1.

a) Markov Decision Processes

$X = \{1, 2, 3, 4\}$

$A = \{$Up, Down, Left, Right, Stay$\}$

b)

$Pr(3 | 1, down) = 0{,}8$

$Pr(1 | 1, down) = 0{,}2$

$Pr(2 | 1, down) = 0$

$Pr(4 | 1, down) = 0$

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$Pr(3 | 2, down) = 0$

$Pr(1 | 2, down) = 0$

$Pr(2 | 2, down) = 0{,}2$

$Pr(4 | 2, down) = 0{,}8$

$Pr(3|3, down) = 1$

$Pr(1|3, down) = 0$

$Pr(2|3, down) = 0$

$Pr(4|3, down) = 0$

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$Pr(4|4, down) = 1$

$Pr(1|4, down) = 0$

$Pr(2|4, down) = 0$

$Pr(3|4, down) = 0$

Transition matrix:

$$P = \begin{bmatrix} 0{,}2 & 0 & 0{,}8 & 0 \\ 0 & 0{,}2 & 0 & 0{,}8 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Cost function :

$$C(\mathcal{X}, A) = \underbrace{\begin{bmatrix} 2 & 0,5 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0,5 & 1 \\ 0,5 & 1 & 2 & 1 & 1 \end{bmatrix}}$$

It verifies the stated conditions ✓

c)

$\gamma = 0,9$ $\boxed{S_x \in \mathcal{X}} \Rightarrow \mathcal{X} = [S_0, S_1, S_2, \dots]$

$$\boxed{U^{\pi}(s) = E\left[Pr([S] \mid S_0 = s_{\pi}) \cdot \left[\sum_{t=0}^{\infty} \gamma C(S,A)\right]\right.}$$

$$\boxed{OC = E\left[\sum_{t=0}^{\infty} \gamma^{t} c_t\right]}$$ → stated in the slides

$$J_{\text{down}}(1) = 0{,}5 + \gamma \times 1 + \gamma^2 \times 1 + \gamma^3 \times 1 \ldots$$

$$= \frac{1}{2} \times \frac{\gamma + 1}{1 - \gamma}$$

$$J_{\text{down}}(2) = 1 + \gamma \times 1 + \gamma^2 \times 1 + \gamma^3 \times 1 \ldots$$

$$= \frac{1}{1 - \gamma}$$

$$J_{\text{down}}(3) = 1 + \gamma \times 1 + \gamma^2 \times 1 + \gamma^3 \times 1 \ldots$$

$$= \frac{1}{1 - \gamma}$$

$$J_{\text{down}}(4) = 1 + \gamma \times 1 + \gamma^2 \times 1 + \gamma^3 \times 1 \ldots$$

$$= \frac{1}{1 - \gamma}$$

$$J_{down}(x) = \begin{bmatrix} \frac{1}{2} \times \frac{1+\gamma}{1-\gamma} \\ \frac{1}{1-\gamma} \\ \frac{1}{1-\gamma} \\ \frac{1}{1-\gamma} \end{bmatrix} = \begin{bmatrix} 9{,}5 \\ 10 \\ 10 \\ 10 \end{bmatrix}$$