 0 | 0.0005 | 0.0001 | 0.0033 | 0 | 0.0005 | 0.0001 |
| 0.21 | -0.0024 | -0.0004 | 0.0016 | -0.0002 | -0.0310 | -0.0003 | 0.0074 | -0.0001 | 0.0015 | 0 | 0.0004 | 0 | 0.0028 | 0 | 0.0005 | 0.0001 |
| 0.23 | -0.0018 | -0.0003 | 0.0014 | -0.0002 | -0.0267 | -0.0003 | 0.0065 | -0.0001 | 0.0013 | 0 | 0.0004 | 0 | 0.0024 | 0 | 0.0004 | 0.0001 |
| 0.25 | -0.0016 | -0.0003 | 0.0013 | -0.0002 | -0.0231 | -0.0002 | 0.0057 | -0.0001 | 0.0012 | 0 | 0.0003 | 0 | 0.0021 | 0 | 0.0004 | 0 |
| 0.27 | -0.0017 | -0.0002 | 0.0013 | -0.0001 | -0.0200 | -0.0002 | 0.0051 | 0 | 0.0011 | 0 | 0.0003 | 0 | 0.0018 | 0 | 0.0003 | 0 |
| 0.29 | -0.0016 | -0.0002 | 0.0012 | -0.0001 | -0.0174 | -0.0002 | 0.0045 | 0 | 0.0011 | 0 | 0.0003 | 0 | 0.0017 | 0 | 0.0003 | 0 |
| 0.31 | -0.0013 | -0.0002 | 0.0011 | -0.0001 | -0.0151 | -0.0002 | 0.0039 | 0 | 0.0010 | 0 | 0.0003 | 0 | 0.0015 | 0 | 0.0003 | 0 |
| 0.33 | -0.0011 | -0.0001 | 0.0009 | -0.0001 | -0.0128 | -0.0002 | 0.0033 | 0 | 0.0009 | 0 | 0.0002 | 0 | 0.0014 | 0 | 0.0003 | 0 |
| 0.35 | -0.0008 | -0.0001 | 0.0006 | 0 | -0.0108 | -0.0001 | 0.0027 | 0 | 0.0009 | 0 | 0.0002 | 0 | 0.0013 | 0 | 0.0003 | 0 |
| 0.37 | -0.0007 | -0.0001 | 0.0005 | -0.0001 | -0.0090 | -0.0001 | 0.0023 | 0 | 0.0009 | 0 | 0.0002 | 0 | 0.0012 | 0 | 0.0003 | 0 |
| 0.39 | -0.0004 | -0.0001 | 0.0006 | -0.0001 | -0.0074 | -0.0001 | 0.0020 | 0 | 0.0009 | 0 | 0.0002 | 0 | 0.0012 | 0 | 0.0003 | 0 |
| 0.41 | -0.0003 | -0.0001 | 0.0005 | -0.0001 | -0.0059 | -0.0001 | 0.0016 | 0 | 0.0008 | 0 | 0.0002 | 0 | 0.0011 | 0 | 0.0003 | 0 |
| 0.43 | -0.0001 | -0.0001 | 0.0003 | 0 | -0.0044 | -0.0001 | 0.0012 | 0 | 0.0008 | 0 | 0.0002 | 0 | 0.0011 | 0 | 0.0002 | 0 |
| 0.45 | 0.0002 | -0.0001 | 0.0001 | 0 | -0.0029 | -0.0001 | 0.0008 | 0 | 0.0008 | 0 | 0.0002 | 0 | 0.0011 | 0 | 0.0002 | 0 |
| 0.47 | 0.0002 | -0.0001 | 0 | 0 | -0.0016 | -0.0001 | 0.0004 | 0 | 0.0008 | 0 | 0.0002 | 0 | 0.0011 | 0 | 0.0002 | 0 |
| 0.49 | 0.0003 | -0.0001 | -0.0001 | 0 | -0.0002 | -0.0001 | 0 | 0 | 0.0008 | 0 | 0.0002 | 0 | 0.0011 | 0 | 0.0002 | 0 |
| 0.51 | 0.0004 | 0 | -0.0002 | 0 | 0.0011 | -0.0001 | -0.0003 | 0 | 0.0008 | 0 | 0.0002 | 0 | 0.0011 | 0 | 0.0002 | 0 |
| 0.53 | 0.0006 | -0.0001 | -0.0003 | 0 | 0.0025 | -0.0001 | -0.0007 | 0.0001 | 0.0008 | 0 | 0.0002 | 0 | 0.0011 | 0 | 0.0002 | 0 |
| 0.55 | 0.0006 | 0 | -0.0004 | 0.0001 | 0.0038 | -0.0001 | -0.0010 | 0.0001 | 0.0008 | 0 | 0.0002 | 0 | 0.0011 | 0 | 0.0002 | 0 |
| 0.57 | 0.0007 | 0 | -0.0004 | 0.0001 | 0.0051 | -0.0001 | -0.0013 | 0.0001 | 0.0008 | 0 | 0.0002 | 0 | 0.0011 | 0 | 0.0003 | 0 |
| 0.59 | 0.0009 | 0 | -0.0004 | 0.0001 | 0.0066 | -0.0001 | -0.0016 | 0.0001 | 0.0008 | 0 | 0.0002 | 0 | 0.0011 | 0 | 0.0003 | 0 |
| 0.61 | 0.0010 | 0 | -0.0004 | 0.0002 | 0.0080 | -0.0001 | -0.0019 | 0.0002 | 0.0009 | 0 | 0.0002 | 0 | 0.0012 | 0 | 0.0003 | 0 |
| 0.63 | 0.0012 | 0 | -0.0005 | 0.0002 | 0.0097 | -0.0001 | -0.0023 | 0.0002 | 0.0009 | 0 | 0.0002 | 0 | 0.0012 | 0 | 0.0003 | 0 |
| 0.65 | 0.0015 | -0.0001 | -0.0006 | 0.0002 | 0.0115 | -0.0001 | -0.0027 | 0.0002 | 0.0009 | 0 | 0.0003 | 0 | 0.0013 | 0 | 0.0003 | 0 |
| 0.67 | 0.0015 | 0 | -0.0008 | 0.0002 | 0.0135 | -0.0001 | -0.0032 | 0.0002 | 0.0010 | 0 | 0.0003 | 0 | 0.0014 | 0 | 0.0003 | 0 |
| 0.69 | 0.0018 | 0 | -0.0008 | 0.0002 | 0.0157 | -0.0001 | -0.0037 | 0.0003 | 0.0010 | 0 | 0.0003 | 0 | 0.0015 | 0 | 0.0003 | 0 |
| 0.71 | 0.0021 | 0 | -0.0010 | 0.0003 | 0.0181 | -0.0001 | -0.0043 | 0.0003 | 0.0011 | 0 | 0.0003 | 0 | 0.0017 | 0 | 0.0003 | 0 |
| 0.73 | 0.0025 | 0 | -0.0011 | 0.0004 | 0.0210 | -0.0001 | -0.0049 | 0.0003 | 0.0011 | 0 | 0.0003 | 0 | 0.0019 | 0 | 0.0004 | 0 |
| 0.75 | 0.0031 | 0 | -0.0013 | 0.0003 | 0.0244 | -0.0001 | -0.0056 | 0.0003 | 0.0012 | 0 | 0.0003 | 0 | 0.0021 | 0 | 0.0004 | 0 |
| 0.77 | 0.0037 | -0.0001 | -0.0015 | 0.0004 | 0.0284 | -0.0001 | -0.0065 | 0.0004 | 0.0014 | 0 | 0.0004 | 0 | 0.0025 | 0 | 0.0004 | 0.0001 |
| 0.79 | 0.0040 | 0 | -0.0017 | 0.0005 | 0.0329 | -0.0002 | -0.0075 | 0.0004 | 0.0015 | 0 | 0.0004 | 0 | 0.0029 | 0 | 0.0005 | 0.0001 |
| 0.81 | 0.0046 | 0 | -0.0018 | 0.0005 | 0.0380 | -0.0001 | -0.0085 | 0.0004 | 0.0017 | 0 | 0.0005 | 0.0001 | 0.0034 | 0 | 0.0005 | 0.0001 |
| 0.83 | 0.0051 | 0 | -0.0019 | 0.0006 | 0.0440 | -0.0001 | -0.0098 | 0.0005 | 0.0019 | 0 | 0.0005 | 0.0001 | 0.0041 | 0.0001 | 0.0006 | 0.0001 |
| 0.85 | 0.0062 | 0 | -0.0024 | 0.0007 | 0.0512 | -0.0001 | -0.0114 | 0.0005 | 0.0022 | 0.0001 | 0.0006 | 0.0001 | 0.0051 | 0.0001 | 0.0007 | 0.0001 |
| 0.87 | 0.0075 | 0.0001 | -0.0029 | 0.0008 | 0.0602 | -0.0001 | -0.0135 | 0.0006 | 0.0026 | 0.0001 | 0.0007 | 0.0001 | 0.0065 | 0.0001 | 0.0009 | 0.0001 |
| 0.89 | 0.0091 | 0.0001 | -0.0036 | 0.0009 | 0.0713 | -0.0001 | -0.0160 | 0.0007 | 0.0033 | 0.0001 | 0.0009 | 0.0001 | 0.0086 | 0.0001 | 0.0011 | 0.0001 |
| 0.91 | 0.0121 | 0.0002 | -0.0046 | 0.0011 | 0.0857 | 0 | -0.0194 | 0.0008 | 0.0045 | 0.0001 | 0.0012 | 0.0002 | 0.0118 | 0.0001 | 0.0015 | 0.0002 |
| 0.93 | 0.0161 | 0.0004 | -0.0062 | 0.0015 | 0.1057 | 0.0001 | -0.0242 | 0.0011 | 0.0064 | 0.0001 | 0.0017 | 0.0002 | 0.0171 | 0.0001 | 0.0021 | 0.0002 |
| 0.95 | 0.0233 | 0.0007 | -0.0095 | 0.0024 | 0.1367 | 0.0004 | -0.0322 | 0.0017 | 0.0105 | 0.0002 | 0.0028 | 0.0004 | 0.0276 | 0.0002 | 0.0035 | 0.0004 |
| 0.97 | 0.0431 | 0.0013 | -0.0173 | 0.0038 | 0.1962 | 0.0009 | -0.0482 | 0.0032 | 0.0242 | 0.0005 | 0.0062 | 0.0008 | 0.0557 | 0.0005 | 0.0072 | 0.0007 |
| 0.99 | 0.1491 | 0.0044 | -0.0564 | 0.0137 | 0.4092 | 0.0038 | -0.1099 | 0.0125 | 0.1665 | 0.0027 | 0.0390 | 0.0058 | 0.2654 | 0.0024 | 0.0395 | 0.0052 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*/2}$. Sample size of $n = 1\,000$. The number of replications is $5,000$. The optimal bandwidth is $\zeta^*/2$.

Table 9: *Model* (2) *defined through Proposition* 2 *when* $\lambda(\tau) = \sqrt{\tau}$*: bias and MSE of estimators*

| | *Bias* | | | | | | *MSE* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | *smoothed* | | | *canonical* | | | *smoothed* | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ |
| 0.01 | 0.0036 | -0.0216 | 0.0565 | -0.1820 | -0.0269 | 0.1256 | 0.0493 | 0.0674 | 0.0682 | 0.0461 | 0.0185 | 0.0335 |
| 0.03 | 0.0118 | -0.0128 | 0.0038 | -0.1119 | -0.0231 | 0.0596 | 0.0259 | 0.0371 | 0.0368 | 0.0232 | 0.0153 | 0.0182 |
| 0.05 | 0.0188 | -0.0147 | -0.0098 | -0.0804 | -0.0207 | 0.0295 | 0.0205 | 0.0285 | 0.0281 | 0.0160 | 0.0135 | 0.0139 |
| 0.07 | 0.0207 | -0.0137 | -0.0147 | -0.0601 | -0.0190 | 0.0103 | 0.0171 | 0.0237 | 0.0233 | 0.0126 | 0.0124 | 0.0122 |
| 0.09 | 0.0228 | -0.0140 | -0.0189 | -0.0455 | -0.0177 | -0.0035 | 0.0157 | 0.0214 | 0.0212 | 0.0106 | 0.0117 | 0.0114 |
| 0.11 | 0.0238 | -0.0143 | -0.0209 | -0.0340 | -0.0166 | -0.0141 | 0.0146 | 0.0195 | 0.0194 | 0.0093 | 0.0111 | 0.0110 |
| 0.13 | 0.0228 | -0.0133 | -0.0212 | -0.0247 | -0.0157 | -0.0225 | 0.0137 | 0.0180 | 0.0182 | 0.0085 | 0.0107 | 0.0110 |
| 0.15 | 0.0231 | -0.0134 | -0.0219 | -0.0169 | -0.0148 | -0.0294 | 0.0133 | 0.0171 | 0.0173 | 0.0080 | 0.0103 | 0.0110 |
| 0.17 | 0.0241 | -0.0143 | -0.0233 | -0.0103 | -0.0141 | -0.0351 | 0.0131 | 0.0167 | 0.0169 | 0.0076 | 0.0100 | 0.0111 |
| 0.19 | 0.0229 | -0.0122 | -0.0237 | -0.0045 | -0.0135 | -0.0398 | 0.0125 | 0.0159 | 0.0164 | 0.0074 | 0.0097 | 0.0112 |
| 0.21 | 0.0230 | -0.0126 | -0.0239 | 0.0006 | -0.0129 | -0.0438 | 0.0122 | 0.0154 | 0.0158 | 0.0073 | 0.0095 | 0.0114 |
| 0.23 | 0.0228 | -0.0123 | -0.0239 | 0.0051 | -0.0124 | -0.0471 | 0.0119 | 0.0148 | 0.0154 | 0.0072 | 0.0093 | 0.0116 |
| 0.25 | 0.0229 | -0.0120 | -0.0247 | 0.0093 | -0.0119 | -0.0496 | 0.0118 | 0.0146 | 0.0153 | 0.0072 | 0.0092 | 0.0117 |
| 0.27 | 0.0229 | -0.0113 | -0.0254 | 0.0132 | -0.0114 | -0.0512 | 0.0116 | 0.0142 | 0.0151 | 0.0072 | 0.0091 | 0.0118 |
| 0.29 | 0.0222 | -0.0115 | -0.0243 | 0.0162 | -0.0109 | -0.0531 | 0.0113 | 0.0138 | 0.0148 | 0.0072 | 0.0090 | 0.0119 |
| 0.31 | 0.0213 | -0.0107 | -0.0239 | 0.0189 | -0.0105 | -0.0545 | 0.0111 | 0.0136 | 0.0146 | 0.0073 | 0.0088 | 0.0120 |
| 0.33 | 0.0215 | -0.0115 | -0.0234 | 0.0214 | -0.0101 | -0.0556 | 0.0110 | 0.0133 | 0.0146 | 0.0074 | 0.0087 | 0.0121 |
| 0.35 | 0.0206 | -0.0113 | -0.0225 | 0.0236 | -0.0098 | -0.0564 | 0.0109 | 0.0132 | 0.0146 | 0.0074 | 0.0087 | 0.0121 |
| 0.37 | 0.0200 | -0.0106 | -0.0222 | 0.0256 | -0.0094 | -0.0568 | 0.0108 | 0.0132 | 0.0141 | 0.0075 | 0.0086 | 0.0121 |
| 0.39 | 0.0195 | -0.0103 | -0.0220 | 0.0274 | -0.0091 | -0.0570 | 0.0108 | 0.0131 | 0.0140 | 0.0076 | 0.0085 | 0.0121 |
| 0.41 | 0.0199 | -0.0106 | -0.0226 | 0.0290 | -0.0088 | -0.0569 | 0.0108 | 0.0132 | 0.0140 | 0.0077 | 0.0084 | 0.0121 |
| 0.43 | 0.0200 | -0.0108 | -0.0228 | 0.0305 | -0.0084 | -0.0566 | 0.0108 | 0.0131 | 0.0140 | 0.0077 | 0.0084 | 0.0120 |
| 0.45 | 0.0190 | -0.0104 | -0.0216 | 0.0317 | -0.0081 | -0.0559 | 0.0107 | 0.0128 | 0.0139 | 0.0078 | 0.0083 | 0.0120 |
| 0.47 | 0.0188 | -0.0101 | -0.0217 | 0.0329 | -0.0079 | -0.0551 | 0.0108 | 0.0128 | 0.0140 | 0.0079 | 0.0083 | 0.0119 |
| 0.49 | 0.0174 | -0.0088 | -0.0207 | 0.0339 | -0.0076 | -0.0540 | 0.0108 | 0.0128 | 0.0140 | 0.0080 | 0.0083 | 0.0118 |
| 0.51 | 0.0171 | -0.0087 | -0.0206 | 0.0347 | -0.0073 | -0.0526 | 0.0107 | 0.0129 | 0.0138 | 0.0081 | 0.0082 | 0.0117 |
| 0.53 | 0.0173 | -0.0089 | -0.0205 | 0.0354 | -0.0071 | -0.0510 | 0.0108 | 0.0128 | 0.0139 | 0.0081 | 0.0082 | 0.0115 |
| 0.55 | 0.0164 | -0.0083 | -0.0199 | 0.0361 | -0.0069 | -0.0492 | 0.0108 | 0.0129 | 0.0138 | 0.0082 | 0.0082 | 0.0114 |
| 0.57 | 0.0154 | -0.0079 | -0.0189 | 0.0365 | -0.0067 | -0.0471 | 0.0109 | 0.0127 | 0.0139 | 0.0083 | 0.0082 | 0.0112 |
| 0.59 | 0.0152 | -0.0081 | -0.0185 | 0.0369 | -0.0065 | -0.0448 | 0.0110 | 0.0127 | 0.0140 | 0.0083 | 0.0082 | 0.0110 |
| 0.61 | 0.0148 | -0.0074 | -0.0186 | 0.0372 | -0.0063 | -0.0421 | 0.0112 | 0.0126 | 0.0143 | 0.0084 | 0.0082 | 0.0108 |
| 0.63 | 0.0153 | -0.0075 | -0.0192 | 0.0374 | -0.0061 | -0.0392 | 0.0112 | 0.0125 | 0.0144 | 0.0085 | 0.0082 | 0.0106 |
| 0.65 | 0.0148 | -0.0071 | -0.0189 | 0.0375 | -0.0059 | -0.0360 | 0.0112 | 0.0124 | 0.0143 | 0.0085 | 0.0082 | 0.0104 |
| 0.67 | 0.0145 | -0.0072 | -0.0185 | 0.0374 | -0.0058 | -0.0324 | 0.0111 | 0.0123 | 0.0141 | 0.0086 | 0.0082 | 0.0103 |
| 0.69 | 0.0141 | -0.0071 | -0.0181 | 0.0373 | -0.0057 | -0.0285 | 0.0112 | 0.0125 | 0.0142 | 0.0086 | 0.0082 | 0.0101 |
| 0.71 | 0.0138 | -0.0072 | -0.0178 | 0.0371 | -0.0055 | -0.0242 | 0.0113 | 0.0126 | 0.0143 | 0.0087 | 0.0083 | 0.0100 |
| 0.73 | 0.0139 | -0.0081 | -0.0174 | 0.0368 | -0.0055 | -0.0195 | 0.0113 | 0.0126 | 0.0143 | 0.0087 | 0.0083 | 0.0098 |
| 0.75 | 0.0123 | -0.0075 | -0.0160 | 0.0369 | -0.0054 | -0.0144 | 0.0113 | 0.0127 | 0.0143 | 0.0088 | 0.0083 | 0.0097 |
| 0.77 | 0.0116 | -0.0078 | -0.0148 | 0.0368 | -0.0054 | -0.0086 | 0.0115 | 0.0128 | 0.0145 | 0.0089 | 0.0084 | 0.0097 |
| 0.79 | 0.0119 | -0.0088 | -0.0144 | 0.0362 | -0.0054 | -0.0022 | 0.0116 | 0.0130 | 0.0148 | 0.0089 | 0.0084 | 0.0098 |
| 0.81 | 0.0109 | -0.0084 | -0.0134 | 0.0356 | -0.0055 | 0.0049 | 0.0119 | 0.0130 | 0.0151 | 0.0090 | 0.0085 | 0.0099 |
| 0.83 | 0.0095 | -0.0083 | -0.0115 | 0.0350 | -0.0056 | 0.0129 | 0.0122 | 0.0133 | 0.0155 | 0.0091 | 0.0086 | 0.0102 |
| 0.85 | 0.0075 | -0.0065 | -0.0103 | 0.0343 | -0.0059 | 0.0219 | 0.0124 | 0.0137 | 0.0159 | 0.0092 | 0.0087 | 0.0107 |
| 0.87 | 0.0063 | -0.0069 | -0.0081 | 0.0336 | -0.0062 | 0.0322 | 0.0127 | 0.0139 | 0.0162 | 0.0094 | 0.0088 | 0.0115 |
| 0.89 | 0.0027 | -0.0047 | -0.0048 | 0.0329 | -0.0067 | 0.0442 | 0.0130 | 0.0143 | 0.0169 | 0.0096 | 0.0090 | 0.0126 |
| 0.91 | -0.0009 | -0.0040 | -0.0003 | 0.0324 | -0.0074 | 0.0583 | 0.0139 | 0.0150 | 0.0179 | 0.0099 | 0.0092 | 0.0144 |
| 0.93 | -0.0045 | -0.0031 | 0.0041 | 0.0323 | -0.0085 | 0.0757 | 0.0151 | 0.0162 | 0.0192 | 0.0104 | 0.0095 | 0.0171 |
| 0.95 | -0.0098 | -0.0022 | 0.0108 | 0.0332 | -0.0101 | 0.0980 | 0.0168 | 0.0176 | 0.0215 | 0.0110 | 0.0098 | 0.0215 |
| 0.97 | -0.0189 | -0.0025 | 0.0231 | 0.0366 | -0.0127 | 0.1299 | 0.0206 | 0.0221 | 0.0268 | 0.0122 | 0.0104 | 0.0295 |
| 0.99 | -0.0608 | -0.0095 | 0.0783 | 0.0514 | -0.0183 | 0.1865 | 0.0331 | 0.0346 | 0.0454 | 0.0154 | 0.0116 | 0.0488 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 10: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \frac{1}{1+e^{-\tau}}$*: bias and MSE of estimators*

| | *Bias* | | | | | | *MSE* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | *smoothed* | | | *canonical* | | | *smoothed* | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ |
| 0.01 | 0.0142 | -0.0283 | 0.0431 | -0.1508 | -0.0625 | 0.0897 | 0.0438 | 0.0610 | 0.0613 | 0.0348 | 0.0198 | 0.0244 |
| 0.03 | 0.0059 | -0.0071 | 0.0082 | -0.1059 | -0.0344 | 0.0479 | 0.0249 | 0.0342 | 0.0352 | 0.0213 | 0.0149 | 0.0164 |
| 0.05 | 0.0035 | 0.0003 | 0.0019 | -0.0849 | -0.0214 | 0.0278 | 0.0194 | 0.0266 | 0.0269 | 0.0164 | 0.0130 | 0.0138 |
| 0.07 | 0.0057 | 0.0002 | -0.0034 | -0.0709 | -0.0132 | 0.0147 | 0.0170 | 0.0231 | 0.0229 | 0.0137 | 0.0120 | 0.0125 |
| 0.09 | 0.0065 | 0.0002 | -0.0058 | -0.0605 | -0.0073 | 0.0052 | 0.0150 | 0.0207 | 0.0201 | 0.0120 | 0.0114 | 0.0118 |
| 0.11 | 0.0090 | -0.0011 | -0.0094 | -0.0521 | -0.0028 | -0.0023 | 0.0139 | 0.0191 | 0.0189 | 0.0108 | 0.0109 | 0.0114 |
| 0.13 | 0.0089 | -0.0003 | -0.0108 | -0.0451 | 0.0007 | -0.0084 | 0.0131 | 0.0182 | 0.0180 | 0.0099 | 0.0106 | 0.0112 |
| 0.15 | 0.0091 | -0.0001 | -0.0119 | -0.0391 | 0.0035 | -0.0133 | 0.0129 | 0.0177 | 0.0178 | 0.0092 | 0.0104 | 0.0111 |
| 0.17 | 0.0091 | 0.0004 | -0.0129 | -0.0338 | 0.0058 | -0.0175 | 0.0123 | 0.0170 | 0.0172 | 0.0087 | 0.0103 | 0.0111 |
| 0.19 | 0.0089 | 0.0005 | -0.0131 | -0.0290 | 0.0077 | -0.0210 | 0.0120 | 0.0167 | 0.0169 | 0.0083 | 0.0102 | 0.0111 |
| 0.21 | 0.0094 | 0 | -0.0138 | -0.0248 | 0.0092 | -0.0239 | 0.0118 | 0.0163 | 0.0168 | 0.0080 | 0.0101 | 0.0111 |
| 0.23 | 0.0099 | -0.0005 | -0.0143 | -0.0209 | 0.0105 | -0.0264 | 0.0119 | 0.0162 | 0.0167 | 0.0078 | 0.0100 | 0.0111 |
| 0.25 | 0.0094 | -0.0010 | -0.0136 | -0.0171 | 0.0115 | -0.0284 | 0.0117 | 0.0158 | 0.0165 | 0.0076 | 0.0099 | 0.0112 |
| 0.27 | 0.0096 | -0.0009 | -0.0143 | -0.0133 | 0.0121 | -0.0299 | 0.0116 | 0.0157 | 0.0164 | 0.0075 | 0.0099 | 0.0113 |
| 0.29 | 0.0093 | -0.0009 | -0.0141 | -0.0104 | 0.0128 | -0.0314 | 0.0116 | 0.0154 | 0.0167 | 0.0074 | 0.0099 | 0.0113 |
| 0.31 | 0.0092 | -0.0012 | -0.0135 | -0.0076 | 0.0133 | -0.0326 | 0.0115 | 0.0151 | 0.0165 | 0.0073 | 0.0099 | 0.0113 |
| 0.33 | 0.0089 | -0.0008 | -0.0141 | -0.0050 | 0.0137 | -0.0335 | 0.0113 | 0.0151 | 0.0164 | 0.0073 | 0.0098 | 0.0114 |
| 0.35 | 0.0089 | -0.0011 | -0.0142 | -0.0026 | 0.0140 | -0.0342 | 0.0112 | 0.0148 | 0.0162 | 0.0073 | 0.0098 | 0.0114 |
| 0.37 | 0.0082 | -0.0010 | -0.0131 | -0.0003 | 0.0141 | -0.0347 | 0.0112 | 0.0148 | 0.0160 | 0.0073 | 0.0098 | 0.0114 |
| 0.39 | 0.0077 | -0.0008 | -0.0128 | 0.0019 | 0.0141 | -0.0351 | 0.0112 | 0.0146 | 0.0158 | 0.0073 | 0.0098 | 0.0114 |
| 0.41 | 0.0071 | -0.0004 | -0.0125 | 0.0039 | 0.0141 | -0.0352 | 0.0112 | 0.0146 | 0.0159 | 0.0073 | 0.0098 | 0.0114 |
| 0.43 | 0.0073 | -0.0007 | -0.0126 | 0.0059 | 0.0139 | -0.0351 | 0.0112 | 0.0145 | 0.0158 | 0.0074 | 0.0098 | 0.0114 |
| 0.45 | 0.0078 | -0.0013 | -0.0131 | 0.0078 | 0.0137 | -0.0349 | 0.0112 | 0.0147 | 0.0157 | 0.0074 | 0.0098 | 0.0114 |
| 0.47 | 0.0075 | -0.0006 | -0.0134 | 0.0096 | 0.0133 | -0.0346 | 0.0114 | 0.0149 | 0.0159 | 0.0075 | 0.0098 | 0.0114 |
| 0.49 | 0.0068 | 0 | -0.0127 | 0.0114 | 0.0129 | -0.0340 | 0.0116 | 0.0150 | 0.0162 | 0.0075 | 0.0099 | 0.0114 |
| 0.51 | 0.0063 | -0.0002 | -0.0121 | 0.0131 | 0.0124 | -0.0333 | 0.0117 | 0.0150 | 0.0163 | 0.0076 | 0.0099 | 0.0114 |
| 0.53 | 0.0062 | 0.0002 | -0.0125 | 0.0147 | 0.0118 | -0.0325 | 0.0118 | 0.0151 | 0.0162 | 0.0077 | 0.0099 | 0.0114 |
| 0.55 | 0.0054 | 0.0010 | -0.0121 | 0.0163 | 0.0111 | -0.0314 | 0.0119 | 0.0152 | 0.0164 | 0.0078 | 0.0099 | 0.0114 |
| 0.57 | 0.0046 | 0.0010 | -0.0109 | 0.0179 | 0.0103 | -0.0302 | 0.0120 | 0.0153 | 0.0164 | 0.0079 | 0.0100 | 0.0114 |
| 0.59 | 0.0053 | 0.0004 | -0.0118 | 0.0194 | 0.0095 | -0.0289 | 0.0123 | 0.0157 | 0.0166 | 0.0080 | 0.0100 | 0.0113 |
| 0.61 | 0.0045 | 0.0006 | -0.0109 | 0.0209 | 0.0085 | -0.0274 | 0.0123 | 0.0158 | 0.0166 | 0.0082 | 0.0101 | 0.0113 |
| 0.63 | 0.0044 | 0.0001 | -0.0105 | 0.0224 | 0.0075 | -0.0257 | 0.0124 | 0.0158 | 0.0169 | 0.0083 | 0.0101 | 0.0113 |
| 0.65 | 0.0039 | 0.0002 | -0.0101 | 0.0239 | 0.0063 | -0.0238 | 0.0125 | 0.0159 | 0.0169 | 0.0084 | 0.0102 | 0.0113 |
| 0.67 | 0.0038 | -0.0001 | -0.0100 | 0.0255 | 0.0050 | -0.0217 | 0.0127 | 0.0161 | 0.0170 | 0.0086 | 0.0102 | 0.0112 |
| 0.69 | 0.0039 | -0.0001 | -0.0102 | 0.0270 | 0.0036 | -0.0194 | 0.0128 | 0.0164 | 0.0172 | 0.0087 | 0.0103 | 0.0112 |
| 0.71 | 0.0040 | -0.0008 | -0.0101 | 0.0286 | 0.0021 | -0.0168 | 0.0129 | 0.0165 | 0.0174 | 0.0089 | 0.0104 | 0.0112 |
| 0.73 | 0.0039 | -0.0005 | -0.0105 | 0.0302 | 0.0004 | -0.0141 | 0.0132 | 0.0167 | 0.0177 | 0.0091 | 0.0105 | 0.0112 |
| 0.75 | 0.0035 | -0.0005 | -0.0099 | 0.0325 | -0.0014 | -0.0110 | 0.0132 | 0.0169 | 0.0178 | 0.0093 | 0.0106 | 0.0112 |
| 0.77 | 0.0029 | -0.0003 | -0.0095 | 0.0347 | -0.0034 | -0.0076 | 0.0133 | 0.0172 | 0.0179 | 0.0096 | 0.0107 | 0.0112 |
| 0.79 | 0.0029 | -0.0007 | -0.0092 | 0.0367 | -0.0057 | -0.0037 | 0.0137 | 0.0175 | 0.0185 | 0.0098 | 0.0109 | 0.0113 |
| 0.81 | 0.0026 | -0.0010 | -0.0089 | 0.0389 | -0.0083 | 0.0005 | 0.0139 | 0.0175 | 0.0187 | 0.0101 | 0.0111 | 0.0115 |
| 0.83 | 0.0013 | -0.0007 | -0.0076 | 0.0414 | -0.0112 | 0.0054 | 0.0143 | 0.0184 | 0.0190 | 0.0105 | 0.0113 | 0.0116 |
| 0.85 | 0.0006 | -0.0008 | -0.0068 | 0.0441 | -0.0145 | 0.0108 | 0.0147 | 0.0190 | 0.0195 | 0.0109 | 0.0116 | 0.0119 |
| 0.87 | -0.0004 | -0.0019 | -0.0048 | 0.0472 | -0.0182 | 0.0171 | 0.0154 | 0.0198 | 0.0204 | 0.0114 | 0.0119 | 0.0123 |
| 0.89 | -0.0006 | -0.0044 | -0.0029 | 0.0510 | -0.0226 | 0.0244 | 0.0162 | 0.0210 | 0.0211 | 0.0120 | 0.0123 | 0.0129 |
| 0.91 | -0.0027 | -0.0043 | -0.0002 | 0.0556 | -0.0279 | 0.0331 | 0.0172 | 0.0222 | 0.0222 | 0.0129 | 0.0129 | 0.0137 |
| 0.93 | -0.0049 | -0.0051 | 0.0029 | 0.0617 | -0.0344 | 0.0439 | 0.0186 | 0.0237 | 0.0239 | 0.0140 | 0.0138 | 0.0149 |
| 0.95 | -0.0083 | -0.0060 | 0.0082 | 0.0705 | -0.0429 | 0.0580 | 0.0208 | 0.0262 | 0.0267 | 0.0157 | 0.0150 | 0.0169 |
| 0.97 | -0.0131 | -0.0086 | 0.0155 | 0.0850 | -0.0553 | 0.0783 | 0.0247 | 0.0320 | 0.0324 | 0.0189 | 0.0170 | 0.0205 |
| 0.99 | -0.0342 | -0.0310 | 0.0499 | 0.1202 | -0.0784 | 0.1154 | 0.0383 | 0.0516 | 0.0526 | 0.0279 | 0.0218 | 0.0294 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 11: *Model* (2) *defined through Proposition* 2 *when* $\lambda(\tau) = \sqrt[3]{\tau}$*: bias and* $MSE$ *of estimators*

| | *Bias* | | | | | | *MSE* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | *smoothed* | | | *canonical* | | | *smoothed* | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ |
| 0.01 | 0.0083 | -0.0230 | 0.0499 | -0.1663 | -0.0411 | 0.1110 | 0.0443 | 0.0637 | 0.0638 | 0.0404 | 0.0185 | 0.0291 |
| 0.03 | 0.0095 | -0.0107 | 0.0061 | -0.1033 | -0.0316 | 0.0509 | 0.0258 | 0.0361 | 0.0356 | 0.0213 | 0.0153 | 0.0166 |
| 0.05 | 0.0134 | -0.0097 | -0.0050 | -0.0754 | -0.0261 | 0.0237 | 0.0201 | 0.0275 | 0.0267 | 0.0153 | 0.0136 | 0.0132 |
| 0.07 | 0.0169 | -0.0108 | -0.0121 | -0.0577 | -0.0222 | 0.0068 | 0.0170 | 0.0234 | 0.0227 | 0.0124 | 0.0125 | 0.0118 |
| 0.09 | 0.0169 | -0.0083 | -0.0158 | -0.0450 | -0.0191 | -0.0052 | 0.0151 | 0.0207 | 0.0201 | 0.0106 | 0.0118 | 0.0111 |
| 0.11 | 0.0167 | -0.0078 | -0.0162 | -0.0350 | -0.0166 | -0.0144 | 0.0143 | 0.0193 | 0.0189 | 0.0095 | 0.0112 | 0.0108 |
| 0.13 | 0.0170 | -0.0084 | -0.0165 | -0.0270 | -0.0145 | -0.0215 | 0.0133 | 0.0181 | 0.0173 | 0.0088 | 0.0108 | 0.0107 |
| 0.15 | 0.0176 | -0.0082 | -0.0178 | -0.0203 | -0.0127 | -0.0273 | 0.0127 | 0.0173 | 0.0165 | 0.0082 | 0.0105 | 0.0107 |
| 0.17 | 0.0181 | -0.0089 | -0.0185 | -0.0145 | -0.0111 | -0.0320 | 0.0125 | 0.0166 | 0.0161 | 0.0079 | 0.0102 | 0.0108 |
| 0.19 | 0.0178 | -0.0077 | -0.0192 | -0.0095 | -0.0097 | -0.0358 | 0.0122 | 0.0161 | 0.0157 | 0.0076 | 0.0100 | 0.0108 |
| 0.21 | 0.0176 | -0.0074 | -0.0191 | -0.0051 | -0.0084 | -0.0389 | 0.0118 | 0.0154 | 0.0153 | 0.0075 | 0.0098 | 0.0109 |
| 0.23 | 0.0167 | -0.0071 | -0.0184 | -0.0013 | -0.0073 | -0.0415 | 0.0116 | 0.0152 | 0.0151 | 0.0074 | 0.0096 | 0.0110 |
| 0.25 | 0.0161 | -0.0071 | -0.0180 | 0.0024 | -0.0063 | -0.0434 | 0.0116 | 0.0151 | 0.0150 | 0.0073 | 0.0095 | 0.0111 |
| 0.27 | 0.0162 | -0.0077 | -0.0176 | 0.0058 | -0.0055 | -0.0446 | 0.0114 | 0.0147 | 0.0149 | 0.0073 | 0.0094 | 0.0112 |
| 0.29 | 0.0154 | -0.0076 | -0.0165 | 0.0085 | -0.0047 | -0.0459 | 0.0115 | 0.0147 | 0.0148 | 0.0073 | 0.0093 | 0.0112 |
| 0.31 | 0.0153 | -0.0072 | -0.0168 | 0.0109 | -0.0039 | -0.0468 | 0.0113 | 0.0146 | 0.0145 | 0.0073 | 0.0092 | 0.0113 |
| 0.33 | 0.0151 | -0.0074 | -0.0166 | 0.0131 | -0.0033 | -0.0475 | 0.0113 | 0.0144 | 0.0145 | 0.0074 | 0.0092 | 0.0113 |
| 0.35 | 0.0145 | -0.0068 | -0.0161 | 0.0151 | -0.0027 | -0.0479 | 0.0112 | 0.0144 | 0.0142 | 0.0074 | 0.0091 | 0.0113 |
| 0.37 | 0.0150 | -0.0070 | -0.0171 | 0.0168 | -0.0021 | -0.0481 | 0.0112 | 0.0141 | 0.0142 | 0.0075 | 0.0091 | 0.0113 |
| 0.39 | 0.0149 | -0.0068 | -0.0173 | 0.0185 | -0.0016 | -0.0480 | 0.0112 | 0.0141 | 0.0142 | 0.0075 | 0.0090 | 0.0113 |
| 0.41 | 0.0145 | -0.0065 | -0.0171 | 0.0200 | -0.0012 | -0.0477 | 0.0111 | 0.0140 | 0.0141 | 0.0076 | 0.0090 | 0.0113 |
| 0.43 | 0.0146 | -0.0073 | -0.0163 | 0.0213 | -0.0008 | -0.0471 | 0.0111 | 0.0140 | 0.0140 | 0.0077 | 0.0090 | 0.0112 |
| 0.45 | 0.0147 | -0.0075 | -0.0164 | 0.0225 | -0.0004 | -0.0464 | 0.0112 | 0.0139 | 0.0140 | 0.0078 | 0.0089 | 0.0112 |
| 0.47 | 0.0149 | -0.0085 | -0.0163 | 0.0236 | -0.0001 | -0.0454 | 0.0114 | 0.0140 | 0.0141 | 0.0079 | 0.0089 | 0.0111 |
| 0.49 | 0.0141 | -0.0083 | -0.0154 | 0.0246 | 0.0001 | -0.0443 | 0.0116 | 0.0140 | 0.0141 | 0.0080 | 0.0089 | 0.0110 |
| 0.51 | 0.0141 | -0.0080 | -0.0160 | 0.0255 | 0.0003 | -0.0430 | 0.0116 | 0.0139 | 0.0140 | 0.0081 | 0.0089 | 0.0110 |
| 0.53 | 0.0137 | -0.0081 | -0.0154 | 0.0264 | 0.0005 | -0.0414 | 0.0116 | 0.0139 | 0.0141 | 0.0082 | 0.0089 | 0.0109 |
| 0.55 | 0.0138 | -0.0076 | -0.0161 | 0.0271 | 0.0006 | -0.0397 | 0.0116 | 0.0138 | 0.0140 | 0.0083 | 0.0089 | 0.0108 |
| 0.57 | 0.0124 | -0.0067 | -0.0150 | 0.0277 | 0.0007 | -0.0377 | 0.0116 | 0.0136 | 0.0140 | 0.0084 | 0.0089 | 0.0107 |
| 0.59 | 0.0118 | -0.0056 | -0.0151 | 0.0283 | 0.0007 | -0.0355 | 0.0118 | 0.0137 | 0.0142 | 0.0085 | 0.0090 | 0.0106 |
| 0.61 | 0.0118 | -0.0052 | -0.0155 | 0.0288 | 0.0007 | -0.0331 | 0.0122 | 0.0139 | 0.0146 | 0.0086 | 0.0090 | 0.0105 |
| 0.63 | 0.0111 | -0.0045 | -0.0151 | 0.0293 | 0.0007 | -0.0305 | 0.0122 | 0.0140 | 0.0147 | 0.0087 | 0.0090 | 0.0104 |
| 0.65 | 0.0110 | -0.0048 | -0.0149 | 0.0297 | 0.0005 | -0.0276 | 0.0122 | 0.0139 | 0.0148 | 0.0088 | 0.0090 | 0.0103 |
| 0.67 | 0.0109 | -0.0051 | -0.0148 | 0.0300 | 0.0004 | -0.0244 | 0.0124 | 0.0139 | 0.0151 | 0.0089 | 0.0091 | 0.0103 |
| 0.69 | 0.0104 | -0.0047 | -0.0145 | 0.0303 | 0.0001 | -0.0210 | 0.0125 | 0.0140 | 0.0152 | 0.0090 | 0.0091 | 0.0102 |
| 0.71 | 0.0096 | -0.0045 | -0.0134 | 0.0306 | -0.0001 | -0.0173 | 0.0126 | 0.0141 | 0.0153 | 0.0091 | 0.0091 | 0.0102 |
| 0.73 | 0.0095 | -0.0052 | -0.0130 | 0.0308 | -0.0005 | -0.0132 | 0.0128 | 0.0143 | 0.0156 | 0.0092 | 0.0092 | 0.0101 |
| 0.75 | 0.0085 | -0.0053 | -0.0116 | 0.0316 | -0.0009 | -0.0087 | 0.0129 | 0.0144 | 0.0158 | 0.0093 | 0.0092 | 0.0101 |
| 0.77 | 0.0079 | -0.0056 | -0.0105 | 0.0321 | -0.0015 | -0.0038 | 0.0132 | 0.0145 | 0.0160 | 0.0095 | 0.0093 | 0.0102 |
| 0.79 | 0.0074 | -0.0049 | -0.0104 | 0.0324 | -0.0022 | 0.0017 | 0.0137 | 0.0147 | 0.0166 | 0.0096 | 0.0094 | 0.0103 |
| 0.81 | 0.0058 | -0.0044 | -0.0086 | 0.0327 | -0.0030 | 0.0078 | 0.0137 | 0.0148 | 0.0168 | 0.0098 | 0.0095 | 0.0105 |
| 0.83 | 0.0050 | -0.0036 | -0.0079 | 0.0331 | -0.0040 | 0.0145 | 0.0139 | 0.0150 | 0.0170 | 0.0100 | 0.0096 | 0.0108 |
| 0.85 | 0.0039 | -0.0037 | -0.0072 | 0.0336 | -0.0052 | 0.0222 | 0.0144 | 0.0154 | 0.0177 | 0.0102 | 0.0097 | 0.0113 |
| 0.87 | 0.0033 | -0.0042 | -0.0063 | 0.0342 | -0.0066 | 0.0309 | 0.0148 | 0.0161 | 0.0182 | 0.0105 | 0.0099 | 0.0119 |
| 0.89 | 0.0017 | -0.0033 | -0.0049 | 0.0352 | -0.0084 | 0.0411 | 0.0151 | 0.0168 | 0.0187 | 0.0108 | 0.0101 | 0.0129 |
| 0.91 | -0.0002 | -0.0035 | -0.0022 | 0.0365 | -0.0107 | 0.0531 | 0.0159 | 0.0175 | 0.0197 | 0.0112 | 0.0104 | 0.0143 |
| 0.93 | -0.0032 | -0.0034 | 0.0016 | 0.0388 | -0.0137 | 0.0678 | 0.0170 | 0.0189 | 0.0212 | 0.0118 | 0.0107 | 0.0165 |
| 0.95 | -0.0079 | -0.0051 | 0.0094 | 0.0425 | -0.0178 | 0.0868 | 0.0189 | 0.0208 | 0.0237 | 0.0127 | 0.0113 | 0.0199 |
| 0.97 | -0.0139 | -0.0090 | 0.0198 | 0.0500 | -0.0239 | 0.1140 | 0.0224 | 0.0254 | 0.0281 | 0.0142 | 0.0122 | 0.0261 |
| 0.99 | -0.0505 | -0.0166 | 0.0671 | 0.0718 | -0.0358 | 0.1634 | 0.0356 | 0.0414 | 0.0472 | 0.0187 | 0.0140 | 0.0412 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 12: *Model (2) defined through Proposition 2 when $\lambda(\tau) = \cos(\tau)$: bias and MSE of estimators*

| | *Bias* | | | | | | *MSE* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | *smoothed* | | | *canonical* | | | *smoothed* | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ |
| 0.01 | 0.0291 | -0.0408 | 0.0284 | -0.1117 | -0.1026 | 0.0524 | 0.0404 | 0.0580 | 0.0551 | 0.0249 | 0.0258 | 0.0178 |
| 0.03 | 0.0125 | -0.0133 | 0.0036 | -0.0808 | -0.0606 | 0.0242 | 0.0235 | 0.0333 | 0.0312 | 0.0168 | 0.0167 | 0.0138 |
| 0.05 | 0.0087 | -0.0063 | 0.0002 | -0.0670 | -0.0404 | 0.0108 | 0.0184 | 0.0254 | 0.0248 | 0.0138 | 0.0135 | 0.0123 |
| 0.07 | 0.0075 | -0.0013 | -0.0048 | -0.0580 | -0.0272 | 0.0025 | 0.0157 | 0.0214 | 0.0214 | 0.0120 | 0.0119 | 0.0115 |
| 0.09 | 0.0073 | -0.0008 | -0.0058 | -0.0515 | -0.0174 | -0.0035 | 0.0143 | 0.0194 | 0.0198 | 0.0109 | 0.0110 | 0.0111 |
| 0.11 | 0.0065 | 0.0012 | -0.0072 | -0.0463 | -0.0097 | -0.0079 | 0.0135 | 0.0176 | 0.0187 | 0.0101 | 0.0104 | 0.0108 |
| 0.13 | 0.0051 | 0.0027 | -0.0070 | -0.0421 | -0.0034 | -0.0113 | 0.0127 | 0.0168 | 0.0175 | 0.0095 | 0.0101 | 0.0106 |
| 0.15 | 0.0043 | 0.0031 | -0.0067 | -0.0385 | 0.0019 | -0.0139 | 0.0123 | 0.0162 | 0.0169 | 0.0090 | 0.0098 | 0.0105 |
| 0.17 | 0.0027 | 0.0050 | -0.0059 | -0.0353 | 0.0064 | -0.0160 | 0.0118 | 0.0156 | 0.0162 | 0.0086 | 0.0097 | 0.0104 |
| 0.19 | 0.0031 | 0.0050 | -0.0070 | -0.0325 | 0.0103 | -0.0176 | 0.0114 | 0.0152 | 0.0158 | 0.0083 | 0.0097 | 0.0103 |
| 0.21 | 0.0033 | 0.0042 | -0.0067 | -0.0301 | 0.0137 | -0.0188 | 0.0111 | 0.0149 | 0.0153 | 0.0080 | 0.0096 | 0.0103 |
| 0.23 | 0.0024 | 0.0048 | -0.0059 | -0.0278 | 0.0166 | -0.0197 | 0.0110 | 0.0146 | 0.0152 | 0.0078 | 0.0096 | 0.0102 |
| 0.25 | 0.0012 | 0.0053 | -0.0047 | -0.0254 | 0.0191 | -0.0203 | 0.0109 | 0.0146 | 0.0151 | 0.0076 | 0.0097 | 0.0102 |
| 0.27 | 0.0006 | 0.0056 | -0.0041 | -0.0230 | 0.0210 | -0.0206 | 0.0108 | 0.0145 | 0.0150 | 0.0075 | 0.0097 | 0.0102 |
| 0.29 | 0.0008 | 0.0053 | -0.0048 | -0.0212 | 0.0229 | -0.0208 | 0.0107 | 0.0143 | 0.0151 | 0.0074 | 0.0098 | 0.0102 |
| 0.31 | 0.0008 | 0.0051 | -0.0049 | -0.0195 | 0.0246 | -0.0209 | 0.0107 | 0.0143 | 0.0149 | 0.0073 | 0.0098 | 0.0102 |
| 0.33 | 0.0005 | 0.0052 | -0.0047 | -0.0179 | 0.0260 | -0.0209 | 0.0105 | 0.0142 | 0.0147 | 0.0072 | 0.0099 | 0.0102 |
| 0.35 | 0.0009 | 0.0046 | -0.0050 | -0.0164 | 0.0272 | -0.0207 | 0.0105 | 0.0141 | 0.0150 | 0.0071 | 0.0100 | 0.0102 |
| 0.37 | 0.0012 | 0.0045 | -0.0056 | -0.0149 | 0.0282 | -0.0205 | 0.0104 | 0.0142 | 0.0151 | 0.0070 | 0.0100 | 0.0101 |
| 0.39 | 0.0013 | 0.0045 | -0.0059 | -0.0134 | 0.0290 | -0.0201 | 0.0105 | 0.0142 | 0.0152 | 0.0070 | 0.0101 | 0.0101 |
| 0.41 | 0.0006 | 0.0050 | -0.0055 | -0.0119 | 0.0296 | -0.0196 | 0.0105 | 0.0145 | 0.0152 | 0.0069 | 0.0101 | 0.0102 |
| 0.43 | -0.0007 | 0.0064 | -0.0046 | -0.0105 | 0.0300 | -0.0190 | 0.0105 | 0.0145 | 0.0151 | 0.0069 | 0.0102 | 0.0102 |
| 0.45 | -0.0008 | 0.0065 | -0.0048 | -0.0090 | 0.0302 | -0.0184 | 0.0107 | 0.0148 | 0.0152 | 0.0069 | 0.0102 | 0.0102 |
| 0.47 | -0.0009 | 0.0069 | -0.0051 | -0.0075 | 0.0303 | -0.0177 | 0.0106 | 0.0147 | 0.0150 | 0.0069 | 0.0103 | 0.0102 |
| 0.49 | -0.0010 | 0.0069 | -0.0052 | -0.0060 | 0.0301 | -0.0169 | 0.0107 | 0.0148 | 0.0153 | 0.0069 | 0.0103 | 0.0102 |
| 0.51 | -0.0010 | 0.0071 | -0.0058 | -0.0045 | 0.0298 | -0.0161 | 0.0108 | 0.0150 | 0.0155 | 0.0069 | 0.0104 | 0.0102 |
| 0.53 | -0.0010 | 0.0070 | -0.0060 | -0.0029 | 0.0294 | -0.0152 | 0.0109 | 0.0151 | 0.0159 | 0.0069 | 0.0104 | 0.0103 |
| 0.55 | -0.0007 | 0.0064 | -0.0059 | -0.0012 | 0.0287 | -0.0143 | 0.0109 | 0.0151 | 0.0160 | 0.0069 | 0.0104 | 0.0103 |
| 0.57 | -0.0008 | 0.0063 | -0.0055 | 0.0005 | 0.0279 | -0.0133 | 0.0110 | 0.0152 | 0.0159 | 0.0070 | 0.0105 | 0.0103 |
| 0.59 | -0.0013 | 0.0068 | -0.0054 | 0.0023 | 0.0268 | -0.0122 | 0.0112 | 0.0155 | 0.0163 | 0.0070 | 0.0105 | 0.0104 |
| 0.61 | -0.0015 | 0.0070 | -0.0054 | 0.0042 | 0.0256 | -0.0111 | 0.0114 | 0.0156 | 0.0165 | 0.0071 | 0.0105 | 0.0105 |
| 0.63 | -0.0020 | 0.0076 | -0.0052 | 0.0062 | 0.0241 | -0.0099 | 0.0116 | 0.0160 | 0.0169 | 0.0072 | 0.0106 | 0.0105 |
| 0.65 | -0.0022 | 0.0069 | -0.0046 | 0.0083 | 0.0224 | -0.0086 | 0.0115 | 0.0161 | 0.0168 | 0.0073 | 0.0106 | 0.0106 |
| 0.67 | -0.0022 | 0.0066 | -0.0045 | 0.0106 | 0.0205 | -0.0073 | 0.0116 | 0.0162 | 0.0169 | 0.0074 | 0.0106 | 0.0107 |
| 0.69 | -0.0014 | 0.0065 | -0.0057 | 0.0130 | 0.0183 | -0.0059 | 0.0117 | 0.0164 | 0.0171 | 0.0075 | 0.0107 | 0.0108 |
| 0.71 | -0.0014 | 0.0069 | -0.0064 | 0.0157 | 0.0159 | -0.0045 | 0.0118 | 0.0166 | 0.0173 | 0.0077 | 0.0107 | 0.0109 |
| 0.73 | -0.0010 | 0.0059 | -0.0063 | 0.0186 | 0.0131 | -0.0029 | 0.0121 | 0.0171 | 0.0177 | 0.0079 | 0.0108 | 0.0110 |
| 0.75 | -0.0012 | 0.0062 | -0.0067 | 0.0222 | 0.0101 | -0.0012 | 0.0123 | 0.0176 | 0.0181 | 0.0081 | 0.0109 | 0.0111 |
| 0.77 | -0.0010 | 0.0054 | -0.0066 | 0.0259 | 0.0066 | 0.0006 | 0.0126 | 0.0181 | 0.0183 | 0.0084 | 0.0110 | 0.0113 |
| 0.79 | -0.0005 | 0.0039 | -0.0065 | 0.0298 | 0.0025 | 0.0025 | 0.0130 | 0.0182 | 0.0190 | 0.0088 | 0.0112 | 0.0115 |
| 0.81 | -0.0006 | 0.0031 | -0.0065 | 0.0342 | -0.0020 | 0.0046 | 0.0131 | 0.0187 | 0.0190 | 0.0092 | 0.0114 | 0.0117 |
| 0.83 | -0.0011 | 0.0031 | -0.0058 | 0.0391 | -0.0072 | 0.0070 | 0.0136 | 0.0196 | 0.0195 | 0.0097 | 0.0117 | 0.0119 |
| 0.85 | -0.0007 | 0.0018 | -0.0060 | 0.0447 | -0.0132 | 0.0095 | 0.0140 | 0.0202 | 0.0201 | 0.0104 | 0.0121 | 0.0122 |
| 0.87 | -0.0003 | 0.0007 | -0.0065 | 0.0512 | -0.0201 | 0.0125 | 0.0146 | 0.0209 | 0.0208 | 0.0113 | 0.0127 | 0.0126 |
| 0.89 | -0.0013 | -0.0006 | -0.0039 | 0.0588 | -0.0282 | 0.0159 | 0.0153 | 0.0218 | 0.0222 | 0.0124 | 0.0134 | 0.0130 |
| 0.91 | -0.0023 | -0.0016 | -0.0024 | 0.0681 | -0.0379 | 0.0201 | 0.0161 | 0.0231 | 0.0235 | 0.0139 | 0.0145 | 0.0136 |
| 0.93 | -0.0006 | -0.0062 | -0.0027 | 0.0800 | -0.0499 | 0.0252 | 0.0172 | 0.0256 | 0.0249 | 0.0160 | 0.0161 | 0.0143 |
| 0.95 | -0.0011 | -0.0121 | 0.0015 | 0.0962 | -0.0657 | 0.0319 | 0.0197 | 0.0293 | 0.0283 | 0.0194 | 0.0186 | 0.0153 |
| 0.97 | -0.0013 | -0.0228 | 0.0079 | 0.1213 | -0.0884 | 0.0418 | 0.0241 | 0.0357 | 0.0343 | 0.0257 | 0.0232 | 0.0170 |
| 0.99 | -0.0069 | -0.0569 | 0.0239 | 0.1762 | -0.1306 | 0.0606 | 0.0363 | 0.0574 | 0.0527 | 0.0436 | 0.0344 | 0.0207 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 13: *Model* (2) *defined through Proposition* 2 *when* $\lambda(\tau) = \sqrt[6]{\tau}$*: bias and MSE of estimators*

| | *Bias* | | | | | | *MSE* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | *smoothed* | | | *canonical* | | | *smoothed* | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ |
| 0.01 | 0.0208 | -0.0353 | 0.0403 | -0.1445 | -0.0651 | 0.0898 | 0.0445 | 0.0619 | 0.0627 | 0.0340 | 0.0213 | 0.0245 |
| 0.03 | 0.0151 | -0.0165 | 0.0024 | -0.0936 | -0.0439 | 0.0404 | 0.0266 | 0.0374 | 0.0347 | 0.0197 | 0.0165 | 0.0156 |
| 0.05 | 0.0136 | -0.0095 | -0.0062 | -0.0711 | -0.0327 | 0.0184 | 0.0204 | 0.0281 | 0.0261 | 0.0151 | 0.0143 | 0.0130 |
| 0.07 | 0.0133 | -0.0072 | -0.0091 | -0.0569 | -0.0251 | 0.0046 | 0.0173 | 0.0237 | 0.0223 | 0.0127 | 0.0131 | 0.0119 |
| 0.09 | 0.0134 | -0.0068 | -0.0101 | -0.0466 | -0.0193 | -0.0051 | 0.0158 | 0.0211 | 0.0203 | 0.0112 | 0.0123 | 0.0114 |
| 0.11 | 0.0131 | -0.0054 | -0.0120 | -0.0387 | -0.0146 | -0.0125 | 0.0150 | 0.0193 | 0.0194 | 0.0102 | 0.0117 | 0.0111 |
| 0.13 | 0.0119 | -0.0040 | -0.0124 | -0.0322 | -0.0107 | -0.0182 | 0.0141 | 0.0185 | 0.0182 | 0.0096 | 0.0113 | 0.0110 |
| 0.15 | 0.0118 | -0.0037 | -0.0128 | -0.0268 | -0.0074 | -0.0227 | 0.0136 | 0.0177 | 0.0173 | 0.0091 | 0.0109 | 0.0110 |
| 0.17 | 0.0117 | -0.0036 | -0.0134 | -0.0221 | -0.0046 | -0.0265 | 0.0133 | 0.0170 | 0.0168 | 0.0087 | 0.0107 | 0.0110 |
| 0.19 | 0.0125 | -0.0041 | -0.0146 | -0.0180 | -0.0022 | -0.0294 | 0.0128 | 0.0164 | 0.0164 | 0.0084 | 0.0105 | 0.0110 |
| 0.21 | 0.0117 | -0.0033 | -0.0143 | -0.0144 | 0 | -0.0319 | 0.0128 | 0.0163 | 0.0163 | 0.0082 | 0.0104 | 0.0111 |
| 0.23 | 0.0101 | -0.0015 | -0.0131 | -0.0112 | 0.0018 | -0.0339 | 0.0125 | 0.0163 | 0.0160 | 0.0080 | 0.0102 | 0.0111 |
| 0.25 | 0.0089 | 0 | -0.0129 | -0.0081 | 0.0034 | -0.0352 | 0.0122 | 0.0159 | 0.0158 | 0.0079 | 0.0102 | 0.0112 |
| 0.27 | 0.0085 | 0 | -0.0121 | -0.0051 | 0.0048 | -0.0361 | 0.0119 | 0.0157 | 0.0154 | 0.0079 | 0.0101 | 0.0112 |
| 0.29 | 0.0086 | -0.0003 | -0.0119 | -0.0028 | 0.0060 | -0.0370 | 0.0119 | 0.0155 | 0.0154 | 0.0078 | 0.0101 | 0.0113 |
| 0.31 | 0.0084 | -0.0004 | -0.0122 | -0.0006 | 0.0071 | -0.0377 | 0.0118 | 0.0154 | 0.0153 | 0.0078 | 0.0100 | 0.0113 |
| 0.33 | 0.0082 | -0.0002 | -0.0118 | 0.0013 | 0.0081 | -0.0381 | 0.0118 | 0.0152 | 0.0152 | 0.0078 | 0.0100 | 0.0113 |
| 0.35 | 0.0079 | 0.0005 | -0.0124 | 0.0032 | 0.0089 | -0.0383 | 0.0117 | 0.0149 | 0.0153 | 0.0078 | 0.0099 | 0.0113 |
| 0.37 | 0.0085 | -0.0001 | -0.0128 | 0.0048 | 0.0096 | -0.0383 | 0.0117 | 0.0150 | 0.0152 | 0.0078 | 0.0099 | 0.0113 |
| 0.39 | 0.0080 | 0.0006 | -0.0129 | 0.0064 | 0.0101 | -0.0381 | 0.0117 | 0.0150 | 0.0152 | 0.0078 | 0.0099 | 0.0113 |
| 0.41 | 0.0077 | 0.0006 | -0.0126 | 0.0079 | 0.0106 | -0.0378 | 0.0118 | 0.0148 | 0.0154 | 0.0078 | 0.0099 | 0.0113 |
| 0.43 | 0.0078 | 0.0004 | -0.0130 | 0.0094 | 0.0109 | -0.0372 | 0.0118 | 0.0146 | 0.0154 | 0.0078 | 0.0099 | 0.0113 |
| 0.45 | 0.0079 | 0.0004 | -0.0133 | 0.0107 | 0.0112 | -0.0366 | 0.0119 | 0.0148 | 0.0154 | 0.0079 | 0.0099 | 0.0112 |
| 0.47 | 0.0077 | 0.0004 | -0.0133 | 0.0120 | 0.0113 | -0.0357 | 0.0120 | 0.0148 | 0.0158 | 0.0079 | 0.0099 | 0.0112 |
| 0.49 | 0.0069 | 0.0010 | -0.0128 | 0.0132 | 0.0114 | -0.0347 | 0.0119 | 0.0150 | 0.0156 | 0.0080 | 0.0099 | 0.0112 |
| 0.51 | 0.0072 | 0.0001 | -0.0129 | 0.0144 | 0.0113 | -0.0335 | 0.0122 | 0.0150 | 0.0159 | 0.0080 | 0.0099 | 0.0111 |
| 0.53 | 0.0071 | -0.0002 | -0.0125 | 0.0155 | 0.0112 | -0.0322 | 0.0123 | 0.0152 | 0.0160 | 0.0081 | 0.0099 | 0.0111 |
| 0.55 | 0.0067 | -0.0004 | -0.0121 | 0.0166 | 0.0110 | -0.0307 | 0.0124 | 0.0153 | 0.0161 | 0.0082 | 0.0099 | 0.0110 |
| 0.57 | 0.0070 | -0.0011 | -0.0117 | 0.0177 | 0.0107 | -0.0291 | 0.0123 | 0.0153 | 0.0158 | 0.0083 | 0.0099 | 0.0110 |
| 0.59 | 0.0059 | -0.0005 | -0.0105 | 0.0187 | 0.0102 | -0.0273 | 0.0123 | 0.0154 | 0.0158 | 0.0083 | 0.0099 | 0.0110 |
| 0.61 | 0.0069 | -0.0021 | -0.0108 | 0.0198 | 0.0097 | -0.0253 | 0.0123 | 0.0154 | 0.0157 | 0.0084 | 0.0099 | 0.0109 |
| 0.63 | 0.0059 | -0.0014 | -0.0101 | 0.0208 | 0.0091 | -0.0231 | 0.0123 | 0.0152 | 0.0159 | 0.0085 | 0.0099 | 0.0109 |
| 0.65 | 0.0061 | -0.0017 | -0.0103 | 0.0218 | 0.0084 | -0.0208 | 0.0124 | 0.0154 | 0.0159 | 0.0086 | 0.0099 | 0.0108 |
| 0.67 | 0.0057 | -0.0010 | -0.0101 | 0.0228 | 0.0076 | -0.0182 | 0.0126 | 0.0155 | 0.0161 | 0.0087 | 0.0100 | 0.0108 |
| 0.69 | 0.0053 | -0.0007 | -0.0099 | 0.0238 | 0.0066 | -0.0154 | 0.0128 | 0.0156 | 0.0162 | 0.0088 | 0.0100 | 0.0108 |
| 0.71 | 0.0048 | -0.0013 | -0.0087 | 0.0249 | 0.0056 | -0.0124 | 0.0130 | 0.0156 | 0.0164 | 0.0089 | 0.0100 | 0.0108 |
| 0.73 | 0.0043 | -0.0015 | -0.0081 | 0.0260 | 0.0043 | -0.0091 | 0.0133 | 0.0156 | 0.0168 | 0.0091 | 0.0101 | 0.0108 |
| 0.75 | 0.0044 | -0.0012 | -0.0087 | 0.0277 | 0.0030 | -0.0056 | 0.0134 | 0.0157 | 0.0169 | 0.0092 | 0.0101 | 0.0109 |
| 0.77 | 0.0040 | -0.0015 | -0.0085 | 0.0292 | 0.0015 | -0.0016 | 0.0135 | 0.0160 | 0.0173 | 0.0094 | 0.0102 | 0.0109 |
| 0.79 | 0.0040 | -0.0027 | -0.0078 | 0.0306 | -0.0002 | 0.0028 | 0.0134 | 0.0162 | 0.0172 | 0.0096 | 0.0103 | 0.0111 |
| 0.81 | 0.0038 | -0.0023 | -0.0080 | 0.0322 | -0.0022 | 0.0077 | 0.0138 | 0.0165 | 0.0177 | 0.0098 | 0.0104 | 0.0113 |
| 0.83 | 0.0029 | -0.0024 | -0.0071 | 0.0340 | -0.0045 | 0.0131 | 0.0141 | 0.0171 | 0.0181 | 0.0101 | 0.0105 | 0.0116 |
| 0.85 | 0.0024 | -0.0019 | -0.0072 | 0.0360 | -0.0071 | 0.0192 | 0.0144 | 0.0174 | 0.0186 | 0.0104 | 0.0107 | 0.0120 |
| 0.87 | 0.0015 | -0.0028 | -0.0054 | 0.0384 | -0.0102 | 0.0262 | 0.0149 | 0.0181 | 0.0194 | 0.0108 | 0.0110 | 0.0126 |
| 0.89 | 0.0009 | -0.0045 | -0.0037 | 0.0413 | -0.0138 | 0.0343 | 0.0157 | 0.0192 | 0.0204 | 0.0113 | 0.0113 | 0.0133 |
| 0.91 | -0.0013 | -0.0038 | -0.0014 | 0.0450 | -0.0182 | 0.0440 | 0.0170 | 0.0206 | 0.0218 | 0.0120 | 0.0118 | 0.0145 |
| 0.93 | -0.0033 | -0.0054 | 0.0022 | 0.0498 | -0.0236 | 0.0559 | 0.0182 | 0.0223 | 0.0235 | 0.0128 | 0.0124 | 0.0161 |
| 0.95 | -0.0070 | -0.0060 | 0.0072 | 0.0569 | -0.0308 | 0.0714 | 0.0199 | 0.0247 | 0.0257 | 0.0142 | 0.0133 | 0.0187 |
| 0.97 | -0.0110 | -0.0101 | 0.0145 | 0.0690 | -0.0413 | 0.0939 | 0.0239 | 0.0296 | 0.0314 | 0.0167 | 0.0149 | 0.0232 |
| 0.99 | -0.0390 | -0.0248 | 0.0542 | 0.0994 | -0.0606 | 0.1349 | 0.0378 | 0.0475 | 0.0515 | 0.0235 | 0.0184 | 0.0343 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 100$. The number of replications is $5{,}000$. The optimal bandwidth is $2\zeta^*$.

Table 14: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \frac{sin(\tau)+1}{2}$*: bias and MSE of estimators*

| | *Bias* | | | | | | *MSE* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | *smoothed* | | | *canonical* | | | *smoothed* | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ |
| 0.01 | 0.0092 | -0.0231 | 0.0500 | -0.1607 | -0.0515 | 0.1010 | 0.0424 | 0.0602 | 0.0592 | 0.0385 | 0.0195 | 0.0264 |
| 0.03 | 0.0047 | -0.0037 | 0.0091 | -0.1119 | -0.0277 | 0.0546 | 0.0250 | 0.0354 | 0.0337 | 0.0232 | 0.0152 | 0.0169 |
| 0.05 | 0.0063 | -0.0013 | -0.0002 | -0.0888 | -0.0168 | 0.0321 | 0.0193 | 0.0277 | 0.0258 | 0.0176 | 0.0135 | 0.0139 |
| 0.07 | 0.0069 | -0.0012 | -0.0038 | -0.0734 | -0.0100 | 0.0175 | 0.0171 | 0.0241 | 0.0224 | 0.0145 | 0.0125 | 0.0124 |
| 0.09 | 0.0082 | -0.0023 | -0.0065 | -0.0619 | -0.0053 | 0.0067 | 0.0152 | 0.0212 | 0.0205 | 0.0126 | 0.0118 | 0.0117 |
| 0.11 | 0.0088 | -0.0019 | -0.0087 | -0.0526 | -0.0017 | -0.0017 | 0.0143 | 0.0200 | 0.0191 | 0.0112 | 0.0114 | 0.0112 |
| 0.13 | 0.0090 | -0.0020 | -0.0095 | -0.0448 | 0.0011 | -0.0085 | 0.0138 | 0.0190 | 0.0183 | 0.0103 | 0.0110 | 0.0110 |
| 0.15 | 0.0089 | -0.0016 | -0.0104 | -0.0382 | 0.0033 | -0.0141 | 0.0133 | 0.0182 | 0.0178 | 0.0095 | 0.0108 | 0.0109 |
| 0.17 | 0.0090 | -0.0014 | -0.0111 | -0.0324 | 0.0051 | -0.0188 | 0.0128 | 0.0174 | 0.0171 | 0.0090 | 0.0106 | 0.0109 |
| 0.19 | 0.0097 | -0.0016 | -0.0124 | -0.0273 | 0.0066 | -0.0228 | 0.0124 | 0.0169 | 0.0166 | 0.0085 | 0.0104 | 0.0109 |
| 0.21 | 0.0095 | -0.0010 | -0.0132 | -0.0226 | 0.0078 | -0.0262 | 0.0123 | 0.0166 | 0.0164 | 0.0082 | 0.0102 | 0.0109 |
| 0.23 | 0.0090 | -0.0002 | -0.0132 | -0.0184 | 0.0087 | -0.0290 | 0.0121 | 0.0161 | 0.0160 | 0.0079 | 0.0101 | 0.0110 |
| 0.25 | 0.0092 | -0.0001 | -0.0141 | -0.0142 | 0.0094 | -0.0313 | 0.0118 | 0.0157 | 0.0157 | 0.0077 | 0.0100 | 0.0111 |
| 0.27 | 0.0097 | -0.0005 | -0.0149 | -0.0103 | 0.0099 | -0.0331 | 0.0117 | 0.0156 | 0.0155 | 0.0076 | 0.0099 | 0.0111 |
| 0.29 | 0.0089 | 0 | -0.0144 | -0.0071 | 0.0104 | -0.0348 | 0.0115 | 0.0152 | 0.0154 | 0.0075 | 0.0098 | 0.0112 |
| 0.31 | 0.0084 | 0.0012 | -0.0148 | -0.0042 | 0.0107 | -0.0362 | 0.0115 | 0.0151 | 0.0155 | 0.0074 | 0.0098 | 0.0113 |
| 0.33 | 0.0081 | 0.0014 | -0.0146 | -0.0014 | 0.0110 | -0.0373 | 0.0115 | 0.0151 | 0.0155 | 0.0073 | 0.0097 | 0.0113 |
| 0.35 | 0.0082 | 0.0008 | -0.0146 | 0.0011 | 0.0111 | -0.0382 | 0.0112 | 0.0148 | 0.0153 | 0.0073 | 0.0097 | 0.0114 |
| 0.37 | 0.0087 | 0.0005 | -0.0154 | 0.0035 | 0.0112 | -0.0388 | 0.0111 | 0.0146 | 0.0154 | 0.0073 | 0.0096 | 0.0114 |
| 0.39 | 0.0093 | 0 | -0.0162 | 0.0057 | 0.0112 | -0.0392 | 0.0112 | 0.0145 | 0.0156 | 0.0073 | 0.0096 | 0.0114 |
| 0.41 | 0.0095 | -0.0004 | -0.0162 | 0.0078 | 0.0111 | -0.0393 | 0.0113 | 0.0146 | 0.0158 | 0.0073 | 0.0095 | 0.0115 |
| 0.43 | 0.0095 | -0.0010 | -0.0160 | 0.0098 | 0.0110 | -0.0393 | 0.0115 | 0.0147 | 0.0158 | 0.0074 | 0.0095 | 0.0115 |
| 0.45 | 0.0095 | -0.0018 | -0.0154 | 0.0117 | 0.0107 | -0.0391 | 0.0114 | 0.0147 | 0.0157 | 0.0074 | 0.0095 | 0.0115 |
| 0.47 | 0.0103 | -0.0020 | -0.0164 | 0.0135 | 0.0105 | -0.0387 | 0.0115 | 0.0146 | 0.0158 | 0.0075 | 0.0095 | 0.0115 |
| 0.49 | 0.0100 | -0.0016 | -0.0162 | 0.0152 | 0.0101 | -0.0380 | 0.0115 | 0.0144 | 0.0158 | 0.0075 | 0.0095 | 0.0115 |
| 0.51 | 0.0095 | -0.0015 | -0.0157 | 0.0168 | 0.0097 | -0.0372 | 0.0115 | 0.0144 | 0.0160 | 0.0076 | 0.0095 | 0.0114 |
| 0.53 | 0.0089 | -0.0016 | -0.0148 | 0.0183 | 0.0092 | -0.0363 | 0.0115 | 0.0144 | 0.0160 | 0.0077 | 0.0095 | 0.0114 |
| 0.55 | 0.0087 | -0.0018 | -0.0143 | 0.0198 | 0.0087 | -0.0351 | 0.0115 | 0.0143 | 0.0159 | 0.0078 | 0.0095 | 0.0114 |
| 0.57 | 0.0086 | -0.0016 | -0.0145 | 0.0212 | 0.0081 | -0.0337 | 0.0115 | 0.0144 | 0.0160 | 0.0079 | 0.0095 | 0.0113 |
| 0.59 | 0.0077 | -0.0006 | -0.0141 | 0.0225 | 0.0074 | -0.0322 | 0.0116 | 0.0144 | 0.0160 | 0.0080 | 0.0095 | 0.0113 |
| 0.61 | 0.0074 | -0.0009 | -0.0136 | 0.0238 | 0.0067 | -0.0304 | 0.0119 | 0.0146 | 0.0164 | 0.0081 | 0.0095 | 0.0113 |
| 0.63 | 0.0064 | -0.0001 | -0.0129 | 0.0251 | 0.0058 | -0.0284 | 0.0121 | 0.0147 | 0.0165 | 0.0082 | 0.0096 | 0.0112 |
| 0.65 | 0.0057 | 0.0001 | -0.0122 | 0.0263 | 0.0050 | -0.0262 | 0.0122 | 0.0147 | 0.0165 | 0.0083 | 0.0096 | 0.0112 |
| 0.67 | 0.0046 | 0.0007 | -0.0112 | 0.0275 | 0.0040 | -0.0238 | 0.0122 | 0.0148 | 0.0165 | 0.0085 | 0.0097 | 0.0112 |
| 0.69 | 0.0038 | 0.0007 | -0.0098 | 0.0287 | 0.0029 | -0.0211 | 0.0123 | 0.0150 | 0.0166 | 0.0086 | 0.0097 | 0.0111 |
| 0.71 | 0.0037 | 0.0007 | -0.0101 | 0.0300 | 0.0017 | -0.0182 | 0.0124 | 0.0152 | 0.0169 | 0.0088 | 0.0098 | 0.0111 |
| 0.73 | 0.0038 | 0.0004 | -0.0101 | 0.0312 | 0.0004 | -0.0150 | 0.0126 | 0.0154 | 0.0171 | 0.0089 | 0.0099 | 0.0111 |
| 0.75 | 0.0035 | 0.0002 | -0.0097 | 0.0330 | -0.0010 | -0.0115 | 0.0128 | 0.0158 | 0.0172 | 0.0091 | 0.0099 | 0.0111 |
| 0.77 | 0.0026 | 0.0007 | -0.0088 | 0.0347 | -0.0026 | -0.0075 | 0.0130 | 0.0160 | 0.0176 | 0.0094 | 0.0100 | 0.0112 |
| 0.79 | 0.0015 | 0.0002 | -0.0068 | 0.0362 | -0.0044 | -0.0032 | 0.0130 | 0.0162 | 0.0177 | 0.0096 | 0.0102 | 0.0113 |
| 0.81 | 0.0018 | -0.0005 | -0.0066 | 0.0379 | -0.0065 | 0.0017 | 0.0136 | 0.0169 | 0.0182 | 0.0099 | 0.0103 | 0.0114 |
| 0.83 | 0.0016 | -0.0011 | -0.0063 | 0.0398 | -0.0088 | 0.0071 | 0.0142 | 0.0176 | 0.0188 | 0.0102 | 0.0105 | 0.0117 |
| 0.85 | 0.0013 | -0.0018 | -0.0053 | 0.0419 | -0.0115 | 0.0133 | 0.0146 | 0.0178 | 0.0197 | 0.0105 | 0.0108 | 0.0120 |
| 0.87 | 0.0008 | -0.0025 | -0.0044 | 0.0444 | -0.0146 | 0.0202 | 0.0155 | 0.0187 | 0.0209 | 0.0110 | 0.0111 | 0.0125 |
| 0.89 | 0.0006 | -0.0037 | -0.0040 | 0.0475 | -0.0183 | 0.0283 | 0.0161 | 0.0195 | 0.0217 | 0.0115 | 0.0114 | 0.0132 |
| 0.91 | -0.0011 | -0.0027 | -0.0027 | 0.0514 | -0.0227 | 0.0379 | 0.0170 | 0.0206 | 0.0230 | 0.0122 | 0.0119 | 0.0142 |
| 0.93 | -0.0032 | -0.0034 | 0.0003 | 0.0566 | -0.0283 | 0.0497 | 0.0179 | 0.0218 | 0.0247 | 0.0132 | 0.0126 | 0.0157 |
| 0.95 | -0.0040 | -0.0077 | 0.0037 | 0.0641 | -0.0357 | 0.0650 | 0.0196 | 0.0247 | 0.0267 | 0.0147 | 0.0136 | 0.0181 |
| 0.97 | -0.0087 | -0.0130 | 0.0132 | 0.0769 | -0.0465 | 0.0869 | 0.0241 | 0.0304 | 0.0327 | 0.0174 | 0.0153 | 0.0223 |
| 0.99 | -0.0329 | -0.0318 | 0.0491 | 0.1089 | -0.0668 | 0.1271 | 0.0383 | 0.0492 | 0.0522 | 0.0252 | 0.0191 | 0.0326 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 15: *Model* (2) *defined through Proposition* 2 *when* $\lambda(\tau) = \sqrt{\tau}$*: bias and MSE of estimators*

| | *Bias* | | | | | | *MSE* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | *smoothed* | | | *canonical* | | | *smoothed* | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ |
| 0.01 | -0.0010 | -0.0043 | 0.0104 | -0.1150 | -0.0094 | 0.0856 | 0.0075 | 0.0118 | 0.0108 | 0.0163 | 0.0049 | 0.0118 |
| 0.03 | 0.0022 | -0.0011 | 0 | -0.0731 | -0.0073 | 0.0443 | 0.0041 | 0.0066 | 0.0057 | 0.0076 | 0.0035 | 0.0052 |
| 0.05 | 0.0035 | -0.0020 | -0.0021 | -0.0543 | -0.0062 | 0.0258 | 0.0032 | 0.0051 | 0.0044 | 0.0049 | 0.0030 | 0.0034 |
| 0.07 | 0.0034 | -0.0018 | -0.0022 | -0.0423 | -0.0054 | 0.0141 | 0.0028 | 0.0044 | 0.0039 | 0.0035 | 0.0027 | 0.0027 |
| 0.09 | 0.0034 | -0.0018 | -0.0026 | -0.0334 | -0.0048 | 0.0055 | 0.0026 | 0.0040 | 0.0036 | 0.0028 | 0.0025 | 0.0024 |
| 0.11 | 0.0034 | -0.0015 | -0.0030 | -0.0264 | -0.0043 | -0.0012 | 0.0025 | 0.0037 | 0.0034 | 0.0023 | 0.0023 | 0.0022 |
| 0.13 | 0.0037 | -0.0019 | -0.0033 | -0.0207 | -0.0039 | -0.0066 | 0.0024 | 0.0036 | 0.0033 | 0.0019 | 0.0022 | 0.0022 |
| 0.15 | 0.0040 | -0.0021 | -0.0038 | -0.0159 | -0.0036 | -0.0109 | 0.0023 | 0.0034 | 0.0032 | 0.0017 | 0.0021 | 0.0022 |
| 0.17 | 0.0045 | -0.0024 | -0.0042 | -0.0118 | -0.0033 | -0.0145 | 0.0022 | 0.0032 | 0.0030 | 0.0016 | 0.0021 | 0.0022 |
| 0.19 | 0.0049 | -0.0029 | -0.0044 | -0.0082 | -0.0030 | -0.0175 | 0.0022 | 0.0031 | 0.0030 | 0.0015 | 0.0020 | 0.0023 |
| 0.21 | 0.0048 | -0.0027 | -0.0044 | -0.0051 | -0.0028 | -0.0200 | 0.0021 | 0.0030 | 0.0029 | 0.0014 | 0.0019 | 0.0024 |
| 0.23 | 0.0047 | -0.0027 | -0.0044 | -0.0024 | -0.0026 | -0.0221 | 0.0021 | 0.0029 | 0.0029 | 0.0014 | 0.0019 | 0.0024 |
| 0.25 | 0.0046 | -0.0025 | -0.0043 | 0.0001 | -0.0024 | -0.0239 | 0.0021 | 0.0028 | 0.0029 | 0.0014 | 0.0018 | 0.0025 |
| 0.27 | 0.0046 | -0.0024 | -0.0043 | 0.0023 | -0.0022 | -0.0253 | 0.0021 | 0.0028 | 0.0028 | 0.0014 | 0.0018 | 0.0025 |
| 0.29 | 0.0043 | -0.0023 | -0.0040 | 0.0043 | -0.0021 | -0.0266 | 0.0021 | 0.0027 | 0.0028 | 0.0014 | 0.0018 | 0.0026 |
| 0.31 | 0.0044 | -0.0026 | -0.0039 | 0.0060 | -0.0019 | -0.0276 | 0.0020 | 0.0026 | 0.0028 | 0.0014 | 0.0017 | 0.0026 |
| 0.33 | 0.0041 | -0.0023 | -0.0038 | 0.0076 | -0.0018 | -0.0284 | 0.0020 | 0.0026 | 0.0028 | 0.0014 | 0.0017 | 0.0027 |
| 0.35 | 0.0040 | -0.0023 | -0.0036 | 0.0091 | -0.0017 | -0.0290 | 0.0020 | 0.0025 | 0.0027 | 0.0014 | 0.0017 | 0.0027 |
| 0.37 | 0.0042 | -0.0026 | -0.0038 | 0.0104 | -0.0015 | -0.0294 | 0.0020 | 0.0025 | 0.0028 | 0.0015 | 0.0017 | 0.0027 |
| 0.39 | 0.0040 | -0.0025 | -0.0036 | 0.0115 | -0.0014 | -0.0296 | 0.0020 | 0.0025 | 0.0028 | 0.0015 | 0.0017 | 0.0027 |
| 0.41 | 0.0038 | -0.0021 | -0.0037 | 0.0126 | -0.0013 | -0.0297 | 0.0020 | 0.0025 | 0.0028 | 0.0015 | 0.0016 | 0.0027 |
| 0.43 | 0.0039 | -0.0021 | -0.0039 | 0.0135 | -0.0012 | -0.0296 | 0.0020 | 0.0024 | 0.0028 | 0.0015 | 0.0016 | 0.0027 |
| 0.45 | 0.0039 | -0.0020 | -0.0041 | 0.0144 | -0.0011 | -0.0294 | 0.0020 | 0.0024 | 0.0028 | 0.0016 | 0.0016 | 0.0027 |
| 0.47 | 0.0041 | -0.0023 | -0.0042 | 0.0151 | -0.0010 | -0.0290 | 0.0020 | 0.0024 | 0.0028 | 0.0016 | 0.0016 | 0.0027 |
| 0.49 | 0.0041 | -0.0024 | -0.0043 | 0.0158 | -0.0009 | -0.0285 | 0.0020 | 0.0024 | 0.0028 | 0.0016 | 0.0016 | 0.0027 |
| 0.51 | 0.0040 | -0.0023 | -0.0044 | 0.0164 | -0.0008 | -0.0279 | 0.0020 | 0.0024 | 0.0027 | 0.0016 | 0.0016 | 0.0027 |
| 0.53 | 0.0039 | -0.0023 | -0.0043 | 0.0169 | -0.0007 | -0.0271 | 0.0020 | 0.0024 | 0.0028 | 0.0017 | 0.0016 | 0.0026 |
| 0.55 | 0.0045 | -0.0027 | -0.0047 | 0.0174 | -0.0006 | -0.0261 | 0.0021 | 0.0024 | 0.0028 | 0.0017 | 0.0016 | 0.0026 |
| 0.57 | 0.0042 | -0.0025 | -0.0045 | 0.0177 | -0.0005 | -0.0251 | 0.0021 | 0.0024 | 0.0028 | 0.0017 | 0.0016 | 0.0025 |
| 0.59 | 0.0040 | -0.0023 | -0.0043 | 0.0180 | -0.0004 | -0.0238 | 0.0021 | 0.0023 | 0.0028 | 0.0017 | 0.0016 | 0.0025 |
| 0.61 | 0.0039 | -0.0024 | -0.0043 | 0.0182 | -0.0003 | -0.0224 | 0.0021 | 0.0023 | 0.0028 | 0.0018 | 0.0016 | 0.0024 |
| 0.63 | 0.0041 | -0.0027 | -0.0043 | 0.0184 | -0.0002 | -0.0207 | 0.0021 | 0.0024 | 0.0028 | 0.0018 | 0.0016 | 0.0024 |
| 0.65 | 0.0043 | -0.0027 | -0.0046 | 0.0184 | -0.0001 | -0.0189 | 0.0021 | 0.0023 | 0.0028 | 0.0018 | 0.0016 | 0.0023 |
| 0.67 | 0.0042 | -0.0027 | -0.0045 | 0.0184 | 0 | -0.0169 | 0.0021 | 0.0023 | 0.0027 | 0.0018 | 0.0016 | 0.0022 |
| 0.69 | 0.0037 | -0.0024 | -0.0041 | 0.0183 | 0.0001 | -0.0146 | 0.0022 | 0.0023 | 0.0028 | 0.0018 | 0.0016 | 0.0022 |
| 0.71 | 0.0036 | -0.0024 | -0.0039 | 0.0182 | 0.0002 | -0.0121 | 0.0021 | 0.0023 | 0.0027 | 0.0018 | 0.0016 | 0.0021 |
| 0.73 | 0.0032 | -0.0024 | -0.0035 | 0.0180 | 0.0003 | -0.0093 | 0.0021 | 0.0023 | 0.0027 | 0.0018 | 0.0016 | 0.0021 |
| 0.75 | 0.0031 | -0.0021 | -0.0036 | 0.0177 | 0.0003 | -0.0063 | 0.0021 | 0.0023 | 0.0027 | 0.0018 | 0.0016 | 0.0020 |
| 0.77 | 0.0028 | -0.0020 | -0.0034 | 0.0174 | 0.0004 | -0.0028 | 0.0022 | 0.0024 | 0.0028 | 0.0018 | 0.0016 | 0.0020 |
| 0.79 | 0.0026 | -0.0020 | -0.0030 | 0.0169 | 0.0004 | 0.0010 | 0.0022 | 0.0024 | 0.0028 | 0.0019 | 0.0016 | 0.0020 |
| 0.81 | 0.0021 | -0.0015 | -0.0027 | 0.0164 | 0.0004 | 0.0053 | 0.0022 | 0.0024 | 0.0028 | 0.0019 | 0.0017 | 0.0021 |
| 0.83 | 0.0019 | -0.0014 | -0.0024 | 0.0159 | 0.0003 | 0.0102 | 0.0023 | 0.0024 | 0.0028 | 0.0019 | 0.0017 | 0.0022 |
| 0.85 | 0.0014 | -0.0010 | -0.0020 | 0.0153 | 0.0002 | 0.0158 | 0.0024 | 0.0025 | 0.0029 | 0.0019 | 0.0017 | 0.0023 |
| 0.87 | 0.0009 | -0.0003 | -0.0020 | 0.0146 | 0 | 0.0223 | 0.0024 | 0.0026 | 0.0030 | 0.0019 | 0.0017 | 0.0026 |
| 0.89 | 0.0011 | -0.0008 | -0.0018 | 0.0139 | -0.0003 | 0.0300 | 0.0025 | 0.0026 | 0.0031 | 0.0019 | 0.0017 | 0.0031 |
| 0.91 | 0.0002 | -0.0003 | -0.0011 | 0.0131 | -0.0008 | 0.0392 | 0.0027 | 0.0027 | 0.0032 | 0.0020 | 0.0018 | 0.0038 |
| 0.93 | -0.0007 | -0.0002 | -0.0001 | 0.0125 | -0.0015 | 0.0507 | 0.0028 | 0.0027 | 0.0034 | 0.0021 | 0.0019 | 0.0049 |
| 0.95 | -0.0024 | 0.0002 | 0.0017 | 0.0127 | -0.0028 | 0.0655 | 0.0031 | 0.0030 | 0.0037 | 0.0022 | 0.0019 | 0.0067 |
| 0.97 | -0.0046 | 0.0008 | 0.0042 | 0.0143 | -0.0048 | 0.0871 | 0.0035 | 0.0035 | 0.0042 | 0.0024 | 0.0021 | 0.0102 |
| 0.99 | -0.0121 | 0.0005 | 0.0148 | 0.0243 | -0.0094 | 0.1271 | 0.0053 | 0.0050 | 0.0066 | 0.0033 | 0.0025 | 0.0192 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 500$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 16: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \frac{1}{1+e^{-\tau}}$*: bias and MSE of estimators*

| | *Bias* | | | | | | *MSE* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | *smoothed* | | | *canonical* | | | *smoothed* | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ |
| 0.01 | 0.0002 | -0.0040 | 0.0082 | -0.0917 | -0.0380 | 0.0616 | 0.0081 | 0.0112 | 0.0104 | 0.0115 | 0.0058 | 0.0077 |
| 0.03 | 0.0008 | 0.0001 | 0.0006 | -0.0649 | -0.0197 | 0.0341 | 0.0046 | 0.0064 | 0.0057 | 0.0066 | 0.0037 | 0.0042 |
| 0.05 | -0.0002 | 0.0018 | 0.0003 | -0.0525 | -0.0113 | 0.0213 | 0.0036 | 0.0051 | 0.0046 | 0.0048 | 0.0031 | 0.0031 |
| 0.07 | 0.0003 | 0.0012 | -0.0007 | -0.0442 | -0.0060 | 0.0129 | 0.0032 | 0.0045 | 0.0042 | 0.0039 | 0.0028 | 0.0026 |
| 0.09 | 0.0005 | 0.0015 | -0.0014 | -0.0380 | -0.0022 | 0.0068 | 0.0029 | 0.0041 | 0.0038 | 0.0033 | 0.0026 | 0.0024 |
| 0.11 | 0.0010 | 0.0010 | -0.0020 | -0.0330 | 0.0007 | 0.0019 | 0.0028 | 0.0038 | 0.0036 | 0.0029 | 0.0025 | 0.0023 |
| 0.13 | 0.0010 | 0.0010 | -0.0020 | -0.0288 | 0.0029 | -0.0019 | 0.0027 | 0.0037 | 0.0034 | 0.0025 | 0.0024 | 0.0022 |
| 0.15 | 0.0013 | 0.0007 | -0.0021 | -0.0251 | 0.0047 | -0.0051 | 0.0026 | 0.0036 | 0.0033 | 0.0023 | 0.0023 | 0.0022 |
| 0.17 | 0.0012 | 0.0008 | -0.0021 | -0.0220 | 0.0061 | -0.0077 | 0.0025 | 0.0035 | 0.0032 | 0.0021 | 0.0023 | 0.0022 |
| 0.19 | 0.0014 | 0.0005 | -0.0023 | -0.0191 | 0.0073 | -0.0099 | 0.0024 | 0.0034 | 0.0032 | 0.0020 | 0.0023 | 0.0022 |
| 0.21 | 0.0011 | 0.0008 | -0.0020 | -0.0166 | 0.0082 | -0.0118 | 0.0023 | 0.0033 | 0.0031 | 0.0019 | 0.0023 | 0.0022 |
| 0.23 | 0.0013 | 0.0007 | -0.0025 | -0.0143 | 0.0090 | -0.0134 | 0.0023 | 0.0032 | 0.0031 | 0.0018 | 0.0022 | 0.0023 |
| 0.25 | 0.0015 | 0.0004 | -0.0025 | -0.0121 | 0.0096 | -0.0147 | 0.0023 | 0.0031 | 0.0031 | 0.0017 | 0.0022 | 0.0023 |
| 0.27 | 0.0015 | 0.0005 | -0.0026 | -0.0102 | 0.0101 | -0.0157 | 0.0023 | 0.0031 | 0.0031 | 0.0017 | 0.0022 | 0.0023 |
| 0.29 | 0.0012 | 0.0010 | -0.0026 | -0.0084 | 0.0105 | -0.0167 | 0.0023 | 0.0031 | 0.0030 | 0.0016 | 0.0022 | 0.0023 |
| 0.31 | 0.0013 | 0.0007 | -0.0024 | -0.0067 | 0.0107 | -0.0174 | 0.0023 | 0.0031 | 0.0030 | 0.0016 | 0.0022 | 0.0023 |
| 0.33 | 0.0012 | 0.0010 | -0.0028 | -0.0051 | 0.0109 | -0.0180 | 0.0023 | 0.0031 | 0.0030 | 0.0016 | 0.0022 | 0.0024 |
| 0.35 | 0.0010 | 0.0011 | -0.0026 | -0.0036 | 0.0111 | -0.0185 | 0.0023 | 0.0030 | 0.0030 | 0.0016 | 0.0022 | 0.0024 |
| 0.37 | 0.0011 | 0.0011 | -0.0027 | -0.0022 | 0.0111 | -0.0188 | 0.0022 | 0.0030 | 0.0030 | 0.0015 | 0.0022 | 0.0024 |
| 0.39 | 0.0011 | 0.0010 | -0.0025 | -0.0009 | 0.0111 | -0.0190 | 0.0022 | 0.0029 | 0.0031 | 0.0015 | 0.0022 | 0.0024 |
| 0.41 | 0.0013 | 0.0008 | -0.0029 | 0.0004 | 0.0110 | -0.0191 | 0.0023 | 0.0030 | 0.0031 | 0.0015 | 0.0022 | 0.0024 |
| 0.43 | 0.0013 | 0.0008 | -0.0030 | 0.0016 | 0.0109 | -0.0191 | 0.0023 | 0.0030 | 0.0031 | 0.0015 | 0.0022 | 0.0024 |
| 0.45 | 0.0014 | 0.0007 | -0.0031 | 0.0027 | 0.0107 | -0.0190 | 0.0023 | 0.0030 | 0.0031 | 0.0016 | 0.0021 | 0.0024 |
| 0.47 | 0.0013 | 0.0007 | -0.0027 | 0.0038 | 0.0105 | -0.0189 | 0.0023 | 0.0029 | 0.0031 | 0.0016 | 0.0021 | 0.0024 |
| 0.49 | 0.0013 | 0.0008 | -0.0029 | 0.0049 | 0.0102 | -0.0186 | 0.0023 | 0.0029 | 0.0031 | 0.0016 | 0.0021 | 0.0024 |
| 0.51 | 0.0016 | 0.0006 | -0.0032 | 0.0059 | 0.0099 | -0.0182 | 0.0023 | 0.0030 | 0.0031 | 0.0016 | 0.0021 | 0.0024 |
| 0.53 | 0.0014 | 0.0007 | -0.0031 | 0.0069 | 0.0095 | -0.0178 | 0.0023 | 0.0030 | 0.0031 | 0.0016 | 0.0021 | 0.0024 |
| 0.55 | 0.0015 | 0.0010 | -0.0036 | 0.0079 | 0.0091 | -0.0173 | 0.0023 | 0.0030 | 0.0031 | 0.0016 | 0.0021 | 0.0024 |
| 0.57 | 0.0015 | 0.0008 | -0.0035 | 0.0089 | 0.0086 | -0.0167 | 0.0023 | 0.0030 | 0.0031 | 0.0017 | 0.0021 | 0.0024 |
| 0.59 | 0.0018 | 0.0006 | -0.0039 | 0.0099 | 0.0081 | -0.0159 | 0.0023 | 0.0030 | 0.0032 | 0.0017 | 0.0021 | 0.0024 |
| 0.61 | 0.0015 | 0.0009 | -0.0035 | 0.0108 | 0.0075 | -0.0151 | 0.0024 | 0.0031 | 0.0032 | 0.0017 | 0.0022 | 0.0024 |
| 0.63 | 0.0013 | 0.0009 | -0.0033 | 0.0117 | 0.0069 | -0.0142 | 0.0024 | 0.0031 | 0.0032 | 0.0018 | 0.0022 | 0.0024 |
| 0.65 | 0.0013 | 0.0010 | -0.0034 | 0.0127 | 0.0062 | -0.0132 | 0.0024 | 0.0031 | 0.0032 | 0.0018 | 0.0022 | 0.0024 |
| 0.67 | 0.0009 | 0.0015 | -0.0032 | 0.0136 | 0.0054 | -0.0121 | 0.0024 | 0.0031 | 0.0033 | 0.0018 | 0.0022 | 0.0024 |
| 0.69 | 0.0009 | 0.0012 | -0.0030 | 0.0146 | 0.0046 | -0.0108 | 0.0025 | 0.0032 | 0.0033 | 0.0019 | 0.0022 | 0.0024 |
| 0.71 | 0.0009 | 0.0011 | -0.0029 | 0.0156 | 0.0037 | -0.0094 | 0.0025 | 0.0032 | 0.0034 | 0.0019 | 0.0022 | 0.0024 |
| 0.73 | 0.0011 | 0.0009 | -0.0031 | 0.0166 | 0.0026 | -0.0078 | 0.0026 | 0.0033 | 0.0034 | 0.0020 | 0.0022 | 0.0023 |
| 0.75 | 0.0016 | 0.0005 | -0.0037 | 0.0177 | 0.0015 | -0.0061 | 0.0026 | 0.0033 | 0.0034 | 0.0021 | 0.0022 | 0.0024 |
| 0.77 | 0.0015 | 0.0004 | -0.0034 | 0.0188 | 0.0002 | -0.0041 | 0.0026 | 0.0034 | 0.0035 | 0.0021 | 0.0023 | 0.0024 |
| 0.79 | 0.0016 | 0 | -0.0032 | 0.0200 | -0.0012 | -0.0019 | 0.0027 | 0.0034 | 0.0036 | 0.0022 | 0.0023 | 0.0024 |
| 0.81 | 0.0016 | -0.0002 | -0.0033 | 0.0213 | -0.0028 | 0.0005 | 0.0028 | 0.0035 | 0.0036 | 0.0023 | 0.0023 | 0.0024 |
| 0.83 | 0.0013 | -0.0001 | -0.0030 | 0.0227 | -0.0045 | 0.0033 | 0.0028 | 0.0035 | 0.0037 | 0.0024 | 0.0024 | 0.0025 |
| 0.85 | 0.0015 | -0.0004 | -0.0031 | 0.0243 | -0.0066 | 0.0065 | 0.0029 | 0.0036 | 0.0038 | 0.0025 | 0.0024 | 0.0025 |
| 0.87 | 0.0012 | -0.0007 | -0.0025 | 0.0261 | -0.0090 | 0.0103 | 0.0030 | 0.0037 | 0.0039 | 0.0026 | 0.0025 | 0.0027 |
| 0.89 | 0.0009 | -0.0005 | -0.0023 | 0.0283 | -0.0117 | 0.0147 | 0.0031 | 0.0038 | 0.0041 | 0.0028 | 0.0026 | 0.0028 |
| 0.91 | 0.0004 | -0.0005 | -0.0019 | 0.0310 | -0.0152 | 0.0201 | 0.0032 | 0.0040 | 0.0042 | 0.0030 | 0.0028 | 0.0031 |
| 0.93 | 0.0007 | -0.0018 | -0.0014 | 0.0347 | -0.0195 | 0.0268 | 0.0035 | 0.0042 | 0.0045 | 0.0033 | 0.0030 | 0.0035 |
| 0.95 | 0.0002 | -0.0018 | -0.0009 | 0.0400 | -0.0251 | 0.0356 | 0.0037 | 0.0046 | 0.0049 | 0.0038 | 0.0034 | 0.0042 |
| 0.97 | -0.0018 | -0.0016 | 0.0015 | 0.0488 | -0.0334 | 0.0486 | 0.0044 | 0.0055 | 0.0056 | 0.0048 | 0.0040 | 0.0055 |
| 0.99 | -0.0033 | -0.0054 | 0.0052 | 0.0713 | -0.0493 | 0.0736 | 0.0062 | 0.0077 | 0.0083 | 0.0079 | 0.0058 | 0.0090 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 500$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 17: *Model* (2) *defined through Proposition* 2 *when* $\lambda(\tau) = \sqrt[3]{\tau}$*: bias and* $MSE$ *of estimators*

| | *Bias* | | | | | | $MSE$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | *smoothed* | | | *canonical* | | | *smoothed* | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ |
| 0.01 | -0.0011 | -0.0044 | 0.0110 | -0.1048 | -0.0205 | 0.0768 | 0.0078 | 0.0118 | 0.0107 | 0.0141 | 0.0050 | 0.0103 |
| 0.03 | 0.0023 | -0.0030 | 0.0011 | -0.0660 | -0.0158 | 0.0382 | 0.0044 | 0.0063 | 0.0058 | 0.0067 | 0.0036 | 0.0046 |
| 0.05 | 0.0028 | -0.0025 | -0.0008 | -0.0491 | -0.0128 | 0.0210 | 0.0035 | 0.0050 | 0.0046 | 0.0045 | 0.0031 | 0.0032 |
| 0.07 | 0.0027 | -0.0020 | -0.0016 | -0.0384 | -0.0105 | 0.0101 | 0.0032 | 0.0045 | 0.0041 | 0.0034 | 0.0028 | 0.0026 |
| 0.09 | 0.0028 | -0.0015 | -0.0025 | -0.0307 | -0.0085 | 0.0024 | 0.0029 | 0.0040 | 0.0037 | 0.0027 | 0.0026 | 0.0024 |
| 0.11 | 0.0029 | -0.0016 | -0.0028 | -0.0246 | -0.0069 | -0.0036 | 0.0027 | 0.0038 | 0.0035 | 0.0023 | 0.0024 | 0.0022 |
| 0.13 | 0.0031 | -0.0017 | -0.0029 | -0.0197 | -0.0056 | -0.0083 | 0.0025 | 0.0035 | 0.0033 | 0.0020 | 0.0023 | 0.0022 |
| 0.15 | 0.0028 | -0.0015 | -0.0025 | -0.0157 | -0.0044 | -0.0120 | 0.0024 | 0.0034 | 0.0032 | 0.0019 | 0.0022 | 0.0022 |
| 0.17 | 0.0029 | -0.0014 | -0.0026 | -0.0122 | -0.0034 | -0.0151 | 0.0024 | 0.0032 | 0.0031 | 0.0017 | 0.0022 | 0.0023 |
| 0.19 | 0.0030 | -0.0012 | -0.0032 | -0.0092 | -0.0025 | -0.0176 | 0.0023 | 0.0031 | 0.0030 | 0.0016 | 0.0021 | 0.0023 |
| 0.21 | 0.0031 | -0.0014 | -0.0036 | -0.0066 | -0.0017 | -0.0196 | 0.0023 | 0.0031 | 0.0030 | 0.0016 | 0.0021 | 0.0023 |
| 0.23 | 0.0034 | -0.0017 | -0.0039 | -0.0043 | -0.0010 | -0.0213 | 0.0023 | 0.0030 | 0.0029 | 0.0015 | 0.0020 | 0.0024 |
| 0.25 | 0.0036 | -0.0019 | -0.0042 | -0.0023 | -0.0004 | -0.0227 | 0.0022 | 0.0029 | 0.0029 | 0.0015 | 0.0020 | 0.0024 |
| 0.27 | 0.0034 | -0.0018 | -0.0041 | -0.0005 | 0.0002 | -0.0238 | 0.0022 | 0.0029 | 0.0029 | 0.0015 | 0.0020 | 0.0025 |
| 0.29 | 0.0033 | -0.0015 | -0.0042 | 0.0012 | 0.0007 | -0.0247 | 0.0021 | 0.0028 | 0.0028 | 0.0015 | 0.0020 | 0.0025 |
| 0.31 | 0.0033 | -0.0014 | -0.0044 | 0.0026 | 0.0012 | -0.0253 | 0.0021 | 0.0028 | 0.0029 | 0.0015 | 0.0020 | 0.0025 |
| 0.33 | 0.0028 | -0.0011 | -0.0041 | 0.0039 | 0.0016 | -0.0258 | 0.0021 | 0.0028 | 0.0028 | 0.0015 | 0.0020 | 0.0025 |
| 0.35 | 0.0028 | -0.0010 | -0.0043 | 0.0051 | 0.0019 | -0.0261 | 0.0021 | 0.0028 | 0.0028 | 0.0015 | 0.0019 | 0.0025 |
| 0.37 | 0.0025 | -0.0008 | -0.0039 | 0.0062 | 0.0023 | -0.0263 | 0.0021 | 0.0027 | 0.0028 | 0.0015 | 0.0019 | 0.0025 |
| 0.39 | 0.0026 | -0.0009 | -0.0039 | 0.0072 | 0.0026 | -0.0263 | 0.0021 | 0.0028 | 0.0027 | 0.0015 | 0.0019 | 0.0025 |
| 0.41 | 0.0026 | -0.0009 | -0.0040 | 0.0081 | 0.0028 | -0.0261 | 0.0021 | 0.0028 | 0.0027 | 0.0015 | 0.0019 | 0.0025 |
| 0.43 | 0.0030 | -0.0011 | -0.0044 | 0.0089 | 0.0030 | -0.0259 | 0.0021 | 0.0028 | 0.0027 | 0.0016 | 0.0019 | 0.0025 |
| 0.45 | 0.0029 | -0.0013 | -0.0043 | 0.0097 | 0.0032 | -0.0255 | 0.0022 | 0.0028 | 0.0028 | 0.0016 | 0.0019 | 0.0025 |
| 0.47 | 0.0030 | -0.0014 | -0.0044 | 0.0104 | 0.0034 | -0.0250 | 0.0022 | 0.0029 | 0.0027 | 0.0016 | 0.0019 | 0.0025 |
| 0.49 | 0.0030 | -0.0015 | -0.0043 | 0.0110 | 0.0035 | -0.0244 | 0.0022 | 0.0028 | 0.0027 | 0.0016 | 0.0019 | 0.0025 |
| 0.51 | 0.0030 | -0.0016 | -0.0042 | 0.0116 | 0.0036 | -0.0236 | 0.0022 | 0.0028 | 0.0027 | 0.0016 | 0.0019 | 0.0024 |
| 0.53 | 0.0030 | -0.0019 | -0.0037 | 0.0121 | 0.0037 | -0.0228 | 0.0022 | 0.0028 | 0.0027 | 0.0017 | 0.0019 | 0.0024 |
| 0.55 | 0.0028 | -0.0017 | -0.0038 | 0.0126 | 0.0037 | -0.0219 | 0.0022 | 0.0028 | 0.0027 | 0.0017 | 0.0019 | 0.0024 |
| 0.57 | 0.0028 | -0.0018 | -0.0037 | 0.0131 | 0.0037 | -0.0208 | 0.0022 | 0.0028 | 0.0027 | 0.0017 | 0.0019 | 0.0023 |
| 0.59 | 0.0027 | -0.0017 | -0.0036 | 0.0135 | 0.0037 | -0.0196 | 0.0023 | 0.0028 | 0.0027 | 0.0017 | 0.0019 | 0.0023 |
| 0.61 | 0.0027 | -0.0015 | -0.0037 | 0.0138 | 0.0036 | -0.0183 | 0.0023 | 0.0027 | 0.0028 | 0.0018 | 0.0019 | 0.0023 |
| 0.63 | 0.0030 | -0.0015 | -0.0042 | 0.0141 | 0.0036 | -0.0168 | 0.0023 | 0.0027 | 0.0028 | 0.0018 | 0.0019 | 0.0022 |
| 0.65 | 0.0029 | -0.0014 | -0.0043 | 0.0144 | 0.0035 | -0.0152 | 0.0023 | 0.0028 | 0.0028 | 0.0018 | 0.0019 | 0.0022 |
| 0.67 | 0.0026 | -0.0011 | -0.0040 | 0.0147 | 0.0034 | -0.0134 | 0.0023 | 0.0028 | 0.0029 | 0.0018 | 0.0020 | 0.0022 |
| 0.69 | 0.0030 | -0.0015 | -0.0044 | 0.0149 | 0.0032 | -0.0114 | 0.0024 | 0.0028 | 0.0029 | 0.0019 | 0.0020 | 0.0021 |
| 0.71 | 0.0027 | -0.0013 | -0.0041 | 0.0151 | 0.0030 | -0.0093 | 0.0024 | 0.0028 | 0.0029 | 0.0019 | 0.0020 | 0.0021 |
| 0.73 | 0.0025 | -0.0013 | -0.0037 | 0.0153 | 0.0027 | -0.0069 | 0.0024 | 0.0028 | 0.0029 | 0.0019 | 0.0020 | 0.0021 |
| 0.75 | 0.0023 | -0.0013 | -0.0036 | 0.0155 | 0.0024 | -0.0043 | 0.0024 | 0.0029 | 0.0030 | 0.0019 | 0.0020 | 0.0021 |
| 0.77 | 0.0027 | -0.0016 | -0.0041 | 0.0157 | 0.0020 | -0.0015 | 0.0025 | 0.0029 | 0.0030 | 0.0020 | 0.0020 | 0.0021 |
| 0.79 | 0.0025 | -0.0012 | -0.0040 | 0.0159 | 0.0015 | 0.0017 | 0.0025 | 0.0029 | 0.0031 | 0.0020 | 0.0020 | 0.0021 |
| 0.81 | 0.0026 | -0.0014 | -0.0038 | 0.0161 | 0.0010 | 0.0053 | 0.0026 | 0.0029 | 0.0032 | 0.0020 | 0.0020 | 0.0022 |
| 0.83 | 0.0024 | -0.0014 | -0.0036 | 0.0163 | 0.0003 | 0.0093 | 0.0026 | 0.0030 | 0.0032 | 0.0021 | 0.0020 | 0.0023 |
| 0.85 | 0.0017 | -0.0010 | -0.0028 | 0.0165 | -0.0006 | 0.0140 | 0.0027 | 0.0030 | 0.0033 | 0.0021 | 0.0020 | 0.0024 |
| 0.87 | 0.0014 | -0.0009 | -0.0026 | 0.0168 | -0.0016 | 0.0194 | 0.0027 | 0.0030 | 0.0033 | 0.0022 | 0.0021 | 0.0027 |
| 0.89 | 0.0012 | -0.0009 | -0.0023 | 0.0172 | -0.0028 | 0.0258 | 0.0028 | 0.0031 | 0.0034 | 0.0022 | 0.0021 | 0.0030 |
| 0.91 | 0.0009 | -0.0006 | -0.0021 | 0.0178 | -0.0044 | 0.0335 | 0.0030 | 0.0032 | 0.0037 | 0.0023 | 0.0022 | 0.0036 |
| 0.93 | -0.0002 | -0.0005 | -0.0008 | 0.0188 | -0.0066 | 0.0430 | 0.0031 | 0.0033 | 0.0039 | 0.0025 | 0.0022 | 0.0044 |
| 0.95 | -0.0014 | -0.0004 | 0.0008 | 0.0210 | -0.0097 | 0.0555 | 0.0035 | 0.0037 | 0.0044 | 0.0027 | 0.0024 | 0.0057 |
| 0.97 | -0.0026 | -0.0028 | 0.0041 | 0.0251 | -0.0143 | 0.0739 | 0.0040 | 0.0042 | 0.0050 | 0.0031 | 0.0026 | 0.0083 |
| 0.99 | -0.0089 | -0.0044 | 0.0137 | 0.0392 | -0.0233 | 0.1085 | 0.0058 | 0.0062 | 0.0074 | 0.0045 | 0.0033 | 0.0151 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 500$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 18: *Model (2) defined through Proposition 2 when $\lambda(\tau) = cos(\tau)$: bias and $MSE$ of estimators*

| | *Bias* | | | | | | *MSE* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | *smoothed* | | | *canonical* | | | *smoothed* | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ |
| 0.01 | 0.0047 | -0.0081 | 0.0041 | -0.0649 | -0.0661 | 0.0360 | 0.0074 | 0.0103 | 0.0098 | 0.0071 | 0.0083 | 0.0050 |
| 0.03 | 0.0020 | -0.0016 | 0.0004 | -0.0471 | -0.0382 | 0.0170 | 0.0042 | 0.0057 | 0.0055 | 0.0044 | 0.0045 | 0.0031 |
| 0.05 | 0.0015 | -0.0005 | -0.0007 | -0.0389 | -0.0252 | 0.0081 | 0.0034 | 0.0047 | 0.0043 | 0.0035 | 0.0033 | 0.0026 |
| 0.07 | 0.0010 | 0.0009 | -0.0017 | -0.0336 | -0.0167 | 0.0024 | 0.0029 | 0.0040 | 0.0038 | 0.0029 | 0.0027 | 0.0024 |
| 0.09 | 0.0010 | 0.0011 | -0.0020 | -0.0297 | -0.0104 | -0.0017 | 0.0028 | 0.0038 | 0.0036 | 0.0026 | 0.0024 | 0.0023 |
| 0.11 | 0.0012 | 0.0012 | -0.0024 | -0.0266 | -0.0055 | -0.0046 | 0.0025 | 0.0035 | 0.0034 | 0.0023 | 0.0023 | 0.0022 |
| 0.13 | 0.0015 | 0.0007 | -0.0025 | -0.0241 | -0.0014 | -0.0069 | 0.0025 | 0.0033 | 0.0033 | 0.0022 | 0.0022 | 0.0022 |
| 0.15 | 0.0013 | 0.0010 | -0.0023 | -0.0220 | 0.0019 | -0.0086 | 0.0023 | 0.0032 | 0.0032 | 0.0020 | 0.0021 | 0.0022 |
| 0.17 | 0.0007 | 0.0016 | -0.0019 | -0.0201 | 0.0047 | -0.0100 | 0.0023 | 0.0031 | 0.0031 | 0.0019 | 0.0021 | 0.0022 |
| 0.19 | 0.0001 | 0.0025 | -0.0017 | -0.0185 | 0.0071 | -0.0110 | 0.0022 | 0.0030 | 0.0030 | 0.0018 | 0.0021 | 0.0022 |
| 0.21 | 0.0005 | 0.0022 | -0.0021 | -0.0170 | 0.0092 | -0.0118 | 0.0022 | 0.0030 | 0.0030 | 0.0018 | 0.0021 | 0.0022 |
| 0.23 | 0.0006 | 0.0022 | -0.0022 | -0.0157 | 0.0110 | -0.0123 | 0.0021 | 0.0029 | 0.0030 | 0.0017 | 0.0021 | 0.0022 |
| 0.25 | 0.0006 | 0.0022 | -0.0022 | -0.0145 | 0.0125 | -0.0127 | 0.0021 | 0.0029 | 0.0029 | 0.0017 | 0.0021 | 0.0022 |
| 0.27 | 0.0003 | 0.0025 | -0.0021 | -0.0133 | 0.0138 | -0.0130 | 0.0021 | 0.0029 | 0.0029 | 0.0016 | 0.0021 | 0.0022 |
| 0.29 | 0.0008 | 0.0020 | -0.0025 | -0.0123 | 0.0149 | -0.0131 | 0.0021 | 0.0029 | 0.0030 | 0.0016 | 0.0022 | 0.0022 |
| 0.31 | 0.0008 | 0.0020 | -0.0026 | -0.0113 | 0.0159 | -0.0131 | 0.0021 | 0.0029 | 0.0030 | 0.0016 | 0.0022 | 0.0022 |
| 0.33 | 0.0007 | 0.0021 | -0.0024 | -0.0104 | 0.0167 | -0.0130 | 0.0021 | 0.0029 | 0.0029 | 0.0015 | 0.0022 | 0.0022 |
| 0.35 | 0.0004 | 0.0020 | -0.0019 | -0.0095 | 0.0174 | -0.0128 | 0.0021 | 0.0029 | 0.0029 | 0.0015 | 0.0022 | 0.0022 |
| 0.37 | 0.0007 | 0.0017 | -0.0022 | -0.0087 | 0.0180 | -0.0126 | 0.0021 | 0.0029 | 0.0029 | 0.0015 | 0.0023 | 0.0022 |
| 0.39 | 0.0007 | 0.0017 | -0.0024 | -0.0079 | 0.0184 | -0.0122 | 0.0021 | 0.0029 | 0.0029 | 0.0015 | 0.0023 | 0.0022 |
| 0.41 | 0.0006 | 0.0020 | -0.0026 | -0.0071 | 0.0187 | -0.0118 | 0.0021 | 0.0029 | 0.0030 | 0.0015 | 0.0023 | 0.0022 |
| 0.43 | 0.0008 | 0.0020 | -0.0029 | -0.0062 | 0.0189 | -0.0114 | 0.0021 | 0.0029 | 0.0030 | 0.0015 | 0.0023 | 0.0022 |
| 0.45 | 0.0005 | 0.0025 | -0.0029 | -0.0054 | 0.0190 | -0.0109 | 0.0021 | 0.0029 | 0.0030 | 0.0015 | 0.0023 | 0.0022 |
| 0.47 | 0.0004 | 0.0025 | -0.0027 | -0.0046 | 0.0190 | -0.0104 | 0.0021 | 0.0029 | 0.0030 | 0.0015 | 0.0023 | 0.0022 |
| 0.49 | 0.0007 | 0.0021 | -0.0029 | -0.0038 | 0.0189 | -0.0098 | 0.0021 | 0.0029 | 0.0030 | 0.0015 | 0.0023 | 0.0022 |
| 0.51 | 0.0005 | 0.0020 | -0.0026 | -0.0030 | 0.0187 | -0.0092 | 0.0021 | 0.0030 | 0.0030 | 0.0015 | 0.0023 | 0.0022 |
| 0.53 | 0.0006 | 0.0018 | -0.0026 | -0.0021 | 0.0184 | -0.0085 | 0.0022 | 0.0030 | 0.0031 | 0.0015 | 0.0023 | 0.0022 |
| 0.55 | 0.0007 | 0.0017 | -0.0027 | -0.0012 | 0.0180 | -0.0078 | 0.0022 | 0.0030 | 0.0031 | 0.0015 | 0.0023 | 0.0022 |
| 0.57 | 0.0008 | 0.0014 | -0.0025 | -0.0003 | 0.0175 | -0.0071 | 0.0022 | 0.0030 | 0.0031 | 0.0015 | 0.0023 | 0.0022 |
| 0.59 | 0.0003 | 0.0016 | -0.0019 | 0.0007 | 0.0168 | -0.0064 | 0.0022 | 0.0031 | 0.0031 | 0.0015 | 0.0023 | 0.0022 |
| 0.61 | 0.0004 | 0.0014 | -0.0019 | 0.0017 | 0.0161 | -0.0056 | 0.0023 | 0.0031 | 0.0032 | 0.0015 | 0.0023 | 0.0022 |
| 0.63 | -0.0001 | 0.0016 | -0.0014 | 0.0028 | 0.0152 | -0.0047 | 0.0023 | 0.0031 | 0.0032 | 0.0015 | 0.0023 | 0.0022 |
| 0.65 | -0.0002 | 0.0019 | -0.0014 | 0.0040 | 0.0142 | -0.0039 | 0.0022 | 0.0031 | 0.0032 | 0.0015 | 0.0023 | 0.0022 |
| 0.67 | 0.0002 | 0.0015 | -0.0017 | 0.0052 | 0.0130 | -0.0030 | 0.0023 | 0.0031 | 0.0033 | 0.0015 | 0.0023 | 0.0022 |
| 0.69 | 0.0002 | 0.0014 | -0.0017 | 0.0066 | 0.0117 | -0.0021 | 0.0023 | 0.0032 | 0.0034 | 0.0016 | 0.0023 | 0.0022 |
| 0.71 | -0.0002 | 0.0015 | -0.0013 | 0.0080 | 0.0102 | -0.0011 | 0.0023 | 0.0032 | 0.0034 | 0.0016 | 0.0023 | 0.0023 |
| 0.73 | -0.0001 | 0.0009 | -0.0010 | 0.0096 | 0.0085 | -0.0001 | 0.0023 | 0.0032 | 0.0034 | 0.0017 | 0.0023 | 0.0023 |
| 0.75 | 0 | 0.0007 | -0.0012 | 0.0114 | 0.0066 | 0.0010 | 0.0024 | 0.0033 | 0.0034 | 0.0017 | 0.0023 | 0.0023 |
| 0.77 | -0.0002 | 0.0008 | -0.0010 | 0.0134 | 0.0045 | 0.0021 | 0.0024 | 0.0034 | 0.0035 | 0.0018 | 0.0023 | 0.0024 |
| 0.79 | -0.0006 | 0.0013 | -0.0007 | 0.0155 | 0.0021 | 0.0033 | 0.0024 | 0.0034 | 0.0036 | 0.0019 | 0.0023 | 0.0024 |
| 0.81 | -0.0007 | 0.0011 | -0.0004 | 0.0180 | -0.0006 | 0.0046 | 0.0025 | 0.0035 | 0.0037 | 0.0020 | 0.0023 | 0.0024 |
| 0.83 | -0.0009 | 0.0008 | 0 | 0.0208 | -0.0038 | 0.0061 | 0.0025 | 0.0036 | 0.0038 | 0.0021 | 0.0024 | 0.0025 |
| 0.85 | -0.0013 | 0.0011 | 0.0004 | 0.0240 | -0.0074 | 0.0076 | 0.0026 | 0.0037 | 0.0039 | 0.0023 | 0.0025 | 0.0026 |
| 0.87 | -0.0014 | 0.0006 | 0.0008 | 0.0277 | -0.0116 | 0.0093 | 0.0027 | 0.0038 | 0.0041 | 0.0025 | 0.0026 | 0.0027 |
| 0.89 | -0.0012 | 0.0004 | 0.0006 | 0.0323 | -0.0165 | 0.0113 | 0.0028 | 0.0040 | 0.0042 | 0.0028 | 0.0028 | 0.0028 |
| 0.91 | -0.0002 | -0.0018 | 0.0001 | 0.0378 | -0.0226 | 0.0137 | 0.0030 | 0.0043 | 0.0045 | 0.0033 | 0.0032 | 0.0029 |
| 0.93 | 0.0001 | -0.0020 | -0.0004 | 0.0449 | -0.0301 | 0.0168 | 0.0031 | 0.0044 | 0.0047 | 0.0039 | 0.0037 | 0.0031 |
| 0.95 | 0.0001 | -0.0025 | -0.0002 | 0.0548 | -0.0400 | 0.0207 | 0.0035 | 0.0048 | 0.0052 | 0.0050 | 0.0045 | 0.0034 |
| 0.97 | 0.0006 | -0.0043 | -0.0001 | 0.0705 | -0.0547 | 0.0267 | 0.0041 | 0.0059 | 0.0060 | 0.0072 | 0.0062 | 0.0040 |
| 0.99 | -0.0004 | -0.0107 | 0.0048 | 0.1059 | -0.0828 | 0.0387 | 0.0062 | 0.0090 | 0.0087 | 0.0138 | 0.0106 | 0.0053 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 500$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 19: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \sqrt[6]{\tau}$*: bias and MSE of estimators*

| | Bias | | | | | | MSE | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | canonical | | | smoothed | | | canonical | | | smoothed | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ |
| 0.01 | 0.0015 | -0.0063 | 0.0092 | -0.0898 | -0.0376 | 0.0631 | 0.0082 | 0.0113 | 0.0110 | 0.0111 | 0.0057 | 0.0082 |
| 0.03 | 0.0029 | -0.0037 | 0.0013 | -0.0579 | -0.0255 | 0.0305 | 0.0046 | 0.0063 | 0.0061 | 0.0057 | 0.0038 | 0.0041 |
| 0.05 | 0.0036 | -0.0031 | -0.0013 | -0.0441 | -0.0190 | 0.0162 | 0.0036 | 0.0049 | 0.0047 | 0.0040 | 0.0031 | 0.0030 |
| 0.07 | 0.0034 | -0.0023 | -0.0018 | -0.0354 | -0.0144 | 0.0073 | 0.0031 | 0.0042 | 0.0040 | 0.0031 | 0.0027 | 0.0026 |
| 0.09 | 0.0023 | -0.0013 | -0.0009 | -0.0291 | -0.0109 | 0.0009 | 0.0028 | 0.0038 | 0.0036 | 0.0026 | 0.0025 | 0.0024 |
| 0.11 | 0.0026 | -0.0019 | -0.0011 | -0.0243 | -0.0080 | -0.0039 | 0.0027 | 0.0036 | 0.0035 | 0.0023 | 0.0024 | 0.0023 |
| 0.13 | 0.0026 | -0.0020 | -0.0012 | -0.0204 | -0.0056 | -0.0076 | 0.0026 | 0.0035 | 0.0033 | 0.0021 | 0.0022 | 0.0022 |
| 0.15 | 0.0026 | -0.0017 | -0.0015 | -0.0171 | -0.0035 | -0.0105 | 0.0025 | 0.0033 | 0.0032 | 0.0019 | 0.0022 | 0.0022 |
| 0.17 | 0.0026 | -0.0016 | -0.0016 | -0.0143 | -0.0018 | -0.0129 | 0.0024 | 0.0032 | 0.0031 | 0.0018 | 0.0021 | 0.0023 |
| 0.19 | 0.0024 | -0.0015 | -0.0016 | -0.0118 | -0.0003 | -0.0149 | 0.0024 | 0.0031 | 0.0031 | 0.0017 | 0.0021 | 0.0023 |
| 0.21 | 0.0025 | -0.0013 | -0.0022 | -0.0097 | 0.0010 | -0.0165 | 0.0023 | 0.0030 | 0.0030 | 0.0017 | 0.0021 | 0.0023 |
| 0.23 | 0.0025 | -0.0011 | -0.0025 | -0.0078 | 0.0021 | -0.0177 | 0.0023 | 0.0030 | 0.0030 | 0.0016 | 0.0020 | 0.0023 |
| 0.25 | 0.0028 | -0.0013 | -0.0028 | -0.0061 | 0.0031 | -0.0188 | 0.0023 | 0.0030 | 0.0030 | 0.0016 | 0.0020 | 0.0024 |
| 0.27 | 0.0027 | -0.0011 | -0.0030 | -0.0045 | 0.0040 | -0.0195 | 0.0023 | 0.0030 | 0.0030 | 0.0016 | 0.0020 | 0.0024 |
| 0.29 | 0.0025 | -0.0006 | -0.0031 | -0.0031 | 0.0047 | -0.0201 | 0.0023 | 0.0029 | 0.0030 | 0.0015 | 0.0020 | 0.0024 |
| 0.31 | 0.0024 | -0.0005 | -0.0030 | -0.0018 | 0.0053 | -0.0206 | 0.0023 | 0.0029 | 0.0030 | 0.0015 | 0.0020 | 0.0024 |
| 0.33 | 0.0024 | -0.0003 | -0.0032 | -0.0007 | 0.0059 | -0.0208 | 0.0023 | 0.0029 | 0.0030 | 0.0015 | 0.0020 | 0.0024 |
| 0.35 | 0.0025 | -0.0008 | -0.0030 | 0.0004 | 0.0064 | -0.0210 | 0.0023 | 0.0029 | 0.0030 | 0.0015 | 0.0020 | 0.0024 |
| 0.37 | 0.0026 | -0.0009 | -0.0032 | 0.0014 | 0.0067 | -0.0210 | 0.0023 | 0.0029 | 0.0030 | 0.0015 | 0.0020 | 0.0024 |
| 0.39 | 0.0027 | -0.0010 | -0.0033 | 0.0024 | 0.0071 | -0.0209 | 0.0023 | 0.0029 | 0.0030 | 0.0015 | 0.0020 | 0.0024 |
| 0.41 | 0.0024 | -0.0009 | -0.0029 | 0.0032 | 0.0073 | -0.0207 | 0.0023 | 0.0028 | 0.0030 | 0.0015 | 0.0020 | 0.0024 |
| 0.43 | 0.0024 | -0.0013 | -0.0027 | 0.0041 | 0.0075 | -0.0203 | 0.0023 | 0.0028 | 0.0030 | 0.0016 | 0.0020 | 0.0024 |
| 0.45 | 0.0026 | -0.0013 | -0.0030 | 0.0048 | 0.0076 | -0.0199 | 0.0023 | 0.0028 | 0.0030 | 0.0016 | 0.0020 | 0.0024 |
| 0.47 | 0.0026 | -0.0015 | -0.0029 | 0.0056 | 0.0077 | -0.0194 | 0.0023 | 0.0029 | 0.0030 | 0.0016 | 0.0020 | 0.0024 |
| 0.49 | 0.0029 | -0.0016 | -0.0033 | 0.0063 | 0.0077 | -0.0188 | 0.0023 | 0.0029 | 0.0030 | 0.0016 | 0.0020 | 0.0024 |
| 0.51 | 0.0025 | -0.0014 | -0.0030 | 0.0070 | 0.0077 | -0.0182 | 0.0023 | 0.0029 | 0.0030 | 0.0016 | 0.0020 | 0.0024 |
| 0.53 | 0.0022 | -0.0012 | -0.0027 | 0.0077 | 0.0076 | -0.0174 | 0.0023 | 0.0029 | 0.0030 | 0.0016 | 0.0020 | 0.0023 |
| 0.55 | 0.0022 | -0.0015 | -0.0025 | 0.0083 | 0.0074 | -0.0166 | 0.0023 | 0.0029 | 0.0030 | 0.0017 | 0.0021 | 0.0023 |
| 0.57 | 0.0019 | -0.0011 | -0.0025 | 0.0089 | 0.0073 | -0.0156 | 0.0023 | 0.0029 | 0.0030 | 0.0017 | 0.0021 | 0.0023 |
| 0.59 | 0.0019 | -0.0011 | -0.0026 | 0.0095 | 0.0070 | -0.0146 | 0.0024 | 0.0030 | 0.0030 | 0.0017 | 0.0021 | 0.0023 |
| 0.61 | 0.0018 | -0.0012 | -0.0025 | 0.0101 | 0.0067 | -0.0134 | 0.0024 | 0.0030 | 0.0031 | 0.0017 | 0.0021 | 0.0023 |
| 0.63 | 0.0016 | -0.0010 | -0.0023 | 0.0107 | 0.0064 | -0.0122 | 0.0024 | 0.0030 | 0.0031 | 0.0017 | 0.0021 | 0.0023 |
| 0.65 | 0.0016 | -0.0009 | -0.0023 | 0.0112 | 0.0060 | -0.0108 | 0.0024 | 0.0030 | 0.0031 | 0.0018 | 0.0021 | 0.0022 |
| 0.67 | 0.0016 | -0.0009 | -0.0024 | 0.0118 | 0.0055 | -0.0093 | 0.0025 | 0.0030 | 0.0032 | 0.0018 | 0.0021 | 0.0022 |
| 0.69 | 0.0017 | -0.0010 | -0.0024 | 0.0124 | 0.0050 | -0.0077 | 0.0025 | 0.0030 | 0.0032 | 0.0018 | 0.0021 | 0.0022 |
| 0.71 | 0.0018 | -0.0014 | -0.0024 | 0.0130 | 0.0043 | -0.0059 | 0.0025 | 0.0030 | 0.0032 | 0.0019 | 0.0021 | 0.0022 |
| 0.73 | 0.0016 | -0.0010 | -0.0025 | 0.0136 | 0.0036 | -0.0040 | 0.0025 | 0.0031 | 0.0032 | 0.0019 | 0.0021 | 0.0022 |
| 0.75 | 0.0013 | -0.0009 | -0.0021 | 0.0143 | 0.0028 | -0.0019 | 0.0025 | 0.0031 | 0.0032 | 0.0019 | 0.0021 | 0.0022 |
| 0.77 | 0.0009 | -0.0006 | -0.0019 | 0.0150 | 0.0019 | 0.0004 | 0.0026 | 0.0032 | 0.0033 | 0.0020 | 0.0021 | 0.0023 |
| 0.79 | 0.0010 | -0.0010 | -0.0017 | 0.0158 | 0.0009 | 0.0030 | 0.0026 | 0.0032 | 0.0033 | 0.0020 | 0.0022 | 0.0023 |
| 0.81 | 0.0011 | -0.0010 | -0.0017 | 0.0166 | -0.0003 | 0.0058 | 0.0027 | 0.0033 | 0.0034 | 0.0021 | 0.0022 | 0.0023 |
| 0.83 | 0.0008 | -0.0011 | -0.0012 | 0.0176 | -0.0017 | 0.0090 | 0.0028 | 0.0033 | 0.0036 | 0.0021 | 0.0022 | 0.0024 |
| 0.85 | 0.0004 | -0.0012 | -0.0005 | 0.0187 | -0.0032 | 0.0127 | 0.0028 | 0.0034 | 0.0036 | 0.0022 | 0.0023 | 0.0026 |
| 0.87 | 0.0006 | -0.0013 | -0.0008 | 0.0200 | -0.0051 | 0.0169 | 0.0029 | 0.0035 | 0.0037 | 0.0023 | 0.0023 | 0.0027 |
| 0.89 | 0.0006 | -0.0015 | -0.0009 | 0.0215 | -0.0073 | 0.0219 | 0.0031 | 0.0037 | 0.0039 | 0.0024 | 0.0024 | 0.0030 |
| 0.91 | 0.0003 | -0.0013 | -0.0008 | 0.0236 | -0.0100 | 0.0278 | 0.0032 | 0.0038 | 0.0041 | 0.0026 | 0.0025 | 0.0034 |
| 0.93 | -0.0002 | -0.0016 | 0.0001 | 0.0265 | -0.0136 | 0.0352 | 0.0033 | 0.0040 | 0.0042 | 0.0028 | 0.0027 | 0.0039 |
| 0.95 | -0.0011 | -0.0011 | 0.0010 | 0.0308 | -0.0183 | 0.0448 | 0.0036 | 0.0043 | 0.0046 | 0.0031 | 0.0029 | 0.0048 |
| 0.97 | -0.0015 | -0.0023 | 0.0022 | 0.0382 | -0.0252 | 0.0591 | 0.0042 | 0.0051 | 0.0054 | 0.0038 | 0.0034 | 0.0065 |
| 0.99 | -0.0055 | -0.0054 | 0.0095 | 0.0576 | -0.0386 | 0.0869 | 0.0061 | 0.0074 | 0.0079 | 0.0062 | 0.0047 | 0.0111 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 500$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 20: *Model* (2) *defined through Proposition* 2 *when* $\lambda(\tau) = \frac{sin(\tau)+1}{2}$: *bias and* $MSE$ *of estimators*

| | *Bias* | | | | | | *MSE* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | *smoothed* | | | *canonical* | | | *smoothed* | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ |
| 0.01 | -0.0002 | -0.0050 | 0.0105 | -0.0980 | -0.0323 | 0.0689 | 0.0081 | 0.0110 | 0.0112 | 0.0126 | 0.0053 | 0.0089 |
| 0.03 | 0.0009 | -0.0013 | 0.0023 | -0.0688 | -0.0166 | 0.0390 | 0.0046 | 0.0063 | 0.0060 | 0.0070 | 0.0035 | 0.0047 |
| 0.05 | 0.0004 | -0.0003 | 0.0015 | -0.0552 | -0.0094 | 0.0250 | 0.0036 | 0.0050 | 0.0048 | 0.0051 | 0.0029 | 0.0034 |
| 0.07 | 0.0008 | 0 | -0.0001 | -0.0462 | -0.0048 | 0.0158 | 0.0031 | 0.0043 | 0.0042 | 0.0040 | 0.0027 | 0.0028 |
| 0.09 | 0.0008 | 0.0008 | -0.0011 | -0.0394 | -0.0016 | 0.0091 | 0.0028 | 0.0040 | 0.0038 | 0.0033 | 0.0025 | 0.0025 |
| 0.11 | 0.0005 | 0.0008 | -0.0010 | -0.0340 | 0.0008 | 0.0038 | 0.0027 | 0.0038 | 0.0036 | 0.0029 | 0.0024 | 0.0023 |
| 0.13 | 0.0005 | 0.0008 | -0.0010 | -0.0293 | 0.0026 | -0.0004 | 0.0026 | 0.0036 | 0.0034 | 0.0025 | 0.0023 | 0.0022 |
| 0.15 | 0.0006 | 0.0003 | -0.0008 | -0.0254 | 0.0041 | -0.0039 | 0.0025 | 0.0035 | 0.0033 | 0.0023 | 0.0023 | 0.0022 |
| 0.17 | 0.0004 | 0.0007 | -0.0009 | -0.0219 | 0.0052 | -0.0068 | 0.0024 | 0.0034 | 0.0032 | 0.0021 | 0.0022 | 0.0022 |
| 0.19 | 0.0002 | 0.0010 | -0.0007 | -0.0189 | 0.0061 | -0.0093 | 0.0024 | 0.0032 | 0.0031 | 0.0019 | 0.0022 | 0.0022 |
| 0.21 | 0.0003 | 0.0007 | -0.0007 | -0.0161 | 0.0068 | -0.0113 | 0.0023 | 0.0032 | 0.0031 | 0.0018 | 0.0022 | 0.0022 |
| 0.23 | 0.0002 | 0.0005 | -0.0005 | -0.0136 | 0.0074 | -0.0131 | 0.0022 | 0.0031 | 0.0030 | 0.0017 | 0.0022 | 0.0022 |
| 0.25 | 0.0003 | 0.0003 | -0.0005 | -0.0113 | 0.0078 | -0.0145 | 0.0022 | 0.0030 | 0.0030 | 0.0017 | 0.0021 | 0.0022 |
| 0.27 | 0.0009 | 0 | -0.0012 | -0.0092 | 0.0082 | -0.0157 | 0.0022 | 0.0030 | 0.0031 | 0.0016 | 0.0021 | 0.0023 |
| 0.29 | 0.0009 | 0.0002 | -0.0014 | -0.0073 | 0.0084 | -0.0168 | 0.0022 | 0.0030 | 0.0030 | 0.0016 | 0.0021 | 0.0023 |
| 0.31 | 0.0009 | 0.0002 | -0.0013 | -0.0056 | 0.0086 | -0.0176 | 0.0022 | 0.0030 | 0.0030 | 0.0015 | 0.0021 | 0.0023 |
| 0.33 | 0.0007 | 0.0004 | -0.0013 | -0.0039 | 0.0087 | -0.0183 | 0.0022 | 0.0030 | 0.0030 | 0.0015 | 0.0021 | 0.0023 |
| 0.35 | 0.0009 | 0.0003 | -0.0016 | -0.0024 | 0.0088 | -0.0188 | 0.0022 | 0.0030 | 0.0029 | 0.0015 | 0.0021 | 0.0024 |
| 0.37 | 0.0006 | 0.0004 | -0.0011 | -0.0010 | 0.0088 | -0.0191 | 0.0022 | 0.0030 | 0.0030 | 0.0015 | 0.0021 | 0.0024 |
| 0.39 | 0.0003 | 0.0007 | -0.0008 | 0.0004 | 0.0088 | -0.0194 | 0.0022 | 0.0030 | 0.0029 | 0.0015 | 0.0021 | 0.0024 |
| 0.41 | 0 | 0.0011 | -0.0007 | 0.0016 | 0.0087 | -0.0195 | 0.0022 | 0.0030 | 0.0029 | 0.0015 | 0.0021 | 0.0024 |
| 0.43 | 0.0001 | 0.0009 | -0.0007 | 0.0028 | 0.0086 | -0.0195 | 0.0022 | 0.0030 | 0.0030 | 0.0015 | 0.0021 | 0.0024 |
| 0.45 | 0.0002 | 0.0009 | -0.0009 | 0.0040 | 0.0084 | -0.0194 | 0.0022 | 0.0030 | 0.0030 | 0.0015 | 0.0021 | 0.0024 |
| 0.47 | 0.0001 | 0.0010 | -0.0009 | 0.0050 | 0.0082 | -0.0192 | 0.0022 | 0.0030 | 0.0030 | 0.0015 | 0.0021 | 0.0024 |
| 0.49 | 0.0003 | 0.0008 | -0.0013 | 0.0060 | 0.0080 | -0.0188 | 0.0022 | 0.0030 | 0.0030 | 0.0016 | 0.0021 | 0.0024 |
| 0.51 | 0.0001 | 0.0010 | -0.0011 | 0.0070 | 0.0077 | -0.0184 | 0.0022 | 0.0030 | 0.0030 | 0.0016 | 0.0021 | 0.0024 |
| 0.53 | 0.0005 | 0.0007 | -0.0014 | 0.0079 | 0.0075 | -0.0179 | 0.0022 | 0.0030 | 0.0030 | 0.0016 | 0.0021 | 0.0024 |
| 0.55 | 0.0006 | 0.0005 | -0.0015 | 0.0088 | 0.0071 | -0.0173 | 0.0023 | 0.0030 | 0.0031 | 0.0016 | 0.0021 | 0.0024 |
| 0.57 | 0.0006 | 0.0002 | -0.0013 | 0.0097 | 0.0068 | -0.0166 | 0.0023 | 0.0030 | 0.0031 | 0.0017 | 0.0021 | 0.0024 |
| 0.59 | 0.0006 | 0.0004 | -0.0015 | 0.0105 | 0.0064 | -0.0157 | 0.0023 | 0.0030 | 0.0031 | 0.0017 | 0.0021 | 0.0023 |
| 0.61 | 0.0005 | 0.0007 | -0.0016 | 0.0113 | 0.0059 | -0.0148 | 0.0024 | 0.0031 | 0.0032 | 0.0017 | 0.0021 | 0.0023 |
| 0.63 | 0.0006 | 0.0004 | -0.0015 | 0.0121 | 0.0054 | -0.0137 | 0.0024 | 0.0031 | 0.0032 | 0.0018 | 0.0021 | 0.0023 |
| 0.65 | 0.0003 | 0.0008 | -0.0013 | 0.0128 | 0.0049 | -0.0125 | 0.0024 | 0.0031 | 0.0032 | 0.0018 | 0.0021 | 0.0023 |
| 0.67 | 0.0002 | 0.0007 | -0.0010 | 0.0136 | 0.0043 | -0.0112 | 0.0025 | 0.0031 | 0.0032 | 0.0018 | 0.0021 | 0.0023 |
| 0.69 | 0.0001 | 0.0005 | -0.0007 | 0.0143 | 0.0037 | -0.0097 | 0.0025 | 0.0032 | 0.0032 | 0.0019 | 0.0021 | 0.0023 |
| 0.71 | 0.0002 | 0.0001 | -0.0005 | 0.0151 | 0.0030 | -0.0081 | 0.0026 | 0.0032 | 0.0032 | 0.0019 | 0.0021 | 0.0023 |
| 0.73 | 0.0007 | -0.0003 | -0.0011 | 0.0158 | 0.0022 | -0.0062 | 0.0026 | 0.0032 | 0.0033 | 0.0020 | 0.0021 | 0.0023 |
| 0.75 | 0.0007 | -0.0002 | -0.0012 | 0.0166 | 0.0014 | -0.0042 | 0.0026 | 0.0033 | 0.0033 | 0.0020 | 0.0022 | 0.0023 |
| 0.77 | 0.0006 | 0.0001 | -0.0017 | 0.0175 | 0.0004 | -0.0020 | 0.0026 | 0.0033 | 0.0033 | 0.0020 | 0.0022 | 0.0023 |
| 0.79 | 0.0004 | 0 | -0.0014 | 0.0183 | -0.0007 | 0.0005 | 0.0027 | 0.0033 | 0.0034 | 0.0021 | 0.0022 | 0.0023 |
| 0.81 | 0.0003 | -0.0001 | -0.0013 | 0.0193 | -0.0019 | 0.0033 | 0.0027 | 0.0033 | 0.0034 | 0.0022 | 0.0022 | 0.0024 |
| 0.83 | 0.0002 | 0.0001 | -0.0014 | 0.0204 | -0.0033 | 0.0064 | 0.0028 | 0.0034 | 0.0035 | 0.0022 | 0.0023 | 0.0024 |
| 0.85 | 0 | 0.0003 | -0.0012 | 0.0216 | -0.0049 | 0.0100 | 0.0029 | 0.0034 | 0.0036 | 0.0023 | 0.0023 | 0.0025 |
| 0.87 | -0.0002 | 0.0003 | -0.0008 | 0.0229 | -0.0068 | 0.0142 | 0.0029 | 0.0036 | 0.0037 | 0.0024 | 0.0023 | 0.0027 |
| 0.89 | -0.0003 | 0.0003 | -0.0006 | 0.0246 | -0.0090 | 0.0191 | 0.0031 | 0.0037 | 0.0039 | 0.0025 | 0.0024 | 0.0029 |
| 0.91 | -0.0008 | 0.0007 | -0.0003 | 0.0268 | -0.0118 | 0.0250 | 0.0032 | 0.0038 | 0.0041 | 0.0027 | 0.0025 | 0.0033 |
| 0.93 | -0.0013 | 0.0007 | 0.0001 | 0.0298 | -0.0154 | 0.0324 | 0.0034 | 0.0040 | 0.0044 | 0.0030 | 0.0027 | 0.0038 |
| 0.95 | -0.0010 | -0.0003 | 0.0003 | 0.0343 | -0.0203 | 0.0421 | 0.0037 | 0.0043 | 0.0048 | 0.0034 | 0.0030 | 0.0046 |
| 0.97 | -0.0020 | -0.0012 | 0.0022 | 0.0420 | -0.0274 | 0.0564 | 0.0044 | 0.0050 | 0.0057 | 0.0042 | 0.0035 | 0.0063 |
| 0.99 | -0.0063 | -0.0043 | 0.0098 | 0.0622 | -0.0413 | 0.0838 | 0.0063 | 0.0072 | 0.0084 | 0.0067 | 0.0048 | 0.0106 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 500$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 21: *Model* (2) *defined through Proposition* 2 *when* $\lambda(\tau) = \sqrt{\tau}$: *bias and* $MSE$ *of estimators*

| | *Bias* | | | | | | *MSE* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | $\widehat{\beta}_n^{2\zeta^*}$ | | | *canonical* | | | $\widehat{\beta}_n^{2\zeta^*}$ | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ |
| 0.01 | -0.0010 | -0.0026 | 0.0067 | -0.0925 | -0.0076 | 0.0716 | 0.0034 | 0.0055 | 0.0052 | 0.0102 | 0.0027 | 0.0077 |
| 0.03 | 0.0016 | -0.0024 | 0.0013 | -0.0580 | -0.0062 | 0.0370 | 0.0019 | 0.0031 | 0.0029 | 0.0045 | 0.0019 | 0.0031 |
| 0.05 | 0.0024 | -0.0019 | -0.0013 | -0.0429 | -0.0052 | 0.0218 | 0.0015 | 0.0025 | 0.0022 | 0.0028 | 0.0016 | 0.0019 |
| 0.07 | 0.0026 | -0.0023 | -0.0015 | -0.0333 | -0.0044 | 0.0120 | 0.0014 | 0.0022 | 0.0019 | 0.0020 | 0.0014 | 0.0014 |
| 0.09 | 0.0022 | -0.0016 | -0.0015 | -0.0263 | -0.0038 | 0.0050 | 0.0013 | 0.0020 | 0.0018 | 0.0015 | 0.0013 | 0.0012 |
| 0.11 | 0.0019 | -0.0014 | -0.0013 | -0.0208 | -0.0033 | -0.0005 | 0.0012 | 0.0018 | 0.0017 | 0.0012 | 0.0012 | 0.0011 |
| 0.13 | 0.0022 | -0.0014 | -0.0018 | -0.0163 | -0.0029 | -0.0048 | 0.0012 | 0.0017 | 0.0016 | 0.0010 | 0.0012 | 0.0011 |
| 0.15 | 0.0020 | -0.0011 | -0.0018 | -0.0125 | -0.0026 | -0.0083 | 0.0011 | 0.0016 | 0.0015 | 0.0009 | 0.0011 | 0.0011 |
| 0.17 | 0.0023 | -0.0013 | -0.0020 | -0.0094 | -0.0023 | -0.0112 | 0.0011 | 0.0016 | 0.0015 | 0.0008 | 0.0011 | 0.0011 |
| 0.19 | 0.0023 | -0.0010 | -0.0024 | -0.0066 | -0.0021 | -0.0135 | 0.0011 | 0.0015 | 0.0014 | 0.0008 | 0.0011 | 0.0012 |
| 0.21 | 0.0025 | -0.0012 | -0.0026 | -0.0042 | -0.0018 | -0.0155 | 0.0011 | 0.0015 | 0.0014 | 0.0007 | 0.0010 | 0.0012 |
| 0.23 | 0.0025 | -0.0015 | -0.0024 | -0.0021 | -0.0017 | -0.0172 | 0.0010 | 0.0015 | 0.0014 | 0.0007 | 0.0010 | 0.0012 |
| 0.25 | 0.0025 | -0.0015 | -0.0024 | -0.0002 | -0.0015 | -0.0185 | 0.0010 | 0.0014 | 0.0014 | 0.0007 | 0.0010 | 0.0013 |
| 0.27 | 0.0023 | -0.0013 | -0.0023 | 0.0014 | -0.0013 | -0.0197 | 0.0010 | 0.0014 | 0.0013 | 0.0007 | 0.0010 | 0.0013 |
| 0.29 | 0.0022 | -0.0013 | -0.0021 | 0.0029 | -0.0012 | -0.0206 | 0.0010 | 0.0014 | 0.0013 | 0.0007 | 0.0010 | 0.0013 |
| 0.31 | 0.0023 | -0.0014 | -0.0021 | 0.0043 | -0.0011 | -0.0214 | 0.0010 | 0.0014 | 0.0013 | 0.0007 | 0.0010 | 0.0014 |
| 0.33 | 0.0021 | -0.0014 | -0.0021 | 0.0055 | -0.0010 | -0.0220 | 0.0010 | 0.0014 | 0.0013 | 0.0007 | 0.0009 | 0.0014 |
| 0.35 | 0.0022 | -0.0014 | -0.0021 | 0.0066 | -0.0008 | -0.0224 | 0.0010 | 0.0013 | 0.0013 | 0.0007 | 0.0009 | 0.0014 |
| 0.37 | 0.0020 | -0.0013 | -0.0020 | 0.0076 | -0.0007 | -0.0228 | 0.0010 | 0.0013 | 0.0013 | 0.0008 | 0.0009 | 0.0014 |
| 0.39 | 0.0021 | -0.0014 | -0.0021 | 0.0085 | -0.0006 | -0.0229 | 0.0010 | 0.0013 | 0.0013 | 0.0008 | 0.0009 | 0.0014 |
| 0.41 | 0.0021 | -0.0015 | -0.0019 | 0.0093 | -0.0005 | -0.0230 | 0.0010 | 0.0013 | 0.0013 | 0.0008 | 0.0009 | 0.0014 |
| 0.43 | 0.0019 | -0.0014 | -0.0018 | 0.0100 | -0.0004 | -0.0230 | 0.0010 | 0.0013 | 0.0013 | 0.0008 | 0.0009 | 0.0014 |
| 0.45 | 0.0020 | -0.0015 | -0.0019 | 0.0107 | -0.0003 | -0.0228 | 0.0010 | 0.0013 | 0.0013 | 0.0008 | 0.0009 | 0.0014 |
| 0.47 | 0.0019 | -0.0013 | -0.0020 | 0.0112 | -0.0002 | -0.0225 | 0.0010 | 0.0013 | 0.0013 | 0.0008 | 0.0009 | 0.0014 |
| 0.49 | 0.0021 | -0.0014 | -0.0021 | 0.0118 | -0.0001 | -0.0222 | 0.0010 | 0.0012 | 0.0013 | 0.0008 | 0.0009 | 0.0014 |
| 0.51 | 0.0019 | -0.0013 | -0.0019 | 0.0122 | 0 | -0.0217 | 0.0010 | 0.0012 | 0.0013 | 0.0009 | 0.0009 | 0.0014 |
| 0.53 | 0.0020 | -0.0015 | -0.0019 | 0.0126 | 0.0001 | -0.0211 | 0.0010 | 0.0012 | 0.0013 | 0.0009 | 0.0009 | 0.0014 |
| 0.55 | 0.0021 | -0.0014 | -0.0021 | 0.0129 | 0.0002 | -0.0204 | 0.0010 | 0.0012 | 0.0013 | 0.0009 | 0.0008 | 0.0013 |
| 0.57 | 0.0020 | -0.0012 | -0.0022 | 0.0132 | 0.0003 | -0.0196 | 0.0010 | 0.0012 | 0.0013 | 0.0009 | 0.0008 | 0.0013 |
| 0.59 | 0.0020 | -0.0011 | -0.0023 | 0.0135 | 0.0004 | -0.0187 | 0.0010 | 0.0012 | 0.0013 | 0.0009 | 0.0008 | 0.0013 |
| 0.61 | 0.0019 | -0.0010 | -0.0022 | 0.0137 | 0.0005 | -0.0176 | 0.0010 | 0.0012 | 0.0013 | 0.0009 | 0.0008 | 0.0013 |
| 0.63 | 0.0019 | -0.0011 | -0.0022 | 0.0138 | 0.0006 | -0.0164 | 0.0010 | 0.0012 | 0.0013 | 0.0009 | 0.0008 | 0.0012 |
| 0.65 | 0.0018 | -0.0011 | -0.0020 | 0.0138 | 0.0007 | -0.0151 | 0.0010 | 0.0012 | 0.0013 | 0.0009 | 0.0008 | 0.0012 |
| 0.67 | 0.0018 | -0.0012 | -0.0019 | 0.0138 | 0.0008 | -0.0135 | 0.0011 | 0.0012 | 0.0014 | 0.0009 | 0.0008 | 0.0011 |
| 0.69 | 0.0018 | -0.0011 | -0.0020 | 0.0138 | 0.0010 | -0.0118 | 0.0011 | 0.0011 | 0.0014 | 0.0010 | 0.0008 | 0.0011 |
| 0.71 | 0.0018 | -0.0011 | -0.0020 | 0.0136 | 0.0011 | -0.0099 | 0.0011 | 0.0011 | 0.0014 | 0.0010 | 0.0008 | 0.0011 |
| 0.73 | 0.0019 | -0.0010 | -0.0022 | 0.0135 | 0.0011 | -0.0078 | 0.0011 | 0.0012 | 0.0013 | 0.0010 | 0.0008 | 0.0010 |
| 0.75 | 0.0016 | -0.0008 | -0.0020 | 0.0132 | 0.0012 | -0.0054 | 0.0011 | 0.0012 | 0.0013 | 0.0010 | 0.0008 | 0.0010 |
| 0.77 | 0.0014 | -0.0006 | -0.0019 | 0.0129 | 0.0013 | -0.0027 | 0.0011 | 0.0012 | 0.0014 | 0.0010 | 0.0008 | 0.0010 |
| 0.79 | 0.0014 | -0.0006 | -0.0019 | 0.0126 | 0.0013 | 0.0003 | 0.0011 | 0.0012 | 0.0014 | 0.0010 | 0.0008 | 0.0010 |
| 0.81 | 0.0013 | -0.0003 | -0.0021 | 0.0122 | 0.0013 | 0.0037 | 0.0011 | 0.0012 | 0.0014 | 0.0010 | 0.0008 | 0.0010 |
| 0.83 | 0.0008 | 0.0003 | -0.0019 | 0.0117 | 0.0013 | 0.0076 | 0.0011 | 0.0012 | 0.0014 | 0.0010 | 0.0008 | 0.0011 |
| 0.85 | 0.0006 | 0.0002 | -0.0015 | 0.0111 | 0.0012 | 0.0121 | 0.0011 | 0.0012 | 0.0014 | 0.0010 | 0.0008 | 0.0012 |
| 0.87 | 0.0006 | 0.0001 | -0.0014 | 0.0105 | 0.0010 | 0.0173 | 0.0012 | 0.0012 | 0.0015 | 0.0010 | 0.0009 | 0.0014 |
| 0.89 | 0.0002 | 0.0004 | -0.0011 | 0.0099 | 0.0007 | 0.0236 | 0.0012 | 0.0012 | 0.0015 | 0.0010 | 0.0009 | 0.0017 |
| 0.91 | -0.0002 | 0.0008 | -0.0009 | 0.0091 | 0.0003 | 0.0313 | 0.0013 | 0.0013 | 0.0016 | 0.0010 | 0.0009 | 0.0021 |
| 0.93 | -0.0005 | 0.0007 | -0.0004 | 0.0085 | -0.0004 | 0.0408 | 0.0014 | 0.0013 | 0.0017 | 0.0011 | 0.0009 | 0.0029 |
| 0.95 | -0.0008 | 0.0004 | 0.0002 | 0.0084 | -0.0016 | 0.0534 | 0.0015 | 0.0014 | 0.0018 | 0.0011 | 0.0010 | 0.0041 |
| 0.97 | -0.0017 | 0.0003 | 0.0014 | 0.0094 | -0.0035 | 0.0717 | 0.0017 | 0.0016 | 0.0021 | 0.0013 | 0.0010 | 0.0065 |
| 0.99 | -0.0056 | 0.0001 | 0.0069 | 0.0174 | -0.0079 | 0.1061 | 0.0025 | 0.0024 | 0.0030 | 0.0017 | 0.0013 | 0.0129 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 1,000$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 22: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \frac{1}{1+e^{-\tau}}$*: bias and MSE of estimators*

| | *Bias* | | | | | | *MSE* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | *smoothed* | | | *canonical* | | | *smoothed* | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ |
| 0.01 | -0.0003 | -0.0019 | 0.0049 | -0.0726 | -0.0321 | 0.0524 | 0.0038 | 0.0053 | 0.0049 | 0.0068 | 0.0033 | 0.0049 |
| 0.03 | 0 | 0.0001 | 0.0009 | -0.0513 | -0.0165 | 0.0291 | 0.0021 | 0.0030 | 0.0029 | 0.0038 | 0.0019 | 0.0025 |
| 0.05 | 0.0001 | 0.0002 | 0.0007 | -0.0412 | -0.0095 | 0.0184 | 0.0018 | 0.0025 | 0.0024 | 0.0027 | 0.0015 | 0.0018 |
| 0.07 | -0.0001 | 0.0006 | 0.0003 | -0.0346 | -0.0050 | 0.0113 | 0.0015 | 0.0022 | 0.0020 | 0.0022 | 0.0014 | 0.0014 |
| 0.09 | 0.0004 | 0.0005 | -0.0004 | -0.0297 | -0.0018 | 0.0062 | 0.0014 | 0.0020 | 0.0019 | 0.0018 | 0.0013 | 0.0013 |
| 0.11 | 0.0006 | 0.0004 | -0.0007 | -0.0258 | 0.0006 | 0.0022 | 0.0013 | 0.0018 | 0.0018 | 0.0015 | 0.0012 | 0.0012 |
| 0.13 | 0.0004 | 0.0006 | -0.0007 | -0.0224 | 0.0024 | -0.0010 | 0.0013 | 0.0018 | 0.0017 | 0.0014 | 0.0012 | 0.0012 |
| 0.15 | 0.0002 | 0.0007 | -0.0006 | -0.0196 | 0.0039 | -0.0036 | 0.0012 | 0.0018 | 0.0016 | 0.0012 | 0.0012 | 0.0012 |
| 0.17 | 0.0004 | 0.0009 | -0.0011 | -0.0171 | 0.0051 | -0.0057 | 0.0012 | 0.0017 | 0.0016 | 0.0011 | 0.0012 | 0.0012 |
| 0.19 | 0.0004 | 0.0007 | -0.0009 | -0.0149 | 0.0060 | -0.0075 | 0.0012 | 0.0016 | 0.0016 | 0.0010 | 0.0012 | 0.0012 |
| 0.21 | 0.0004 | 0.0006 | -0.0010 | -0.0129 | 0.0068 | -0.0089 | 0.0012 | 0.0016 | 0.0016 | 0.0010 | 0.0012 | 0.0012 |
| 0.23 | 0.0004 | 0.0007 | -0.0009 | -0.0112 | 0.0074 | -0.0102 | 0.0012 | 0.0016 | 0.0016 | 0.0009 | 0.0012 | 0.0012 |
| 0.25 | 0.0003 | 0.0007 | -0.0007 | -0.0095 | 0.0079 | -0.0112 | 0.0012 | 0.0016 | 0.0015 | 0.0009 | 0.0011 | 0.0012 |
| 0.27 | -0.0001 | 0.0010 | -0.0005 | -0.0080 | 0.0083 | -0.0120 | 0.0011 | 0.0016 | 0.0015 | 0.0009 | 0.0011 | 0.0012 |
| 0.29 | 0 | 0.0010 | -0.0005 | -0.0066 | 0.0085 | -0.0127 | 0.0011 | 0.0016 | 0.0015 | 0.0008 | 0.0011 | 0.0012 |
| 0.31 | 0.0003 | 0.0008 | -0.0010 | -0.0053 | 0.0087 | -0.0133 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0012 |
| 0.33 | 0.0006 | 0.0005 | -0.0012 | -0.0041 | 0.0089 | -0.0137 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0013 |
| 0.35 | 0.0007 | 0.0004 | -0.0013 | -0.0030 | 0.0090 | -0.0141 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0013 |
| 0.37 | 0.0006 | 0.0005 | -0.0014 | -0.0020 | 0.0090 | -0.0143 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0013 |
| 0.39 | 0.0005 | 0.0007 | -0.0015 | -0.0010 | 0.0090 | -0.0144 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0013 |
| 0.41 | 0.0006 | 0.0008 | -0.0017 | 0 | 0.0089 | -0.0145 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0013 |
| 0.43 | 0.0007 | 0.0006 | -0.0017 | 0.0009 | 0.0088 | -0.0145 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0013 |
| 0.45 | 0.0006 | 0.0006 | -0.0016 | 0.0018 | 0.0086 | -0.0144 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0013 |
| 0.47 | 0.0007 | 0.0005 | -0.0016 | 0.0026 | 0.0085 | -0.0142 | 0.0011 | 0.0015 | 0.0016 | 0.0008 | 0.0011 | 0.0013 |
| 0.49 | 0.0006 | 0.0005 | -0.0014 | 0.0034 | 0.0082 | -0.0140 | 0.0011 | 0.0015 | 0.0016 | 0.0008 | 0.0011 | 0.0013 |
| 0.51 | 0.0005 | 0.0006 | -0.0015 | 0.0042 | 0.0080 | -0.0137 | 0.0012 | 0.0015 | 0.0016 | 0.0008 | 0.0011 | 0.0013 |
| 0.53 | 0.0005 | 0.0008 | -0.0016 | 0.0049 | 0.0077 | -0.0133 | 0.0012 | 0.0015 | 0.0016 | 0.0008 | 0.0011 | 0.0013 |
| 0.55 | 0.0004 | 0.0007 | -0.0014 | 0.0056 | 0.0074 | -0.0129 | 0.0012 | 0.0015 | 0.0016 | 0.0008 | 0.0011 | 0.0013 |
| 0.57 | 0.0006 | 0.0005 | -0.0016 | 0.0063 | 0.0070 | -0.0124 | 0.0012 | 0.0015 | 0.0016 | 0.0009 | 0.0011 | 0.0013 |
| 0.59 | 0.0005 | 0.0007 | -0.0016 | 0.0071 | 0.0066 | -0.0118 | 0.0012 | 0.0015 | 0.0016 | 0.0009 | 0.0011 | 0.0013 |
| 0.61 | 0.0004 | 0.0005 | -0.0013 | 0.0077 | 0.0062 | -0.0112 | 0.0012 | 0.0015 | 0.0016 | 0.0009 | 0.0011 | 0.0013 |
| 0.63 | 0.0004 | 0.0004 | -0.0014 | 0.0084 | 0.0057 | -0.0105 | 0.0012 | 0.0015 | 0.0016 | 0.0009 | 0.0011 | 0.0013 |
| 0.65 | 0.0001 | 0.0008 | -0.0014 | 0.0091 | 0.0052 | -0.0096 | 0.0012 | 0.0015 | 0.0016 | 0.0009 | 0.0011 | 0.0012 |
| 0.67 | 0.0003 | 0.0008 | -0.0015 | 0.0098 | 0.0046 | -0.0087 | 0.0012 | 0.0015 | 0.0017 | 0.0009 | 0.0011 | 0.0012 |
| 0.69 | 0.0004 | 0.0006 | -0.0016 | 0.0105 | 0.0040 | -0.0077 | 0.0012 | 0.0015 | 0.0017 | 0.0010 | 0.0011 | 0.0012 |
| 0.71 | 0.0005 | 0.0005 | -0.0015 | 0.0112 | 0.0033 | -0.0066 | 0.0012 | 0.0016 | 0.0017 | 0.0010 | 0.0011 | 0.0012 |
| 0.73 | 0.0004 | 0.0006 | -0.0015 | 0.0119 | 0.0025 | -0.0053 | 0.0013 | 0.0016 | 0.0017 | 0.0010 | 0.0011 | 0.0012 |
| 0.75 | 0.0001 | 0.0005 | -0.0009 | 0.0127 | 0.0017 | -0.0040 | 0.0013 | 0.0016 | 0.0017 | 0.0010 | 0.0011 | 0.0012 |
| 0.77 | 0.0001 | 0.0004 | -0.0009 | 0.0135 | 0.0007 | -0.0024 | 0.0013 | 0.0016 | 0.0017 | 0.0011 | 0.0011 | 0.0012 |
| 0.79 | -0.0002 | 0.0007 | -0.0007 | 0.0144 | -0.0003 | -0.0007 | 0.0013 | 0.0017 | 0.0017 | 0.0011 | 0.0011 | 0.0012 |
| 0.81 | -0.0002 | 0.0008 | -0.0010 | 0.0153 | -0.0016 | 0.0013 | 0.0013 | 0.0017 | 0.0018 | 0.0012 | 0.0012 | 0.0013 |
| 0.83 | -0.0003 | 0.0009 | -0.0011 | 0.0164 | -0.0030 | 0.0035 | 0.0014 | 0.0017 | 0.0018 | 0.0012 | 0.0012 | 0.0013 |
| 0.85 | -0.0003 | 0.0008 | -0.0010 | 0.0176 | -0.0046 | 0.0061 | 0.0014 | 0.0018 | 0.0019 | 0.0013 | 0.0012 | 0.0013 |
| 0.87 | -0.0001 | 0.0003 | -0.0009 | 0.0190 | -0.0064 | 0.0090 | 0.0015 | 0.0018 | 0.0019 | 0.0013 | 0.0013 | 0.0014 |
| 0.89 | 0 | 0.0001 | -0.0010 | 0.0206 | -0.0086 | 0.0126 | 0.0015 | 0.0019 | 0.0020 | 0.0014 | 0.0013 | 0.0015 |
| 0.91 | -0.0003 | 0 | -0.0006 | 0.0227 | -0.0114 | 0.0170 | 0.0016 | 0.0020 | 0.0021 | 0.0016 | 0.0014 | 0.0017 |
| 0.93 | 0.0003 | -0.0006 | -0.0009 | 0.0256 | -0.0150 | 0.0225 | 0.0018 | 0.0021 | 0.0023 | 0.0017 | 0.0015 | 0.0019 |
| 0.95 | -0.0001 | -0.0012 | 0.0001 | 0.0298 | -0.0199 | 0.0298 | 0.0018 | 0.0022 | 0.0024 | 0.0020 | 0.0018 | 0.0024 |
| 0.97 | -0.0008 | -0.0010 | 0.0008 | 0.0368 | -0.0269 | 0.0407 | 0.0021 | 0.0025 | 0.0028 | 0.0026 | 0.0022 | 0.0033 |
| 0.99 | -0.0024 | -0.0029 | 0.0043 | 0.0551 | -0.0409 | 0.0621 | 0.0030 | 0.0035 | 0.0041 | 0.0045 | 0.0034 | 0.0058 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 1,000$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 23: *Model (2) defined through Proposition 2 when $\lambda(\tau) = cos(\tau)$: bias and $MSE$ of estimators*

| | *Bias* | | | | | | *MSE* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | *smoothed* | | | *canonical* | | | *smoothed* | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ |
| 0.01 | 0.0024 | -0.0037 | 0.0017 | -0.0502 | -0.0553 | 0.0301 | 0.0036 | 0.0051 | 0.0045 | 0.0041 | 0.0052 | 0.0029 |
| 0.03 | 0.0011 | -0.0005 | -0.0007 | -0.0361 | -0.0323 | 0.0143 | 0.0020 | 0.0028 | 0.0027 | 0.0024 | 0.0026 | 0.0017 |
| 0.05 | 0.0014 | -0.0008 | -0.0009 | -0.0298 | -0.0216 | 0.0071 | 0.0016 | 0.0022 | 0.0022 | 0.0019 | 0.0018 | 0.0014 |
| 0.07 | 0.0012 | -0.0007 | -0.0009 | -0.0257 | -0.0145 | 0.0024 | 0.0014 | 0.0019 | 0.0019 | 0.0016 | 0.0015 | 0.0012 |
| 0.09 | 0.0006 | -0.0001 | -0.0005 | -0.0227 | -0.0092 | -0.0008 | 0.0013 | 0.0018 | 0.0017 | 0.0014 | 0.0013 | 0.0011 |
| 0.11 | 0.0004 | -0.0001 | -0.0002 | -0.0205 | -0.0050 | -0.0031 | 0.0012 | 0.0017 | 0.0016 | 0.0012 | 0.0012 | 0.0011 |
| 0.13 | 0.0003 | 0.0002 | -0.0004 | -0.0187 | -0.0016 | -0.0049 | 0.0012 | 0.0016 | 0.0016 | 0.0011 | 0.0011 | 0.0011 |
| 0.15 | 0 | 0.0002 | 0.0001 | -0.0171 | 0.0012 | -0.0063 | 0.0011 | 0.0016 | 0.0015 | 0.0011 | 0.0011 | 0.0011 |
| 0.17 | -0.0001 | 0.0007 | -0.0003 | -0.0157 | 0.0035 | -0.0073 | 0.0011 | 0.0015 | 0.0015 | 0.0010 | 0.0011 | 0.0011 |
| 0.19 | 0 | 0.0007 | -0.0007 | -0.0145 | 0.0055 | -0.0081 | 0.0011 | 0.0015 | 0.0015 | 0.0010 | 0.0011 | 0.0011 |
| 0.21 | 0.0001 | 0.0006 | -0.0009 | -0.0135 | 0.0072 | -0.0087 | 0.0011 | 0.0015 | 0.0015 | 0.0009 | 0.0011 | 0.0011 |
| 0.23 | 0.0002 | 0.0006 | -0.0009 | -0.0125 | 0.0086 | -0.0092 | 0.0011 | 0.0015 | 0.0015 | 0.0009 | 0.0011 | 0.0011 |
| 0.25 | 0.0001 | 0.0006 | -0.0010 | -0.0116 | 0.0098 | -0.0095 | 0.0011 | 0.0014 | 0.0015 | 0.0009 | 0.0011 | 0.0011 |
| 0.27 | 0.0001 | 0.0007 | -0.0010 | -0.0108 | 0.0109 | -0.0096 | 0.0011 | 0.0015 | 0.0015 | 0.0009 | 0.0012 | 0.0011 |
| 0.29 | 0.0001 | 0.0008 | -0.0010 | -0.0100 | 0.0118 | -0.0097 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0012 | 0.0011 |
| 0.31 | 0 | 0.0008 | -0.0008 | -0.0093 | 0.0126 | -0.0097 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0012 | 0.0011 |
| 0.33 | -0.0002 | 0.0010 | -0.0006 | -0.0086 | 0.0133 | -0.0096 | 0.0011 | 0.0014 | 0.0015 | 0.0008 | 0.0012 | 0.0011 |
| 0.35 | -0.0003 | 0.0012 | -0.0007 | -0.0079 | 0.0138 | -0.0094 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0012 | 0.0011 |
| 0.37 | -0.0002 | 0.0013 | -0.0008 | -0.0073 | 0.0142 | -0.0092 | 0.0011 | 0.0014 | 0.0015 | 0.0008 | 0.0012 | 0.0011 |
| 0.39 | -0.0003 | 0.0012 | -0.0006 | -0.0067 | 0.0146 | -0.0090 | 0.0011 | 0.0014 | 0.0015 | 0.0008 | 0.0012 | 0.0011 |
| 0.41 | -0.0003 | 0.0012 | -0.0007 | -0.0060 | 0.0148 | -0.0087 | 0.0011 | 0.0014 | 0.0015 | 0.0008 | 0.0013 | 0.0011 |
| 0.43 | -0.0004 | 0.0013 | -0.0005 | -0.0054 | 0.0150 | -0.0083 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0013 | 0.0011 |
| 0.45 | -0.0003 | 0.0013 | -0.0008 | -0.0048 | 0.0151 | -0.0080 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0013 | 0.0011 |
| 0.47 | -0.0003 | 0.0013 | -0.0008 | -0.0042 | 0.0151 | -0.0076 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0013 | 0.0011 |
| 0.49 | -0.0003 | 0.0014 | -0.0009 | -0.0035 | 0.0150 | -0.0071 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0013 | 0.0011 |
| 0.51 | -0.0003 | 0.0014 | -0.0010 | -0.0028 | 0.0148 | -0.0067 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0013 | 0.0011 |
| 0.53 | -0.0002 | 0.0013 | -0.0010 | -0.0022 | 0.0146 | -0.0062 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0013 | 0.0011 |
| 0.55 | -0.0001 | 0.0012 | -0.0011 | -0.0015 | 0.0143 | -0.0057 | 0.0011 | 0.0016 | 0.0015 | 0.0008 | 0.0013 | 0.0011 |
| 0.57 | 0 | 0.0010 | -0.0010 | -0.0007 | 0.0139 | -0.0052 | 0.0011 | 0.0016 | 0.0015 | 0.0008 | 0.0013 | 0.0011 |
| 0.59 | 0 | 0.0010 | -0.0010 | 0 | 0.0134 | -0.0047 | 0.0011 | 0.0016 | 0.0015 | 0.0008 | 0.0013 | 0.0011 |
| 0.61 | 0 | 0.0010 | -0.0011 | 0.0009 | 0.0129 | -0.0041 | 0.0011 | 0.0016 | 0.0016 | 0.0008 | 0.0013 | 0.0011 |
| 0.63 | 0 | 0.0009 | -0.0011 | 0.0017 | 0.0122 | -0.0035 | 0.0011 | 0.0016 | 0.0016 | 0.0008 | 0.0013 | 0.0011 |
| 0.65 | -0.0001 | 0.0012 | -0.0012 | 0.0027 | 0.0114 | -0.0029 | 0.0011 | 0.0016 | 0.0016 | 0.0008 | 0.0013 | 0.0011 |
| 0.67 | 0.0001 | 0.0011 | -0.0013 | 0.0036 | 0.0106 | -0.0023 | 0.0012 | 0.0016 | 0.0016 | 0.0008 | 0.0013 | 0.0012 |
| 0.69 | 0.0003 | 0.0010 | -0.0015 | 0.0047 | 0.0095 | -0.0016 | 0.0012 | 0.0016 | 0.0017 | 0.0008 | 0.0012 | 0.0012 |
| 0.71 | 0.0001 | 0.0010 | -0.0013 | 0.0059 | 0.0084 | -0.0010 | 0.0012 | 0.0017 | 0.0017 | 0.0009 | 0.0012 | 0.0012 |
| 0.73 | 0 | 0.0010 | -0.0012 | 0.0072 | 0.0071 | -0.0002 | 0.0012 | 0.0017 | 0.0017 | 0.0009 | 0.0012 | 0.0012 |
| 0.75 | 0.0002 | 0.0007 | -0.0012 | 0.0086 | 0.0056 | 0.0005 | 0.0012 | 0.0017 | 0.0018 | 0.0009 | 0.0012 | 0.0012 |
| 0.77 | 0 | 0.0008 | -0.0010 | 0.0101 | 0.0040 | 0.0014 | 0.0013 | 0.0017 | 0.0018 | 0.0010 | 0.0012 | 0.0012 |
| 0.79 | -0.0001 | 0.0009 | -0.0008 | 0.0118 | 0.0021 | 0.0023 | 0.0013 | 0.0018 | 0.0018 | 0.0010 | 0.0012 | 0.0013 |
| 0.81 | 0.0002 | 0.0004 | -0.0009 | 0.0137 | 0 | 0.0032 | 0.0013 | 0.0018 | 0.0019 | 0.0011 | 0.0013 | 0.0013 |
| 0.83 | 0.0001 | 0.0006 | -0.0008 | 0.0159 | -0.0025 | 0.0043 | 0.0014 | 0.0019 | 0.0020 | 0.0012 | 0.0013 | 0.0013 |
| 0.85 | 0.0004 | -0.0001 | -0.0009 | 0.0185 | -0.0054 | 0.0055 | 0.0014 | 0.0019 | 0.0020 | 0.0013 | 0.0013 | 0.0014 |
| 0.87 | 0.0004 | -0.0005 | -0.0006 | 0.0215 | -0.0087 | 0.0069 | 0.0015 | 0.0020 | 0.0021 | 0.0014 | 0.0014 | 0.0014 |
| 0.89 | 0.0002 | -0.0007 | -0.0003 | 0.0250 | -0.0128 | 0.0084 | 0.0015 | 0.0021 | 0.0021 | 0.0016 | 0.0016 | 0.0015 |
| 0.91 | 0 | -0.0005 | -0.0001 | 0.0294 | -0.0177 | 0.0104 | 0.0016 | 0.0022 | 0.0022 | 0.0019 | 0.0018 | 0.0016 |
| 0.93 | -0.0002 | -0.0006 | 0.0002 | 0.0351 | -0.0239 | 0.0130 | 0.0017 | 0.0023 | 0.0024 | 0.0023 | 0.0021 | 0.0017 |
| 0.95 | -0.0002 | -0.0010 | 0.0004 | 0.0429 | -0.0321 | 0.0163 | 0.0018 | 0.0025 | 0.0026 | 0.0030 | 0.0026 | 0.0019 |
| 0.97 | -0.0004 | -0.0016 | 0.0008 | 0.0557 | -0.0443 | 0.0213 | 0.0021 | 0.0029 | 0.0030 | 0.0043 | 0.0036 | 0.0022 |
| 0.99 | -0.0006 | -0.0045 | 0.0024 | 0.0847 | -0.0682 | 0.0316 | 0.0029 | 0.0040 | 0.0042 | 0.0086 | 0.0066 | 0.0031 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 1,000$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 24: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \frac{sin(\tau)+1}{2}$*: bias and MSE of estimators*

| | *Bias* | | | | | | *MSE* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *canonical* | | | *smoothed* | | | *canonical* | | | *smoothed* | | |
| $\tau$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ | $\alpha_0$ | $\alpha_1$ | $\theta_1$ |
| 0.01 | 0.0001 | -0.0030 | 0.0060 | -0.0763 | -0.0280 | 0.0562 | 0.0038 | 0.0054 | 0.0049 | 0.0074 | 0.0031 | 0.0054 |
| 0.03 | 0.0018 | -0.0023 | 0.0007 | -0.0529 | -0.0147 | 0.0312 | 0.0022 | 0.0031 | 0.0029 | 0.0040 | 0.0019 | 0.0026 |
| 0.05 | 0.0014 | -0.0010 | -0.0006 | -0.0420 | -0.0087 | 0.0196 | 0.0018 | 0.0025 | 0.0023 | 0.0028 | 0.0016 | 0.0018 |
| 0.07 | 0.0013 | -0.0007 | -0.0009 | -0.0349 | -0.0048 | 0.0120 | 0.0016 | 0.0022 | 0.0021 | 0.0022 | 0.0014 | 0.0015 |
| 0.09 | 0.0015 | -0.0008 | -0.0010 | -0.0295 | -0.0020 | 0.0064 | 0.0014 | 0.0020 | 0.0019 | 0.0018 | 0.0013 | 0.0013 |
| 0.11 | 0.0013 | -0.0007 | -0.0010 | -0.0251 | -0.0001 | 0.0021 | 0.0013 | 0.0018 | 0.0018 | 0.0015 | 0.0012 | 0.0012 |
| 0.13 | 0.0012 | -0.0004 | -0.0011 | -0.0215 | 0.0014 | -0.0014 | 0.0013 | 0.0017 | 0.0017 | 0.0013 | 0.0012 | 0.0012 |
| 0.15 | 0.0015 | -0.0006 | -0.0014 | -0.0183 | 0.0025 | -0.0043 | 0.0012 | 0.0017 | 0.0016 | 0.0012 | 0.0012 | 0.0012 |
| 0.17 | 0.0014 | -0.0003 | -0.0015 | -0.0156 | 0.0035 | -0.0066 | 0.0012 | 0.0017 | 0.0016 | 0.0011 | 0.0012 | 0.0012 |
| 0.19 | 0.0013 | -0.0003 | -0.0015 | -0.0132 | 0.0042 | -0.0086 | 0.0012 | 0.0016 | 0.0016 | 0.0010 | 0.0011 | 0.0012 |
| 0.21 | 0.0012 | -0.0001 | -0.0015 | -0.0111 | 0.0048 | -0.0102 | 0.0012 | 0.0016 | 0.0016 | 0.0009 | 0.0011 | 0.0012 |
| 0.23 | 0.0012 | -0.0002 | -0.0015 | -0.0092 | 0.0052 | -0.0116 | 0.0011 | 0.0015 | 0.0016 | 0.0009 | 0.0011 | 0.0012 |
| 0.25 | 0.0011 | -0.0002 | -0.0013 | -0.0074 | 0.0056 | -0.0127 | 0.0011 | 0.0015 | 0.0015 | 0.0009 | 0.0011 | 0.0012 |
| 0.27 | 0.0009 | -0.0001 | -0.0012 | -0.0058 | 0.0058 | -0.0137 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0013 |
| 0.29 | 0.0011 | -0.0001 | -0.0015 | -0.0043 | 0.0060 | -0.0144 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0013 |
| 0.31 | 0.0012 | -0.0001 | -0.0015 | -0.0030 | 0.0061 | -0.0150 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0013 |
| 0.33 | 0.0012 | -0.0002 | -0.0017 | -0.0017 | 0.0062 | -0.0155 | 0.0011 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0013 |
| 0.35 | 0.0015 | -0.0004 | -0.0020 | -0.0006 | 0.0062 | -0.0159 | 0.0012 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0013 |
| 0.37 | 0.0015 | -0.0004 | -0.0018 | 0.0005 | 0.0062 | -0.0162 | 0.0012 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0013 |
| 0.39 | 0.0014 | -0.0005 | -0.0017 | 0.0016 | 0.0062 | -0.0163 | 0.0012 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0013 |
| 0.41 | 0.0015 | -0.0005 | -0.0018 | 0.0025 | 0.0061 | -0.0164 | 0.0012 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0013 |
| 0.43 | 0.0015 | -0.0007 | -0.0018 | 0.0034 | 0.0060 | -0.0163 | 0.0012 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0013 |
| 0.45 | 0.0015 | -0.0006 | -0.0018 | 0.0043 | 0.0058 | -0.0162 | 0.0012 | 0.0015 | 0.0015 | 0.0008 | 0.0011 | 0.0013 |
| 0.47 | 0.0016 | -0.0007 | -0.0017 | 0.0050 | 0.0057 | -0.0160 | 0.0012 | 0.0015 | 0.0016 | 0.0008 | 0.0011 | 0.0013 |
| 0.49 | 0.0013 | -0.0006 | -0.0014 | 0.0058 | 0.0055 | -0.0157 | 0.0012 | 0.0015 | 0.0016 | 0.0009 | 0.0011 | 0.0013 |
| 0.51 | 0.0013 | -0.0007 | -0.0013 | 0.0065 | 0.0053 | -0.0153 | 0.0012 | 0.0015 | 0.0016 | 0.0009 | 0.0011 | 0.0013 |
| 0.53 | 0.0013 | -0.0006 | -0.0014 | 0.0072 | 0.0050 | -0.0149 | 0.0012 | 0.0015 | 0.0016 | 0.0009 | 0.0011 | 0.0013 |
| 0.55 | 0.0013 | -0.0005 | -0.0014 | 0.0079 | 0.0048 | -0.0144 | 0.0012 | 0.0015 | 0.0016 | 0.0009 | 0.0011 | 0.0013 |
| 0.57 | 0.0012 | -0.0006 | -0.0012 | 0.0085 | 0.0045 | -0.0138 | 0.0012 | 0.0015 | 0.0016 | 0.0009 | 0.0011 | 0.0013 |
| 0.59 | 0.0013 | -0.0008 | -0.0012 | 0.0091 | 0.0042 | -0.0131 | 0.0012 | 0.0015 | 0.0016 | 0.0009 | 0.0011 | 0.0013 |
| 0.61 | 0.0014 | -0.0009 | -0.0012 | 0.0097 | 0.0038 | -0.0123 | 0.0012 | 0.0015 | 0.0016 | 0.0009 | 0.0011 | 0.0013 |
| 0.63 | 0.0016 | -0.0012 | -0.0014 | 0.0102 | 0.0035 | -0.0115 | 0.0012 | 0.0015 | 0.0016 | 0.0010 | 0.0011 | 0.0013 |
| 0.65 | 0.0013 | -0.0011 | -0.0013 | 0.0108 | 0.0031 | -0.0105 | 0.0012 | 0.0015 | 0.0016 | 0.0010 | 0.0011 | 0.0013 |
| 0.67 | 0.0015 | -0.0012 | -0.0013 | 0.0113 | 0.0026 | -0.0094 | 0.0012 | 0.0015 | 0.0016 | 0.0010 | 0.0011 | 0.0012 |
| 0.69 | 0.0013 | -0.0010 | -0.0011 | 0.0119 | 0.0022 | -0.0082 | 0.0012 | 0.0015 | 0.0016 | 0.0010 | 0.0011 | 0.0012 |
| 0.71 | 0.0011 | -0.0009 | -0.0008 | 0.0124 | 0.0016 | -0.0069 | 0.0012 | 0.0015 | 0.0016 | 0.0010 | 0.0011 | 0.0012 |
| 0.73 | 0.0008 | -0.0007 | -0.0006 | 0.0129 | 0.0011 | -0.0055 | 0.0013 | 0.0015 | 0.0017 | 0.0011 | 0.0011 | 0.0012 |
| 0.75 | 0.0006 | -0.0007 | -0.0004 | 0.0135 | 0.0004 | -0.0039 | 0.0013 | 0.0016 | 0.0017 | 0.0011 | 0.0011 | 0.0012 |
| 0.77 | 0.0004 | -0.0003 | -0.0004 | 0.0141 | -0.0003 | -0.0021 | 0.0013 | 0.0016 | 0.0017 | 0.0011 | 0.0011 | 0.0012 |
| 0.79 | 0.0004 | -0.0002 | -0.0006 | 0.0147 | -0.0011 | -0.0001 | 0.0013 | 0.0016 | 0.0018 | 0.0011 | 0.0011 | 0.0012 |
| 0.81 | 0.0003 | -0.0002 | -0.0004 | 0.0154 | -0.0020 | 0.0021 | 0.0014 | 0.0017 | 0.0018 | 0.0012 | 0.0012 | 0.0012 |
| 0.83 | 0.0005 | -0.0004 | -0.0005 | 0.0161 | -0.0031 | 0.0046 | 0.0014 | 0.0017 | 0.0018 | 0.0012 | 0.0012 | 0.0013 |
| 0.85 | 0.0004 | -0.0003 | -0.0006 | 0.0170 | -0.0044 | 0.0075 | 0.0014 | 0.0018 | 0.0019 | 0.0013 | 0.0012 | 0.0013 |
| 0.87 | 0.0004 | -0.0003 | -0.0005 | 0.0180 | -0.0058 | 0.0109 | 0.0015 | 0.0019 | 0.0020 | 0.0013 | 0.0013 | 0.0014 |
| 0.89 | 0.0003 | -0.0004 | -0.0004 | 0.0192 | -0.0075 | 0.0149 | 0.0015 | 0.0019 | 0.0020 | 0.0014 | 0.0013 | 0.0016 |
| 0.91 | 0.0003 | -0.0005 | -0.0005 | 0.0207 | -0.0097 | 0.0198 | 0.0016 | 0.0020 | 0.0020 | 0.0015 | 0.0014 | 0.0018 |
| 0.93 | 0.0001 | -0.0006 | -0.0002 | 0.0230 | -0.0127 | 0.0259 | 0.0017 | 0.0021 | 0.0022 | 0.0016 | 0.0015 | 0.0021 |
| 0.95 | -0.0004 | -0.0004 | 0.0003 | 0.0264 | -0.0168 | 0.0339 | 0.0018 | 0.0022 | 0.0023 | 0.0018 | 0.0016 | 0.0027 |
| 0.97 | -0.0013 | -0.0001 | 0.0013 | 0.0323 | -0.0226 | 0.0459 | 0.0021 | 0.0025 | 0.0027 | 0.0023 | 0.0020 | 0.0037 |
| 0.99 | -0.0029 | -0.0025 | 0.0049 | 0.0484 | -0.0345 | 0.0695 | 0.0029 | 0.0035 | 0.0038 | 0.0038 | 0.0029 | 0.0067 |

*Notes*: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 1,000$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.