analise_modal

June 21, 2023

[25]:
```
import numpy as np
import matplotlib.pyplot as plt
from sympy import Matrix, symbols, atan2, pi, exp, sin, cos, sqrt, lambdify,␣
↪pprint, simplify
from numpy import array, arange
import scienceplots

np.set_printoptions(precision=3, suppress=True)

plt.style.use([
    'grid',
    'retro'
])
```

[26]:
```
m  = 840  # kg
ix = 820  # kg*meter**2
iy = 1100 # kg*meter**2
mf = 53   # kg
mr = 76   # kg

M = np.diag([m, ix, iy, mf, mf, mr, mr])

a1 = 1.4  # meter
a2 = 1.47 # meter
b1 = 0.7  # meter
b2 = 0.75 # meter

# # # # # # # # # # # #

kf  = 1e4   # N/meter
kr  = 1.3e4 # N/meter
ktf = 2e5   # N/meter
ktr = 2e5   # N/meter
kR  = 1e4   # (N*meter/rad)
w   = b1+b2

k11 = 2*kf+2*kr
```

```python
k21 = (b1-b2)*kf+(b2-b1)*kr
k31 = 2*(a2*kr-a1*kf)

k22 = kR + (b1**2+b2**2)*kf+(b1**2+b2**2)*kr
k32 = (a1*b2-a1*b1)*kf+(a2*b2-a2*b1)*kr
k42 = -b1*kf - (1/w)*kR
k52 = b2*kf+(1/2)*kR
k33 = 2*kf*a1**2+2*kr*a2**2
k44 = kf+ktf+(1/(w**2))*kR
k55 = kf+ktf+(1/(w**2))*kR

K = np.array([
    [k11,  k21,    k31,      -kf,      -kf,    -kr,    -kr],
    [k21,  k22,    k32,      k42,      k52,  b1*kr, -b2*kr],
    [k31,  k32,    k33,    a1*kf,    a1*kf, -a2*kr, -a2*kr],
    [-kf,  k42,  a1*kf,      k44, -kR/w**2,      0,      0],
    [-kf,  k52,  a1*kf, -kR/w**w,      k55,      0,      0],
    [-kr, b1*kr, -a2*kr,       0,        0, kr+ktr,      0],
    [-kr,-b2*kr, -a2*kr,       0,        0,      0, kr+ktr]
])

# # # # # # # # # # # #

cf = 500 # N*s/meter
cr = 500 # N*s/meter

c11 = 2*cf+2*cr
c21 = (b1-b2)*cf+(b2-b1)*cr
c31 = 2*a2*cr-2*a1*cf
c22 = (b1**2+b2**2)*cf+(b1**2+b2**2)*cr
c32 = (a1*b2-a1*b1)*cf+(a2*b2-a2*b1)*cr
c33 = 2*cf*a1**2+2*cr*a2**2

C = np.array([
    [c11,   c21,   c31,    -cf,   -cf,   -cr,    -cr],
    [c21,   c22,   c32, -b1*cf, b2*cf, b1*cr, -b2*cr],
    [c31,   c32,   c33,  a1*cf, a1*cf,-a2*cr, -a2*cr],
    [-cf,-b1*cf, a1*cf,     cf,     0,     0,      0],
    [-cf, b2*cf, a1*cf,      0,    cf,     0,      0],
    [-cr, b1*cr,-a2*cr,      0,     0,    cr,      0],
    [-cr,-b2*cr, -a2*cr,     0,     0,     0,     cr]
])

F = np.array([
    [0, 0, 0, ktf, ktf, ktr, ktr]
]).T
```

```
X0 = array([
    [0],
    [0],
    [0],
    [0],
    [0],
    [0],
    [1]
])

V0 = array([
    [0],
    [0],
    [0],
    [0],
    [0],
    [0],
    [0],
])

N = M.shape[0]
```

[27]:
```
# Calculando M á menos meio
M_minus_half = np.vectorize(lambda x: 1/np.sqrt(x) if x != 0 else 0)(M)
M_half       = np.vectorize(lambda x: np.sqrt(x))(M)



# Calculando Ktil
K_til = M_minus_half @ K @ M_minus_half
C_til = M_minus_half @ C @ M_minus_half

wn, P = np.linalg.eig(K_til)
wn    = np.sqrt(wn)

S     = M_minus_half @ P
S_inv = P.T@M_half

Y     = P.T@C_til@P
Y     = Y / (2*wn)
zetas = array([Y[i,i] for i in range(N)])

wds   = wn * np.sqrt(1-zetas**2)
wds   = np.real(wds)
```

[28]:
```
print('wn: ', wn/2/np.pi)
print('wd: ', wds)
print('zeta: ', zetas)
```

```python
print(' ')
print('P: \n', P)
print(' ')
print('S: \n', S)
print(' ')
print('S_inv: \n', S_inv)
```

```
wn:  [ 1.464  1.113  0.989 10.01   10.257  8.433  8.427]
wd:  [ 9.053  6.907  6.192 62.705 64.27  52.874 52.845]
zeta:  [0.177 0.158 0.089 0.077 0.074 0.066 0.063]

P:
 [[ 0.278  0.96   0.032 -0.017 -0.001  0.027  0.   ]
 [ 0.037 -0.043  0.998 -0.     0.022  0.001 -0.019]
 [ 0.959 -0.277 -0.048  0.021  0.001  0.034  0.   ]
 [-0.01   0.015  0.017  0.669 -0.67  -0.001 -0.001]
 [-0.012  0.016 -0.013  0.742  0.742 -0.001  0.001]
 [ 0.028  0.012 -0.014 -0.     0.001 -0.7   -0.713]
 [ 0.029  0.011  0.014 -0.    -0.001 -0.713  0.701]]

S:
 [[ 0.01   0.033  0.001 -0.001 -0.     0.001  0.   ]
 [ 0.001 -0.002  0.035 -0.     0.001  0.    -0.001]
 [ 0.029 -0.008 -0.001  0.001  0.     0.001  0.   ]
 [-0.001  0.002  0.002  0.092 -0.092 -0.    -0.   ]
 [-0.002  0.002 -0.002  0.102  0.102 -0.     0.   ]
 [ 0.003  0.001 -0.002 -0.     0.    -0.08  -0.082]
 [ 0.003  0.001  0.002 -0.    -0.    -0.082  0.08 ]]

S_inv:
 [[ 8.058  1.071 31.802 -0.075 -0.084  0.245  0.254]
 [27.811 -1.232 -9.182  0.109  0.12   0.103  0.093]
 [ 0.922 28.576 -1.602  0.126 -0.097 -0.12   0.118]
 [-0.495 -0.     0.701  4.873  5.405 -0.004 -0.004]
 [-0.024  0.621  0.034 -4.879  5.401  0.005 -0.006]
 [ 0.77   0.025  1.141 -0.004 -0.005 -6.103 -6.213]
 [ 0.007 -0.552  0.01  -0.007  0.006 -6.218  6.108]]
```

[29]:
```python
# condições iniciais inversas modais
r_0     = (S_inv@X0)
r_dot_0 = (S_inv@V0)

pprint(r_0)
print(' ')
pprint(r_dot_0)
```

```
[[ 0.254]
 [ 0.093]
```

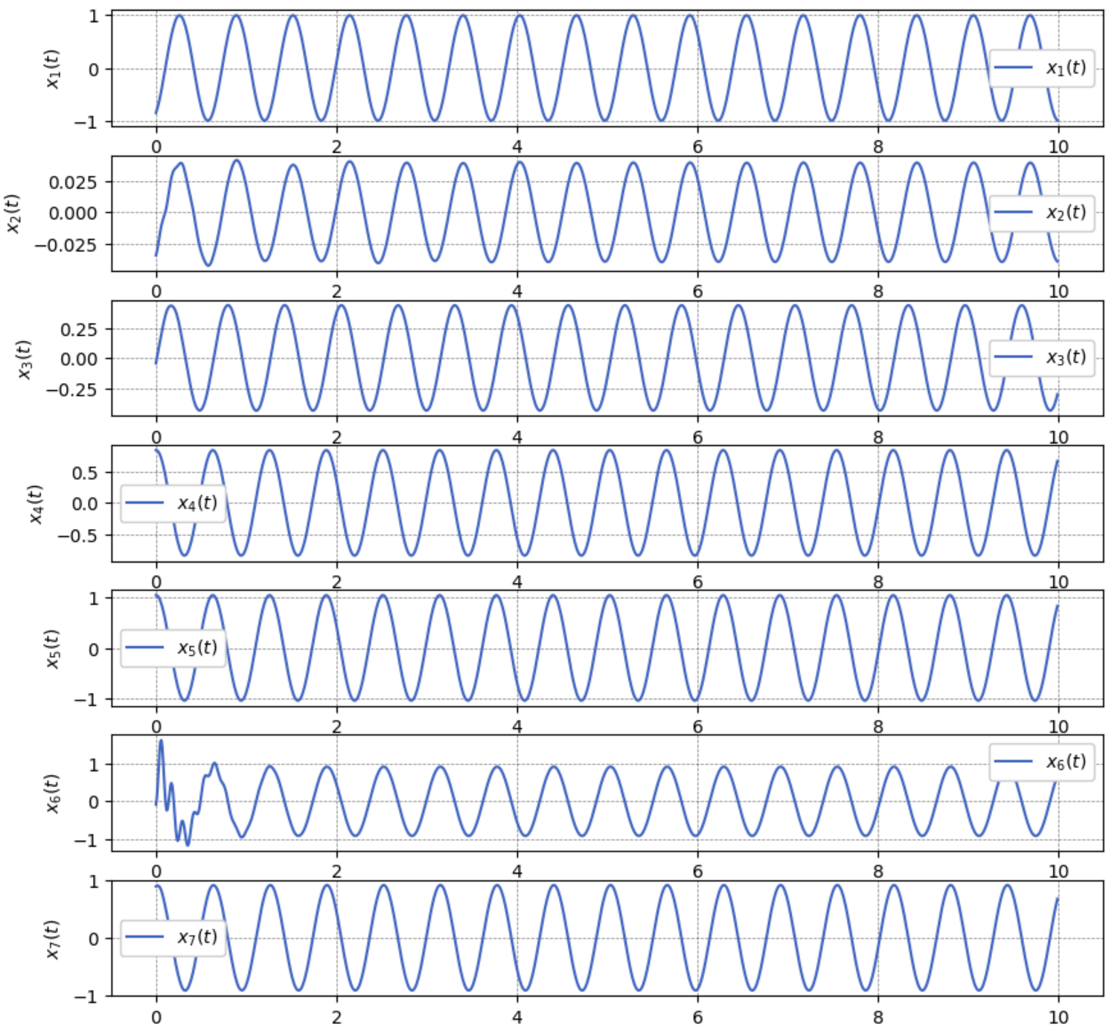
```
 [ 0.118]
 [-0.004]
 [-0.006]
 [-6.213]
 [ 6.108]]

[[0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]]
```

[30]:
```python
# Calculando as soluções modais desacopladas
t = symbols('t')

r = []

B = np.eye(N)

F = P.T @ M_minus_half @ B @ F
wf = 10

for i in range(N):
    w    = wn[i]
    wd   = wds[i]
    zeta = zetas[i]

    r_zero    = r_0    [i, 0]
    rdot_zero = r_dot_0[i, 0]

    Ai = sqrt(
        ( (r_zero * wd)**2 + (rdot_zero + zeta*w*r_zero)**2 ) / (wd**2)
    )

    phi_i = atan2(
        (r_zero*wd),(rdot_zero + zeta*w*r_zero)
    )

    if rdot_zero == 0:
        phi_i = pi/2

    expr_r = Ai*exp(-zeta*w*t)*sin(wd*t + phi_i)

    f0 = F[i]
    A0 = f0 / sqrt( (w**2 - wf**2)**2 + (2*zeta*w*wf)**2 )
```

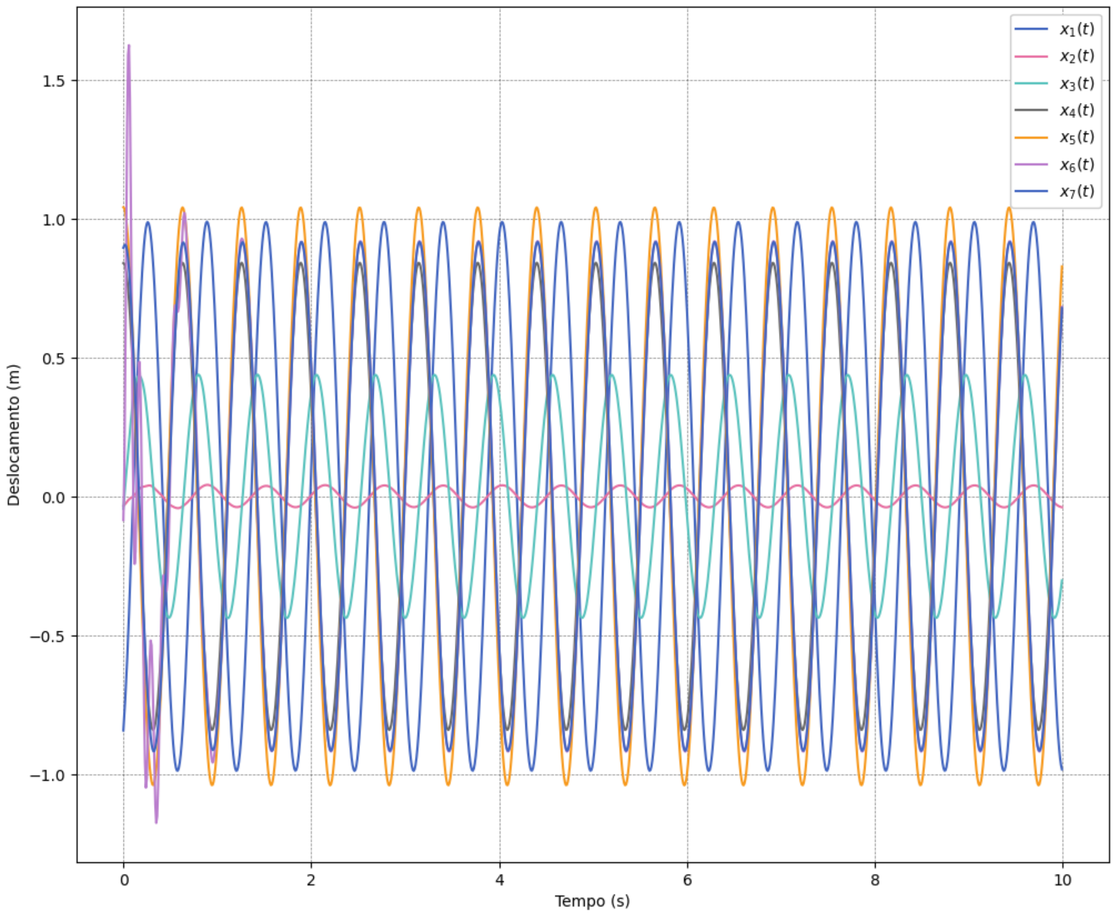
```
    theta = atan2( (2*zeta*w*wf), (w**2 - wf**2) )

    if (w**2 - wf**2) == 0:
        theta = pi/2

    expr_r += A0*cos(wf*t - theta)

    r.append(
        expr_r
    )

R = Matrix(r)
x_t = S@R
```

[31]:

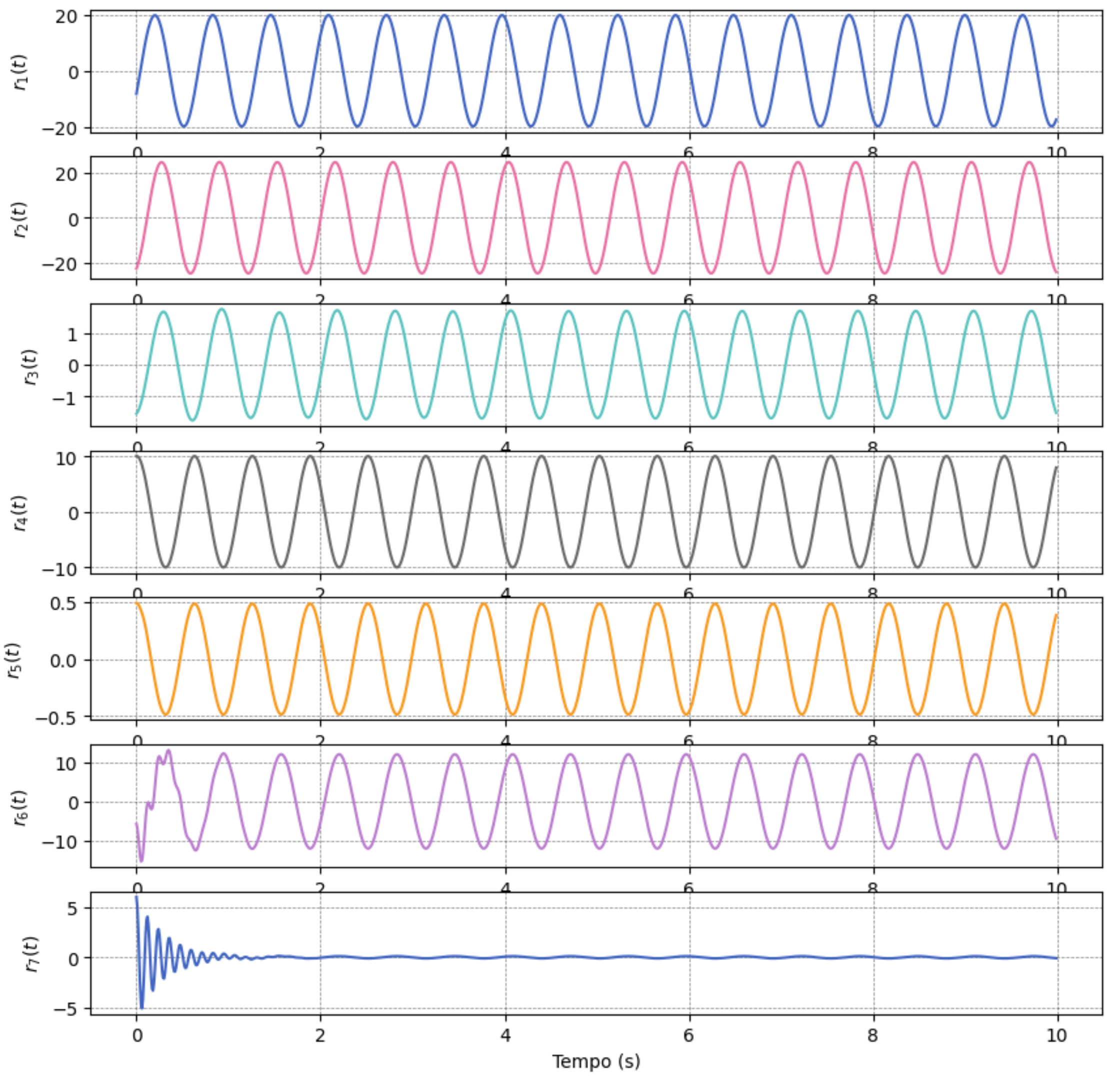
```
# Simulação

sampling_period = 1e-2
samples = 10000

time = arange(0, samples*sampling_period, sampling_period)

x = [lambdify(t, x_t[i,0], 'numpy') for i in range(N)]

xi = [x[i](time) for i in range(N)]

fig, ax = plt.subplots(N, 1, figsize=(10, 10))
plt.subplots_adjust(hspace=0.25)

idx = 1000

for i in range(N):
    ax[i].plot(time[:idx], xi[i][:idx], label=f'$x_{i+1}(t)$')
    ax[i].set_ylabel(f'$x_{i+1}(t)$')
    ax[i].legend()

plt.savefig('respsota_forcada_wf_10.png', dpi=300)
```



[32]:
```
plt.figure()

for i in range(N):
    plt.plot(time[:idx], xi[i][:idx], label=f'$x_{i+1}(t)$')

plt.xlabel('Tempo (s)')
plt.ylabel('Deslocamento (m)')

plt.legend()
plt.savefig('respsota_forcada_wf_10_unico.png', dpi=300)
```

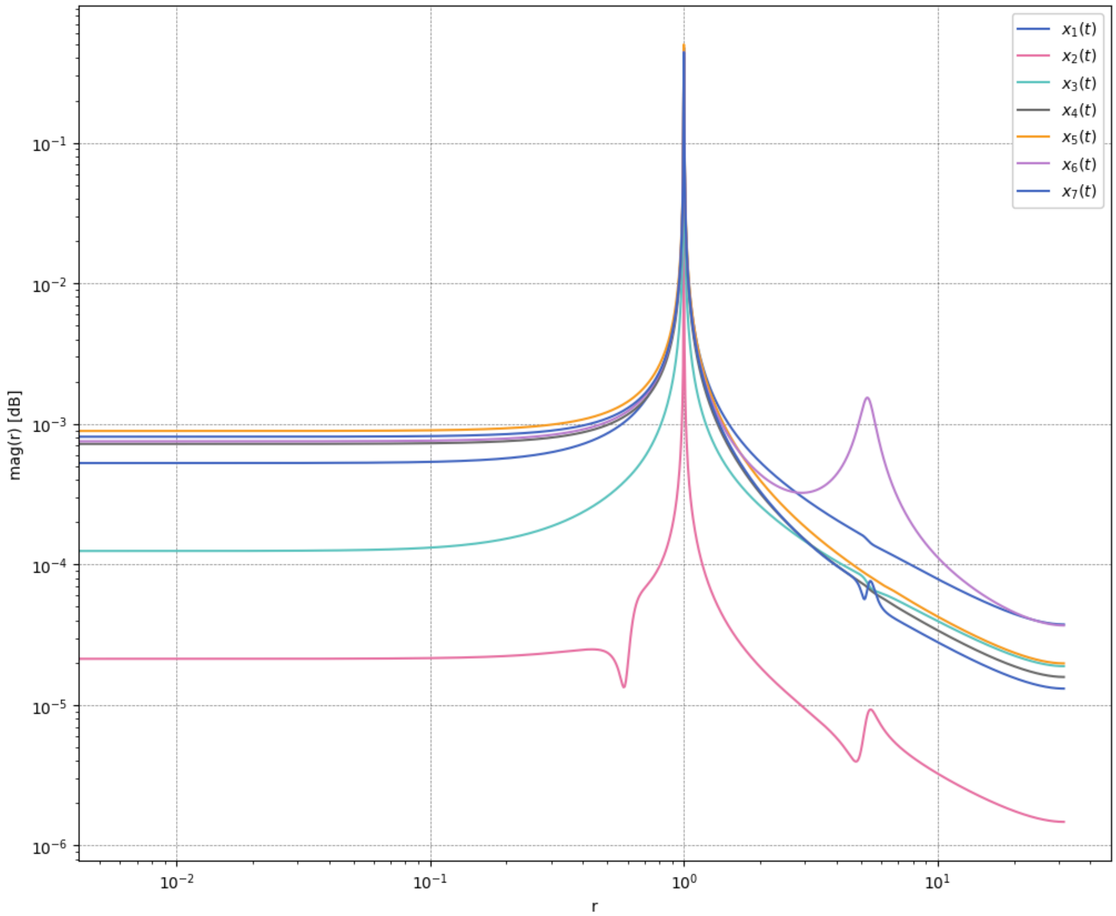


[33]:
```
# plot formas modais

fig, axs = plt.subplots(N, 1, figsize=(10, 10))

for i in range(N):
    R_lm = lambdify(t, R[i,0], 'numpy')

    r_i = R_lm(time)

    axs[i].plot(time[:idx], r_i[:idx], label=f'$r_{i+1}(t)$', color='C'+str(i))
    axs[i].set_ylabel(f'$r_{i+1}(t)$')

axs[-1].set_xlabel('Tempo (s)')
plt.savefig('formas_modais_wf_10.png', dpi=300)
```



[34]:
```
def fft(signal, sampling_time):
    fft = np.fft.fft(signal)
    N = signal.shape[0]

    f = np.fft.fftfreq(len(signal), sampling_time)

    K = N // 2

    freqs = f[:K]*2*np.pi
    amplitudes = np.abs(fft)[:K] * (1 / N)

    phase = np.rad2deg(np.angle(fft)[:K])

    return freqs, amplitudes, phase
```

```
ffts   = [fft(xi[i], sampling_period) for i in range(N)]

maximum_amp = np.max([
    np.max(ffts[i][1]) for i in range(N)
])

maximum_freq = np.max(
    [ffts[i][0][np.argmax(ffts[i][1])] for i in range(N)]
)

plt.figure()
plt.loglog()

for i in range(N):

    plt.plot(
        ffts[i][0]/maximum_freq,
        ffts[i][1],
        label=f'$x_{i+1}(t)$'
    )

plt.xlabel('r')
plt.ylabel('mag(r) [dB]')
plt.legend()
plt.savefig('fft_forcada_wf_10.png', dpi=300)
```



[35]:

```
# for each eigenvalue ( natural frequency ), plot an bar
# of the energy of the system at that frequency with the eigenvector


colors = ['C6', 'C1', 'C2', 'C3', 'C4', 'C5', 'C7', 'C8', 'C9', 'C0']

wn = sorted(wn, reverse=True)

plt.rcParams['figure.figsize'] = (12, 10)
plt.figure()
for i in range(len(wn)):
    w = wn[i]
    p = np.abs(P[:, i])**2

    for j in range(len(p)):
        plt.bar((i+1), p[j], bottom=np.sum(p[:j]), width=0.1, color=colors[j])
```

```
plt.xlabel(r'$\omega_n (H_z)$')
plt.ylabel('Energia')

# y ticks in percentage
plt.gca().yaxis.set_major_formatter(lambda x, _: f'{x*100:.0f}%')
plt.xticks(range(N, 0, -1), np.round((wn), 3))

# plt.legend([r'$m$', r'$i_x$', r'$i_y$', r'$m_f$', r'$m_r$'])
plt.savefig('energyhc.png', dpi=300)
```