# Modelagem Estatística - Lista 4 Mestrado PPGEst UFRGS

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```
# install.packages("olsrr")
# install.packages("tidyverse")
                                 3 pacotes para rodar mais facil os métodos de selecão de variáveis
# install.packages("leaps")
library(olsrr)
##
## Attaching package: 'olsrr'
## The following object is masked from 'package:datasets':
##
##
       rivers
library(tidyverse)
## -- Attaching packages -----
## v ggplot2 3.3.2
                       v purrr
                                 0.3.4
## v tibble 3.0.2
                       v dplyr
                                 1.0.1
## v tidyr
            1.1.1
                       v stringr 1.4.0
## v readr
             1.3.1
                       v forcats 0.5.0
## -- Conflicts -----
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()
                     masks stats::lag()
library(leaps)
library(MPV)
## Loading required package: KernSmooth
## KernSmooth 2.23 loaded
## Copyright M. P. Wand 1997-2009
##
## Attaching package: 'MPV'
## The following object is masked from 'package:olsrr':
##
##
       cement
```

## Ex 1:

a)

```
# forward regression
model_fw \leftarrow lm(y \sim ., data = table.b2)
ols_step_forward_aic(model_fw, details = T)
## Forward Selection Method
## -----
##
## Candidate Terms:
##
## 1 . x1
## 2 . x2
## 3 . x3
## 4 . x4
## 5 . x5
##
## Step 0: AIC = 266.8837
## y ~ 1
##
## Variable DF AIC Sum Sq RSS R-Sq Adj. R-Sq
       1 231.913 10578.685 4103.244 0.721 0.710
1 254.361 5783.780 8898.148 0.394 0.371
## x4
## x1
## x5
           1 265.068 1810.081 12871.848 0.123
                                                     0.091
           1 268.517
                         184.471 14497.458 0.013
## x3
                                                     -0.024
                       153.807 14528.121 0.010
## x2
           1 268.578
                                                     -0.026
##
## - x4
##
##
## Step 1 : AIC = 231.9133
## y \sim x4
##
## Variable DF AIC Sum Sq RSS R-Sq Adj. R-Sq
## -----
       1 214.131 2028.914 2074.330 0.859
## x3
                                                     0.848
           1 221.121 1463.527 2639.717 0.820
## x5
                                                     0.806
           1 232.495 195.805
                                  3907.439 0.734
## x1
                                                     0.713
           1 233.912
                        0.115 4103.128 0.721
                                                    0.699
## -----
##
## - x3
##
##
## Step 2 : AIC = 214.1313
## y \sim x4 + x3
##
```

```
## Variable DF AIC Sum Sq RSS R-Sq Adj. R-Sq
## -----
            1
                212.782
                         226.269
                                           0.874
                                 1848.062

    1
    214.857
    89.172
    1985.158
    0.865

    1
    215.076
    74.099
    2000.232
    0.864

                                                    0.849
                                                    0.847
## - x2
##
##
## Step 3 : AIC = 212.7817
## y \sim x4 + x3 + x2
## -----
## Variable DF AIC Sum Sq RSS R-Sq Adj. R-Sq
         1 210.636
                         246.163 1601.899 0.891
                                                    0.873
           1 214.607 11.108 1836.953 0.875
                                                    0.854
## -----
##
## - x1
##
##
## Step 4 : AIC = 210.6363
## y \sim x4 + x3 + x2 + x1
##
## Variable DF AIC Sum Sq RSS R-Sq Adj. R-Sq
        1 210.466 115.504 1486.395 0.899 0.877
##
## - x5
##
##
## Variables Entered:
##
## - x4
## - x3
## - x2
## - x1
## - x5
##
## Final Model Output
##
                    Model Summary
                   0.948 RMSE
0.899 Coef. Var
0.877 MSE
0.788 MAE
## R
                                            8.039
## R-Squared
                                            3.220
## Adj. R-Squared
                                           64.626
## Pred R-Squared
                                            5.440
```

```
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
                            ANOVA
##
                Sum of
##
               Squares
                           DF Mean Square
                          _____
##
  ______
                          5
## Regression 13195.533
                                   2639.107
                                             40.837 0.0000
## Residual
              1486.395
                            23
                                    64.626
            14681.928
## Total
                           28
##
##
                                Parameter Estimates
##
                                                        Sig
                Beta Std. Error Std. Beta
      model
                                                t
                                                                lower
                                                                         upper
## (Intercept)
               325.436
                           96.127
                                                3.385
                                                       0.003
                                                               126.582
                                                                        524.290
##
    x4 -22.949
                           2.704
                                     -0.910 -8.488 0.000 -28.542 -17.357
##
        x3
               3.800
                           1.461
                                      0.308 2.601 0.016
                                                               0.778
                                                                        6.823
                                      0.158 2.044 0.053
               2.552
                           1.248
                                                               -0.030
                                                                         5.134
##
        x2
                                              2.329
                                                               0.008
                0.068
                           0.029
                                      0.235
                                                     0.029
        x1
                                                                         0.128
         x5
                2.417
                           1.808
                                      0.206
                                              1.337 0.194
                                                               -1.323
                                                                        6.158
##
##
                        Selection Summary
              AIC
                                   RSS
                                            R-Sq
## Variable
                       Sum Sq
                                                    Adj. R-Sq
## x4
             231.913
                      10578.685 4103.244
                                           0.72052
                                                      0.71017
            214.131 12607.598 2074.330
## x3
                                           0.85872
                                                     0.84785
                      12833.867 1848.062
## x2
            212.782
                                           0.87413
                                                     0.85902
                     13080.030 1601.899
            210.636
## x1
                                           0.89089
                                                      0.87271
## x5
            210.466
                      13195.533 1486.395
                                           0.89876
                                                      0.87675
## -----
# backward regression
model_bw \leftarrow lm(y \sim ., data = table.b2)
ols_step_backward_aic(model_bw, details = T )
## Backward Elimination Method
##
## Candidate Terms:
##
## 1 . x1
## 2 . x2
## 3 . x3
## 4 . x4
## 5 . x5
## Step 0: AIC = 210.466
```

```
## y \sim x1 + x2 + x3 + x4 + x5
##
## -----
                     AIC
                               Sum Sq RSS R-Sq Adj. R-Sq
## Variable
              DF
## -----

      1
      210.636
      115.504
      1601.899
      0.891

      1
      213.308
      270.126
      1756.521
      0.880

      1
      214.607
      350.558
      1836.953
      0.875

      1
      215.943
      437.152
      1923.547
      0.869

      1
      249.616
      4656.562
      6142.957
      0.582

                                                       0.891
                                                                  0.873
                                                                  0.860
## x1
                                                                  0.854
                                                                  0.847
## x3
## x4
                                                                  0.512
##
##
## Variables Removed:
##
##
## No more variables to be removed.
## Variables Removed:
##
##
##
## Final Model Output
## -----
##
                          Model Summary
## -----
                         0.948 RMSE
0.899 Coef. Var
0.877 MSE
## R
                                                         8.039
## R-Squared
                                                        3.220
## Adj. R-Squared
                        0.877
                                                        64,626
                      0.788
## Pred R-Squared
                                     MAE
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
##
                                ANOVA
## -----
##
                  Sum of
##
                Squares
                               DF Mean Square F Sig.
## -----
                               5
                                        2639.107
## Regression 13195.533
                                                     40.837 0.0000
                              23
                                         64.626
## Residual
                1486.395
## Total
               14681.928
                                28
##
                                     Parameter Estimates
## ------
       model
                                                               Sig
                  Beta Std. Error
                                         Std. Beta
                                                       t
                                                                           lower
                                                                                     upper
## ------

      96.127
      3.385
      0.003
      126.582
      524.290

      0.029
      0.235
      2.329
      0.029
      0.008
      0.128

      1.248
      0.158
      2.044
      0.053
      -0.030
      5.134

      1.461
      0.308
      2.601
      0.016
      0.778
      6.823

## (Intercept) 325.436
## x1 0.068
##
          x2
                 2.552
      x3 3.800
##
```

```
-28.542
                        ##
          x4
                -22.949
                                                                             -17.357
          x5
                             1.808
                                                                   -1.323
##
                 2.417
                                        0.206 1.337 0.194
                                                                             6.158
## [1] "No variables have been removed from the model."
# stepwise aic regression
model_step \leftarrow lm(y \sim ., data = table.b2)
ols_step_both_aic(model_step, details = T)
## Stepwise Selection Method
## -----
##
## Candidate Terms:
##
## 1 . x1
## 2 . x2
## 3 . x3
## 4 . x4
## 5 . x5
##
## Step 0: AIC = 266.8837
## y ~ 1
##
##
## Variables Entered/Removed:
##
##
                          Enter New Variables
                             Sum Sq
                                            RSS
                                                     R-Sq Adj. R-Sq
## Variable
            DF
                    AIC
## -----
             1 231.913 10578.685 4103.244 0.721 0.710

    1
    254.361
    5783.780
    8898.148
    0.394

    1
    265.068
    1810.081
    12871.848
    0.123

    1
    268.517
    184.471
    14497.458
    0.013

    1
    268.578
    153.807
    14528.121
    0.010

                                                               0.371
                                                               0.091
## x5
## x3
                                                               -0.024
                                                               -0.026
##
## - x4 added
##
## Step 1 : AIC = 231.9133
## y \sim x4
##
                          Enter New Variables
## Variable DF AIC
                                         RSS
                             Sum Sq
                                                    R-Sq Adj. R-Sq
## -----
             1
                   214.131 12607.598 2074.330
                                                    0.859
                                                               0.848
                             12042.212 2639.717
## x5
              1
                   221.121
                                                    0.820
                                                                0.806
## x1
             1 232.495 10774.490 3907.439 0.734
                                                              0.713
              1 233.912 10578.800 4103.128
                                                    0.721
##
## - x3 added
```

```
##
##
## Step 2 : AIC = 214.1313
## y \sim x4 + x3
##
                  Remove Existing Variables
                              RSS R-Sq
        DF AIC Sum Sq
         1 231.913 10578.685
                                4103.244 0.721
## x3
                                                 0.710
          1 268.517 184.471 14497.458 0.013
                                                 -0.024
                     Enter New Variables
## Variable DF AIC
                             RSS R-Sq Adj. R-Sq
                      Sum Sq
         1 212.782
                       12833.867
                                1848.062 0.874
                                                  0.859
          1 214.857 12696.770 1985.158 0.865
                                                 0.849
          1 215.076 12681.697 2000.232 0.864
                                                  0.847
## - x2 added
##
##
## Step 3 : AIC = 212.7817
## y \sim x4 + x3 + x2
##
                   Remove Existing Variables
                              RSS R-Sq
## Variable DF AIC Sum Sq
                                2074.330 0.859
          1 214.131 12607.598
                                                  0.848
          1 233.912 10578.800
                                4103.128 0.721
                                                  0.699
           1 269.872
                       503.284 14178.644 0.034
                                                  -0.040
##
                     Enter New Variables
## -----
## Variable DF AIC
                       Sum Sq
                                 RSS
                                         R-Sq
           1
               210.636 13080.030 1601.899
                                         0.891
                                                  0.873
          1 214.607 12844.975 1836.953 0.875
                                                 0.854
## - x1 added
##
##
## Step 4 : AIC = 210.6363
## y \sim x4 + x3 + x2 + x1
##
                 Remove Existing Variables
                               RSS R-Sq Adj. R-Sq
## Variable DF AIC Sum Sq
```

```
1 212.782 12833.867 1848.062 0.874
                                          0.859
## x1
         1 214.857 12696.770 1985.158 0.865
                                            0.849
## x3
          1 234.244
                           3873.777
                                     0.736
                                             0.704
                    10808.151
          1
             251.007
                     7777.001 6904.928 0.530
                                             0.473
 ______
                   Enter New Variables
  ______
         DF AIC
                              RSS
                                     R-Sq
                    Sum Sq
         1 210.466 13195.533 1486.395
                                    0.899
                                             0.877
## - x5 added
##
##
## Step 5 : AIC = 210.466
## y \sim x4 + x3 + x2 + x1 + x5
##
##
                Remove Existing Variables
## -----
## Variable DF AIC
                    Sum Sq
                             RSS
                                     R-Sq Adj. R-Sq
         1 210.636
                    13080.030
                             1601.899 0.891
                                             0.873
         1 213.308 12925.407 1756.521 0.880
                                             0.860
          1 214.607 12844.975 1836.953
                                    0.875
                                             0.854
## x1
          1 215.943 12758.382 1923.547 0.869
## x3
                                            0.847
         1 249.616 8538.971 6142.957 0.582
                                            0.512
##
##
## Final Model Output
##
                 Model Summary
## -----
                     RMSE
Coef. Var
## R
                 0.948
                                     8.039
                0.899
## R-Squared
                                     3.220
## Adj. R-Squared
                0.877
                       MSE
                                    64.626
## Pred R-Squared
                0.788
                       MAE
                                     5.440
## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
##
                     ANOVA
##
            Sum of
           Squares DF Mean Square F Sig.
##
## -----
                    5 2639.107 40.837 0.0000
23 64.626
## Regression 13195.533
## Residual 1486.395
```

	Total	14681.928							
##				Paramete	er Estima	ates			
## ## ##	model	Beta	Std. Er					lower	
	(Intercept)		96.3		0.010	3.385			
##	x3	3.800	1.4	461	0.308	2.601	0.016	0.778	6.823
##	x1		0.0	029	0.235	2.329	0.029	-0.030 0.008	0.128
## ##	x5							-1.323 	
## ##									
## ##				Stepwise Su 					
## ##	Variable	Method	AIC	RSS	Sum	Sq	R-Sq	Adj. R-Sq	
##	x4	addition addition	231.913	4103.244 2074.330	10578	8.685 7.598	0.72052	0.71017	
##	x2	addition	212.782	1848.062	1283	3.867	0.87413	0.85902	
##	x5	addition addition	210.636	1601.899 1486.395		5.533	0.89089 0.89876		
444									

b) ModeloS Forward e Stepwise ficaram iguais. Modelo Backward ficou diferente.

## Ex 2:

## 10 . x10 ## 11 . x11

a)

```
# forward regression
model_fw <- lm(y ~ ., data = table.b3)
ols_step_forward_aic(model_fw, details = T)

## Forward Selection Method
## ------
##
## Candidate Terms:
##
## 1 . x1
## 2 . x2
## 3 . x3
## 4 . x4
## 5 . x5
## 6 . x6
## 7 . x7
## 8 . x8
## 9 . x9</pre>
```

```
##
## Step 0: AIC = 211.7768
##
          DF AIC
                           Sum Sq RSS R-Sq Adj. R-Sq
## Variable
            1
                166.430
## x1
                           955.720
                                     281.824
                                              0.772
                                                         0.765
                                  316.016
                                                        0.736
## x10
             1
                170.094
                           921.528
                                              0.745
## x3
            1
                161.857
                           822.213
                                  316.892
                                            0.722
                                                        0.712
## x2
            1 180.086
                           805.709
                                  431.835
                                            0.651
                                                        0.639
                         739.680 497.864 0.598
704.724 532.820 0.569
                 184.639
                                                        0.584
## x9
             1
## x8
             1
                186.810
                                                        0.555
## x11
                187.860 686.965 550.579 0.555
            1
                                                        0.540
## x7
                 190.204
                                  592.421 0.521
                                                        0.505
             1
                           645.123
                                            0.354
## x5
             1
                 199.806
                         437.807
                                    799.737
                                                        0.332
## x6
                  205.114
                           293.504
                                    944.040
                                            0.237
             1
                                                        0.212
                  209.426
                           157.325
                                    1080.219
                                                        0.098
                                              0.127
##
## - x3
##
##
## Step 1 : AIC = 161.8573
## y ~ x3
##
                AIC
                                   RSS
## Variable
          DF
                           Sum Sq
            1
                                   263.505 0.769
## x1
                 158.323
                           53.388
                                                       0.752
## x10
             1
                161.155 27.296
                                   289.597 0.746
                                                       0.727
## x4
            1
                161.191 26.948
                                   289.944 0.745
                                                      0.727
            1
                                          0.745
## x9
                161.265
                           26.234
                                   290.659
                                                       0.726
                         14.005
                                           0.734
## x7
             1
                 162.501
                                   302.887
                                                       0.714
## x6
            1
                162.530 13.713
                                   303.179 0.734
                                                       0.714
## x5
            1
                163.063 8.285
                                   308.608 0.729
                                                       0.709
## x2
            1
                 163.069 8.223
                                   308.669 0.729
                                                       0.709
                 163.358 5.226
163.708 1.568
## x11
                                   311.667
                                          0.726
             1
                                                       0.706
## x8
                                   315.325
                                          0.723
                                                       0.703
             1
##
## - x1
##
## Step 2 : AIC = 158.3225
## y \sim x3 + x1
##
                AIC
                           Sum Sq RSS
                                           R-Sq Adj. R-Sq
## Variable DF
            1 158.479 15.708 247.796 0.782
                                                       0.757
## x9
            1 158.678 14.058
                                   249.447 0.781
                                                       0.756
                                   251.317 0.779
## x6
            1
                158.902
                           12.187
                                                       0.754
```

```
      1
      159.497
      7.151
      256.353
      0.775

      1
      159.634
      5.975
      257.530
      0.774

      1
      160.131
      1.675
      261.830
      0.770

      1
      160.294
      0.252
      263.252
      0.769

      1
      160.301
      0.191
      263.314
      0.769

      1
      160.305
      0.152
      263.352
      0.769

## x10
                                                                  0.749
## x5
                                                                  0.748
                                                                  0.744
## x7
## x8
                                                                  0.742
                                                                  0.742
## x11
                                                                  0.742
##
## No more variables to be added.
## Variables Entered:
## - x3
## - x1
##
##
## Final Model Output
##
                          Model Summary
                          0.877 RMSE
0.769 Coef. Var
## R
                                                         3.124
## R-Squared
                                                        15.590
## Adj. R-Squared
                         0.752
                                    MSE
                                                         9.759
## Pred R-Squared
                         0.698
                                    MAE
                                                          2.356
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
                                ANOVA
##
                 Sum of
                               DF Mean Square
                                                     F
                 Squares
                                                                Sig.
## ------
## Regression 875.600
                                2
                                      437.800 44.859
                                                               0.0000
## Residual
                263.505
                                27
                                           9.759
## Total
                1139.105
                                29
##
                                   Parameter Estimates
  ______
               Beta Std. Error Std. Beta
       {\tt model}
## (Intercept) 32.769
                               1.709
                                                      19.176 0.000
                                                                         29.263
                                                                                   36.276
                               хЗ
##
               0.046
                                                                         -0.052
                                                                                  0.144
##
                 -0.080
          x1
                                                                         -0.149
                                                                                   -0.010
##
                         Selection Summary
## Variable AIC Sum Sq
                                      RSS R-Sq Adj. R-Sq
```

```
161.857 822.213 316.892 0.72181
## x3
                                                             0.71187
              158.323 875.600 263.505 0.76867
# backward regression
model_bw \leftarrow lm(y \sim ., data = table.b3)
ols_step_backward_aic(model_bw, details = T)
## Backward Elimination Method
##
## Candidate Terms:
## 1 . x1
## 2 . x2
## 3 . x3
## 4 . x4
## 5 . x5
## 6 . x6
## 7 . x7
## 8 . x8
## 9 . x9
## 10 . x10
## 11 . x11
##
## Step 0: AIC = 166.0979
## y \sim x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11
## ------
## Variable DF AIC
                                           RSS
                                                       R-Sq
                                 Sum Sq
                                                               Adj. R-Sq
## -----
                                                       0.831
## x3
              1
                     174.866 -76.824
                                          209.016
                                                                     0.751
              1
                                           187.866
## x11
                     164.172
                                  0.465
                                                       0.835
                                                                     0.748
                                                      0.835
## x6
              1
                     164.199
                                  0.632
                                          188.033
                                                                     0.748
## x4
              1
                    164.483
                                 2.418 189.819 0.833
                                                                     0.746
## x2
              1
                     165.115
                                 6.462 193.862 0.830
                                                                     0.740

      1
      165.327
      7.836
      195.236
      0.829

      1
      165.819
      11.065
      198.466
      0.826

## x10
                                                                     0.738
## x7
                                                                    0.734
## x9
               1
                    166.520 15.758 203.159 0.822
                                                                   0.728

      1
      166.957
      18.736
      206.136
      0.819

      1
      167.147
      20.047
      207.448
      0.818

      1
      169.351
      35.864
      223.264
      0.804

## x1
                                                                     0.724
## x8
                                                                     0.722
## x5
                                                                    0.701
##
## Variables Removed:
##
## - x11
##
##
## Step 1 : AIC = 164.1722
## y \sim x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10
##
## -----
```

```
## Variable DF AIC Sum Sq
                               RSS R-Sq Adj. R-Sq
## -----
        1
## x3
               172.891 -77.121
                               209.184
                                        0.831
                                                   0.762
                162.258
                         0.536
                                        0.835
## x6
           1
                               188.401
                                                  0.760
## x4
           1
                162.547
                         2.363
                                190.228
                                        0.833
                                                  0.758
                                                  0.752
## x2
               163.215
                        6.642 194.508 0.829
          1
                        7.985 195.850
## x10
          1
               163.421
                                       0.828
                                                  0.751
                       14.124
                                       0.823
## x7
           1
               164.347
                                201.990
                                                  0.743
                       16.914
                                       0.820
## x9
           1
               164.758
                                204.779
                                                  0.739
## x1
           1
               164.958 18.280 206.145 0.819
                                                  0.738
## x8
               165.250 20.301
                                208.166 0.817
                                                  0.735
           1
                                224.235 0.803
               167.481 36.370
           1
                                                  0.715
##
## - x6
##
##
  Step 2 : AIC = 162.2576
## y \sim x1 + x2 + x3 + x4 + x5 + x7 + x8 + x9 + x10
##
## -----
## Variable
         DF AIC
                      Sum Sq
                                RSS R-Sq Adj. R-Sq
## -----
                               209.376
                                       0.831
               170.921
                      -77.464
## x3
          1
                                                  0.772
## x4
                                                 0.767
           1
               160.802
                        3.451 191.853 0.832
## x2
          1
               161.342
                        6.932 195.334 0.829
                                                 0.763
               161.711
                         9.351
                               197.752
                                        0.826
                                                  0.760
## x10
           1
                       14.473
                               202.874
                                       0.822
## x7
           1
               162.478
                                                  0.754
               163.01618.146206.5470.819163.10818.780207.1810.818
## x9
           1
                                                  0.750
                                                  0.749
## x1
           1
                       21.244
                                       0.816
## x8
           1
                163.463
                                209.645
                                                  0.746
## x5
               165.946
                        39.332
                                227.733 0.800
                                                  0.724
           1
##
## - x4
##
##
##
  Step 3 : AIC = 160.8022
  y \sim x1 + x2 + x3 + x5 + x7 + x8 + x9 + x10
##
##
                AIC
## Variable DF
                              RSS
                                         R-Sq Adj. R-Sq
                        Sum Sq
## -----
                                                  0.775
               169.837
                      -74.833
                               215.458
                                        0.826
## x3
          1
## x2
          1
               160.442
                       10.780
                               202.632
                                       0.822
                                                   0.766
                       11.113
                                202.966
                                                  0.765
## x7
           1
               160.491
                                        0.822
                       14.988
## x10
           1
               161.059
                               206.841
                                       0.818
                                                  0.761
## x1
           1
               161.292 16.602 208.455 0.817
                                                  0.759
## x9
               161.503 18.072
                                209.924
                                       0.816
                                                  0.757
           1

      163.003
      28.835
      220.688
      0.806

      164.525
      40.323
      232.176
      0.796

## x8
           1
                                                  0.745
## x5
           1
                                                  0.731
## -----
##
## - x2
```

```
##
##
##
     Step 4 : AIC = 160.4422
## y \sim x1 + x3 + x5 + x7 + x8 + x9 + x10
## -----
## Variable DF AIC
                                        Sum Sq RSS R-Sq Adj. R-Sq
## -----
                         167.847
                                      -85.544
## x3
                 1
                                                     215.528
                                                                   0.826
                                                                                  0.784
## x7
                 1
                         159.952
                                      10.457
                                                   213.090
                                                                 0.813
                                                                                 0.764
## x1
                 1
                        160.168
                                      11.998 214.631
                                                                 0.812
                                                                                 0.762

    1
    160.258
    12.643
    215.275
    0.811
    0.762

    1
    160.431
    13.887
    216.520
    0.810
    0.760

    1
    162.281
    27.665
    230.297
    0.798
    0.745

    1
    162.609
    30.191
    232.823
    0.796
    0.742

                        160.258
## x9
## x10
## x8
## x5
##
## - x7
##
##
##
    Step 5 : AIC = 159.9518
## y \sim x1 + x3 + x5 + x8 + x9 + x10
##
## Variable DF AIC Sum Sq RSS R-Sq Adj. R-Sq
                  1
                                        -92.116
                                                      219.413
                                                                   0.823
                                                                                   0.789
                          166.419

    1
    158.676
    5.205
    218.294
    0.808

    1
    158.692
    5.321
    218.411
    0.808

    1
    160.432
    18.368
    231.457
    0.797

    1
    161.071
    23.346
    236.435
    0.792

    1
    161.409
    26.032
    239.121
    0.790

## x9
                                                                                   0.768
                                                                                   0.768
## x1
## x10
                                                                                  0.754
## x5
                                                                                  0.749
## x8
                                                                                  0.746
##
## - x9
##
##
##
   Step 6 : AIC = 158.6757
## y \sim x1 + x3 + x5 + x8 + x10
##
## Variable DF AIC Sum Sq RSS
                                                                   R-Sq Adj. R-Sq
## -----
## x3
               1
                         165.531 -89.559 227.174
                                                                 0.816
                                                                                  0.789

    1
    157.205
    3.888
    222.182
    0.805

    1
    159.855
    24.407
    242.701
    0.787

    1
    160.214
    27.322
    245.617
    0.784

    1
    161.585
    38.811
    257.105
    0.774

## x1
                 1
                                                                                  0.774
                                                                                  0.753
## x5
## x8
                                                                                 0.750
## x10
                                                                                  0.738
##
## - x1
##
##
##
     Step 7 : AIC = 157.2053
```

```
## y \sim x3 + x5 + x8 + x10
##
## -----
            DF
                   AIC Sum Sq RSS
                                              R-Sq Adj. R-Sq
## Variable
## -----

      1
      163.767
      -91.772
      228.850
      0.815
      0.795

      1
      159.464
      33.887
      256.070
      0.775
      0.749

      1
      161.310
      50.144
      272.326
      0.761
      0.733

      1
      164.869
      84.439
      306.621
      0.731
      0.700

## x8
        1
## x10
##
## No more variables to be removed.
## Variables Removed:
##
## - x11
## - x6
## - x4
## - x2
## - x7
## - x9
## - x1
##
##
## Final Model Output
##
                       Model Summary
                       0.897 RMSE
0.805 Coef. Var
## R
                                                  2.981
## R-Squared
                                                 14.877
## Adj. R-Squared
                               MSE
                     0.774
                                                 8.887
## Pred R-Squared
                      0.710
                               MAE
                                                  2.344
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
                            ANOVA
##
  ______
##
               Sum of
             Squares DF Mean Square
                                              F
                                                       Sig.
  ______
## Regression 916.923 4
## Residual 222.182 25
                                  229.231
                                              25.793 0.0000
                                     8.887
## Total
              1139.105
                            29
##
##
                               Parameter Estimates
      model Beta Std. Error Std. Beta
## (Intercept) 1.890 13.511
                                                0.140 0.890 -25.937 29.716
```

```
x3 0.011 0.025 0.145 0.430 0.671 -0.041 0.063
x5 2.931 1.501 0.247 1.953 0.062 -0.160 6.022
                                                                               0.063
##
##
##
          x8
                 0.237
                              0.100
                                          0.795
                                                    2.375 0.026
                                                                        0.032
                                                                                 0.443
                                          -1.606 -3.082 0.005
##
          x10 -0.011
                              0.003
                                                                       -0.018 -0.004
##
##
##
                     Backward Elimination Summary
## -----
## Variable AIC RSS
                                   Sum Sq R-Sq Adj. R-Sq
## Full Model 166.098 187.401 951.704 0.83548
                                                           0.73495
## x11 164.172 187.866 951.240 0.83508
                                                           0.74827

    162.258
    188.401
    950.704
    0.83461

    160.802
    191.853
    947.252
    0.83158

    160.442
    202.632
    936.473
    0.82211

    159.952
    213.090
    926.016
    0.81293

    159.676
    218.204
    202.811
    0.80236

## x6
                                                           0.76018
                                                           0.76741
## x4
                                                           0.76551
## x2
## x7
                                                           0.76413
## x9
              158.676 218.294 920.811 0.80836
                                                           0.76844
            157.205 222.182 916.923 0.80495 0.77374
## x1
# stepwise aic regression
model_step <- lm(y ~ ., data = table.b3)</pre>
ols_step_both_aic(model_step, details = T)
## Stepwise Selection Method
##
## Candidate Terms:
##
## 1 . x1
## 2 . x2
## 3 . x3
## 4 . x4
## 5 . x5
## 6 . x6
## 7 . x7
## 8 . x8
## 9 . x9
## 10 . x10
## 11 . x11
##
## Step 0: AIC = 211.7768
## y ~ 1
##
## Variables Entered/Removed:
##
                        Enter New Variables
                     AIC
## Variable DF
                               Sum Sq
                                                    R-Sq
                                            RSS
                                                             Adj. R-Sq
## -----
## x1 1 166.430 955.720 281.824 0.772
## x10 1 170.094 921.528 316.016 0.745
                                                                  0.765
                                                                 0.736
```

```
161.857822.213316.8920.722180.086805.709431.8350.651184.639739.680497.8640.598
## x3
                                                      0.712
           1
## x2
                                                      0.639
             1
## x9
            1
                                                      0.584

    1
    186.810
    704.724
    532.820
    0.569

    1
    187.860
    686.965
    550.579
    0.555

    1
    190.204
    645.123
    592.421
    0.521

## x8
                                                      0.555
## x11
                                                      0.540
## x7
                                                      0.505
                199.806 437.807 799.737 0.354
## x5
            1
                                                      0.332
                                 944.040 0.237
                                                      0.212
                 205.114 293.504
## x6
            1
## x4
             1
                209.426 157.325
                                 1080.219 0.127
                                                      0.098
## - x3 added
##
##
## Step 1 : AIC = 161.8573
## y ~ x3
##
##
                      Enter New Variables
                 AIC
                          Sum Sq
                                   RSS
                                           R-Sq Adj. R-Sq
## Variable
           DF
## -----
                158.323 875.600
                                          0.769
            1
                                 263.505
                                                     0.752
            1 161.155 849.508 289.597 0.746
## x10
                                                     0.727
            1 161.191 849.161 289.944
                                                    0.727
## x4
                                          0.745
## x9
            1 161.265 848.446 290.659 0.745
                                                    0.726
## x7
            1 162.501 836.218 302.887
                                          0.734
                                                    0.714
            1
                162.530 835.926 303.179
                                          0.734
## x6
                                                     0.714
            1
## x5
               163.063 830.497 308.608 0.729
                                                      0.709
## x2
            1 163.069 830.436 308.669 0.729
                                                     0.709
                163.358 827.438 311.667 0.726
## x11
                                                     0.706
            1
                163.708 823.780 315.325 0.723
## x8
             1
                                                      0.703
##
## - x1 added
##
##
## Step 2 : AIC = 158.3225
## y ~ x3 + x1
##
##
                   Remove Existing Variables
## -----
                                   RSS
## Variable DF
                  AIC
                                           R-Sq Adj. R-Sq
                         Sum Sq
## -----
            1 166.430 955.720 281.824 0.772
## x3
                                                    0.765
            1 161.857 822.213 316.892 0.722
##
##
                      Enter New Variables
## Variable DF AIC Sum Sq RSS R-Sq Adj. R-Sq
           1 158.479 891.309 247.796 0.782
## x4
                                                   0.757
## x9
            1 158.678 889.658 249.447 0.781
                                                    0.756
                                 251.317 0.779 0.754
            1 158.902
## x6
                          887.788
```

x10	1 159	.497	882.75	52 2!	56.353	0.775	5	0.7	49	
x5	1 159	.634	881.57	75 25	57.530	0.774	Ļ	0.7	48	
x7						0.770			44	
x8									42	
x2									42	
x11		.305	875.75	53 26	33.352	0.769	)	0.7	42	
No more var:	Output	e added	or rem	noved.						
		Model		-						
R				RMSE		3	3.124			
							5.590			
Adj. R-Squa:										
Pred R-Squar							2.356			
MSE: Mean MAE: Mean	Absolute Er	ror	ANOV!							
	Sum of									
Regression Residual Total	875.600 263.505 1139.105		27 29	9	9.759	44.859	9 0.	.0000	•	
			Pa	aramete	r Estima	ates			· ·	
model		Std.	Error	Std	. Beta	t 		Sig	lower	ирр
(Intercept)	32.769		1.709		·== <b>=</b> ·			0.000		
х3			0.048		0.609	0.96	66 (	0.343	-0.052	0.1
x1	-0.080		0.034		-1.475 	-2.33	39 (	0.027	-0.149	-0.0
				<i>i</i> ise Sur	nmary					
Variable				RSS	Sum	Sq	R-Sq		Adj. R-Sq	
	addition addition	161.85				.213 .600			0.71187 0.75154	

b) ModeloS Forward, Backward e Stepwise ficaram iguais.

#### Ex 3:

a) Obs: mesmo procedimento das questões anteriores. Tirei o argumento 'details = TRUE' pra poupar espaço no pdf.

```
# forward regression
model_fw \leftarrow lm(y \sim ., data = table.b4)
ols_step_forward_aic(model_fw)
##
##
                     Selection Summary
## ------
## Variable AIC Sum Sq RSS R-Sq Adj. R-Sq
## -----
      124.127 636.156 192.891 0.76733
122.191 665.332 163.714 0.80253
## x1
                                                 0.75676
## x2
                                                 0.78372
# backward regression
model_bw \leftarrow lm(y \sim ., data = table.b4)
ols_step_backward_aic(model_bw)
##
##
##
                 Backward Elimination Summary
## -----
## Variable AIC RSS Sum Sq R-Sq Adj. R-Sq
## Full Model 129.083 121.748 707.298 0.85315 0.75874
## x6 127.134 122.007 707.040 0.85283 0.77435
## x3 125.308 122.896 706.150 0.85176 0.78691
## x8 123.933 126.141 702.906 0.84785 0.79415
## x4 122.854 131.075 697.971 0.84190 0.79798
             121.848 136.615 692.431 0.83521 0.80052
## x9
## -----
# stepwise aic regression
model_step <- lm(y ~ ., data = table.b4)</pre>
ols_step_both_aic(model_step)
##
##
                          Stepwise Summary
## ------
## Variable
            Method AIC RSS Sum Sq
                                               R-Sq
  ______
       addition 124.127 192.891 636.156 0.76733
addition 122.191 163.714 665.332 0.80253
## x1
                                                           0.75676
                                                         0.78372
```

## b) ModeloS Forward e Stepwise ficaram iguais. Modelo Backward ficou diferente.

#### Ex 4:

```
# Table.b15 - Air Pollution and Mortality Data
MORT<-c(790.73, 823.76, 839.71, 844.05, 857.62, 860.1, 861.44, 861.83, 871.34, 871.77, 874.28, 887.47,
        , 899.26, 899.53, 904.16, 911.7, 911.82, 912.2, 912.35, 919.73, 921.87, 923.23, 929.15, 934.7,
        946.18, 950.67, 952.53, 953.56, 954.44, 958.84, 959.22, 961.01, 962.35, 967.8, 968.66, 970.47,
        985.95, 989.27, 991.29, 994.65, 997.88, 1001.9, 1003.5, 1006.49,
        1015.02, 1017.61, 1024.89, 1025.5, 1030.38, 1071.29, 1113.06)
PRECIP<-c(13, 28, 10, 43, 25, 35, 60, 11, 31, 15, 32, 43, 31, 37, 45, 35, 45, 45, 18, 42, 40, 36, 35, 3
          30, 41, 38, 46, 34, 38, 37, 31, 45, 44, 41, 39, 40, 42, 31, 47, 35, 30, 36, 42, 35, 36, 45, 5
          43, 54)
EDUC<-c(12.2, 12.1, 12.1, 9.5, 12.10, 11.8, 11.5, 12.1, 10.9, 12.2, 11.1, 11.5, 11.4, 12, 11.1, 12.2, 1
        9, 10.3, 10.7, 12, 11.4, 11.3, 11.1, 12.1, 11.4, 10.1, 10.8, 9.6, 11.4, 11.4, 9.70, 10.7, 11.9,
        12.3, 11.4, 10.2, 10.4, 10.7, 11.1, 11.10, 9.9, 10.6, 10.7, 11, 10.5, 11.3, 10.4, 10.5, 9.6, 10
NONWHITE<-c(3,7.5, 5.9, 2.9, 3, 14.8, 11.5, 7.8, 5.1, 4.7, 5, 7.2, 11.5, 3.6, 1, 5.7, 5.3, 3.4, 13.7, 4
        8.8, 5.8, 3.5, 12.4, 2.2, 13.1, 2.7, 3.8, 21, 17.2, 11.7, 13.1, 15.8, 21, 0.8, 25.9, 15.6, 13,
        13.1, 8.1, 11.3, 3.5, 8.1, 12.1, 36.7, 17.5, 22.2, 16.3, 28.6, 38.5, 24.4, 31.4)
NOX<-c(32, 2, 66, 7, 11, 1, 1, 319, 3, 8, 4, 3, 1, 21, 3, 7, 4, 4, 171, 8, 2, 7, 4, 15, 3, 23, 32, 4, 4
       9, 35, 14, 6, 28, 7, 26, 3, 7, 8, 21, 37, 59, 26, 10, 12, 11, 18, 32, 8, 63, 9, 32, 38, 17)
S02<-c(3, 1, 20, 32, 26, 1, 1, 130, 10, 28, 18, 10, 1, 44, 8, 20, 4, 20, 86, 49, 11, 20, 4, 59, 8, 125,
       25, 1, 68, 39, 15, 124, 78, 33, 102, 33, 146, 5, 25, 24, 64, 193, 263, 108, 39, 37, 42, 34, 161,
       72, 206, 1)
table.b15 <- data.frame(MORT,PRECIP, EDUC, NONWHITE, NOX, SO2)
```

#### a)

```
model_apr = lm(MORT ~ PRECIP + EDUC + NONWHITE + SO2, data = table.b15)
ols_step_all_possible(model_apr)
```

```
##
      Index N
                            Predictors R-Square Adj. R-Square Mallow's Cp
## 3
          1 1
                              NONWHITE 0.4192508
                                                      0.4092379
                                                                  41.000347
## 2
          2 1
                                  EDUC 0.2611045
                                                      0.2483649
                                                                  67.414934
## 1
          3 1
                                PRECIP 0.2595825
                                                      0.2468166
                                                                  67.669151
## 4
          4 1
                                   SO2 0.1814358
                                                      0.1673226
                                                                  80.721691
## 8
          5 2
                         EDUC NONWHITE 0.5667779
                                                      0.5515771
                                                                  18.359459
## 10
          6 2
                          NONWHITE SO2 0.5251109
                                                      0.5084481
                                                                  25.318932
## 7
          7 2
                            PRECIP SO2 0.4930645
                                                      0.4752773
                                                                  30.671523
          8 2
## 6
                       PRECIP NONWHITE 0.4929597
                                                      0.4751688
                                                                  30.689035
## 9
         9 2
                              EDUC SO2 0.3603149
                                                      0.3378698
                                                                  52.844186
## 5
         10 2
                           PRECIP EDUC 0.3493561
                                                      0.3265264
                                                                  54.674604
         11 3
                   PRECIP NONWHITE SO2 0.6405644
                                                      0.6213090
                                                                   8.035165
## 13
## 14
         12 3
                     EDUC NONWHITE SO2 0.6298005
                                                      0.6099684
                                                                   9.833024
        13 3
                  PRECIP EDUC NONWHITE 0.5786719
## 11
                                                      0.5561008
                                                                  18.372838
## 12
                       PRECIP EDUC SO2 0.5149803
                                                      0.4889971
                                                                  29.011017
## 15
         15 4 PRECIP EDUC NONWHITE SO2 0.6707104
                                                      0.6467621
                                                                   5.000000
```

```
ols_step_best_subset(model_apr) # resumo do 'best subset model'
         Best Subsets Regression
## -----
## Model Index Predictors
##
                NONWHITE
       1
##
              EDUC NONWHITE
##
              PRECIP NONWHITE SO2
##
               PRECIP EDUC NONWHITE SO2
## -----
##
##
                                                    Subsets Regression Summary
##
##
                                 Pred
                      Adj.
                                          C(p)
## Model R-Square R-Square R-Square
                                                     AIC
                                                                SBIC SBC
## -----
## 1 0.4193 0.4092 0.3765 41.0003 638.3043 466.2148 644.5873 137144.8
## 2 0.5668 0.5516 0.5122 18.3595 622.7202 451.3643 631.0975 104132.9
## 3 0.6406 0.6213 0.5808 8.0352 613.5172 443.2399 623.9889 87967.8
## 4 0.6707 0.6468 0.598 5.0000 610.2614 440.8813 622.8274 82082.3
## AIC: Akaike Information Criteria
## SBIC: Sawa's Bayesian Information Criteria
## SBC: Schwarz Bayesian Criteria
## MSEP: Estimated error of prediction, assuming multivariate normality
## FPE: Final Prediction Error
## HSP: Hocking's Sp
## APC: Amemiya Prediction Criteria
b)
# forward regression
model_fw <- lm(MORT ~ ., data = table.b15)</pre>
ols_step_forward_aic(model_fw, details = T)
## Forward Selection Method
##
## Candidate Terms:
##
## 1 . PRECIP
## 2 . EDUC
## 3 . NONWHITE
## 4 . NOX
## 5 . SO2
##
## Step 0: AIC = 668.9104
## MORT ~ 1
## Variable DF AIC
                          Sum Sq
                                            RSS
                                                R-Sq
                                                             Adj. R-Sq
## -----
## NONWHITE 1 638.304 95704.645 132570.751 0.419
                                                                0.409
```

MSEP

```
      1
      652.755
      59603.726
      168671.670
      0.261

      1
      652.878
      59256.287
      169019.109
      0.260

                                                      0.248
## EDUC 1 652.755
## PRECIP
                                                         0.247
            1 658.898 41417.329 186858.066 0.181
## SO2
                                                         0.167
            1 670.550
                          1366.959 226908.436 0.006
## NOX
                                                         -0.011
##
## - NONWHITE
##
##
## Step 1 : AIC = 638.3043
## MORT ~ NONWHITE
## -----
## Variable DF AIC Sum Sq
                                 RSS R-Sq Adj. R-Sq
                          33676.797 98893.954 0.567 0.552
## EDUC
         1 622.720
## SO2
            1 628.230 24165.258 108405.493 0.525
                                                         0.508
## PRECIP
            1 632.161 16825.918 115744.834 0.493
                                                         0.475
                          1856.217 130714.534 0.427
## NOX
            1 639.458
                                                          0.407
## - EDUC
##
##
## Step 2 : AIC = 622.7202
## MORT ~ NONWHITE + EDUC
## -----
## Variable DF AIC Sum Sq RSS R-Sq Adj. R-Sq
                         14386.520
## SO2
           1 615.288
                                     84507.435
                                               0.630
                                                         0.610
## PRECIP
            1 623.050 2715.123 96178.831 0.579
                                                         0.556
            1 624.720
                          0.038 98893.916 0.567
                                                         0.544
##
## - S02
##
##
## Step 3 : AIC = 615.2876
## MORT ~ NONWHITE + EDUC + SO2
##
## Variable DF AIC Sum Sq RSS R-Sq Adj.R-Sq
           1 610.261
                          9338.721 75168.714
## PRECIP
                                              0.671
                                                       0.647
                          4429.527 80077.907
             1 614.057
                                              0.649
                                                        0.624
## - PRECIP
##
##
## Step 4 : AIC = 610.2614
## MORT ~ NONWHITE + EDUC + SO2 + PRECIP
```

```
##
## -----
                          Sum Sq
                 AIC
            1 611.588 839.375 74329.339 0.674
                                                       0.644
##
## No more variables to be added.
##
## Variables Entered:
##
## - NONWHITE
## - EDUC
## - S02
## - PRECIP
##
##
## Final Model Output
## -----
##
##
                     Model Summary
## -----
                     0.819 RMSE
                                               36.969
## R-Squared
                     0.671
                             Coef. Var
                                               3.931
                             MSE
## Adj. R-Squared
                     0.647
                                             1366.704
                     0.598
## Pred R-Squared
                             MAE
                                              26.016
  RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
                           ANOVA
##
##
                Sum of
                          DF Mean Square F
               Squares
  ______
## Regression 153106.682
                         4
                                38276.670
                                            28.007 0.0000
## Residual
            75168.714
                           55
                                 1366.704
## Total
             228275.396
                          59
##
                              Parameter Estimates
      model Beta Std. Error Std. Beta
## (Intercept) 995.822
                                           10.902 0.000 812.773 1178.871
                         91.340
                                   0.444 5.100 0.000 1.882 4.318
##
    NONWHITE 3.100
                         0.608
                                   -0.212 -2.244 0.029 -29.475
##
     EDUC -15.570
                         6.939
                                                                    -1.664
     S02 0.326
PRECIP 1.635

      0.333
      3.921
      0.000
      0.160
      0.493

      0.262
      2.614
      0.012
      0.382
      2.889

##
                          0.083
                         0.625
```

##

```
Selection Summary
## -----
                      Sum Sq
             AIC
                                   RSS
                                            R-Sq
## -----
## NONWHITE 638.304
## EDUC 622.720
                     95704.645 132570.751
                                           0.41925
                                                      0.40924
                   129381.442 98893.954 0.56678
                                                     0.55158
## SO2
           615.288
                   143767.961
                                84507.435 0.62980
                                                      0.60997
        610.261 153106.682
                               75168.714 0.67071
                                                   0.64676
## PRECIP
# backward regression
model_bw <- lm(MORT ~ ., data = table.b15)</pre>
ols_step_backward_aic(model_bw, details = T)
## Backward Elimination Method
##
## Candidate Terms:
## 1 . PRECIP
## 2 . EDUC
## 3 . NONWHITE
## 4 . NOX
## 5 . SO2
##
## Step 0: AIC = 611.5876
## MORT ~ PRECIP + EDUC + NONWHITE + NOX + SO2
##
## Variable DF AIC Sum Sq
                                       RSS R-Sq Adj. R-Sq
                          839.375 75168.714 0.671
## NOX
           1 610.261
                                                          0.647
## PRECIP
           1 614.057 5748.568 80077.907 0.649
                                                         0.624
           1 614.384 6186.444
1 624.484 20946.720
                          6186.444
                                    80515.784
                                              0.647
## EDUC
                                                          0.622
## SO2
                                    95276.059 0.583
                                                          0.552
## NONWHITE
           1 633.461 36323.799 110653.138 0.515
                                                         0.480
##
##
## Variables Removed:
##
## - NOX
##
##
   Step 1 : AIC = 610.2614
##
## MORT ~ PRECIP + EDUC + NONWHITE + SO2
##
## -----
## Variable DF AIC
                                              R-Sq Adj. R-Sq
                          Sum Sq
                                       RSS
## EDUC 1 613.517 6881.579 82050.293 0.641
## PRECIP 1 615.288 9338.721 84507.435 0.630
## SO2 1 623.050 21010.118 96178.831 0.579
                                                          0.621
                                                          0.610
                623.050
                          21010.118 96178.831 0.579
## SO2
            1
                                                          0.556
## NONWHITE
           1
                631.496
                        35549.358 110718.071 0.515
                                                          0.489
```

```
##
##
## No more variables to be removed.
## Variables Removed:
##
## - NOX
##
##
## Final Model Output
##
##
                     Model Summary
## -----
                    0.819 RMSE
0.671 Coef. Var
                                              36.969
## R-Squared
                                             3.931
                           MSE
## Adj. R-Squared
                   0.647
                                           1366.704
                   0.598
                            MAE
## Pred R-Squared
                                            26.016
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
                          ANOVA
  ______
               Sum of
##
               Squares
                         DF Mean Square
                                           F
                                                   Sig.
                        4
55
                             38276.670
                                          28.007 0.0000
## Regression 153106.682
## Residual 75168.714
## Total 228275.396
                                1366.704
                       59
## Total
             228275.396
##
##
                             Parameter Estimates
      model Beta Std. Error
                                Std. Beta
                                                   Sig lower upper
             995.822
                        91.340
                                          10.902
                                                 0.000 812.773
## (Intercept)
                                                                  1178.871
                        0.625
                                 0.262 2.614 0.012 0.382
##
  PRECIP
             1.635
                                                                  2.889
                                 -0.212 -2.244 0.029 -29.475
##
     EDUC
            -15.570
                        6.939
                                                                  -1.664
  NONWHITE
             3.100
                        0.608
                                  0.444 5.100 0.000
                                                         1.882
                                                                  4.318
                                   0.333 3.921 0.000 0.160
     S02
              0.326
                         0.083
                                                                   0.493
##
##
                  Backward Elimination Summary
                                Sum Sq R-Sq Adj. R-Sq
## Full Model 611.588
                     74329.339
                             153946.057 0.67439
                                                  0.64424
## NOX 610.261 75168.714 153106.682 0.67071
                                                   0.64676
```

```
# stepwise aic regression
model_step <- lm(MORT ~ ., data = table.b15)</pre>
ols step both aic(model step, details = T)
## Stepwise Selection Method
## Candidate Terms:
##
## 1 . PRECIP
## 2 . EDUC
## 3 . NONWHITE
## 4 . NOX
## 5 . SO2
##
## Step 0: AIC = 668.9104
## MORT ~ 1
##
## Variables Entered/Removed:
##
##
                        Enter New Variables
## -----
           DF
                 AIC
                         Sum Sq
                                      RSS
## Variable
                                              R-Sq
## -----
## NONWHITE 1 638.304 95704.645 132570.751 0.419
## EDUC 1 652.755 59603.726 168671.670 0.261
                         95704.645 132570.751 0.419
                                                        0.409
                                                       0.248
## PRECIP
           1 652.878 59256.287 169019.109 0.260
                                                        0.247
            1 658.898 41417.329 186858.066
                                             0.181
## SO2
                                                        0.167
## NOX
            1 670.550
                         1366.959 226908.436
                                               0.006
                                                       -0.011
## - NONWHITE added
##
##
## Step 1 : AIC = 638.3043
## MORT ~ NONWHITE
##
##
                        Enter New Variables
## Variable DF AIC
                          Sum Sq
                                      RSS
                                               R-Sq Adj. R-Sq
## ------
           1 622.720 129381.442 98893.954
## EDUC
                                               0.567
            1 628.230 119869.903 108405.493
## SO2
                                               0.525
                                                         0.508
                        112530.562 115744.834 0.493
## PRECIP
            1 632.161
                                                         0.475
## NOX
            1 639.458 97560.861 130714.534 0.427
                                                         0.407
##
## - EDUC added
##
##
## Step 2 : AIC = 622.7202
## MORT ~ NONWHITE + EDUC
##
```

Remove Existing Variables												
: : Variable :			Sum Sq	RSS	R-Sq	Adj. R-Sq						
EDUC	1 1	638.304 652.755	95704.645 59603.726	132570.751 168671.670		0.409 0.248						
: : :			Enter New Var	riables								
			Sum Sq	RSS	R-Sq	Adj. R-Sq						
SO2 PRECIP	1 1	615.288 623.050	132096.564	84507.435 96178.831 98893.916	0.579	0.556						
NOX 1 624.720 129381.480 98893.916 0.567 0.544												
MORT ~ NONWHITE + EDUC + SO2  Remove Existing Variables												
		AIC	Sum Sq	RSS	R-Sq	Adj. R-S						
EDUC	1 1	628.230		98893.954 0.567 0. 108405.493 0.525 0. 1146024.365 0.360 0.								
Enter New Variables												
Variable	DF	AIC	Sum Sq	RSS	R-Sq	Adj. R-Sq						
NOX		614.057	148197.489	75168.714 80077.907		0.647 0.624						
NOX 1 614.057 148197.489 80077.907 0.649 0.624												
Step 4 : A		10.2614 + EDUC + SO	2 + PRECIP									
		Rem	ove Existing \	/ariables 								
Variable	DF AIC		Sum Sq	RSS	R-Sq							
EDUC PRECIP	1 1	1 613.517 146225.103 82050.293 1 615.288 143767.961 84507.435 1 623.050 132096.564 96178.831 1 631.496 117557.324 110718.071				0.61						

# # ,,					inter Ne								
# 1	 Variable 	DF	AIC		Sum S	lq.	RS	S	R-Sc	a A	dj.	R-Sq	
# ]		1	611.5	88	153946.	057	74329	.339	0.67	74	C	0.644	
#													
# # ]	No more var:	iables	to be	added	or remo	ved.							
‡ ‡ ]	Final Model	Outpu	t										
‡ · ‡			_										
#					Summar	•							
4 1	 R			010	D	MCE			26 (	060			
; ; ; ]	n R-Squared Adj. R-Squa Pred R-Squa			0.671	C	coef. V	ar		3.9	931			
‡ .	Adj. R-Squan	red		0.647	M	ISE			1366.7	704			
<b>‡</b> ]	Pred R-Squar	0.598	M	IAE			26.0	)16					
‡ ·													
	RMSE: Root		_	Error									
	MSE: Mean S	-		r									
#	TIAL. Hear I	nDSOIU	ce Liio	1									
#					ANOVA	L							
# -													
#			Sum of										
# #		<b>`</b>	Squares		DF	Mean	Square		F	Si	.g.		
	 Regression									0.00	000		
# ]	Residual	75	168.714		55	130	66.704	2	.0.001	0.00	,00		
‡ '	Total	228	275.396		59								
‡													
#					P	aramet	er Est	imate	s				
# · #					Error					_		lower	upp
# · #	 (Intercept)				91.340				.0.902			812.773	1178.8
#	NONWHITE	;	3.100				0.444					1.882	
#	EDUC	-1	5.570		6.939	•	-0.212	-	2.244	0.02	29	-29.475	-1.6
#	S02	(	0.326		0.083		0.333		3.921	0.00	0	0.160	0.4
# # ·	PRECIP		1.635 		0.625		0.262		2.614	0.01	.2	0.382	2.8
#													
# #					Ste	pwise	Summar	v					
<b>#</b> -													
	Variable 					RSS		Sum 	ı Sq 	K−S	6q 	Adj. R 	,−Sq 
	NONWHITE					32570.7	51	9570	4.645	0.41	925	0.40	924

```
## EDUC addition 622.720 98893.954 129381.442 0.56678 0.55158 ## SO2 addition 615.288 84507.435 143767.961 0.62980 0.60997
           addition 610.261 75168.714 153106.682 0.67071
                                                              0.64676
## PRECIP
\mathbf{c}
# forward regression
model_fw <- lm(MORT ~ ., data = table.b15)</pre>
ols_step_forward_p(model_fw, details = T)
## Forward Selection Method
## Candidate Terms:
## 1. PRECIP
## 2. EDUC
## 3. NONWHITE
## 4. NOX
## 5. SO2
##
## We are selecting variables based on p value...
##
##
## Forward Selection: Step 1
## - NONWHITE
##
                      Model Summary
## -----
## R.
                     0.647
                             RMSE
                                               47.809
## R-Squared
                    0.419
                             Coef. Var
                                                5.084
## Adj. R-Squared
                    0.409
                                              2285.703
                              MSE
                    0.376
## Pred R-Squared
                               MAE
                                               37.873
## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
##
                           ANOVA
## -----
##
                Sum of
                Squares DF Mean Square F Sig.
##
## Residual 132570.751 58 2285.703
## Total 228275.396
##
                               Parameter Estimates
                                           t Sig
                                                             lower upper
   model Beta Std. Error Std. Beta
```

```
85.803 0.000 866.035
## (Intercept) 886.722 10.334
                                                                                    907.408
                               0.699 0.647 6.471 0.000 3.125 5.925
   NONWHITE
               4.525
##
##
##
## Forward Selection: Step 2
## - EDUC
##
                           Model Summary
## --
                      0.753 RMSE
0.567 Coef. Var
0.552 MSE
0.512 MAE
                                                          41.653
## R-Squared
                                                           4.430
## Adj. R-Squared
                                                        1734.982
## Pred R-Squared
                                                        31.977
   RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
##
##
                    Sum of
##
                             DF Mean Square
                                                      F
                  Squares
  ______
                               2
## Regression 129381.442
## Regress:
## Residual 98895...
228275.396
                                         64690.721
                                                      37.286 0.0000
                                57
                                         1734.982
                                59
##
                                       Parameter Estimates
        model
                    Beta Std. Error Std. Beta
                                                                  Sig
                                                        t
                                                                             lower
                                                                                        upper
                                                       16.343 0.000 1062.337
## (Intercept) 1210.681 74.081
                                                                                      1359.025

      0.623
      0.565
      6.342
      0.000
      2.703
      5.199

      6.560
      -0.393
      -4.406
      0.000
      -42.039
      -15.766

   NONWHITE 3.951
##
      EDUC -28.902
##
##
## Forward Selection: Step 3
##
## - SO2
##
                           Model Summary
                                     RMSE
## R
                          0.794
                                                           38.847
## R-Squared 0.630 Coef. Var
## Adj. R-Squared 0.610 MSE
## Pred R-Squared 0.572 MAE
                                                           4.131
                                                       1509.061
```

## RMSE: Root Mean Square Error

MSE: Mean So MAE: Mean Al	0001400 21101						
		ANOV	<i>\</i>				
	Sum of						
			Mean Square				
Residual Total	84507.435 228275.396	56 59					
			Parameter Estin	nates			
model	Beta	Std. Erro	Std. Beta	t	Sig	lower	upp
(Intercept)	1155.479	71.36	 5	16.191	0.000	1012.518	1298.4
NONWHITE	3.736	0.58	0.535 -0.338 0.260	6.385	0.000	2.564	4.9
EDUC	-24.890	6.25	-0.338	-3.979	0.000	-37.420	-12.3
S02	0.255	0.083	3 0.260	3.088	0.003	0.090	0.4
Forward Select		Model Summa	cv				
- PRECIP	 0			36.9			
- PRECIPR R-Squared	0	.819 I	RMSE Coef. Var	36.90 3.93	31		
- PRECIPR R-Squared Adj. R-Square	0 0 0	.819 I .671 (	RMSE Coef. Var MSE	36.90 3.93 1366.70	31 04		
- PRECIP  R R-Squared Adj. R-Square	0 0 0 ed 0	.819 I .671 ( .647 I .598 I	RMSE Coef. Var MSE MAE	36.90 3.93	31 04		
- PRECIP  R R-Squared Adj. R-Square Pred R-Square RMSE: Root 1	o o ed o ed o ed Tean Square En	.819 I .671 ( .647 I .598 I	RMSE Coef. Var MSE MAE	36.90 3.93 1366.70	31 04		
- PRECIP  R R-Squared Adj. R-Square Pred R-Square RMSE: Root 1	ed 0 ed 0 Mean Square Enquare Error	.819 I .671 ( .647 I .598 I	RMSE Coef. Var MSE MAE	36.90 3.93 1366.70	31 04		
- PRECIP  R R-Squared Adj. R-Square Pred R-Square RMSE: Root 1	ed 0 ed 0 Mean Square Enquare Error psolute Error	.819 1 .671 0 .647 1 .598 1	RMSE Coef. Var ASE AAE	36.9 3.9; 1366.7 26.0;	31 04 16 		
- PRECIP  R R-Squared Adj. R-Square Pred R-Square RMSE: Root N MSE: Mean So MAE: Mean AN	ed 0 ed 0 Mean Square Enquare Error osolute Error Sum of Squares	.819   1 .671   0 .647   1 .598   1 rror	MSE Coef. Var MSE MAE  Mean Square	36.9 3.9 1366.7 26.0	31 04 16  Sig.		
- PRECIP  R R-Squared Adj. R-Square Pred R-Square RMSE: Root N MSE: Mean So MAE: Mean AN	ed 0 ed 0 Mean Square Enquare Error osolute Error Sum of Squares	.819   1 .671   0 .647   1 .598   1 rror	MSE Coef. Var MSE MAE  Mean Square	36.9 3.9 1366.7 26.0	31 04 16  Sig.		
- PRECIP  R R-Squared Adj. R-Square Pred R-Square RMSE: Root N MSE: Mean So MAE: Mean AN	ed 0 ed 0 Mean Square Enquare Error osolute Error Sum of Squares	.819   1 .671   0 .647   1 .598   1 rror	MSE Coef. Var MSE MAE  Mean Square	36.9 3.9 1366.7 26.0	31 04 16  Sig.		
- PRECIP  R R-Squared Adj. R-Square Pred R-Square RMSE: Root N MSE: Mean So MAE: Mean AN	ed 0 ed 0 Mean Square Enquare Error osolute Error Sum of Squares	.819   1 .671   0 .647   1 .598   1 rror	RMSE Coef. Var MSE MAE  MAE  Mean Square	36.9 3.9 1366.7 26.0	31 04 16  Sig.		
- PRECIP  R R-Squared Adj. R-Square Pred R-Square RMSE: Root N MSE: Mean So MAE: Mean AN	0 0 ed 0 Mean Square Enquare Error osolute Error Sum of Squares 153106.682 75168.714	.819   1   671   6   647   1   598   1   6   6   6   6   6   6   6   6   6	Mean Square 38276.670 1366.704	36.9 3.9 1366.7 26.0	31 04 16  Sig.		
- PRECIP  R R-Squared Adj. R-Square Pred R-Square RMSE: Root I MSE: Mean Add MAE: Mean Add Regression Residual Total	0 0 ed 0 Mean Square Enquare Error osolute Error Sum of Squares 153106.682 75168.714	.819   1   671   6   647   1   598   1   6   6   6   6   6   6   6   6   6	Mean Square 38276.670 1366.704	36.9 3.9 1366.7 26.0	31 04 16  Sig.		

```
91.340
                                          10.902
                                                  0.000 812.773
## (Intercept) 995.822
                                                                  1178.871
                        0.608
                                  0.444 5.100 0.000 1.882 4.318
##
   NONWHITE 3.100
    EDUC -15.570
                         6.939
                                                                   -1.664
##
                                  -0.212 -2.244 0.029 -29.475
                                   0.333
##
       S02
              0.326
                         0.083
                                           3.921
                                                  0.000
                                                          0.160
                                                                    0.493
     PRECIP
              1.635
                         0.625
                                   0.262
                                           2.614 0.012 0.382
                                                                     2.889
## ------
##
##
## No more variables to be added.
## Variables Entered:
## + NONWHITE
## + EDUC
## + SO2
## + PRECIP
##
##
## Final Model Output
##
                     Model Summary
## R
                            RMSE
                    0.819
                                              36.969
## R-Squared
                    0.671
                             Coef. Var
                                               3.931
## Adj. R-Squared
                    0.647
                              MSE
                                            1366.704
## Pred R-Squared
                    0.598
                              MAE
                                              26.016
  RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
##
                          ANOVA
##
##
               Sum of
               Squares
                       DF Mean Square F
## Regression
                         4
                                38276.670
            153106.682
                                           28.007
                                                 0.0000
## Residual
            75168.714
                                 1366.704
                         55
            228275.396
                         59
##
                             Parameter Estimates
     model
             Beta Std. Error
                                Std. Beta
##
                                                    Sig
                                                           lower
  (Intercept) 995.822
                                          10.902 0.000 812.773
                         91.340
                                                                 1178.871
##
    NONWHITE
             3.100
                         0.608
                                   0.444
                                          5.100 0.000 1.882
                                                                   4.318
                                   -0.212
                                           -2.244 0.029
##
     EDUC
            -15.570
                          6.939
                                                          -29.475
                                                                    -1.664
##
       S02
                          0.083
                                   0.333 3.921 0.000
             0.326
                                                          0.160
                                                                    0.493
     PRECIP
              1.635
                          0.625
                                   0.262
                                          2.614 0.012
                                                          0.382
                                                                     2.889
```

```
##
                        Selection Summary
##
        Variable
                             Adj.
        Entered R-Square R-Square C(p) AIC
## Step
                                                         RMSE
  ______
      NONWHITE 0.4193 0.4092 40.3122
EDUC 0.5668 0.5516 17.8461
S02 0.6298 0.6100 9.3943
                                              638.3043
                                                      47.8090
    2
##
                                              622.7202
                                                        41.6531
##
    3 SO2
                                              615.2876
                                                        38.8466
    4 PRECIP
##
                  0.6707
                            0.6468
                                      4.6098
                                              610.2614
                                                        36.9690
# backward regression
model_bw <- lm(MORT ~ ., data = table.b15)</pre>
ols_step_backward_p(model_bw, details = T)
## Backward Elimination Method
##
## Candidate Terms:
##
## 1 . PRECIP
## 2 . EDUC
## 3 . NONWHITE
## 4 . NOX
## 5 . SO2
## We are eliminating variables based on p value...
## - NOX
##
## Backward Elimination: Step 1
## Variable NOX Removed
                      Model Summary
                             RMSE
## R
                     0.819
                                                36.969
## R-Squared
                     0.671
                                                 3.931
                              Coef. Var
## Adj. R-Squared
                    0.647
                               MSE
                                              1366.704
## Pred R-Squared
                    0.598
                               MAE
                                                26.016
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
##
                            ANOVA
## -----
##
                Sum of
                         DF Mean Square F
                Squares
                                                    Sig.
## -----
## Regression 153106.682
                         4 38276.670 28.007 0.0000
                          55
## Residual
             75168.714
                                  1366.704
                        59
## Total
             228275.396
```

## ##									
## ##	model		Std. Erro	r Std.	Beta	t	Sig	lower	upper
## ## ## ##	(Intercept) PRECIP		91.34	5 (		10.902 2.614 -2.244	0.012		2.889
##								1.882	
##	S02		0.08					0.160	0.493
## ## ## ## ## ##	No more varia	ables satisfy	the condi	tion of p	value	= 0.3			
	Variables Rem	noved:							
##	- NOX								
##	NOX								
##									
	Final Model (	_							
##			Model Cumm						
##			Model Summa	ary 					
##	R	O	.819	RMSE		36.9	969		
##	R-Squared	O	.671	Coef. Var	r	3.9	931		
	Adj. R-Square			MSE		1366.7			
	Pred R-Square	ed 0	.598	MAE		26.0	016		
## ## ##	MSE: Mean So	Mean Square E quare Error osolute Error							
##									
##			ANO	VA					
##									
## ##		Sum of Squares	DF	Mean So	nuaro	F	Sig		
##	Regression	153106.682	4	38276	6.670	28.007	0.0000		
	Residual			1366	6.704				
	Total	228275.396							
## ##									
##				Parameter	r Estin	nates			
##									
##		Beta					_		upper
##		005 000							1170 074
## ##	(Intercept)	995.822 1.635	91.34		0.262			812.773	
##	EDUC	-15.570	6.93	9 -(	0.212	-2.244	0.029	0.382 -29.475	-1.664
##		3.100				5.100			

```
S02 0.326 0.083 0.333 3.921 0.000 0.160 0.493
##
##
##
                    Elimination Summary
       Variable
                         Adj.
       Removed R-Square R-Square C(p)
## Step
                                        AIC
## -----
       NOX
           ## -----
# stepwise regression
model_step <- lm(MORT ~ ., data = table.b15)</pre>
ols_step_both_p(model_step, details = T)
## Stepwise Selection Method
##
## Candidate Terms:
##
## 1. PRECIP
## 2. EDUC
## 3. NONWHITE
## 4. NOX
## 5. SO2
##
## We are selecting variables based on p value...
##
## Stepwise Selection: Step 1
## - NONWHITE added
##
##
                   Model Summary
## -----
                  0.647
                          RMSE
## R
                                         47.809
## R-Squared 0.419 Coef. Var
## Adj. R-Squared 0.409 MSE
## Pred R-Squared 0.376 MAE
                                          5.084
                                        2285.703
## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
                        ANOVA
## -----
##
              Sum of
             Squares DF Mean Square F Sig.
           95704.645 1 95704.645 41.871 0.0000
## Regression
## Residual 132570.751
## Total 228275.396
                       58
                             2285.703
                    59
```

model	Beta	Std.	Error	Std. Beta	t 	Sig	lower	upper
(Intercent)	886.722 4.525	1	0.334	0.647	85.803 6.471	0.000	866.035 3.125	907.408 5.925
Stepwise Selec	ction: Step	2						
- EDUC added								
			Summary					
R		0.753	RM	ISE	41.6			
				ef. Var				
Adj. R-Squared Pred R-Squared					1734.9			
	- 			 				
RMSE: Root Me	_	Error						
MSE: Mean Squ MAE: Mean Abs		r						
MAE. Medii ADS	solute Ello	1						
			ANOVA					
	Sum of							
			DF	Mean Square	F	Sig.		
Regression Residual					37.286	0.0000		
Total	228275.396		59	1734.902				
				arameter Esti	mates			
model				Std. Beta			lower	ирр
(Intercept)				<b>_</b>			1062.337	1359.0
NONWHITE	3.951		0.623	0.565				
EDUC	-28.902 		6.560 	-0.393 	-4.406 	0.000	-42.039 	-15.7 
		Model	Summary					
 R		 0.753			41.6	 53		
		0.567	Co	0.567 Coef. Var 0.552 MSE				
R-Squared Adj. R-Squared Pred R-Squared	l			E	4.4 1734.9 31.9	82		

##		-	ror						
##	MSE: Mean Squ								
##	MAE: Mean Abs	solute Error							
## ##				ANOVA					
## ##		Sum of Squares		DF	Mean Square	F	Sig.		
##									
##	Regression Residual	98893.954		2 57	1734.982	31.200	0.0000		
	10041	220210.000		00					
## ##					Parameter Esti	mates			
## ##	model	Beta 	Std.	Error	Std. Beta	t 	Sig 	lower	upper
##	(Intercept) NONWHITE EDUC	3.951		0.623	0.565	6.342	0.000	2.703	5.199
##	EDUC	-28.902		6.560	-0.393	-4.406	0.000	-42.039	-15.766
##									
##									
##									
## ##	Stepwise Selec	ction: Step 3							
	- SO2 added								
##									
##				Summar					
##	P.	0	70/	R	MSE	32.2	 17		
## ##	R-Squared	0.	630	C	oef. Var	4.1	31		
##	Adj. R-Squared	i 0.	610	М	SE	1509.0	61		
##	R-Squared Adj. R-Squared Pred R-Squared	i 0.	572	М	AE	28.7	72		
	RMSE: Root Me								
##	MSE: Mean Squ								
##	MAE: Mean Abs	solute Error							
## ##				ANOVA					
##									
##		Sum of				_	_		
## ##					Mean Square				
##	Regression	143767.961		3	47922.654				
##	Residual	84507.435		56	1509.061				
		228275.396		59					
## ##									
## ##					Parameter Esti	mates			
##								1	
## ##	model	веtа 	Std.	Error	Std. Beta 	t 	•	lower 	upper
##									

EDUC	3.736 -24.890 0.255	0 6 0						4.90
		Model Sur	nmary					
 R		 ).794			38.84	 17		
R-Squared				f. Var	4.13			
Adj. R-Square								
Pred R-Square	d (	0.572	MAE		28.77			
RMSE: Root M MSE: Mean Sq MAE: Mean Ab	ean Square E uare Error	Error	JOVA					
	Sum of							
	Squares 	Di 	· Mo	ean Square	F 	Sig.		
Regression								
Residual Total	84507.435	56	3					
model	Beta	Std. E	ror	rameter Esti  Std. Beta	t	Sig	lower	uppe
(Intercept)					16.191		1012.518	1298 44
	3.736	0	. 585	0.535	6.385	0.000	2.564	4.90
EDUC	-24.890	6	. 255	-0.338	-3.979	0.000	2.564 -37.420 0.090	-12.36
S02	0.255	0	.083	0.260	3.088	0.003	0.090	0.42
Stepwise Sele	_	4						
11,2021 4440		Model Sur						
					36.96	· <u>-</u>		
R		0.819	RMS1	Ł	00.00	, ,		
	(	0.819 0.671		f. Var	3.93			
R	(			f. Var		31		

##	MAE: Mean Ab	solute Erro	r						
##				ANOV	A				
## ##		Sum of							
## ##		Squares		DF 	Mean Square	F 	Sig. 		
	Regression Residual			4		28.007	0.0000		
##	Total	228275.396		59	1300.704				
## ##									
## ##					Parameter Esti	mates 			
##	model				Std. Beta	t	Sig	lower	upper
##	(Intercept)	995.822		91.340			0.000		
## ##		3.100 -15.570		0.608 6.939	0.444 -0.212	5.100 -2 244	0.000	1.882 -29.475	4.318 -1.664
##		0.326						0.160	
##	PRECIP	1.635		0.625		2.614		0.382	2.889
##									
## ##									
##									
## ##			Model	L Summa	ry				
##	R.		0.819	]	RMSE	36.9	969		
	R-Squared				Coef. Var	3.9			
	Adj. R-Square			I		1366.			
	Pred R-Square				ЛАЕ 	26.0	016		
	RMSE: Root M								
##		_							
##	MAE: Mean Ab	solute Erro	r						
## ##				ANOV	۸				
				ANUV	· 				
##		Sum of							
## ##		Squares			Mean Square	F	Sig.		
					38276.670	28.007	0.0000		
		75168.714			1366.704				
	Total			59					
## ##									
##				]	Parameter Esti	mates			
##									
## ##	model	Beta	Std.	. Error	Std. Beta	t 	Sig	lower	upper
	(Intercept)			91.340		10.902	0.000	812.773	1178.871
##	NONWHITE	3.100			0.444				
##	EDUC				-0.212				-1.664
##	S02	0.326		0.083	0.333	3.921	0.000	0.160	0.493

## ##		1.635			0.262	2.614	0.012	0.382	2.889
##									
## ##									
	No more vari	ables to be	e added/rem	oved.					
## ##									
##	Final Model	_							
## ##									
##			Model Su						
## ##							 0		
	R-Squared		0.819 0.671			36.96 3.93			
##	Adj. R-Squar	ed	0.647	MSE		1366.70	4		
##	Pred R-Squar	ed	0.598	MAE		26.01			
	RMSE: Root								
	MSE: Mean S	quare Erron	<u>-</u>						
##	MAE: Mean A	bsolute Eri	cor						
## ##			A	NOVA					
##									
##		Sum o		E Maa	- C	P	Q:		
		Square	es ມ 	r меа 	n Square 	r 	Sig.		
	Regression					28.007	0.0000		
	Residual Total			5 9	1366.704				
##									
##				Param 	eter Estima <sup>1</sup>	tes 			
##		Beta	Std. Er	ror S	td. Beta		Sig	lower	upper
	(Intercept)					10.902	0.000	812.773	1178.871
##	NONWHITE	3.100		608	0.444	5.100	0.000	1.882	
## ##	EDUC SO2	-15.570 0.326		939 083	-0.212 0.333	-2.244 3.921	0.029	-29.475 0.160	
##	PRECIP	1.635		625	0.333		0.000	0.100	
##									
##									
## ##			Ste	pwise Se	lection Summ	mary			
##		Ac	 lded/		Adj.				
##	Step Vari			-Square		C(p)	А	IC	RMSE
## ##		UTTE ~42	 lition	0.419	0.409	40.312		.3043	47.8090
	1 N(∩N().			0.419	0.409	40.312	U 038		T1.0030
##	1 NONW 2 ED						0 622	.7202	41.6531
## ## ##	2 ED 3 SO	UC add	lition lition lition	0.567 0.630 0.671	0.552	17.846 9.394	0 615	.2876	41.6531 38.8466 36.9690

## Ex 5:

```
attach(cement)
cement
       y x1 x2 x3 x4
## 1
     78.5 7 26 6 60
     74.3 1 29 15 52
## 3 104.3 11 56 8 20
     87.6 11 31 8 47
## 4
## 5 95.9 7 52 6 33
## 6 109.2 11 55 9 22
## 7 102.7 3 71 17 6
## 8
     72.5 1 31 22 44
## 9 93.1 2 54 18 22
## 10 115.9 21 47 4 26
## 11 83.8 1 40 23 34
## 12 113.3 11 66 9 12
## 13 109.4 10 68 8 12
model_apr = lm(y \sim ., data = cement)
ols_step_all_possible(model_apr)
     Index N Predictors R-Square Adj. R-Square Mallow's Cp
##
## 4
       1 1
                  x4 0.6745420
                                   0.6449549 138.730833
## 2
        2 1
                   x2 0.6662683
                                   0.6359290 142.486407
## 1
        3 1
                   x1 0.5339480
                                   0.4915797
                                             202.548769
## 3
                   x3 0.2858727
        4 1
                                   0.2209521 315.154284
## 5
       5 2
                x1 x2 0.9786784
                                   0.9744140
                                             2.678242
## 7
        6 2
                x1 x4 0.9724710
                                   0.9669653
                                              5.495851
## 10
        7 2
                x3 x4 0.9352896
                                   0.9223476
                                              22.373112
## 8
       8 2
                x2 x3 0.8470254
                                   0.8164305
                                              62.437716
## 9
       9 2
                x2 x4 0.6800604
                                   0.6160725 138.225920
      10 2
                x1 x3 0.5481667
                                   0.4578001 198.094653
## 6
             x1 x2 x4 0.9823355
## 12
     11 3
                                   0.9764473
                                              3.018233
## 11
     12 3
            x1 x2 x3 0.9822847
                                  0.9763796
                                             3.041280
## 13
     13 3
              x1 x3 x4 0.9812811
                                   0.9750415 3.496824
       14 3
              x2 x3 x4 0.9728200
                                   0.9637599
                                               7.337474
## 14
       15 4 x1 x2 x3 x4 0.9823756
## 15
                                   0.9735634
                                               5.000000
ols_step_best_subset(model_apr)
## Best Subsets Regression
## -----
## Model Index
               Predictors
##
      1
               x4
##
               x1 x2
               x1 x2 x4
##
      3
##
               x1 x2 x3 x4
##
##
                                                 Subsets Regression Summary
## ------
##
                      Adj.
                                 Pred
```

## ##	Model	R-Square	R-Square	R-Square	C(p)	AIC	SBIC	SBC	MSEP
	1	0 6745	0.6450	0 5602	120 7200	07 7440	FF F401	00 4300	1047 0402
##	1	0.6745	0.6450	0.5603	138.7308	97.7440	55.5401	99.4389	1047.0423
##	2	0.9787	0.9744	0.9654	2.6782	64.3124	29.2437	66.5722	76.2162
##	3	0.9823	0.9764	0.9686	3.0182	63.8663	31.1723	66.6910	71.0365
##	4	0.9824	0.9736	0.9594	5.0000	65.8367	34.4130	69.2264	81.0000
шш									

## AIC: Akaike Information Criteria

## SBIC: Sawa's Bayesian Information Criteria

## SBC: Schwarz Bayesian Criteria

## MSEP: Estimated error of prediction, assuming multivariate normality

## FPE: Final Prediction Error

## HSP: Hocking's Sp

## APC: Amemiya Prediction Criteria