1.

a)

$$\mathcal{X} = \{1, 2a, 2b, 3, 4\}$$

$$A = \{a, b, c\}$$

$$Z = \{O_1, O_2, O_3, O_4\}$$

b)

Transition matrix:

Action a:

$$T(x'|a, x) = P_a = \begin{bmatrix} 0 & 0,5 & 0,5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Action b:

$$T(x'|a, x) = P_b = \begin{bmatrix} 0 & 0,5 & 0,5 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Action c:

$$T(x' | c, x) = P_c = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Observation Matrix:

$$O_{\#} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Cost function:

$$C(x, a) = C = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.5 & 1.0 & 0.0 \\ 0.5 & 0.0 & 1.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

c)

$$b_{t+1} = \frac{1}{c} b_t \times P_a, \quad a \in A \qquad c \to \text{fator de normalização}$$

↳ General formula to update agent belief

For action a at time step t:

$$b_{t+1} = \frac{1}{c} [0, 0 \ 0,5 \ 0,5 \ 0,0 \ 0,0] \times \begin{bmatrix} 0 & 0,5 & 0,5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$= \frac{1}{c} [0 \ 0 \ 0 \ 0,5 \ 0,5]$$

$$= [0 \ 0 \ 0 \ 0,5 \ 0,5] \ \text{//}$$

For action b at time step t:

$$b_{t+1} = \frac{1}{c} \times [0,0 \ 0,5 \ 0,5 \ 0,0 \ 0,0] \times \begin{bmatrix} 0 & 0,5 & 0,5 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$= [1 \ 0 \ 0 \ 0 \ 0] \ \text{//}$$

For action c at time step t:

$$b_{t+1} = \frac{1}{c} [0,0 \ 0,5 \ 0,5 \ 0,0 \ 0,0] \times \begin{bmatrix} 0 & 0,5 & 0,5 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$= [1 \ 0 \ 0 \ 0 \ 0] \ \text{//}$$