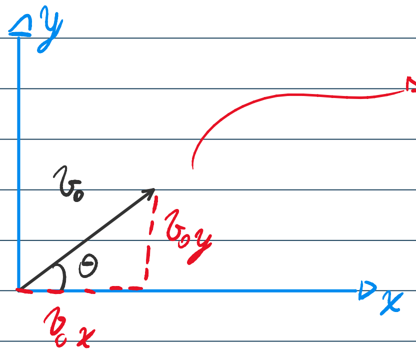
Lançamento obliquo:

$v_x(t) = v_{0x} + \overset{\to 0}{\cancel{at}}$ $\qquad$ $v_y(t) = v_{0y} - at$ $\to g = 9{,}81\ m/s^2$

$v_x(t) = v_{0x}$ $\qquad$ $y(t) = v_{0y}.t + \frac{1}{2}at^2$

$x(t) = v_{0x}.t$



$v_{0x} = v_0.\cos\theta$ $\qquad$ $v_x(t) = v_0.\cos\theta$ $\qquad$ $x(t) = (v_{0x}\cos\theta)t$

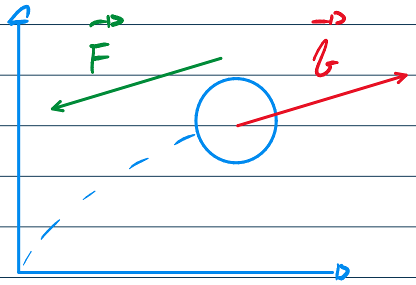
$v_{0y} = v_0.\sin\theta$ $\qquad$ $v_y(t) = (v_0.\sin\theta) - gT$

$y(t) = (v_0.\sin\theta)t + \frac{1}{2}gt^2$

considerando a força de atrito com direção contrária à velocidade e módulo em função de $|\vec{v}|$.

$\hat{v} = \frac{\vec{v}}{v}$ $\qquad$ $v = |\vec{v}|$

$\vec{F} = -\hat{v}.f(v)$



$f(v) = b_0 + b_1.v + b_2.v^2 \dots$

Para simplificações, irei considerar que:

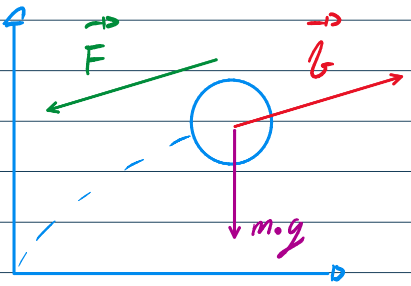
$f(v) = b.v$ $\to$ $\vec{F} = -\hat{v}.bv$ $\to$ $F = -bv$

Segunda Lei de Newton:

$\sum F_i = \frac{dQ}{dT}$ $\qquad$ $Q = m.\frac{dx}{dt}$ $\qquad$ $\frac{dQ}{dt} = m.\frac{dv}{dt}$

$\to$ constante

$\sum F_i = m.\frac{dv}{dt}$



$$m \cdot \frac{dv}{dt} = -b \cdot v - m \cdot g$$

$$m v' = -bv - mg$$

$$v' = \frac{-b}{m} v - g \quad \Big\} \text{ EDO } 1^{a} \text{ ordem}$$

Para x: $v_x' = \frac{-b_x}{m} v_x$ $\qquad v_x' + \frac{b_x}{m} v_x = 0$ $\qquad \frac{dv}{dt} + \frac{b}{m} v = 0$

Para y: $v_y' = -\frac{b_y}{m} v_y - g$ $\qquad v_y' + \frac{b_y}{m} v_y + g = 0$ $\qquad \frac{dv}{dt} + \frac{b}{m} v + g = 0$

$$u = e^{\int p(x)dx} \qquad \frac{dv_x}{dt} + \frac{b}{m} v_x(t) = 0$$

$$\frac{dv_x}{dt} = \frac{-b}{m} v_x(t) \qquad \frac{v_x'}{\frac{v_x(t)}{m}} = -b$$

$$\frac{dv_x}{dt} \cdot \frac{m}{v_x(t)} = \frac{m\, dv_x}{v_x(t)\, dt} \qquad \int \frac{m\, dv_x}{v_x(t)\cancel{dt}} \cancel{dt} = \int -b\, dt$$

$$v_x = c_1 e^{\frac{-bt}{m}}$$

$$\int \frac{m\, dv_x}{v_x(t)} = -bt$$

$$v_x^{m} = e^{-bt}$$

$$m \int \frac{dv_x}{v_x(t)} = -bt$$

$$m \cdot \ln v_x = -bt$$

$$\ln x = \frac{-bt}{m} \qquad v_x = e^{\frac{-bt}{m}}$$

$$\frac{dv_y}{dt} + \frac{b}{m} v_y + g = 0 \qquad \frac{dv_y}{dt} = \frac{-b}{m} v_y - g$$

$$z = -g - \frac{b\, v_y}{m} \qquad \frac{v_y'}{z} = \frac{z}{z} \qquad \frac{v'}{z} = 1 \qquad \int \frac{v'}{z} = t$$

$m \quad 2 \quad 2 \quad 2 \quad \int 2$

$$\int \frac{\frac{dv_y}{dt}}{\frac{-g-bv_y}{m}}\, dt = \int \frac{dv_y}{\left(-g-\frac{bv_y}{m}\right)\cancel{dt}}\,\cancel{dt}$$

$$\int \frac{dv_y}{-g-\frac{bv_y}{m}} \qquad u = -\frac{bv_y}{m} - g \qquad du = -\frac{b}{m}\cdot dv_y$$

$$\int \frac{du}{\frac{-b}{m}u} = -\frac{m}{b}\int \frac{du}{u} = -\frac{m}{b}\cdot \ln u \rightsquigarrow -\frac{m}{b}\cdot \ln\left(\frac{-b\cdot v_y}{m} - g\right)$$

$$\hookrightarrow \frac{-m\cdot \ln\left(\frac{-bv_y+gm}{m}\right)}{b}$$

$$\frac{-m\cdot \ln\left(\frac{-bv_y+gm}{m}\right)}{b} = t$$

$$-m\cdot \ln\left(bv_y+gm\right) = bT \qquad \ln\left(bv_y+gm\right) = -\frac{bT}{m}$$

$$bv_y + gm = e^{\frac{-bt}{m}}$$

$$bv_y = e^{\frac{-bt}{m}} - gm$$

$$v_y = \frac{e^{\frac{-bt}{m}}}{b} - \frac{gm}{b} \qquad v_y = C_2\, e^{\frac{-bt}{m}} - gm\cdot\frac{1}{b}$$

$$\ln\left(\frac{-b\cdot v_y}{m} - g\right) = \frac{-bt}{m}$$

$$\frac{-b}{m}\cdot v_y - g = \frac{-bt}{m}$$

$$-\frac{b}{m}\cdot v_y - g = e^{\frac{-bt}{m}}$$

$$-\frac{b}{m}\cdot v_y = e^{\frac{-bt}{m}} + g$$

$$-b\cdot v_y = m\cdot e^{\frac{-bt}{m}} + \frac{g}{m}$$

$$v_y = \frac{m}{b}\cdot e^{\frac{-bt}{m}} + \frac{g}{mb} \qquad v_y = c_2 e^{\frac{-bt}{m}} + \frac{g}{mb}$$

$$v_x = c_1 e^{\frac{-bt}{m}} \qquad v_{xo} = v_o \cos\theta$$

$$v_y = c_2 e^{\frac{-bt}{m}} - gm\cdot\frac{1}{b} \qquad v_{yo} = v_o \sin\theta$$

$$v_x(t) = c_1 e^{-\frac{b}{m}\cdot t} \qquad c_1 e^{-\frac{b}{m}\cdot 0} = v_o\cos\theta \qquad c_1 = v_o\cos\theta$$

$$v_y(t) = c_2\, e^{-\frac{b}{m}\cdot t} - \frac{gm}{b} \qquad c_2\, e^{-\frac{b}{m}\cdot 0} - \frac{gm}{b} = v_o \sin\theta \qquad c_2 = v_o\sin\theta + \frac{mg}{b}$$

$$v_x(t) = (v_o\cos\theta)\, e^{-\frac{b}{m}\cdot t}$$

$$v_y(t) = -\frac{m\cdot g}{b} + \left(v_o \sin\theta + \frac{m\cdot g}{b}\right) e^{-\frac{b}{m}\cdot t}$$

$$v_x(t) = (v_{ox})\, e^{-\frac{b}{m}\cdot t}$$

$$v_y(t) = -\frac{m\cdot g}{b} + \left(v_{oy} + \frac{m\cdot g}{b}\right) e^{-\frac{b}{m}\cdot t}$$

$$\vec{v}(t) = v_x(t)\,\hat{i} + v_y(t)\,\hat{j}$$

$$\|\vec{v}\| = \left(v_x^2 + v_y^2 \right)^{\frac{1}{2}}$$

$$v_x(t) = (v_{0x})\, e^{\frac{-b}{m}\cdot t}$$

$$v_y(t) = -\frac{m\cdot g}{b} + \left(v_{0y} + \frac{m\cdot g}{b} \right) e^{\frac{-b}{m}\cdot t}$$

$$\int (v_{0x})\, e^{\frac{-b}{m}\cdot t}\, dt = v_{0x} \int e^{\frac{-b}{m}\cdot t}\, dt = \frac{-m\cdot v_{0x}}{b}\, e^{\frac{-b}{m}\cdot t} + c_3$$

$$\int -\frac{m\cdot g}{b} + \left(v_{0y} + \frac{m\cdot g}{b} \right) e^{\frac{-b}{m}\cdot t}\, dt =$$

$$\int -\frac{m\cdot g}{b}\, dt + \int v_{0y}\, e^{\frac{-b}{m}\cdot t}\, dt + \int \frac{m\cdot g}{b}\, e^{\frac{-b}{m}\cdot t}\, dt =$$

$$-\frac{m\cdot g}{b}\, t - \frac{m\cdot v_{0y}}{b}\, e^{\frac{-b}{m}\cdot t} - \frac{m^2 g}{b^2}\, e^{\frac{-b}{m}\cdot t} + c_4$$

$$x(0) = x_0$$

$$y(0) = y_0$$

$$x_0 = \frac{-m\cdot v_{0x}}{b}\, e^{\frac{-b}{m}\cdot 0} + c_3 \rightsquigarrow x_0 = \frac{-m\cdot v_{0x}}{b} + c_3$$

$$c_3 = x_0 + \frac{m}{b}\, v_0 x$$

$$y_0 = -\frac{m\cdot g}{b}\cdot 0 - \frac{m\cdot v_{0y}}{b}\, e^{\frac{-b}{m}\cdot 0} - \frac{m^2 g}{b^2}\, e^{\frac{-b}{m}\cdot 0} + c_4$$

$$y_0 = 0 - \frac{m\cdot v_{0y}}{b} - \frac{m^2 g}{b^2} + c_4$$

$$y_0 = 0 - \frac{m \cdot v_{0y}}{b} - \frac{m g}{b^2} + C_4$$

$$C_4 = y_0 + \frac{m \cdot v_{0y}}{b} + \frac{m^2 g}{b^2}$$

$$x(t) = \frac{-m \cdot v_{0x}}{b} e^{\frac{-b}{m} \cdot t} + x_0 + \frac{m}{b} v_{0x}$$

$$y(t) = -\frac{m \cdot g}{b} t - \frac{m \cdot v_{0y}}{b} e^{\frac{-b}{m} \cdot t} - \frac{m^2 g}{b^2} e^{\frac{-b}{m} \cdot t} + y_0 + \frac{m \cdot v_{0y}}{b} + \frac{m^2 g}{b^2}$$