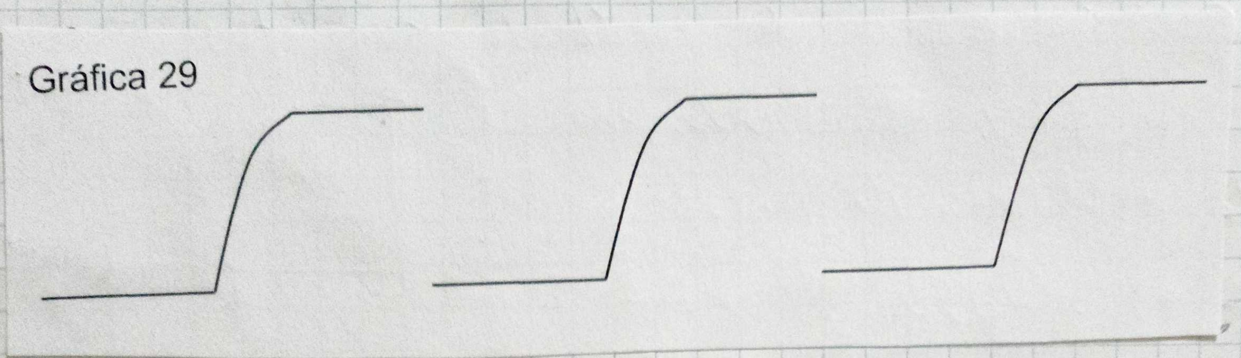
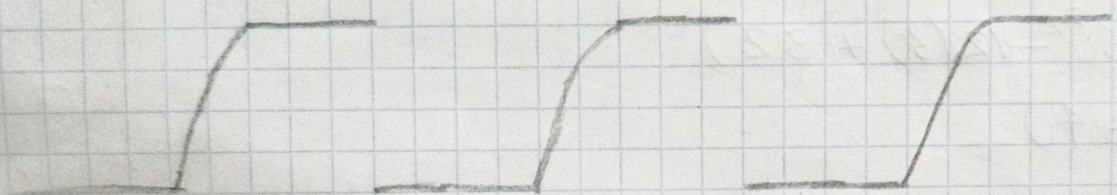
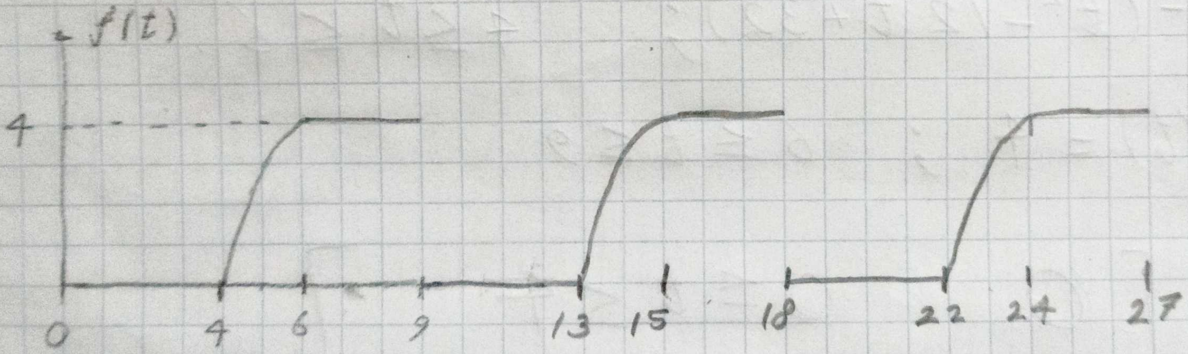
1)

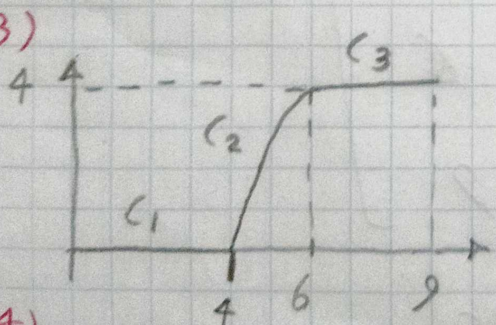


2)





3)

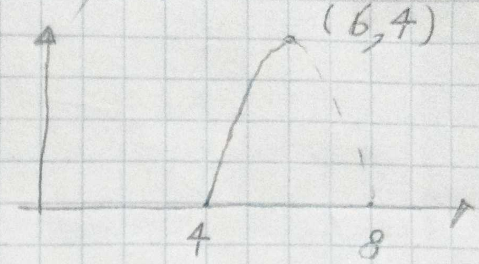


4)

c_1: recta horizontal con ordenada en cero.

$f(t) = 0$ para $0 \le t \le 4$

4) C_2: imaginando la parábola completa: raíces en 4 y 8. Siendo una parábola negativa. Por tanto:



$y = -(x-4)(x-8)a$ $\quad x = 4$

$\quad x = 8$

$y = -a(x^2 - 12x + 32)$

Usamos el vértice de la parábola para hallar la constante a:

$4 = -a(6^2 - 12(6) + 32)$

$4 = -a(-4)$

$a = 1$

$f(t) = -(t^2 - 12t + 32); \quad 4 \leq t \leq 6$

C_3: $f(t) = 4 \quad ; \quad 6 \leq t \leq 9$

5)
$$f(t) = \begin{bmatrix} 0 & 0 \leq t \leq 4 \\ -(t^2 - 12t + 32) & 4 \leq t \leq 6 \\ 4 & 6 \leq t \leq 9 \end{bmatrix}$$

La función no es par ni impar

Periodo: 9

6) $$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos n w_0 t + b_n \operatorname{sen} n w_0 t\right]$$

donde :

$$w_0 = \frac{2\pi}{T} \qquad a_0 = \frac{2}{T}\int_0^T f(t)\,dt \qquad a_n = \frac{2}{T}\int_0^T f(t)\cos n w_0 t\,dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\operatorname{sen} n w_0 t\,dt$$

7)

$$a_0 = \frac{2}{9}\int_0^9 f(t)\,dt = \frac{2}{9}\left[\int_0^4 0\,dt - \int_4^6 (t^2 - 12t + 32)\,dt + \int_6^9 4\,dt\right]$$

$$a_0 = \frac{2}{9}\left[0 + \left[\frac{t^3}{3} - \frac{12}{2}t^2 + 32t\right]_6^4 + 4t\Big|_6^9\right]$$

$$a_0 = \frac{2}{9}\left[\left(\frac{160}{3} - 48\right) + 12\right] = \frac{104}{27}$$

$$a_n = \cancel{\frac{2}{T}\int_0^4 0 \cos n\omega_0 t \, dt} - \frac{2}{T}\int_4^6 (t^2 - 12t + 32)\cos n\omega_0 t \, dt$$

$$+ \frac{2}{T}\int_6^9 4\cos n\omega_0 t \, dt$$

$$= \frac{2}{9}\int_6^4 t^2 \cos n\omega_0 t \, dt + \frac{2}{9}\int_6^4 -12t \cos n\omega_0 t \, dt + \frac{2}{9}\int_6^4 32\cos n\omega_0 t \, dt$$

$$+ \frac{2}{9}\int_6^9 4\cos n\omega_0 t \, dt$$

$$\frac{2}{9}\int_6^4 t^2 \cos n\omega_0 t \, dt = \frac{2}{9}\frac{1}{(n\omega_0)^3}\left[\left[(n\omega_0 t)^2 - 2\right]\operatorname{sen} n\omega_0 t + 2n\omega_0 t \cos n\omega_0 t\right]_6^4$$

$$\omega_0 = \frac{2\pi}{9} \qquad \frac{1}{\omega_0} = \frac{9}{2\pi} \qquad 4\omega_0 = \frac{8\pi}{9} \qquad 6\omega_0 = \frac{4}{3}\pi$$

$$= \frac{2}{9}\frac{1}{(n\omega_0)^3}\left[\left[\left(n\frac{8\pi}{9}\right)^2 - 2\right)\operatorname{sen}\frac{8\pi}{9}n + \frac{16\pi}{9}n\cos\frac{8\pi}{9}n\right.$$

$$\left. - \left[\left(n\frac{4}{3}\pi\right)^2 - 2\right]\operatorname{sen}\frac{4}{3}\pi n - \frac{8}{3}\pi n \cos\frac{4}{3}\pi n\right]$$

$$-\frac{24}{9}\int_6^4 t\cos n\omega_0 t\,dt = \frac{8}{3}\int_4^6 t\cos n\omega_0 t\,dt$$

$$= \frac{8}{3}\frac{1}{n\omega_0}\left[t\,\mathrm{sen}\,n\omega_0 t + \frac{1}{n\omega_0}\cos n\omega_0 t\right]\Big|_4^6$$

$\omega_0 = \frac{2\pi}{9}$ $\quad 6\omega_0 = \frac{4}{3}\pi$ $\quad 4\omega_0 = \frac{8}{9}\pi$ $\quad 9\omega_0 = 2\pi$

$$= \frac{8}{3}\frac{1}{n\omega_0}\left[6\,\mathrm{sen}\frac{4}{3}\pi n + \frac{1}{n\omega_0}\cos\frac{4}{3}\pi n - 4\,\mathrm{sen}\frac{8}{9}\pi n - \frac{1}{n\omega_0}\cos\frac{8}{9}\pi n\right]$$

$$\frac{2}{9}\int_6^4 32\cos n\omega_0 t\,dt = \frac{64}{9}\int_6^4 \cos n\omega_0 t\,dt$$

$$= \frac{64}{9}\frac{1}{n\omega_0}\left[\mathrm{sen}\,n\omega_0 t\right]_6^4 = \frac{64}{9}\frac{1}{n\omega_0}\left[\mathrm{sen}\frac{8}{9}\pi n - \mathrm{sen}\frac{4}{3}\pi n\right]$$

$$\frac{2}{9}\int_6^9 4\cos n\omega_0 t\,dt = \frac{8}{9}\int_6^9 \cos n\omega_0 t\,dt = \frac{8}{9}\frac{1}{n\omega_0}\left[\mathrm{sen}\,n\omega_0 t\right]_6^9$$

$$= \frac{8}{9}\frac{1}{n\omega_0}\left[\cancel{\mathrm{sen}\,2\pi n}^{\,0} - \mathrm{sen}\frac{4}{3}\pi\right]$$

$$= -\frac{8}{9}\frac{1}{n\omega_0}\,\mathrm{sen}\frac{4}{3}\pi n$$

$$a_n = \operatorname{sen}\frac{4}{3}\pi n\left[-\frac{2}{9}\frac{\left[\left(n\frac{4}{3}\pi\right)^2-2\right]}{(nw_0)^3}+\frac{16}{nw_0}-\frac{64}{9}\frac{1}{nw_0}\right.$$

$$\left.-\frac{8}{9}\frac{1}{nw_0}\right]+\operatorname{sen}\frac{8}{9}\pi n\left[\frac{2}{9}\frac{\left[\left(n\frac{8}{9}\pi\right)^2-2\right]}{(nw_0)^3}-\frac{32}{3}\frac{1}{nw_0}\right.$$

$$\left.+\frac{64}{9}\frac{1}{nw_0}\right]+\cos\frac{4}{3}\pi n\left[-\frac{16}{27}\frac{\pi n}{(nw_0)^3}+\frac{8}{3}\frac{1}{(nw_0)^2}\right]$$

$$+\cos\frac{8}{9}\pi n\left[\frac{32}{81}\frac{\pi n}{(nw_0)^3}-\frac{8}{3}\frac{1}{(nw_0)^2}\right]$$

$$a_n = \left[\frac{8}{nw_0}-\frac{2}{9}\frac{\left[\left(n\frac{4}{3}\pi\right)^2-2\right]}{(nw_0)^3}\right]\operatorname{sen}\frac{4}{3}\pi n$$

$$+\left[\frac{2}{9}\frac{\left[\left(\frac{8}{9}\pi n\right)^2-2\right]}{(nw_0)^3}-\frac{32}{9}\frac{1}{nw_0}\right]\operatorname{sen}\frac{8}{9}\pi n$$

$$+\frac{8}{3}\frac{1}{(nw_0)^2}\left[1-\frac{2}{9}\frac{\pi n}{nw_0}\right]\cos\frac{4}{3}\pi n$$

$$+\frac{8}{3}\frac{1}{(nw_0)^2}\left[\frac{4}{27}\frac{\pi n}{nw_0}-1\right]\cos\frac{8}{9}\pi n$$

Reduciendo la ecuación:

$$a_n = \frac{8}{3}\frac{1}{(nw_0)^2}\left[\frac{9nw_0-2\pi n}{9nw_0}\cos\frac{4}{3}\pi n+\frac{4\pi n-27nw_0}{27nw_0}\cos\frac{8}{9}\pi n\right]$$

$$+\frac{2}{9nw_0}\left[\left(36-\frac{\left[\left(\frac{4}{3}\pi n\right)^2-2\right]}{(nw_0)^2}\right)\operatorname{sen}\frac{4}{3}\pi n+\left(\frac{\left[\left(\frac{8}{9}\pi n\right)^2-2\right]}{(nw_0)^2}-16\right)\operatorname{sen}\frac{8}{9}\pi n\right]$$

$$a_n = \frac{8}{27(nw_0)^3}\left[n\left(9\frac{2\pi}{9}-2\pi\right)\cos\frac{4}{3}\pi n+\frac{n}{3}(4\pi-6\pi)\cos\frac{8}{9}\pi n\right]$$

$$+\frac{2}{9(nw_0)^3}\left[\left(36(nw_0)^2-\left[\left(\frac{4}{3}\pi n\right)^2-2\right]\right)\operatorname{sen}\frac{4}{3}\pi n+\left(\left[\left(\frac{8}{9}\pi n\right)^2-2\right]-16(nw_0)^2\right)\operatorname{sen}\frac{8}{9}\pi n\right]$$

$$a_n = \frac{2}{9(n\omega_0)^3}\left[\frac{4}{3}\cdot\frac{n}{3}(-2\pi)\cos\frac{8}{9}\pi n\right.$$

$$\left. + \left(\cancel{\frac{16}{9}\pi^2 n^2} - \cancel{\frac{16}{9}\pi^2 n^2} + 2\right)\operatorname{sen}\frac{4}{3}\pi n + \left(\cancel{\frac{64}{81}\pi^2 n^2} - 2 - \cancel{\frac{64}{81}\pi^2 n^2}\right)\operatorname{sen}\frac{8}{9}\pi n\right]$$

$$a_n = \frac{2}{9(n\omega_0)^3}\left[-\frac{8}{9}\pi n\cos\frac{8}{9}\pi n + 2\operatorname{sen}\frac{4}{3}\pi n - 2\operatorname{sen}\frac{8}{9}\pi n\right]$$

$$\omega_0^3 = \left(\frac{2\pi}{9}\right)^3 = \frac{8\pi^3}{729} \qquad \frac{2\cdot 729}{9\,n^3\,8\pi^3} = \frac{81}{4\pi^3 n^3}$$

$$a_n = \frac{81}{4\pi^3 n^3}\left[-\frac{8}{9}\pi n\cos\frac{8}{9}\pi n + 2\left(\operatorname{sen}\frac{4}{3}\pi n - \operatorname{sen}\frac{8}{9}\pi n\right)\right]$$

$$b_n = \frac{2}{T}\cancel{\int_{9}^{4} 0 \,\text{sen}\, n w_0 t \, dt} - \frac{2}{T}\int_{4}^{6} (t^2 - 12t + 32)\,\text{sen}\, n w_0 t \, dt$$

$$+ \frac{2}{T}\int_{6} 4 \,\text{sen}\, n w_0 t \, dt$$

$$= \frac{2}{9}\int_{6}^{4} t^2 \,\text{sen}\, n w_0 t \, dt - \frac{24}{9}\int_{6}^{4} t \,\text{sen}\, n w_0 t \, dt + \frac{64}{9}\int_{6}^{4} \text{sen}\, n w_0 t \, dt$$

$$+ \frac{8}{9}\int_{6}^{9} \text{sen}\, n w_0 t \, dt$$

$$\frac{2}{9}\int_{6}^{4} t^2 \,\text{sen}\, n w_0 t \, dt = -\frac{2}{9}\frac{1}{(n w_0)^3}\Big[\left[(n w_0 t)^2 - 2\right]\cos n w_0 t$$

$$- 2 n w_0 t \,\text{sen}\, n w_0 t\Big]\Big|_{6}^{4}$$

$$w_0 = \frac{2\pi}{9} \qquad 4 w_0 = \frac{8}{9}\pi$$

$$6 w_0 = \frac{4}{3}\pi$$

$$= -\frac{2}{9}\frac{1}{(n w_0)^3}\Big[\left[\left(\frac{8}{9}\pi n\right)^2 - 2\right]\cos \frac{8}{9}\pi n - \frac{16}{9}\pi n \,\text{sen}\, \frac{8}{9}\pi n$$

$$- \left[\left(\frac{4}{3}\pi n\right)^2 - 2\right]\cos \frac{4}{3}\pi n + \frac{8}{3}\pi n \,\text{sen}\, \frac{4}{3}\pi n\Big]$$

$$-\frac{24}{9}\int_6^4 t \operatorname{sen} n\omega_0 t \, dt = \frac{8}{3}\int_4^6 t \operatorname{sen} n\omega_0 t \, dt$$

$$\omega_0 = \frac{2\pi}{9} \qquad 6\omega_0 = \frac{4}{3}\pi \qquad 4\omega_0 = \frac{8}{9}\pi$$

$$\frac{8}{3}\int_4^6 t \operatorname{sen} n\omega_0 t \, dt = -\frac{8}{3}\frac{1}{n\omega_0}\left[t\cos n\omega_0 t - \frac{1}{n\omega_0}\operatorname{sen} n\omega_0 t\right]\Bigg|_4^6$$

$$= -\frac{8}{3}\frac{1}{n\omega_0}\left[6\cos\frac{4}{3}\pi n - \frac{1}{n\omega_0}\operatorname{sen}\frac{4}{3}\pi n - 4\cos\frac{8}{9}\pi n + \frac{1}{n\omega_0}\operatorname{sen}\frac{8}{9}\pi n\right]$$

$$= -\frac{16}{n\omega_0}\cos\frac{4}{3}\pi n + \frac{8}{3(n\omega_0)^2}\operatorname{sen}\frac{4}{3}\pi n + \frac{32}{3}\frac{1}{n\omega_0}\cos\frac{8}{9}\pi n - \frac{8}{3(n\omega_0)^2}\operatorname{sen}\frac{8}{9}\pi n$$

$$\frac{64}{9}\int_6^4 \operatorname{sen} n\omega_0 t \, dt = -\frac{64}{9}\frac{1}{n\omega_0}\Big[\cos n\omega_0 t\Big]_6^4$$

$$= -\frac{64}{9}\frac{1}{n\omega_0}\left[\cos\frac{8}{9}\pi n - \cos\frac{4}{3}\pi n\right]$$

$$= \frac{64}{9}\frac{1}{n\omega_0}\left[\cos\frac{4}{3}\pi n - \cos\frac{8}{9}\pi n\right]$$

$$\frac{8}{9}\int_{6}^{9} \text{sen}\, n\omega_0 t\, dt = -\frac{8}{9}\frac{1}{n\omega_0}\Big[\cos n\omega_0 t\Big]_{6}^{9}$$

$$= -\frac{8}{9}\frac{1}{n\omega_0}\left[\cos 2\pi n - \cos\frac{4}{3}\pi n\right]$$

$$= \frac{8}{9}\frac{1}{n\omega_0}\left[\cos\frac{4}{3}\pi n - 1\right]$$

$$b_n = \text{sen}\frac{4}{3}\pi n\left[-\frac{16}{27}\frac{\pi n}{(n\omega_0)^3} + \frac{8}{3(n\omega_0)^2}\right]$$

$$\text{sen}\frac{8}{9}\pi n\left[\frac{32}{81}\frac{\pi n}{(n\omega_0)^3} - \frac{8}{3(n\omega_0)^2}\right]$$

$$+\cos\frac{4}{3}\pi n\left[\frac{2}{9}\frac{\left[\left(\frac{4}{3}\pi n\right)^2-2\right]}{(n\omega_0)^3} - \frac{16}{n\omega_0} + \frac{64}{9}\frac{1}{n\omega_0} + \frac{8}{9}\frac{1}{n\omega_0}\right]$$

$$+\cos\frac{8}{9}\pi n\left[-\frac{2}{9}\frac{\left[\left(\frac{8}{9}\pi n\right)^2-2\right]}{(n\omega_0)^3} + \frac{32}{3(n\omega_0)} - \frac{64}{9(n\omega_0)}\right]$$

$$-\frac{8}{9(n\omega_0)}$$

$$b_n = \frac{8}{3}\frac{1}{(n\omega_0)^2}\left[1 - \frac{2}{9}\frac{\pi n}{n\omega_0}\right]\text{sen}\frac{4}{3}\pi n + \frac{8}{3}\frac{1}{(n\omega_0)^2}\left[\frac{4\, n\pi}{27\, n\omega_0} - 1\right]\text{sen}\frac{8}{9}\pi n$$

$$+\left[\frac{2}{9}\frac{\left[\left(\frac{4}{3}\pi n\right)^2-2\right]}{(n\omega_0)^3} - 8\frac{1}{n\omega_0}\right]\cos\frac{4}{3}\pi n$$

$$-\left[\frac{2}{9}\frac{\left[\left(\frac{8}{9}\pi n\right)^2-2\right]}{(n\omega_0)^3} - \frac{32}{9}\frac{1}{n\omega_0}\right]\cos\frac{8}{9}\pi n - \frac{8}{9}\frac{1}{n\omega_0}$$

$$b_n = \frac{8}{3(nw_0)^2}\left[\frac{1}{9nw_0}\left[\left(9nw_0 - 2\pi n\right)\text{sen}\,\frac{4}{3}\pi n + \left(\frac{4\pi}{3} - \frac{27}{3}\left(\frac{2\pi}{9}\right)\right)\text{sen}\,\frac{8}{9}\pi n\right]\right]$$

$$+ \frac{2}{9(nw_0)^3}\left[\left(\frac{4}{3}\pi n\right)^2 - 2 - 36\left(\frac{2\pi}{9}n\right)^2\right]\cos\frac{4}{3}\pi n$$

$$- \frac{2}{9(nw_0)^3}\left[\left(\frac{8}{9}\pi n\right)^2 - 2 - 16\left(\frac{2\pi}{9}n\right)^2\right]\cos\frac{8}{9}\pi n - \frac{8}{9}\frac{1}{nw_0}$$

$$b_n = \frac{8}{27(nw_0)^3}\left[-\frac{2}{3}\pi\,\text{sen}\,\frac{8}{9}\pi n\right] - \frac{4}{9(nw_0)^3}\cos\frac{4}{3}\pi n$$

$$+ \frac{4}{9(nw_0)^3}\cos\frac{8}{9}\pi n - \frac{8}{9}\frac{1}{nw_0}$$

$$b_n = \frac{4}{9(nw_0)^3}\left[\cos\frac{8}{9}\pi n - \cos\frac{4}{3}\pi n - \frac{4}{9}\pi n\,\text{sen}\,\frac{8}{9}\pi n\right] - \frac{8}{9}\frac{1}{nw_0}$$

Simulación de la gráfica por series de Fourier

Función propuesta: $\begin{bmatrix} 0 & 0 \le t \le 4 \\ -\left(t^2 - 12\ t + 32\right) & 4 \le t \le 6 \\ 4 & 6 \le t \le 9 \end{bmatrix}$

$t_1 := 0, 0.1 .. 4$

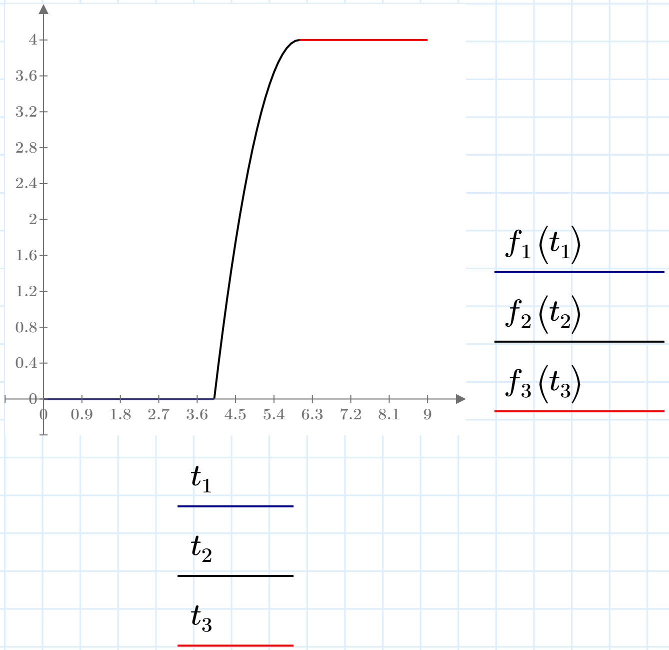
$t_2 := 4, 4.1 .. 6$

$t_3 := 6, 6.1 .. 9$

$f_1(t_1) := 0$

$f_2(t_2) := -\left({t_2}^2 - 12\ t_2 + 32\right)$

$f_3(t_3) := 4$



$T := 9 \qquad k := 10000 \qquad d_1 := 2 \qquad d_2 := 3 \qquad r := 0.01$

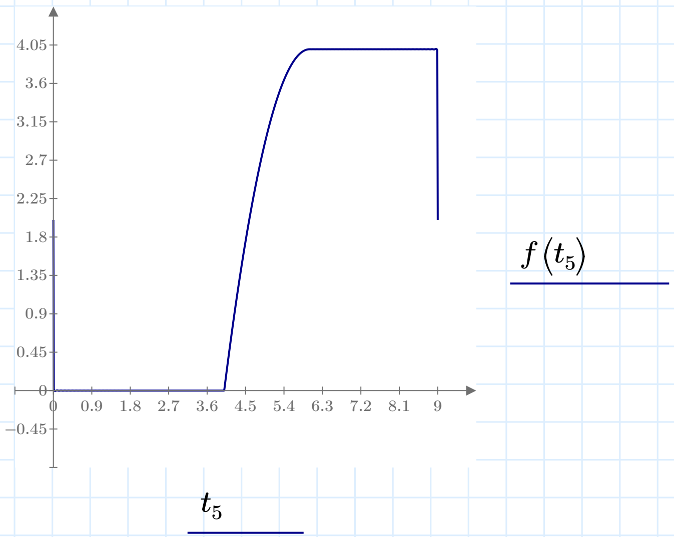
$t_4 := -d_1 \cdot T, -d_1 \cdot T + r .. d_2 \cdot T \qquad t_5 := 0, r .. T$

$w := \frac{2\ \pi}{T} \qquad a_0 := \frac{104}{27} \qquad n := 1, 2 .. k$

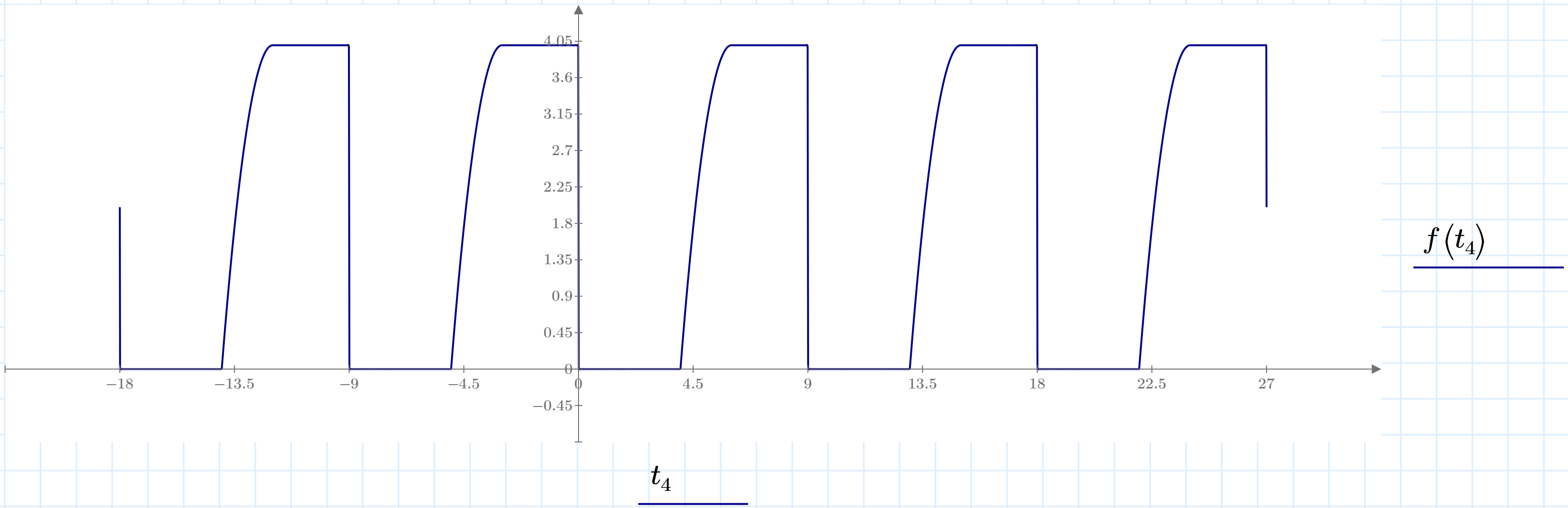
$$a_n(n) := \frac{81}{4 \cdot \pi^3 \cdot n^3} \left(\frac{-8}{9}\ \pi \cdot n \cdot \cos\left(\frac{8}{9}\ \pi \cdot n\right) + 2\ \left(\sin\left(\frac{4}{3} \cdot \pi \cdot n\right) - \sin\left(\frac{8}{9} \cdot \pi \cdot n\right)\right)\right)$$

$$b_n(n) := \frac{4}{9 \cdot (n \cdot w)^3} \left(\cos\left(\frac{8}{9}\ \pi \cdot n\right) - \cos\left(\frac{4}{3}\ \pi \cdot n\right) - \frac{4}{9}\ \pi \cdot n \cdot \sin\left(\frac{8}{9}\ \pi \cdot n\right)\right) - \frac{8}{9 \cdot n \cdot w}$$

$$f(t) := \frac{a_0}{2} + \sum_{n=1}^{k} \left(a_n(n) \cdot \cos(n \cdot w \cdot t) + b_n(n) \cdot \sin(n \cdot w \cdot t)\right)$$



4.05
3.6
3.15
2.7
2.25
1.8
1.35
0.9
0.45
0
−0.45
0
0.9
1.8
2.7
3.6
4.5
5.4
6.3
7.2
8.1
9
$f\left(t_5\right)$
t_5



4.05
3.6
3.15
2.7
2.25
1.8
1.35
0.9
0.45
0
−0.45
−18
−13.5
−9
−4.5
0
4.5
9
13.5
18
22.5
27
$f\left(t_4\right)$
t_4

$m := 1, 2 .. 15$

$$c(m) := \frac{1}{2} \cdot \sqrt[2]{\left(a_n(|m|)\right)^2 + \left(b_n(|m|)\right)^2} \qquad \phi(m) := \operatorname{atan}\left(\frac{-b_n(|m|)}{a_n(|m|)}\right)$$

$\omega(m) := m \cdot w$