Homework 1

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(1)

(a) Markov chain specified by the pair (X, P)

states: $X = \{R, A, B, C, D, E, F\}$

Probabilities: $P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 1/4 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 & 0 & 1/2 \\ 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 0 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 1/2 & 0 & 0 & 1/2 \\ 0 & 0 & 1/3 & 0 & 1/3 & 1/3 & 0 \end{bmatrix}$

(b) $P^2 = \begin{bmatrix} 0.25 & 0 & 0.25 & 0.25 & 0.25 & 0 & 0 \\ 0 & 0.625 & 0 & 0 & 0 & 0.125 & 0.25 \\ 0.125 & 0 & 0.29 & 0.125 & 0.29 & 0.165 & 0 \\ 0.125 & 0 & 0.125 & 0.375 & 0.125 & 0 & 0.25 \\ 0.125 & 0 & 0.29 & 0.125 & 0.29 & 0.165 & 0 \\ 0 & 0.25 & 0.165 & 0 & 0.165 & 0.415 & 0 \\ 0 & 0.33 & 0 & 0.165 & 0 & 0 & 0.495 \end{bmatrix}$

$\mu_0 P^2 = \begin{bmatrix} 0.25 & 0 & 0.25 & 0.25 & 0.25 & 0 & 0 \end{bmatrix}$

As in Markov Decision Problems the transition probabilities in an instant t are independent from previous transitions. This result was to be expected, as in $t=1$, the garbage truck will be "forced" to go through stop A.

(e)

T_{ij} = expected between garbage truck travelling from i to j
min

$$T_{AA} = \frac{1}{6}(T_{AA} + T_{AA}) + \frac{1}{6}(T_{AB} + T_{BA}) + \frac{1}{6}(T_{AC} + T_{CA}) +$$

$$+ \frac{1}{6}(T_{AD} + T_{DA}) + \frac{1}{6}(T_{AE} + T_{EA}) + \frac{1}{6}(T_{AF} + T_{FA}) =$$

$$= \frac{1}{6}(2 \times 30) + \frac{1}{6}(2 \times (30 + T_{AB})) + \frac{1}{6}(2 \times (30 + T_{AC})) +$$

$$+ \frac{1}{6}(2 \times (30 + T_{AD})) + \frac{1}{6}(2 \times (30 + T_{AE})) + \frac{1}{6}(2 \times (30 + T_{AF}))$$

$T_{AA} + T_{AB}$

$T_{AB} = 40$; $T_{AC} = 55$; $T_{AD} = 70$; (1h 10 mins)

$T_{AE} = T_{AC} + T_{CE} =$
$= 55 + 55 =$
$= 110$ mins $=$
$= (1\text{h } 50 \text{ mins})$

$$T_{AF} = \frac{1}{3}(T_{AB} + T_{BF}) +$$

$$+ \frac{1}{3}(T_{AC} + T_{CE} + T_{EF}) +$$

$$+ \frac{1}{3}(T_{AD} + T_{DF})$$

$$T_{AF} = \frac{1}{3}(40 + 80) + \frac{1}{3}(110 + 20) + \frac{1}{3}(70 + 70) =$$

$$= 40 + 43.33 + 46.67 = 130 \text{ mins} = (2\text{h } 10 \text{ mins})$$

$$T_{AA} = \frac{1}{6}(2 \times 30) + \frac{1}{6}(2 \times (30 + 40)) + \frac{1}{6}(2 \times (30 + 55)) +$$

$$+ \frac{1}{6}(2 \times (30 + 70)) + \frac{1}{6}(2 \times (30 + 110)) + \frac{1}{6}((2 \times (30 + 130)) =$$

$$= 10 + 23.33 + 28.33 + 33,33 + 46.67 + 53.33 =$$

$$= 194.99 \text{ mins} \approx (3\text{h } 15 \text{ mins})$$