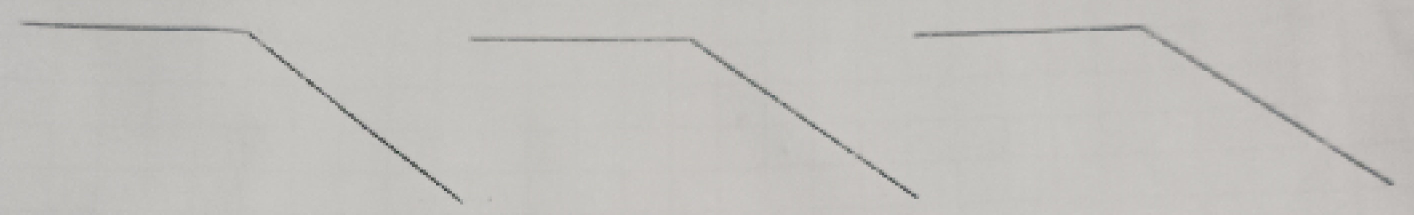
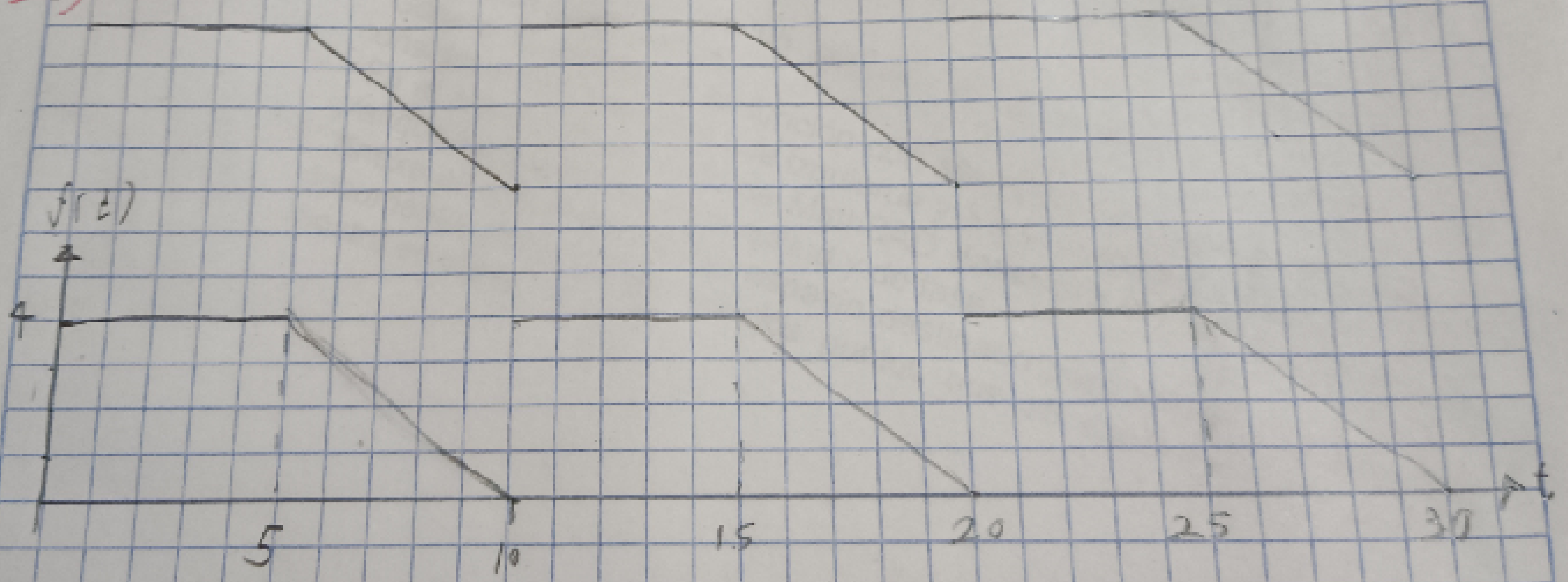
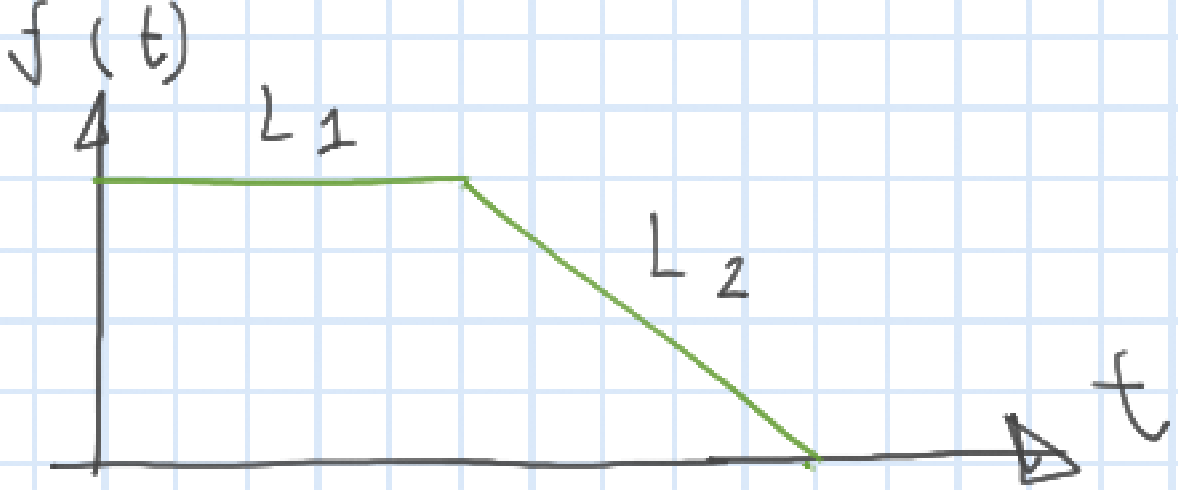
1)

Gráfica 1



2)





Para L_1:

Recta horizontal que cruza eje y en la ordenada 4

Para L_2:

$$m = -\frac{4}{5} \quad \therefore \quad y = -\frac{4}{5}x + b$$

$$0 = -\frac{4}{5}(10) + b$$

$$b = 8 \quad \therefore \quad \underline{y = -\frac{4}{5}x + 8}$$

$$f(t) = \begin{cases} 4 & 0 \leq t \leq 5 \\ -\frac{4}{5}t + 8 & 5 \leq t \leq 10 \end{cases}$$

Periodo = 10 segundos

La función no es par ni impar.

Calculando coeficientes:

$$a_0 : \frac{2}{T}\int_0^T f(t)dt \quad \therefore \quad a_0 = \frac{2}{10}\left[\int_0^5 4\,dt + \int_5^{10}\left[-\frac{4}{5}t+8\right]dt\right]$$

$$a_0 = \frac{1}{5}\left[4t\Big|_0^5 - \frac{4}{5}\frac{1}{2}t^2\Big|_5^{10} + 8t\Big|_5^{10}\right] = \frac{1}{5}\left[20 - \frac{2}{5}\left[10^2-5^2\right] + 8\left[10-5\right]\right]$$

$$= \frac{1}{5}\left[20 - 30 + 40\right] = 6$$

$$a_0 = 6$$

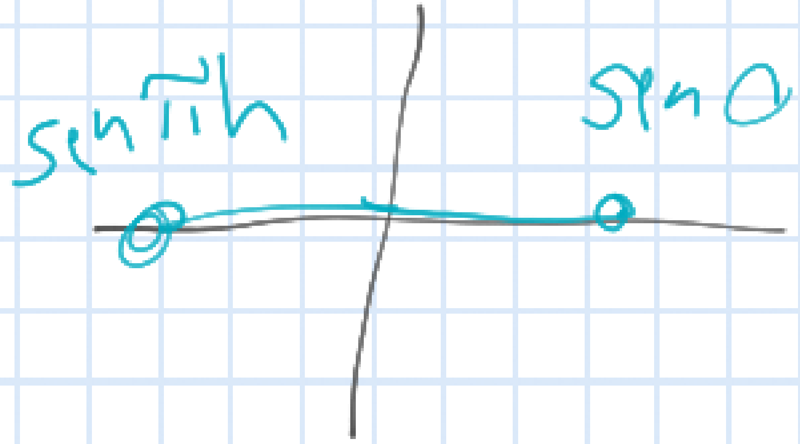
$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\,dt$$

donde $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{10} = \frac{\pi}{5}$

$$a_n = \frac{1}{5}\int_0^5 4\cos n\omega_0 t\,dt + \frac{1}{5}\int_5^{10}\left(-\frac{4}{5}t+8\right)\cos n\omega_0 t\,dt$$

$$= \frac{4}{5}\int_0^5 \cos n\omega_0 t\,dt - \frac{4}{25}\int_5^{10} t\cos n\omega_0 t\,dt + \frac{8}{5}\int_5^{10}\cos n\omega_0 t\,dt$$

$$\frac{4}{5}\int_0^5 \cos n\omega_0 t\,dt = \frac{4}{5}\,\frac{1}{n\omega_0}\,\operatorname{sen}\omega_0 n t\,\Big|_0^5 = \frac{4}{5n}\,\frac{5}{\pi}\left[\operatorname{sen}\pi n - \operatorname{sen} 0\right]$$



$$5\omega_0 = \frac{\pi}{5}\cdot 5 = \pi$$

$$\frac{4}{5}\int_0^5 \cos n\omega_0 t\,dt = 0$$

Podemos concluir $\displaystyle \frac{8}{5}\int_5^{10}\cos n\omega_0 t\,dt = 0$

$$a_n = -\frac{4}{25}\int_5^{10} t\cos n w_0 t\, dt = \frac{4}{25}\int_{10}^{5} t\cos n w_0 t = \frac{4}{25}\,\frac{5}{n\pi}\left[t\,\mathrm{sen}\,\frac{\pi}{5}nt + \frac{5}{\pi n}\cos\frac{\pi}{5}nt\right]\Bigg|_{10}^{5}$$

$$a_n = \frac{4}{5n\pi}\left[\frac{5}{\pi n}\cos\pi n - \frac{5}{\pi n}\cos 2\pi n\right] = \frac{4}{5n\pi}\,\frac{5}{n\pi}\left[\cos\pi n - \cos 2\pi n\right]$$

$$a_n = \frac{4}{(n\pi)^2}\left[\cos\pi n - 1\right]$$

$$\int At\cos Kt\, dt = \frac{A}{K}\left[t\,\mathrm{sen}Kt + \frac{1}{K}\cos Kt\right] + C$$

$$b_n = \frac{2}{T}\int_0^T f(t)\,\mathrm{Sen}\,\omega_0 n t\,dt$$

$$b_n = \frac{4}{5}\int_0^5 \mathrm{sen}\,\omega_0 n t\,dt + \frac{4}{25}\int_{10}^{5} t\,\mathrm{Sen}\,\omega_0 n t\,dt + \frac{8}{5}\int_5^{10} \mathrm{sen}\,\omega_0 n t\,dt$$

$$\frac{4}{5}\int_0^5 \mathrm{sen}\,\omega_0 n t\,dt = -\frac{4}{5}\,\frac{5}{n\pi}\left[\cos n\frac{\pi}{5}t\right]\Big|_0^5 = -\frac{4}{n\pi}\left[\cos n\pi - 1\right] = \frac{4}{n\pi}\left[1 - \cos\pi n\right]$$

$$\frac{8}{5}\int_5^{10} \mathrm{sen}\,\frac{\pi}{5}nt\,dt = -\frac{8}{5}\,\frac{5}{n\pi}\left[\cos\frac{\pi}{5}nt\right]\Big|_5^{10} = -\frac{8}{n\pi}\left[\cos 2\pi n - \cos\pi n\right] = \frac{8}{n\pi}\left[\cos\pi n - 1\right]$$

$$b_n = \frac{4}{n\pi}\left[\cos\pi n - 1\right] + \frac{4}{25}\int_{10}^{5} t\,\mathrm{Sen}\,\omega_0 n t\,dt$$

$$\frac{4}{25}\int_{10}^{5} t \operatorname{sen} w_0 n t \, dt = -\frac{4}{25}\frac{5}{n\pi}\left[t \cos\frac{\pi}{5}nt - \frac{5}{n\pi}\operatorname{sen}\frac{\pi}{5}nt\right]\Bigg|_{10}^{5}$$

$$= -\frac{4}{5n\pi}\left[5\cos\pi n - 10\cos 2\pi n\right]$$

$$= \frac{4}{5\pi n}\left[10 - 5\cos\pi n\right] = \frac{4}{n\pi}\left[2 - \cos\pi n\right]$$

$$\int A t \operatorname{sen} Kt \, dt = -\frac{A}{K}\left[t\cos Kt - \frac{1}{K}\operatorname{sen} Kt\right] + C$$

$$b_n = \frac{4}{n\pi}\left[\cos \pi n - 1\right] + \frac{4}{n\pi}\left[2 - \cos \pi n\right] = \frac{4}{\pi n}\left[\cos \pi n - \cos \pi n + 2 - 1\right]$$

$$b_n = \frac{4}{\pi n}$$

Resumen:

$$a_0 = 6$$

$$b_n = \frac{4}{\pi n}$$

$$a_n = \left[\frac{2}{n\pi}\right]^2 \left[\cos \pi n - 1\right]$$

$$T = 10$$

$$w_0 = \frac{\pi}{5}$$

Simulación de la gráfica por series de Fourier

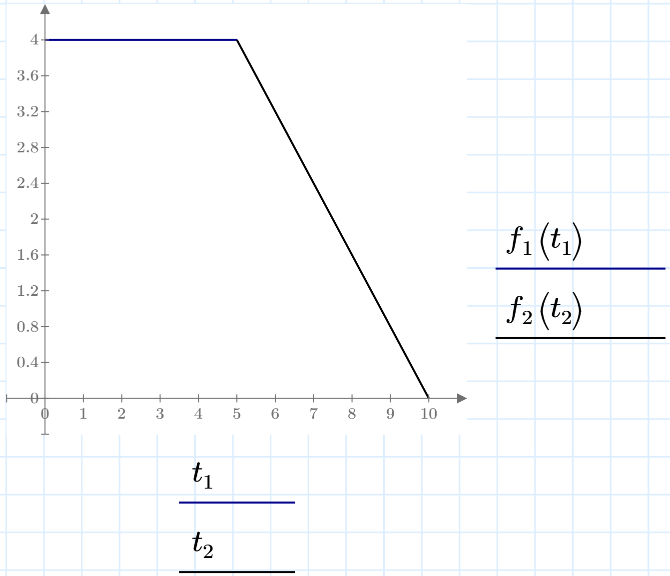
Función propuesta: $\begin{bmatrix} 4 & 0 \le t \le 5 \\ \dfrac{-4}{5}\ t+8 & 5 \le t \le 10 \end{bmatrix}$

$t_1 := 0, 0.1 .. 5$

$t_2 := 5, 5.1 .. 10$

$f_1(t_1) := 4$

$f_2(t_2) := \dfrac{-4}{5}\ t_2 + 8$



$T := 10$ $\quad k := 10000$ $\quad d_1 := 2$ $\quad d_2 := 3$ $\quad r := 0.01$

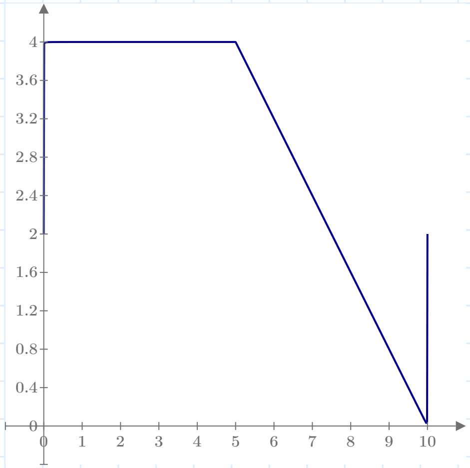
$t_4 := -d_1 \cdot T, -d_1 \cdot T + r .. d_2 \cdot T$ $\qquad t_5 := 0, r .. T$

$w := \dfrac{2\,\pi}{T}$ $\qquad a_0 := 6$ $\qquad n := 1, 2 .. k$

$$a_n(n) := \left(\frac{2}{n \cdot \pi}\right)^2 \cdot (\cos(\pi \cdot n) - 1)$$

$$b_n(n) := \frac{4}{n \cdot \pi}$$

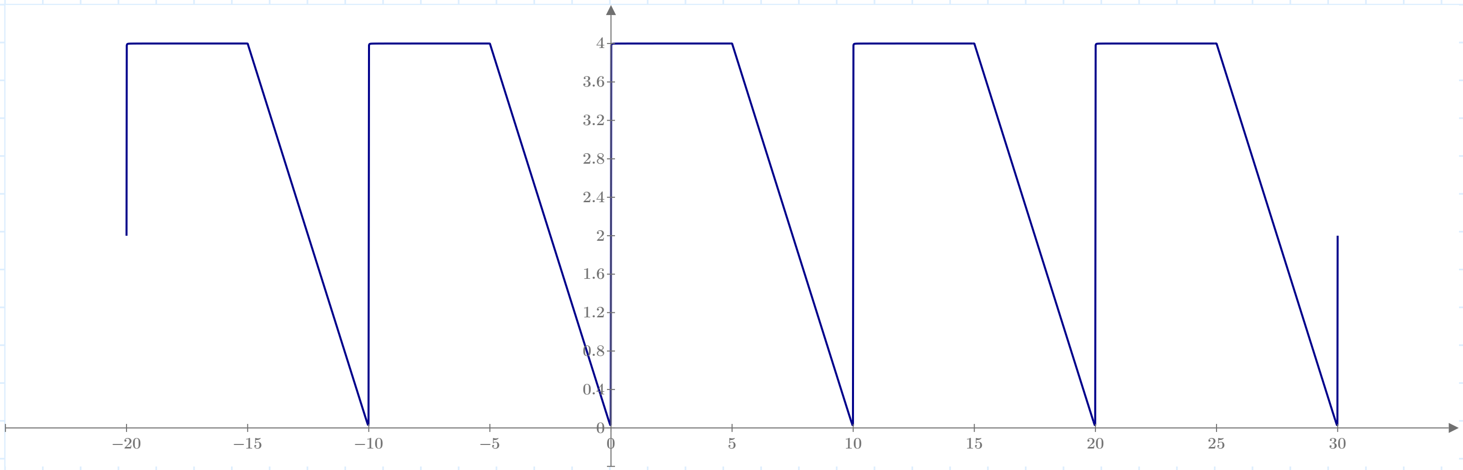
$$f(t) := \frac{a_0}{2} + \sum_{n=1}^{k} \left(a_n(n) \cdot \cos(n \cdot w \cdot t) + b_n(n) \cdot \sin(n \cdot w \cdot t)\right)$$



4
3.6
3.2
2.8
2.4
2
1.6
1.2
0.8
0.4
0
0 1 2 3 4 5 6 7 8 9 10

$f(t_5)$

t_5



4
3.6
3.2
2.8
2.4
2
1.6
1.2
0.8
0.4
0
−20 −15 −10 −5 0 5 10 15 20 25 30

$f(t_4)$

t_4

$$m := 1, 2 .. 15$$

$$c(m) := \frac{1}{2} \cdot \sqrt[2]{\left(a_n(|m|)\right)^2 + \left(b_n(|m|)\right)^2}$$

$$\phi(m) := \operatorname{atan}\left(\frac{-b_n(|m|)}{a_n(|m|)}\right)$$

$$\omega(m) := m \cdot w$$