Testing Causality between Monetary policy and GDP growth in Brazil: A Vector Autoregression approach

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Abstract. This work aims to verify the existence of a causal relationship between monetary policy conduction and GDP growth in Brazil during the period 2012-2019. It is argued that changes in interest rates may have an impact on several macroeconomic aggregates, specially on GDP. Therefore, it is carried out an investigation of causality trough the modeling of a vector autoregression (VAR) and the application of the Toda Yamamoto procedure for testing Granger Causality from two main time series: the nominal interest rate and the GDP growth. It is concluded that there was a causal relationship (in the sense of Granger) between monetary policy and GDP growth in Brazil during the period 2012-2019.

Keywords: VAR, Multivariate Time Series, Monetary Policy, GDP, Granger Causality

1. Introduction

It is argued that changes in interest rates may have an impact on several macroeconomic aggregates. In fact, we have seen that a considerable number of times through the years: economic crisis, inflation targeting and high unemployment rates are usually associated with decisions taken by policy makers, which sometimes are reflected by the monetary policy.

In this sense, the present work aims to verify the existence of a causal relation between monetary policy conduction and GDP growth in Brazil during the period 2012-2019. The years between 2012 and 2019 and the time lapse were specifically chosen due to the influence of a previous work by Libânio (2010). In his paper, the author uses a Structural Vector Autoregression (SVAR) to analyze the relation between monetary policy and economic performance in Brazil during the period 1999-2006, so that it could be discussed the growth effects of the inflation targeting regime through its effects on aggregate demand. In this paper, however, the focus is something else entirely. While the referred author is concerned about the effects of the inflation targeting regime on Brazil's GDP - arguing that monetary policy is said to react in a procyclical and asymmetric way to fluctuations in economic activity under inflation targeting - we focused the analsyis on testing if there was a causal relation between the nominal interest rate levels and the GDP growth.

Regarding the methodology of the time series analysis, the investigation of causality was carried out through the modeling of a Vector Autoregression (VAR) and the application of the Granger Causality Test from two main time series: the nominal interest rate (which in Brazil is represented by

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the SELIC rate) and the brazilian GDP growth provided by the American Central Bank (that is, the Federal Reserve). It is worth mentioning that the this work applies the Toda and Yamamoto (1995) procedure, whose functionality is to allow causality testing without pre assuming cointegration or stationarity of the time series. A more detailed presentation of this procedure is shown in the next section.

Henceforth, this work is divided in four parts. The first one finishes with this brief introduction. The second is dedicated to the explanation of both the Vector Autorregression and the Granger Causality Test methodologies. The third part presents the results obtained throughout the econometric modeling. The fourth and last one summarizes the conclusions about the statistical causality test.

2. Methodology

Since the seminal paper written by Sims (1980), econometric modeling has evolved in a significant way. The development of the mathematical and statistical theory needed in econometrics was often linked to problems in real life. Therefore, its applications used to propose solutions and innovations that improved the economic science and last until today. This is certainly the case of *'Macroeconomics and Reality'*, the most comprehensive and influential in a series of papers by Christopher Sims in the early eighties of the 20^{th} century. According to Christiano (2012), Vector Autoregression provided key empirical input into substantive economic debates, and they continue to do so today. Furthermore, research on technical questions raised by these models proceeds at a brisk pace.

The Vector Autoregression model (henceforth: **VAR**) represented a huge advance in macroeconometric and time series theory. Not only it helped policy makers to perform more accurate analysis (specially in therms of monetary policy), but also it allowed an alternative approach for multivariate time series modeling. As Greene (2003) argues, the VAR was not just the reduced form of some structural model. For purposes of analyzing and forecasting macroeconomic activity, researchers have found that VARs have proved as good as or better than structural equation systems. In addition to forecasting, VARs have been used for two primary functions: testing Granger causality and studying the effects of policy through impulse response characteristics.

2.1. **Vector Autoregression (VAR).** According to Greene (2003), we can write a VAR equation system as follows:

$$\mathbf{y_t} = \mu + \mathbf{\Gamma_1 y_{t-1}} + ... + \mathbf{\Gamma_p y_{t-p}} + \varepsilon_\mathbf{t}, \tag{2.1}$$

where ε_t is a vector of nonautocorrelated innovations with zero means and contemporaneous covariance matrix $E[\varepsilon_t \varepsilon_t'] = \Omega$. Equation (2.1) may also be written as

$$\mathbf{\Gamma}(L)\mathbf{y_t} = \mu + \varepsilon_\mathbf{t}$$

where $\Gamma(L)$ is a matrix of polynomial in the lag operator. The correspondent individuals equations are

$$y_{mt} = \mu_m + \sum_{j=1}^{p} (\Gamma_j)_{m1} y_{1,t\,-j} + \sum_{j=1}^{p} (\Gamma_j)_{m2} y_{2,t\,-j} y_{t-p} + ... + \sum_{j=1}^{p} (\Gamma_j)_{mM} y_{M,t\,-j} + \varepsilon_{mt},$$

where $(\Gamma_j)_{lm}$ indicates the (l, m) element of Γ_j.

It is also useful to know that we can write any pth order VAR as a first-order VAR by augmenting it, if necessary, with additional identity equations. For example, the model

$$\mathbf{y_t} = \mu + \mathbf{\Gamma_1 y_{t-1}} + \mathbf{\Gamma_2 y_{t-2}} + \mathbf{v_t}$$

can be written

$$\begin{bmatrix} \mathbf{y_t} \\ \mathbf{y_{t-1}} \end{bmatrix} = \begin{bmatrix} \mu \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \Gamma_1 & \Gamma_2 \\ \mathbb{I} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{y_{t-1}} \\ \mathbf{y_{t-2}} \end{bmatrix} + \begin{bmatrix} \mathbf{v_t} \\ \mathbf{0} \end{bmatrix},$$

which is a first-order model. We can study the dynamic characteristics of the model in either form. The second is usually more convenient, though. Because VARs may also be used with the intention to estimate the **impulse response function**, its representation might vary. There are other aplicable forms of the VAR model in a macroeconomic context, such as the **SVAR**, where the $'S'$ stands for Structural VAR. In this paper, however, we are specifically concerned about the causality test. For this reason, we proceeded the estimation of the model in its reduced form.

2.2. **Granger Causality and Toda Yamamoto procedure.** Does movement of the interest rate *cause* movements in brazilian GDP in the Granger sense? That is the question we are trying to answer. Let $\mathbf{y_t} = [\mathbf{GDP},\ \mathbf{interest\ rate\ level}]'_{\mathbf{t}}$.Then, a simple VAR would be

$$\mathbf{y_t} = \begin{bmatrix} \mu_\mathbf{1} \\ \mu_\mathbf{2} \end{bmatrix} + \begin{bmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{bmatrix} \mathbf{y_{t-1}} + \begin{bmatrix} \varepsilon_\mathbf{1t} \\ \varepsilon_\mathbf{2t} \end{bmatrix}.$$

To assert a causal relationship between interest rate levels and GDP, we must find that α_2 is not zero. Despite the real simplicity and lack of information of this example, it helps us to build an intuiton for what is coming next.

Another way to understand this method would be the following one: if past values of a variable X are found predicting Y, then X is said to cause Y in the Granger sense, or simply $X \longrightarrow Y$. Suppose that the following equation describes the relation between X and Y:

$$Y_t = \sum_{i=1}^{n} \alpha_i X_{t-i} + \sum_{i=1}^{n} \beta_i Y_{t-i} + \varepsilon_{1t} \tag{2.2}$$

If $\sum\limits_{i=1}^{n} \alpha_i \neq 0$, that is, if all α_i are jointly different from zero, then we have that $X \longrightarrow Y$. On the other hand, we could also have that $Y \longrightarrow X$. In this case, the method would be analogous. This idea was first introduced by Granger (1969).

Time series theory is first thought and developed under the assumption of stationarity. As the modeling evolves, a question frequently rises: what if the series are not stationary? In case of non-stationarity of one or more series involved in the Granger Test, it is possible that the results found are spurious.

In order to solve this issue, Toda and Yamamoto (1995) established a comprehensive procedure to investigate causality in the Granger sense. Mishra (2014) argues that their method of Granger causality test is relatively more efficient in small sample data sizes and is particularly appropriate for time series for which the order of integration is not known or may not be necessarily the same, or the order of integration is more than two. Its advantage lies on the fact that it does not require the pretesting of the time series for cointegration properties, so long as the order of integration of the process does not exceed the true lag length of the model. The procedure basically consists of the following steps:

- Check the order of integration of variables through unit root and stationarity tests;
- Define the maximum order (m) of integration between the variables;
- Build the VAR model in levels;
- Specify the order of lag of the VAR (p) by the traditional information criteria;
- Check the stability of the model, in particular auto-correlation issues;
- If everything is right, add m lags to the VAR, so that we will have a VAR ($p + m$);
- Run the Wald test with p coefficients and p degrees of freedom.

Toda and Yamamoto (1995) methodology of Granger causality consists on performing the test directly on the coefficients of the levels VAR, minimises the risk associated with possibly wrongly identifying the orders of integration of the series and the presence of cointegration relationship, as described by Mavrotas and Kelly (2001). As the steps listed above might suggest, the basic idea in the procedure is artificially augmenting the correct VAR order.

3. Econometric Modeling

First of all, we need to take a look at our data. We can plot both series as follows:

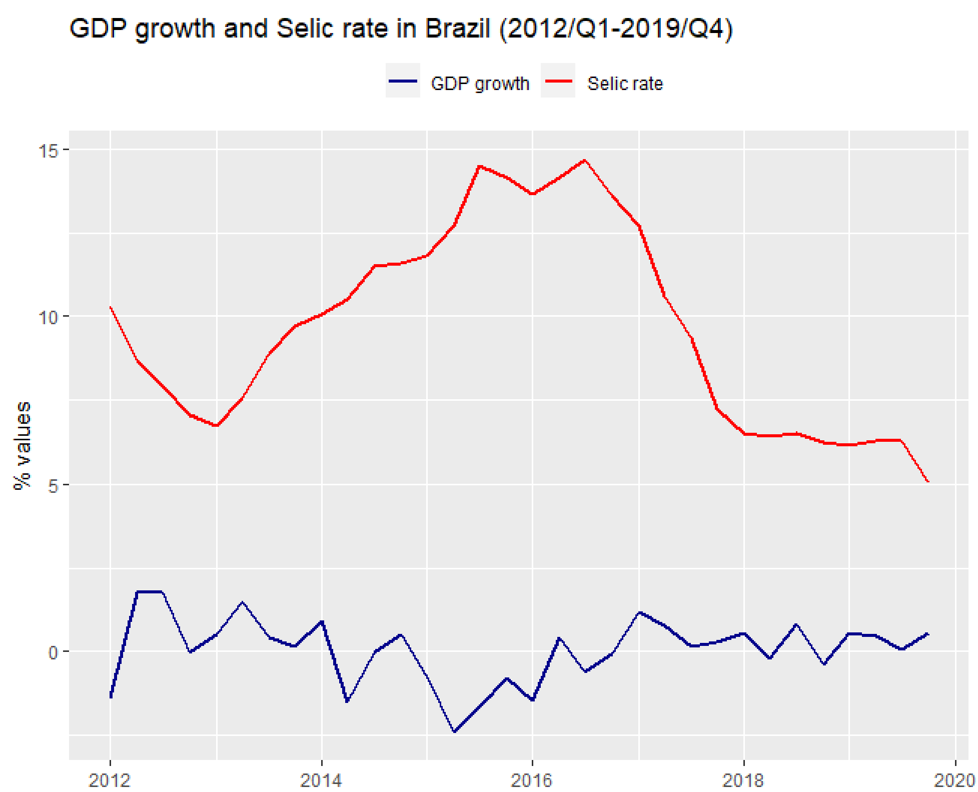

Figure 1. Joint graph - GDP growth and Selic rate in Brazil (2012-2019; quarterly data).

The websites from where the series were collected are http://ipeadata.gov.br/beta3 (Selic) and https://fred.stlouisfed.org/series/NAEXKP01BRQ657S (Brazilian GDP growth). The econometric modeling was carried out following the steps discussed in the last section. We already plotted both series togheter. Now, in order to investigate stationarity and determine their order of integration, let's look at them separately:

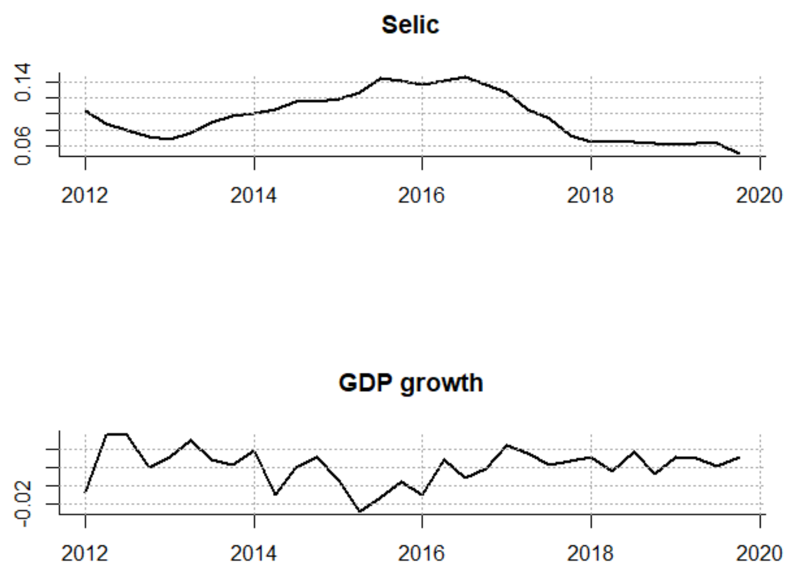

Figure 2. Selic rate and GDP growth in Brazil (2012-2019; quarterly data).

When looking at these series, one might suspect that they are not stationary. Stationarity is a quite rare phenomenum when dealing with economic data. Nevertheless, we do need to test this hyphotesis with proper inference methods. To proceed that, we apply the Augmented Dickey-Fuller Test (for which the null hypothesis is non-stationarity) in both series. All computations in this article were performed using the R statistical environment.

Order of Integration. Unit root tests were first considered as a tool to determine the order of integration:

Table 1. ADF Unit Root Test Results

	ADF_{level}	p-value	ADF diff_1	p-value	ADF diff_2	p-value	ADF diff_3	p-value
Selic	-1.82	0.64	-2.54	0.36	-2.29	0.46	-3.66	0.04
GDP	-1.68	0.70	-2.81	0.26	-4.44	0.01	-5.44	0.01

Both series had to be differentiated so that we could find the m argument, that is, the maximum order of integration between the variables. The GDP series became stationary with 2 differences. On the other hand, for the Selic rate series we could only reject the null-hyphotesis of non-stationarity when incrementing 3 differences. Therefore, we cosidered the GDP growth to be $I(2)$ (i.e., integrated of order 2) and the Selic rate, representing the changes in monetary policy, to be $I(3)$ (integrated of order 3). In all tests it was assumed the default significance level $\alpha = 0.05$. That said, we got $m = 3$.

VAR in the levels. Following the procedure established in the literature by Toda and Yamamoto (1995), one must estimate the VAR model in the levels of the data, regardless of the orders of integration of the various time-series. To specify the order of lag of the VAR (p), the usual way is to find the minimun information criteria:

Table 2. Information Criteria for VAR order selection

AIC(n)	HQ(n)	SC(n)	FPE(n)
2	2	1	2

Based on the above table, the *pth* order selected was $p = 2$. The next step is to make sure that the VAR is well-specified. As this work analyzes quarterly data comprehending eight years (2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019), $n = 32$. This sample size matches the assumption that the Toda and Yamamoto method of Granger causality test is relatively more efficient in small samples. Finishing this step, we ensured that there was no serial correlation in the residuals of the model and checked its stability:

Table 3. Portmanteau Test (asymptotic) - Residuals of VAR(2)

Chi-squared	p-value	H_0
50.074	0.6976	absence of autocorrelation

The above table shows the result of the Portmanteau Test: one must not reject the absence of autocorrelation in the residuals. So, the modeling goes as expected.

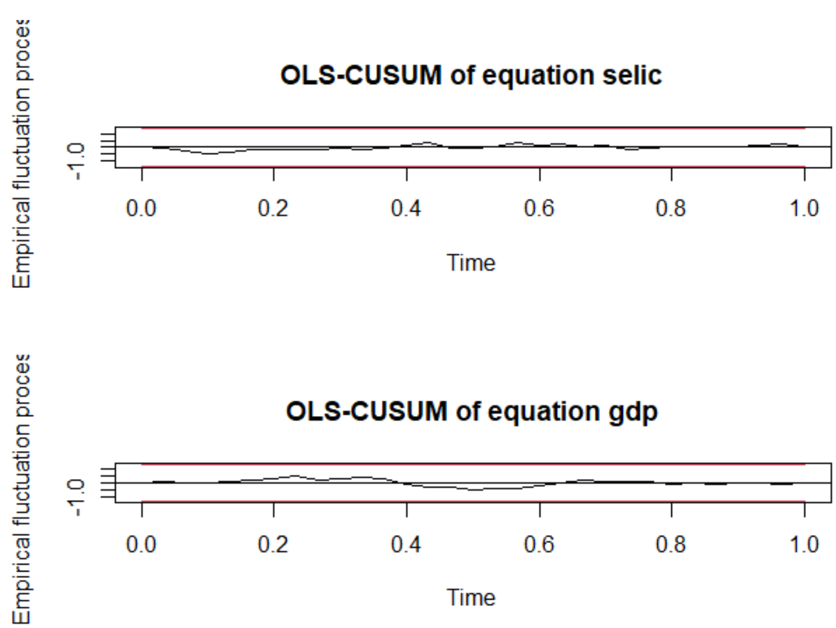

Figure 3. Checking the stability of the model: VAR(2).

VAR(p+m). As everything seemed to be right, it was time to add m lags to the VAR, so that we would have a VAR$(p+m)$. We first determined $m = 3$ and then estimated a VAR(2). Hence, we had to estimate a VAR(5) to proceed with the modified Wald test. We obtained the following outputs for these tests:

Table 4. Wald Test 1: SELIC does not granger cause GDP growth

Chi-squared test	p-value	H_0: Null Hypothesis	Decision
7.2	0.028	SELIC does not granger cause GDP growth	**Reject**

Table 5. Wald Test 2: GDP growth does not granger cause SELIC

Chi-squared test	p-value	H_0: Null Hypothesis	Decision
0.68	0.71	GDP growth does not granger cause SELIC	**Not Reject**

As a last remark, it is worth noticing that Toda and Yamamoto (1995) only guarantee the non-necessity of pretesting cointegration if the order of integration of the process does not exceed the true lag length of the model. Once we found $m = 3 > p = 2$, we should perform the Johansen Cointegration Test to validate our methodology and our results. These inference methods are well described in both Johansen (1991) and Johansen (1995). If we are able to reject the null hypothesis of no cointegration, then our results are valid, indeed. This result appears as follows:

Table 6. Johansen-Procedure: Eigenvalue Test

	Test	$\alpha = 10\%$	$\alpha = 5\%$	H_0: Null Hypothesis	Decision
$r \leq 1$	1.55	6.50	8.18	No Cointegration	**Not Reject**
$r \leq 0$	15.33	12.91	14.90	No Cointegration	**Reject**

In the Johansen Cointegration Test, the alternative hypothesis for the eigenvalue test is that there are $r + 1$ cointegration relations. This test is sequential, that is, we should test first for $r = 0$, then $r = 1$ and so on, where r represents the rank condition. The test concludes when the value of r when fails to reject H_0 for the first time. In our case, the test fails to reject the null hypothesis for the first time when $r = 1$. Therefore, we are able to reject the null hypothesis of non-cointegration at 5% of significance, as we have at least one cointegration relationship. The results above and their respective conclusions are discussed in the next section.

4. Results and Conclusions

As showed in Tables 4 and 5, we estimated a VAR(2) and then a VAR(5) in order to apply the modified Wald Test for testing Granger causality as Toda and Yamamoto (1995) recommended. We actually executed two different Wald tests: the first testing if movements on the nominal interest rate level, which represented monetary policy conduction, caused, in the sense of Granger, the changes in the GDP growth during the period 2012-2019; the second testing the contrary, i.e., if changes in the GDP growth rate caused, in the sense of Granger, movements on the nominal interest level, which represented the monetary policy conduction.

Regarding the decision presented in Table 4, we rejected the null hypothesis assuming that the changes on the nominal interest rate level did not Granger cause the changes in the GDP growth. This is our main result, because it confirms our primary theoretical hypotheis: monetary police conduction did affect the GDP growth rate in Brazil between the years of 2012 and 2019. For the decision presented in Table 5, we are not able to reject the null hypothesis assuming that the changes on the nominal interest rate level did not Granger cause the changes in the GDP growth. Although we could have found a bidirectional causal relationship between the studied time series, the latter decision was less important, considering that our macroeconomic assumption was that monetary policy affects GDP, not the contrary. Finally, we performed the Johansen procedure for testing Cointegration between the studied time series. This last step was necessary because Toda and Yamamoto procedure only guarantees the non-necessity of pretesting cointegration if the order of integration of the process does not exceed the true lag length of the model. Rejecting the null hypothesis of non-cointegration for $r = 0$ allowed us to infer at least one cointegration relationship, which ended up validating the whole procedure.

This work aimed to verify the existence of a causal relationship between monetary policy conduction and GDP growth in Brazil during the period 2012-2019. It is argued that changes in interest rates may have an impact on several macroeconomic aggregates, specially on GDP. Therefore, it was carried out an investigation of causality trough the modeling of a vector autoregression (VAR) and the application of the Toda Yamamoto procedure for testing Granger Causality between the nominal interest rate and the GDP growth. It is concluded that there has been indeed a causal relationship (in the sense of Granger) between monetary policy conduction and GDP growth in Brazil during the period 2012-2019.

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