Proposition 1. Let $v_{ij}\colon (0,1) \to [0,1]$, $i,j \in \{0,1\}$ be non-decreasing, left-continuous functions with $v_{11} + v_{00} = v_{10} + v_{01}$. Let additionally Y_0 and X_0 be random variables supported on the unit interval, and let $U_1, U_2, \ldots$ and $V_1, V_2, \ldots$ be *iid* sequences of random variables in the unit interval, with both sequences mutually independent and also independent from Y_0 and X_0. Define moreover, for $t \geqslant 1$,

$$\begin{aligned} Y_t &\coloneqq \alpha_0(U_t) + \alpha_1(U_t)Y_{t-1} + \theta_1(U_t)X_{t-1} \\ X_t &\coloneqq Q_t(V_t, X_{t-1}, \ldots, X_0, Y_t, Y_{t-1}, \ldots, Y_0) \end{aligned} \tag{1}$$

where, for all $t \geqslant 1$, the measurable mapping $Q_t\colon (0,1) \times [0,1]^{2t+1} \to [0,1]$ is a quantile function of its first argument, and where $\alpha_0 \coloneqq v_{00}$, $\alpha_1 \coloneqq v_{10} - v_{00}$ and $\theta_1 \coloneqq v_{01} - v_{00}$. Then the process $(X_t, Y_t)_{t\in\mathbb{Z}}$ has state space contained in the unit square $[0,1]^2$ and satisfies

$$Q_{Y_t}(\tau|\mathscr{F}_{t-1}) = \alpha_0(\tau) + \alpha_1(\tau)Y_{t-1} + \theta_1(\tau)X_{t-1} \tag{2}$$

for all $0 < \tau < 1$ and all $t \geqslant 1$, where $\mathscr{F}_t \coloneqq \sigma\{(X_s, Y_s)\colon\ s \leqslant t\}$.

Proposition 2. Let $v_{10}, v_{01}\colon (0,1) \to \mathbb{R}$ be any two non-decreasing, continuously differentiable functions, with $v_{10}(0+) = v_{01}(0+) = 0$ and $v_{10}(1-) = v_{01}(1-) = 1$. Also, let $\lambda\colon (0,1) \to \mathbb{R}$ be any non-negative, continuous function satisfying the requirement that $0 < \int_0^1 (v'_{10}(u) + v'_{01}(u))\lambda(u)\,\mathrm{d}u < \infty$. Then the following holds:

1. The function $v_{00}\colon (0,1) \to \mathbb{R}$ defined through

$$v_{00}(\tau) \coloneqq \frac{\int_0^\tau (v'_{10}(u) + v'_{01}(u))\lambda(u)\,\mathrm{d}u}{\int_0^1 (v'_{10}(u) + v'_{01}(u))\lambda(u)\,\mathrm{d}u}, \quad 0 < \tau < 1, \tag{3}$$

   is continuously differentiable and non-decreasing, with $v_{00}(0+) = 0$ and $v_{00}(1-) = 1$.
2. The function $v_{11} \coloneqq v_{10} + v_{01} - v_{00}$ is continuously differentiable, satisfies $v_{11}(0+) = 0$, $v_{11}(1-) = 1$ and, for $0 < \tau < 1$,

$$v'_{11}(\tau) = (v'_{10}(\tau) + v'_{01}(\tau))\left(1 - \frac{\lambda(\tau)}{\int_0^1 (v'_{10}(u) + v'_{01}(u))\lambda(u)\,\mathrm{d}u}\right). \tag{4}$$

   In particular, in order that v_{11} be non-decreasing, it is necessary and sufficient that the bound

$$\lambda(\tau) \leqslant \int_0^1 (v'_{10}(u) + v'_{01}(u))\lambda(u)\mathrm{d}u$$

   holds for each τ such that $v'_{10}(\tau) + v'_{01}(\tau) \neq 0$.

Table 1: *Location-shift model 1: bias and MSE of estimators*

	Bias								*MSE*							
	canonical				*smoothed*				*canonical*				*smoothed*			
τ	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2
0.01	0.2615	-0.0144	-0.0028	0.0060	-0.0910	-0.0146	-0.0012	0.0074	0.0684	0.0002	0	0	0.0083	0.0002	0	0.0001
0.03	0.0585	-0.0148	-0.0010	0.0088	-0.1168	-0.0156	-0.0001	0.0085	0.0034	0.0002	0	0.0001	0.0137	0.0002	0	0.0001
0.05	0.0325	-0.0144	-0.0003	0.0095	-0.1082	-0.0163	0.0006	0.0088	0.0011	0.0002	0	0.0001	0.0117	0.0003	0	0.0001
0.07	0.0245	-0.0156	0.0018	0.0092	-0.0985	-0.0167	0.0007	0.0089	0.0006	0.0002	0	0.0001	0.0097	0.0003	0	0.0001
0.09	0.0207	-0.0173	0.0019	0.0098	-0.0899	-0.0168	0.0007	0.0088	0.0004	0.0003	0	0.0001	0.0081	0.0003	0	0.0001
0.11	0.0181	-0.0179	0.0008	0.0092	-0.0822	-0.0170	0.0006	0.0086	0.0003	0.0003	0	0.0001	0.0068	0.0003	0	0.0001
0.13	0.0153	-0.0179	0.0013	0.0081	-0.0755	-0.0171	0.0007	0.0085	0.0002	0.0003	0	0.0001	0.0057	0.0003	0	0.0001
0.15	0.0132	-0.0178	0.0003	0.0087	-0.0694	-0.0172	0.0006	0.0085	0.0002	0.0003	0	0.0001	0.0048	0.0003	0	0.0001
0.17	0.0120	-0.0174	0.0003	0.0078	-0.0639	-0.0172	0.0007	0.0085	0.0001	0.0003	0	0.0001	0.0041	0.0003	0	0.0001
0.19	0.0107	-0.0175	0.0005	0.0082	-0.0586	-0.0172	0.0008	0.0085	0.0001	0.0003	0	0.0001	0.0034	0.0003	0	0.0001
0.21	0.0093	-0.0174	0.0004	0.0086	-0.0537	-0.0172	0.0009	0.0085	0.0001	0.0003	0	0.0001	0.0029	0.0003	0	0.0001
0.23	0.0086	-0.0174	0.0005	0.0083	-0.0490	-0.0172	0.0011	0.0086	0.0001	0.0003	0	0.0001	0.0024	0.0003	0	0.0001
0.25	0.0073	-0.0171	0.0006	0.0083	-0.0429	-0.0172	0.0011	0.0086	0.0001	0.0003	0	0.0001	0.0018	0.0003	0	0.0001
0.27	0.0078	-0.0172	0.0011	0.0081	-0.0364	-0.0172	0.0012	0.0087	0.0001	0.0003	0	0.0001	0.0013	0.0003	0	0.0001
0.29	0.0066	-0.0169	0.0015	0.0084	-0.0328	-0.0171	0.0013	0.0087	0	0.0003	0	0.0001	0.0011	0.0003	0	0.0001
0.31	0.0060	-0.0173	0.0015	0.0082	-0.0294	-0.0171	0.0014	0.0087	0	0.0003	0	0.0001	0.0009	0.0003	0	0.0001
0.33	0.0053	-0.0169	0.0014	0.0081	-0.0261	-0.0171	0.0015	0.0087	0	0.0003	0	0.0001	0.0007	0.0003	0	0.0001
0.35	0.0047	-0.0167	0.0016	0.0086	-0.0229	-0.0171	0.0016	0.0088	0	0.0003	0	0.0001	0.0005	0.0003	0	0.0001
0.37	0.0039	-0.0171	0.0017	0.0093	-0.0197	-0.0170	0.0016	0.0088	0	0.0003	0	0.0001	0.0004	0.0003	0	0.0001
0.39	0.0030	-0.0172	0.0022	0.0097	-0.0166	-0.0170	0.0018	0.0089	0	0.0003	0	0.0001	0.0003	0.0003	0	0.0001
0.41	0.0023	-0.0170	0.0028	0.0096	-0.0135	-0.0170	0.0019	0.0089	0	0.0003	0	0.0001	0.0002	0.0003	0	0.0001
0.43	0.0017	-0.0169	0.0028	0.0098	-0.0104	-0.0169	0.0020	0.0089	0	0.0003	0	0.0001	0.0001	0.0003	0	0.0001
0.45	0.0019	-0.0168	0.0026	0.0098	-0.0073	-0.0169	0.0021	0.0089	0	0.0003	0	0.0001	0.0001	0.0003	0	0.0001
0.47	0.0010	-0.0166	0.0025	0.0097	-0.0043	-0.0169	0.0021	0.0088	0	0.0003	0	0.0001	0	0.0003	0	0.0001
0.49	0.0008	-0.0164	0.0028	0.0087	-0.0012	-0.0169	0.0022	0.0088	0	0.0003	0	0.0001	0	0.0003	0	0.0001
0.51	0	-0.0163	0.0030	0.0086	0.0018	-0.0168	0.0023	0.0088	0	0.0003	0	0.0001	0	0.0003	0	0.0001
0.53	-0.0008	-0.0165	0.0031	0.0094	0.0049	-0.0168	0.0023	0.0087	0	0.0003	0	0.0001	0	0.0003	0	0.0001
0.55	-0.0010	-0.0168	0.0029	0.0093	0.0079	-0.0167	0.0023	0.0087	0	0.0003	0	0.0001	0.0001	0.0003	0	0.0001
0.57	-0.0015	-0.0170	0.0027	0.0091	0.0109	-0.0167	0.0023	0.0086	0	0.0003	0	0.0001	0.0001	0.0003	0	0.0001
0.59	-0.0014	-0.0169	0.0030	0.0090	0.0139	-0.0166	0.0023	0.0085	0	0.0003	0	0.0001	0.0002	0.0003	0	0.0001
0.61	-0.0019	-0.0171	0.0030	0.0091	0.0169	-0.0166	0.0023	0.0084	0	0.0003	0	0.0001	0.0003	0.0003	0	0.0001
0.63	-0.0026	-0.0171	0.0025	0.0088	0.0200	-0.0165	0.0022	0.0083	0	0.0003	0	0.0001	0.0004	0.0003	0	0.0001
0.65	-0.0037	-0.0168	0.0028	0.0082	0.0231	-0.0164	0.0022	0.0082	0	0.0003	0	0.0001	0.0005	0.0003	0	0.0001
0.67	-0.0046	-0.0164	0.0025	0.0076	0.0262	-0.0163	0.0021	0.0081	0	0.0003	0	0.0001	0.0007	0.0003	0	0.0001
0.69	-0.0064	-0.0160	0.0018	0.0074	0.0294	-0.0162	0.0021	0.0080	0	0.0003	0	0.0001	0.0009	0.0003	0	0.0001
0.71	-0.0069	-0.0157	0.0013	0.0073	0.0327	-0.0161	0.0019	0.0078	0	0.0002	0	0.0001	0.0011	0.0003	0	0.0001
0.73	-0.0074	-0.0151	0.0011	0.0070	0.0363	-0.0160	0.0018	0.0077	0.0001	0.0002	0	0	0.0013	0.0003	0	0.0001
0.75	-0.0083	-0.0154	0.0010	0.0068	0.0425	-0.0159	0.0016	0.0076	0.0001	0.0002	0	0	0.0018	0.0003	0	0.0001
0.77	-0.0097	-0.0154	0.0011	0.0071	0.0485	-0.0158	0.0014	0.0074	0.0001	0.0002	0	0.0001	0.0024	0.0002	0	0.0001
0.79	-0.0110	-0.0151	0.0009	0.0063	0.0531	-0.0156	0.0012	0.0073	0.0001	0.0002	0	0	0.0028	0.0002	0	0.0001
0.81	-0.0118	-0.0149	0.0008	0.0065	0.0578	-0.0155	0.0010	0.0071	0.0001	0.0002	0	0	0.0033	0.0002	0	0.0001
0.83	-0.0131	-0.0147	-0.0001	0.0070	0.0629	-0.0153	0.0008	0.0069	0.0002	0.0002	0	0	0.0040	0.0002	0	0
0.85	-0.0148	-0.0145	-0.0008	0.0062	0.0683	-0.0152	0.0006	0.0067	0.0002	0.0002	0	0	0.0047	0.0002	0	0
0.87	-0.0171	-0.0141	-0.0002	0.0054	0.0742	-0.0150	0.0004	0.0066	0.0003	0.0002	0	0	0.0055	0.0002	0	0
0.89	-0.0188	-0.0148	0.0002	0.0052	0.0808	-0.0148	0.0002	0.0065	0.0004	0.0002	0	0	0.0065	0.0002	0	0
0.91	-0.0218	-0.0140	-0.0009	0.0056	0.0880	-0.0147	-0.0001	0.0066	0.0005	0.0002	0	0	0.0077	0.0002	0	0
0.93	-0.0272	-0.0143	-0.0015	0.0065	0.0961	-0.0145	-0.0001	0.0069	0.0007	0.0002	0	0	0.0092	0.0002	0	0
0.95	-0.0380	-0.0144	-0.0007	0.0077	0.1053	-0.0143	0.0002	0.0074	0.0014	0.0002	0	0.0001	0.0111	0.0002	0	0.0001
0.97	-0.0596	-0.0132	0.0020	0.0068	0.1140	-0.0141	0.0008	0.0075	0.0036	0.0002	0	0	0.0130	0.0002	0	0.0001
0.99	-0.2651	-0.0108	-0.0001	0.0069	0.0869	-0.0132	0.0015	0.0075	0.0703	0.0001	0	0	0.0075	0.0002	0	0.0001

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{\zeta^*/2}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $\zeta^*/2$.

Table 2: *Location-shift model 2: bias and MSE of estimators*

	Bias								*MSE*							
	canonical				*smoothed*				*canonical*				*smoothed*			
τ	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2
0.01	-0.1268	0.0033	0.0113	-0.0101	-0.2165	-0.0018	0.0080	-0.0043	7.6610	0.2630	0.7827	0.7171	7.2056	0.1815	0.5577	0.5048
0.03	-0.0695	-0.0202	0.0046	0.0077	-0.1221	-0.0213	0.0042	0.0074	0.7263	0.0708	0.1734	0.1734	0.6582	0.0555	0.1364	0.1363
0.05	-0.0559	-0.0215	0.0023	0.0077	-0.1004	-0.0221	0.0023	0.0087	0.2981	0.0377	0.0875	0.0911	0.2729	0.0307	0.0720	0.0742
0.07	-0.0465	-0.0200	0.0007	0.0037	-0.0852	-0.0194	-0.0001	0.0047	0.1800	0.0256	0.0580	0.0609	0.1624	0.0208	0.0477	0.0491
0.09	-0.0354	-0.0180	-0.0010	0.0028	-0.0729	-0.0171	-0.0008	0.0034	0.1232	0.0186	0.0415	0.0440	0.1116	0.0154	0.0346	0.0364
0.11	-0.0276	-0.0155	-0.0008	0.0033	-0.0640	-0.0159	-0.0006	0.0040	0.0911	0.0145	0.0325	0.0348	0.0836	0.0122	0.0273	0.0288
0.13	-0.0216	-0.0151	-0.0006	0.0032	-0.0566	-0.0151	-0.0001	0.0040	0.0724	0.0120	0.0269	0.0283	0.0663	0.0101	0.0225	0.0237
0.15	-0.0159	-0.0147	0.0009	0.0033	-0.0505	-0.0143	-0.0001	0.0041	0.0597	0.0103	0.0228	0.0240	0.0549	0.0087	0.0190	0.0202
0.17	-0.0139	-0.0139	0.0005	0.0046	-0.0455	-0.0136	-0.0001	0.0039	0.0516	0.0092	0.0193	0.0212	0.0471	0.0077	0.0163	0.0176
0.19	-0.0123	-0.0131	-0.0001	0.0043	-0.0411	-0.0127	-0.0001	0.0036	0.0442	0.0082	0.0171	0.0188	0.0411	0.0069	0.0144	0.0157
0.21	-0.0112	-0.0121	-0.0010	0.0044	-0.0371	-0.0118	-0.0002	0.0032	0.0396	0.0074	0.0152	0.0169	0.0365	0.0062	0.0130	0.0142
0.23	-0.0097	-0.0118	-0.0004	0.0027	-0.0335	-0.0111	-0.0002	0.0029	0.0360	0.0068	0.0137	0.0153	0.0329	0.0057	0.0119	0.0129
0.25	-0.0088	-0.0112	0	0.0022	-0.0288	-0.0105	-0.0002	0.0027	0.0332	0.0062	0.0129	0.0141	0.0301	0.0053	0.0110	0.0119
0.27	-0.0060	-0.0101	-0.0001	0.0014	-0.0244	-0.0100	-0.0002	0.0026	0.0304	0.0057	0.0121	0.0129	0.0278	0.0050	0.0104	0.0111
0.29	-0.0048	-0.0091	-0.0006	0.0018	-0.0219	-0.0096	-0.0002	0.0026	0.0284	0.0053	0.0113	0.0120	0.0259	0.0047	0.0098	0.0105
0.31	-0.0047	-0.0088	-0.0007	0.0021	-0.0197	-0.0092	-0.0002	0.0026	0.0273	0.0051	0.0108	0.0115	0.0243	0.0044	0.0093	0.0099
0.33	-0.0043	-0.0087	-0.0004	0.0024	-0.0176	-0.0089	-0.0003	0.0027	0.0257	0.0048	0.0102	0.0110	0.0229	0.0042	0.0089	0.0095
0.35	-0.0026	-0.0087	-0.0013	0.0032	-0.0156	-0.0087	-0.0003	0.0028	0.0245	0.0046	0.0098	0.0107	0.0217	0.0041	0.0086	0.0091
0.37	-0.0020	-0.0083	-0.0006	0.0035	-0.0135	-0.0085	-0.0003	0.0029	0.0232	0.0045	0.0096	0.0102	0.0207	0.0039	0.0083	0.0088
0.39	-0.0011	-0.0079	-0.0005	0.0029	-0.0114	-0.0084	-0.0003	0.0031	0.0225	0.0043	0.0093	0.0099	0.0199	0.0038	0.0081	0.0086
0.41	-0.0012	-0.0085	0.0004	0.0043	-0.0093	-0.0083	-0.0002	0.0033	0.0217	0.0043	0.0090	0.0097	0.0192	0.0037	0.0079	0.0084
0.43	-0.0013	-0.0081	-0.0002	0.0037	-0.0072	-0.0082	-0.0001	0.0034	0.0213	0.0042	0.0089	0.0095	0.0187	0.0036	0.0078	0.0082
0.45	-0.0008	-0.0081	0	0.0038	-0.0052	-0.0081	0	0.0035	0.0208	0.0041	0.0089	0.0094	0.0183	0.0036	0.0077	0.0081
0.47	-0.0004	-0.0081	0.0003	0.0040	-0.0030	-0.0081	0.0001	0.0036	0.0205	0.0041	0.0089	0.0092	0.0180	0.0035	0.0076	0.0080
0.49	-0.0002	-0.0078	0.0002	0.0036	-0.0008	-0.0080	0.0002	0.0037	0.0205	0.0040	0.0087	0.0091	0.0179	0.0035	0.0076	0.0080
0.51	0.0011	-0.0080	0.0003	0.0039	0.0015	-0.0079	0.0003	0.0037	0.0207	0.0041	0.0086	0.0091	0.0178	0.0035	0.0076	0.0080
0.53	0.0014	-0.0078	0.0002	0.0035	0.0038	-0.0078	0.0004	0.0037	0.0206	0.0041	0.0087	0.0093	0.0179	0.0035	0.0076	0.0080
0.55	0.0007	-0.0079	0.0007	0.0038	0.0061	-0.0078	0.0004	0.0038	0.0207	0.0041	0.0089	0.0093	0.0181	0.0036	0.0077	0.0081
0.57	0.0015	-0.0078	0.0007	0.0037	0.0085	-0.0079	0.0004	0.0038	0.0210	0.0042	0.0090	0.0094	0.0183	0.0036	0.0078	0.0082
0.59	0.0031	-0.0081	0.0007	0.0040	0.0108	-0.0079	0.0004	0.0038	0.0214	0.0043	0.0092	0.0096	0.0187	0.0037	0.0080	0.0084
0.61	0.0032	-0.0082	0.0006	0.0046	0.0131	-0.0079	0.0004	0.0038	0.0218	0.0044	0.0094	0.0097	0.0193	0.0038	0.0082	0.0085
0.63	0.0039	-0.0079	0.0004	0.0042	0.0155	-0.0080	0.0004	0.0038	0.0227	0.0045	0.0097	0.0101	0.0200	0.0039	0.0084	0.0088
0.65	0.0062	-0.0073	0.0003	0.0032	0.0177	-0.0080	0.0004	0.0037	0.0231	0.0045	0.0100	0.0104	0.0208	0.0040	0.0087	0.0091
0.67	0.0068	-0.0075	0.0006	0.0034	0.0198	-0.0081	0.0004	0.0036	0.0243	0.0047	0.0105	0.0110	0.0219	0.0041	0.0090	0.0094
0.69	0.0068	-0.0080	0.0003	0.0033	0.0220	-0.0083	0.0004	0.0036	0.0257	0.0049	0.0108	0.0115	0.0232	0.0043	0.0094	0.0098
0.71	0.0069	-0.0083	0.0004	0.0037	0.0242	-0.0085	0.0004	0.0035	0.0274	0.0052	0.0113	0.0120	0.0248	0.0045	0.0098	0.0104
0.73	0.0083	-0.0085	0.0001	0.0036	0.0267	-0.0088	0.0005	0.0035	0.0295	0.0055	0.0120	0.0129	0.0267	0.0048	0.0104	0.0110
0.75	0.0098	-0.0089	0.0001	0.0026	0.0313	-0.0093	0.0006	0.0034	0.0318	0.0059	0.0127	0.0138	0.0290	0.0051	0.0110	0.0117
0.77	0.0123	-0.0096	0.0005	0.0021	0.0366	-0.0097	0.0008	0.0034	0.0346	0.0065	0.0138	0.0148	0.0321	0.0055	0.0119	0.0125
0.79	0.0154	-0.0107	0.0011	0.0036	0.0410	-0.0102	0.0009	0.0035	0.0388	0.0072	0.0153	0.0162	0.0359	0.0061	0.0130	0.0137
0.81	0.0181	-0.0108	0.0005	0.0031	0.0455	-0.0107	0.0011	0.0035	0.0441	0.0080	0.0170	0.0177	0.0407	0.0067	0.0145	0.0152
0.83	0.0192	-0.0108	0.0007	0.0027	0.0505	-0.0113	0.0012	0.0039	0.0504	0.0090	0.0191	0.0203	0.0469	0.0076	0.0164	0.0172
0.85	0.0228	-0.0114	0.0011	0.0042	0.0559	-0.0120	0.0017	0.0044	0.0592	0.0103	0.0227	0.0236	0.0554	0.0087	0.0189	0.0199
0.87	0.0256	-0.0127	0.0018	0.0047	0.0616	-0.0133	0.0021	0.0055	0.0727	0.0122	0.0266	0.0281	0.0677	0.0103	0.0224	0.0235
0.89	0.0345	-0.0159	0.0013	0.0073	0.0693	-0.0152	0.0021	0.0074	0.0944	0.0152	0.0323	0.0347	0.0863	0.0125	0.0273	0.0286
0.91	0.0419	-0.0175	0.0020	0.0102	0.0798	-0.0172	0.0023	0.0099	0.1280	0.0195	0.0425	0.0444	0.1161	0.0158	0.0345	0.0366
0.93	0.0521	-0.0216	0.0008	0.0131	0.0925	-0.0196	0.0022	0.0121	0.1870	0.0261	0.0565	0.0605	0.1707	0.0213	0.0465	0.0501
0.95	0.0638	-0.0209	0.0012	0.0159	0.1114	-0.0210	0.0004	0.0144	0.3184	0.0391	0.0886	0.0928	0.2923	0.0316	0.0705	0.0758
0.97	0.0976	-0.0236	-0.0045	0.0225	0.1472	-0.0234	-0.0017	0.0206	0.7632	0.0722	0.1654	0.1762	0.7028	0.0567	0.1307	0.1388
0.99	0.1356	-0.0165	-0.0083	0.0133	0.2290	-0.0215	-0.0052	0.0172	7.9011	0.2421	0.6875	0.7477	7.5178	0.1673	0.5041	0.5491

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{\zeta^*/2}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $\zeta^*/2$.

Table 3: *Location-shift-scale model 1: bias and MSE of estimators*

	Bias								*MSE*							
	canonical				*smoothed*				*canonical*				*smoothed*			
τ	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2
0.01	-0.2014	-0.0020	0.1110	0.0024	-0.4127	-0.0022	0.1353	0.0015	0.2473	0.0046	0.0272	0.0054	3.7711	0.0028	0.0283	0.0033
0.03	-0.1006	-0.0025	0.0445	0.0003	-0.2537	-0.0024	0.0686	0.0010	0.0986	0.0024	0.0115	0.0027	2.7073	0.0018	0.0127	0.0020
0.05	-0.0627	-0.0029	0.0292	0.0005	-0.1851	-0.0023	0.0490	0.0009	0.0608	0.0016	0.0076	0.0017	2.1744	0.0013	0.0081	0.0014
0.07	-0.0441	-0.0025	0.0210	0.0005	-0.1465	-0.0020	0.0384	0.0007	0.0446	0.0012	0.0058	0.0013	1.8029	0.0010	0.0062	0.0011
0.09	-0.0369	-0.0017	0.0169	0.0001	-0.1201	-0.0017	0.0316	0.0004	0.0365	0.0010	0.0048	0.0011	1.5208	0.0009	0.0051	0.0010
0.11	-0.0291	-0.0014	0.0142	-0.0003	-0.1005	-0.0015	0.0269	0.0003	0.0304	0.0008	0.0042	0.0009	1.2952	0.0008	0.0044	0.0008
0.13	-0.0235	-0.0015	0.0120	-0.0001	-0.0850	-0.0014	0.0230	0.0004	0.0263	0.0007	0.0037	0.0008	1.1089	0.0007	0.0039	0.0007
0.15	-0.0195	-0.0014	0.0104	0	-0.0724	-0.0014	0.0200	0.0005	0.0234	0.0007	0.0034	0.0007	0.9518	0.0006	0.0036	0.0007
0.17	-0.0163	-0.0014	0.0090	0.0001	-0.0616	-0.0014	0.0173	0.0005	0.0213	0.0006	0.0032	0.0007	0.8177	0.0006	0.0033	0.0006
0.19	-0.0141	-0.0014	0.0078	0.0002	-0.0525	-0.0014	0.0151	0.0005	0.0201	0.0006	0.0030	0.0006	0.7012	0.0005	0.0031	0.0006
0.21	-0.0131	-0.0012	0.0069	0.0004	-0.0447	-0.0014	0.0131	0.0005	0.0186	0.0005	0.0028	0.0006	0.5992	0.0005	0.0029	0.0006
0.23	-0.0103	-0.0011	0.0058	0.0003	-0.0378	-0.0014	0.0113	0.0005	0.0178	0.0005	0.0027	0.0005	0.5095	0.0005	0.0028	0.0005
0.25	-0.0076	-0.0011	0.0047	0.0003	-0.0311	-0.0014	0.0097	0.0004	0.0169	0.0005	0.0026	0.0005	0.4312	0.0005	0.0027	0.0005
0.27	-0.0052	-0.0013	0.0040	0.0003	-0.0252	-0.0014	0.0084	0.0004	0.0161	0.0004	0.0025	0.0005	0.3620	0.0004	0.0026	0.0005
0.29	-0.0037	-0.0013	0.0038	0.0002	-0.0208	-0.0014	0.0075	0.0003	0.0156	0.0004	0.0025	0.0005	0.2998	0.0004	0.0025	0.0005
0.31	-0.0023	-0.0013	0.0036	0.0002	-0.0168	-0.0014	0.0066	0.0003	0.0151	0.0004	0.0024	0.0005	0.2452	0.0004	0.0024	0.0005
0.33	-0.0009	-0.0014	0.0033	0.0002	-0.0133	-0.0014	0.0058	0.0003	0.0147	0.0004	0.0023	0.0005	0.1974	0.0004	0.0024	0.0005
0.35	0.0001	-0.0013	0.0028	0.0003	-0.0102	-0.0015	0.0049	0.0003	0.0146	0.0004	0.0023	0.0005	0.1559	0.0004	0.0024	0.0005
0.37	0.0018	-0.0015	0.0023	0.0004	-0.0071	-0.0015	0.0041	0.0004	0.0144	0.0004	0.0023	0.0005	0.1204	0.0004	0.0023	0.0005
0.39	0.0038	-0.0016	0.0016	0.0004	-0.0042	-0.0015	0.0032	0.0004	0.0141	0.0004	0.0023	0.0005	0.0905	0.0004	0.0023	0.0005
0.41	0.0043	-0.0015	0.0011	0.0004	-0.0015	-0.0016	0.0023	0.0005	0.0141	0.0004	0.0022	0.0005	0.0658	0.0004	0.0023	0.0004
0.43	0.0051	-0.0016	0.0007	0.0005	0.0011	-0.0016	0.0016	0.0005	0.0140	0.0004	0.0022	0.0005	0.0461	0.0004	0.0022	0.0004
0.45	0.0063	-0.0016	0.0004	0.0006	0.0037	-0.0016	0.0009	0.0005	0.0139	0.0004	0.0022	0.0004	0.0312	0.0004	0.0022	0.0004
0.47	0.0069	-0.0015	-0.0002	0.0005	0.0061	-0.0017	0.0001	0.0005	0.0137	0.0004	0.0022	0.0004	0.0210	0.0004	0.0022	0.0004
0.49	0.0075	-0.0015	-0.0006	0.0005	0.0084	-0.0016	-0.0006	0.0005	0.0136	0.0004	0.0022	0.0004	0.0154	0.0004	0.0022	0.0004
0.51	0.0089	-0.0015	-0.0012	0.0006	0.0109	-0.0017	-0.0014	0.0005	0.0136	0.0004	0.0021	0.0004	0.0145	0.0004	0.0022	0.0004
0.53	0.0103	-0.0015	-0.0018	0.0006	0.0134	-0.0016	-0.0022	0.0005	0.0138	0.0004	0.0022	0.0004	0.0181	0.0004	0.0022	0.0004
0.55	0.0115	-0.0016	-0.0022	0.0006	0.0157	-0.0016	-0.0029	0.0005	0.0139	0.0004	0.0022	0.0004	0.0264	0.0004	0.0022	0.0004
0.57	0.0124	-0.0016	-0.0023	0.0007	0.0182	-0.0016	-0.0036	0.0005	0.0142	0.0004	0.0022	0.0004	0.0395	0.0004	0.0023	0.0004
0.59	0.0136	-0.0016	-0.0029	0.0006	0.0206	-0.0016	-0.0043	0.0005	0.0144	0.0004	0.0022	0.0004	0.0575	0.0004	0.0023	0.0004
0.61	0.0141	-0.0014	-0.0036	0.0005	0.0229	-0.0016	-0.0049	0.0005	0.0143	0.0004	0.0023	0.0004	0.0805	0.0004	0.0023	0.0004
0.63	0.0147	-0.0014	-0.0036	0.0004	0.0253	-0.0016	-0.0055	0.0005	0.0143	0.0004	0.0023	0.0004	0.1090	0.0004	0.0023	0.0004
0.65	0.0156	-0.0014	-0.0039	0.0004	0.0277	-0.0016	-0.0062	0.0004	0.0147	0.0004	0.0023	0.0005	0.1431	0.0004	0.0024	0.0004
0.67	0.0158	-0.0012	-0.0040	0.0004	0.0304	-0.0016	-0.0069	0.0004	0.0150	0.0004	0.0024	0.0005	0.1831	0.0004	0.0024	0.0005
0.69	0.0168	-0.0012	-0.0045	0.0004	0.0334	-0.0015	-0.0076	0.0005	0.0154	0.0004	0.0025	0.0005	0.2296	0.0004	0.0025	0.0005
0.71	0.0182	-0.0013	-0.0050	0.0006	0.0370	-0.0015	-0.0085	0.0005	0.0162	0.0004	0.0025	0.0005	0.2829	0.0004	0.0026	0.0005
0.73	0.0196	-0.0012	-0.0056	0.0006	0.0412	-0.0016	-0.0097	0.0006	0.0169	0.0004	0.0026	0.0005	0.3433	0.0004	0.0026	0.0005
0.75	0.0213	-0.0012	-0.0064	0.0007	0.0471	-0.0016	-0.0112	0.0006	0.0176	0.0005	0.0026	0.0005	0.4108	0.0004	0.0027	0.0005
0.77	0.0222	-0.0012	-0.0073	0.0008	0.0536	-0.0016	-0.0129	0.0007	0.0185	0.0005	0.0028	0.0005	0.4875	0.0005	0.0029	0.0005
0.79	0.0239	-0.0011	-0.0080	0.0008	0.0603	-0.0016	-0.0145	0.0007	0.0194	0.0005	0.0029	0.0006	0.5755	0.0005	0.0030	0.0006
0.81	0.0253	-0.0009	-0.0090	0.0009	0.0676	-0.0016	-0.0164	0.0008	0.0212	0.0005	0.0031	0.0006	0.6763	0.0005	0.0032	0.0006
0.83	0.0284	-0.0010	-0.0102	0.0010	0.0763	-0.0015	-0.0186	0.0008	0.0226	0.0006	0.0033	0.0007	0.7914	0.0005	0.0034	0.0006
0.85	0.0316	-0.0010	-0.0119	0.0011	0.0863	-0.0014	-0.0212	0.0009	0.0245	0.0006	0.0036	0.0007	0.9248	0.0006	0.0037	0.0007
0.87	0.0346	-0.0008	-0.0132	0.0011	0.0988	-0.0014	-0.0243	0.0009	0.0272	0.0007	0.0039	0.0008	1.0795	0.0006	0.0041	0.0007
0.89	0.0387	-0.0004	-0.0152	0.0008	0.1145	-0.0013	-0.0283	0.0009	0.0303	0.0008	0.0044	0.0009	1.2623	0.0007	0.0046	0.0008
0.91	0.0464	-0.0005	-0.0186	0.0009	0.1349	-0.0012	-0.0335	0.0009	0.0360	0.0009	0.0050	0.0010	1.4832	0.0008	0.0053	0.0009
0.93	0.0596	-0.0007	-0.0236	0.0015	0.1627	-0.0013	-0.0405	0.0009	0.0455	0.0011	0.0060	0.0013	1.7583	0.0009	0.0065	0.0011
0.95	0.0810	-0.0008	-0.0318	0.0013	0.2041	-0.0014	-0.0515	0.0010	0.0642	0.0014	0.0078	0.0017	2.1182	0.0012	0.0084	0.0014
0.97	0.1203	-0.0004	-0.0484	0.0013	0.2765	-0.0017	-0.0719	0.0012	0.1119	0.0024	0.0121	0.0027	2.6344	0.0017	0.0128	0.0020
0.99	0.2264	-0.0025	-0.1145	0.0013	0.4403	-0.0028	-0.1379	0.0010	0.2751	0.0048	0.0283	0.0057	3.6724	0.0028	0.0293	0.0033

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{\zeta^*/2}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $\zeta^*/2$.

Table 4: *Location-shift-scale model 2: bias and MSE of estimators*

	Bias								*MSE*							
	canonical				*smoothed*				*canonical*				*smoothed*			
τ	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2
0.01	-0.7681	-0.0125	0.2905	-0.0105	-1.1328	-0.0129	0.3420	-0.0076	6.6734	0.0482	0.4797	0.1216	6.0112	0.0346	0.4022	0.0844
0.03	-0.2569	-0.0157	0.0885	-0.0102	-0.4655	-0.0147	0.1190	-0.0076	0.6452	0.0147	0.1179	0.0270	0.6305	0.0117	0.1063	0.0223
0.05	-0.1350	-0.0117	0.0534	-0.0043	-0.2893	-0.0117	0.0778	-0.0028	0.2675	0.0068	0.0410	0.0116	0.2723	0.0058	0.0385	0.0099
0.07	-0.0814	-0.0097	0.0358	-0.0015	-0.2096	-0.0092	0.0566	-0.0012	0.1487	0.0039	0.0236	0.0053	0.1600	0.0035	0.0228	0.0047
0.09	-0.0580	-0.0076	0.0271	-0.0015	-0.1632	-0.0076	0.0447	-0.0007	0.1006	0.0026	0.0162	0.0035	0.1100	0.0024	0.0157	0.0031
0.11	-0.0404	-0.0065	0.0221	-0.0012	-0.1318	-0.0064	0.0367	-0.0003	0.0736	0.0020	0.0117	0.0025	0.0821	0.0018	0.0117	0.0023
0.13	-0.0313	-0.0057	0.0182	-0.0004	-0.1087	-0.0057	0.0307	0	0.0583	0.0016	0.0092	0.0020	0.0645	0.0015	0.0093	0.0018
0.15	-0.0248	-0.0050	0.0152	0	-0.0907	-0.0051	0.0259	0.0003	0.0475	0.0013	0.0076	0.0016	0.0529	0.0012	0.0078	0.0015
0.17	-0.0196	-0.0044	0.0124	0	-0.0769	-0.0047	0.0222	0.0005	0.0404	0.0011	0.0065	0.0014	0.0448	0.0010	0.0066	0.0013
0.19	-0.0146	-0.0044	0.0104	0.0004	-0.0657	-0.0044	0.0193	0.0008	0.0353	0.0010	0.0057	0.0012	0.0390	0.0009	0.0058	0.0011
0.21	-0.0116	-0.0040	0.0091	0.0004	-0.0561	-0.0041	0.0167	0.0010	0.0316	0.0008	0.0051	0.0010	0.0349	0.0008	0.0052	0.0010
0.23	-0.0093	-0.0037	0.0078	0.0007	-0.0479	-0.0039	0.0145	0.0011	0.0289	0.0008	0.0046	0.0009	0.0316	0.0008	0.0047	0.0009
0.25	-0.0080	-0.0034	0.0066	0.0007	-0.0402	-0.0036	0.0127	0.0012	0.0263	0.0007	0.0041	0.0009	0.0288	0.0007	0.0042	0.0008
0.27	-0.0068	-0.0031	0.0062	0.0008	-0.0334	-0.0034	0.0112	0.0012	0.0244	0.0006	0.0038	0.0008	0.0264	0.0006	0.0039	0.0008
0.29	-0.0053	-0.0030	0.0055	0.0009	-0.0285	-0.0032	0.0101	0.0013	0.0228	0.0006	0.0036	0.0008	0.0245	0.0006	0.0037	0.0007
0.31	-0.0037	-0.0029	0.0052	0.0011	-0.0240	-0.0031	0.0091	0.0014	0.0214	0.0006	0.0034	0.0007	0.0230	0.0006	0.0035	0.0007
0.33	-0.0026	-0.0028	0.0048	0.0012	-0.0200	-0.0029	0.0081	0.0014	0.0200	0.0005	0.0033	0.0007	0.0218	0.0005	0.0034	0.0007
0.35	-0.0017	-0.0025	0.0045	0.0011	-0.0163	-0.0028	0.0071	0.0014	0.0190	0.0005	0.0031	0.0006	0.0207	0.0005	0.0032	0.0006
0.37	-0.0005	-0.0023	0.0035	0.0009	-0.0127	-0.0026	0.0061	0.0013	0.0184	0.0005	0.0030	0.0006	0.0199	0.0005	0.0031	0.0006
0.39	0.0012	-0.0022	0.0031	0.0009	-0.0091	-0.0025	0.0052	0.0013	0.0176	0.0005	0.0029	0.0006	0.0193	0.0005	0.0030	0.0006
0.41	0.0022	-0.0022	0.0028	0.0011	-0.0060	-0.0024	0.0044	0.0013	0.0174	0.0005	0.0028	0.0006	0.0188	0.0005	0.0029	0.0006
0.43	0.0033	-0.0021	0.0023	0.0010	-0.0029	-0.0024	0.0036	0.0013	0.0170	0.0004	0.0027	0.0006	0.0184	0.0004	0.0028	0.0006
0.45	0.0050	-0.0021	0.0018	0.0011	0.0002	-0.0023	0.0028	0.0014	0.0169	0.0004	0.0027	0.0005	0.0182	0.0004	0.0028	0.0005
0.47	0.0063	-0.0021	0.0013	0.0012	0.0033	-0.0023	0.0019	0.0014	0.0168	0.0004	0.0027	0.0005	0.0180	0.0004	0.0028	0.0005
0.49	0.0074	-0.0021	0.0008	0.0013	0.0063	-0.0023	0.0011	0.0015	0.0167	0.0004	0.0027	0.0005	0.0180	0.0004	0.0028	0.0005
0.51	0.0085	-0.0021	0.0003	0.0014	0.0094	-0.0023	0.0003	0.0015	0.0170	0.0004	0.0027	0.0005	0.0180	0.0004	0.0028	0.0005
0.53	0.0097	-0.0020	-0.0003	0.0014	0.0126	-0.0022	-0.0005	0.0015	0.0172	0.0004	0.0027	0.0005	0.0182	0.0004	0.0028	0.0005
0.55	0.0110	-0.0020	-0.0007	0.0015	0.0158	-0.0022	-0.0014	0.0015	0.0171	0.0004	0.0028	0.0005	0.0184	0.0004	0.0028	0.0006
0.57	0.0116	-0.0019	-0.0012	0.0015	0.0191	-0.0022	-0.0022	0.0016	0.0173	0.0004	0.0028	0.0006	0.0187	0.0004	0.0028	0.0006
0.59	0.0137	-0.0020	-0.0017	0.0017	0.0224	-0.0022	-0.0030	0.0017	0.0178	0.0004	0.0028	0.0006	0.0191	0.0004	0.0029	0.0006
0.61	0.0154	-0.0021	-0.0023	0.0018	0.0259	-0.0022	-0.0039	0.0018	0.0182	0.0004	0.0028	0.0006	0.0197	0.0004	0.0029	0.0006
0.63	0.0157	-0.0019	-0.0021	0.0017	0.0296	-0.0022	-0.0047	0.0018	0.0188	0.0005	0.0029	0.0006	0.0204	0.0005	0.0030	0.0006
0.65	0.0167	-0.0017	-0.0028	0.0016	0.0336	-0.0022	-0.0057	0.0018	0.0197	0.0005	0.0031	0.0006	0.0213	0.0005	0.0031	0.0006
0.67	0.0187	-0.0017	-0.0037	0.0018	0.0381	-0.0021	-0.0068	0.0018	0.0202	0.0005	0.0032	0.0007	0.0223	0.0005	0.0033	0.0007
0.69	0.0207	-0.0016	-0.0043	0.0016	0.0428	-0.0021	-0.0079	0.0018	0.0215	0.0005	0.0034	0.0007	0.0235	0.0005	0.0035	0.0007
0.71	0.0232	-0.0016	-0.0050	0.0018	0.0480	-0.0021	-0.0091	0.0019	0.0229	0.0005	0.0036	0.0007	0.0250	0.0005	0.0037	0.0007
0.73	0.0253	-0.0017	-0.0056	0.0020	0.0538	-0.0020	-0.0104	0.0020	0.0243	0.0006	0.0039	0.0008	0.0269	0.0006	0.0040	0.0008
0.75	0.0273	-0.0016	-0.0064	0.0022	0.0618	-0.0020	-0.0123	0.0022	0.0263	0.0006	0.0042	0.0009	0.0295	0.0006	0.0043	0.0009
0.77	0.0292	-0.0014	-0.0076	0.0025	0.0704	-0.0019	-0.0142	0.0024	0.0289	0.0007	0.0046	0.0010	0.0328	0.0007	0.0047	0.0009
0.79	0.0306	-0.0010	-0.0085	0.0026	0.0791	-0.0017	-0.0163	0.0025	0.0317	0.0008	0.0051	0.0011	0.0364	0.0007	0.0052	0.0010
0.81	0.0361	-0.0011	-0.0102	0.0030	0.0895	-0.0016	-0.0188	0.0027	0.0359	0.0008	0.0057	0.0012	0.0411	0.0008	0.0058	0.0012
0.83	0.0397	-0.0008	-0.0118	0.0034	0.1020	-0.0015	-0.0217	0.0031	0.0404	0.0010	0.0066	0.0014	0.0474	0.0009	0.0067	0.0013
0.85	0.0462	-0.0006	-0.0141	0.0036	0.1171	-0.0014	-0.0252	0.0035	0.0491	0.0011	0.0078	0.0017	0.0568	0.0011	0.0079	0.0015
0.87	0.0556	-0.0007	-0.0172	0.0045	0.1369	-0.0013	-0.0297	0.0040	0.0600	0.0013	0.0095	0.0020	0.0700	0.0013	0.0095	0.0019
0.89	0.0671	-0.0005	-0.0202	0.0054	0.1626	-0.0013	-0.0353	0.0049	0.0786	0.0017	0.0119	0.0027	0.0902	0.0016	0.0118	0.0024
0.91	0.0885	-0.0005	-0.0264	0.0068	0.1979	-0.0013	-0.0434	0.0060	0.1084	0.0024	0.0160	0.0037	0.1213	0.0021	0.0155	0.0032
0.93	0.1197	-0.0009	-0.0339	0.0094	0.2492	-0.0012	-0.0547	0.0083	0.1642	0.0035	0.0239	0.0058	0.1778	0.0030	0.0229	0.0050
0.95	0.1767	0.0001	-0.0498	0.0125	0.3350	-0.0008	-0.0737	0.0117	0.2996	0.0062	0.0421	0.0102	0.3144	0.0051	0.0403	0.0091
0.97	0.3089	0.0006	-0.0871	0.0203	0.5229	0.0001	-0.1188	0.0173	0.7262	0.0137	0.1098	0.0271	0.7215	0.0111	0.0970	0.0222
0.99	0.8397	-0.0048	-0.3015	0.0367	1.2038	-0.0036	-0.3501	0.0300	4.7482	0.0552	0.4263	0.5343	4.5949	0.0396	0.3672	0.4058

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{\zeta^*/2}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $\zeta^*/2$.

Table 5: *Location-shift model 1: bias and MSE of estimators*

	Bias								*MSE*							
	canonical				*smoothed*				*canonical*				*smoothed*			
τ	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2
0.01	2.2266	-0.0006	-0.0015	0.0019	2.2063	-0.0006	-0.0006	0.0020	0.0143	0.0066	0.0138	0.0158	0.0117	0.0052	0.0108	0.0124
0.03	1.0763	-0.0016	0.0001	0.0027	1.0597	-0.0015	-0.0002	0.0024	0.0065	0.0030	0.0061	0.0073	0.0056	0.0025	0.0051	0.0060
0.05	0.7123	-0.0016	-0.0001	0.0021	0.6985	-0.0014	-0.0002	0.0017	0.0046	0.0022	0.0044	0.0051	0.0041	0.0018	0.0038	0.0043
0.07	0.5220	-0.0015	-0.0006	0.0011	0.5096	-0.0013	-0.0001	0.0013	0.0037	0.0017	0.0036	0.0042	0.0034	0.0015	0.0031	0.0036
0.09	0.4034	-0.0013	0.0004	0.0009	0.3915	-0.0013	0.0001	0.0011	0.0032	0.0015	0.0031	0.0036	0.0029	0.0013	0.0027	0.0031
0.11	0.3207	-0.0015	0.0003	0.0014	0.3103	-0.0014	0.0001	0.0011	0.0028	0.0013	0.0027	0.0031	0.0026	0.0011	0.0024	0.0027
0.13	0.2609	-0.0014	0.0003	0.0011	0.2511	-0.0013	0.0002	0.0010	0.0026	0.0012	0.0025	0.0028	0.0024	0.0010	0.0022	0.0025
0.15	0.2153	-0.0011	0.0002	0.0005	0.2061	-0.0013	0.0002	0.0010	0.0024	0.0011	0.0023	0.0026	0.0022	0.0010	0.0020	0.0023
0.17	0.1794	-0.0014	0.0002	0.0011	0.1709	-0.0013	0.0001	0.0011	0.0022	0.0010	0.0022	0.0025	0.0021	0.0009	0.0019	0.0022
0.19	0.1499	-0.0011	-0.0003	0.0012	0.1426	-0.0013	-0.0001	0.0012	0.0022	0.0010	0.0020	0.0023	0.0020	0.0009	0.0018	0.0021
0.21	0.1265	-0.0012	-0.0003	0.0014	0.1196	-0.0014	-0.0002	0.0013	0.0021	0.0009	0.0020	0.0023	0.0019	0.0008	0.0017	0.0020
0.23	0.1070	-0.0015	-0.0005	0.0011	0.1007	-0.0014	-0.0003	0.0014	0.0020	0.0009	0.0019	0.0022	0.0018	0.0008	0.0017	0.0019
0.25	0.0906	-0.0015	-0.0005	0.0015	0.0849	-0.0015	-0.0003	0.0015	0.0019	0.0009	0.0018	0.0021	0.0018	0.0008	0.0016	0.0019
0.27	0.0766	-0.0016	-0.0004	0.0016	0.0717	-0.0016	-0.0004	0.0016	0.0019	0.0008	0.0018	0.0021	0.0017	0.0008	0.0016	0.0018
0.29	0.0650	-0.0016	-0.0004	0.0016	0.0604	-0.0017	-0.0004	0.0016	0.0018	0.0008	0.0017	0.0020	0.0017	0.0007	0.0015	0.0018
0.31	0.0551	-0.0018	-0.0004	0.0016	0.0508	-0.0017	-0.0004	0.0016	0.0018	0.0008	0.0017	0.0020	0.0016	0.0007	0.0015	0.0018
0.33	0.0461	-0.0018	-0.0004	0.0018	0.0425	-0.0018	-0.0004	0.0017	0.0017	0.0008	0.0016	0.0019	0.0016	0.0007	0.0015	0.0017
0.35	0.0385	-0.0018	-0.0002	0.0017	0.0353	-0.0018	-0.0004	0.0016	0.0017	0.0008	0.0016	0.0019	0.0015	0.0007	0.0014	0.0017
0.37	0.0319	-0.0018	-0.0003	0.0015	0.0289	-0.0018	-0.0004	0.0016	0.0017	0.0008	0.0016	0.0019	0.0015	0.0007	0.0014	0.0017
0.39	0.0256	-0.0019	-0.0005	0.0015	0.0233	-0.0018	-0.0004	0.0016	0.0017	0.0008	0.0016	0.0018	0.0015	0.0007	0.0014	0.0016
0.41	0.0202	-0.0018	-0.0004	0.0015	0.0182	-0.0019	-0.0004	0.0017	0.0016	0.0007	0.0016	0.0018	0.0015	0.0007	0.0014	0.0016
0.43	0.0152	-0.0018	-0.0005	0.0016	0.0136	-0.0019	-0.0004	0.0017	0.0016	0.0007	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016
0.45	0.0105	-0.0018	-0.0004	0.0017	0.0093	-0.0019	-0.0004	0.0017	0.0016	0.0007	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016
0.47	0.0059	-0.0018	-0.0003	0.0018	0.0053	-0.0019	-0.0005	0.0018	0.0016	0.0007	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016
0.49	0.0016	-0.0018	-0.0004	0.0018	0.0013	-0.0019	-0.0005	0.0018	0.0016	0.0007	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016
0.51	-0.0029	-0.0020	-0.0004	0.0020	-0.0026	-0.0020	-0.0005	0.0019	0.0016	0.0007	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016
0.53	-0.0071	-0.0021	-0.0006	0.0019	-0.0065	-0.0020	-0.0005	0.0019	0.0016	0.0008	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016
0.55	-0.0117	-0.0022	-0.0006	0.0021	-0.0106	-0.0020	-0.0005	0.0019	0.0016	0.0008	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016
0.57	-0.0166	-0.0023	-0.0006	0.0019	-0.0149	-0.0021	-0.0005	0.0019	0.0016	0.0008	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016
0.59	-0.0218	-0.0020	-0.0005	0.0018	-0.0195	-0.0021	-0.0005	0.0019	0.0016	0.0008	0.0016	0.0018	0.0014	0.0007	0.0014	0.0016
0.61	-0.0270	-0.0020	-0.0005	0.0018	-0.0246	-0.0020	-0.0005	0.0018	0.0016	0.0008	0.0016	0.0018	0.0014	0.0007	0.0014	0.0017
0.63	-0.0329	-0.0020	-0.0005	0.0018	-0.0301	-0.0020	-0.0005	0.0018	0.0016	0.0008	0.0016	0.0019	0.0014	0.0007	0.0015	0.0017
0.65	-0.0395	-0.0020	-0.0004	0.0018	-0.0364	-0.0019	-0.0005	0.0018	0.0016	0.0008	0.0017	0.0019	0.0015	0.0007	0.0015	0.0017
0.67	-0.0472	-0.0018	-0.0005	0.0019	-0.0435	-0.0018	-0.0005	0.0017	0.0017	0.0008	0.0017	0.0019	0.0015	0.0007	0.0015	0.0017
0.69	-0.0561	-0.0016	-0.0004	0.0017	-0.0517	-0.0017	-0.0005	0.0017	0.0017	0.0008	0.0017	0.0020	0.0015	0.0007	0.0015	0.0018
0.71	-0.0657	-0.0015	-0.0004	0.0017	-0.0613	-0.0016	-0.0005	0.0016	0.0017	0.0008	0.0018	0.0020	0.0016	0.0007	0.0016	0.0018
0.73	-0.0778	-0.0016	-0.0005	0.0014	-0.0724	-0.0016	-0.0005	0.0016	0.0018	0.0009	0.0018	0.0021	0.0016	0.0008	0.0016	0.0018
0.75	-0.0914	-0.0015	-0.0006	0.0012	-0.0855	-0.0015	-0.0005	0.0015	0.0018	0.0009	0.0018	0.0021	0.0017	0.0008	0.0017	0.0019
0.77	-0.1073	-0.0013	-0.0007	0.0013	-0.1011	-0.0014	-0.0005	0.0015	0.0019	0.0009	0.0019	0.0022	0.0017	0.0008	0.0017	0.0020
0.79	-0.1267	-0.0013	-0.0003	0.0014	-0.1198	-0.0013	-0.0005	0.0015	0.0020	0.0010	0.0020	0.0023	0.0018	0.0008	0.0018	0.0020
0.81	-0.1500	-0.0013	-0.0002	0.0015	-0.1426	-0.0013	-0.0004	0.0014	0.0021	0.0010	0.0021	0.0024	0.0019	0.0009	0.0018	0.0021
0.83	-0.1787	-0.0013	-0.0002	0.0013	-0.1707	-0.0013	-0.0003	0.0013	0.0022	0.0010	0.0022	0.0025	0.0020	0.0009	0.0019	0.0022
0.85	-0.2146	-0.0010	-0.0001	0.0013	-0.2060	-0.0012	-0.0001	0.0013	0.0024	0.0011	0.0023	0.0027	0.0022	0.0010	0.0020	0.0023
0.87	-0.2610	-0.0013	0.0003	0.0013	-0.2511	-0.0013	0.0001	0.0013	0.0026	0.0012	0.0025	0.0028	0.0023	0.0011	0.0021	0.0025
0.89	-0.3214	-0.0016	0.0005	0.0014	-0.3104	-0.0013	0.0003	0.0013	0.0028	0.0014	0.0027	0.0031	0.0025	0.0012	0.0023	0.0027
0.91	-0.4026	-0.0014	0.0006	0.0011	-0.3914	-0.0013	0.0003	0.0011	0.0032	0.0015	0.0030	0.0035	0.0028	0.0013	0.0026	0.0030
0.93	-0.5215	-0.0011	0.0002	0.0008	-0.5091	-0.0012	0.0002	0.0009	0.0036	0.0018	0.0036	0.0041	0.0032	0.0015	0.0031	0.0035
0.95	-0.7121	-0.0011	0.0002	0.0007	-0.6978	-0.0011	0.0002	0.0008	0.0044	0.0022	0.0045	0.0051	0.0039	0.0019	0.0038	0.0043
0.97	-1.0750	-0.0012	0.0006	0.0009	-1.0587	-0.0011	0.0008	0.0004	0.0063	0.0031	0.0063	0.0070	0.0054	0.0026	0.0053	0.0058
0.99	-2.2276	-0.0004	0.0029	-0.0019	-2.2067	-0.0008	0.0024	-0.0012	0.0141	0.0070	0.0137	0.0155	0.0116	0.0055	0.0107	0.0121

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{\zeta^*/2}$. Sample size of $n = 1\,000$. The number of replications is $5,000$. The optimal bandwidth is $\zeta^*/2$.

Table 6: *Location-shift model 2: bias and MSE of estimators*

	Bias								*MSE*							
	canonical				*smoothed*				*canonical*				*smoothed*			
τ	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2
0.01	-0.1268	0.0033	0.0113	-0.0101	-0.2165	-0.0018	0.0080	-0.0043	7.6610	0.2630	0.7827	0.7171	7.2056	0.1815	0.5577	0.5048
0.03	-0.0695	-0.0202	0.0046	0.0077	-0.1221	-0.0213	0.0042	0.0074	0.7263	0.0708	0.1734	0.1734	0.6582	0.0555	0.1364	0.1363
0.05	-0.0559	-0.0215	0.0023	0.0077	-0.1004	-0.0221	0.0023	0.0087	0.2981	0.0377	0.0875	0.0911	0.2729	0.0307	0.0720	0.0742
0.07	-0.0465	-0.0200	0.0007	0.0037	-0.0852	-0.0194	-0.0001	0.0047	0.1800	0.0256	0.0580	0.0609	0.1624	0.0208	0.0477	0.0491
0.09	-0.0354	-0.0180	-0.0010	0.0028	-0.0729	-0.0171	-0.0008	0.0034	0.1232	0.0186	0.0415	0.0440	0.1116	0.0154	0.0346	0.0364
0.11	-0.0276	-0.0155	-0.0008	0.0033	-0.0640	-0.0159	-0.0006	0.0040	0.0911	0.0145	0.0325	0.0348	0.0836	0.0122	0.0273	0.0288
0.13	-0.0216	-0.0151	-0.0006	0.0032	-0.0566	-0.0151	-0.0001	0.0040	0.0724	0.0120	0.0269	0.0283	0.0663	0.0101	0.0225	0.0237
0.15	-0.0159	-0.0147	0.0009	0.0033	-0.0505	-0.0143	-0.0001	0.0041	0.0597	0.0103	0.0228	0.0240	0.0549	0.0087	0.0190	0.0202
0.17	-0.0139	-0.0139	0.0005	0.0046	-0.0455	-0.0136	-0.0001	0.0039	0.0516	0.0092	0.0193	0.0212	0.0471	0.0077	0.0163	0.0176
0.19	-0.0123	-0.0131	-0.0001	0.0043	-0.0411	-0.0127	-0.0001	0.0036	0.0442	0.0082	0.0171	0.0188	0.0411	0.0069	0.0144	0.0157
0.21	-0.0112	-0.0121	-0.0010	0.0044	-0.0371	-0.0118	-0.0002	0.0032	0.0396	0.0074	0.0152	0.0169	0.0365	0.0062	0.0130	0.0142
0.23	-0.0097	-0.0118	-0.0004	0.0027	-0.0335	-0.0111	-0.0002	0.0029	0.0360	0.0068	0.0137	0.0153	0.0329	0.0057	0.0119	0.0129
0.25	-0.0088	-0.0112	0	0.0022	-0.0288	-0.0105	-0.0002	0.0027	0.0332	0.0062	0.0129	0.0141	0.0301	0.0053	0.0110	0.0119
0.27	-0.0060	-0.0101	-0.0001	0.0014	-0.0244	-0.0100	-0.0002	0.0026	0.0304	0.0057	0.0121	0.0129	0.0278	0.0050	0.0104	0.0111
0.29	-0.0048	-0.0091	-0.0006	0.0018	-0.0219	-0.0096	-0.0002	0.0026	0.0284	0.0053	0.0113	0.0120	0.0259	0.0047	0.0098	0.0105
0.31	-0.0047	-0.0088	-0.0007	0.0021	-0.0197	-0.0092	-0.0002	0.0026	0.0273	0.0051	0.0108	0.0115	0.0243	0.0044	0.0093	0.0099
0.33	-0.0043	-0.0087	-0.0004	0.0024	-0.0176	-0.0089	-0.0003	0.0027	0.0257	0.0048	0.0102	0.0110	0.0229	0.0042	0.0089	0.0095
0.35	-0.0026	-0.0087	-0.0013	0.0032	-0.0156	-0.0087	-0.0003	0.0028	0.0245	0.0046	0.0098	0.0107	0.0217	0.0041	0.0086	0.0091
0.37	-0.0020	-0.0083	-0.0006	0.0035	-0.0135	-0.0085	-0.0003	0.0029	0.0232	0.0045	0.0096	0.0102	0.0207	0.0039	0.0083	0.0088
0.39	-0.0011	-0.0079	-0.0005	0.0029	-0.0114	-0.0084	-0.0003	0.0031	0.0225	0.0043	0.0093	0.0099	0.0199	0.0038	0.0081	0.0086
0.41	-0.0012	-0.0085	0.0004	0.0043	-0.0093	-0.0083	-0.0002	0.0033	0.0217	0.0043	0.0090	0.0097	0.0192	0.0037	0.0079	0.0084
0.43	-0.0013	-0.0081	-0.0002	0.0037	-0.0072	-0.0082	-0.0001	0.0034	0.0213	0.0042	0.0089	0.0095	0.0187	0.0036	0.0078	0.0082
0.45	-0.0008	-0.0081	0	0.0038	-0.0052	-0.0081	0	0.0035	0.0208	0.0041	0.0089	0.0094	0.0183	0.0036	0.0077	0.0081
0.47	-0.0004	-0.0081	0.0003	0.0040	-0.0030	-0.0081	0.0001	0.0036	0.0205	0.0041	0.0089	0.0092	0.0180	0.0035	0.0076	0.0080
0.49	-0.0002	-0.0078	0.0002	0.0036	-0.0008	-0.0080	0.0002	0.0037	0.0205	0.0040	0.0087	0.0091	0.0179	0.0035	0.0076	0.0080
0.51	0.0011	-0.0080	0.0003	0.0039	0.0015	-0.0079	0.0003	0.0037	0.0207	0.0041	0.0086	0.0091	0.0178	0.0035	0.0076	0.0080
0.53	0.0014	-0.0078	0.0002	0.0035	0.0038	-0.0078	0.0004	0.0037	0.0206	0.0041	0.0087	0.0093	0.0179	0.0035	0.0076	0.0080
0.55	0.0007	-0.0079	0.0007	0.0038	0.0061	-0.0078	0.0004	0.0038	0.0207	0.0041	0.0089	0.0093	0.0181	0.0036	0.0077	0.0081
0.57	0.0015	-0.0078	0.0007	0.0037	0.0085	-0.0079	0.0004	0.0038	0.0210	0.0042	0.0090	0.0094	0.0183	0.0036	0.0078	0.0082
0.59	0.0031	-0.0081	0.0007	0.0040	0.0108	-0.0079	0.0004	0.0038	0.0214	0.0043	0.0092	0.0096	0.0187	0.0037	0.0080	0.0084
0.61	0.0032	-0.0082	0.0006	0.0046	0.0131	-0.0079	0.0004	0.0038	0.0218	0.0044	0.0094	0.0097	0.0193	0.0038	0.0082	0.0085
0.63	0.0039	-0.0079	0.0004	0.0042	0.0155	-0.0080	0.0004	0.0038	0.0227	0.0045	0.0097	0.0101	0.0200	0.0039	0.0084	0.0088
0.65	0.0062	-0.0073	0.0003	0.0032	0.0177	-0.0080	0.0004	0.0037	0.0231	0.0045	0.0100	0.0104	0.0208	0.0040	0.0087	0.0091
0.67	0.0068	-0.0075	0.0006	0.0034	0.0198	-0.0081	0.0004	0.0036	0.0243	0.0047	0.0105	0.0110	0.0219	0.0041	0.0090	0.0094
0.69	0.0068	-0.0080	0.0003	0.0033	0.0220	-0.0083	0.0004	0.0036	0.0257	0.0049	0.0108	0.0115	0.0232	0.0043	0.0094	0.0098
0.71	0.0069	-0.0083	0.0004	0.0037	0.0242	-0.0085	0.0004	0.0035	0.0274	0.0052	0.0113	0.0120	0.0248	0.0045	0.0098	0.0104
0.73	0.0083	-0.0085	0.0001	0.0036	0.0267	-0.0088	0.0005	0.0035	0.0295	0.0055	0.0120	0.0129	0.0267	0.0048	0.0104	0.0110
0.75	0.0098	-0.0089	0.0001	0.0026	0.0313	-0.0093	0.0006	0.0034	0.0318	0.0059	0.0127	0.0138	0.0290	0.0051	0.0110	0.0117
0.77	0.0123	-0.0096	0.0005	0.0021	0.0366	-0.0097	0.0008	0.0034	0.0346	0.0065	0.0138	0.0148	0.0321	0.0055	0.0119	0.0125
0.79	0.0154	-0.0107	0.0011	0.0036	0.0410	-0.0102	0.0009	0.0035	0.0388	0.0072	0.0153	0.0162	0.0359	0.0061	0.0130	0.0137
0.81	0.0181	-0.0108	0.0005	0.0031	0.0455	-0.0107	0.0011	0.0035	0.0441	0.0080	0.0170	0.0177	0.0407	0.0067	0.0145	0.0152
0.83	0.0192	-0.0108	0.0007	0.0027	0.0505	-0.0113	0.0012	0.0039	0.0504	0.0090	0.0191	0.0203	0.0469	0.0076	0.0164	0.0172
0.85	0.0228	-0.0114	0.0011	0.0042	0.0559	-0.0120	0.0017	0.0044	0.0592	0.0103	0.0227	0.0236	0.0554	0.0087	0.0189	0.0199
0.87	0.0256	-0.0127	0.0018	0.0047	0.0616	-0.0133	0.0021	0.0055	0.0727	0.0122	0.0266	0.0281	0.0677	0.0103	0.0224	0.0235
0.89	0.0345	-0.0159	0.0013	0.0073	0.0693	-0.0152	0.0021	0.0074	0.0944	0.0152	0.0323	0.0347	0.0863	0.0125	0.0273	0.0286
0.91	0.0419	-0.0175	0.0020	0.0102	0.0798	-0.0172	0.0023	0.0099	0.1280	0.0195	0.0425	0.0444	0.1161	0.0158	0.0345	0.0366
0.93	0.0521	-0.0216	0.0008	0.0131	0.0925	-0.0196	0.0022	0.0121	0.1870	0.0261	0.0565	0.0605	0.1707	0.0213	0.0465	0.0501
0.95	0.0638	-0.0209	0.0012	0.0159	0.1114	-0.0210	0.0004	0.0144	0.3184	0.0391	0.0886	0.0928	0.2923	0.0316	0.0705	0.0758
0.97	0.0976	-0.0236	-0.0045	0.0225	0.1472	-0.0234	-0.0017	0.0206	0.7632	0.0722	0.1654	0.1762	0.7028	0.0567	0.1307	0.1388
0.99	0.1356	-0.0165	-0.0083	0.0133	0.2290	-0.0215	-0.0052	0.0172	7.9011	0.2421	0.6875	0.7477	7.5178	0.1673	0.5041	0.5491

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{\zeta^*/2}$. Sample size of $n = 1\,000$. The number of replications is $5,000$. The optimal bandwidth is $\zeta^*/2$.

Table 7: *Location-shift-scale model 1: bias and MSE of estimators*

	Bias								*MSE*							
	canonical				*smoothed*				*canonical*				*smoothed*			
τ	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2
0.01	-0.0290	-0.0014	0.0156	-0.0016	-0.1912	-0.0010	0.0440	-0.0009	0.0098	0.0002	0.0020	0.0003	4.5652	0.0002	0.0034	0.0003
0.03	-0.0115	-0.0008	0.0061	-0.0007	-0.1177	-0.0004	0.0266	-0.0004	0.0036	0.0001	0.0008	0.0001	3.1117	0.0001	0.0014	0.0001
0.05	-0.0069	-0.0006	0.0039	-0.0004	-0.0879	-0.0004	0.0200	-0.0002	0.0023	0.0001	0.0006	0.0001	2.4267	0.0001	0.0009	0.0001
0.07	-0.0049	-0.0005	0.0030	-0.0003	-0.0698	-0.0004	0.0161	-0.0001	0.0018	0	0.0004	0.0001	1.9788	0.0001	0.0007	0.0001
0.09	-0.0035	-0.0005	0.0021	-0.0002	-0.0574	-0.0003	0.0133	-0.0001	0.0015	0	0.0004	0	1.6489	0	0.0005	0.0001
0.11	-0.0028	-0.0004	0.0018	-0.0002	-0.0483	-0.0003	0.0113	-0.0001	0.0014	0	0.0003	0	1.3899	0	0.0005	0
0.13	-0.0028	-0.0003	0.0016	-0.0001	-0.0412	-0.0002	0.0097	-0.0001	0.0012	0	0.0003	0	1.1792	0	0.0004	0
0.15	-0.0026	-0.0002	0.0014	-0.0001	-0.0353	-0.0002	0.0084	-0.0001	0.0012	0	0.0003	0	1.0036	0	0.0004	0
0.17	-0.0022	-0.0002	0.0014	-0.0002	-0.0304	-0.0001	0.0073	-0.0001	0.0011	0	0.0003	0	0.8547	0	0.0003	0
0.19	-0.0020	-0.0002	0.0011	-0.0001	-0.0261	-0.0001	0.0064	0	0.0010	0	0.0003	0	0.7267	0	0.0003	0
0.21	-0.0019	-0.0001	0.0009	-0.0001	-0.0225	-0.0001	0.0055	0	0.0010	0	0.0002	0	0.6157	0	0.0003	0
0.23	-0.0015	-0.0001	0.0008	-0.0001	-0.0194	-0.0001	0.0048	0	0.0009	0	0.0002	0	0.5188	0	0.0003	0
0.25	-0.0013	-0.0001	0.0007	-0.0001	-0.0165	-0.0001	0.0041	0	0.0009	0	0.0002	0	0.4340	0	0.0003	0
0.27	-0.0009	-0.0002	0.0006	0	-0.0141	-0.0001	0.0035	0	0.0009	0	0.0002	0	0.3595	0	0.0002	0
0.29	-0.0008	-0.0001	0.0004	0	-0.0121	-0.0001	0.0030	0	0.0008	0	0.0002	0	0.2941	0	0.0002	0
0.31	-0.0005	-0.0002	0.0004	0	-0.0103	-0.0001	0.0026	0	0.0008	0	0.0002	0	0.2368	0	0.0002	0
0.33	-0.0004	-0.0002	0.0003	0	-0.0089	-0.0001	0.0022	0	0.0008	0	0.0002	0	0.1868	0	0.0002	0
0.35	-0.0004	-0.0001	0.0002	0	-0.0078	-0.0001	0.0019	0	0.0008	0	0.0002	0	0.1435	0	0.0002	0
0.37	-0.0004	-0.0001	0.0001	0	-0.0068	-0.0001	0.0016	0	0.0008	0	0.0002	0	0.1066	0	0.0002	0
0.39	-0.0004	-0.0001	0.0002	0	-0.0060	-0.0001	0.0014	0	0.0007	0	0.0002	0	0.0757	0	0.0002	0
0.41	-0.0002	-0.0001	0	0	-0.0052	-0.0001	0.0012	0	0.0007	0	0.0002	0	0.0504	0	0.0002	0
0.43	-0.0001	-0.0001	0.0001	0	-0.0042	-0.0001	0.0010	0	0.0007	0	0.0002	0	0.0306	0	0.0002	0
0.45	0	-0.0001	0.0001	0	-0.0031	-0.0001	0.0007	0	0.0007	0	0.0002	0	0.0160	0	0.0002	0
0.47	0.0002	-0.0001	0	0	-0.0018	-0.0001	0.0004	0	0.0007	0	0.0002	0	0.0064	0	0.0002	0
0.49	0.0003	-0.0001	-0.0001	0.0001	-0.0003	-0.0001	0.0001	0	0.0007	0	0.0002	0	0.0016	0	0.0002	0
0.51	0.0004	-0.0001	0	0	0.0012	-0.0001	-0.0002	0	0.0007	0	0.0002	0	0.0015	0	0.0002	0
0.53	0.0005	-0.0001	-0.0001	0	0.0027	-0.0001	-0.0005	0	0.0007	0	0.0002	0	0.0062	0	0.0002	0
0.55	0.0008	-0.0001	-0.0003	0	0.0040	-0.0001	-0.0009	0	0.0007	0	0.0002	0	0.0158	0	0.0002	0
0.57	0.0009	-0.0001	-0.0003	0	0.0050	-0.0001	-0.0011	0	0.0007	0	0.0002	0	0.0303	0	0.0002	0
0.59	0.0010	-0.0001	-0.0003	0.0001	0.0060	-0.0001	-0.0014	0	0.0007	0	0.0002	0	0.0501	0	0.0002	0
0.61	0.0012	-0.0001	-0.0004	0.0001	0.0069	-0.0001	-0.0016	0	0.0007	0	0.0002	0	0.0752	0	0.0002	0
0.63	0.0014	-0.0001	-0.0003	0	0.0077	-0.0001	-0.0017	0	0.0008	0	0.0002	0	0.1061	0	0.0002	0
0.65	0.0012	0	-0.0002	0	0.0086	-0.0001	-0.0019	0	0.0008	0	0.0002	0	0.1429	0	0.0002	0
0.67	0.0012	0	-0.0002	0.0001	0.0096	-0.0001	-0.0021	0.0001	0.0008	0	0.0002	0	0.1862	0	0.0002	0
0.69	0.0011	0	-0.0002	0.0001	0.0110	-0.0001	-0.0024	0.0001	0.0008	0	0.0002	0	0.2361	0	0.0002	0
0.71	0.0014	0	-0.0003	0.0001	0.0127	-0.0001	-0.0029	0.0001	0.0008	0	0.0002	0	0.2934	0	0.0002	0
0.73	0.0013	0	-0.0004	0.0002	0.0146	0	-0.0033	0.0001	0.0008	0	0.0002	0	0.3589	0	0.0002	0
0.75	0.0012	0.0001	-0.0004	0.0002	0.0170	0	-0.0039	0.0001	0.0009	0	0.0002	0	0.4334	0	0.0002	0
0.77	0.0016	0.0001	-0.0007	0.0001	0.0197	0	-0.0046	0.0001	0.0009	0	0.0002	0	0.5183	0	0.0003	0
0.79	0.0016	0.0001	-0.0008	0.0002	0.0228	0	-0.0054	0.0001	0.0009	0	0.0002	0	0.6153	0	0.0003	0
0.81	0.0020	0.0001	-0.0009	0.0002	0.0263	0	-0.0062	0.0001	0.0010	0	0.0002	0	0.7265	0	0.0003	0
0.83	0.0023	0.0001	-0.0012	0.0002	0.0305	0.0001	-0.0072	0.0001	0.0011	0	0.0003	0	0.8545	0	0.0003	0
0.85	0.0024	0.0002	-0.0015	0.0003	0.0355	0.0001	-0.0085	0.0001	0.0011	0	0.0003	0	1.0032	0	0.0004	0
0.87	0.0026	0.0003	-0.0017	0.0002	0.0414	0.0001	-0.0098	0.0001	0.0012	0	0.0003	0	1.1786	0	0.0004	0
0.89	0.0034	0.0003	-0.0020	0.0002	0.0487	0.0002	-0.0114	0.0001	0.0014	0	0.0003	0	1.3888	0	0.0005	0
0.91	0.0042	0.0003	-0.0024	0.0003	0.0581	0.0002	-0.0134	0.0001	0.0015	0	0.0004	0	1.6471	0	0.0006	0.0001
0.93	0.0053	0.0004	-0.0029	0.0003	0.0704	0.0002	-0.0161	0.0001	0.0018	0.0001	0.0004	0.0001	1.9772	0.0001	0.0007	0.0001
0.95	0.0072	0.0005	-0.0037	0.0004	0.0885	0.0003	-0.0200	0.0002	0.0023	0.0001	0.0006	0.0001	2.4249	0.0001	0.0009	0.0001
0.97	0.0122	0.0006	-0.0061	0.0006	0.1184	0.0004	-0.0268	0.0003	0.0036	0.0001	0.0008	0.0001	3.1093	0.0001	0.0014	0.0001
0.99	0.0320	0.0012	-0.0161	0.0014	0.1933	0.0008	-0.0447	0.0007	0.0104	0.0002	0.0021	0.0003	4.5564	0.0002	0.0034	0.0002

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 1\,000$. The number of replications is $5,000$. The optimal bandwidth is $\zeta^*/2$.

Table 8: *Location-shift-scale model 2: bias and MSE of estimators*

	Bias								*MSE*							
	canonical				*smoothed*				*canonical*				*smoothed*			
τ	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2	α_0	α_1	θ_1	θ_2
0.01	-0.1517	-0.0060	0.0587	-0.0127	-0.4102	-0.0055	0.1107	-0.0108	0.1734	0.0030	0.0406	0.0058	0.2719	0.0027	0.0407	0.0051
0.03	-0.0422	-0.0020	0.0167	-0.0032	-0.1954	-0.0016	0.0474	-0.0022	0.0237	0.0005	0.0064	0.0008	0.0559	0.0005	0.0074	0.0007
0.05	-0.0220	-0.0011	0.0094	-0.0020	-0.1353	-0.0009	0.0323	-0.0013	0.0103	0.0002	0.0029	0.0003	0.0272	0.0002	0.0036	0.0003
0.07	-0.0140	-0.0009	0.0069	-0.0013	-0.1038	-0.0006	0.0249	-0.0009	0.0062	0.0001	0.0018	0.0002	0.0166	0.0002	0.0022	0.0002
0.09	-0.0094	-0.0007	0.0050	-0.0010	-0.0834	-0.0005	0.0197	-0.0006	0.0043	0.0001	0.0012	0.0001	0.0113	0.0001	0.0015	0.0002
0.11	-0.0078	-0.0005	0.0037	-0.0007	-0.0691	-0.0004	0.0161	-0.0004	0.0032	0.0001	0.0009	0.0001	0.0083	0.0001	0.0012	0.0001
0.13	-0.0057	-0.0005	0.0028	-0.0005	-0.0581	-0.0003	0.0134	-0.0003	0.0025	0.0001	0.0007	0.0001	0.0063	0.0001	0.0009	0.0001
0.15	-0.0046	-0.0004	0.0022	-0.0004	-0.0492	-0.0003	0.0114	-0.0002	0.0022	0.0001	0.0006	0.0001	0.0049	0.0001	0.0007	0.0001
0.17	-0.0037	-0.0004	0.0021	-0.0003	-0.0420	-0.0003	0.0098	-0.0001	0.0019	0	0.0005	0.0001	0.0040	0.0001	0.0006	0.0001
0.19	-0.0029	-0.0004	0.0018	-0.0002	-0.0361	-0.0003	0.0085	-0.0001	0.0016	0	0.0005	0.0001	0.0033	0	0.0005	0.0001
0.21	-0.0024	-0.0004	0.0016	-0.0002	-0.0310	-0.0003	0.0074	-0.0001	0.0015	0	0.0004	0	0.0028	0	0.0005	0.0001
0.23	-0.0018	-0.0003	0.0014	-0.0002	-0.0267	-0.0003	0.0065	-0.0001	0.0013	0	0.0004	0	0.0024	0	0.0004	0.0001
0.25	-0.0016	-0.0003	0.0013	-0.0002	-0.0231	-0.0002	0.0057	-0.0001	0.0012	0	0.0003	0	0.0021	0	0.0004	0
0.27	-0.0017	-0.0002	0.0013	-0.0001	-0.0200	-0.0002	0.0051	0	0.0011	0	0.0003	0	0.0018	0	0.0003	0
0.29	-0.0016	-0.0002	0.0012	-0.0001	-0.0174	-0.0002	0.0045	0	0.0011	0	0.0003	0	0.0017	0	0.0003	0
0.31	-0.0013	-0.0002	0.0011	-0.0001	-0.0151	-0.0002	0.0039	0	0.0010	0	0.0003	0	0.0015	0	0.0003	0
0.33	-0.0011	-0.0001	0.0009	-0.0001	-0.0128	-0.0002	0.0033	0	0.0009	0	0.0002	0	0.0014	0	0.0003	0
0.35	-0.0008	-0.0001	0.0006	0	-0.0108	-0.0001	0.0027	0	0.0009	0	0.0002	0	0.0013	0	0.0003	0
0.37	-0.0007	-0.0001	0.0005	-0.0001	-0.0090	-0.0001	0.0023	0	0.0009	0	0.0002	0	0.0012	0	0.0003	0
0.39	-0.0004	-0.0001	0.0006	-0.0001	-0.0074	-0.0001	0.0020	0	0.0009	0	0.0002	0	0.0012	0	0.0003	0
0.41	-0.0003	-0.0001	0.0005	-0.0001	-0.0059	-0.0001	0.0016	0	0.0008	0	0.0002	0	0.0011	0	0.0003	0
0.43	-0.0001	-0.0001	0.0003	0	-0.0044	-0.0001	0.0012	0	0.0008	0	0.0002	0	0.0011	0	0.0002	0
0.45	0.0002	-0.0001	0.0001	0	-0.0029	-0.0001	0.0008	0	0.0008	0	0.0002	0	0.0011	0	0.0002	0
0.47	0.0002	-0.0001	0	0	-0.0016	-0.0001	0.0004	0	0.0008	0	0.0002	0	0.0011	0	0.0002	0
0.49	0.0003	-0.0001	-0.0001	0	-0.0002	-0.0001	0	0	0.0008	0	0.0002	0	0.0011	0	0.0002	0
0.51	0.0004	0	-0.0002	0	0.0011	-0.0001	-0.0003	0	0.0008	0	0.0002	0	0.0011	0	0.0002	0
0.53	0.0006	-0.0001	-0.0003	0	0.0025	-0.0001	-0.0007	0.0001	0.0008	0	0.0002	0	0.0011	0	0.0002	0
0.55	0.0006	0	-0.0004	0.0001	0.0038	-0.0001	-0.0010	0.0001	0.0008	0	0.0002	0	0.0011	0	0.0002	0
0.57	0.0007	0	-0.0004	0.0001	0.0051	-0.0001	-0.0013	0.0001	0.0008	0	0.0002	0	0.0011	0	0.0003	0
0.59	0.0009	0	-0.0004	0.0001	0.0066	-0.0001	-0.0016	0.0001	0.0008	0	0.0002	0	0.0011	0	0.0003	0
0.61	0.0010	0	-0.0004	0.0002	0.0080	-0.0001	-0.0019	0.0002	0.0009	0	0.0002	0	0.0012	0	0.0003	0
0.63	0.0012	0	-0.0005	0.0002	0.0097	-0.0001	-0.0023	0.0002	0.0009	0	0.0002	0	0.0012	0	0.0003	0
0.65	0.0015	-0.0001	-0.0006	0.0002	0.0115	-0.0001	-0.0027	0.0002	0.0009	0	0.0003	0	0.0013	0	0.0003	0
0.67	0.0015	0	-0.0008	0.0002	0.0135	-0.0001	-0.0032	0.0002	0.0010	0	0.0003	0	0.0014	0	0.0003	0
0.69	0.0018	0	-0.0008	0.0002	0.0157	-0.0001	-0.0037	0.0003	0.0010	0	0.0003	0	0.0015	0	0.0003	0
0.71	0.0021	0	-0.0010	0.0003	0.0181	-0.0001	-0.0043	0.0003	0.0011	0	0.0003	0	0.0017	0	0.0003	0
0.73	0.0025	0	-0.0011	0.0004	0.0210	-0.0001	-0.0049	0.0003	0.0011	0	0.0003	0	0.0019	0	0.0004	0
0.75	0.0031	0	-0.0013	0.0003	0.0244	-0.0001	-0.0056	0.0003	0.0012	0	0.0003	0	0.0021	0	0.0004	0
0.77	0.0037	-0.0001	-0.0015	0.0004	0.0284	-0.0001	-0.0065	0.0004	0.0014	0	0.0004	0	0.0025	0	0.0004	0.0001
0.79	0.0040	0	-0.0017	0.0005	0.0329	-0.0002	-0.0075	0.0004	0.0015	0	0.0004	0	0.0029	0	0.0005	0.0001
0.81	0.0046	0	-0.0018	0.0005	0.0380	-0.0001	-0.0085	0.0004	0.0017	0	0.0005	0.0001	0.0034	0	0.0005	0.0001
0.83	0.0051	0	-0.0019	0.0006	0.0440	-0.0001	-0.0098	0.0005	0.0019	0	0.0005	0.0001	0.0041	0.0001	0.0006	0.0001
0.85	0.0062	0	-0.0024	0.0007	0.0512	-0.0001	-0.0114	0.0005	0.0022	0.0001	0.0006	0.0001	0.0051	0.0001	0.0007	0.0001
0.87	0.0075	0.0001	-0.0029	0.0008	0.0602	-0.0001	-0.0135	0.0006	0.0026	0.0001	0.0007	0.0001	0.0065	0.0001	0.0009	0.0001
0.89	0.0091	0.0001	-0.0036	0.0009	0.0713	-0.0001	-0.0160	0.0007	0.0033	0.0001	0.0009	0.0001	0.0086	0.0001	0.0011	0.0001
0.91	0.0121	0.0002	-0.0046	0.0011	0.0857	0	-0.0194	0.0008	0.0045	0.0001	0.0012	0.0002	0.0118	0.0001	0.0015	0.0002
0.93	0.0161	0.0004	-0.0062	0.0015	0.1057	0.0001	-0.0242	0.0011	0.0064	0.0001	0.0017	0.0002	0.0171	0.0001	0.0021	0.0002
0.95	0.0233	0.0007	-0.0095	0.0024	0.1367	0.0004	-0.0322	0.0017	0.0105	0.0002	0.0028	0.0004	0.0276	0.0002	0.0035	0.0004
0.97	0.0431	0.0013	-0.0173	0.0038	0.1962	0.0009	-0.0482	0.0032	0.0242	0.0005	0.0062	0.0008	0.0557	0.0005	0.0072	0.0007
0.99	0.1491	0.0044	-0.0564	0.0137	0.4092	0.0038	-0.1099	0.0125	0.1665	0.0027	0.0390	0.0058	0.2654	0.0024	0.0395	0.0052

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*/2}$. Sample size of $n = 1\,000$. The number of replications is $5,000$. The optimal bandwidth is $\zeta^*/2$.

Table 9: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \sqrt{\tau}$*: bias and MSE of estimators*

	Bias						*MSE*					
	canonical			*smoothed*			*canonical*			*smoothed*		
τ	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1
0.01	0.0036	-0.0216	0.0565	-0.1820	-0.0269	0.1256	0.0493	0.0674	0.0682	0.0461	0.0185	0.0335
0.03	0.0118	-0.0128	0.0038	-0.1119	-0.0231	0.0596	0.0259	0.0371	0.0368	0.0232	0.0153	0.0182
0.05	0.0188	-0.0147	-0.0098	-0.0804	-0.0207	0.0295	0.0205	0.0285	0.0281	0.0160	0.0135	0.0139
0.07	0.0207	-0.0137	-0.0147	-0.0601	-0.0190	0.0103	0.0171	0.0237	0.0233	0.0126	0.0124	0.0122
0.09	0.0228	-0.0140	-0.0189	-0.0455	-0.0177	-0.0035	0.0157	0.0214	0.0212	0.0106	0.0117	0.0114
0.11	0.0238	-0.0143	-0.0209	-0.0340	-0.0166	-0.0141	0.0146	0.0195	0.0194	0.0093	0.0111	0.0110
0.13	0.0228	-0.0133	-0.0212	-0.0247	-0.0157	-0.0225	0.0137	0.0180	0.0182	0.0085	0.0107	0.0110
0.15	0.0231	-0.0134	-0.0219	-0.0169	-0.0148	-0.0294	0.0133	0.0171	0.0173	0.0080	0.0103	0.0110
0.17	0.0241	-0.0143	-0.0233	-0.0103	-0.0141	-0.0351	0.0131	0.0167	0.0169	0.0076	0.0100	0.0111
0.19	0.0229	-0.0122	-0.0237	-0.0045	-0.0135	-0.0398	0.0125	0.0159	0.0164	0.0074	0.0097	0.0112
0.21	0.0230	-0.0126	-0.0239	0.0006	-0.0129	-0.0438	0.0122	0.0154	0.0158	0.0073	0.0095	0.0114
0.23	0.0228	-0.0123	-0.0239	0.0051	-0.0124	-0.0471	0.0119	0.0148	0.0154	0.0072	0.0093	0.0116
0.25	0.0229	-0.0120	-0.0247	0.0093	-0.0119	-0.0496	0.0118	0.0146	0.0153	0.0072	0.0092	0.0117
0.27	0.0229	-0.0113	-0.0254	0.0132	-0.0114	-0.0512	0.0116	0.0142	0.0151	0.0072	0.0091	0.0118
0.29	0.0222	-0.0115	-0.0243	0.0162	-0.0109	-0.0531	0.0113	0.0138	0.0148	0.0072	0.0090	0.0119
0.31	0.0213	-0.0107	-0.0239	0.0189	-0.0105	-0.0545	0.0111	0.0136	0.0146	0.0073	0.0088	0.0120
0.33	0.0215	-0.0115	-0.0234	0.0214	-0.0101	-0.0556	0.0110	0.0133	0.0146	0.0074	0.0087	0.0121
0.35	0.0206	-0.0113	-0.0225	0.0236	-0.0098	-0.0564	0.0109	0.0132	0.0146	0.0074	0.0087	0.0121
0.37	0.0200	-0.0106	-0.0222	0.0256	-0.0094	-0.0568	0.0108	0.0132	0.0141	0.0075	0.0086	0.0121
0.39	0.0195	-0.0103	-0.0220	0.0274	-0.0091	-0.0570	0.0108	0.0131	0.0140	0.0076	0.0085	0.0121
0.41	0.0199	-0.0106	-0.0226	0.0290	-0.0088	-0.0569	0.0108	0.0132	0.0140	0.0077	0.0084	0.0121
0.43	0.0200	-0.0108	-0.0228	0.0305	-0.0084	-0.0566	0.0108	0.0131	0.0140	0.0077	0.0084	0.0120
0.45	0.0190	-0.0104	-0.0216	0.0317	-0.0081	-0.0559	0.0107	0.0128	0.0139	0.0078	0.0083	0.0120
0.47	0.0188	-0.0101	-0.0217	0.0329	-0.0079	-0.0551	0.0108	0.0128	0.0140	0.0079	0.0083	0.0119
0.49	0.0174	-0.0088	-0.0207	0.0339	-0.0076	-0.0540	0.0108	0.0128	0.0140	0.0080	0.0083	0.0118
0.51	0.0171	-0.0087	-0.0206	0.0347	-0.0073	-0.0526	0.0107	0.0129	0.0138	0.0081	0.0082	0.0117
0.53	0.0173	-0.0089	-0.0205	0.0354	-0.0071	-0.0510	0.0108	0.0128	0.0139	0.0081	0.0082	0.0115
0.55	0.0164	-0.0083	-0.0199	0.0361	-0.0069	-0.0492	0.0108	0.0129	0.0138	0.0082	0.0082	0.0114
0.57	0.0154	-0.0079	-0.0189	0.0365	-0.0067	-0.0471	0.0109	0.0127	0.0139	0.0083	0.0082	0.0112
0.59	0.0152	-0.0081	-0.0185	0.0369	-0.0065	-0.0448	0.0110	0.0127	0.0140	0.0083	0.0082	0.0110
0.61	0.0148	-0.0074	-0.0186	0.0372	-0.0063	-0.0421	0.0112	0.0126	0.0143	0.0084	0.0082	0.0108
0.63	0.0153	-0.0075	-0.0192	0.0374	-0.0061	-0.0392	0.0112	0.0125	0.0144	0.0085	0.0082	0.0106
0.65	0.0148	-0.0071	-0.0189	0.0375	-0.0059	-0.0360	0.0112	0.0124	0.0143	0.0085	0.0082	0.0104
0.67	0.0145	-0.0072	-0.0185	0.0374	-0.0058	-0.0324	0.0111	0.0123	0.0141	0.0086	0.0082	0.0103
0.69	0.0141	-0.0071	-0.0181	0.0373	-0.0057	-0.0285	0.0112	0.0125	0.0142	0.0086	0.0082	0.0101
0.71	0.0138	-0.0072	-0.0178	0.0371	-0.0055	-0.0242	0.0113	0.0126	0.0143	0.0087	0.0083	0.0100
0.73	0.0139	-0.0081	-0.0174	0.0368	-0.0055	-0.0195	0.0113	0.0126	0.0143	0.0087	0.0083	0.0098
0.75	0.0123	-0.0075	-0.0160	0.0369	-0.0054	-0.0144	0.0113	0.0127	0.0143	0.0088	0.0083	0.0097
0.77	0.0116	-0.0078	-0.0148	0.0368	-0.0054	-0.0086	0.0115	0.0128	0.0145	0.0089	0.0084	0.0097
0.79	0.0119	-0.0088	-0.0144	0.0362	-0.0054	-0.0022	0.0116	0.0130	0.0148	0.0089	0.0084	0.0098
0.81	0.0109	-0.0084	-0.0134	0.0356	-0.0055	0.0049	0.0119	0.0130	0.0151	0.0090	0.0085	0.0099
0.83	0.0095	-0.0083	-0.0115	0.0350	-0.0056	0.0129	0.0122	0.0133	0.0155	0.0091	0.0086	0.0102
0.85	0.0075	-0.0065	-0.0103	0.0343	-0.0059	0.0219	0.0124	0.0137	0.0159	0.0092	0.0087	0.0107
0.87	0.0063	-0.0069	-0.0081	0.0336	-0.0062	0.0322	0.0127	0.0139	0.0162	0.0094	0.0088	0.0115
0.89	0.0027	-0.0047	-0.0048	0.0329	-0.0067	0.0442	0.0130	0.0143	0.0169	0.0096	0.0090	0.0126
0.91	-0.0009	-0.0040	-0.0003	0.0324	-0.0074	0.0583	0.0139	0.0150	0.0179	0.0099	0.0092	0.0144
0.93	-0.0045	-0.0031	0.0041	0.0323	-0.0085	0.0757	0.0151	0.0162	0.0192	0.0104	0.0095	0.0171
0.95	-0.0098	-0.0022	0.0108	0.0332	-0.0101	0.0980	0.0168	0.0176	0.0215	0.0110	0.0098	0.0215
0.97	-0.0189	-0.0025	0.0231	0.0366	-0.0127	0.1299	0.0206	0.0221	0.0268	0.0122	0.0104	0.0295
0.99	-0.0608	-0.0095	0.0783	0.0514	-0.0183	0.1865	0.0331	0.0346	0.0454	0.0154	0.0116	0.0488

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 10: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \frac{1}{1+e^{-\tau}}$*: bias and MSE of estimators*

	Bias						*MSE*					
	canonical			*smoothed*			*canonical*			*smoothed*		
τ	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1
0.01	0.0142	-0.0283	0.0431	-0.1508	-0.0625	0.0897	0.0438	0.0610	0.0613	0.0348	0.0198	0.0244
0.03	0.0059	-0.0071	0.0082	-0.1059	-0.0344	0.0479	0.0249	0.0342	0.0352	0.0213	0.0149	0.0164
0.05	0.0035	0.0003	0.0019	-0.0849	-0.0214	0.0278	0.0194	0.0266	0.0269	0.0164	0.0130	0.0138
0.07	0.0057	0.0002	-0.0034	-0.0709	-0.0132	0.0147	0.0170	0.0231	0.0229	0.0137	0.0120	0.0125
0.09	0.0065	0.0002	-0.0058	-0.0605	-0.0073	0.0052	0.0150	0.0207	0.0201	0.0120	0.0114	0.0118
0.11	0.0090	-0.0011	-0.0094	-0.0521	-0.0028	-0.0023	0.0139	0.0191	0.0189	0.0108	0.0109	0.0114
0.13	0.0089	-0.0003	-0.0108	-0.0451	0.0007	-0.0084	0.0131	0.0182	0.0180	0.0099	0.0106	0.0112
0.15	0.0091	-0.0001	-0.0119	-0.0391	0.0035	-0.0133	0.0129	0.0177	0.0178	0.0092	0.0104	0.0111
0.17	0.0091	0.0004	-0.0129	-0.0338	0.0058	-0.0175	0.0123	0.0170	0.0172	0.0087	0.0103	0.0111
0.19	0.0089	0.0005	-0.0131	-0.0290	0.0077	-0.0210	0.0120	0.0167	0.0169	0.0083	0.0102	0.0111
0.21	0.0094	0	-0.0138	-0.0248	0.0092	-0.0239	0.0118	0.0163	0.0168	0.0080	0.0101	0.0111
0.23	0.0099	-0.0005	-0.0143	-0.0209	0.0105	-0.0264	0.0119	0.0162	0.0167	0.0078	0.0100	0.0111
0.25	0.0094	-0.0010	-0.0136	-0.0171	0.0115	-0.0284	0.0117	0.0158	0.0165	0.0076	0.0099	0.0112
0.27	0.0096	-0.0009	-0.0143	-0.0133	0.0121	-0.0299	0.0116	0.0157	0.0164	0.0075	0.0099	0.0113
0.29	0.0093	-0.0009	-0.0141	-0.0104	0.0128	-0.0314	0.0116	0.0154	0.0167	0.0074	0.0099	0.0113
0.31	0.0092	-0.0012	-0.0135	-0.0076	0.0133	-0.0326	0.0115	0.0151	0.0165	0.0073	0.0099	0.0113
0.33	0.0089	-0.0008	-0.0141	-0.0050	0.0137	-0.0335	0.0113	0.0151	0.0164	0.0073	0.0098	0.0114
0.35	0.0089	-0.0011	-0.0142	-0.0026	0.0140	-0.0342	0.0112	0.0148	0.0162	0.0073	0.0098	0.0114
0.37	0.0082	-0.0010	-0.0131	-0.0003	0.0141	-0.0347	0.0112	0.0148	0.0160	0.0073	0.0098	0.0114
0.39	0.0077	-0.0008	-0.0128	0.0019	0.0141	-0.0351	0.0112	0.0146	0.0158	0.0073	0.0098	0.0114
0.41	0.0071	-0.0004	-0.0125	0.0039	0.0141	-0.0352	0.0112	0.0146	0.0159	0.0073	0.0098	0.0114
0.43	0.0073	-0.0007	-0.0126	0.0059	0.0139	-0.0351	0.0112	0.0145	0.0158	0.0074	0.0098	0.0114
0.45	0.0078	-0.0013	-0.0131	0.0078	0.0137	-0.0349	0.0112	0.0147	0.0157	0.0074	0.0098	0.0114
0.47	0.0075	-0.0006	-0.0134	0.0096	0.0133	-0.0346	0.0114	0.0149	0.0159	0.0075	0.0098	0.0114
0.49	0.0068	0	-0.0127	0.0114	0.0129	-0.0340	0.0116	0.0150	0.0162	0.0075	0.0099	0.0114
0.51	0.0063	-0.0002	-0.0121	0.0131	0.0124	-0.0333	0.0117	0.0150	0.0163	0.0076	0.0099	0.0114
0.53	0.0062	0.0002	-0.0125	0.0147	0.0118	-0.0325	0.0118	0.0151	0.0162	0.0077	0.0099	0.0114
0.55	0.0054	0.0010	-0.0121	0.0163	0.0111	-0.0314	0.0119	0.0152	0.0164	0.0078	0.0099	0.0114
0.57	0.0046	0.0010	-0.0109	0.0179	0.0103	-0.0302	0.0120	0.0153	0.0164	0.0079	0.0100	0.0114
0.59	0.0053	0.0004	-0.0118	0.0194	0.0095	-0.0289	0.0123	0.0157	0.0166	0.0080	0.0100	0.0113
0.61	0.0045	0.0006	-0.0109	0.0209	0.0085	-0.0274	0.0123	0.0158	0.0166	0.0082	0.0101	0.0113
0.63	0.0044	0.0001	-0.0105	0.0224	0.0075	-0.0257	0.0124	0.0158	0.0169	0.0083	0.0101	0.0113
0.65	0.0039	0.0002	-0.0101	0.0239	0.0063	-0.0238	0.0125	0.0159	0.0169	0.0084	0.0102	0.0113
0.67	0.0038	-0.0001	-0.0100	0.0255	0.0050	-0.0217	0.0127	0.0161	0.0170	0.0086	0.0102	0.0112
0.69	0.0039	-0.0001	-0.0102	0.0270	0.0036	-0.0194	0.0128	0.0164	0.0172	0.0087	0.0103	0.0112
0.71	0.0040	-0.0008	-0.0101	0.0286	0.0021	-0.0168	0.0129	0.0165	0.0174	0.0089	0.0104	0.0112
0.73	0.0039	-0.0005	-0.0105	0.0302	0.0004	-0.0141	0.0132	0.0167	0.0177	0.0091	0.0105	0.0112
0.75	0.0035	-0.0005	-0.0099	0.0325	-0.0014	-0.0110	0.0132	0.0169	0.0178	0.0093	0.0106	0.0112
0.77	0.0029	-0.0003	-0.0095	0.0347	-0.0034	-0.0076	0.0133	0.0172	0.0179	0.0096	0.0107	0.0112
0.79	0.0029	-0.0007	-0.0092	0.0367	-0.0057	-0.0037	0.0137	0.0175	0.0185	0.0098	0.0109	0.0113
0.81	0.0026	-0.0010	-0.0089	0.0389	-0.0083	0.0005	0.0139	0.0175	0.0187	0.0101	0.0111	0.0115
0.83	0.0013	-0.0007	-0.0076	0.0414	-0.0112	0.0054	0.0143	0.0184	0.0190	0.0105	0.0113	0.0116
0.85	0.0006	-0.0008	-0.0068	0.0441	-0.0145	0.0108	0.0147	0.0190	0.0195	0.0109	0.0116	0.0119
0.87	-0.0004	-0.0019	-0.0048	0.0472	-0.0182	0.0171	0.0154	0.0198	0.0204	0.0114	0.0119	0.0123
0.89	-0.0006	-0.0044	-0.0029	0.0510	-0.0226	0.0244	0.0162	0.0210	0.0211	0.0120	0.0123	0.0129
0.91	-0.0027	-0.0043	-0.0002	0.0556	-0.0279	0.0331	0.0172	0.0222	0.0222	0.0129	0.0129	0.0137
0.93	-0.0049	-0.0051	0.0029	0.0617	-0.0344	0.0439	0.0186	0.0237	0.0239	0.0140	0.0138	0.0149
0.95	-0.0083	-0.0060	0.0082	0.0705	-0.0429	0.0580	0.0208	0.0262	0.0267	0.0157	0.0150	0.0169
0.97	-0.0131	-0.0086	0.0155	0.0850	-0.0553	0.0783	0.0247	0.0320	0.0324	0.0189	0.0170	0.0205
0.99	-0.0342	-0.0310	0.0499	0.1202	-0.0784	0.1154	0.0383	0.0516	0.0526	0.0279	0.0218	0.0294

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 11: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \sqrt[3]{\tau}$*: bias and MSE of estimators*

	Bias						*MSE*					
	canonical			*smoothed*			*canonical*			*smoothed*		
τ	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1
0.01	0.0083	-0.0230	0.0499	-0.1663	-0.0411	0.1110	0.0443	0.0637	0.0638	0.0404	0.0185	0.0291
0.03	0.0095	-0.0107	0.0061	-0.1033	-0.0316	0.0509	0.0258	0.0361	0.0356	0.0213	0.0153	0.0166
0.05	0.0134	-0.0097	-0.0050	-0.0754	-0.0261	0.0237	0.0201	0.0275	0.0267	0.0153	0.0136	0.0132
0.07	0.0169	-0.0108	-0.0121	-0.0577	-0.0222	0.0068	0.0170	0.0234	0.0227	0.0124	0.0125	0.0118
0.09	0.0169	-0.0083	-0.0158	-0.0450	-0.0191	-0.0052	0.0151	0.0207	0.0201	0.0106	0.0118	0.0111
0.11	0.0167	-0.0078	-0.0162	-0.0350	-0.0166	-0.0144	0.0143	0.0193	0.0189	0.0095	0.0112	0.0108
0.13	0.0170	-0.0084	-0.0165	-0.0270	-0.0145	-0.0215	0.0133	0.0181	0.0173	0.0088	0.0108	0.0107
0.15	0.0176	-0.0082	-0.0178	-0.0203	-0.0127	-0.0273	0.0127	0.0173	0.0165	0.0082	0.0105	0.0107
0.17	0.0181	-0.0089	-0.0185	-0.0145	-0.0111	-0.0320	0.0125	0.0166	0.0161	0.0079	0.0102	0.0108
0.19	0.0178	-0.0077	-0.0192	-0.0095	-0.0097	-0.0358	0.0122	0.0161	0.0157	0.0076	0.0100	0.0108
0.21	0.0176	-0.0074	-0.0191	-0.0051	-0.0084	-0.0389	0.0118	0.0154	0.0153	0.0075	0.0098	0.0109
0.23	0.0167	-0.0071	-0.0184	-0.0013	-0.0073	-0.0415	0.0116	0.0152	0.0151	0.0074	0.0096	0.0110
0.25	0.0161	-0.0071	-0.0180	0.0024	-0.0063	-0.0434	0.0116	0.0151	0.0150	0.0073	0.0095	0.0111
0.27	0.0162	-0.0077	-0.0176	0.0058	-0.0055	-0.0446	0.0114	0.0147	0.0149	0.0073	0.0094	0.0112
0.29	0.0154	-0.0076	-0.0165	0.0085	-0.0047	-0.0459	0.0115	0.0147	0.0148	0.0073	0.0093	0.0112
0.31	0.0153	-0.0072	-0.0168	0.0109	-0.0039	-0.0468	0.0113	0.0146	0.0145	0.0073	0.0092	0.0113
0.33	0.0151	-0.0074	-0.0166	0.0131	-0.0033	-0.0475	0.0113	0.0144	0.0145	0.0074	0.0092	0.0113
0.35	0.0145	-0.0068	-0.0161	0.0151	-0.0027	-0.0479	0.0112	0.0144	0.0142	0.0074	0.0091	0.0113
0.37	0.0150	-0.0070	-0.0171	0.0168	-0.0021	-0.0481	0.0112	0.0141	0.0142	0.0075	0.0091	0.0113
0.39	0.0149	-0.0068	-0.0173	0.0185	-0.0016	-0.0480	0.0112	0.0141	0.0142	0.0075	0.0090	0.0113
0.41	0.0145	-0.0065	-0.0171	0.0200	-0.0012	-0.0477	0.0111	0.0140	0.0141	0.0076	0.0090	0.0113
0.43	0.0146	-0.0073	-0.0163	0.0213	-0.0008	-0.0471	0.0111	0.0140	0.0140	0.0077	0.0090	0.0112
0.45	0.0147	-0.0075	-0.0164	0.0225	-0.0004	-0.0464	0.0112	0.0139	0.0140	0.0078	0.0089	0.0112
0.47	0.0149	-0.0085	-0.0163	0.0236	-0.0001	-0.0454	0.0114	0.0140	0.0141	0.0079	0.0089	0.0111
0.49	0.0141	-0.0083	-0.0154	0.0246	0.0001	-0.0443	0.0116	0.0140	0.0141	0.0080	0.0089	0.0110
0.51	0.0141	-0.0080	-0.0160	0.0255	0.0003	-0.0430	0.0116	0.0139	0.0140	0.0081	0.0089	0.0110
0.53	0.0137	-0.0081	-0.0154	0.0264	0.0005	-0.0414	0.0116	0.0139	0.0141	0.0082	0.0089	0.0109
0.55	0.0138	-0.0076	-0.0161	0.0271	0.0006	-0.0397	0.0116	0.0138	0.0140	0.0083	0.0089	0.0108
0.57	0.0124	-0.0067	-0.0150	0.0277	0.0007	-0.0377	0.0116	0.0136	0.0140	0.0084	0.0089	0.0107
0.59	0.0118	-0.0056	-0.0151	0.0283	0.0007	-0.0355	0.0118	0.0137	0.0142	0.0085	0.0090	0.0106
0.61	0.0118	-0.0052	-0.0155	0.0288	0.0007	-0.0331	0.0122	0.0139	0.0146	0.0086	0.0090	0.0105
0.63	0.0111	-0.0045	-0.0151	0.0293	0.0007	-0.0305	0.0122	0.0140	0.0147	0.0087	0.0090	0.0104
0.65	0.0110	-0.0048	-0.0149	0.0297	0.0005	-0.0276	0.0122	0.0139	0.0148	0.0088	0.0090	0.0103
0.67	0.0109	-0.0051	-0.0148	0.0300	0.0004	-0.0244	0.0124	0.0139	0.0151	0.0089	0.0091	0.0103
0.69	0.0104	-0.0047	-0.0145	0.0303	0.0001	-0.0210	0.0125	0.0140	0.0152	0.0090	0.0091	0.0102
0.71	0.0096	-0.0045	-0.0134	0.0306	-0.0001	-0.0173	0.0126	0.0141	0.0153	0.0091	0.0091	0.0102
0.73	0.0095	-0.0052	-0.0130	0.0308	-0.0005	-0.0132	0.0128	0.0143	0.0156	0.0092	0.0092	0.0101
0.75	0.0085	-0.0053	-0.0116	0.0316	-0.0009	-0.0087	0.0129	0.0144	0.0158	0.0093	0.0092	0.0101
0.77	0.0079	-0.0056	-0.0105	0.0321	-0.0015	-0.0038	0.0132	0.0145	0.0160	0.0095	0.0093	0.0102
0.79	0.0074	-0.0049	-0.0104	0.0324	-0.0022	0.0017	0.0137	0.0147	0.0166	0.0096	0.0094	0.0103
0.81	0.0058	-0.0044	-0.0086	0.0327	-0.0030	0.0078	0.0137	0.0148	0.0168	0.0098	0.0095	0.0105
0.83	0.0050	-0.0036	-0.0079	0.0331	-0.0040	0.0145	0.0139	0.0150	0.0170	0.0100	0.0096	0.0108
0.85	0.0039	-0.0037	-0.0072	0.0336	-0.0052	0.0222	0.0144	0.0154	0.0177	0.0102	0.0097	0.0113
0.87	0.0033	-0.0042	-0.0063	0.0342	-0.0066	0.0309	0.0148	0.0161	0.0182	0.0105	0.0099	0.0119
0.89	0.0017	-0.0033	-0.0049	0.0352	-0.0084	0.0411	0.0151	0.0168	0.0187	0.0108	0.0101	0.0129
0.91	-0.0002	-0.0035	-0.0022	0.0365	-0.0107	0.0531	0.0159	0.0175	0.0197	0.0112	0.0104	0.0143
0.93	-0.0032	-0.0034	0.0016	0.0388	-0.0137	0.0678	0.0170	0.0189	0.0212	0.0118	0.0107	0.0165
0.95	-0.0079	-0.0051	0.0094	0.0425	-0.0178	0.0868	0.0189	0.0208	0.0237	0.0127	0.0113	0.0199
0.97	-0.0139	-0.0090	0.0198	0.0500	-0.0239	0.1140	0.0224	0.0254	0.0281	0.0142	0.0122	0.0261
0.99	-0.0505	-0.0166	0.0671	0.0718	-0.0358	0.1634	0.0356	0.0414	0.0472	0.0187	0.0140	0.0412

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 12: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \cos(\tau)$*: bias and MSE of estimators*

	Bias						*MSE*					
	canonical			*smoothed*			*canonical*			*smoothed*		
τ	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1
0.01	0.0291	-0.0408	0.0284	-0.1117	-0.1026	0.0524	0.0404	0.0580	0.0551	0.0249	0.0258	0.0178
0.03	0.0125	-0.0133	0.0036	-0.0808	-0.0606	0.0242	0.0235	0.0333	0.0312	0.0168	0.0167	0.0138
0.05	0.0087	-0.0063	0.0002	-0.0670	-0.0404	0.0108	0.0184	0.0254	0.0248	0.0138	0.0135	0.0123
0.07	0.0075	-0.0013	-0.0048	-0.0580	-0.0272	0.0025	0.0157	0.0214	0.0214	0.0120	0.0119	0.0115
0.09	0.0073	-0.0008	-0.0058	-0.0515	-0.0174	-0.0035	0.0143	0.0194	0.0198	0.0109	0.0110	0.0111
0.11	0.0065	0.0012	-0.0072	-0.0463	-0.0097	-0.0079	0.0135	0.0176	0.0187	0.0101	0.0104	0.0108
0.13	0.0051	0.0027	-0.0070	-0.0421	-0.0034	-0.0113	0.0127	0.0168	0.0175	0.0095	0.0101	0.0106
0.15	0.0043	0.0031	-0.0067	-0.0385	0.0019	-0.0139	0.0123	0.0162	0.0169	0.0090	0.0098	0.0105
0.17	0.0027	0.0050	-0.0059	-0.0353	0.0064	-0.0160	0.0118	0.0156	0.0162	0.0086	0.0097	0.0104
0.19	0.0031	0.0050	-0.0070	-0.0325	0.0103	-0.0176	0.0114	0.0152	0.0158	0.0083	0.0097	0.0103
0.21	0.0033	0.0042	-0.0067	-0.0301	0.0137	-0.0188	0.0111	0.0149	0.0153	0.0080	0.0096	0.0103
0.23	0.0024	0.0048	-0.0059	-0.0278	0.0166	-0.0197	0.0110	0.0146	0.0152	0.0078	0.0096	0.0102
0.25	0.0012	0.0053	-0.0047	-0.0254	0.0191	-0.0203	0.0109	0.0146	0.0151	0.0076	0.0097	0.0102
0.27	0.0006	0.0056	-0.0041	-0.0230	0.0210	-0.0206	0.0108	0.0145	0.0150	0.0075	0.0097	0.0102
0.29	0.0008	0.0053	-0.0048	-0.0212	0.0229	-0.0208	0.0107	0.0143	0.0151	0.0074	0.0098	0.0102
0.31	0.0008	0.0051	-0.0049	-0.0195	0.0246	-0.0209	0.0107	0.0143	0.0149	0.0073	0.0098	0.0102
0.33	0.0005	0.0052	-0.0047	-0.0179	0.0260	-0.0209	0.0105	0.0142	0.0147	0.0072	0.0099	0.0102
0.35	0.0009	0.0046	-0.0050	-0.0164	0.0272	-0.0207	0.0105	0.0141	0.0150	0.0071	0.0100	0.0102
0.37	0.0012	0.0045	-0.0056	-0.0149	0.0282	-0.0205	0.0104	0.0142	0.0151	0.0070	0.0100	0.0101
0.39	0.0013	0.0045	-0.0059	-0.0134	0.0290	-0.0201	0.0105	0.0142	0.0152	0.0070	0.0101	0.0101
0.41	0.0006	0.0050	-0.0055	-0.0119	0.0296	-0.0196	0.0105	0.0145	0.0152	0.0069	0.0101	0.0102
0.43	-0.0007	0.0064	-0.0046	-0.0105	0.0300	-0.0190	0.0105	0.0145	0.0151	0.0069	0.0102	0.0102
0.45	-0.0008	0.0065	-0.0048	-0.0090	0.0302	-0.0184	0.0107	0.0148	0.0152	0.0069	0.0102	0.0102
0.47	-0.0009	0.0069	-0.0051	-0.0075	0.0303	-0.0177	0.0106	0.0147	0.0150	0.0069	0.0103	0.0102
0.49	-0.0010	0.0069	-0.0052	-0.0060	0.0301	-0.0169	0.0107	0.0148	0.0153	0.0069	0.0103	0.0102
0.51	-0.0010	0.0071	-0.0058	-0.0045	0.0298	-0.0161	0.0108	0.0150	0.0155	0.0069	0.0104	0.0102
0.53	-0.0010	0.0070	-0.0060	-0.0029	0.0294	-0.0152	0.0109	0.0151	0.0159	0.0069	0.0104	0.0103
0.55	-0.0007	0.0064	-0.0059	-0.0012	0.0287	-0.0143	0.0109	0.0151	0.0160	0.0069	0.0104	0.0103
0.57	-0.0008	0.0063	-0.0055	0.0005	0.0279	-0.0133	0.0110	0.0152	0.0159	0.0070	0.0105	0.0103
0.59	-0.0013	0.0068	-0.0054	0.0023	0.0268	-0.0122	0.0112	0.0155	0.0163	0.0070	0.0105	0.0104
0.61	-0.0015	0.0070	-0.0054	0.0042	0.0256	-0.0111	0.0114	0.0156	0.0165	0.0071	0.0105	0.0105
0.63	-0.0020	0.0076	-0.0052	0.0062	0.0241	-0.0099	0.0116	0.0160	0.0169	0.0072	0.0106	0.0105
0.65	-0.0022	0.0069	-0.0046	0.0083	0.0224	-0.0086	0.0115	0.0161	0.0168	0.0073	0.0106	0.0106
0.67	-0.0022	0.0066	-0.0045	0.0106	0.0205	-0.0073	0.0116	0.0162	0.0169	0.0074	0.0106	0.0107
0.69	-0.0014	0.0065	-0.0057	0.0130	0.0183	-0.0059	0.0117	0.0164	0.0171	0.0075	0.0107	0.0108
0.71	-0.0014	0.0069	-0.0064	0.0157	0.0159	-0.0045	0.0118	0.0166	0.0173	0.0077	0.0107	0.0109
0.73	-0.0010	0.0059	-0.0063	0.0186	0.0131	-0.0029	0.0121	0.0171	0.0177	0.0079	0.0108	0.0110
0.75	-0.0012	0.0062	-0.0067	0.0222	0.0101	-0.0012	0.0123	0.0176	0.0181	0.0081	0.0109	0.0111
0.77	-0.0010	0.0054	-0.0066	0.0259	0.0066	0.0006	0.0126	0.0181	0.0183	0.0084	0.0110	0.0113
0.79	-0.0005	0.0039	-0.0065	0.0298	0.0025	0.0025	0.0130	0.0182	0.0190	0.0088	0.0112	0.0115
0.81	-0.0006	0.0031	-0.0065	0.0342	-0.0020	0.0046	0.0131	0.0187	0.0190	0.0092	0.0114	0.0117
0.83	-0.0011	0.0031	-0.0058	0.0391	-0.0072	0.0070	0.0136	0.0196	0.0195	0.0097	0.0117	0.0119
0.85	-0.0007	0.0018	-0.0060	0.0447	-0.0132	0.0095	0.0140	0.0202	0.0201	0.0104	0.0121	0.0122
0.87	-0.0003	0.0007	-0.0065	0.0512	-0.0201	0.0125	0.0146	0.0209	0.0208	0.0113	0.0127	0.0126
0.89	-0.0013	-0.0006	-0.0039	0.0588	-0.0282	0.0159	0.0153	0.0218	0.0222	0.0124	0.0134	0.0130
0.91	-0.0023	-0.0016	-0.0024	0.0681	-0.0379	0.0201	0.0161	0.0231	0.0235	0.0139	0.0145	0.0136
0.93	-0.0006	-0.0062	-0.0027	0.0800	-0.0499	0.0252	0.0172	0.0256	0.0249	0.0160	0.0161	0.0143
0.95	-0.0011	-0.0121	0.0015	0.0962	-0.0657	0.0319	0.0197	0.0293	0.0283	0.0194	0.0186	0.0153
0.97	-0.0013	-0.0228	0.0079	0.1213	-0.0884	0.0418	0.0241	0.0357	0.0343	0.0257	0.0232	0.0170
0.99	-0.0069	-0.0569	0.0239	0.1762	-0.1306	0.0606	0.0363	0.0574	0.0527	0.0436	0.0344	0.0207

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 13: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \sqrt[6]{\tau}$*: bias and MSE of estimators*

	Bias						*MSE*					
	canonical			*smoothed*			*canonical*			*smoothed*		
τ	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1
0.01	0.0208	-0.0353	0.0403	-0.1445	-0.0651	0.0898	0.0445	0.0619	0.0627	0.0340	0.0213	0.0245
0.03	0.0151	-0.0165	0.0024	-0.0936	-0.0439	0.0404	0.0266	0.0374	0.0347	0.0197	0.0165	0.0156
0.05	0.0136	-0.0095	-0.0062	-0.0711	-0.0327	0.0184	0.0204	0.0281	0.0261	0.0151	0.0143	0.0130
0.07	0.0133	-0.0072	-0.0091	-0.0569	-0.0251	0.0046	0.0173	0.0237	0.0223	0.0127	0.0131	0.0119
0.09	0.0134	-0.0068	-0.0101	-0.0466	-0.0193	-0.0051	0.0158	0.0211	0.0203	0.0112	0.0123	0.0114
0.11	0.0131	-0.0054	-0.0120	-0.0387	-0.0146	-0.0125	0.0150	0.0193	0.0194	0.0102	0.0117	0.0111
0.13	0.0119	-0.0040	-0.0124	-0.0322	-0.0107	-0.0182	0.0141	0.0185	0.0182	0.0096	0.0113	0.0110
0.15	0.0118	-0.0037	-0.0128	-0.0268	-0.0074	-0.0227	0.0136	0.0177	0.0173	0.0091	0.0109	0.0110
0.17	0.0117	-0.0036	-0.0134	-0.0221	-0.0046	-0.0265	0.0133	0.0170	0.0168	0.0087	0.0107	0.0110
0.19	0.0125	-0.0041	-0.0146	-0.0180	-0.0022	-0.0294	0.0128	0.0164	0.0164	0.0084	0.0105	0.0110
0.21	0.0117	-0.0033	-0.0143	-0.0144	0	-0.0319	0.0128	0.0163	0.0163	0.0082	0.0104	0.0111
0.23	0.0101	-0.0015	-0.0131	-0.0112	0.0018	-0.0339	0.0125	0.0163	0.0160	0.0080	0.0102	0.0111
0.25	0.0089	0	-0.0129	-0.0081	0.0034	-0.0352	0.0122	0.0159	0.0158	0.0079	0.0102	0.0112
0.27	0.0085	0	-0.0121	-0.0051	0.0048	-0.0361	0.0119	0.0157	0.0154	0.0079	0.0101	0.0112
0.29	0.0086	-0.0003	-0.0119	-0.0028	0.0060	-0.0370	0.0119	0.0155	0.0154	0.0078	0.0101	0.0113
0.31	0.0084	-0.0004	-0.0122	-0.0006	0.0071	-0.0377	0.0118	0.0154	0.0153	0.0078	0.0100	0.0113
0.33	0.0082	-0.0002	-0.0118	0.0013	0.0081	-0.0381	0.0118	0.0152	0.0152	0.0078	0.0100	0.0113
0.35	0.0079	0.0005	-0.0124	0.0032	0.0089	-0.0383	0.0117	0.0149	0.0153	0.0078	0.0099	0.0113
0.37	0.0085	-0.0001	-0.0128	0.0048	0.0096	-0.0383	0.0117	0.0150	0.0152	0.0078	0.0099	0.0113
0.39	0.0080	0.0006	-0.0129	0.0064	0.0101	-0.0381	0.0117	0.0150	0.0152	0.0078	0.0099	0.0113
0.41	0.0077	0.0006	-0.0126	0.0079	0.0106	-0.0378	0.0118	0.0148	0.0154	0.0078	0.0099	0.0113
0.43	0.0078	0.0004	-0.0130	0.0094	0.0109	-0.0372	0.0118	0.0146	0.0154	0.0078	0.0099	0.0113
0.45	0.0079	0.0004	-0.0133	0.0107	0.0112	-0.0366	0.0119	0.0148	0.0154	0.0079	0.0099	0.0112
0.47	0.0077	0.0004	-0.0133	0.0120	0.0113	-0.0357	0.0120	0.0148	0.0158	0.0079	0.0099	0.0112
0.49	0.0069	0.0010	-0.0128	0.0132	0.0114	-0.0347	0.0119	0.0150	0.0156	0.0080	0.0099	0.0112
0.51	0.0072	0.0001	-0.0129	0.0144	0.0113	-0.0335	0.0122	0.0150	0.0159	0.0080	0.0099	0.0111
0.53	0.0071	-0.0002	-0.0125	0.0155	0.0112	-0.0322	0.0123	0.0152	0.0160	0.0081	0.0099	0.0111
0.55	0.0067	-0.0004	-0.0121	0.0166	0.0110	-0.0307	0.0124	0.0153	0.0161	0.0082	0.0099	0.0110
0.57	0.0070	-0.0011	-0.0117	0.0177	0.0107	-0.0291	0.0123	0.0153	0.0158	0.0083	0.0099	0.0110
0.59	0.0059	-0.0005	-0.0105	0.0187	0.0102	-0.0273	0.0123	0.0154	0.0158	0.0083	0.0099	0.0110
0.61	0.0069	-0.0021	-0.0108	0.0198	0.0097	-0.0253	0.0123	0.0154	0.0157	0.0084	0.0099	0.0109
0.63	0.0059	-0.0014	-0.0101	0.0208	0.0091	-0.0231	0.0123	0.0152	0.0159	0.0085	0.0099	0.0109
0.65	0.0061	-0.0017	-0.0103	0.0218	0.0084	-0.0208	0.0124	0.0154	0.0159	0.0086	0.0099	0.0108
0.67	0.0057	-0.0010	-0.0101	0.0228	0.0076	-0.0182	0.0126	0.0155	0.0161	0.0087	0.0100	0.0108
0.69	0.0053	-0.0007	-0.0099	0.0238	0.0066	-0.0154	0.0128	0.0156	0.0162	0.0088	0.0100	0.0108
0.71	0.0048	-0.0013	-0.0087	0.0249	0.0056	-0.0124	0.0130	0.0156	0.0164	0.0089	0.0100	0.0108
0.73	0.0043	-0.0015	-0.0081	0.0260	0.0043	-0.0091	0.0133	0.0156	0.0168	0.0091	0.0101	0.0108
0.75	0.0044	-0.0012	-0.0087	0.0277	0.0030	-0.0056	0.0134	0.0157	0.0169	0.0092	0.0101	0.0109
0.77	0.0040	-0.0015	-0.0085	0.0292	0.0015	-0.0016	0.0135	0.0160	0.0173	0.0094	0.0102	0.0109
0.79	0.0040	-0.0027	-0.0078	0.0306	-0.0002	0.0028	0.0134	0.0162	0.0172	0.0096	0.0103	0.0111
0.81	0.0038	-0.0023	-0.0080	0.0322	-0.0022	0.0077	0.0138	0.0165	0.0177	0.0098	0.0104	0.0113
0.83	0.0029	-0.0024	-0.0071	0.0340	-0.0045	0.0131	0.0141	0.0171	0.0181	0.0101	0.0105	0.0116
0.85	0.0024	-0.0019	-0.0072	0.0360	-0.0071	0.0192	0.0144	0.0174	0.0186	0.0104	0.0107	0.0120
0.87	0.0015	-0.0028	-0.0054	0.0384	-0.0102	0.0262	0.0149	0.0181	0.0194	0.0108	0.0110	0.0126
0.89	0.0009	-0.0045	-0.0037	0.0413	-0.0138	0.0343	0.0157	0.0192	0.0204	0.0113	0.0113	0.0133
0.91	-0.0013	-0.0038	-0.0014	0.0450	-0.0182	0.0440	0.0170	0.0206	0.0218	0.0120	0.0118	0.0145
0.93	-0.0033	-0.0054	0.0022	0.0498	-0.0236	0.0559	0.0182	0.0223	0.0235	0.0128	0.0124	0.0161
0.95	-0.0070	-0.0060	0.0072	0.0569	-0.0308	0.0714	0.0199	0.0247	0.0257	0.0142	0.0133	0.0187
0.97	-0.0110	-0.0101	0.0145	0.0690	-0.0413	0.0939	0.0239	0.0296	0.0314	0.0167	0.0149	0.0232
0.99	-0.0390	-0.0248	0.0542	0.0994	-0.0606	0.1349	0.0378	0.0475	0.0515	0.0235	0.0184	0.0343

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 14: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \frac{sin(\tau)+1}{2}$*: bias and MSE of estimators*

	Bias						*MSE*					
	canonical			*smoothed*			*canonical*			*smoothed*		
τ	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1
0.01	0.0092	-0.0231	0.0500	-0.1607	-0.0515	0.1010	0.0424	0.0602	0.0592	0.0385	0.0195	0.0264
0.03	0.0047	-0.0037	0.0091	-0.1119	-0.0277	0.0546	0.0250	0.0354	0.0337	0.0232	0.0152	0.0169
0.05	0.0063	-0.0013	-0.0002	-0.0888	-0.0168	0.0321	0.0193	0.0277	0.0258	0.0176	0.0135	0.0139
0.07	0.0069	-0.0012	-0.0038	-0.0734	-0.0100	0.0175	0.0171	0.0241	0.0224	0.0145	0.0125	0.0124
0.09	0.0082	-0.0023	-0.0065	-0.0619	-0.0053	0.0067	0.0152	0.0212	0.0205	0.0126	0.0118	0.0117
0.11	0.0088	-0.0019	-0.0087	-0.0526	-0.0017	-0.0017	0.0143	0.0200	0.0191	0.0112	0.0114	0.0112
0.13	0.0090	-0.0020	-0.0095	-0.0448	0.0011	-0.0085	0.0138	0.0190	0.0183	0.0103	0.0110	0.0110
0.15	0.0089	-0.0016	-0.0104	-0.0382	0.0033	-0.0141	0.0133	0.0182	0.0178	0.0095	0.0108	0.0109
0.17	0.0090	-0.0014	-0.0111	-0.0324	0.0051	-0.0188	0.0128	0.0174	0.0171	0.0090	0.0106	0.0109
0.19	0.0097	-0.0016	-0.0124	-0.0273	0.0066	-0.0228	0.0124	0.0169	0.0166	0.0085	0.0104	0.0109
0.21	0.0095	-0.0010	-0.0132	-0.0226	0.0078	-0.0262	0.0123	0.0166	0.0164	0.0082	0.0102	0.0109
0.23	0.0090	-0.0002	-0.0132	-0.0184	0.0087	-0.0290	0.0121	0.0161	0.0160	0.0079	0.0101	0.0110
0.25	0.0092	-0.0001	-0.0141	-0.0142	0.0094	-0.0313	0.0118	0.0157	0.0157	0.0077	0.0100	0.0111
0.27	0.0097	-0.0005	-0.0149	-0.0103	0.0099	-0.0331	0.0117	0.0156	0.0155	0.0076	0.0099	0.0111
0.29	0.0089	0	-0.0144	-0.0071	0.0104	-0.0348	0.0115	0.0152	0.0154	0.0075	0.0098	0.0112
0.31	0.0084	0.0012	-0.0148	-0.0042	0.0107	-0.0362	0.0115	0.0151	0.0155	0.0074	0.0098	0.0113
0.33	0.0081	0.0014	-0.0146	-0.0014	0.0110	-0.0373	0.0115	0.0151	0.0155	0.0073	0.0097	0.0113
0.35	0.0082	0.0008	-0.0146	0.0011	0.0111	-0.0382	0.0112	0.0148	0.0153	0.0073	0.0097	0.0114
0.37	0.0087	0.0005	-0.0154	0.0035	0.0112	-0.0388	0.0111	0.0146	0.0154	0.0073	0.0096	0.0114
0.39	0.0093	0	-0.0162	0.0057	0.0112	-0.0392	0.0112	0.0145	0.0156	0.0073	0.0096	0.0114
0.41	0.0095	-0.0004	-0.0162	0.0078	0.0111	-0.0393	0.0113	0.0146	0.0158	0.0073	0.0095	0.0115
0.43	0.0095	-0.0010	-0.0160	0.0098	0.0110	-0.0393	0.0115	0.0147	0.0158	0.0074	0.0095	0.0115
0.45	0.0095	-0.0018	-0.0154	0.0117	0.0107	-0.0391	0.0114	0.0147	0.0157	0.0074	0.0095	0.0115
0.47	0.0103	-0.0020	-0.0164	0.0135	0.0105	-0.0387	0.0115	0.0146	0.0158	0.0075	0.0095	0.0115
0.49	0.0100	-0.0016	-0.0162	0.0152	0.0101	-0.0380	0.0115	0.0144	0.0158	0.0075	0.0095	0.0115
0.51	0.0095	-0.0015	-0.0157	0.0168	0.0097	-0.0372	0.0115	0.0144	0.0160	0.0076	0.0095	0.0114
0.53	0.0089	-0.0016	-0.0148	0.0183	0.0092	-0.0363	0.0115	0.0144	0.0160	0.0077	0.0095	0.0114
0.55	0.0087	-0.0018	-0.0143	0.0198	0.0087	-0.0351	0.0115	0.0143	0.0159	0.0078	0.0095	0.0114
0.57	0.0086	-0.0016	-0.0145	0.0212	0.0081	-0.0337	0.0115	0.0144	0.0160	0.0079	0.0095	0.0113
0.59	0.0077	-0.0006	-0.0141	0.0225	0.0074	-0.0322	0.0116	0.0144	0.0160	0.0080	0.0095	0.0113
0.61	0.0074	-0.0009	-0.0136	0.0238	0.0067	-0.0304	0.0119	0.0146	0.0164	0.0081	0.0095	0.0113
0.63	0.0064	-0.0001	-0.0129	0.0251	0.0058	-0.0284	0.0121	0.0147	0.0165	0.0082	0.0096	0.0112
0.65	0.0057	0.0001	-0.0122	0.0263	0.0050	-0.0262	0.0122	0.0147	0.0165	0.0083	0.0096	0.0112
0.67	0.0046	0.0007	-0.0112	0.0275	0.0040	-0.0238	0.0122	0.0148	0.0165	0.0085	0.0097	0.0112
0.69	0.0038	0.0007	-0.0098	0.0287	0.0029	-0.0211	0.0123	0.0150	0.0166	0.0086	0.0097	0.0111
0.71	0.0037	0.0007	-0.0101	0.0300	0.0017	-0.0182	0.0124	0.0152	0.0169	0.0088	0.0098	0.0111
0.73	0.0038	0.0004	-0.0101	0.0312	0.0004	-0.0150	0.0126	0.0154	0.0171	0.0089	0.0099	0.0111
0.75	0.0035	0.0002	-0.0097	0.0330	-0.0010	-0.0115	0.0128	0.0158	0.0172	0.0091	0.0099	0.0111
0.77	0.0026	0.0007	-0.0088	0.0347	-0.0026	-0.0075	0.0130	0.0160	0.0176	0.0094	0.0100	0.0112
0.79	0.0015	0.0002	-0.0068	0.0362	-0.0044	-0.0032	0.0130	0.0162	0.0177	0.0096	0.0102	0.0113
0.81	0.0018	-0.0005	-0.0066	0.0379	-0.0065	0.0017	0.0136	0.0169	0.0182	0.0099	0.0103	0.0114
0.83	0.0016	-0.0011	-0.0063	0.0398	-0.0088	0.0071	0.0142	0.0176	0.0188	0.0102	0.0105	0.0117
0.85	0.0013	-0.0018	-0.0053	0.0419	-0.0115	0.0133	0.0146	0.0178	0.0197	0.0105	0.0108	0.0120
0.87	0.0008	-0.0025	-0.0044	0.0444	-0.0146	0.0202	0.0155	0.0187	0.0209	0.0110	0.0111	0.0125
0.89	0.0006	-0.0037	-0.0040	0.0475	-0.0183	0.0283	0.0161	0.0195	0.0217	0.0115	0.0114	0.0132
0.91	-0.0011	-0.0027	-0.0027	0.0514	-0.0227	0.0379	0.0170	0.0206	0.0230	0.0122	0.0119	0.0142
0.93	-0.0032	-0.0034	0.0003	0.0566	-0.0283	0.0497	0.0179	0.0218	0.0247	0.0132	0.0126	0.0157
0.95	-0.0040	-0.0077	0.0037	0.0641	-0.0357	0.0650	0.0196	0.0247	0.0267	0.0147	0.0136	0.0181
0.97	-0.0087	-0.0130	0.0132	0.0769	-0.0465	0.0869	0.0241	0.0304	0.0327	0.0174	0.0153	0.0223
0.99	-0.0329	-0.0318	0.0491	0.1089	-0.0668	0.1271	0.0383	0.0492	0.0522	0.0252	0.0191	0.0326

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 100$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 15: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \sqrt{\tau}$*: bias and MSE of estimators*

	Bias						*MSE*					
	canonical			*smoothed*			*canonical*			*smoothed*		
τ	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1
0.01	-0.0010	-0.0043	0.0104	-0.1150	-0.0094	0.0856	0.0075	0.0118	0.0108	0.0163	0.0049	0.0118
0.03	0.0022	-0.0011	0	-0.0731	-0.0073	0.0443	0.0041	0.0066	0.0057	0.0076	0.0035	0.0052
0.05	0.0035	-0.0020	-0.0021	-0.0543	-0.0062	0.0258	0.0032	0.0051	0.0044	0.0049	0.0030	0.0034
0.07	0.0034	-0.0018	-0.0022	-0.0423	-0.0054	0.0141	0.0028	0.0044	0.0039	0.0035	0.0027	0.0027
0.09	0.0034	-0.0018	-0.0026	-0.0334	-0.0048	0.0055	0.0026	0.0040	0.0036	0.0028	0.0025	0.0024
0.11	0.0034	-0.0015	-0.0030	-0.0264	-0.0043	-0.0012	0.0025	0.0037	0.0034	0.0023	0.0023	0.0022
0.13	0.0037	-0.0019	-0.0033	-0.0207	-0.0039	-0.0066	0.0024	0.0036	0.0033	0.0019	0.0022	0.0022
0.15	0.0040	-0.0021	-0.0038	-0.0159	-0.0036	-0.0109	0.0023	0.0034	0.0032	0.0017	0.0021	0.0022
0.17	0.0045	-0.0024	-0.0042	-0.0118	-0.0033	-0.0145	0.0022	0.0032	0.0030	0.0016	0.0021	0.0022
0.19	0.0049	-0.0029	-0.0044	-0.0082	-0.0030	-0.0175	0.0022	0.0031	0.0030	0.0015	0.0020	0.0023
0.21	0.0048	-0.0027	-0.0044	-0.0051	-0.0028	-0.0200	0.0021	0.0030	0.0029	0.0014	0.0019	0.0024
0.23	0.0047	-0.0027	-0.0044	-0.0024	-0.0026	-0.0221	0.0021	0.0029	0.0029	0.0014	0.0019	0.0024
0.25	0.0046	-0.0025	-0.0043	0.0001	-0.0024	-0.0239	0.0021	0.0028	0.0029	0.0014	0.0018	0.0025
0.27	0.0046	-0.0024	-0.0043	0.0023	-0.0022	-0.0253	0.0021	0.0028	0.0028	0.0014	0.0018	0.0025
0.29	0.0043	-0.0023	-0.0040	0.0043	-0.0021	-0.0266	0.0021	0.0027	0.0028	0.0014	0.0018	0.0026
0.31	0.0044	-0.0026	-0.0039	0.0060	-0.0019	-0.0276	0.0020	0.0026	0.0028	0.0014	0.0017	0.0026
0.33	0.0041	-0.0023	-0.0038	0.0076	-0.0018	-0.0284	0.0020	0.0026	0.0028	0.0014	0.0017	0.0027
0.35	0.0040	-0.0023	-0.0036	0.0091	-0.0017	-0.0290	0.0020	0.0025	0.0027	0.0014	0.0017	0.0027
0.37	0.0042	-0.0026	-0.0038	0.0104	-0.0015	-0.0294	0.0020	0.0025	0.0028	0.0015	0.0017	0.0027
0.39	0.0040	-0.0025	-0.0036	0.0115	-0.0014	-0.0296	0.0020	0.0025	0.0028	0.0015	0.0017	0.0027
0.41	0.0038	-0.0021	-0.0037	0.0126	-0.0013	-0.0297	0.0020	0.0025	0.0028	0.0015	0.0016	0.0027
0.43	0.0039	-0.0021	-0.0039	0.0135	-0.0012	-0.0296	0.0020	0.0024	0.0028	0.0015	0.0016	0.0027
0.45	0.0039	-0.0020	-0.0041	0.0144	-0.0011	-0.0294	0.0020	0.0024	0.0028	0.0016	0.0016	0.0027
0.47	0.0041	-0.0023	-0.0042	0.0151	-0.0010	-0.0290	0.0020	0.0024	0.0028	0.0016	0.0016	0.0027
0.49	0.0041	-0.0024	-0.0043	0.0158	-0.0009	-0.0285	0.0020	0.0024	0.0028	0.0016	0.0016	0.0027
0.51	0.0040	-0.0023	-0.0044	0.0164	-0.0008	-0.0279	0.0020	0.0024	0.0027	0.0016	0.0016	0.0027
0.53	0.0039	-0.0023	-0.0043	0.0169	-0.0007	-0.0271	0.0020	0.0024	0.0028	0.0017	0.0016	0.0026
0.55	0.0045	-0.0027	-0.0047	0.0174	-0.0006	-0.0261	0.0021	0.0024	0.0028	0.0017	0.0016	0.0026
0.57	0.0042	-0.0025	-0.0045	0.0177	-0.0005	-0.0251	0.0021	0.0024	0.0028	0.0017	0.0016	0.0025
0.59	0.0040	-0.0023	-0.0043	0.0180	-0.0004	-0.0238	0.0021	0.0023	0.0028	0.0017	0.0016	0.0025
0.61	0.0039	-0.0024	-0.0043	0.0182	-0.0003	-0.0224	0.0021	0.0023	0.0028	0.0018	0.0016	0.0024
0.63	0.0041	-0.0027	-0.0043	0.0184	-0.0002	-0.0207	0.0021	0.0024	0.0028	0.0018	0.0016	0.0024
0.65	0.0043	-0.0027	-0.0046	0.0184	-0.0001	-0.0189	0.0021	0.0023	0.0028	0.0018	0.0016	0.0023
0.67	0.0042	-0.0027	-0.0045	0.0184	0	-0.0169	0.0021	0.0023	0.0027	0.0018	0.0016	0.0022
0.69	0.0037	-0.0024	-0.0041	0.0183	0.0001	-0.0146	0.0022	0.0023	0.0028	0.0018	0.0016	0.0022
0.71	0.0036	-0.0024	-0.0039	0.0182	0.0002	-0.0121	0.0021	0.0023	0.0027	0.0018	0.0016	0.0021
0.73	0.0032	-0.0024	-0.0035	0.0180	0.0003	-0.0093	0.0021	0.0023	0.0027	0.0018	0.0016	0.0021
0.75	0.0031	-0.0021	-0.0036	0.0177	0.0003	-0.0063	0.0021	0.0023	0.0027	0.0018	0.0016	0.0020
0.77	0.0028	-0.0020	-0.0034	0.0174	0.0004	-0.0028	0.0022	0.0024	0.0028	0.0018	0.0016	0.0020
0.79	0.0026	-0.0020	-0.0030	0.0169	0.0004	0.0010	0.0022	0.0024	0.0028	0.0019	0.0016	0.0020
0.81	0.0021	-0.0015	-0.0027	0.0164	0.0004	0.0053	0.0022	0.0024	0.0028	0.0019	0.0017	0.0021
0.83	0.0019	-0.0014	-0.0024	0.0159	0.0003	0.0102	0.0023	0.0024	0.0028	0.0019	0.0017	0.0022
0.85	0.0014	-0.0010	-0.0020	0.0153	0.0002	0.0158	0.0024	0.0025	0.0029	0.0019	0.0017	0.0023
0.87	0.0009	-0.0003	-0.0020	0.0146	0	0.0223	0.0024	0.0026	0.0030	0.0019	0.0017	0.0026
0.89	0.0011	-0.0008	-0.0018	0.0139	-0.0003	0.0300	0.0025	0.0026	0.0031	0.0019	0.0017	0.0031
0.91	0.0002	-0.0003	-0.0011	0.0131	-0.0008	0.0392	0.0027	0.0027	0.0032	0.0020	0.0018	0.0038
0.93	-0.0007	-0.0002	-0.0001	0.0125	-0.0015	0.0507	0.0028	0.0027	0.0034	0.0021	0.0019	0.0049
0.95	-0.0024	0.0002	0.0017	0.0127	-0.0028	0.0655	0.0031	0.0030	0.0037	0.0022	0.0019	0.0067
0.97	-0.0046	0.0008	0.0042	0.0143	-0.0048	0.0871	0.0035	0.0035	0.0042	0.0024	0.0021	0.0102
0.99	-0.0121	0.0005	0.0148	0.0243	-0.0094	0.1271	0.0053	0.0050	0.0066	0.0033	0.0025	0.0192

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 500$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 16: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \frac{1}{1+e^{-\tau}}$*: bias and MSE of estimators*

	Bias						*MSE*					
	canonical			*smoothed*			*canonical*			*smoothed*		
τ	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1
0.01	0.0002	-0.0040	0.0082	-0.0917	-0.0380	0.0616	0.0081	0.0112	0.0104	0.0115	0.0058	0.0077
0.03	0.0008	0.0001	0.0006	-0.0649	-0.0197	0.0341	0.0046	0.0064	0.0057	0.0066	0.0037	0.0042
0.05	-0.0002	0.0018	0.0003	-0.0525	-0.0113	0.0213	0.0036	0.0051	0.0046	0.0048	0.0031	0.0031
0.07	0.0003	0.0012	-0.0007	-0.0442	-0.0060	0.0129	0.0032	0.0045	0.0042	0.0039	0.0028	0.0026
0.09	0.0005	0.0015	-0.0014	-0.0380	-0.0022	0.0068	0.0029	0.0041	0.0038	0.0033	0.0026	0.0024
0.11	0.0010	0.0010	-0.0020	-0.0330	0.0007	0.0019	0.0028	0.0038	0.0036	0.0029	0.0025	0.0023
0.13	0.0010	0.0010	-0.0020	-0.0288	0.0029	-0.0019	0.0027	0.0037	0.0034	0.0025	0.0024	0.0022
0.15	0.0013	0.0007	-0.0021	-0.0251	0.0047	-0.0051	0.0026	0.0036	0.0033	0.0023	0.0023	0.0022
0.17	0.0012	0.0008	-0.0021	-0.0220	0.0061	-0.0077	0.0025	0.0035	0.0032	0.0021	0.0023	0.0022
0.19	0.0014	0.0005	-0.0023	-0.0191	0.0073	-0.0099	0.0024	0.0034	0.0032	0.0020	0.0023	0.0022
0.21	0.0011	0.0008	-0.0020	-0.0166	0.0082	-0.0118	0.0023	0.0033	0.0031	0.0019	0.0023	0.0022
0.23	0.0013	0.0007	-0.0025	-0.0143	0.0090	-0.0134	0.0023	0.0032	0.0031	0.0018	0.0022	0.0023
0.25	0.0015	0.0004	-0.0025	-0.0121	0.0096	-0.0147	0.0023	0.0031	0.0031	0.0017	0.0022	0.0023
0.27	0.0015	0.0005	-0.0026	-0.0102	0.0101	-0.0157	0.0023	0.0031	0.0031	0.0017	0.0022	0.0023
0.29	0.0012	0.0010	-0.0026	-0.0084	0.0105	-0.0167	0.0023	0.0031	0.0030	0.0016	0.0022	0.0023
0.31	0.0013	0.0007	-0.0024	-0.0067	0.0107	-0.0174	0.0023	0.0031	0.0030	0.0016	0.0022	0.0023
0.33	0.0012	0.0010	-0.0028	-0.0051	0.0109	-0.0180	0.0023	0.0031	0.0030	0.0016	0.0022	0.0024
0.35	0.0010	0.0011	-0.0026	-0.0036	0.0111	-0.0185	0.0023	0.0030	0.0030	0.0016	0.0022	0.0024
0.37	0.0011	0.0011	-0.0027	-0.0022	0.0111	-0.0188	0.0022	0.0030	0.0030	0.0015	0.0022	0.0024
0.39	0.0011	0.0010	-0.0025	-0.0009	0.0111	-0.0190	0.0022	0.0029	0.0031	0.0015	0.0022	0.0024
0.41	0.0013	0.0008	-0.0029	0.0004	0.0110	-0.0191	0.0023	0.0030	0.0031	0.0015	0.0022	0.0024
0.43	0.0013	0.0008	-0.0030	0.0016	0.0109	-0.0191	0.0023	0.0030	0.0031	0.0015	0.0022	0.0024
0.45	0.0014	0.0007	-0.0031	0.0027	0.0107	-0.0190	0.0023	0.0030	0.0031	0.0016	0.0021	0.0024
0.47	0.0013	0.0007	-0.0027	0.0038	0.0105	-0.0189	0.0023	0.0029	0.0031	0.0016	0.0021	0.0024
0.49	0.0013	0.0008	-0.0029	0.0049	0.0102	-0.0186	0.0023	0.0029	0.0031	0.0016	0.0021	0.0024
0.51	0.0016	0.0006	-0.0032	0.0059	0.0099	-0.0182	0.0023	0.0030	0.0031	0.0016	0.0021	0.0024
0.53	0.0014	0.0007	-0.0031	0.0069	0.0095	-0.0178	0.0023	0.0030	0.0031	0.0016	0.0021	0.0024
0.55	0.0015	0.0010	-0.0036	0.0079	0.0091	-0.0173	0.0023	0.0030	0.0031	0.0016	0.0021	0.0024
0.57	0.0015	0.0008	-0.0035	0.0089	0.0086	-0.0167	0.0023	0.0030	0.0031	0.0017	0.0021	0.0024
0.59	0.0018	0.0006	-0.0039	0.0099	0.0081	-0.0159	0.0023	0.0030	0.0032	0.0017	0.0021	0.0024
0.61	0.0015	0.0009	-0.0035	0.0108	0.0075	-0.0151	0.0024	0.0031	0.0032	0.0017	0.0022	0.0024
0.63	0.0013	0.0009	-0.0033	0.0117	0.0069	-0.0142	0.0024	0.0031	0.0032	0.0018	0.0022	0.0024
0.65	0.0013	0.0010	-0.0034	0.0127	0.0062	-0.0132	0.0024	0.0031	0.0032	0.0018	0.0022	0.0024
0.67	0.0009	0.0015	-0.0032	0.0136	0.0054	-0.0121	0.0024	0.0031	0.0033	0.0018	0.0022	0.0024
0.69	0.0009	0.0012	-0.0030	0.0146	0.0046	-0.0108	0.0025	0.0032	0.0033	0.0019	0.0022	0.0024
0.71	0.0009	0.0011	-0.0029	0.0156	0.0037	-0.0094	0.0025	0.0032	0.0034	0.0019	0.0022	0.0024
0.73	0.0011	0.0009	-0.0031	0.0166	0.0026	-0.0078	0.0026	0.0033	0.0034	0.0020	0.0022	0.0023
0.75	0.0016	0.0005	-0.0037	0.0177	0.0015	-0.0061	0.0026	0.0033	0.0034	0.0021	0.0022	0.0024
0.77	0.0015	0.0004	-0.0034	0.0188	0.0002	-0.0041	0.0026	0.0034	0.0035	0.0021	0.0023	0.0024
0.79	0.0016	0	-0.0032	0.0200	-0.0012	-0.0019	0.0027	0.0034	0.0036	0.0022	0.0023	0.0024
0.81	0.0016	-0.0002	-0.0033	0.0213	-0.0028	0.0005	0.0028	0.0035	0.0036	0.0023	0.0023	0.0024
0.83	0.0013	-0.0001	-0.0030	0.0227	-0.0045	0.0033	0.0028	0.0035	0.0037	0.0024	0.0024	0.0025
0.85	0.0015	-0.0004	-0.0031	0.0243	-0.0066	0.0065	0.0029	0.0036	0.0038	0.0025	0.0024	0.0025
0.87	0.0012	-0.0007	-0.0025	0.0261	-0.0090	0.0103	0.0030	0.0037	0.0039	0.0026	0.0025	0.0027
0.89	0.0009	-0.0005	-0.0023	0.0283	-0.0117	0.0147	0.0031	0.0038	0.0041	0.0028	0.0026	0.0028
0.91	0.0004	-0.0005	-0.0019	0.0310	-0.0152	0.0201	0.0032	0.0040	0.0042	0.0030	0.0028	0.0031
0.93	0.0007	-0.0018	-0.0014	0.0347	-0.0195	0.0268	0.0035	0.0042	0.0045	0.0033	0.0030	0.0035
0.95	0.0002	-0.0018	-0.0009	0.0400	-0.0251	0.0356	0.0037	0.0046	0.0049	0.0038	0.0034	0.0042
0.97	-0.0018	-0.0016	0.0015	0.0488	-0.0334	0.0486	0.0044	0.0055	0.0056	0.0048	0.0040	0.0055
0.99	-0.0033	-0.0054	0.0052	0.0713	-0.0493	0.0736	0.0062	0.0077	0.0083	0.0079	0.0058	0.0090

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 500$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 17: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \sqrt[3]{\tau}$*: bias and MSE of estimators*

	Bias						*MSE*					
	canonical			*smoothed*			*canonical*			*smoothed*		
τ	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1
0.01	-0.0011	-0.0044	0.0110	-0.1048	-0.0205	0.0768	0.0078	0.0118	0.0107	0.0141	0.0050	0.0103
0.03	0.0023	-0.0030	0.0011	-0.0660	-0.0158	0.0382	0.0044	0.0063	0.0058	0.0067	0.0036	0.0046
0.05	0.0028	-0.0025	-0.0008	-0.0491	-0.0128	0.0210	0.0035	0.0050	0.0046	0.0045	0.0031	0.0032
0.07	0.0027	-0.0020	-0.0016	-0.0384	-0.0105	0.0101	0.0032	0.0045	0.0041	0.0034	0.0028	0.0026
0.09	0.0028	-0.0015	-0.0025	-0.0307	-0.0085	0.0024	0.0029	0.0040	0.0037	0.0027	0.0026	0.0024
0.11	0.0029	-0.0016	-0.0028	-0.0246	-0.0069	-0.0036	0.0027	0.0038	0.0035	0.0023	0.0024	0.0022
0.13	0.0031	-0.0017	-0.0029	-0.0197	-0.0056	-0.0083	0.0025	0.0035	0.0033	0.0020	0.0023	0.0022
0.15	0.0028	-0.0015	-0.0025	-0.0157	-0.0044	-0.0120	0.0024	0.0034	0.0032	0.0019	0.0022	0.0022
0.17	0.0029	-0.0014	-0.0026	-0.0122	-0.0034	-0.0151	0.0024	0.0032	0.0031	0.0017	0.0022	0.0023
0.19	0.0030	-0.0012	-0.0032	-0.0092	-0.0025	-0.0176	0.0023	0.0031	0.0030	0.0016	0.0021	0.0023
0.21	0.0031	-0.0014	-0.0036	-0.0066	-0.0017	-0.0196	0.0023	0.0031	0.0030	0.0016	0.0021	0.0023
0.23	0.0034	-0.0017	-0.0039	-0.0043	-0.0010	-0.0213	0.0023	0.0030	0.0029	0.0015	0.0020	0.0024
0.25	0.0036	-0.0019	-0.0042	-0.0023	-0.0004	-0.0227	0.0022	0.0029	0.0029	0.0015	0.0020	0.0024
0.27	0.0034	-0.0018	-0.0041	-0.0005	0.0002	-0.0238	0.0022	0.0029	0.0029	0.0015	0.0020	0.0025
0.29	0.0033	-0.0015	-0.0042	0.0012	0.0007	-0.0247	0.0021	0.0028	0.0028	0.0015	0.0020	0.0025
0.31	0.0033	-0.0014	-0.0044	0.0026	0.0012	-0.0253	0.0021	0.0028	0.0029	0.0015	0.0020	0.0025
0.33	0.0028	-0.0011	-0.0041	0.0039	0.0016	-0.0258	0.0021	0.0028	0.0028	0.0015	0.0020	0.0025
0.35	0.0028	-0.0010	-0.0043	0.0051	0.0019	-0.0261	0.0021	0.0028	0.0028	0.0015	0.0019	0.0025
0.37	0.0025	-0.0008	-0.0039	0.0062	0.0023	-0.0263	0.0021	0.0027	0.0028	0.0015	0.0019	0.0025
0.39	0.0026	-0.0009	-0.0039	0.0072	0.0026	-0.0263	0.0021	0.0028	0.0027	0.0015	0.0019	0.0025
0.41	0.0026	-0.0009	-0.0040	0.0081	0.0028	-0.0261	0.0021	0.0028	0.0027	0.0015	0.0019	0.0025
0.43	0.0030	-0.0011	-0.0044	0.0089	0.0030	-0.0259	0.0021	0.0028	0.0027	0.0016	0.0019	0.0025
0.45	0.0029	-0.0013	-0.0043	0.0097	0.0032	-0.0255	0.0022	0.0028	0.0028	0.0016	0.0019	0.0025
0.47	0.0030	-0.0014	-0.0044	0.0104	0.0034	-0.0250	0.0022	0.0029	0.0027	0.0016	0.0019	0.0025
0.49	0.0030	-0.0015	-0.0043	0.0110	0.0035	-0.0244	0.0022	0.0028	0.0027	0.0016	0.0019	0.0025
0.51	0.0030	-0.0016	-0.0042	0.0116	0.0036	-0.0236	0.0022	0.0028	0.0027	0.0016	0.0019	0.0024
0.53	0.0030	-0.0019	-0.0037	0.0121	0.0037	-0.0228	0.0022	0.0028	0.0027	0.0017	0.0019	0.0024
0.55	0.0028	-0.0017	-0.0038	0.0126	0.0037	-0.0219	0.0022	0.0028	0.0027	0.0017	0.0019	0.0024
0.57	0.0028	-0.0018	-0.0037	0.0131	0.0037	-0.0208	0.0022	0.0028	0.0027	0.0017	0.0019	0.0023
0.59	0.0027	-0.0017	-0.0036	0.0135	0.0037	-0.0196	0.0023	0.0028	0.0027	0.0017	0.0019	0.0023
0.61	0.0027	-0.0015	-0.0037	0.0138	0.0036	-0.0183	0.0023	0.0027	0.0028	0.0018	0.0019	0.0023
0.63	0.0030	-0.0015	-0.0042	0.0141	0.0036	-0.0168	0.0023	0.0027	0.0028	0.0018	0.0019	0.0022
0.65	0.0029	-0.0014	-0.0043	0.0144	0.0035	-0.0152	0.0023	0.0028	0.0028	0.0018	0.0019	0.0022
0.67	0.0026	-0.0011	-0.0040	0.0147	0.0034	-0.0134	0.0023	0.0028	0.0029	0.0018	0.0020	0.0022
0.69	0.0030	-0.0015	-0.0044	0.0149	0.0032	-0.0114	0.0024	0.0028	0.0029	0.0019	0.0020	0.0021
0.71	0.0027	-0.0013	-0.0041	0.0151	0.0030	-0.0093	0.0024	0.0028	0.0029	0.0019	0.0020	0.0021
0.73	0.0025	-0.0013	-0.0037	0.0153	0.0027	-0.0069	0.0024	0.0028	0.0029	0.0019	0.0020	0.0021
0.75	0.0023	-0.0013	-0.0036	0.0155	0.0024	-0.0043	0.0024	0.0029	0.0030	0.0019	0.0020	0.0021
0.77	0.0027	-0.0016	-0.0041	0.0157	0.0020	-0.0015	0.0025	0.0029	0.0030	0.0020	0.0020	0.0021
0.79	0.0025	-0.0012	-0.0040	0.0159	0.0015	0.0017	0.0025	0.0029	0.0031	0.0020	0.0020	0.0021
0.81	0.0026	-0.0014	-0.0038	0.0161	0.0010	0.0053	0.0026	0.0029	0.0032	0.0020	0.0020	0.0022
0.83	0.0024	-0.0014	-0.0036	0.0163	0.0003	0.0093	0.0026	0.0030	0.0032	0.0021	0.0020	0.0023
0.85	0.0017	-0.0010	-0.0028	0.0165	-0.0006	0.0140	0.0027	0.0030	0.0033	0.0021	0.0020	0.0024
0.87	0.0014	-0.0009	-0.0026	0.0168	-0.0016	0.0194	0.0027	0.0030	0.0033	0.0022	0.0021	0.0027
0.89	0.0012	-0.0009	-0.0023	0.0172	-0.0028	0.0258	0.0028	0.0031	0.0034	0.0022	0.0021	0.0030
0.91	0.0009	-0.0006	-0.0021	0.0178	-0.0044	0.0335	0.0030	0.0032	0.0037	0.0023	0.0022	0.0036
0.93	-0.0002	-0.0005	-0.0008	0.0188	-0.0066	0.0430	0.0031	0.0033	0.0039	0.0025	0.0022	0.0044
0.95	-0.0014	-0.0004	0.0008	0.0210	-0.0097	0.0555	0.0035	0.0037	0.0044	0.0027	0.0024	0.0057
0.97	-0.0026	-0.0028	0.0041	0.0251	-0.0143	0.0739	0.0040	0.0042	0.0050	0.0031	0.0026	0.0083
0.99	-0.0089	-0.0044	0.0137	0.0392	-0.0233	0.1085	0.0058	0.0062	0.0074	0.0045	0.0033	0.0151

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 500$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 18: *Model (2) defined through Proposition 2 when $\lambda(\tau) = cos(\tau)$: bias and MSE of estimators*

	Bias						*MSE*					
	canonical			*smoothed*			*canonical*			*smoothed*		
τ	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1
0.01	0.0047	-0.0081	0.0041	-0.0649	-0.0661	0.0360	0.0074	0.0103	0.0098	0.0071	0.0083	0.0050
0.03	0.0020	-0.0016	0.0004	-0.0471	-0.0382	0.0170	0.0042	0.0057	0.0055	0.0044	0.0045	0.0031
0.05	0.0015	-0.0005	-0.0007	-0.0389	-0.0252	0.0081	0.0034	0.0047	0.0043	0.0035	0.0033	0.0026
0.07	0.0010	0.0009	-0.0017	-0.0336	-0.0167	0.0024	0.0029	0.0040	0.0038	0.0029	0.0027	0.0024
0.09	0.0010	0.0011	-0.0020	-0.0297	-0.0104	-0.0017	0.0028	0.0038	0.0036	0.0026	0.0024	0.0023
0.11	0.0012	0.0012	-0.0024	-0.0266	-0.0055	-0.0046	0.0025	0.0035	0.0034	0.0023	0.0023	0.0022
0.13	0.0015	0.0007	-0.0025	-0.0241	-0.0014	-0.0069	0.0025	0.0033	0.0033	0.0022	0.0022	0.0022
0.15	0.0013	0.0010	-0.0023	-0.0220	0.0019	-0.0086	0.0023	0.0032	0.0032	0.0020	0.0021	0.0022
0.17	0.0007	0.0016	-0.0019	-0.0201	0.0047	-0.0100	0.0023	0.0031	0.0031	0.0019	0.0021	0.0022
0.19	0.0001	0.0025	-0.0017	-0.0185	0.0071	-0.0110	0.0022	0.0030	0.0030	0.0018	0.0021	0.0022
0.21	0.0005	0.0022	-0.0021	-0.0170	0.0092	-0.0118	0.0022	0.0030	0.0030	0.0018	0.0021	0.0022
0.23	0.0006	0.0022	-0.0022	-0.0157	0.0110	-0.0123	0.0021	0.0029	0.0030	0.0017	0.0021	0.0022
0.25	0.0006	0.0022	-0.0022	-0.0145	0.0125	-0.0127	0.0021	0.0029	0.0029	0.0017	0.0021	0.0022
0.27	0.0003	0.0025	-0.0021	-0.0133	0.0138	-0.0130	0.0021	0.0029	0.0029	0.0016	0.0021	0.0022
0.29	0.0008	0.0020	-0.0025	-0.0123	0.0149	-0.0131	0.0021	0.0029	0.0030	0.0016	0.0022	0.0022
0.31	0.0008	0.0020	-0.0026	-0.0113	0.0159	-0.0131	0.0021	0.0029	0.0030	0.0016	0.0022	0.0022
0.33	0.0007	0.0021	-0.0024	-0.0104	0.0167	-0.0130	0.0021	0.0029	0.0029	0.0015	0.0022	0.0022
0.35	0.0004	0.0020	-0.0019	-0.0095	0.0174	-0.0128	0.0021	0.0029	0.0029	0.0015	0.0022	0.0022
0.37	0.0007	0.0017	-0.0022	-0.0087	0.0180	-0.0126	0.0021	0.0029	0.0029	0.0015	0.0023	0.0022
0.39	0.0007	0.0017	-0.0024	-0.0079	0.0184	-0.0122	0.0021	0.0029	0.0029	0.0015	0.0023	0.0022
0.41	0.0006	0.0020	-0.0026	-0.0071	0.0187	-0.0118	0.0021	0.0029	0.0030	0.0015	0.0023	0.0022
0.43	0.0008	0.0020	-0.0029	-0.0062	0.0189	-0.0114	0.0021	0.0029	0.0030	0.0015	0.0023	0.0022
0.45	0.0005	0.0025	-0.0029	-0.0054	0.0190	-0.0109	0.0021	0.0029	0.0030	0.0015	0.0023	0.0022
0.47	0.0004	0.0025	-0.0027	-0.0046	0.0190	-0.0104	0.0021	0.0029	0.0030	0.0015	0.0023	0.0022
0.49	0.0007	0.0021	-0.0029	-0.0038	0.0189	-0.0098	0.0021	0.0029	0.0030	0.0015	0.0023	0.0022
0.51	0.0005	0.0020	-0.0026	-0.0030	0.0187	-0.0092	0.0021	0.0030	0.0030	0.0015	0.0023	0.0022
0.53	0.0006	0.0018	-0.0026	-0.0021	0.0184	-0.0085	0.0022	0.0030	0.0031	0.0015	0.0023	0.0022
0.55	0.0007	0.0017	-0.0027	-0.0012	0.0180	-0.0078	0.0022	0.0030	0.0031	0.0015	0.0023	0.0022
0.57	0.0008	0.0014	-0.0025	-0.0003	0.0175	-0.0071	0.0022	0.0030	0.0031	0.0015	0.0023	0.0022
0.59	0.0003	0.0016	-0.0019	0.0007	0.0168	-0.0064	0.0022	0.0031	0.0031	0.0015	0.0023	0.0022
0.61	0.0004	0.0014	-0.0019	0.0017	0.0161	-0.0056	0.0023	0.0031	0.0032	0.0015	0.0023	0.0022
0.63	-0.0001	0.0016	-0.0014	0.0028	0.0152	-0.0047	0.0023	0.0031	0.0032	0.0015	0.0023	0.0022
0.65	-0.0002	0.0019	-0.0014	0.0040	0.0142	-0.0039	0.0022	0.0031	0.0032	0.0015	0.0023	0.0022
0.67	0.0002	0.0015	-0.0017	0.0052	0.0130	-0.0030	0.0023	0.0031	0.0033	0.0015	0.0023	0.0022
0.69	0.0002	0.0014	-0.0017	0.0066	0.0117	-0.0021	0.0023	0.0032	0.0034	0.0016	0.0023	0.0022
0.71	-0.0002	0.0015	-0.0013	0.0080	0.0102	-0.0011	0.0023	0.0032	0.0034	0.0016	0.0023	0.0023
0.73	-0.0001	0.0009	-0.0010	0.0096	0.0085	-0.0001	0.0023	0.0032	0.0034	0.0017	0.0023	0.0023
0.75	0	0.0007	-0.0012	0.0114	0.0066	0.0010	0.0024	0.0033	0.0034	0.0017	0.0023	0.0023
0.77	-0.0002	0.0008	-0.0010	0.0134	0.0045	0.0021	0.0024	0.0034	0.0035	0.0018	0.0023	0.0024
0.79	-0.0006	0.0013	-0.0007	0.0155	0.0021	0.0033	0.0024	0.0034	0.0036	0.0019	0.0023	0.0024
0.81	-0.0007	0.0011	-0.0004	0.0180	-0.0006	0.0046	0.0025	0.0035	0.0037	0.0020	0.0023	0.0024
0.83	-0.0009	0.0008	0	0.0208	-0.0038	0.0061	0.0025	0.0036	0.0038	0.0021	0.0024	0.0025
0.85	-0.0013	0.0011	0.0004	0.0240	-0.0074	0.0076	0.0026	0.0037	0.0039	0.0023	0.0025	0.0026
0.87	-0.0014	0.0006	0.0008	0.0277	-0.0116	0.0093	0.0027	0.0038	0.0041	0.0025	0.0026	0.0027
0.89	-0.0012	0.0004	0.0006	0.0323	-0.0165	0.0113	0.0028	0.0040	0.0042	0.0028	0.0028	0.0028
0.91	-0.0002	-0.0018	0.0001	0.0378	-0.0226	0.0137	0.0030	0.0043	0.0045	0.0033	0.0032	0.0029
0.93	0.0001	-0.0020	-0.0004	0.0449	-0.0301	0.0168	0.0031	0.0044	0.0047	0.0039	0.0037	0.0031
0.95	0.0001	-0.0025	-0.0002	0.0548	-0.0400	0.0207	0.0035	0.0048	0.0052	0.0050	0.0045	0.0034
0.97	0.0006	-0.0043	-0.0001	0.0705	-0.0547	0.0267	0.0041	0.0059	0.0060	0.0072	0.0062	0.0040
0.99	-0.0004	-0.0107	0.0048	0.1059	-0.0828	0.0387	0.0062	0.0090	0.0087	0.0138	0.0106	0.0053

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 500$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 19: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \sqrt[6]{\tau}$*: bias and MSE of estimators*

	Bias						*MSE*					
	canonical			*smoothed*			*canonical*			*smoothed*		
τ	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1
0.01	0.0015	-0.0063	0.0092	-0.0898	-0.0376	0.0631	0.0082	0.0113	0.0110	0.0111	0.0057	0.0082
0.03	0.0029	-0.0037	0.0013	-0.0579	-0.0255	0.0305	0.0046	0.0063	0.0061	0.0057	0.0038	0.0041
0.05	0.0036	-0.0031	-0.0013	-0.0441	-0.0190	0.0162	0.0036	0.0049	0.0047	0.0040	0.0031	0.0030
0.07	0.0034	-0.0023	-0.0018	-0.0354	-0.0144	0.0073	0.0031	0.0042	0.0040	0.0031	0.0027	0.0026
0.09	0.0023	-0.0013	-0.0009	-0.0291	-0.0109	0.0009	0.0028	0.0038	0.0036	0.0026	0.0025	0.0024
0.11	0.0026	-0.0019	-0.0011	-0.0243	-0.0080	-0.0039	0.0027	0.0036	0.0035	0.0023	0.0024	0.0023
0.13	0.0026	-0.0020	-0.0012	-0.0204	-0.0056	-0.0076	0.0026	0.0035	0.0033	0.0021	0.0022	0.0022
0.15	0.0026	-0.0017	-0.0015	-0.0171	-0.0035	-0.0105	0.0025	0.0033	0.0032	0.0019	0.0022	0.0022
0.17	0.0026	-0.0016	-0.0016	-0.0143	-0.0018	-0.0129	0.0024	0.0032	0.0031	0.0018	0.0021	0.0023
0.19	0.0024	-0.0015	-0.0016	-0.0118	-0.0003	-0.0149	0.0024	0.0031	0.0031	0.0017	0.0021	0.0023
0.21	0.0025	-0.0013	-0.0022	-0.0097	0.0010	-0.0165	0.0023	0.0030	0.0030	0.0017	0.0021	0.0023
0.23	0.0025	-0.0011	-0.0025	-0.0078	0.0021	-0.0177	0.0023	0.0030	0.0030	0.0016	0.0020	0.0023
0.25	0.0028	-0.0013	-0.0028	-0.0061	0.0031	-0.0188	0.0023	0.0030	0.0030	0.0016	0.0020	0.0024
0.27	0.0027	-0.0011	-0.0030	-0.0045	0.0040	-0.0195	0.0023	0.0030	0.0030	0.0016	0.0020	0.0024
0.29	0.0025	-0.0006	-0.0031	-0.0031	0.0047	-0.0201	0.0023	0.0029	0.0030	0.0015	0.0020	0.0024
0.31	0.0024	-0.0005	-0.0030	-0.0018	0.0053	-0.0206	0.0023	0.0029	0.0030	0.0015	0.0020	0.0024
0.33	0.0024	-0.0003	-0.0032	-0.0007	0.0059	-0.0208	0.0023	0.0029	0.0030	0.0015	0.0020	0.0024
0.35	0.0025	-0.0008	-0.0030	0.0004	0.0064	-0.0210	0.0023	0.0029	0.0030	0.0015	0.0020	0.0024
0.37	0.0026	-0.0009	-0.0032	0.0014	0.0067	-0.0210	0.0023	0.0029	0.0030	0.0015	0.0020	0.0024
0.39	0.0027	-0.0010	-0.0033	0.0024	0.0071	-0.0209	0.0023	0.0029	0.0030	0.0015	0.0020	0.0024
0.41	0.0024	-0.0009	-0.0029	0.0032	0.0073	-0.0207	0.0023	0.0028	0.0030	0.0015	0.0020	0.0024
0.43	0.0024	-0.0013	-0.0027	0.0041	0.0075	-0.0203	0.0023	0.0028	0.0030	0.0016	0.0020	0.0024
0.45	0.0026	-0.0013	-0.0030	0.0048	0.0076	-0.0199	0.0023	0.0028	0.0030	0.0016	0.0020	0.0024
0.47	0.0026	-0.0015	-0.0029	0.0056	0.0077	-0.0194	0.0023	0.0029	0.0030	0.0016	0.0020	0.0024
0.49	0.0029	-0.0016	-0.0033	0.0063	0.0077	-0.0188	0.0023	0.0029	0.0030	0.0016	0.0020	0.0024
0.51	0.0025	-0.0014	-0.0030	0.0070	0.0077	-0.0182	0.0023	0.0029	0.0030	0.0016	0.0020	0.0024
0.53	0.0022	-0.0012	-0.0027	0.0077	0.0076	-0.0174	0.0023	0.0029	0.0030	0.0016	0.0020	0.0023
0.55	0.0022	-0.0015	-0.0025	0.0083	0.0074	-0.0166	0.0023	0.0029	0.0030	0.0017	0.0021	0.0023
0.57	0.0019	-0.0011	-0.0025	0.0089	0.0073	-0.0156	0.0023	0.0029	0.0030	0.0017	0.0021	0.0023
0.59	0.0019	-0.0011	-0.0026	0.0095	0.0070	-0.0146	0.0024	0.0030	0.0030	0.0017	0.0021	0.0023
0.61	0.0018	-0.0012	-0.0025	0.0101	0.0067	-0.0134	0.0024	0.0030	0.0031	0.0017	0.0021	0.0023
0.63	0.0016	-0.0010	-0.0023	0.0107	0.0064	-0.0122	0.0024	0.0030	0.0031	0.0017	0.0021	0.0023
0.65	0.0016	-0.0009	-0.0023	0.0112	0.0060	-0.0108	0.0024	0.0030	0.0031	0.0018	0.0021	0.0022
0.67	0.0016	-0.0009	-0.0024	0.0118	0.0055	-0.0093	0.0025	0.0030	0.0032	0.0018	0.0021	0.0022
0.69	0.0017	-0.0010	-0.0024	0.0124	0.0050	-0.0077	0.0025	0.0030	0.0032	0.0018	0.0021	0.0022
0.71	0.0018	-0.0014	-0.0024	0.0130	0.0043	-0.0059	0.0025	0.0030	0.0032	0.0019	0.0021	0.0022
0.73	0.0016	-0.0010	-0.0025	0.0136	0.0036	-0.0040	0.0025	0.0031	0.0032	0.0019	0.0021	0.0022
0.75	0.0013	-0.0009	-0.0021	0.0143	0.0028	-0.0019	0.0025	0.0031	0.0032	0.0019	0.0021	0.0022
0.77	0.0009	-0.0006	-0.0019	0.0150	0.0019	0.0004	0.0026	0.0032	0.0033	0.0020	0.0021	0.0023
0.79	0.0010	-0.0010	-0.0017	0.0158	0.0009	0.0030	0.0026	0.0032	0.0033	0.0020	0.0022	0.0023
0.81	0.0011	-0.0010	-0.0017	0.0166	-0.0003	0.0058	0.0027	0.0033	0.0034	0.0021	0.0022	0.0023
0.83	0.0008	-0.0011	-0.0012	0.0176	-0.0017	0.0090	0.0028	0.0033	0.0036	0.0021	0.0022	0.0024
0.85	0.0004	-0.0012	-0.0005	0.0187	-0.0032	0.0127	0.0028	0.0034	0.0036	0.0022	0.0023	0.0026
0.87	0.0006	-0.0013	-0.0008	0.0200	-0.0051	0.0169	0.0029	0.0035	0.0037	0.0023	0.0023	0.0027
0.89	0.0006	-0.0015	-0.0009	0.0215	-0.0073	0.0219	0.0031	0.0037	0.0039	0.0024	0.0024	0.0030
0.91	0.0003	-0.0013	-0.0008	0.0236	-0.0100	0.0278	0.0032	0.0038	0.0041	0.0026	0.0025	0.0034
0.93	-0.0002	-0.0016	0.0001	0.0265	-0.0136	0.0352	0.0033	0.0040	0.0042	0.0028	0.0027	0.0039
0.95	-0.0011	-0.0011	0.0010	0.0308	-0.0183	0.0448	0.0036	0.0043	0.0046	0.0031	0.0029	0.0048
0.97	-0.0015	-0.0023	0.0022	0.0382	-0.0252	0.0591	0.0042	0.0051	0.0054	0.0038	0.0034	0.0065
0.99	-0.0055	-0.0054	0.0095	0.0576	-0.0386	0.0869	0.0061	0.0074	0.0079	0.0062	0.0047	0.0111

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 500$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 20: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \frac{sin(\tau)+1}{2}$*: bias and MSE of estimators*

	Bias						*MSE*					
	canonical			*smoothed*			*canonical*			*smoothed*		
τ	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1
0.01	-0.0002	-0.0050	0.0105	-0.0980	-0.0323	0.0689	0.0081	0.0110	0.0112	0.0126	0.0053	0.0089
0.03	0.0009	-0.0013	0.0023	-0.0688	-0.0166	0.0390	0.0046	0.0063	0.0060	0.0070	0.0035	0.0047
0.05	0.0004	-0.0003	0.0015	-0.0552	-0.0094	0.0250	0.0036	0.0050	0.0048	0.0051	0.0029	0.0034
0.07	0.0008	0	-0.0001	-0.0462	-0.0048	0.0158	0.0031	0.0043	0.0042	0.0040	0.0027	0.0028
0.09	0.0008	0.0008	-0.0011	-0.0394	-0.0016	0.0091	0.0028	0.0040	0.0038	0.0033	0.0025	0.0025
0.11	0.0005	0.0008	-0.0010	-0.0340	0.0008	0.0038	0.0027	0.0038	0.0036	0.0029	0.0024	0.0023
0.13	0.0005	0.0008	-0.0010	-0.0293	0.0026	-0.0004	0.0026	0.0036	0.0034	0.0025	0.0023	0.0022
0.15	0.0006	0.0003	-0.0008	-0.0254	0.0041	-0.0039	0.0025	0.0035	0.0033	0.0023	0.0023	0.0022
0.17	0.0004	0.0007	-0.0009	-0.0219	0.0052	-0.0068	0.0024	0.0034	0.0032	0.0021	0.0022	0.0022
0.19	0.0002	0.0010	-0.0007	-0.0189	0.0061	-0.0093	0.0024	0.0032	0.0031	0.0019	0.0022	0.0022
0.21	0.0003	0.0007	-0.0007	-0.0161	0.0068	-0.0113	0.0023	0.0032	0.0031	0.0018	0.0022	0.0022
0.23	0.0002	0.0005	-0.0005	-0.0136	0.0074	-0.0131	0.0022	0.0031	0.0030	0.0017	0.0022	0.0022
0.25	0.0003	0.0003	-0.0005	-0.0113	0.0078	-0.0145	0.0022	0.0030	0.0030	0.0017	0.0021	0.0022
0.27	0.0009	0	-0.0012	-0.0092	0.0082	-0.0157	0.0022	0.0030	0.0031	0.0016	0.0021	0.0023
0.29	0.0009	0.0002	-0.0014	-0.0073	0.0084	-0.0168	0.0022	0.0030	0.0030	0.0016	0.0021	0.0023
0.31	0.0009	0.0002	-0.0013	-0.0056	0.0086	-0.0176	0.0022	0.0030	0.0030	0.0015	0.0021	0.0023
0.33	0.0007	0.0004	-0.0013	-0.0039	0.0087	-0.0183	0.0022	0.0030	0.0030	0.0015	0.0021	0.0023
0.35	0.0009	0.0003	-0.0016	-0.0024	0.0088	-0.0188	0.0022	0.0030	0.0029	0.0015	0.0021	0.0024
0.37	0.0006	0.0004	-0.0011	-0.0010	0.0088	-0.0191	0.0022	0.0030	0.0030	0.0015	0.0021	0.0024
0.39	0.0003	0.0007	-0.0008	0.0004	0.0088	-0.0194	0.0022	0.0030	0.0029	0.0015	0.0021	0.0024
0.41	0	0.0011	-0.0007	0.0016	0.0087	-0.0195	0.0022	0.0030	0.0029	0.0015	0.0021	0.0024
0.43	0.0001	0.0009	-0.0007	0.0028	0.0086	-0.0195	0.0022	0.0030	0.0030	0.0015	0.0021	0.0024
0.45	0.0002	0.0009	-0.0009	0.0040	0.0084	-0.0194	0.0022	0.0030	0.0030	0.0015	0.0021	0.0024
0.47	0.0001	0.0010	-0.0009	0.0050	0.0082	-0.0192	0.0022	0.0030	0.0030	0.0015	0.0021	0.0024
0.49	0.0003	0.0008	-0.0013	0.0060	0.0080	-0.0188	0.0022	0.0030	0.0030	0.0016	0.0021	0.0024
0.51	0.0001	0.0010	-0.0011	0.0070	0.0077	-0.0184	0.0022	0.0030	0.0030	0.0016	0.0021	0.0024
0.53	0.0005	0.0007	-0.0014	0.0079	0.0075	-0.0179	0.0022	0.0030	0.0030	0.0016	0.0021	0.0024
0.55	0.0006	0.0005	-0.0015	0.0088	0.0071	-0.0173	0.0023	0.0030	0.0031	0.0016	0.0021	0.0024
0.57	0.0006	0.0002	-0.0013	0.0097	0.0068	-0.0166	0.0023	0.0030	0.0031	0.0017	0.0021	0.0024
0.59	0.0006	0.0004	-0.0015	0.0105	0.0064	-0.0157	0.0023	0.0030	0.0031	0.0017	0.0021	0.0023
0.61	0.0005	0.0007	-0.0016	0.0113	0.0059	-0.0148	0.0024	0.0031	0.0032	0.0017	0.0021	0.0023
0.63	0.0006	0.0004	-0.0015	0.0121	0.0054	-0.0137	0.0024	0.0031	0.0032	0.0018	0.0021	0.0023
0.65	0.0003	0.0008	-0.0013	0.0128	0.0049	-0.0125	0.0024	0.0031	0.0032	0.0018	0.0021	0.0023
0.67	0.0002	0.0007	-0.0010	0.0136	0.0043	-0.0112	0.0025	0.0031	0.0032	0.0018	0.0021	0.0023
0.69	0.0001	0.0005	-0.0007	0.0143	0.0037	-0.0097	0.0025	0.0032	0.0032	0.0019	0.0021	0.0023
0.71	0.0002	0.0001	-0.0005	0.0151	0.0030	-0.0081	0.0026	0.0032	0.0032	0.0019	0.0021	0.0023
0.73	0.0007	-0.0003	-0.0011	0.0158	0.0022	-0.0062	0.0026	0.0032	0.0033	0.0020	0.0021	0.0023
0.75	0.0007	-0.0002	-0.0012	0.0166	0.0014	-0.0042	0.0026	0.0033	0.0033	0.0020	0.0022	0.0023
0.77	0.0006	0.0001	-0.0017	0.0175	0.0004	-0.0020	0.0026	0.0033	0.0033	0.0020	0.0022	0.0023
0.79	0.0004	0	-0.0014	0.0183	-0.0007	0.0005	0.0027	0.0033	0.0034	0.0021	0.0022	0.0023
0.81	0.0003	-0.0001	-0.0013	0.0193	-0.0019	0.0033	0.0027	0.0033	0.0034	0.0022	0.0022	0.0024
0.83	0.0002	0.0001	-0.0014	0.0204	-0.0033	0.0064	0.0028	0.0034	0.0035	0.0022	0.0023	0.0024
0.85	0	0.0003	-0.0012	0.0216	-0.0049	0.0100	0.0029	0.0034	0.0036	0.0023	0.0023	0.0025
0.87	-0.0002	0.0003	-0.0008	0.0229	-0.0068	0.0142	0.0029	0.0036	0.0037	0.0024	0.0023	0.0027
0.89	-0.0003	0.0003	-0.0006	0.0246	-0.0090	0.0191	0.0031	0.0037	0.0039	0.0025	0.0024	0.0029
0.91	-0.0008	0.0007	-0.0003	0.0268	-0.0118	0.0250	0.0032	0.0038	0.0041	0.0027	0.0025	0.0033
0.93	-0.0013	0.0007	0.0001	0.0298	-0.0154	0.0324	0.0034	0.0040	0.0044	0.0030	0.0027	0.0038
0.95	-0.0010	-0.0003	0.0003	0.0343	-0.0203	0.0421	0.0037	0.0043	0.0048	0.0034	0.0030	0.0046
0.97	-0.0020	-0.0012	0.0022	0.0420	-0.0274	0.0564	0.0044	0.0050	0.0057	0.0042	0.0035	0.0063
0.99	-0.0063	-0.0043	0.0098	0.0622	-0.0413	0.0838	0.0063	0.0072	0.0084	0.0067	0.0048	0.0106

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 500$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 21: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \sqrt{\tau}$*: bias and MSE of estimators*

	Bias						*MSE*					
	canonical			$\widehat{\beta}_n^{2\zeta^*}$			*canonical*			$\widehat{\beta}_n^{2\zeta^*}$		
τ	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1
0.01	-0.0010	-0.0026	0.0067	-0.0925	-0.0076	0.0716	0.0034	0.0055	0.0052	0.0102	0.0027	0.0077
0.03	0.0016	-0.0024	0.0013	-0.0580	-0.0062	0.0370	0.0019	0.0031	0.0029	0.0045	0.0019	0.0031
0.05	0.0024	-0.0019	-0.0013	-0.0429	-0.0052	0.0218	0.0015	0.0025	0.0022	0.0028	0.0016	0.0019
0.07	0.0026	-0.0023	-0.0015	-0.0333	-0.0044	0.0120	0.0014	0.0022	0.0019	0.0020	0.0014	0.0014
0.09	0.0022	-0.0016	-0.0015	-0.0263	-0.0038	0.0050	0.0013	0.0020	0.0018	0.0015	0.0013	0.0012
0.11	0.0019	-0.0014	-0.0013	-0.0208	-0.0033	-0.0005	0.0012	0.0018	0.0017	0.0012	0.0012	0.0011
0.13	0.0022	-0.0014	-0.0018	-0.0163	-0.0029	-0.0048	0.0012	0.0017	0.0016	0.0010	0.0012	0.0011
0.15	0.0020	-0.0011	-0.0018	-0.0125	-0.0026	-0.0083	0.0011	0.0016	0.0015	0.0009	0.0011	0.0011
0.17	0.0023	-0.0013	-0.0020	-0.0094	-0.0023	-0.0112	0.0011	0.0016	0.0015	0.0008	0.0011	0.0011
0.19	0.0023	-0.0010	-0.0024	-0.0066	-0.0021	-0.0135	0.0011	0.0015	0.0014	0.0008	0.0011	0.0012
0.21	0.0025	-0.0012	-0.0026	-0.0042	-0.0018	-0.0155	0.0011	0.0015	0.0014	0.0007	0.0010	0.0012
0.23	0.0025	-0.0015	-0.0024	-0.0021	-0.0017	-0.0172	0.0010	0.0015	0.0014	0.0007	0.0010	0.0012
0.25	0.0025	-0.0015	-0.0024	-0.0002	-0.0015	-0.0185	0.0010	0.0014	0.0014	0.0007	0.0010	0.0013
0.27	0.0023	-0.0013	-0.0023	0.0014	-0.0013	-0.0197	0.0010	0.0014	0.0013	0.0007	0.0010	0.0013
0.29	0.0022	-0.0013	-0.0021	0.0029	-0.0012	-0.0206	0.0010	0.0014	0.0013	0.0007	0.0010	0.0013
0.31	0.0023	-0.0014	-0.0021	0.0043	-0.0011	-0.0214	0.0010	0.0014	0.0013	0.0007	0.0010	0.0014
0.33	0.0021	-0.0014	-0.0021	0.0055	-0.0010	-0.0220	0.0010	0.0014	0.0013	0.0007	0.0009	0.0014
0.35	0.0022	-0.0014	-0.0021	0.0066	-0.0008	-0.0224	0.0010	0.0013	0.0013	0.0007	0.0009	0.0014
0.37	0.0020	-0.0013	-0.0020	0.0076	-0.0007	-0.0228	0.0010	0.0013	0.0013	0.0008	0.0009	0.0014
0.39	0.0021	-0.0014	-0.0021	0.0085	-0.0006	-0.0229	0.0010	0.0013	0.0013	0.0008	0.0009	0.0014
0.41	0.0021	-0.0015	-0.0019	0.0093	-0.0005	-0.0230	0.0010	0.0013	0.0013	0.0008	0.0009	0.0014
0.43	0.0019	-0.0014	-0.0018	0.0100	-0.0004	-0.0230	0.0010	0.0013	0.0013	0.0008	0.0009	0.0014
0.45	0.0020	-0.0015	-0.0019	0.0107	-0.0003	-0.0228	0.0010	0.0013	0.0013	0.0008	0.0009	0.0014
0.47	0.0019	-0.0013	-0.0020	0.0112	-0.0002	-0.0225	0.0010	0.0013	0.0013	0.0008	0.0009	0.0014
0.49	0.0021	-0.0014	-0.0021	0.0118	-0.0001	-0.0222	0.0010	0.0012	0.0013	0.0008	0.0009	0.0014
0.51	0.0019	-0.0013	-0.0019	0.0122	0	-0.0217	0.0010	0.0012	0.0013	0.0009	0.0009	0.0014
0.53	0.0020	-0.0015	-0.0019	0.0126	0.0001	-0.0211	0.0010	0.0012	0.0013	0.0009	0.0009	0.0014
0.55	0.0021	-0.0014	-0.0021	0.0129	0.0002	-0.0204	0.0010	0.0012	0.0013	0.0009	0.0008	0.0013
0.57	0.0020	-0.0012	-0.0022	0.0132	0.0003	-0.0196	0.0010	0.0012	0.0013	0.0009	0.0008	0.0013
0.59	0.0020	-0.0011	-0.0023	0.0135	0.0004	-0.0187	0.0010	0.0012	0.0013	0.0009	0.0008	0.0013
0.61	0.0019	-0.0010	-0.0022	0.0137	0.0005	-0.0176	0.0010	0.0012	0.0013	0.0009	0.0008	0.0013
0.63	0.0019	-0.0011	-0.0022	0.0138	0.0006	-0.0164	0.0010	0.0012	0.0013	0.0009	0.0008	0.0012
0.65	0.0018	-0.0011	-0.0020	0.0138	0.0007	-0.0151	0.0010	0.0012	0.0013	0.0009	0.0008	0.0012
0.67	0.0018	-0.0012	-0.0019	0.0138	0.0008	-0.0135	0.0011	0.0012	0.0014	0.0009	0.0008	0.0011
0.69	0.0018	-0.0011	-0.0020	0.0138	0.0010	-0.0118	0.0011	0.0011	0.0014	0.0010	0.0008	0.0011
0.71	0.0018	-0.0011	-0.0020	0.0136	0.0011	-0.0099	0.0011	0.0011	0.0014	0.0010	0.0008	0.0011
0.73	0.0019	-0.0010	-0.0022	0.0135	0.0011	-0.0078	0.0011	0.0012	0.0013	0.0010	0.0008	0.0010
0.75	0.0016	-0.0008	-0.0020	0.0132	0.0012	-0.0054	0.0011	0.0012	0.0013	0.0010	0.0008	0.0010
0.77	0.0014	-0.0006	-0.0019	0.0129	0.0013	-0.0027	0.0011	0.0012	0.0014	0.0010	0.0008	0.0010
0.79	0.0014	-0.0006	-0.0019	0.0126	0.0013	0.0003	0.0011	0.0012	0.0014	0.0010	0.0008	0.0010
0.81	0.0013	-0.0003	-0.0021	0.0122	0.0013	0.0037	0.0011	0.0012	0.0014	0.0010	0.0008	0.0010
0.83	0.0008	0.0003	-0.0019	0.0117	0.0013	0.0076	0.0011	0.0012	0.0014	0.0010	0.0008	0.0011
0.85	0.0006	0.0002	-0.0015	0.0111	0.0012	0.0121	0.0011	0.0012	0.0014	0.0010	0.0008	0.0012
0.87	0.0006	0.0001	-0.0014	0.0105	0.0010	0.0173	0.0012	0.0012	0.0015	0.0010	0.0009	0.0014
0.89	0.0002	0.0004	-0.0011	0.0099	0.0007	0.0236	0.0012	0.0012	0.0015	0.0010	0.0009	0.0017
0.91	-0.0002	0.0008	-0.0009	0.0091	0.0003	0.0313	0.0013	0.0013	0.0016	0.0010	0.0009	0.0021
0.93	-0.0005	0.0007	-0.0004	0.0085	-0.0004	0.0408	0.0014	0.0013	0.0017	0.0011	0.0009	0.0029
0.95	-0.0008	0.0004	0.0002	0.0084	-0.0016	0.0534	0.0015	0.0014	0.0018	0.0011	0.0010	0.0041
0.97	-0.0017	0.0003	0.0014	0.0094	-0.0035	0.0717	0.0017	0.0016	0.0021	0.0013	0.0010	0.0065
0.99	-0.0056	0.0001	0.0069	0.0174	-0.0079	0.1061	0.0025	0.0024	0.0030	0.0017	0.0013	0.0129

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 1,000$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 22: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \frac{1}{1+e^{-\tau}}$*: bias and MSE of estimators*

	Bias						*MSE*					
	canonical			*smoothed*			*canonical*			*smoothed*		
τ	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1
0.01	-0.0003	-0.0019	0.0049	-0.0726	-0.0321	0.0524	0.0038	0.0053	0.0049	0.0068	0.0033	0.0049
0.03	0	0.0001	0.0009	-0.0513	-0.0165	0.0291	0.0021	0.0030	0.0029	0.0038	0.0019	0.0025
0.05	0.0001	0.0002	0.0007	-0.0412	-0.0095	0.0184	0.0018	0.0025	0.0024	0.0027	0.0015	0.0018
0.07	-0.0001	0.0006	0.0003	-0.0346	-0.0050	0.0113	0.0015	0.0022	0.0020	0.0022	0.0014	0.0014
0.09	0.0004	0.0005	-0.0004	-0.0297	-0.0018	0.0062	0.0014	0.0020	0.0019	0.0018	0.0013	0.0013
0.11	0.0006	0.0004	-0.0007	-0.0258	0.0006	0.0022	0.0013	0.0018	0.0018	0.0015	0.0012	0.0012
0.13	0.0004	0.0006	-0.0007	-0.0224	0.0024	-0.0010	0.0013	0.0018	0.0017	0.0014	0.0012	0.0012
0.15	0.0002	0.0007	-0.0006	-0.0196	0.0039	-0.0036	0.0012	0.0018	0.0016	0.0012	0.0012	0.0012
0.17	0.0004	0.0009	-0.0011	-0.0171	0.0051	-0.0057	0.0012	0.0017	0.0016	0.0011	0.0012	0.0012
0.19	0.0004	0.0007	-0.0009	-0.0149	0.0060	-0.0075	0.0012	0.0016	0.0016	0.0010	0.0012	0.0012
0.21	0.0004	0.0006	-0.0010	-0.0129	0.0068	-0.0089	0.0012	0.0016	0.0016	0.0010	0.0012	0.0012
0.23	0.0004	0.0007	-0.0009	-0.0112	0.0074	-0.0102	0.0012	0.0016	0.0016	0.0009	0.0012	0.0012
0.25	0.0003	0.0007	-0.0007	-0.0095	0.0079	-0.0112	0.0012	0.0016	0.0015	0.0009	0.0011	0.0012
0.27	-0.0001	0.0010	-0.0005	-0.0080	0.0083	-0.0120	0.0011	0.0016	0.0015	0.0009	0.0011	0.0012
0.29	0	0.0010	-0.0005	-0.0066	0.0085	-0.0127	0.0011	0.0016	0.0015	0.0008	0.0011	0.0012
0.31	0.0003	0.0008	-0.0010	-0.0053	0.0087	-0.0133	0.0011	0.0015	0.0015	0.0008	0.0011	0.0012
0.33	0.0006	0.0005	-0.0012	-0.0041	0.0089	-0.0137	0.0011	0.0015	0.0015	0.0008	0.0011	0.0013
0.35	0.0007	0.0004	-0.0013	-0.0030	0.0090	-0.0141	0.0011	0.0015	0.0015	0.0008	0.0011	0.0013
0.37	0.0006	0.0005	-0.0014	-0.0020	0.0090	-0.0143	0.0011	0.0015	0.0015	0.0008	0.0011	0.0013
0.39	0.0005	0.0007	-0.0015	-0.0010	0.0090	-0.0144	0.0011	0.0015	0.0015	0.0008	0.0011	0.0013
0.41	0.0006	0.0008	-0.0017	0	0.0089	-0.0145	0.0011	0.0015	0.0015	0.0008	0.0011	0.0013
0.43	0.0007	0.0006	-0.0017	0.0009	0.0088	-0.0145	0.0011	0.0015	0.0015	0.0008	0.0011	0.0013
0.45	0.0006	0.0006	-0.0016	0.0018	0.0086	-0.0144	0.0011	0.0015	0.0015	0.0008	0.0011	0.0013
0.47	0.0007	0.0005	-0.0016	0.0026	0.0085	-0.0142	0.0011	0.0015	0.0016	0.0008	0.0011	0.0013
0.49	0.0006	0.0005	-0.0014	0.0034	0.0082	-0.0140	0.0011	0.0015	0.0016	0.0008	0.0011	0.0013
0.51	0.0005	0.0006	-0.0015	0.0042	0.0080	-0.0137	0.0012	0.0015	0.0016	0.0008	0.0011	0.0013
0.53	0.0005	0.0008	-0.0016	0.0049	0.0077	-0.0133	0.0012	0.0015	0.0016	0.0008	0.0011	0.0013
0.55	0.0004	0.0007	-0.0014	0.0056	0.0074	-0.0129	0.0012	0.0015	0.0016	0.0008	0.0011	0.0013
0.57	0.0006	0.0005	-0.0016	0.0063	0.0070	-0.0124	0.0012	0.0015	0.0016	0.0009	0.0011	0.0013
0.59	0.0005	0.0007	-0.0016	0.0071	0.0066	-0.0118	0.0012	0.0015	0.0016	0.0009	0.0011	0.0013
0.61	0.0004	0.0005	-0.0013	0.0077	0.0062	-0.0112	0.0012	0.0015	0.0016	0.0009	0.0011	0.0013
0.63	0.0004	0.0004	-0.0014	0.0084	0.0057	-0.0105	0.0012	0.0015	0.0016	0.0009	0.0011	0.0013
0.65	0.0001	0.0008	-0.0014	0.0091	0.0052	-0.0096	0.0012	0.0015	0.0016	0.0009	0.0011	0.0012
0.67	0.0003	0.0008	-0.0015	0.0098	0.0046	-0.0087	0.0012	0.0015	0.0017	0.0009	0.0011	0.0012
0.69	0.0004	0.0006	-0.0016	0.0105	0.0040	-0.0077	0.0012	0.0015	0.0017	0.0010	0.0011	0.0012
0.71	0.0005	0.0005	-0.0015	0.0112	0.0033	-0.0066	0.0012	0.0016	0.0017	0.0010	0.0011	0.0012
0.73	0.0004	0.0006	-0.0015	0.0119	0.0025	-0.0053	0.0013	0.0016	0.0017	0.0010	0.0011	0.0012
0.75	0.0001	0.0005	-0.0009	0.0127	0.0017	-0.0040	0.0013	0.0016	0.0017	0.0010	0.0011	0.0012
0.77	0.0001	0.0004	-0.0009	0.0135	0.0007	-0.0024	0.0013	0.0016	0.0017	0.0011	0.0011	0.0012
0.79	-0.0002	0.0007	-0.0007	0.0144	-0.0003	-0.0007	0.0013	0.0017	0.0017	0.0011	0.0011	0.0012
0.81	-0.0002	0.0008	-0.0010	0.0153	-0.0016	0.0013	0.0013	0.0017	0.0018	0.0012	0.0012	0.0013
0.83	-0.0003	0.0009	-0.0011	0.0164	-0.0030	0.0035	0.0014	0.0017	0.0018	0.0012	0.0012	0.0013
0.85	-0.0003	0.0008	-0.0010	0.0176	-0.0046	0.0061	0.0014	0.0018	0.0019	0.0013	0.0012	0.0013
0.87	-0.0001	0.0003	-0.0009	0.0190	-0.0064	0.0090	0.0015	0.0018	0.0019	0.0013	0.0013	0.0014
0.89	0	0.0001	-0.0010	0.0206	-0.0086	0.0126	0.0015	0.0019	0.0020	0.0014	0.0013	0.0015
0.91	-0.0003	0	-0.0006	0.0227	-0.0114	0.0170	0.0016	0.0020	0.0021	0.0016	0.0014	0.0017
0.93	0.0003	-0.0006	-0.0009	0.0256	-0.0150	0.0225	0.0018	0.0021	0.0023	0.0017	0.0015	0.0019
0.95	-0.0001	-0.0012	0.0001	0.0298	-0.0199	0.0298	0.0018	0.0022	0.0024	0.0020	0.0018	0.0024
0.97	-0.0008	-0.0010	0.0008	0.0368	-0.0269	0.0407	0.0021	0.0025	0.0028	0.0026	0.0022	0.0033
0.99	-0.0024	-0.0029	0.0043	0.0551	-0.0409	0.0621	0.0030	0.0035	0.0041	0.0045	0.0034	0.0058

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 1,000$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 23: *Model (2) defined through Proposition 2 when $\lambda(\tau) = cos(\tau)$: bias and MSE of estimators*

	Bias						*MSE*					
	canonical			*smoothed*			*canonical*			*smoothed*		
τ	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1
0.01	0.0024	-0.0037	0.0017	-0.0502	-0.0553	0.0301	0.0036	0.0051	0.0045	0.0041	0.0052	0.0029
0.03	0.0011	-0.0005	-0.0007	-0.0361	-0.0323	0.0143	0.0020	0.0028	0.0027	0.0024	0.0026	0.0017
0.05	0.0014	-0.0008	-0.0009	-0.0298	-0.0216	0.0071	0.0016	0.0022	0.0022	0.0019	0.0018	0.0014
0.07	0.0012	-0.0007	-0.0009	-0.0257	-0.0145	0.0024	0.0014	0.0019	0.0019	0.0016	0.0015	0.0012
0.09	0.0006	-0.0001	-0.0005	-0.0227	-0.0092	-0.0008	0.0013	0.0018	0.0017	0.0014	0.0013	0.0011
0.11	0.0004	-0.0001	-0.0002	-0.0205	-0.0050	-0.0031	0.0012	0.0017	0.0016	0.0012	0.0012	0.0011
0.13	0.0003	0.0002	-0.0004	-0.0187	-0.0016	-0.0049	0.0012	0.0016	0.0016	0.0011	0.0011	0.0011
0.15	0	0.0002	0.0001	-0.0171	0.0012	-0.0063	0.0011	0.0016	0.0015	0.0011	0.0011	0.0011
0.17	-0.0001	0.0007	-0.0003	-0.0157	0.0035	-0.0073	0.0011	0.0015	0.0015	0.0010	0.0011	0.0011
0.19	0	0.0007	-0.0007	-0.0145	0.0055	-0.0081	0.0011	0.0015	0.0015	0.0010	0.0011	0.0011
0.21	0.0001	0.0006	-0.0009	-0.0135	0.0072	-0.0087	0.0011	0.0015	0.0015	0.0009	0.0011	0.0011
0.23	0.0002	0.0006	-0.0009	-0.0125	0.0086	-0.0092	0.0011	0.0015	0.0015	0.0009	0.0011	0.0011
0.25	0.0001	0.0006	-0.0010	-0.0116	0.0098	-0.0095	0.0011	0.0014	0.0015	0.0009	0.0011	0.0011
0.27	0.0001	0.0007	-0.0010	-0.0108	0.0109	-0.0096	0.0011	0.0015	0.0015	0.0009	0.0012	0.0011
0.29	0.0001	0.0008	-0.0010	-0.0100	0.0118	-0.0097	0.0011	0.0015	0.0015	0.0008	0.0012	0.0011
0.31	0	0.0008	-0.0008	-0.0093	0.0126	-0.0097	0.0011	0.0015	0.0015	0.0008	0.0012	0.0011
0.33	-0.0002	0.0010	-0.0006	-0.0086	0.0133	-0.0096	0.0011	0.0014	0.0015	0.0008	0.0012	0.0011
0.35	-0.0003	0.0012	-0.0007	-0.0079	0.0138	-0.0094	0.0011	0.0015	0.0015	0.0008	0.0012	0.0011
0.37	-0.0002	0.0013	-0.0008	-0.0073	0.0142	-0.0092	0.0011	0.0014	0.0015	0.0008	0.0012	0.0011
0.39	-0.0003	0.0012	-0.0006	-0.0067	0.0146	-0.0090	0.0011	0.0014	0.0015	0.0008	0.0012	0.0011
0.41	-0.0003	0.0012	-0.0007	-0.0060	0.0148	-0.0087	0.0011	0.0014	0.0015	0.0008	0.0013	0.0011
0.43	-0.0004	0.0013	-0.0005	-0.0054	0.0150	-0.0083	0.0011	0.0015	0.0015	0.0008	0.0013	0.0011
0.45	-0.0003	0.0013	-0.0008	-0.0048	0.0151	-0.0080	0.0011	0.0015	0.0015	0.0008	0.0013	0.0011
0.47	-0.0003	0.0013	-0.0008	-0.0042	0.0151	-0.0076	0.0011	0.0015	0.0015	0.0008	0.0013	0.0011
0.49	-0.0003	0.0014	-0.0009	-0.0035	0.0150	-0.0071	0.0011	0.0015	0.0015	0.0008	0.0013	0.0011
0.51	-0.0003	0.0014	-0.0010	-0.0028	0.0148	-0.0067	0.0011	0.0015	0.0015	0.0008	0.0013	0.0011
0.53	-0.0002	0.0013	-0.0010	-0.0022	0.0146	-0.0062	0.0011	0.0015	0.0015	0.0008	0.0013	0.0011
0.55	-0.0001	0.0012	-0.0011	-0.0015	0.0143	-0.0057	0.0011	0.0016	0.0015	0.0008	0.0013	0.0011
0.57	0	0.0010	-0.0010	-0.0007	0.0139	-0.0052	0.0011	0.0016	0.0015	0.0008	0.0013	0.0011
0.59	0	0.0010	-0.0010	0	0.0134	-0.0047	0.0011	0.0016	0.0015	0.0008	0.0013	0.0011
0.61	0	0.0010	-0.0011	0.0009	0.0129	-0.0041	0.0011	0.0016	0.0016	0.0008	0.0013	0.0011
0.63	0	0.0009	-0.0011	0.0017	0.0122	-0.0035	0.0011	0.0016	0.0016	0.0008	0.0013	0.0011
0.65	-0.0001	0.0012	-0.0012	0.0027	0.0114	-0.0029	0.0011	0.0016	0.0016	0.0008	0.0013	0.0011
0.67	0.0001	0.0011	-0.0013	0.0036	0.0106	-0.0023	0.0012	0.0016	0.0016	0.0008	0.0013	0.0012
0.69	0.0003	0.0010	-0.0015	0.0047	0.0095	-0.0016	0.0012	0.0016	0.0017	0.0008	0.0012	0.0012
0.71	0.0001	0.0010	-0.0013	0.0059	0.0084	-0.0010	0.0012	0.0017	0.0017	0.0009	0.0012	0.0012
0.73	0	0.0010	-0.0012	0.0072	0.0071	-0.0002	0.0012	0.0017	0.0017	0.0009	0.0012	0.0012
0.75	0.0002	0.0007	-0.0012	0.0086	0.0056	0.0005	0.0012	0.0017	0.0018	0.0009	0.0012	0.0012
0.77	0	0.0008	-0.0010	0.0101	0.0040	0.0014	0.0013	0.0017	0.0018	0.0010	0.0012	0.0012
0.79	-0.0001	0.0009	-0.0008	0.0118	0.0021	0.0023	0.0013	0.0018	0.0018	0.0010	0.0012	0.0013
0.81	0.0002	0.0004	-0.0009	0.0137	0	0.0032	0.0013	0.0018	0.0019	0.0011	0.0013	0.0013
0.83	0.0001	0.0006	-0.0008	0.0159	-0.0025	0.0043	0.0014	0.0019	0.0020	0.0012	0.0013	0.0013
0.85	0.0004	-0.0001	-0.0009	0.0185	-0.0054	0.0055	0.0014	0.0019	0.0020	0.0013	0.0013	0.0014
0.87	0.0004	-0.0005	-0.0006	0.0215	-0.0087	0.0069	0.0015	0.0020	0.0021	0.0014	0.0014	0.0014
0.89	0.0002	-0.0007	-0.0003	0.0250	-0.0128	0.0084	0.0015	0.0021	0.0021	0.0016	0.0016	0.0015
0.91	0	-0.0005	-0.0001	0.0294	-0.0177	0.0104	0.0016	0.0022	0.0022	0.0019	0.0018	0.0016
0.93	-0.0002	-0.0006	0.0002	0.0351	-0.0239	0.0130	0.0017	0.0023	0.0024	0.0023	0.0021	0.0017
0.95	-0.0002	-0.0010	0.0004	0.0429	-0.0321	0.0163	0.0018	0.0025	0.0026	0.0030	0.0026	0.0019
0.97	-0.0004	-0.0016	0.0008	0.0557	-0.0443	0.0213	0.0021	0.0029	0.0030	0.0043	0.0036	0.0022
0.99	-0.0006	-0.0045	0.0024	0.0847	-0.0682	0.0316	0.0029	0.0040	0.0042	0.0086	0.0066	0.0031

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 1,000$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.

Table 24: *Model (2) defined through Proposition 2 when* $\lambda(\tau) = \frac{sin(\tau)+1}{2}$*: bias and MSE of estimators*

	Bias						*MSE*					
	canonical			*smoothed*			*canonical*			*smoothed*		
τ	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1	α_0	α_1	θ_1
0.01	0.0001	-0.0030	0.0060	-0.0763	-0.0280	0.0562	0.0038	0.0054	0.0049	0.0074	0.0031	0.0054
0.03	0.0018	-0.0023	0.0007	-0.0529	-0.0147	0.0312	0.0022	0.0031	0.0029	0.0040	0.0019	0.0026
0.05	0.0014	-0.0010	-0.0006	-0.0420	-0.0087	0.0196	0.0018	0.0025	0.0023	0.0028	0.0016	0.0018
0.07	0.0013	-0.0007	-0.0009	-0.0349	-0.0048	0.0120	0.0016	0.0022	0.0021	0.0022	0.0014	0.0015
0.09	0.0015	-0.0008	-0.0010	-0.0295	-0.0020	0.0064	0.0014	0.0020	0.0019	0.0018	0.0013	0.0013
0.11	0.0013	-0.0007	-0.0010	-0.0251	-0.0001	0.0021	0.0013	0.0018	0.0018	0.0015	0.0012	0.0012
0.13	0.0012	-0.0004	-0.0011	-0.0215	0.0014	-0.0014	0.0013	0.0017	0.0017	0.0013	0.0012	0.0012
0.15	0.0015	-0.0006	-0.0014	-0.0183	0.0025	-0.0043	0.0012	0.0017	0.0016	0.0012	0.0012	0.0012
0.17	0.0014	-0.0003	-0.0015	-0.0156	0.0035	-0.0066	0.0012	0.0017	0.0016	0.0011	0.0012	0.0012
0.19	0.0013	-0.0003	-0.0015	-0.0132	0.0042	-0.0086	0.0012	0.0016	0.0016	0.0010	0.0011	0.0012
0.21	0.0012	-0.0001	-0.0015	-0.0111	0.0048	-0.0102	0.0012	0.0016	0.0016	0.0009	0.0011	0.0012
0.23	0.0012	-0.0002	-0.0015	-0.0092	0.0052	-0.0116	0.0011	0.0015	0.0016	0.0009	0.0011	0.0012
0.25	0.0011	-0.0002	-0.0013	-0.0074	0.0056	-0.0127	0.0011	0.0015	0.0015	0.0009	0.0011	0.0012
0.27	0.0009	-0.0001	-0.0012	-0.0058	0.0058	-0.0137	0.0011	0.0015	0.0015	0.0008	0.0011	0.0013
0.29	0.0011	-0.0001	-0.0015	-0.0043	0.0060	-0.0144	0.0011	0.0015	0.0015	0.0008	0.0011	0.0013
0.31	0.0012	-0.0001	-0.0015	-0.0030	0.0061	-0.0150	0.0011	0.0015	0.0015	0.0008	0.0011	0.0013
0.33	0.0012	-0.0002	-0.0017	-0.0017	0.0062	-0.0155	0.0011	0.0015	0.0015	0.0008	0.0011	0.0013
0.35	0.0015	-0.0004	-0.0020	-0.0006	0.0062	-0.0159	0.0012	0.0015	0.0015	0.0008	0.0011	0.0013
0.37	0.0015	-0.0004	-0.0018	0.0005	0.0062	-0.0162	0.0012	0.0015	0.0015	0.0008	0.0011	0.0013
0.39	0.0014	-0.0005	-0.0017	0.0016	0.0062	-0.0163	0.0012	0.0015	0.0015	0.0008	0.0011	0.0013
0.41	0.0015	-0.0005	-0.0018	0.0025	0.0061	-0.0164	0.0012	0.0015	0.0015	0.0008	0.0011	0.0013
0.43	0.0015	-0.0007	-0.0018	0.0034	0.0060	-0.0163	0.0012	0.0015	0.0015	0.0008	0.0011	0.0013
0.45	0.0015	-0.0006	-0.0018	0.0043	0.0058	-0.0162	0.0012	0.0015	0.0015	0.0008	0.0011	0.0013
0.47	0.0016	-0.0007	-0.0017	0.0050	0.0057	-0.0160	0.0012	0.0015	0.0016	0.0008	0.0011	0.0013
0.49	0.0013	-0.0006	-0.0014	0.0058	0.0055	-0.0157	0.0012	0.0015	0.0016	0.0009	0.0011	0.0013
0.51	0.0013	-0.0007	-0.0013	0.0065	0.0053	-0.0153	0.0012	0.0015	0.0016	0.0009	0.0011	0.0013
0.53	0.0013	-0.0006	-0.0014	0.0072	0.0050	-0.0149	0.0012	0.0015	0.0016	0.0009	0.0011	0.0013
0.55	0.0013	-0.0005	-0.0014	0.0079	0.0048	-0.0144	0.0012	0.0015	0.0016	0.0009	0.0011	0.0013
0.57	0.0012	-0.0006	-0.0012	0.0085	0.0045	-0.0138	0.0012	0.0015	0.0016	0.0009	0.0011	0.0013
0.59	0.0013	-0.0008	-0.0012	0.0091	0.0042	-0.0131	0.0012	0.0015	0.0016	0.0009	0.0011	0.0013
0.61	0.0014	-0.0009	-0.0012	0.0097	0.0038	-0.0123	0.0012	0.0015	0.0016	0.0009	0.0011	0.0013
0.63	0.0016	-0.0012	-0.0014	0.0102	0.0035	-0.0115	0.0012	0.0015	0.0016	0.0010	0.0011	0.0013
0.65	0.0013	-0.0011	-0.0013	0.0108	0.0031	-0.0105	0.0012	0.0015	0.0016	0.0010	0.0011	0.0013
0.67	0.0015	-0.0012	-0.0013	0.0113	0.0026	-0.0094	0.0012	0.0015	0.0016	0.0010	0.0011	0.0012
0.69	0.0013	-0.0010	-0.0011	0.0119	0.0022	-0.0082	0.0012	0.0015	0.0016	0.0010	0.0011	0.0012
0.71	0.0011	-0.0009	-0.0008	0.0124	0.0016	-0.0069	0.0012	0.0015	0.0016	0.0010	0.0011	0.0012
0.73	0.0008	-0.0007	-0.0006	0.0129	0.0011	-0.0055	0.0013	0.0015	0.0017	0.0011	0.0011	0.0012
0.75	0.0006	-0.0007	-0.0004	0.0135	0.0004	-0.0039	0.0013	0.0016	0.0017	0.0011	0.0011	0.0012
0.77	0.0004	-0.0003	-0.0004	0.0141	-0.0003	-0.0021	0.0013	0.0016	0.0017	0.0011	0.0011	0.0012
0.79	0.0004	-0.0002	-0.0006	0.0147	-0.0011	-0.0001	0.0013	0.0016	0.0018	0.0011	0.0011	0.0012
0.81	0.0003	-0.0002	-0.0004	0.0154	-0.0020	0.0021	0.0014	0.0017	0.0018	0.0012	0.0012	0.0012
0.83	0.0005	-0.0004	-0.0005	0.0161	-0.0031	0.0046	0.0014	0.0017	0.0018	0.0012	0.0012	0.0013
0.85	0.0004	-0.0003	-0.0006	0.0170	-0.0044	0.0075	0.0014	0.0018	0.0019	0.0013	0.0012	0.0013
0.87	0.0004	-0.0003	-0.0005	0.0180	-0.0058	0.0109	0.0015	0.0019	0.0020	0.0013	0.0013	0.0014
0.89	0.0003	-0.0004	-0.0004	0.0192	-0.0075	0.0149	0.0015	0.0019	0.0020	0.0014	0.0013	0.0016
0.91	0.0003	-0.0005	-0.0005	0.0207	-0.0097	0.0198	0.0016	0.0020	0.0020	0.0015	0.0014	0.0018
0.93	0.0001	-0.0006	-0.0002	0.0230	-0.0127	0.0259	0.0017	0.0021	0.0022	0.0016	0.0015	0.0021
0.95	-0.0004	-0.0004	0.0003	0.0264	-0.0168	0.0339	0.0018	0.0022	0.0023	0.0018	0.0016	0.0027
0.97	-0.0013	-0.0001	0.0013	0.0323	-0.0226	0.0459	0.0021	0.0025	0.0027	0.0023	0.0020	0.0037
0.99	-0.0029	-0.0025	0.0049	0.0484	-0.0345	0.0695	0.0029	0.0035	0.0038	0.0038	0.0029	0.0067

Notes: Bias and MSE of $\widehat{\beta}_n$ and $\widehat{\beta}_n^{2\zeta^*}$. Sample size of $n = 1,000$. The number of replications is $5,000$. The optimal bandwidth is $2\zeta^*$.