

**Proposition 1.** Let  $v_{ij}: (0, 1) \rightarrow [0, 1]$ ,  $i, j \in \{0, 1\}$  be non-decreasing, left-continuous functions with  $v_{11} + v_{00} = v_{10} + v_{01}$ . Let additionally  $Y_0$  and  $X_0$  be random variables supported on the unit interval, and let  $U_1, U_2, \dots$  and  $V_1, V_2, \dots$  be *iid* sequences of random variables in the unit interval, with both sequences mutually independent and also independent from  $Y_0$  and  $X_0$ . Define moreover, for  $t \geq 1$ ,

$$\begin{aligned} Y_t &:= \alpha_0(U_t) + \alpha_1(U_t)Y_{t-1} + \theta_1(U_t)X_{t-1} \\ X_t &:= Q_t(V_t, X_{t-1}, \dots, X_0, Y_t, Y_{t-1}, \dots, Y_0) \end{aligned} \quad (1)$$

where, for all  $t \geq 1$ , the measurable mapping  $Q_t: (0, 1) \times [0, 1]^{2t+1} \rightarrow [0, 1]$  is a quantile function of its first argument, and where  $\alpha_0 := v_{00}$ ,  $\alpha_1 := v_{10} - v_{00}$  and  $\theta_1 := v_{01} - v_{00}$ . Then the process  $(X_t, Y_t)_{t \in \mathbb{Z}}$  has state space contained in the unit square  $[0, 1]^2$  and satisfies

$$Q_{Y_t}(\tau | \mathcal{F}_{t-1}) = \alpha_0(\tau) + \alpha_1(\tau)Y_{t-1} + \theta_1(\tau)X_{t-1} \quad (2)$$

for all  $0 < \tau < 1$  and all  $t \geq 1$ , where  $\mathcal{F}_t := \sigma\{(X_s, Y_s): s \leq t\}$ .

**Proposition 2.** Let  $v_{10}, v_{01}: (0, 1) \rightarrow \mathbb{R}$  be any two non-decreasing, continuously differentiable functions, with  $v_{10}(0+) = v_{01}(0+) = 0$  and  $v_{10}(1-) = v_{01}(1-) = 1$ . Also, let  $\lambda: (0, 1) \rightarrow \mathbb{R}$  be any non-negative, continuous function satisfying the requirement that  $0 < \int_0^1 (v'_{10}(u) + v'_{01}(u))\lambda(u) du < \infty$ . Then the following holds:

1. The function  $v_{00}: (0, 1) \rightarrow \mathbb{R}$  defined through

$$v_{00}(\tau) := \frac{\int_0^\tau (v'_{10}(u) + v'_{01}(u))\lambda(u) du}{\int_0^1 (v'_{10}(u) + v'_{01}(u))\lambda(u) du}, \quad 0 < \tau < 1, \quad (3)$$

is continuously differentiable and non-decreasing, with  $v_{00}(0+) = 0$  and  $v_{00}(1-) = 1$ .

2. The function  $v_{11} := v_{10} + v_{01} - v_{00}$  is continuously differentiable, satisfies  $v_{11}(0+) = 0$ ,  $v_{11}(1-) = 1$  and, for  $0 < \tau < 1$ ,

$$v'_{11}(\tau) = (v'_{10}(\tau) + v'_{01}(\tau)) \left( 1 - \frac{\lambda(\tau)}{\int_0^1 (v'_{10}(u) + v'_{01}(u))\lambda(u) du} \right). \quad (4)$$

In particular, in order that  $v_{11}$  be non-decreasing, it is necessary and sufficient that the bound

$$\lambda(\tau) \leq \int_0^1 (v'_{10}(u) + v'_{01}(u))\lambda(u) du$$

holds for each  $\tau$  such that  $v'_{10}(\tau) + v'_{01}(\tau) \neq 0$ .

Table 1: Location-shift model 1: bias and MSE of estimators

$\tau$	Bias										MSE									
	canonical					smoothed					canonical					smoothed				
	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$
0.01	0.2615	-0.0144	-0.0028	0.0060	-0.0910	-0.0146	-0.0012	0.0074	0.0684	0.0002	0	0	0.0083	0.0002	0	0	0.0083	0.0002	0	0.0001
0.03	0.0585	-0.0148	-0.0010	0.0088	-0.1168	-0.0156	-0.0001	0.0085	0.0034	0.0002	0	0.0001	0.0137	0.0002	0	0.0001	0.0137	0.0002	0	0.0001
0.05	0.0325	-0.0144	-0.0003	0.0095	-0.1082	-0.0163	0.0006	0.0088	0.0011	0.0002	0	0.0001	0.0117	0.0003	0	0.0001	0.0117	0.0003	0	0.0001
0.07	0.0245	-0.0156	0.0018	0.0092	-0.0985	-0.0167	0.0007	0.0089	0.0006	0.0002	0	0.0001	0.0097	0.0003	0	0.0001	0.0097	0.0003	0	0.0001
0.09	0.0207	-0.0173	0.0019	0.0098	-0.0899	-0.0168	0.0007	0.0088	0.0004	0.0003	0	0.0001	0.0081	0.0003	0	0.0001	0.0081	0.0003	0	0.0001
0.11	0.0181	-0.0179	0.0008	0.0092	-0.0822	-0.0170	0.0006	0.0086	0.0003	0.0003	0	0.0001	0.0068	0.0003	0	0.0001	0.0068	0.0003	0	0.0001
0.13	0.0153	-0.0179	0.0013	0.0081	-0.0755	-0.0171	0.0007	0.0085	0.0002	0.0003	0	0.0001	0.0057	0.0003	0	0.0001	0.0057	0.0003	0	0.0001
0.15	0.0132	-0.0178	0.0003	0.0087	-0.0694	-0.0172	0.0006	0.0085	0.0002	0.0003	0	0.0001	0.0048	0.0003	0	0.0001	0.0048	0.0003	0	0.0001
0.17	0.0120	-0.0174	0.0003	0.0078	-0.0639	-0.0172	0.0007	0.0085	0.0001	0.0003	0	0.0001	0.0041	0.0003	0	0.0001	0.0041	0.0003	0	0.0001
0.19	0.0107	-0.0175	0.0005	0.0082	-0.0586	-0.0172	0.0008	0.0085	0.0001	0.0003	0	0.0001	0.0034	0.0003	0	0.0001	0.0034	0.0003	0	0.0001
0.21	0.0093	-0.0174	0.0004	0.0086	-0.0537	-0.0172	0.0009	0.0085	0.0001	0.0003	0	0.0001	0.0029	0.0003	0	0.0001	0.0029	0.0003	0	0.0001
0.23	0.0086	-0.0174	0.0005	0.0083	-0.0490	-0.0172	0.0011	0.0086	0.0001	0.0003	0	0.0001	0.0024	0.0003	0	0.0001	0.0024	0.0003	0	0.0001
0.25	0.0073	-0.0171	0.0006	0.0083	-0.0429	-0.0172	0.0011	0.0086	0.0001	0.0003	0	0.0001	0.0018	0.0003	0	0.0001	0.0018	0.0003	0	0.0001
0.27	0.0078	-0.0172	0.0011	0.0081	-0.0364	-0.0172	0.0012	0.0087	0.0001	0.0003	0	0.0001	0.0013	0.0003	0	0.0001	0.0013	0.0003	0	0.0001
0.29	0.0066	-0.0169	0.0015	0.0084	-0.0328	-0.0171	0.0013	0.0087	0	0.0003	0	0.0001	0.0011	0.0003	0	0.0001	0.0011	0.0003	0	0.0001
0.31	0.0060	-0.0173	0.0015	0.0082	-0.0294	-0.0171	0.0014	0.0087	0	0.0003	0	0.0001	0.0009	0.0003	0	0.0001	0.0009	0.0003	0	0.0001
0.33	0.0053	-0.0169	0.0014	0.0081	-0.0261	-0.0171	0.0015	0.0087	0	0.0003	0	0.0001	0.0007	0.0003	0	0.0001	0.0007	0.0003	0	0.0001
0.35	0.0047	-0.0167	0.0016	0.0086	-0.0229	-0.0171	0.0016	0.0088	0	0.0003	0	0.0001	0.0005	0.0003	0	0.0001	0.0005	0.0003	0	0.0001
0.37	0.0039	-0.0171	0.0017	0.0093	-0.0197	-0.0170	0.0016	0.0088	0	0.0003	0	0.0001	0.0004	0.0003	0	0.0001	0.0004	0.0003	0	0.0001
0.39	0.0030	-0.0172	0.0022	0.0097	-0.0166	-0.0170	0.0018	0.0089	0	0.0003	0	0.0001	0.0003	0.0003	0	0.0001	0.0003	0.0003	0	0.0001
0.41	0.0023	-0.0170	0.0028	0.0096	-0.0135	-0.0170	0.0019	0.0089	0	0.0003	0	0.0001	0.0002	0.0003	0	0.0001	0.0002	0.0003	0	0.0001
0.43	0.0017	-0.0169	0.0028	0.0098	-0.0104	-0.0169	0.0020	0.0089	0	0.0003	0	0.0001	0.0001	0.0003	0	0.0001	0.0001	0.0003	0	0.0001
0.45	0.0019	-0.0168	0.0026	0.0098	-0.0073	-0.0169	0.0021	0.0089	0	0.0003	0	0.0001	0.0001	0.0003	0	0.0001	0.0001	0.0003	0	0.0001
0.47	0.0010	-0.0166	0.0025	0.0097	-0.0043	-0.0169	0.0021	0.0088	0	0.0003	0	0.0001	0	0.0003	0	0.0001	0	0.0003	0	0.0001
0.49	0.0008	-0.0164	0.0028	0.0087	-0.0012	-0.0169	0.0022	0.0088	0	0.0003	0	0.0001	0	0.0003	0	0.0001	0	0.0003	0	0.0001
0.51	0	-0.0163	0.0030	0.0086	0.0018	-0.0168	0.0023	0.0088	0	0.0003	0	0.0001	0	0.0003	0	0.0001	0	0.0003	0	0.0001
0.53	-0.0008	-0.0165	0.0031	0.0094	0.0049	-0.0168	0.0023	0.0087	0	0.0003	0	0.0001	0	0.0003	0	0.0001	0	0.0003	0	0.0001
0.55	-0.0010	-0.0168	0.0029	0.0093	0.0079	-0.0167	0.0023	0.0087	0	0.0003	0	0.0001	0.0001	0.0003	0	0.0001	0.0001	0.0003	0	0.0001
0.57	-0.0015	-0.0170	0.0027	0.0091	0.0109	-0.0167	0.0023	0.0086	0	0.0003	0	0.0001	0.0001	0.0003	0	0.0001	0.0001	0.0003	0	0.0001
0.59	-0.0014	-0.0169	0.0030	0.0090	0.0139	-0.0166	0.0023	0.0085	0	0.0003	0	0.0001	0.0001	0.0003	0	0.0001	0.0001	0.0003	0	0.0001
0.61	-0.0019	-0.0171	0.0030	0.0091	0.0169	-0.0166	0.0023	0.0084	0	0.0003	0	0.0001	0.0001	0.0003	0	0.0001	0.0001	0.0003	0	0.0001
0.63	-0.0026	-0.0171	0.0025	0.0088	0.0200	-0.0165	0.0022	0.0083	0	0.0003	0	0.0001	0.0001	0.0003	0	0.0001	0.0001	0.0003	0	0.0001
0.65	-0.0037	-0.0168	0.0028	0.0082	0.0231	-0.0164	0.0022	0.0082	0	0.0003	0	0.0001	0.0001	0.0003	0	0.0001	0.0001	0.0003	0	0.0001
0.67	-0.0046	-0.0164	0.0025	0.0076	0.0262	-0.0163	0.0021	0.0081	0	0.0003	0	0.0001	0.0001	0.0003	0	0.0001	0.0001	0.0003	0	0.0001
0.69	-0.0064	-0.0160	0.0018	0.0074	0.0294	-0.0162	0.0021	0.0080	0	0.0003	0	0.0001	0.0001	0.0003	0	0.0001	0.0001	0.0003	0	0.0001
0.71	-0.0069	-0.0157	0.0013	0.0073	0.0327	-0.0161	0.0019	0.0078	0	0.0002	0	0.0001	0.0001	0.0003	0	0.0001	0.0001	0.0003	0	0.0001
0.73	-0.0074	-0.0151	0.0011	0.0070	0.0363	-0.0160	0.0018	0.0077	0.0001	0.0002	0	0	0.0013	0.0003	0	0.0001	0.0013	0.0003	0	0.0001
0.75	-0.0083	-0.0154	0.0010	0.0068	0.0425	-0.0159	0.0016	0.0076	0.0001	0.0002	0	0	0.0018	0.0003	0	0.0001	0.0018	0.0003	0	0.0001
0.77	-0.0097	-0.0154	0.0011	0.0071	0.0485	-0.0158	0.0014	0.0074	0.0001	0.0002	0	0.0001	0.0024	0.0002	0	0.0001	0.0024	0.0002	0	0.0001
0.79	-0.0110	-0.0151	0.0009	0.0063	0.0531	-0.0156	0.0012	0.0073	0.0001	0.0002	0	0	0.0028	0.0002	0	0.0001	0.0028	0.0002	0	0.0001
0.81	-0.0118	-0.0149	0.0008	0.0065	0.0578	-0.0155	0.0010	0.0071	0.0001	0.0002	0	0	0.0033	0.0002	0	0.0001	0.0033	0.0002	0	0.0001
0.83	-0.0131	-0.0147	-0.0001	0.0070	0.0629	-0.0153	0.0008	0.0069	0.0002	0.0002	0	0	0.0040	0.0002	0	0	0.0040	0.0002	0	0
0.85	-0.0148	-0.0145	-0.0008	0.0062	0.0683	-0.0152	0.0006	0.0067	0.0002	0.0002	0	0	0.0047	0.0002	0	0	0.0047	0.0002	0	0
0.87	-0.0171	-0.0141	-0.0002	0.0054	0.0742	-0.0150	0.0004	0.0066	0.0003	0.0002	0	0	0.0055	0.0002	0	0	0.0055	0.0002	0	0
0.89	-0.0188	-0.0148	0.0002	0.0052	0.0808	-0.0148	0.0002	0.0065	0.0004	0.0002	0	0	0.0065	0.0002	0	0	0.0065	0.0002	0	0
0.91	-0.0218	-0.0140	-0.0009	0.0056	0.0880	-0.0147	-0.0001	0.0066	0.0005	0.0002	0	0	0.0077	0.0002	0	0	0.0077	0.0002	0	0
0.93	-0.0272	-0.0143	-0.0015	0.0065	0.0961	-0.0145	-0.0001	0.0069	0.0007	0.0002	0	0	0.0092	0.0002	0	0	0.0092	0.0002	0	0
0.95	-0.0380	-0.0144	-0.0007	0.0077	0.1053	-0.0143	0.0002	0.0074	0.0014	0.0002	0	0.0001	0.0111	0.0002	0	0.0001	0.0111	0.0002	0	0.0001
0.97	-0.0596	-0.0132	0.0020	0.0068	0.1140	-0.0141	0.0008	0.0075	0.0036	0.0002	0	0	0.0130	0.0002	0	0.0001	0.0130	0.0002	0	0.0001
0.99	-0.2651	-0.0108	-0.0001	0.0069	0.0869	-0.0132	0.0015	0.0075	0.0703	0.0001	0	0	0.0075	0.0002	0	0.0001	0.0075	0.0002	0	0.0001

Notes: Bias and MSE of  $\hat{\beta}_n$  and  $\hat{\beta}_n^{\zeta^*/2}$ . Sample size of  $n = 100$ . The number of replications is 5,000. The optimal bandwidth is  $\zeta^*/2$ .

Table 2: Location-shift model 2: bias and MSE of estimators

$\tau$	Bias					smoothed					canonical					smoothed					MSE					
	canonical			$\theta_2$		smoothed			$\theta_2$		canonical			$\theta_2$		smoothed			$\theta_2$		MSE					
	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	
0.01	-0.1268	0.0033	0.0113	-0.0101	-0.0101	-0.2165	-0.0018	0.0080	-0.0043	7.6610	0.2630	0.7827	0.7171	7.2056	0.1815	0.5577	0.7171	7.2056	0.1815	0.5577	0.7171	7.2056	0.1815	0.5577	0.7171	7.2056
0.03	-0.0695	-0.0202	0.0046	0.0077	0.0077	-0.1221	-0.0213	0.0042	0.0074	0.7263	0.0708	0.1734	0.1734	0.6582	0.0555	0.1364	0.1734	0.6582	0.0555	0.1364	0.1734	0.6582	0.0555	0.1364	0.1734	0.6582
0.05	-0.0559	-0.0200	0.0023	0.0077	0.0077	-0.1004	-0.0221	0.0023	0.0087	0.2981	0.0377	0.0875	0.0911	0.2729	0.0307	0.0720	0.0911	0.2729	0.0307	0.0720	0.0911	0.2729	0.0307	0.0720	0.0911	0.2729
0.07	-0.0465	-0.0200	0.0007	0.0037	0.0037	-0.0852	-0.0194	-0.0001	0.0047	0.1800	0.0256	0.0580	0.0609	0.1624	0.0208	0.0477	0.0609	0.1624	0.0208	0.0477	0.0609	0.1624	0.0208	0.0477	0.0609	0.1624
0.09	-0.0354	-0.0180	-0.0010	0.0028	0.0028	-0.0729	-0.0171	-0.0008	0.0034	0.1232	0.0186	0.0415	0.0440	0.1116	0.0154	0.0346	0.0440	0.1116	0.0154	0.0346	0.0440	0.1116	0.0154	0.0346	0.0440	0.1116
0.11	-0.0276	-0.0155	-0.0008	0.0033	0.0033	-0.0640	-0.0159	-0.0006	0.0040	0.0911	0.0145	0.0325	0.0348	0.0836	0.0122	0.0273	0.0348	0.0836	0.0122	0.0273	0.0348	0.0836	0.0122	0.0273	0.0348	0.0836
0.13	-0.0216	-0.0151	-0.0006	0.0032	0.0032	-0.0566	-0.0151	-0.0001	0.0040	0.0724	0.0120	0.0269	0.0283	0.0663	0.0101	0.0225	0.0283	0.0663	0.0101	0.0225	0.0283	0.0663	0.0101	0.0225	0.0283	0.0663
0.15	-0.0159	-0.0147	0.0009	0.0033	0.0033	-0.0505	-0.0143	-0.0001	0.0041	0.0597	0.0103	0.0228	0.0240	0.0549	0.0087	0.0190	0.0240	0.0549	0.0087	0.0190	0.0240	0.0549	0.0087	0.0190	0.0240	0.0549
0.17	-0.0139	-0.0139	0.0005	0.0046	0.0046	-0.0455	-0.0136	-0.0001	0.0039	0.0516	0.0092	0.0193	0.0212	0.0471	0.0077	0.0163	0.0212	0.0471	0.0077	0.0163	0.0212	0.0471	0.0077	0.0163	0.0212	0.0471
0.19	-0.0123	-0.0131	-0.0001	0.0043	0.0043	-0.0411	-0.0127	-0.0001	0.0036	0.0442	0.0082	0.0171	0.0188	0.0411	0.0069	0.0144	0.0188	0.0411	0.0069	0.0144	0.0188	0.0411	0.0069	0.0144	0.0188	0.0411
0.21	-0.0112	-0.0121	-0.0010	0.0044	0.0044	-0.0371	-0.0118	-0.0002	0.0032	0.0396	0.0074	0.0152	0.0169	0.0365	0.0062	0.0130	0.0169	0.0365	0.0062	0.0130	0.0169	0.0365	0.0062	0.0130	0.0169	0.0365
0.23	-0.0097	-0.0118	-0.0004	0.0027	0.0027	-0.0335	-0.0111	-0.0002	0.0029	0.0360	0.0068	0.0137	0.0153	0.0329	0.0057	0.0119	0.0153	0.0329	0.0057	0.0119	0.0153	0.0329	0.0057	0.0119	0.0153	0.0329
0.25	-0.0088	-0.0112	0	0.0022	0.0022	-0.0288	-0.0105	-0.0002	0.0027	0.0332	0.0062	0.0129	0.0141	0.0301	0.0053	0.0110	0.0141	0.0301	0.0053	0.0110	0.0141	0.0301	0.0053	0.0110	0.0141	0.0301
0.27	-0.0060	-0.0101	-0.0001	0.0014	0.0014	-0.0244	-0.0100	-0.0002	0.0026	0.0304	0.0057	0.0121	0.0129	0.0278	0.0050	0.0104	0.0121	0.0278	0.0050	0.0104	0.0121	0.0278	0.0050	0.0104	0.0121	0.0278
0.29	-0.0048	-0.0091	-0.0006	0.0018	0.0018	-0.0219	-0.0096	-0.0002	0.0026	0.0284	0.0053	0.0113	0.0120	0.0259	0.0047	0.0098	0.0113	0.0259	0.0047	0.0098	0.0113	0.0259	0.0047	0.0098	0.0113	0.0259
0.31	-0.0047	-0.0088	-0.0007	0.0021	0.0021	-0.0197	-0.0092	-0.0002	0.0026	0.0273	0.0051	0.0108	0.0115	0.0243	0.0044	0.0093	0.0108	0.0243	0.0044	0.0093	0.0108	0.0243	0.0044	0.0093	0.0108	0.0243
0.33	-0.0043	-0.0087	-0.0004	0.0024	0.0024	-0.0176	-0.0089	-0.0003	0.0027	0.0257	0.0048	0.0102	0.0110	0.0229	0.0042	0.0089	0.0102	0.0229	0.0042	0.0089	0.0102	0.0229	0.0042	0.0089	0.0102	0.0229
0.35	-0.0026	-0.0087	-0.0013	0.0032	0.0032	-0.0156	-0.0087	-0.0003	0.0028	0.0245	0.0046	0.0098	0.0107	0.0217	0.0041	0.0086	0.0098	0.0217	0.0041	0.0086	0.0107	0.0217	0.0041	0.0086	0.0107	0.0217
0.37	-0.0020	-0.0083	-0.0006	0.0035	0.0035	-0.0135	-0.0085	-0.0003	0.0029	0.0232	0.0045	0.0096	0.0102	0.0207	0.0039	0.0083	0.0096	0.0207	0.0039	0.0083	0.0102	0.0207	0.0039	0.0083	0.0102	0.0207
0.39	-0.0011	-0.0079	-0.0005	0.0029	0.0029	-0.0114	-0.0084	-0.0003	0.0031	0.0225	0.0043	0.0093	0.0099	0.0199	0.0038	0.0081	0.0093	0.0199	0.0038	0.0081	0.0099	0.0199	0.0038	0.0081	0.0099	0.0199
0.41	-0.0012	-0.0085	0.0004	0.0043	0.0043	-0.0093	-0.0083	-0.0002	0.0033	0.0217	0.0043	0.0090	0.0097	0.0192	0.0037	0.0079	0.0090	0.0192	0.0037	0.0079	0.0097	0.0192	0.0037	0.0079	0.0097	0.0192
0.43	-0.0013	-0.0081	-0.0002	0.0037	0.0037	-0.0072	-0.0082	-0.0001	0.0034	0.0213	0.0042	0.0089	0.0095	0.0187	0.0036	0.0078	0.0089	0.0187	0.0036	0.0078	0.0095	0.0187	0.0036	0.0078	0.0095	0.0187
0.45	-0.0008	-0.0081	0	0.0038	0.0038	-0.0052	-0.0081	0	0.0035	0.0208	0.0041	0.0089	0.0094	0.0183	0.0036	0.0077	0.0089	0.0183	0.0036	0.0077	0.0094	0.0183	0.0036	0.0077	0.0094	0.0183
0.47	-0.0004	-0.0081	0.0003	0.0040	0.0040	-0.0030	-0.0081	0.0001	0.0036	0.0205	0.0041	0.0089	0.0092	0.0180	0.0035	0.0076	0.0092	0.0180	0.0035	0.0076	0.0094	0.0180	0.0035	0.0076	0.0094	0.0180
0.49	-0.0002	-0.0078	0.0002	0.0036	0.0036	-0.0008	-0.0080	0.0002	0.0037	0.0205	0.0040	0.0087	0.0091	0.0179	0.0035	0.0076	0.0087	0.0179	0.0035	0.0076	0.0091	0.0179	0.0035	0.0076	0.0091	0.0179
0.51	0.0011	-0.0080	0.0003	0.0039	0.0039	0.0015	-0.0079	0.0003	0.0037	0.0207	0.0041	0.0086	0.0091	0.0178	0.0035	0.0076	0.0091	0.0178	0.0035	0.0076	0.0091	0.0178	0.0035	0.0076	0.0091	0.0178
0.53	0.0014	-0.0078	0.0002	0.0035	0.0035	0.0038	-0.0078	0.0004	0.0037	0.0206	0.0041	0.0087	0.0093	0.0179	0.0035	0.0076	0.0093	0.0179	0.0035	0.0076	0.0093	0.0179	0.0035	0.0076	0.0093	0.0179
0.55	0.0007	-0.0079	0.0007	0.0038	0.0038	0.0061	-0.0078	0.0004	0.0038	0.0207	0.0041	0.0089	0.0093	0.0181	0.0036	0.0077	0.0093	0.0181	0.0036	0.0077	0.0093	0.0181	0.0036	0.0077	0.0093	0.0181
0.57	0.0015	-0.0078	0.0007	0.0037	0.0037	0.0085	-0.0079	0.0004	0.0038	0.0210	0.0042	0.0090	0.0094	0.0183	0.0036	0.0078	0.0094	0.0183	0.0036	0.0078	0.0094	0.0183	0.0036	0.0078	0.0094	0.0183
0.59	0.0031	-0.0081	0.0007	0.0040	0.0040	0.0108	-0.0079	0.0004	0.0038	0.0214	0.0043	0.0092	0.0096	0.0187	0.0037	0.0080	0.0096	0.0187	0.0037	0.0080	0.0096	0.0187	0.0037	0.0080	0.0096	0.0187
0.61	0.0032	-0.0082	0.0006	0.0046	0.0046	0.0131	-0.0079	0.0004	0.0038	0.0218	0.0044	0.0094	0.0097	0.0193	0.0038	0.0082	0.0094	0.0193	0.0038	0.0082	0.0097	0.0193	0.0038	0.0082	0.0097	0.0193
0.63	0.0039	-0.0079	0.0004	0.0042	0.0042	0.0155	-0.0080	0.0004	0.0038	0.0227	0.0045	0.0097	0.0101	0.0200	0.0039	0.0084	0.0101	0.0200	0.0039	0.0084	0.0101	0.0200	0.0039	0.0084	0.0101	0.0200
0.65	0.0062	-0.0073	0.0003	0.0032	0.0032	0.0177	-0.0080	0.0004	0.0037	0.0231	0.0045	0.0100	0.0104	0.0208	0.0040	0.0087	0.0104	0.0208	0.0040	0.0087	0.0104	0.0208	0.0040	0.0087	0.0104	0.0208

Table 3: Location-shift-scale model 1: bias and MSE of estimators

$\tau$	Bias						MSE					
	canonical			smoothed			canonical			smoothed		
	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\theta_1$
0.01	-0.2014	-0.0020	0.1110	0.0024	-0.4127	-0.0022	0.1353	0.0015	0.0272	0.0054	3.7711	0.0283
0.03	-0.1006	-0.0025	0.0445	0.0003	-0.2537	-0.0024	0.0686	0.0010	0.0115	0.0027	2.7073	0.0127
0.05	-0.0627	-0.0029	0.0292	0.0005	-0.1851	-0.0023	0.0490	0.0009	0.0076	0.0017	2.1744	0.0081
0.07	-0.0441	-0.0025	0.0210	0.0005	-0.1465	-0.0020	0.0384	0.0007	0.0058	0.0013	1.8029	0.0062
0.09	-0.0369	-0.0017	0.0169	0.0001	-0.1201	-0.0017	0.0316	0.0004	0.0048	0.0011	1.5208	0.0051
0.11	-0.0291	-0.0014	0.0142	-0.0003	-0.1005	-0.0015	0.0269	0.0003	0.0042	0.0009	1.2952	0.0044
0.13	-0.0235	-0.0015	0.0120	-0.0001	-0.0850	-0.0014	0.0230	0.0004	0.0037	0.0008	1.1089	0.0039
0.15	-0.0195	-0.0014	0.0104	0	-0.0724	-0.0014	0.0200	0.0005	0.0034	0.0007	0.9518	0.0036
0.17	-0.0163	-0.0014	0.0090	0.0001	-0.0616	-0.0014	0.0173	0.0005	0.0032	0.0007	0.8177	0.0033
0.19	-0.0141	-0.0014	0.0078	0.0002	-0.0525	-0.0014	0.0151	0.0005	0.0030	0.0006	0.7012	0.0031
0.21	-0.0131	-0.0012	0.0069	0.0004	-0.0447	-0.0014	0.0131	0.0005	0.0028	0.0006	0.5992	0.0029
0.23	-0.0103	-0.0011	0.0058	0.0003	-0.0378	-0.0014	0.0113	0.0005	0.0027	0.0005	0.5095	0.0028
0.25	-0.0076	-0.0011	0.0047	0.0003	-0.0311	-0.0014	0.0097	0.0004	0.0026	0.0005	0.4312	0.0027
0.27	-0.0052	-0.0013	0.0040	0.0003	-0.0252	-0.0014	0.0084	0.0004	0.0025	0.0005	0.3620	0.0026
0.29	-0.0037	-0.0013	0.0038	0.0002	-0.0208	-0.0014	0.0075	0.0003	0.0025	0.0005	0.2988	0.0025
0.31	-0.0023	-0.0013	0.0036	0.0002	-0.0168	-0.0014	0.0066	0.0003	0.0024	0.0005	0.2452	0.0024
0.33	-0.0009	-0.0014	0.0033	0.0002	-0.0133	-0.0014	0.0058	0.0003	0.0023	0.0005	0.1974	0.0024
0.35	0.0001	-0.0013	0.0028	0.0003	-0.0102	-0.0015	0.0049	0.0003	0.0023	0.0005	0.1559	0.0024
0.37	0.0018	-0.0015	0.0023	0.0004	-0.0071	-0.0015	0.0041	0.0004	0.0023	0.0005	0.1204	0.0023
0.39	0.0038	-0.0016	0.0016	0.0004	-0.0042	-0.0015	0.0032	0.0004	0.0023	0.0005	0.0905	0.0023
0.41	0.0043	-0.0015	0.0011	0.0004	-0.0015	-0.0016	0.0023	0.0005	0.0022	0.0005	0.0658	0.0023
0.43	0.0051	-0.0016	0.0007	0.0005	0.0011	-0.0016	0.0016	0.0005	0.0022	0.0005	0.0461	0.0022
0.45	0.0063	-0.0016	0.0004	0.0006	0.0037	-0.0016	0.0009	0.0005	0.0022	0.0004	0.0312	0.0022
0.47	0.0069	-0.0015	-0.0002	0.0005	0.0061	-0.0017	0.0001	0.0005	0.0022	0.0004	0.0210	0.0022
0.49	0.0075	-0.0015	-0.0006	0.0005	0.0084	-0.0016	-0.0006	0.0005	0.0022	0.0004	0.0154	0.0022
0.51	0.0089	-0.0015	-0.0012	0.0006	0.0109	-0.0017	-0.0014	0.0005	0.0021	0.0004	0.0145	0.0022
0.53	0.0103	-0.0015	-0.0018	0.0006	0.0134	-0.0016	-0.0022	0.0005	0.0022	0.0004	0.0181	0.0022
0.55	0.0115	-0.0016	-0.0022	0.0006	0.0157	-0.0016	-0.0029	0.0005	0.0022	0.0004	0.0264	0.0022
0.57	0.0124	-0.0016	-0.0023	0.0007	0.0182	-0.0016	-0.0036	0.0005	0.0022	0.0004	0.0395	0.0023
0.59	0.0136	-0.0016	-0.0029	0.0006	0.0206	-0.0016	-0.0043	0.0005	0.0022	0.0004	0.0575	0.0023
0.61	0.0141	-0.0014	-0.0036	0.0005	0.0229	-0.0016	-0.0049	0.0005	0.0023	0.0004	0.0805	0.0023
0.63	0.0147	-0.0014	-0.0036	0.0004	0.0253	-0.0016	-0.0055	0.0005	0.0023	0.0004	0.1090	0.0023
0.65	0.0156	-0.0014	-0.0039	0.0004	0.0277	-0.0016	-0.0062	0.0004	0.0023	0.0005	0.1431	0.0024
0.67	0.0158	-0.0012	-0.0040	0.0004	0.0304	-0.0016	-0.0069	0.0004	0.0024	0.0005	0.1831	0.0024
0.69	0.0168	-0.0012	-0.0045	0.0004	0.0334	-0.0015	-0.0076	0.0005	0.0025	0.0005	0.2296	0.0025
0.71	0.0182	-0.0013	-0.0050	0.0006	0.0370	-0.0015	-0.0085	0.0005	0.0025	0.0005	0.2829	0.0026
0.73	0.0196	-0.0012	-0.0056	0.0006	0.0412	-0.0016	-0.0097	0.0006	0.0026	0.0005	0.3433	0.0026
0.75	0.0213	-0.0012	-0.0064	0.0007	0.0471	-0.0016	-0.0112	0.0007	0.0026	0.0005	0.4108	0.0027
0.77	0.0222	-0.0012	-0.0073	0.0008	0.0536	-0.0016	-0.0129	0.0007	0.0028	0.0005	0.4875	0.0029
0.79	0.0239	-0.0011	-0.0080	0.0008	0.0603	-0.0016	-0.0145	0.0007	0.0029	0.0006	0.5755	0.0030
0.81	0.0253	-0.0009	-0.0090	0.0009	0.0676	-0.0016	-0.0164	0.0008	0.0031	0.0006	0.6763	0.0032
0.83	0.0284	-0.0010	-0.0102	0.0010	0.0763	-0.0015	-0.0186	0.0008	0.0033	0.0007	0.7914	0.0034
0.85	0.0316	-0.0010	-0.0119	0.0011	0.0863	-0.0014	-0.0212	0.0009	0.0036	0.0007	0.9248	0.0037
0.87	0.0346	-0.0008	-0.0132	0.0011	0.0988	-0.0014	-0.0243	0.0009	0.0039	0.0008	1.0795	0.0041
0.89	0.0387	-0.0004	-0.0152	0.0008	0.1145	-0.0013	-0.0283	0.0009	0.0044	0.0009	1.2623	0.0046
0.91	0.0464	-0.0005	-0.0186	0.0009	0.1349	-0.0012	-0.0335	0.0009	0.0050	0.0010	1.4832	0.0053
0.93	0.0596	-0.0007	-0.0236	0.0015	0.1627	-0.0013	-0.0405	0.0009	0.0060	0.0013	1.7583	0.0065
0.95	0.0810	-0.0008	-0.0318	0.0013	0.2041	-0.0014	-0.0515	0.0010	0.0078	0.0017	2.1182	0.0084
0.97	0.1203	-0.0004	-0.0484	0.0013	0.2765	-0.0017	-0.0719	0.0012	0.0121	0.0027	2.6344	0.0128
0.99	0.2264	-0.0025	-0.1145	0.0013	0.4403	-0.0028	-0.1379	0.0010	0.0283	0.0057	3.6724	0.0293

Notes: Bias and MSE of  $\hat{\beta}_n$  and  $\hat{\beta}_n^{**/2}$ . Sample size of  $n = 100$ . The number of replications is 5,000. The optimal bandwidth is  $\zeta^*/2$ .

Table 4: *Location-shift-scale model 2: bias and MSE of estimators*

$\tau$	Bias						MSE						
	canonical			smoothed			canonical			smoothed			
	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	
0.01	-0.7681	-0.0125	0.2905	-0.0105	-1.1328	-0.0129	0.3420	-0.0076	6.6734	0.0482	0.4797	0.1216	0.0346
0.03	-0.2569	-0.0157	0.0885	-0.0102	-0.4655	-0.0147	0.1190	-0.0076	0.6452	0.0147	0.1179	0.0270	0.0117
0.05	-0.1350	-0.0117	0.0534	-0.0043	-0.2893	-0.0117	0.0778	-0.0028	0.2675	0.0068	0.0410	0.0116	0.0058
0.07	-0.0814	-0.0097	0.0358	-0.0015	-0.2096	-0.0092	0.0566	-0.0012	0.1487	0.0039	0.0236	0.0053	0.0035
0.09	-0.0580	-0.0076	0.0271	-0.0015	-0.1632	-0.0076	0.0447	-0.0007	0.1006	0.0026	0.0162	0.0035	0.0028
0.11	-0.0404	-0.0065	0.0221	-0.0012	-0.1318	-0.0064	0.0367	-0.0003	0.0736	0.0020	0.0117	0.0025	0.0023
0.13	-0.0313	-0.0057	0.0182	-0.0004	-0.1087	-0.0057	0.0307	0	0.0583	0.0016	0.0092	0.0020	0.0018
0.15	-0.0248	-0.0050	0.0152	0	-0.0907	-0.0051	0.0259	0.0003	0.0475	0.0013	0.0076	0.0016	0.0015
0.17	-0.0196	-0.0044	0.0124	0	-0.0769	-0.0047	0.0222	0.0005	0.0404	0.0011	0.0065	0.0014	0.0013
0.19	-0.0146	-0.0044	0.0104	0.0004	-0.0657	-0.0044	0.0193	0.0008	0.0353	0.0010	0.0057	0.0012	0.0011
0.21	-0.0116	-0.0040	0.0091	0.0004	-0.0561	-0.0041	0.0167	0.0010	0.0316	0.0008	0.0051	0.0010	0.0010
0.23	-0.0093	-0.0037	0.0078	0.0007	-0.0479	-0.0039	0.0145	0.0011	0.0289	0.0008	0.0046	0.0009	0.0008
0.25	-0.0080	-0.0034	0.0066	0.0007	-0.0402	-0.0036	0.0127	0.0012	0.0263	0.0007	0.0041	0.0009	0.0008
0.27	-0.0068	-0.0031	0.0062	0.0008	-0.0334	-0.0034	0.0112	0.0012	0.0244	0.0006	0.0038	0.0008	0.0008
0.29	-0.0053	-0.0030	0.0055	0.0009	-0.0285	-0.0032	0.0101	0.0013	0.0228	0.0006	0.0036	0.0008	0.0007
0.31	-0.0037	-0.0029	0.0052	0.0011	-0.0240	-0.0031	0.0091	0.0014	0.0214	0.0006	0.0034	0.0007	0.0007
0.33	-0.0026	-0.0028	0.0048	0.0012	-0.0200	-0.0029	0.0081	0.0014	0.0200	0.0005	0.0033	0.0007	0.0007
0.35	-0.0017	-0.0025	0.0045	0.0011	-0.0163	-0.0028	0.0071	0.0014	0.0190	0.0005	0.0031	0.0006	0.0006
0.37	-0.0005	-0.0023	0.0035	0.0009	-0.0127	-0.0026	0.0061	0.0013	0.0184	0.0005	0.0030	0.0006	0.0006
0.39	0.0012	-0.0022	0.0031	0.0009	-0.0091	-0.0025	0.0052	0.0013	0.0176	0.0005	0.0029	0.0006	0.0006
0.41	0.0022	-0.0022	0.0028	0.0011	-0.0060	-0.0024	0.0044	0.0013	0.0174	0.0005	0.0028	0.0006	0.0006
0.43	0.0033	-0.0021	0.0023	0.0010	-0.0029	-0.0024	0.0036	0.0013	0.0170	0.0004	0.0027	0.0006	0.0006
0.45	0.0050	-0.0021	0.0018	0.0011	0.0002	-0.0023	0.0028	0.0014	0.0169	0.0004	0.0027	0.0005	0.0005
0.47	0.0063	-0.0021	0.0013	0.0012	0.0033	-0.0023	0.0019	0.0015	0.0168	0.0004	0.0027	0.0005	0.0005
0.49	0.0074	-0.0021	0.0008	0.0013	0.0063	-0.0023	0.0011	0.0015	0.0167	0.0004	0.0027	0.0005	0.0005
0.51	0.0085	-0.0021	0.0003	0.0014	0.0094	-0.0023	0.0003	0.0015	0.0170	0.0004	0.0027	0.0005	0.0005
0.53	0.0097	-0.0020	-0.0003	0.0014	0.0126	-0.0022	-0.0005	0.0015	0.0172	0.0004	0.0027	0.0005	0.0005
0.55	0.0110	-0.0020	-0.0007	0.0015	0.0158	-0.0022	-0.0014	0.0015	0.0171	0.0004	0.0028	0.0005	0.0005
0.57	0.0116	-0.0019	-0.0012	0.0015	0.0191	-0.0022	-0.0022	0.0016	0.0173	0.0004	0.0028	0.0006	0.0006
0.59	0.0137	-0.0020	-0.0017	0.0017	0.0224	-0.0022	-0.0030	0.0017	0.0178	0.0004	0.0028	0.0006	0.0006
0.61	0.0154	-0.0021	-0.0023	0.0018	0.0259	-0.0022	-0.0039	0.0018	0.0182	0.0004	0.0028	0.0006	0.0006
0.63	0.0157	-0.0019	-0.0021	0.0017	0.0296	-0.0022	-0.0047	0.0018	0.0188	0.0005	0.0029	0.0006	0.0006
0.65	0.0167	-0.0017	-0.0028	0.0016	0.0336	-0.0022	-0.0057	0.0018	0.0197	0.0005	0.0031	0.0006	0.0006
0.67	0.0187	-0.0017	-0.0037	0.0018	0.0381	-0.0021	-0.0068	0.0018	0.0202	0.0005	0.0032	0.0007	0.0007
0.69	0.0207	-0.0016	-0.0043	0.0016	0.0428	-0.0021	-0.0079	0.0018	0.0215	0.0005	0.0034	0.0007	0.0007
0.71	0.0232	-0.0016	-0.0050	0.0018	0.0480	-0.0021	-0.0091	0.0019	0.0229	0.0005	0.0036	0.0007	0.0007
0.73	0.0253	-0.0017	-0.0056	0.0020	0.0538	-0.0020	-0.0104	0.0020	0.0243	0.0006	0.0039	0.0008	0.0008
0.75	0.0273	-0.0016	-0.0064	0.0022	0.0618	-0.0020	-0.0123	0.0022	0.0263	0.0006	0.0042	0.0009	0.0009
0.77	0.0292	-0.0014	-0.0076	0.0025	0.0704	-0.0019	-0.0142	0.0024	0.0289	0.0007	0.0046	0.0010	0.0009
0.79	0.0306	-0.0010	-0.0085	0.0026	0.0791	-0.0017	-0.0163	0.0025	0.0317	0.0008	0.0051	0.0011	0.0009
0.81	0.0361	-0.0011	-0.0102	0.0030	0.0895	-0.0016	-0.0188	0.0027	0.0359	0.0008	0.0057	0.0012	0.0012
0.83	0.0397	-0.0008	-0.0118	0.0034	0.1020	-0.0015	-0.0217	0.0031	0.0404	0.0010	0.0066	0.0014	0.0013
0.85	0.0462	-0.0006	-0.0141	0.0036	0.1171	-0.0014	-0.0252	0.0035	0.0491	0.0011	0.0078	0.0017	0.0015
0.87	0.0556	-0.0007	-0.0172	0.0045	0.1369	-0.0013	-0.0297	0.0040	0.0600	0.0013	0.0095	0.0020	0.0019
0.89	0.0671	-0.0005	-0.0202	0.0054	0.1626	-0.0013	-0.0353	0.0049	0.0786	0.0017	0.0119	0.0027	0.0024
0.91	0.0885	-0.0005	-0.0264	0.0068	0.1979	-0.0013	-0.0434	0.0060	0.1043	0.0024	0.0160	0.0037	0.0032
0.93	0.1197	-0.0009	-0.0339	0.0094	0.2492	-0.0012	-0.0547	0.0083	0.1642	0.0035	0.0239	0.0058	0.0050
0.95	0.1767	0.0001	-0.0498	0.0125	0.3350	-0.0008	-0.0737	0.0117	0.2996	0.0062	0.0421	0.0102	0.0091
0.97	0.3089	0.0006	-0.0871	0.0203	0.5229	0.0001	-0.1188	0.0173	0.7262	0.0137	0.1098	0.0271	0.0222
0.99	0.8397	-0.0048	-0.3015	0.0367	1.2038	-0.0036	-0.3501	0.0300	4.7482	0.0552	0.4263	0.5343	0.4058

Notes: Bias and MSE of  $\hat{\beta}_n$  and  $\hat{\beta}_n^{**/2}$ . Sample size of  $n = 100$ . The number of replications is 5,000. The optimal bandwidth is  $\zeta^*/2$ .

Table 5: *Location-shift model 1: bias and MSE of estimators*

$\tau$	Bias						MSE																	
	canonical						smoothed						canonical						smoothed					
	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$				
0.01	2.2266	-0.0006	-0.0015	0.0019	2.2063	-0.0006	-0.0006	0.0020	0.0143	0.0066	0.0138	0.0158	0.0117	0.0052	0.0108	0.0124	0.0158	0.0052	0.0108	0.0124				
0.03	1.0763	-0.0016	0.0001	0.0027	1.0597	-0.0015	-0.0002	0.0024	0.0065	0.0030	0.0061	0.0073	0.0056	0.0025	0.0051	0.0060	0.0073	0.0025	0.0051	0.0060				
0.05	0.7123	-0.0016	-0.0001	0.0021	0.6985	-0.0014	-0.0002	0.0017	0.0046	0.0022	0.0044	0.0051	0.0041	0.0018	0.0038	0.0043	0.0051	0.0018	0.0038	0.0043				
0.07	0.5220	-0.0015	-0.0006	0.0011	0.5096	-0.0013	-0.0001	0.0013	0.0037	0.0017	0.0036	0.0042	0.0034	0.0015	0.0031	0.0036	0.0042	0.0015	0.0031	0.0036				
0.09	0.4034	-0.0013	0.0004	0.0009	0.3915	-0.0013	0.0001	0.0011	0.0032	0.0015	0.0031	0.0036	0.0029	0.0013	0.0027	0.0031	0.0036	0.0013	0.0027	0.0031				
0.11	0.3207	-0.0015	0.0003	0.0014	0.3103	-0.0014	0.0001	0.0011	0.0028	0.0013	0.0027	0.0031	0.0026	0.0011	0.0024	0.0027	0.0031	0.0011	0.0024	0.0027				
0.13	0.2609	-0.0014	0.0003	0.0011	0.2511	-0.0013	0.0002	0.0010	0.0026	0.0012	0.0025	0.0028	0.0024	0.0010	0.0022	0.0024	0.0028	0.0010	0.0022	0.0024				
0.15	0.2153	-0.0011	0.0002	0.0005	0.2061	-0.0013	0.0002	0.0010	0.0024	0.0011	0.0023	0.0026	0.0022	0.0010	0.0020	0.0022	0.0026	0.0010	0.0020	0.0022				
0.17	0.1794	-0.0014	0.0002	0.0011	0.1709	-0.0013	0.0001	0.0011	0.0022	0.0010	0.0022	0.0025	0.0021	0.0009	0.0019	0.0022	0.0025	0.0009	0.0019	0.0022				
0.19	0.1499	-0.0011	-0.0003	0.0012	0.1426	-0.0013	-0.0001	0.0012	0.0022	0.0010	0.0020	0.0023	0.0020	0.0009	0.0018	0.0021	0.0023	0.0009	0.0018	0.0021				
0.21	0.1265	-0.0012	-0.0003	0.0014	0.1196	-0.0014	-0.0002	0.0013	0.0021	0.0009	0.0020	0.0023	0.0019	0.0008	0.0017	0.0020	0.0023	0.0008	0.0017	0.0020				
0.23	0.1070	-0.0015	-0.0005	0.0011	0.1007	-0.0014	-0.0003	0.0014	0.0020	0.0009	0.0019	0.0022	0.0018	0.0008	0.0017	0.0019	0.0022	0.0008	0.0017	0.0019				
0.25	0.0906	-0.0015	-0.0005	0.0015	0.0849	-0.0015	-0.0003	0.0015	0.0019	0.0009	0.0018	0.0021	0.0018	0.0008	0.0016	0.0019	0.0021	0.0008	0.0016	0.0019				
0.27	0.0766	-0.0016	-0.0004	0.0016	0.0717	-0.0016	-0.0004	0.0016	0.0019	0.0008	0.0018	0.0021	0.0017	0.0008	0.0016	0.0018	0.0021	0.0008	0.0016	0.0018				
0.29	0.0650	-0.0016	-0.0004	0.0016	0.0604	-0.0017	-0.0004	0.0016	0.0018	0.0008	0.0017	0.0020	0.0017	0.0007	0.0015	0.0018	0.0020	0.0007	0.0015	0.0018				
0.31	0.0551	-0.0018	-0.0004	0.0016	0.0508	-0.0017	-0.0004	0.0016	0.0018	0.0008	0.0017	0.0020	0.0016	0.0007	0.0015	0.0018	0.0020	0.0007	0.0015	0.0018				
0.33	0.0461	-0.0018	-0.0004	0.0018	0.0425	-0.0018	-0.0004	0.0017	0.0017	0.0008	0.0016	0.0019	0.0016	0.0007	0.0014	0.0017	0.0020	0.0007	0.0014	0.0017				
0.35	0.0385	-0.0018	-0.0002	0.0017	0.0353	-0.0018	-0.0004	0.0016	0.0017	0.0008	0.0016	0.0019	0.0015	0.0007	0.0014	0.0017	0.0020	0.0007	0.0014	0.0017				
0.37	0.0319	-0.0018	-0.0003	0.0015	0.0289	-0.0018	-0.0004	0.0016	0.0017	0.0008	0.0016	0.0018	0.0015	0.0007	0.0014	0.0017	0.0019	0.0007	0.0014	0.0017				
0.39	0.0256	-0.0019	-0.0005	0.0015	0.0233	-0.0018	-0.0004	0.0016	0.0017	0.0008	0.0016	0.0018	0.0015	0.0007	0.0014	0.0016	0.0018	0.0007	0.0014	0.0016				
0.41	0.0202	-0.0018	-0.0004	0.0015	0.0182	-0.0019	-0.0004	0.0017	0.0016	0.0007	0.0016	0.0018	0.0015	0.0007	0.0014	0.0016	0.0018	0.0007	0.0014	0.0016				
0.43	0.0152	-0.0018	-0.0005	0.0016	0.0136	-0.0019	-0.0004	0.0017	0.0016	0.0007	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016	0.0018	0.0007	0.0014	0.0016				
0.45	0.0105	-0.0018	-0.0004	0.0017	0.0093	-0.0019	-0.0005	0.0017	0.0016	0.0007	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016	0.0018	0.0007	0.0014	0.0016				
0.47	0.0059	-0.0018	-0.0003	0.0018	0.0053	-0.0019	-0.0005	0.0018	0.0016	0.0007	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016	0.0018	0.0007	0.0014	0.0016				
0.49	0.0016	-0.0018	-0.0004	0.0018	0.0013	-0.0019	-0.0005	0.0018	0.0016	0.0007	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016	0.0018	0.0007	0.0014	0.0016				
0.51	-0.0029	-0.0020	-0.0004	0.0020	-0.0026	-0.0020	-0.0005	0.0019	0.0016	0.0007	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016	0.0018	0.0007	0.0014	0.0016				
0.53	-0.0071	-0.0021	-0.0006	0.0019	-0.0065	-0.0020	-0.0005	0.0019	0.0016	0.0008	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016	0.0018	0.0007	0.0014	0.0016				
0.55	-0.0117	-0.0022	-0.0006	0.0021	-0.0106	-0.0020	-0.0005	0.0019	0.0016	0.0008	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016	0.0018	0.0007	0.0014	0.0016				
0.57	-0.0166	-0.0023	-0.0006	0.0019	-0.0149	-0.0021	-0.0005	0.0019	0.0016	0.0008	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016	0.0018	0.0007	0.0014	0.0016				
0.59	-0.0218	-0.0020	-0.0005	0.0018	-0.0195	-0.0021	-0.0005	0.0019	0.0016	0.0008	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016	0.0018	0.0007	0.0014	0.0016				
0.61	-0.0270	-0.0020	-0.0005	0.0018	-0.0246	-0.0020	-0.0005	0.0018	0.0016	0.0008	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016	0.0018	0.0007	0.0014	0.0016				
0.63	-0.0329	-0.0020	-0.0005	0.0018	-0.0301	-0.0020	-0.0005	0.0018	0.0016	0.0008	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016	0.0018	0.0007	0.0014	0.0016				
0.65	-0.0395	-0.0020	-0.0004	0.0018	-0.0364	-0.0019	-0.0005	0.0018	0.0016	0.0008	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016	0.0018	0.0007	0.0014	0.0016				
0.67	-0.0472	-0.0018	-0.0005	0.0019	-0.0435	-0.0018	-0.0005	0.0017	0.0017	0.0008	0.0017	0.0019	0.0015	0.0007	0.0015	0.0017	0.0019	0.0007	0.0015	0.0017				
0.69	-0.0561	-0.0016	-0.0004	0.0017	-0.0517	-0.0017	-0.0005	0.0017	0.0017	0.0008	0.0017	0.0019	0.0015	0.0007	0.0015	0.0017	0.0019	0.0007	0.0015	0.0017				
0.71	-0.0637	-0.0015	-0.0004	0.0017	-0.0613	-0.0016	-0.0005	0.0016	0.0017	0.0008	0.0018	0.0020	0.0015	0.0007	0.0015	0.0017	0.0020	0.0007	0.0015	0.0017				
0.73	-0.0778	-0.0016	-0.0005	0.0014	-0.0724	-0.0016	-0.0005	0.0016	0.0014	0.0009	0.0018	0.0021	0.0016	0.0007	0.0016	0.0018	0.0021	0.0007	0.0016	0.0018				
0.75	-0.0914	-0.0015	-0.0006	0.0012	-0.0855	-0.0015	-0.0005	0.0015	0.0018	0.0009	0.0018	0.0021	0.0017	0.0008	0.0017	0.0019	0.0021	0.0008	0.0017	0.0019				
0.77	-0.1073	-0.0013	-0.0007	0.0013	-0.1011	-0.0014	-0.0005	0.0015	0.0019	0.0009	0.0019	0.0022	0.0017	0.0008	0.0017	0.0020	0.0022	0.0008	0.0017	0.0020				
0.79	-0.1267	-0.0013	-0.0003	0.0014	-0.1198	-0.0013	-0.0005	0.0015	0.0020	0.0010	0.0020	0.0023	0.0018	0.0008	0.0018	0.0020	0.0023	0.0008	0.0018	0.0020				
0.81	-0.1500	-0.0013	-0.0002	0.0015	-0.1426	-0.0013	-0.0004	0.0014	0.0021	0.0010	0.0021	0.0024	0.0019	0.0009	0.0018	0.0021	0.0024	0.0009	0.0018	0.0021				
0.83	-0.1787	-0.0013	-0.0002	0.0013	-0.1707	-0.0013	-0.0003	0.0013	0.0022	0.0010	0.0022	0.0025	0.0020	0.0009	0.0019	0.0022	0.0025	0.0009	0.0019	0.0022				
0.85	-0.2146	-0.0010	-0.0001	0.0013	-0.2060	-0.0012	-0.0001	0.0013	0.0024	0.0011	0.0023	0.0027	0.0023	0.0010	0.0020	0.0027	0.0027	0.0010	0.0020	0.0023				
0.87	-0.2610	-0.0013	0.0003	0.0013	-0.2511	-0.0013	0.0001	0.0013	0.0026	0.0012	0.0025	0.0028	0.0023	0.0011	0.0021	0.0030	0.0028	0.0011	0.0021	0.0025				
0.89	-0.3214	-0.0016	0.0005	0.0014	-0.3104	-0.0013	0.0003	0.0013	0.0028	0.0014	0.0027	0.0031	0.0025	0.0012	0.0023	0.0030	0.0031	0.0012	0.0023					

Notes: Bias and MSE of  $\hat{\beta}_n$  and  $\hat{\beta}_n^{**/2}$ . Sample size of  $n = 1000$ . The number of replications is 5,000. The optimal bandwidth is  $\zeta^*/2$ .

Table 6: *Location-shift model 2: bias and MSE of estimators*

$\tau$	Bias						MSE					
	canonical			smoothed			canonical			smoothed		
	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\theta_1$
0.01	-0.1268	0.0033	0.0113	-0.0101	-0.2165	-0.0018	7.6610	0.2630	0.7827	0.7171	7.2056	0.1815
0.03	-0.0695	-0.0202	0.0046	0.0077	-0.1221	-0.0213	0.7263	0.0708	0.1734	0.1734	0.6582	0.0555
0.05	-0.0559	-0.0215	0.0023	0.0077	-0.1004	-0.0221	0.2981	0.0377	0.0875	0.0911	0.2729	0.0307
0.07	-0.0465	-0.0200	0.0007	0.0037	-0.0200	-0.0194	0.1800	0.0256	0.0580	0.0609	0.1624	0.0208
0.09	-0.0354	-0.0180	-0.0010	0.0028	-0.0729	-0.0171	0.1232	0.0186	0.0415	0.0440	0.1116	0.0154
0.11	-0.0276	-0.0155	-0.0008	0.0033	-0.0640	-0.0159	0.0911	0.0145	0.0325	0.0348	0.0836	0.0122
0.13	-0.0216	-0.0151	-0.0006	0.0032	-0.0566	-0.0151	0.0724	0.0120	0.0269	0.0283	0.0663	0.0101
0.15	-0.0159	-0.0147	0.0009	0.0033	-0.0505	-0.0143	0.0597	0.0103	0.0228	0.0240	0.0549	0.0087
0.17	-0.0139	-0.0139	0.0005	0.0046	-0.0455	-0.0136	0.0516	0.0092	0.0193	0.0212	0.0471	0.0077
0.19	-0.0123	-0.0131	-0.0001	0.0043	-0.0411	-0.0127	0.0442	0.0082	0.0171	0.0188	0.0411	0.0069
0.21	-0.0112	-0.0121	-0.0010	0.0044	-0.0371	-0.0118	0.0396	0.0074	0.0152	0.0169	0.0365	0.0062
0.23	-0.0097	-0.0118	-0.0004	0.0027	-0.0335	-0.0111	0.0360	0.0068	0.0137	0.0153	0.0329	0.0057
0.25	-0.0088	-0.0112	0	0.0022	-0.0288	-0.0105	0.0332	0.0062	0.0129	0.0141	0.0301	0.0053
0.27	-0.0060	-0.0101	-0.0001	0.0014	-0.0244	-0.0100	0.0304	0.0057	0.0121	0.0129	0.0278	0.0050
0.29	-0.0048	-0.0091	-0.0006	0.0018	-0.0219	-0.0096	0.0284	0.0053	0.0113	0.0120	0.0259	0.0047
0.31	-0.0047	-0.0088	-0.0007	0.0021	-0.0197	-0.0092	0.0273	0.0051	0.0108	0.0115	0.0243	0.0044
0.33	-0.0043	-0.0087	-0.0004	0.0024	-0.0176	-0.0089	0.0257	0.0048	0.0102	0.0110	0.0229	0.0042
0.35	-0.0026	-0.0087	-0.0013	0.0032	-0.0156	-0.0087	0.0245	0.0046	0.0098	0.0107	0.0217	0.0041
0.37	-0.0020	-0.0083	-0.0006	0.0035	-0.0135	-0.0085	0.0232	0.0045	0.0096	0.0102	0.0207	0.0039
0.39	-0.0011	-0.0079	-0.0005	0.0029	-0.0114	-0.0084	0.0225	0.0043	0.0093	0.0099	0.0199	0.0038
0.41	-0.0012	-0.0085	0.0004	0.0043	-0.0093	-0.0083	0.0217	0.0043	0.0090	0.0097	0.0192	0.0037
0.43	-0.0013	-0.0081	-0.0002	0.0037	-0.0072	-0.0082	0.0213	0.0042	0.0089	0.0095	0.0187	0.0036
0.45	-0.0008	-0.0081	0	0.0038	-0.0052	-0.0081	0.0208	0.0041	0.0089	0.0094	0.0183	0.0036
0.47	-0.0004	-0.0081	0.0003	0.0040	-0.0030	-0.0081	0.0205	0.0041	0.0089	0.0092	0.0180	0.0035
0.49	-0.0002	-0.0078	0.0002	0.0036	-0.0008	-0.0080	0.0205	0.0040	0.0087	0.0091	0.0179	0.0035
0.51	0.0011	-0.0080	0.0003	0.0039	0.0015	-0.0079	0.0207	0.0041	0.0086	0.0091	0.0178	0.0035
0.53	0.0014	-0.0078	0.0002	0.0035	0.0038	-0.0078	0.0206	0.0041	0.0087	0.0093	0.0179	0.0035
0.55	0.0007	-0.0079	0.0007	0.0038	0.0061	-0.0078	0.0207	0.0041	0.0089	0.0093	0.0181	0.0036
0.57	0.0015	-0.0078	0.0007	0.0037	0.0085	-0.0079	0.0210	0.0042	0.0090	0.0094	0.0183	0.0036
0.59	0.0031	-0.0081	0.0007	0.0040	0.0108	-0.0079	0.0214	0.0043	0.0092	0.0096	0.0187	0.0037
0.61	0.0032	-0.0082	0.0006	0.0046	0.0131	-0.0080	0.0218	0.0044	0.0094	0.0097	0.0193	0.0038
0.63	0.0039	-0.0079	0.0004	0.0042	0.0155	-0.0080	0.0227	0.0045	0.0097	0.0101	0.0200	0.0039
0.65	0.0062	-0.0073	0.0003	0.0032	0.0177	-0.0080	0.0231	0.0045	0.0100	0.0104	0.0208	0.0040
0.67	0.0068	-0.0075	0.0006	0.0034	0.0198	-0.0081	0.0243	0.0047	0.0105	0.0110	0.0219	0.0041
0.69	0.0068	-0.0080	0.0003	0.0033	0.0220	-0.0083	0.0257	0.0049	0.0108	0.0115	0.0232	0.0043
0.71	0.0069	-0.0083	0.0004	0.0037	0.0242	-0.0085	0.0274	0.0052	0.0113	0.0120	0.0248	0.0045
0.73	0.0083	-0.0085	0.0001	0.0036	0.0267	-0.0088	0.0295	0.0055	0.0120	0.0129	0.0267	0.0048
0.75	0.0098	-0.0089	0.0001	0.0026	0.0313	-0.0093	0.0318	0.0059	0.0127	0.0138	0.0290	0.0051
0.77	0.0123	-0.0096	0.0005	0.0021	0.0366	-0.0097	0.0346	0.0065	0.0138	0.0148	0.0321	0.0055
0.79	0.0154	-0.0107	0.0011	0.0036	0.0410	-0.0102	0.0388	0.0072	0.0153	0.0162	0.0359	0.0061
0.81	0.0181	-0.0108	0.0005	0.0031	0.0455	-0.0107	0.0441	0.0080	0.0170	0.0177	0.0407	0.0067
0.83	0.0192	-0.0108	0.0007	0.0027	0.0505	-0.0113	0.0504	0.0090	0.0191	0.0203	0.0469	0.0076
0.85	0.0228	-0.0114	0.0011	0.0042	0.0559	-0.0120	0.0592	0.0103	0.0227	0.0236	0.0554	0.0087
0.87	0.0256	-0.0127	0.0018	0.0047	0.0616	-0.0133	0.0616	0.0122	0.0266	0.0281	0.0677	0.0103
0.89	0.0345	-0.0159	0.0013	0.0073	0.0693	-0.0152	0.0693	0.0152	0.0323	0.0347	0.0863	0.0125
0.91	0.0419	-0.0175	0.0020	0.0102	0.0798	-0.0172	0.0809	0.0195	0.0425	0.0444	0.1161	0.0158
0.93	0.0521	-0.0216	0.0008	0.0131	0.0925	-0.0196	0.0925	0.0261	0.0565	0.0605	0.1707	0.0213
0.95	0.0638	-0.0209	0.0012	0.0159	0.1114	-0.0210	0.1114	0.3184	0.0886	0.0928	0.2923	0.0316
0.97	0.0976	-0.0236	-0.0045	0.0225	0.1472	-0.0234	0.1472	0.7632	0.1654	0.1762	0.7028	0.0567
0.99	0.1356	-0.0165	-0.0083	0.0133	0.2290	-0.0215	0.2290	7.9011	0.6875	0.7477	7.5178	0.1673

Notes: Bias and MSE of  $\hat{\beta}_n$  and  $\hat{\beta}_n^{\zeta^*/2}$ . Sample size of  $n = 1000$ . The number of replications is 5,000. The optimal bandwidth is  $\zeta^*/2$ .

Table 7: Location-shift-scale model 1: bias and MSE of estimators

$\tau$	Bias						MSE						
	canonical			smoothed			canonical			smoothed			
	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	
0.01	-0.0290	-0.0014	0.0156	-0.0016	-0.1912	-0.0010	0.0440	-0.0009	0.0098	0.0002	0.0020	0.0003	4.5652
0.03	-0.0115	-0.0008	0.0061	-0.0007	-0.1177	-0.0004	0.0266	-0.0004	0.0036	0.0001	0.0008	0.0001	3.1117
0.05	-0.0069	-0.0006	0.0039	-0.0004	-0.0879	-0.0004	0.0200	-0.0002	0.0023	0.0001	0.0006	0.0001	2.4677
0.07	-0.0049	-0.0005	0.0030	-0.0003	-0.0698	-0.0004	0.0161	-0.0001	0.0018	0	0.0004	0.0001	1.9788
0.09	-0.0035	-0.0005	0.0021	-0.0002	-0.0574	-0.0003	0.0133	-0.0001	0.0015	0	0.0004	0	1.6489
0.11	-0.0028	-0.0004	0.0018	-0.0002	-0.0483	-0.0003	0.0113	-0.0001	0.0014	0	0.0003	0	1.3899
0.13	-0.0028	-0.0003	0.0016	-0.0001	-0.0412	-0.0002	0.0097	-0.0001	0.0012	0	0.0003	0	1.1792
0.15	-0.0026	-0.0002	0.0014	-0.0001	-0.0353	-0.0002	0.0084	-0.0001	0.0012	0	0.0003	0	1.0036
0.17	-0.0022	-0.0002	0.0014	-0.0002	-0.0304	-0.0001	0.0073	-0.0001	0.0011	0	0.0003	0	0.8547
0.19	-0.0020	-0.0002	0.0011	-0.0001	-0.0261	-0.0001	0.0064	0	0.0010	0	0.0003	0	0.7267
0.21	-0.0019	-0.0001	0.0009	-0.0001	-0.0225	-0.0001	0.0055	0	0.0010	0	0.0002	0	0.6157
0.23	-0.0015	-0.0001	0.0008	-0.0001	-0.0194	-0.0001	0.0048	0	0.0009	0	0.0002	0	0.5188
0.25	-0.0013	-0.0001	0.0007	-0.0001	-0.0165	-0.0001	0.0041	0	0.0009	0	0.0002	0	0.4340
0.27	-0.0009	-0.0002	0.0006	0	-0.0141	-0.0001	0.0035	0	0.0009	0	0.0002	0	0.3595
0.29	-0.0008	-0.0001	0.0004	0	-0.0121	-0.0001	0.0030	0	0.0008	0	0.0002	0	0.2941
0.31	-0.0005	-0.0002	0.0004	0	-0.0103	-0.0001	0.0026	0	0.0008	0	0.0002	0	0.2368
0.33	-0.0004	-0.0002	0.0003	0	-0.0089	-0.0001	0.0022	0	0.0008	0	0.0002	0	0.1868
0.35	-0.0004	-0.0001	0.0002	0	-0.0078	-0.0001	0.0019	0	0.0008	0	0.0002	0	0.1435
0.37	-0.0004	-0.0001	0.0001	0	-0.0068	-0.0001	0.0016	0	0.0008	0	0.0002	0	0.1066
0.39	-0.0004	-0.0001	0.0002	0	-0.0060	-0.0001	0.0014	0	0.0007	0	0.0002	0	0.0757
0.41	-0.0002	-0.0001	0	0	-0.0052	-0.0001	0.0014	0	0.0007	0	0.0002	0	0.0504
0.43	-0.0001	-0.0001	0.0001	0	-0.0042	-0.0001	0.0010	0	0.0007	0	0.0002	0	0.0306
0.45	0	-0.0001	0.0001	0	-0.0031	-0.0001	0.0007	0	0.0007	0	0.0002	0	0.0160
0.47	0.0002	-0.0001	0	0	-0.0018	-0.0001	0.0004	0	0.0007	0	0.0002	0	0.0064
0.49	0.0003	-0.0001	-0.0001	0.0001	-0.0003	-0.0001	0.0001	0	0.0007	0	0.0002	0	0.0016
0.51	0.0004	-0.0001	0	0	0.0012	-0.0001	-0.0002	0	0.0007	0	0.0002	0	0.0015
0.53	0.0005	-0.0001	-0.0001	0	0.0027	-0.0001	-0.0005	0	0.0007	0	0.0002	0	0.0062
0.55	0.0008	-0.0001	-0.0003	0	0.0040	-0.0001	-0.0009	0	0.0007	0	0.0002	0	0.0158
0.57	0.0009	-0.0001	-0.0003	0	0.0050	-0.0001	-0.0011	0	0.0007	0	0.0002	0	0.0303
0.59	0.0010	-0.0001	-0.0003	0.0001	0.0060	-0.0001	-0.0014	0	0.0007	0	0.0002	0	0.0501
0.61	0.0012	-0.0001	-0.0004	0.0001	0.0069	-0.0001	-0.0016	0	0.0007	0	0.0002	0	0.0752
0.63	0.0014	-0.0001	-0.0003	0	0.0077	-0.0001	-0.0017	0	0.0008	0	0.0002	0	0.1061
0.65	0.0012	0	-0.0002	0	0.0086	-0.0001	-0.0019	0	0.0008	0	0.0002	0	0.1429
0.67	0.0012	0	-0.0002	0.0001	0.0096	-0.0001	-0.0021	0.0001	0.0008	0	0.0002	0	0.1862
0.69	0.0011	0	-0.0002	0.0001	0.0110	-0.0001	-0.0024	0.0001	0.0008	0	0.0002	0	0.2361
0.71	0.0014	0	-0.0003	0.0001	0.0127	-0.0001	-0.0029	0.0001	0.0008	0	0.0002	0	0.2934
0.73	0.0013	0	-0.0004	0.0002	0.0146	0	-0.0033	0.0001	0.0008	0	0.0002	0	0.3589
0.75	0.0012	0.0001	-0.0004	0.0002	0.0170	0	-0.0039	0.0001	0.0009	0	0.0002	0	0.4334
0.77	0.0016	0.0001	-0.0007	0.0001	0.0197	0	-0.0046	0.0001	0.0009	0	0.0002	0	0.5183
0.79	0.0016	0.0001	-0.0008	0.0002	0.0228	0	-0.0054	0.0001	0.0009	0	0.0002	0	0.6153
0.81	0.0020	0.0001	-0.0009	0.0002	0.0263	0	-0.0062	0.0001	0.0010	0	0.0002	0	0.7265
0.83	0.0023	0.0001	-0.0012	0.0002	0.0305	0.0001	-0.0072	0.0001	0.0011	0	0.0003	0	0.8545
0.85	0.0024	0.0002	-0.0015	0.0003	0.0355	0.0001	-0.0085	0.0001	0.0011	0	0.0003	0	1.0032
0.87	0.0026	0.0003	-0.0017	0.0002	0.0414	0.0001	-0.0098	0.0001	0.0012	0	0.0003	0	1.1786
0.89	0.0034	0.0003	-0.0020	0.0002	0.0487	0.0002	-0.0114	0.0001	0.0014	0	0.0003	0	1.3888
0.91	0.0042	0.0003	-0.0024	0.0003	0.0581	0.0002	-0.0134	0.0001	0.0015	0	0.0004	0	1.6471
0.93	0.0053	0.0004	-0.0029	0.0003	0.0704	0.0002	-0.0161	0.0001	0.0018	0.0001	0.0004	0.0001	1.9772
0.95	0.0072	0.0005	-0.0037	0.0004	0.0885	0.0003	-0.0200	0.0002	0.0023	0.0001	0.0006	0.0001	2.4249
0.97	0.0122	0.0006	-0.0061	0.0006	0.1184	0.0004	-0.0268	0.0003	0.0036	0.0001	0.0008	0.0001	3.1093
0.99	0.0320	0.0012	-0.0161	0.0014	0.1933	0.0008	-0.0447	0.0007	0.0104	0.0002	0.0021	0.0003	4.5564
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Notes: Bias and MSE of  $\hat{\beta}_n$  and  $\hat{\rho}_n^{2\zeta^*}$ . Sample size of  $n = 1000$ . The number of replications is 5,000. The optimal bandwidth is  $\zeta^*/2$ .



Table 8: Location-shift-scale model 2: bias and MSE of estimators

$\tau$	Bias						smoothed						canonical						MSE					
	canonical			smoothed			smoothed			canonical			smoothed			canonical			MSE					
	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$				
0.01	-0.1517	-0.0060	0.0587	-0.0127	-0.4102	-0.0055	0.1107	-0.0108	0.1734	0.0030	0.0406	0.0058	0.2719	0.0027	0.0407	0.0051								
0.03	-0.0422	-0.0020	0.0167	-0.0032	-0.1954	-0.0016	0.0474	-0.0022	0.0237	0.0005	0.0064	0.0008	0.0559	0.0005	0.0074	0.0007								
0.05	-0.0220	-0.0011	0.0094	-0.0020	-0.1353	-0.0009	0.0323	-0.0013	0.0103	0.0002	0.0029	0.0003	0.0272	0.0002	0.0036	0.0003								
0.07	-0.0140	-0.0009	0.0069	-0.0013	-0.1038	-0.0006	0.0249	-0.0009	0.0062	0.0001	0.0018	0.0002	0.0166	0.0002	0.0022	0.0002								
0.09	-0.0094	-0.0007	0.0050	-0.0010	-0.0834	-0.0005	0.0197	-0.0006	0.0043	0.0001	0.0012	0.0001	0.0113	0.0001	0.0015	0.0002								
0.11	-0.0078	-0.0005	0.0037	-0.0007	-0.0691	-0.0004	0.0161	-0.0004	0.0032	0.0001	0.0009	0.0001	0.0083	0.0001	0.0012	0.0001								
0.13	-0.0057	-0.0005	0.0028	-0.0005	-0.0581	-0.0003	0.0134	-0.0003	0.0025	0.0001	0.0007	0.0001	0.0063	0.0001	0.0009	0.0001								
0.15	-0.0046	-0.0004	0.0022	-0.0004	-0.0492	-0.0003	0.0114	-0.0002	0.0022	0.0001	0.0006	0.0001	0.0049	0.0001	0.0007	0.0001								
0.17	-0.0037	-0.0004	0.0021	-0.0003	-0.0420	-0.0003	0.0098	-0.0001	0.0019	0	0.0005	0.0001	0.0040	0.0001	0.0006	0.0001								
0.19	-0.0029	-0.0004	0.0018	-0.0002	-0.0361	-0.0003	0.0085	-0.0001	0.0016	0	0.0005	0.0001	0.0033	0	0.0005	0.0001								
0.21	-0.0024	-0.0004	0.0016	-0.0002	-0.0310	-0.0003	0.0074	-0.0001	0.0015	0	0.0004	0	0.0028	0	0.0005	0.0001								
0.23	-0.0018	-0.0003	0.0014	-0.0002	-0.0267	-0.0003	0.0065	-0.0001	0.0013	0	0.0004	0	0.0024	0	0.0004	0.0001								
0.25	-0.0016	-0.0003	0.0013	-0.0002	-0.0231	-0.0002	0.0057	-0.0001	0.0012	0	0.0003	0	0.0021	0	0.0004	0								
0.27	-0.0017	-0.0002	0.0013	-0.0001	-0.0200	-0.0002	0.0051	0	0.0011	0	0.0003	0	0.0018	0	0.0003	0								
0.29	-0.0016	-0.0002	0.0012	-0.0001	-0.0174	-0.0002	0.0045	0	0.0011	0	0.0003	0	0.0017	0	0.0003	0								
0.31	-0.0013	-0.0002	0.0011	-0.0001	-0.0151	-0.0002	0.0039	0	0.0010	0	0.0003	0	0.0015	0	0.0003	0								
0.33	-0.0011	-0.0001	0.0009	-0.0001	-0.0128	-0.0002	0.0033	0	0.0009	0	0.0002	0	0.0014	0	0.0003	0								
0.35	-0.0008	-0.0001	0.0006	0	-0.0108	-0.0001	0.0027	0	0.0009	0	0.0002	0	0.0013	0	0.0003	0								
0.37	-0.0007	-0.0001	0.0005	-0.0001	-0.0090	-0.0001	0.0023	0	0.0009	0	0.0002	0	0.0012	0	0.0003	0								
0.39	-0.0004	-0.0001	0.0006	-0.0001	-0.0074	-0.0001	0.0020	0	0.0009	0	0.0002	0	0.0012	0	0.0003	0								
0.41	-0.0003	-0.0001	0.0005	-0.0001	-0.0059	-0.0001	0.0016	0	0.0008	0	0.0002	0	0.0011	0	0.0003	0								
0.43	-0.0001	-0.0001	0.0003	0	-0.0044	-0.0001	0.0012	0	0.0008	0	0.0002	0	0.0011	0	0.0002	0								
0.45	0.0002	-0.0001	0.0001	0	-0.0029	-0.0001	0.0008	0	0.0008	0	0.0002	0	0.0011	0	0.0002	0								
0.47	0.0002	-0.0001	0	0	-0.0016	-0.0001	0.0004	0	0.0008	0	0.0002	0	0.0011	0	0.0002	0								
0.49	0.0003	-0.0001	-0.0001	0	-0.0002	-0.0001	0	0	0.0008	0	0.0002	0	0.0011	0	0.0002	0								
0.51	0.0004	0	-0.0002	0	0.0011	-0.0001	-0.0003	0	0.0008	0	0.0002	0	0.0011	0	0.0002	0								
0.53	0.0006	-0.0001	-0.0003	0	0.0025	-0.0001	-0.0007	0.0001	0.0008	0	0.0002	0	0.0011	0	0.0002	0								
0.55	0.0006	0	-0.0004	0.0001	0.0038	-0.0001	-0.0010	0.0001	0.0008	0	0.0002	0	0.0011	0	0.0002	0								
0.57	0.0007	0	-0.0004	0.0001	0.0051	-0.0001	-0.0013	0.0001	0.0008	0	0.0002	0	0.0011	0	0.0003	0								
0.59	0.0009	0	-0.0004	0.0001	0.0066	-0.0001	-0.0016	0.0001	0.0008	0	0.0002	0	0.0011	0	0.0003	0								
0.61	0.0010	0	-0.0004	0.0002	0.0080	-0.0001	-0.0019	0.0002	0.0009	0	0.0002	0	0.0012	0	0.0003	0								
0.63	0.0012	0	-0.0005	0.0002	0.0097	-0.0001	-0.0023	0.0002	0.0009	0	0.0002	0	0.0012	0	0.0003	0								
0.65	0.0015	-0.0001	-0.0006	0.0002	0.0115	-0.0001	-0.0027	0.0002	0.0009	0	0.0003	0	0.0013	0	0.0003	0								
0.67	0.0015	0	-0.0008	0.0002	0.0135	-0.0001	-0.0032	0.0002	0.0010	0	0.0003	0	0.0014	0	0.0003	0								
0.69	0.0018	0	-0.0008	0.0002	0.0157	-0.0001	-0.0037	0.0003	0.0010	0	0.0003	0	0.0015	0	0.0003	0								
0.71	0.0021	0	-0.0010	0.0003	0.0181	-0.0001	-0.0043	0.0003	0.0011	0	0.0003	0	0.0017	0	0.0003	0								
0.73	0.0025	0	-0.0011	0.0004	0.0210	-0.0001	-0.0049	0.0003	0.0011	0	0.0003	0	0.0019	0	0.0004	0								
0.75	0.0031	0	-0.0013	0.0003	0.0244	-0.0001	-0.0056	0.0003	0.0012	0	0.0003	0	0.0021	0	0.0004	0								
0.77	0.0037	-0.0001	-0.0015	0.0004	0.0284	-0.0001	-0.0065	0.0004	0.0014	0	0.0004	0	0.0025	0	0.0004	0.0001								
0.79	0.0040	0	-0.0017	0.0005	0.0329	-0.0002	-0.0075	0.0004	0.0015	0	0.0005	0.0001	0.0029	0	0.0005	0.0001								
0.81	0.0046	0	-0.0018	0.0005	0.0380	-0.0001	-0.0085	0.0004	0.0017	0	0.0005	0.0001	0.0034	0	0.0005	0.0001								
0.83	0.0051	0	-0.0019	0.0006	0.0440	-0.0001	-0.0098	0.0005	0.0019	0	0.0005	0.0001	0.0041	0.0001	0.0006	0.0001								
0.85	0.0062	0	-0.0024	0.0007	0.0512	-0.0001	-0.0114	0.0005	0.0022	0.0001	0.0006	0.0001	0.0051	0.0001	0.0007	0.0001								
0.87	0.0075	0.0001	-0.0029	0.0008	0.0602	-0.0001	-0.0135	0.0006	0.0026	0.0001	0.0007	0.0001	0.0065	0.0001	0.0009	0.0001								
0.89	0.0091	0.0001	-0.0036	0.0009	0.0713	-0.0001	-0.0160	0.0007	0.0033	0.0001	0.0009	0.0001	0.0086	0.0001	0.0011	0.0001								
0.91	0.0121	0.0002	-0.0046	0.0011	0.0857	0	-0.0194	0.0008	0.0045	0.0001	0.0012	0.0002	0.0118	0.0001	0.0015	0.0002								
0.93	0.0161	0.0004	-0.0062	0.0015	0.1057	0.0001	-0.0242	0.0011	0.0064	0.0001	0.0017	0.0002	0.0171	0.0001	0.0021	0.0002								
0.95	0.0233	0.0007	-0.0095	0.0024	0.1367	0.0004	-0.0322	0.0017	0.0105	0.0002	0.0028	0.0004	0.0276	0.0002	0.0035	0.0004								
0.97	0.0431	0.0013	-0.0173	0.0038	0.1962	0.0009	-0.0482	0.0032	0.0242	0.0005	0.0062	0.0008	0.0557	0.0005	0.0072	0.0007								
0.99	0.1491	0.0044	-0.0564	0.0137	0.4092	0.0038	-0.1099	0.0125	0.1665	0.0027	0.0390	0.0058	0.2654	0.0024	0.0395	0.0052								

Notes: Bias and MSE of  $\hat{\beta}_n$  and  $\hat{\rho}_n^{2\zeta^*}/2$ . Sample size of  $n = 1000$ . The number of replications is 5,000. The optimal bandwidth is  $\zeta^*/2$ .

Table 9: *Model (2) defined through Proposition 2 when  $\lambda(\tau) = \sqrt{\tau}$ : bias and MSE of estimators*

$\tau$	Bias			MSE					
	canonical			smoothed			canonical		
	$\alpha_0$	$\alpha_1$	$\theta_1$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\alpha_0$	$\alpha_1$	$\theta_1$
0.01	0.0036	-0.0216	0.0565	-0.1820	-0.0269	0.1256	0.0493	0.0674	0.0682
0.03	0.0118	-0.0128	0.0038	-0.1119	-0.0231	0.0596	0.0259	0.0371	0.0368
0.05	0.0188	-0.0147	-0.0098	-0.0804	-0.0207	0.0295	0.0205	0.0285	0.0281
0.07	0.0207	-0.0137	-0.0147	-0.0601	-0.0190	-0.0103	0.0171	0.0237	0.0233
0.09	0.0228	-0.0140	-0.0189	-0.0455	-0.0177	-0.0035	0.0157	0.0214	0.0212
0.11	0.0238	-0.0143	-0.0209	-0.0340	-0.0166	-0.0141	0.0146	0.0195	0.0194
0.13	0.0228	-0.0133	-0.0212	-0.0247	-0.0157	-0.0225	0.0137	0.0180	0.0182
0.15	0.0231	-0.0134	-0.0219	-0.0169	-0.0148	-0.0294	0.0133	0.0171	0.0173
0.17	0.0241	-0.0143	-0.0233	-0.0103	-0.0141	-0.0351	0.0131	0.0167	0.0169
0.19	0.0229	-0.0122	-0.0237	-0.0045	-0.0135	-0.0398	0.0125	0.0159	0.0164
0.21	0.0230	-0.0126	-0.0239	0.0006	-0.0129	-0.0438	0.0122	0.0154	0.0158
0.23	0.0228	-0.0123	-0.0239	0.0051	-0.0124	-0.0471	0.0119	0.0148	0.0154
0.25	0.0229	-0.0120	-0.0247	0.0093	-0.0119	-0.0496	0.0118	0.0146	0.0153
0.27	0.0229	-0.0113	-0.0254	0.0132	-0.0114	-0.0512	0.0116	0.0142	0.0151
0.29	0.0222	-0.0115	-0.0243	0.0162	-0.0109	-0.0531	0.0113	0.0138	0.0148
0.31	0.0213	-0.0107	-0.0239	0.0189	-0.0105	-0.0545	0.0111	0.0136	0.0146
0.33	0.0215	-0.0115	-0.0234	0.0214	-0.0101	-0.0556	0.0110	0.0133	0.0146
0.35	0.0206	-0.0113	-0.0225	0.0236	-0.0098	-0.0564	0.0109	0.0132	0.0146
0.37	0.0200	-0.0106	-0.0222	0.0256	-0.0094	-0.0568	0.0108	0.0132	0.0141
0.39	0.0195	-0.0103	-0.0220	0.0274	-0.0091	-0.0570	0.0108	0.0131	0.0140
0.41	0.0199	-0.0106	-0.0226	0.0290	-0.0088	-0.0569	0.0108	0.0132	0.0140
0.43	0.0200	-0.0108	-0.0228	0.0305	-0.0084	-0.0566	0.0108	0.0131	0.0140
0.45	0.0190	-0.0104	-0.0216	0.0317	-0.0081	-0.0559	0.0107	0.0128	0.0139
0.47	0.0188	-0.0101	-0.0217	0.0329	-0.0079	-0.0551	0.0108	0.0128	0.0140
0.49	0.0174	-0.0088	-0.0207	0.0339	-0.0076	-0.0540	0.0108	0.0128	0.0140
0.51	0.0171	-0.0087	-0.0206	0.0347	-0.0073	-0.0526	0.0107	0.0129	0.0138
0.53	0.0173	-0.0089	-0.0205	0.0354	-0.0071	-0.0510	0.0108	0.0128	0.0139
0.55	0.0164	-0.0083	-0.0199	0.0361	-0.0069	-0.0492	0.0108	0.0129	0.0138
0.57	0.0154	-0.0079	-0.0189	0.0365	-0.0067	-0.0471	0.0109	0.0127	0.0139
0.59	0.0152	-0.0081	-0.0185	0.0369	-0.0065	-0.0448	0.0110	0.0127	0.0140
0.61	0.0148	-0.0074	-0.0186	0.0372	-0.0063	-0.0421	0.0112	0.0126	0.0143
0.63	0.0153	-0.0075	-0.0192	0.0374	-0.0061	-0.0392	0.0112	0.0125	0.0144
0.65	0.0148	-0.0071	-0.0189	0.0375	-0.0059	-0.0360	0.0112	0.0124	0.0143
0.67	0.0145	-0.0072	-0.0185	0.0374	-0.0058	-0.0324	0.0111	0.0123	0.0141
0.69	0.0141	-0.0071	-0.0181	0.0373	-0.0057	-0.0285	0.0112	0.0125	0.0142
0.71	0.0138	-0.0072	-0.0178	0.0371	-0.0055	-0.0242	0.0113	0.0126	0.0143
0.73	0.0139	-0.0081	-0.0174	0.0368	-0.0055	-0.0195	0.0113	0.0126	0.0143
0.75	0.0123	-0.0075	-0.0160	0.0369	-0.0054	-0.0144	0.0113	0.0127	0.0143
0.77	0.0116	-0.0078	-0.0148	0.0368	-0.0054	-0.0086	0.0115	0.0128	0.0145
0.79	0.0119	-0.0088	-0.0144	0.0362	-0.0054	-0.0022	0.0116	0.0130	0.0148
0.81	0.0109	-0.0084	-0.0134	0.0356	-0.0055	0.0049	0.0119	0.0130	0.0151
0.83	0.0095	-0.0083	-0.0115	0.0350	-0.0056	0.0129	0.0122	0.0133	0.0155
0.85	0.0075	-0.0065	-0.0103	0.0343	-0.0059	0.0219	0.0124	0.0137	0.0159
0.87	0.0063	-0.0069	-0.0081	0.0336	-0.0062	0.0322	0.0127	0.0139	0.0162
0.89	0.0027	-0.0047	-0.0048	0.0329	-0.0067	0.0442	0.0130	0.0143	0.0169
0.91	-0.0009	-0.0040	-0.0003	0.0324	-0.0074	0.0583	0.0139	0.0150	0.0179
0.93	-0.0045	-0.0031	0.0041	0.0323	-0.0085	0.0757	0.0151	0.0162	0.0192
0.95	-0.0098	-0.0022	0.0108	0.0332	-0.0101	0.0980	0.0168	0.0176	0.0215
0.97	-0.0189	-0.0025	0.0231	0.0366	-0.0127	0.1299	0.0206	0.0221	0.0268
0.99	-0.0608	-0.0095	0.0783	0.0514	-0.0183	0.1865	0.0331	0.0346	0.0454

Notes: Bias and MSE of  $\hat{\beta}_n$  and  $\hat{\beta}_n^{2\zeta^*}$ . Sample size of  $n = 100$ . The number of replications is 5,000. The optimal bandwidth is  $2\zeta^*$ .

Table 10: Model (2) defined through Proposition 2 when  $\lambda(\tau) = \frac{1}{1+e^{-\tau}}$ : bias and MSE of estimators

$\tau$	Bias			MSE					
	canonical			smoothed			canonical		
	$\alpha_0$	$\alpha_1$	$\theta_1$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\alpha_0$	$\alpha_1$	$\theta_1$
0.01	0.0142	-0.0283	0.0431	-0.1508	-0.0625	0.0897	0.0438	0.0610	0.0613
0.03	0.0059	-0.0071	0.0082	-0.1059	-0.0344	0.0479	0.0249	0.0342	0.0352
0.05	0.0035	0.0003	0.0019	-0.0849	-0.0214	0.0278	0.0194	0.0266	0.0269
0.07	0.0057	0.0002	-0.0034	-0.0709	-0.0132	0.0147	0.0170	0.0231	0.0229
0.09	0.0065	0.0002	-0.0058	-0.0605	-0.0073	0.0052	0.0150	0.0207	0.0201
0.11	0.0090	-0.0011	-0.0094	-0.0521	-0.0028	-0.0023	0.0139	0.0191	0.0189
0.13	0.0089	-0.0003	-0.0108	-0.0451	0.0007	-0.0084	0.0131	0.0182	0.0180
0.15	0.0091	-0.0001	-0.0119	-0.0391	0.0035	-0.0133	0.0129	0.0177	0.0178
0.17	0.0091	0.0004	-0.0129	-0.0338	0.0058	-0.0175	0.0123	0.0170	0.0172
0.19	0.0089	0.0005	-0.0131	-0.0290	0.0077	-0.0210	0.0120	0.0167	0.0169
0.21	0.0094	0	-0.0138	-0.0248	0.0092	-0.0239	0.0118	0.0163	0.0168
0.23	0.0099	-0.0005	-0.0143	-0.0209	0.0105	-0.0264	0.0119	0.0162	0.0167
0.25	0.0094	-0.0010	-0.0136	-0.0171	0.0115	-0.0284	0.0117	0.0158	0.0165
0.27	0.0096	-0.0009	-0.0143	-0.0133	0.0121	-0.0299	0.0116	0.0157	0.0164
0.29	0.0093	-0.0009	-0.0141	-0.0104	0.0128	-0.0314	0.0116	0.0154	0.0167
0.31	0.0092	-0.0012	-0.0135	-0.0076	0.0133	-0.0326	0.0115	0.0151	0.0165
0.33	0.0089	-0.0008	-0.0141	-0.0050	0.0137	-0.0335	0.0113	0.0151	0.0164
0.35	0.0089	-0.0011	-0.0142	-0.0026	0.0140	-0.0342	0.0112	0.0148	0.0162
0.37	0.0082	-0.0010	-0.0131	-0.0003	0.0141	-0.0347	0.0112	0.0148	0.0160
0.39	0.0077	-0.0008	-0.0128	0.0019	0.0141	-0.0351	0.0112	0.0146	0.0158
0.41	0.0071	-0.0004	-0.0125	0.0039	0.0141	-0.0352	0.0112	0.0146	0.0159
0.43	0.0073	-0.0007	-0.0126	0.0059	0.0139	-0.0351	0.0112	0.0145	0.0158
0.45	0.0078	-0.0013	-0.0131	0.0078	0.0137	-0.0349	0.0112	0.0147	0.0157
0.47	0.0075	-0.0006	-0.0134	0.0096	0.0133	-0.0346	0.0114	0.0149	0.0159
0.49	0.0068	0	-0.0127	0.0114	0.0129	-0.0340	0.0116	0.0150	0.0162
0.51	0.0063	-0.0002	-0.0121	0.0131	0.0124	-0.0333	0.0117	0.0150	0.0163
0.53	0.0062	0.0002	-0.0125	0.0147	0.0118	-0.0325	0.0118	0.0152	0.0162
0.55	0.0054	0.0010	-0.0121	0.0163	0.0111	-0.0314	0.0119	0.0152	0.0164
0.57	0.0046	0.0010	-0.0109	0.0179	0.0103	-0.0302	0.0120	0.0153	0.0164
0.59	0.0053	0.0004	-0.0118	0.0194	0.0095	-0.0289	0.0123	0.0157	0.0166
0.61	0.0045	0.0006	-0.0109	0.0209	0.0085	-0.0274	0.0123	0.0158	0.0166
0.63	0.0044	0.0001	-0.0105	0.0224	0.0075	-0.0257	0.0124	0.0158	0.0169
0.65	0.0039	0.0002	-0.0101	0.0239	0.0063	-0.0238	0.0125	0.0159	0.0169
0.67	0.0038	-0.0001	-0.0100	0.0255	0.0050	-0.0217	0.0127	0.0161	0.0170
0.69	0.0039	-0.0001	-0.0102	0.0270	0.0036	-0.0194	0.0128	0.0164	0.0172
0.71	0.0040	-0.0008	-0.0101	0.0286	0.0021	-0.0168	0.0129	0.0165	0.0174
0.73	0.0039	-0.0005	-0.0105	0.0302	0.0004	-0.0141	0.0132	0.0167	0.0177
0.75	0.0035	-0.0005	-0.0099	0.0325	-0.0014	-0.0110	0.0132	0.0169	0.0178
0.77	0.0029	-0.0003	-0.0095	0.0347	-0.0034	-0.0076	0.0133	0.0172	0.0179
0.79	0.0029	-0.0007	-0.0092	0.0367	-0.0057	-0.0037	0.0137	0.0175	0.0185
0.81	0.0026	-0.0010	-0.0089	0.0389	-0.0083	0.0005	0.0139	0.0175	0.0187
0.83	0.0013	-0.0007	-0.0076	0.0414	-0.0112	0.0054	0.0143	0.0184	0.0190
0.85	0.0006	-0.0008	-0.0068	0.0441	-0.0145	0.0108	0.0147	0.0190	0.0195
0.87	-0.0004	-0.0019	-0.0048	0.0472	-0.0182	0.0171	0.0154	0.0198	0.0204
0.89	-0.0006	-0.0044	-0.0029	0.0510	-0.0226	0.0244	0.0162	0.0210	0.0211
0.91	-0.0027	-0.0043	-0.0002	0.0556	-0.0279	0.0331	0.0172	0.0222	0.0222
0.93	-0.0049	-0.0051	0.0029	0.0617	-0.0344	0.0439	0.0186	0.0237	0.0239
0.95	-0.0083	-0.0060	0.0082	0.0705	-0.0429	0.0580	0.0208	0.0262	0.0267
0.97	-0.0131	-0.0086	0.0155	0.0850	-0.0553	0.0783	0.0247	0.0320	0.0324
0.99	-0.0342	-0.0310	0.0499	0.1202	-0.0784	0.1154	0.0383	0.0516	0.0526

Notes: Bias and MSE of  $\hat{\beta}_n$  and  $\hat{\beta}_n^{2\zeta^*}$ . Sample size of  $n = 100$ . The number of replications is 5,000. The optimal bandwidth is  $2\zeta^*$ .

Table 11: Model (2) defined through Proposition 2 when  $\lambda(\tau) = \sqrt[3]{\tau}$ : bias and MSE of estimators

$\tau$	Bias			MSE					
	canonical			smoothed			canonical		
	$\alpha_0$	$\alpha_1$	$\theta_1$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\alpha_0$	$\alpha_1$	$\theta_1$
0.01	0.0083	-0.0230	0.0499	-0.1663	-0.0411	0.1110	0.0443	0.0637	0.0638
0.03	0.0095	-0.0107	0.0061	-0.1033	-0.0316	0.0509	0.0258	0.0361	0.0356
0.05	0.0134	-0.0097	-0.0050	-0.0754	-0.0261	0.0237	0.0201	0.0275	0.0267
0.07	0.0169	-0.0108	-0.0121	-0.0577	-0.0222	0.0068	0.0170	0.0234	0.0227
0.09	0.0169	-0.0083	-0.0158	-0.0450	-0.0191	-0.0052	0.0151	0.0207	0.0201
0.11	0.0167	-0.0078	-0.0162	-0.0350	-0.0166	-0.0144	0.0143	0.0193	0.0189
0.13	0.0170	-0.0084	-0.0165	-0.0270	-0.0145	-0.0215	0.0133	0.0181	0.0173
0.15	0.0176	-0.0082	-0.0178	-0.0203	-0.0127	-0.0273	0.0127	0.0173	0.0165
0.17	0.0181	-0.0089	-0.0185	-0.0145	-0.0111	-0.0320	0.0125	0.0166	0.0161
0.19	0.0178	-0.0077	-0.0192	-0.0095	-0.0097	-0.0358	0.0122	0.0161	0.0157
0.21	0.0176	-0.0074	-0.0191	-0.0051	-0.0084	-0.0389	0.0118	0.0154	0.0153
0.23	0.0167	-0.0071	-0.0184	-0.0013	-0.0073	-0.0415	0.0116	0.0152	0.0151
0.25	0.0161	-0.0071	-0.0180	0.0024	-0.0063	-0.0434	0.0116	0.0151	0.0150
0.27	0.0162	-0.0077	-0.0176	0.0058	-0.0055	-0.0446	0.0114	0.0147	0.0149
0.29	0.0154	-0.0076	-0.0165	0.0085	-0.0047	-0.0459	0.0115	0.0147	0.0148
0.31	0.0153	-0.0072	-0.0168	0.0109	-0.0039	-0.0468	0.0113	0.0146	0.0145
0.33	0.0151	-0.0074	-0.0166	0.0131	-0.0033	-0.0475	0.0113	0.0144	0.0145
0.35	0.0145	-0.0068	-0.0161	0.0151	-0.0027	-0.0479	0.0112	0.0144	0.0142
0.37	0.0150	-0.0070	-0.0171	0.0168	-0.0021	-0.0481	0.0112	0.0141	0.0142
0.39	0.0149	-0.0068	-0.0173	0.0185	-0.0016	-0.0480	0.0112	0.0141	0.0142
0.41	0.0145	-0.0065	-0.0171	0.0200	-0.0012	-0.0477	0.0111	0.0140	0.0141
0.43	0.0146	-0.0073	-0.0163	0.0213	-0.0008	-0.0471	0.0111	0.0140	0.0140
0.45	0.0147	-0.0075	-0.0164	0.0225	-0.0004	-0.0464	0.0112	0.0139	0.0140
0.47	0.0149	-0.0085	-0.0163	0.0236	-0.0001	-0.0454	0.0114	0.0140	0.0141
0.49	0.0141	-0.0083	-0.0154	0.0246	0.0001	-0.0443	0.0116	0.0140	0.0141
0.51	0.0141	-0.0080	-0.0160	0.0255	0.0003	-0.0430	0.0116	0.0139	0.0140
0.53	0.0137	-0.0081	-0.0154	0.0264	0.0005	-0.0414	0.0116	0.0139	0.0141
0.55	0.0138	-0.0076	-0.0161	0.0271	0.0006	-0.0397	0.0116	0.0138	0.0140
0.57	0.0124	-0.0067	-0.0150	0.0277	0.0007	-0.0377	0.0116	0.0136	0.0140
0.59	0.0118	-0.0056	-0.0151	0.0283	0.0007	-0.0355	0.0118	0.0137	0.0142
0.61	0.0118	-0.0052	-0.0155	0.0288	0.0007	-0.0331	0.0122	0.0139	0.0146
0.63	0.0111	-0.0045	-0.0151	0.0293	0.0007	-0.0305	0.0122	0.0140	0.0147
0.65	0.0110	-0.0048	-0.0149	0.0297	0.0005	-0.0276	0.0122	0.0139	0.0148
0.67	0.0109	-0.0051	-0.0148	0.0300	0.0004	-0.0244	0.0124	0.0139	0.0151
0.69	0.0104	-0.0047	-0.0145	0.0303	0.0001	-0.0210	0.0125	0.0140	0.0152
0.71	0.0096	-0.0045	-0.0134	0.0306	-0.0001	-0.0173	0.0126	0.0141	0.0153
0.73	0.0095	-0.0052	-0.0130	0.0308	-0.0005	-0.0132	0.0128	0.0143	0.0156
0.75	0.0085	-0.0053	-0.0116	0.0316	-0.0009	-0.0087	0.0129	0.0144	0.0158
0.77	0.0079	-0.0056	-0.0105	0.0321	-0.0015	-0.0038	0.0132	0.0145	0.0160
0.79	0.0074	-0.0049	-0.0104	0.0324	-0.0022	0.0017	0.0137	0.0147	0.0166
0.81	0.0058	-0.0044	-0.0086	0.0327	-0.0030	0.0078	0.0137	0.0148	0.0168
0.83	0.0050	-0.0036	-0.0079	0.0331	-0.0040	0.0145	0.0139	0.0150	0.0170
0.85	0.0039	-0.0037	-0.0072	0.0336	-0.0052	0.0222	0.0144	0.0154	0.0177
0.87	0.0033	-0.0042	-0.0063	0.0342	-0.0066	0.0309	0.0148	0.0161	0.0182
0.89	0.0017	-0.0033	-0.0049	0.0352	-0.0084	0.0411	0.0151	0.0168	0.0187
0.91	-0.0002	-0.0035	-0.0022	0.0365	-0.0107	0.0531	0.0159	0.0175	0.0197
0.93	-0.0032	-0.0034	0.0016	0.0388	-0.0137	0.0678	0.0170	0.0189	0.0212
0.95	-0.0079	-0.0051	0.0094	0.0425	-0.0178	0.0868	0.0189	0.0208	0.0237
0.97	-0.0139	-0.0090	0.0198	0.0500	-0.0239	0.1140	0.0224	0.0254	0.0281
0.99	-0.0505	-0.0166	0.0671	0.0718	-0.0358	0.1634	0.0356	0.0414	0.0472

Notes: Bias and MSE of  $\hat{\beta}_n$  and  $\hat{\beta}_n^{2\zeta^*}$ . Sample size of  $n = 100$ . The number of replications is 5,000. The optimal bandwidth is  $2\zeta^*$ .

Table 12: *Model (2) defined through Proposition 2 when  $\lambda(\tau) = \cos(\tau)$ : bias and MSE of estimators*

$\tau$	Bias			MSE					
	canonical			smoothed			canonical		
	$\alpha_0$	$\alpha_1$	$\theta_1$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\alpha_0$	$\alpha_1$	$\theta_1$
0.01	0.0291	-0.0408	0.0284	-0.1117	-0.1026	0.0524	0.0404	0.0580	0.0551
0.03	0.0125	-0.0133	0.0036	-0.0808	-0.0606	0.0242	0.0235	0.0333	0.0312
0.05	0.0087	-0.0063	0.0002	-0.0670	-0.0404	0.0108	0.0184	0.0254	0.0248
0.07	0.0075	-0.0013	-0.0048	-0.0580	-0.0272	0.0025	0.0157	0.0214	0.0214
0.09	0.0073	-0.0008	-0.0058	-0.0515	-0.0174	-0.0035	0.0143	0.0194	0.0198
0.11	0.0065	0.0012	-0.0072	-0.0463	-0.0097	-0.0079	0.0135	0.0176	0.0187
0.13	0.0051	0.0027	-0.0070	-0.0421	-0.0034	-0.0113	0.0127	0.0168	0.0175
0.15	0.0043	0.0031	-0.0067	-0.0385	0.0019	-0.0139	0.0123	0.0162	0.0169
0.17	0.0027	0.0050	-0.0059	-0.0353	0.0064	-0.0160	0.0118	0.0156	0.0162
0.19	0.0031	0.0050	-0.0070	-0.0325	0.0103	-0.0176	0.0114	0.0152	0.0158
0.21	0.0033	0.0042	-0.0067	-0.0301	0.0137	-0.0188	0.0111	0.0149	0.0153
0.23	0.0024	0.0048	-0.0059	-0.0278	0.0166	-0.0197	0.0110	0.0146	0.0152
0.25	0.0012	0.0053	-0.0047	-0.0254	0.0191	-0.0203	0.0109	0.0146	0.0151
0.27	0.0006	0.0056	-0.0041	-0.0230	0.0210	-0.0206	0.0108	0.0145	0.0150
0.29	0.0008	0.0053	-0.0048	-0.0212	0.0229	-0.0208	0.0107	0.0143	0.0151
0.31	0.0008	0.0051	-0.0049	-0.0195	0.0246	-0.0209	0.0107	0.0143	0.0149
0.33	0.0005	0.0052	-0.0047	-0.0179	0.0260	-0.0209	0.0105	0.0142	0.0147
0.35	0.0009	0.0046	-0.0050	-0.0164	0.0272	-0.0207	0.0105	0.0141	0.0150
0.37	0.0012	0.0045	-0.0056	-0.0149	0.0282	-0.0205	0.0104	0.0141	0.0151
0.39	0.0013	0.0045	-0.0059	-0.0134	0.0290	-0.0201	0.0105	0.0142	0.0152
0.41	0.0006	0.0050	-0.0055	-0.0119	0.0296	-0.0196	0.0105	0.0145	0.0152
0.43	-0.0007	0.0064	-0.0046	-0.0105	0.0300	-0.0190	0.0105	0.0145	0.0151
0.45	-0.0008	0.0065	-0.0048	-0.0090	0.0302	-0.0184	0.0107	0.0148	0.0152
0.47	-0.0009	0.0069	-0.0051	-0.0075	0.0303	-0.0177	0.0106	0.0147	0.0150
0.49	-0.0010	0.0069	-0.0052	-0.0060	0.0301	-0.0169	0.0107	0.0148	0.0153
0.51	-0.0010	0.0071	-0.0058	-0.0045	0.0298	-0.0161	0.0108	0.0150	0.0155
0.53	-0.0010	0.0070	-0.0060	-0.0029	0.0294	-0.0152	0.0109	0.0151	0.0159
0.55	-0.0007	0.0064	-0.0059	-0.0012	0.0287	-0.0143	0.0109	0.0151	0.0160
0.57	-0.0008	0.0063	-0.0055	0.0005	0.0279	-0.0133	0.0110	0.0152	0.0159
0.59	-0.0013	0.0068	-0.0054	0.0023	0.0268	-0.0122	0.0112	0.0155	0.0163
0.61	-0.0015	0.0070	-0.0054	0.0042	0.0256	-0.0111	0.0114	0.0156	0.0165
0.63	-0.0020	0.0076	-0.0052	0.0062	0.0241	-0.0099	0.0116	0.0160	0.0169
0.65	-0.0022	0.0069	-0.0046	0.0083	0.0224	-0.0086	0.0115	0.0161	0.0168
0.67	-0.0022	0.0066	-0.0045	0.0106	0.0205	-0.0073	0.0116	0.0162	0.0169
0.69	-0.0014	0.0065	-0.0057	0.0130	0.0183	-0.0059	0.0117	0.0164	0.0171
0.71	-0.0010	0.0069	-0.0063	0.0157	0.0159	-0.0045	0.0118	0.0166	0.0173
0.73	-0.0010	0.0059	-0.0063	0.0186	0.0131	-0.0029	0.0121	0.0171	0.0177
0.75	-0.0012	0.0062	-0.0067	0.0222	0.0101	-0.0012	0.0123	0.0176	0.0181
0.77	-0.0010	0.0054	-0.0066	0.0259	0.0066	0.0006	0.0126	0.0181	0.0183
0.79	-0.0005	0.0039	-0.0065	0.0298	0.0025	0.0025	0.0130	0.0182	0.0190
0.81	-0.0006	0.0031	-0.0065	0.0342	-0.0020	0.0046	0.0131	0.0187	0.0190
0.83	-0.0011	0.0031	-0.0058	0.0391	-0.0072	0.0070	0.0136	0.0196	0.0195
0.85	-0.0007	0.0018	-0.0060	0.0447	-0.0132	0.0095	0.0140	0.0202	0.0201
0.87	-0.0003	0.0007	-0.0065	0.0512	-0.0201	0.0125	0.0146	0.0209	0.0208
0.89	-0.0013	-0.0006	-0.0039	0.0588	-0.0282	0.0159	0.0153	0.0218	0.0222
0.91	-0.0023	-0.0016	-0.0024	0.0681	-0.0379	0.0201	0.0161	0.0231	0.0235
0.93	-0.0006	-0.0062	-0.0027	0.0800	-0.0499	0.0252	0.0172	0.0256	0.0249
0.95	-0.0011	-0.0121	0.0015	0.0962	-0.0657	0.0319	0.0197	0.0293	0.0283
0.97	-0.0013	-0.0228	0.0079	0.1213	-0.0884	0.0418	0.0241	0.0357	0.0343
0.99	-0.0069	-0.0569	0.0239	0.1762	-0.1306	0.0606	0.0363	0.0574	0.0527

Notes: Bias and MSE of  $\hat{\beta}_n$  and  $\hat{\beta}_n^{2\zeta^*}$ . Sample size of  $n = 100$ . The number of replications is 5,000. The optimal bandwidth is  $2\zeta^*$ .









Table 16: *Model (2) defined through Proposition 2 when  $\lambda(\tau) = \frac{1}{1+e^{-\tau}}$ : bias and MSE of estimators*

$\tau$	Bias			MSE					
	canonical			smoothed			canonical		
	$\alpha_0$	$\alpha_1$	$\theta_1$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\alpha_0$	$\alpha_1$	$\theta_1$
0.01	0.0002	-0.0040	0.0082	-0.0917	-0.0380	0.0616	0.0081	0.0112	0.0104
0.03	0.0008	0.0001	0.0006	-0.0649	-0.0197	0.0341	0.0046	0.0064	0.0057
0.05	-0.0002	0.0018	0.0003	-0.0525	-0.0113	0.0213	0.0036	0.0051	0.0046
0.07	0.0003	0.0012	-0.0007	-0.0442	-0.0060	0.0129	0.0032	0.0045	0.0042
0.09	0.0005	0.0015	-0.0014	-0.0380	-0.0022	0.0068	0.0029	0.0041	0.0038
0.11	0.0010	0.0010	-0.0020	-0.0330	0.0007	0.0019	0.0028	0.0038	0.0036
0.13	0.0010	0.0010	-0.0020	-0.0288	0.0029	-0.0019	0.0027	0.0037	0.0034
0.15	0.0013	0.0007	-0.0021	-0.0251	0.0047	-0.0051	0.0026	0.0036	0.0033
0.17	0.0012	0.0008	-0.0021	-0.0220	0.0061	-0.0077	0.0025	0.0035	0.0032
0.19	0.0014	0.0005	-0.0023	-0.0191	0.0073	-0.0099	0.0024	0.0034	0.0032
0.21	0.0011	0.0008	-0.0020	-0.0166	0.0082	-0.0118	0.0023	0.0033	0.0031
0.23	0.0013	0.0007	-0.0025	-0.0143	0.0090	-0.0134	0.0023	0.0032	0.0031
0.25	0.0015	0.0004	-0.0025	-0.0121	0.0096	-0.0147	0.0023	0.0031	0.0031
0.27	0.0015	0.0005	-0.0026	-0.0102	0.0101	-0.0157	0.0023	0.0031	0.0031
0.29	0.0012	0.0010	-0.0026	-0.0084	0.0105	-0.0167	0.0023	0.0031	0.0030
0.31	0.0013	0.0007	-0.0024	-0.0067	0.0107	-0.0174	0.0023	0.0031	0.0030
0.33	0.0012	0.0010	-0.0028	-0.0051	0.0109	-0.0180	0.0023	0.0031	0.0030
0.35	0.0010	0.0011	-0.0026	-0.0036	0.0111	-0.0185	0.0023	0.0030	0.0030
0.37	0.0011	0.0011	-0.0027	-0.0022	0.0111	-0.0188	0.0022	0.0030	0.0030
0.39	0.0011	0.0010	-0.0025	-0.0009	0.0111	-0.0190	0.0022	0.0029	0.0031
0.41	0.0013	0.0008	-0.0029	0.0004	0.0110	-0.0191	0.0023	0.0030	0.0031
0.43	0.0013	0.0008	-0.0030	0.0016	0.0109	-0.0191	0.0023	0.0030	0.0031
0.45	0.0014	0.0007	-0.0031	0.0027	0.0107	-0.0190	0.0023	0.0030	0.0031
0.47	0.0013	0.0007	-0.0027	0.0038	0.0105	-0.0189	0.0023	0.0029	0.0031
0.49	0.0013	0.0008	-0.0029	0.0049	0.0102	-0.0186	0.0023	0.0029	0.0031
0.51	0.0016	0.0006	-0.0032	0.0059	0.0099	-0.0182	0.0023	0.0030	0.0031
0.53	0.0014	0.0007	-0.0031	0.0069	0.0095	-0.0178	0.0023	0.0030	0.0031
0.55	0.0015	0.0010	-0.0036	0.0079	0.0091	-0.0173	0.0023	0.0030	0.0031
0.57	0.0015	0.0008	-0.0035	0.0089	0.0086	-0.0167	0.0023	0.0030	0.0031
0.59	0.0018	0.0006	-0.0039	0.0099	0.0081	-0.0159	0.0023	0.0030	0.0032
0.61	0.0015	0.0009	-0.0035	0.0108	0.0075	-0.0151	0.0024	0.0031	0.0032
0.63	0.0013	0.0009	-0.0033	0.0117	0.0069	-0.0142	0.0024	0.0031	0.0032
0.65	0.0013	0.0010	-0.0034	0.0127	0.0062	-0.0132	0.0024	0.0031	0.0032
0.67	0.0009	0.0015	-0.0032	0.0136	0.0054	-0.0121	0.0024	0.0031	0.0033
0.69	0.0009	0.0012	-0.0030	0.0146	0.0046	-0.0108	0.0025	0.0032	0.0033
0.71	0.0009	0.0011	-0.0029	0.0156	0.0037	-0.0094	0.0025	0.0032	0.0034
0.73	0.0011	0.0009	-0.0031	0.0166	0.0026	-0.0078	0.0026	0.0033	0.0034
0.75	0.0016	0.0005	-0.0037	0.0177	0.0015	-0.0061	0.0026	0.0033	0.0034
0.77	0.0015	0.0004	-0.0034	0.0188	0.0002	-0.0041	0.0026	0.0034	0.0035
0.79	0.0016	0	-0.0032	0.0200	-0.0012	-0.0019	0.0027	0.0034	0.0036
0.81	0.0016	-0.0002	-0.0033	0.0213	-0.0028	0.0005	0.0028	0.0035	0.0036
0.83	0.0013	-0.0001	-0.0030	0.0227	-0.0045	0.0003	0.0028	0.0035	0.0037
0.85	0.0015	-0.0004	-0.0031	0.0243	-0.0066	0.0065	0.0029	0.0036	0.0038
0.87	0.0012	-0.0007	-0.0025	0.0261	-0.0090	0.0103	0.0030	0.0037	0.0039
0.89	0.0009	-0.0005	-0.0023	0.0283	-0.0117	0.0147	0.0031	0.0038	0.0041
0.91	0.0004	-0.0005	-0.0019	0.0310	-0.0152	0.0201	0.0032	0.0040	0.0042
0.93	0.0007	-0.0018	-0.0014	0.0347	-0.0195	0.0268	0.0035	0.0042	0.0045
0.95	0.0002	-0.0018	-0.0009	0.0400	-0.0251	0.0356	0.0037	0.0046	0.0049
0.97	-0.0018	-0.0016	0.0015	0.0488	-0.0334	0.0486	0.0044	0.0055	0.0056
0.99	-0.0033	-0.0054	0.0052	0.0713	-0.0493	0.0736	0.0062	0.0077	0.0083

Notes: Bias and MSE of  $\hat{\beta}_n$  and  $\hat{\beta}_n^{2\zeta^*}$ . Sample size of  $n = 500$ . The number of replications is 5,000. The optimal bandwidth is  $2\zeta^*$ .





Table 19: *Model (2) defined through Proposition 2 when  $\lambda(\tau) = \sqrt[6]{\tau}$ : bias and MSE of estimators*

$\tau$	Bias			MSE					
	canonical			smoothed			canonical		
	$\alpha_0$	$\alpha_1$	$\theta_1$	$\alpha_0$	$\alpha_1$	$\theta_1$	$\alpha_0$	$\alpha_1$	$\theta_1$
0.01	0.0015	-0.0063	0.0092	-0.0898	-0.0376	0.0631	0.0082	0.0113	0.0110
0.03	0.0029	-0.0037	0.0013	-0.0579	-0.0255	0.0305	0.0046	0.0063	0.0061
0.05	0.0036	-0.0031	-0.0013	-0.0441	-0.0190	0.0162	0.0036	0.0049	0.0047
0.07	0.0034	-0.0023	-0.0018	-0.0354	-0.0144	0.0073	0.0031	0.0042	0.0040
0.09	0.0023	-0.0013	-0.0009	-0.0291	-0.0109	0.0009	0.0028	0.0038	0.0036
0.11	0.0026	-0.0019	-0.0011	-0.0243	-0.0080	-0.0039	0.0027	0.0036	0.0035
0.13	0.0026	-0.0020	-0.0012	-0.0204	-0.0056	-0.0076	0.0026	0.0035	0.0033
0.15	0.0026	-0.0017	-0.0015	-0.0171	-0.0035	-0.0105	0.0025	0.0033	0.0032
0.17	0.0026	-0.0016	-0.0016	-0.0143	-0.0018	-0.0129	0.0024	0.0032	0.0031
0.19	0.0024	-0.0015	-0.0016	-0.0118	-0.0003	-0.0149	0.0024	0.0031	0.0031
0.21	0.0025	-0.0013	-0.0022	-0.0097	0.0010	-0.0165	0.0023	0.0030	0.0030
0.23	0.0025	-0.0011	-0.0025	-0.0078	0.0021	-0.0177	0.0023	0.0030	0.0030
0.25	0.0028	-0.0013	-0.0028	-0.0061	0.0031	-0.0188	0.0023	0.0030	0.0030
0.27	0.0027	-0.0011	-0.0030	-0.0045	0.0040	-0.0195	0.0023	0.0030	0.0030
0.29	0.0025	-0.0006	-0.0031	-0.0031	0.0047	-0.0201	0.0023	0.0029	0.0030
0.31	0.0024	-0.0005	-0.0030	-0.0018	0.0053	-0.0206	0.0023	0.0029	0.0030
0.33	0.0024	-0.0003	-0.0032	-0.0007	0.0059	-0.0208	0.0023	0.0029	0.0030
0.35	0.0025	-0.0008	-0.0030	0.0004	0.0064	-0.0210	0.0023	0.0029	0.0030
0.37	0.0026	-0.0009	-0.0032	0.0014	0.0067	-0.0210	0.0023	0.0029	0.0030
0.39	0.0027	-0.0010	-0.0033	0.0024	0.0071	-0.0209	0.0023	0.0029	0.0030
0.41	0.0024	-0.0009	-0.0029	0.0032	0.0073	-0.0207	0.0023	0.0028	0.0030
0.43	0.0024	-0.0013	-0.0027	0.0041	0.0075	-0.0203	0.0023	0.0028	0.0030
0.45	0.0026	-0.0013	-0.0030	0.0048	0.0076	-0.0199	0.0023	0.0028	0.0030
0.47	0.0026	-0.0015	-0.0029	0.0056	0.0077	-0.0194	0.0023	0.0029	0.0030
0.49	0.0029	-0.0016	-0.0033	0.0063	0.0077	-0.0188	0.0023	0.0029	0.0030
0.51	0.0025	-0.0014	-0.0030	0.0070	0.0077	-0.0182	0.0023	0.0029	0.0030
0.53	0.0022	-0.0012	-0.0027	0.0077	0.0076	-0.0174	0.0023	0.0029	0.0030
0.55	0.0022	-0.0015	-0.0025	0.0083	0.0074	-0.0166	0.0023	0.0029	0.0030
0.57	0.0019	-0.0011	-0.0025	0.0089	0.0073	-0.0156	0.0023	0.0029	0.0030
0.59	0.0019	-0.0011	-0.0026	0.0095	0.0070	-0.0146	0.0024	0.0030	0.0030
0.61	0.0018	-0.0012	-0.0025	0.0101	0.0067	-0.0134	0.0024	0.0030	0.0031
0.63	0.0016	-0.0010	-0.0023	0.0107	0.0064	-0.0122	0.0024	0.0030	0.0031
0.65	0.0016	-0.0009	-0.0023	0.0112	0.0060	-0.0108	0.0024	0.0030	0.0031
0.67	0.0016	-0.0009	-0.0024	0.0118	0.0055	-0.0093	0.0025	0.0030	0.0032
0.69	0.0017	-0.0010	-0.0024	0.0124	0.0050	-0.0077	0.0025	0.0030	0.0032
0.71	0.0018	-0.0014	-0.0024	0.0130	0.0043	-0.0059	0.0025	0.0030	0.0032
0.73	0.0016	-0.0010	-0.0025	0.0136	0.0036	-0.0040	0.0025	0.0031	0.0032
0.75	0.0013	-0.0009	-0.0021	0.0143	0.0028	-0.0019	0.0025	0.0031	0.0032
0.77	0.0009	-0.0006	-0.0019	0.0150	0.0019	0.0004	0.0026	0.0032	0.0033
0.79	0.0010	-0.0010	-0.0017	0.0158	0.0009	0.0030	0.0026	0.0032	0.0033
0.81	0.0011	-0.0010	-0.0017	0.0166	-0.0003	0.0058	0.0027	0.0033	0.0034
0.83	0.0008	-0.0011	-0.0012	0.0176	-0.0017	0.0090	0.0028	0.0033	0.0036
0.85	0.0004	-0.0012	-0.0005	0.0187	-0.0032	0.0127	0.0028	0.0034	0.0036
0.87	0.0006	-0.0013	-0.0008	0.0200	-0.0051	0.0169	0.0029	0.0035	0.0037
0.89	0.0006	-0.0015	-0.0009	0.0215	-0.0073	0.0219	0.0031	0.0037	0.0039
0.91	0.0003	-0.0013	-0.0008	0.0236	-0.0100	0.0278	0.0032	0.0038	0.0041
0.93	-0.0002	-0.0016	0.0001	0.0265	-0.0136	0.0352	0.0033	0.0040	0.0042
0.95	-0.0011	-0.0011	0.0010	0.0308	-0.0183	0.0448	0.0036	0.0043	0.0046
0.97	-0.0015	-0.0023	0.0022	0.0382	-0.0252	0.0591	0.0042	0.0051	0.0054
0.99	-0.0055	-0.0054	0.0095	0.0576	-0.0386	0.0869	0.0061	0.0074	0.0079

Notes: Bias and MSE of  $\hat{\beta}_n$  and  $\hat{\beta}_n^{2\zeta^*}$ . Sample size of  $n = 500$ . The number of replications is 5,000. The optimal bandwidth is  $2\zeta^*$ .

