Notas

sábado, 21 de maio de 2022 19:12

Coeficiente de arrasto: C_D

$$C_D = \frac{2 \cdot F_D}{\rho v^2 A}$$

ρ = Densidade de massa do fluido

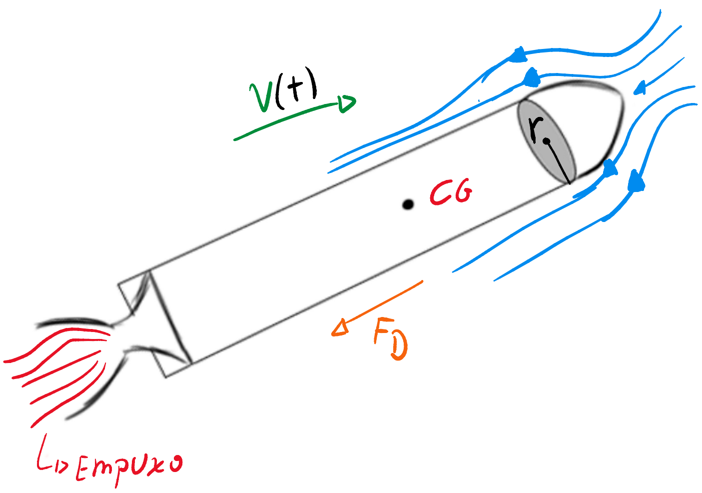
v = Velocidade do objeto em relação ao fluido.

A = Área Projetada na Direção // ao escoamento

$$F_D = (C_D \rho v^2 A) \cdot \frac{1}{2}$$

Densidade do Ar: 1,1839 kg/m^3 (25 °C)

Geometria do foguete:



CG: centro de Gravidade

Bico: Alta Pressão

Cauda: Baixa Pressão

Geometria do bico: - cone

Área " ": $\pi r (r + r_g)$

Aproximação: $F_D = \frac{1}{2}(C_D \rho v^2 A)$

$\rho = 1{,}1839\ kg/m^3$

$A = \pi r (r + r_g)$ *cone

Reação vinagre com bicarbonato:

$1CH_3COOH_{(aq)} + 1NaHCO_{3(s)} \rightarrow 1CO_{2(g)} + 1NaCH_3COO_{(aq)} + 1H_2O_{(L)}$

$n = \frac{m}{M}$

1 mol ↳ - $6{,}023 \cdot 10^{23}$
- 22,4 Litros

1	22,4
0,47	x

$CH_3COOH \rightsquigarrow$ 60,052 g/mol (vinagre)

$NaHCO_3 \rightsquigarrow$ 84,007 g/mol (Bicarbonato de sódio)

Quantidades utilizadas:

- 750 mL (vinagre) 0,75L
- 40 g (Bicarbonato de sódio) 0,04 Kg

Vinagre: 1 mL = 0,972 g $\rightsquigarrow$ 750 mL = 729 g

mols	massa (g)
1	60,052
x	729

x = 12,1394791 mols

Bicarbonato:

mols	massa (g)
1	84,007
12,1394791	x

x = 1019,8 g

O corpo tem velocidade
no sentido contrário do peso.
Velocidade inicial: V_0 (para cima) ; Peso: $-mg$
Resistência do ar: $-bv^2$;

$t \leq \frac{V_T}{g} \tan^{-1}\left(\frac{V_0}{V_T}\right) \equiv T_{max}$; $V_T = \frac{\sqrt{mg}}{\sqrt{b}}$

$$F_R = -mg - bv^2 = ma = m \cdot \dot{v} = m \frac{dv}{dy}\frac{dy}{dt} = mv \frac{dv}{dy}$$

$$mv \frac{dv}{dy} = -mg - bv^2 \rightarrow dy = \frac{-mv}{mg + bv^2} dv$$

$$\int_0^y dy' = \int_{V_0}^{V} \frac{-mv'}{mg + bv'^2} dv'$$

$$\int_{V_0}^{V} \frac{-mv'}{mg + bv'^2} dv' = -m \int_{V_0}^{V} \frac{v'}{mg + bv'^2} dv'$$

$$u = mg + bv'^2 \qquad du = 2bv' \, dv'$$

$$-m \int_{V_0}^{V} \frac{\cancel{v'}}{u} \frac{du}{2b\cancel{v'}} = \frac{m}{2b}\left[\ln(mg + bV^2) - \ln(mg + bV_0^2) \right]$$

$$y(V) = \frac{m}{2b} \ln\left(\frac{mg + bV_0^2}{mg + bV^2} \right) \rightarrow \text{Posição em função da velocidade}$$

$$y_{max} = \frac{m}{2b} \ln\left(1 + \frac{bV_0^2}{mg} \right)$$

$$m \frac{dv}{dt} = -mg - bv^2 \rightarrow \frac{-m\,dv}{mg + bv^2} = dt$$

$$\int_{V_0}^{V} \frac{-m\,dv'}{mg + bv'^2} = \int_0^T dt'$$

$$-m \int^{V} \frac{dv'}{mg + bv'^2} = T$$

$$-m\int_{V_0}^{v}\frac{dv'}{mg+bv'^2} = t$$

$$mg+bv'^2 = mg\left(\frac{bv'^2}{mg}+1\right)$$

$$-m\int_{V_0}^{V}\frac{dv'}{mg\left(\frac{bv'^2}{mg}+1\right)} = t$$

$$-\frac{1}{g}\int_{V_0}^{V}\frac{dv'}{\frac{bv'^2}{mg}+1} \qquad u=\frac{\sqrt{b}\,v'}{\sqrt{mg}} \qquad du=\left(\frac{b}{mg}\right)^{\frac{1}{2}}dv'$$

$$-\frac{\sqrt{m}}{\sqrt{bg}}\int_{V_0}^{V}\frac{du}{u^2+1} = \tan^{-1}(u)$$

$$\frac{\sqrt{m}}{\sqrt{bg}}\left[\tan^{-1}\left(\frac{\sqrt{b}\,v^2}{\sqrt{mg}}\right)-\tan^{-1}\left(\frac{\sqrt{b}\,V_0^2}{\sqrt{mg}}\right)\right] = t$$

$$v(t) = V_t\tan\left[\tan^{-1}\left(\frac{V_0}{V_t}\right)-\frac{g}{V_t}\cdot t\right]$$

integrando para encontrar $y(t)$

$$\int_0^t V_t\tan\left[\tan^{-1}\left(\frac{V_0}{V_t}\right)-\frac{g}{V_t}\cdot t'\right]dt' = \int_0^t -V_t\tan\left[\frac{g}{V_t}\cdot t'-\tan^{-1}\left(\frac{V_0}{V_t}\right)\right]dt'$$

$$\int\tan x\,dx = -\ln|\cos x|+c \qquad -V_t\int^t\tan\left[\frac{g}{\ }\cdot t'-\tan^{-1}\left(\frac{V_0}{V}\right)\right]dt'$$

$$\int \tan x \, dx = -\ln|\cos x| + c \qquad -V_t \int_0^t \tan\left[\frac{g}{V_t} \cdot t' - \tan^{-1}\left(\frac{V_0}{V_t}\right)\right] dt'$$

$$u = \frac{g}{V_t} \cdot t' - \tan^{-1}\left(\frac{V_0}{V_t}\right) \qquad du = \frac{g}{V_t} \, dv'$$

$$-V_t \int_0^t \tan u \, \frac{du \, V_t}{g} \;\rightarrow\; \cancel{-}\frac{V_t^2}{g} \cancel{-}\ln|\cos u|$$

$$y(t) = y(0) + \frac{V_t^2}{g}\left[\frac{\cos(\tan^{-1}(V_0 / V_t))}{\cos(\tan^{-1}(V_0 / V_t)) - \frac{g t}{V_t}}\right]$$

Essas soluções são válidas para $t \leq t_{max}$

$v(t) > 0$.

Para o caso do corpo caindo, tem-se:

$F = -mg + bv^2$, pois a velocidade está na mesma direção do peso.

Logo, integraremos de: $v_0 = 0$ até V, pois o objeto começa a cair quando $v(t) = 0$.

$$dy = \frac{-V/g}{1 - \left(\frac{v}{V_t}\right)^2} \, dv$$

$$\int^y dy' = \int^v \frac{-v'}{\quad} \, dv'$$

$$\int_{y_{max}}^{y} dy' = \int_0^v \frac{-v'}{g\left(1-\left(\frac{v'}{v_T}\right)^2\right)} dv'$$

$$\frac{-1}{g}\int_0^v \frac{v'}{1-\left(\frac{v'}{v_T}\right)^2} dv' \quad \rightsquigarrow \quad u = 1-\frac{v'^2}{v_T^2} \qquad du = \frac{-2v'}{v_T^2} dv'$$

$$\frac{-1}{g}\int_0^v \frac{\cancel{v'}}{u}\frac{du}{-2\cancel{v'}\cdot v_T^2} = \frac{v_T^2}{2g} \ln(u)\Big|_0^v$$

$$y - y_{max} = \frac{v_T^2}{2g}\ln\left(1-\left(\frac{v}{v_T}\right)^2\right)$$

$$dt = \frac{m\,dv}{bv^2 - mg} \qquad v_t = \left(\frac{mg}{b}\right)^{\frac{1}{2}}$$

$$dt = \frac{dv}{g\left(1-\left(\frac{v}{v_t}\right)^2\right)}$$

$$\int_{t_{max}}^{T} dt' = \int_0^v \frac{dv'}{g\left(1-\left(\frac{v}{v_t}\right)^2\right)}$$

$$\int^v \frac{dv'}{1-\left(\frac{v'}{v_T}\right)^2} \qquad x = \frac{v'}{v_T}$$

$$\int_0 \frac{dv'}{1-\left(\frac{v'}{v_t}\right)^2} \qquad x = \frac{v}{v_t}$$

$$\int_0^{\frac{v}{v_t}} \frac{dx'}{1-x'^2} \qquad \int \frac{dx}{1-x^2} \qquad \frac{dx}{1-x^2} = (1-x)(1+x)$$

$$\int \frac{dx}{1-x^2} = \int \frac{1}{(1-x)(1+x)} = \left(\frac{1}{(1-x)} + \frac{1}{(1+x)}\right)\frac{1}{2}$$

$$\to \frac{V_t}{2}\left[\ln\left|\frac{1+\frac{V}{V_t}}{1-\frac{V}{V_t}}\right|\right]$$

$$\frac{V_t}{2g}\left[\ln\left|\frac{1+\frac{V}{V_t}}{1-\frac{V}{V_t}}\right|\right] = t - t_{max}$$

$$0 > V > -V_t$$

$$\to V = V_t \frac{1-\exp(2(t-t_{max})\frac{g}{V_t})}{1+\exp(2(t-t_{max})\frac{g}{V_t})}$$

$$= -V_t \tanh\left((t-t_{max})\frac{g}{V_t}\right) \qquad y_{max} = \tanh\left(-t_{max}\cdot\frac{g}{V_t}\right)$$

$$\int_0^t -V_t \tanh\left((t'-t_{max})\frac{g}{V_t}\right)dt' = -V_t\int_0^t \tanh\left((t'-t_{max})\frac{g}{V_t}\right)dt'$$

$$\int \tanh x\, dx = \ln(\cosh x) + c \qquad \begin{cases} u = t' - t_{max} \\ du = dt' \end{cases}$$

$$\int \tanh x\, dx = \ln(\cosh x) + c \quad \begin{cases} u = t' - t_{max} \\ du = dt' \end{cases}$$

$$-V_t \int \tanh\left(\frac{u g}{V_t}\right) \quad s = \frac{u g}{V_t}$$

$$ds = \frac{g}{V_t}\, du$$

$$-\frac{V_t^2}{g}\int \tanh s \; ds = -\frac{V_t^2}{g} \ln\left(\cosh\left(\frac{g}{V_t}(t - t_{max})\right)\right)$$

$$y(t) = y_{max} - \frac{V_t^2}{g} \ln\left(\cosh\left(\frac{g}{V_t}(t - t_{max})\right)\right)$$

Para x:

$$\frac{dV}{dt} = \frac{-b\, V(t)^2}{m} \quad (1)$$

Velocidade terminal

$$-b V_t^2 = 0$$

$$V_t = \sqrt{b}$$

$$\frac{dV}{V(t)^2} = \frac{-b}{m}\, dt$$

$$\int_{V_0}^{V} \frac{dV'}{V'(t)^2} = \int_0^t \frac{-b}{m}\, dt'$$

$$\int_{V_0}^{V} \frac{dV'}{V'(t)^2} = \frac{-b}{m}\, t' \Big|_0^t$$

$$-1 \Big|^{V} = -b \quad +$$

v_0

$$\left.\frac{-1}{v'}\right|_{v_0}^{v} = \frac{-b}{m} t$$

$$\frac{1}{v_0} - \frac{1}{v} = \frac{-b}{m} t$$

$$\frac{1}{v} = \frac{b}{m} t + \frac{1}{v_0} = \frac{1 + \frac{b v_0 t}{m}}{v_0} = \frac{1 + t/\tau}{v_0}$$

$$\frac{m}{b v_0} = \tau \qquad v(t) = \frac{v_0}{1 + t/\tau} = \frac{dx}{dt}$$

$$\int_0^t \frac{v_0}{1 + t'/\tau} dt' = \int_{x_0}^{x} dx'$$

$$x - x_0 = \tau \int_0^t \frac{\frac{1}{\tau} v_0}{1 + t'/\tau}$$

$$x - x_0 = v_0 \tau \ln\left(1 + \frac{t'}{\tau}\right)\Big|_0^t$$

$$x - x_0 = v_0 \tau \ln\left(1 + \frac{t}{\tau}\right)$$

$$x = v_0 \tau \ln\left(1 + \frac{t}{\tau}\right) + x_0$$