



SOLUÇÕES DOS EXERCÍCIOS PROPOSTOS DA FICHA DE EXERCÍCIOS 2

1. (a) $x^3 + \frac{5}{2}x^2 + 7x + c, \quad c \in \mathbb{R}$
(b) $\frac{3}{4}\sqrt[3]{x^4} + c, \quad c \in \mathbb{R}$
(c) $\frac{x^7}{7} + \frac{x^4}{2} + x + c, \quad c \in \mathbb{R}$
(d) $\frac{(\operatorname{arctg} x)^2}{2} + c, \quad c \in \mathbb{R}$
(e) $\ln |1 + x^3| + c, \quad c \in \mathbb{R}$
(f) $-\frac{1}{6x^6} + c, \quad c \in \mathbb{R}$
(g) $\frac{1}{8} \ln(2 + 4x^2) + \frac{\sqrt{2}}{4} \operatorname{arctg}(\sqrt{2}x) + c, \quad c \in \mathbb{R}$
(h) $\operatorname{sen} x^4 + c, \quad c \in \mathbb{R}$
(i) $-\sqrt{1 - x^2} + c, \quad c \in \mathbb{R}$
(j) $-\frac{\cos^6 x}{6} + c, \quad c \in \mathbb{R}$
(k) $-\ln |\cos x| + c, \quad c \in \mathbb{R}$
(l) $\frac{(\ln x)^2}{2} + c, \quad c \in \mathbb{R}$
(m) $e^{\operatorname{tg} x} + c, \quad c \in \mathbb{R}$
(n) $\frac{1}{2 \ln 7} 7^{x^2} + c, \quad c \in \mathbb{R}$
(o) $-\frac{\sqrt{2}}{2} \cos(\sqrt{2}x) + c, \quad c \in \mathbb{R}$
(p) $\frac{x^2}{2} + \ln |x| + c, \quad c \in \mathbb{R}$
(q) $-\frac{1}{5\sqrt{7+5x^2}} + c, \quad c \in \mathbb{R}$
(r) $\frac{1}{4} \operatorname{arctg}(x^4) + c, \quad c \in \mathbb{R}$
(s) $\frac{5}{3} \operatorname{arcsen}(x^3) + c, \quad c \in \mathbb{R}$
(t) $\frac{\sqrt{7}}{7} \operatorname{arctg}\left(\frac{x}{\sqrt{7}}\right) + c, \quad c \in \mathbb{R}$
2. $F(x) = 2 \ln |x| - \frac{3}{x} - 2$
3. $\frac{\pi}{8}(\sqrt{2} - 2)$
4. $F(x) = -\frac{1}{x} + x - \frac{3}{2}$
5. $g(x) = \operatorname{arctg}(\operatorname{arctg} x) - \operatorname{arctg}(\pi/2)$
6. (a) $x \operatorname{sen} x + \cos x + c, \quad c \in \mathbb{R}$
(b) $x^2 \operatorname{sen} x + 2x \cos x - 2 \operatorname{sen} x + c, \quad c \in \mathbb{R}$
(c) $-\frac{2x+3}{3} e^{-3x} - \frac{2}{9} e^{-3x} + c, \quad c \in \mathbb{R}$
(d) $x(\ln^2 x - 2 \ln x + 2) + c, \quad c \in \mathbb{R}$
(e) $\frac{-e^{2x} \cos x + 2e^{2x} \operatorname{sen} x}{5} + c, \quad c \in \mathbb{R}$
(f) $\frac{x \operatorname{sen}(\ln x) - x \cos(\ln x)}{2} + c, \quad c \in \mathbb{R}$
(g) $x \operatorname{arcsen} x + \sqrt{1 - x^2} + c, \quad c \in \mathbb{R}$
(h) $\frac{x^2}{2} \operatorname{arcsen}(x^2) + \frac{1}{2} \sqrt{1 - x^4} + c, \quad c \in \mathbb{R}$

- (i) $x \operatorname{arctg} x - \frac{1}{2} \ln(1+x^2) + c, \quad c \in \mathbb{R}$
(j) $x \operatorname{arctg} \frac{1}{x} + \frac{1}{2} \ln(1+x^2) + c, \quad c \in \mathbb{R}$
(k) $\frac{2}{3} \sqrt{x^3} \ln x - \frac{4}{9} \sqrt{x^3} + c, \quad c \in \mathbb{R}$
(l) $\frac{\operatorname{sen}^2 x}{2} + c, \quad c \in \mathbb{R}$
7. (a) $-\ln |\operatorname{cosec} x + \cotg x| + c, \quad c \in \mathbb{R}$
(b) $\frac{\operatorname{tg}^4 x}{4} + c, \quad c \in \mathbb{R}$
(c) $-\cotg x - x + c, \quad c \in \mathbb{R}$
(d) $\frac{1}{2} \theta + \frac{1}{4} \operatorname{sen}(2\theta) + c, \quad c \in \mathbb{R}$
(e) $\frac{1}{2} x - \frac{1}{4} \operatorname{sen}(2x) + c, \quad c \in \mathbb{R}$
(f) $-\cos t + \frac{1}{3} \cos^3 t + c, \quad c \in \mathbb{R}$
(g) $\frac{\operatorname{tg}^3 x}{3} + \operatorname{tg} x - x + c, \quad c \in \mathbb{R}$
(h) $-\frac{1}{3} \cos(3x) + \frac{1}{5} \operatorname{sen}(5x) + c, \quad c \in \mathbb{R}$
(i) $\frac{\operatorname{tg}^2 x}{2} + c, \quad c \in \mathbb{R}$
(j) $-\frac{\cos^3 x}{3} + \frac{2}{5} \cos^5 x - \frac{\cos^7 x}{7} + c, \quad c \in \mathbb{R}$
(k) $\frac{1}{16} x - \frac{1}{64} \operatorname{sen}(4x) + \frac{1}{48} \operatorname{sen}^3(2x) + c, \quad c \in \mathbb{R}$
(l) $\frac{1}{12} \operatorname{sen}(6x) + \frac{1}{8} \operatorname{sen}(4x) + c, \quad c \in \mathbb{R}$
(m) $\operatorname{sen}(\ln x) + c, \quad c \in \mathbb{R}$
(n) $-\frac{\cos(x^6)}{6} + c, \quad c \in \mathbb{R}$
(o) $-\frac{1}{2} (\arccos x)^2 + \sqrt{1-x^2} + c, \quad c \in \mathbb{R}$
(p) $\frac{1}{2} \operatorname{sen}(\ln(x^2)) + c, \quad c \in \mathbb{R}$
8. (a) $\frac{3}{7} \ln|x-1| + \frac{4}{7} |x+6| + c, \quad c \in \mathbb{R}$
(b) $\frac{1}{8} \ln|x-1| - \frac{1}{8} \ln|x+1| + \frac{1}{4(x+1)} + \frac{1}{4(x+1)^2} + c, \quad c \in \mathbb{R}$
(c) $\frac{1}{12} \ln|x+2| - \frac{1}{24} \ln(x^2-2x+4) + \frac{\sqrt{3}}{12} \operatorname{arctg}\left(\frac{x-1}{\sqrt{3}}\right) + c, \quad c \in \mathbb{R}$
(d) $\frac{x^3}{3} + 5x + 8 \ln\left|\frac{x-3}{x+3}\right| + c, \quad c \in \mathbb{R}$
(e) $-\frac{3}{4} \ln|x| - \frac{1}{4x} + \frac{13}{16} \ln|x-2| + \frac{15}{16} \ln|x+2| + c, \quad c \in \mathbb{R}$
(f) $\frac{1}{4} (4x + \ln|x-1| - \ln|x+1| - 2 \operatorname{arctg} x) + c, \quad c \in \mathbb{R}$
(g) $\ln|x| - \frac{1}{2} \ln(1+x^2) + \frac{1}{2(x^2+1)} + c, \quad c \in \mathbb{R}$
(h) $\frac{1}{2} \ln(x^2+4x+5) - \operatorname{arctg}(x+2) + c, \quad c \in \mathbb{R}$
9. (a) $-\frac{2}{3} (1-x) \sqrt{1-x} - \frac{2}{7} (1-x)^3 \sqrt{1-x} + \frac{4}{5} (1-x)^2 \sqrt{1-x} + c, \quad c \in \mathbb{R}$
(b) $\frac{6}{7} x \sqrt[6]{x} - \frac{6}{5} \sqrt[6]{x^5} + 2\sqrt{x} - 6\sqrt[6]{x} + 6 \operatorname{arctg} \sqrt[6]{x} + c, \quad c \in \mathbb{R}$
(c) $\frac{1}{48} (2x+5)^{12} - \frac{5}{44} (2x+5)^{11} + c, \quad c \in \mathbb{R}$
(d) $-\frac{\sqrt{9-x^2}}{9x} + c, \quad c \in \mathbb{R}$
(e) $\arccos \frac{1}{x} + c, \quad c \in \mathbb{R}$
(f) $-\frac{1}{2} \ln \left| \frac{\sqrt{x^2+4}}{x} + \frac{2}{x} \right| + c, \quad c \in \mathbb{R}$
(g) $\frac{3\sqrt{2}}{4} \operatorname{arcsen}\left(\sqrt{\frac{2}{3}} x\right) + \frac{1}{2} x \sqrt{3-2x^2} + c, \quad c \in \mathbb{R}$
(h) $2 \operatorname{arcsen} \frac{x+1}{\sqrt{2}} - \frac{(x+1) \sqrt{2-(x+1)^2}}{2} + 2 \sqrt{2-(x+1)^2} + c, \quad c \in \mathbb{R}$
(i) $\frac{\sqrt{x^2-7}}{7x} + c, \quad c \in \mathbb{R}$

- (j) $\frac{3}{2}\sqrt[3]{2x+3} - 3\sqrt[6]{2x+3} + 3\ln(\sqrt[6]{2x+3} + 1) + c, \quad c \in \mathbb{R}$
- (k) $2e^{\sqrt{x}}(\sqrt{x} - 1) + c, \quad c \in \mathbb{R}$
- (l) $\frac{2}{3}(\sqrt{1+\ln x})^3 - 2\sqrt{1+\ln x} + c, \quad c \in \mathbb{R}$
10. (a) $-\sqrt{3-x^2} + \arcsen \frac{x}{\sqrt{3}} + c, \quad c \in \mathbb{R}$
- (b) $\frac{3}{8}x - \frac{1}{4}\text{sen}(2x) + \frac{1}{32}\text{sen}(4x) + c, \quad c \in \mathbb{R}$
- (c) $\frac{1}{2}\text{arctg}\left(\frac{x+1}{2}\right) + c, \quad c \in \mathbb{R}$
- (d) $\ln \left| \sqrt{\frac{2+x^2}{2}} + \frac{x}{\sqrt{2}} \right| + c, \quad c \in \mathbb{R}$
- (e) $-2\cos \sqrt{x} + c, \quad c \in \mathbb{R}$
- (f) $3\ln|x-3| - 2\ln|x-2| + c, \quad c \in \mathbb{R}$
- (g) $\arcsen(x-1) + c, \quad c \in \mathbb{R}$
- (h) $\frac{(1+x^2)^2\sqrt{1+x^2}}{5} + c, \quad c \in \mathbb{R}$
- (i) $x - 2\sqrt{x} + 2\ln(1+\sqrt{x}) + c, \quad c \in \mathbb{R}$
- (j) $\frac{x^2}{2}\ln x - \frac{x^2}{4} + c, \quad c \in \mathbb{R}$
- (k) $\frac{1}{4}x - \frac{1}{8}\ln(e^{2x} + 4) + \frac{1}{2}\text{arctg} \frac{e^x}{2} + c, \quad c \in \mathbb{R}$
- (l) $\frac{x^2+1}{2}\text{arctg} x - \frac{1}{2}x + c, \quad c \in \mathbb{R}$
- (m) $-\frac{1}{2(1-\cos x)^2} + c, \quad c \in \mathbb{R}$
- (n) $(\frac{2}{3}x^3 + 3x)\text{arctg} x - \frac{1}{3}x^2 - \frac{7}{6}\ln(1+x^2) + c, \quad c \in \mathbb{R}$
- (o) $\ln \left| \frac{x+1+\sqrt{(x+1)^2-4}}{2} \right| + c, \quad c \in \mathbb{R}$
- (p) $2\sqrt{1+e^x} + \ln|\sqrt{1+e^x} - 1| - \ln(\sqrt{1+e^x} + 1) + c, \quad c \in \mathbb{R}$
- (q) $2\text{arctg} \sqrt{e^x - 1} + c, \quad c \in \mathbb{R}$
- (r) $-2\sqrt{\cos x} + \frac{2}{5}\sqrt{\cos^5 x} + c, \quad c \in \mathbb{R}$
- (s) $\frac{1}{2}\ln(\ln^2 x + 1) + c, \quad c \in \mathbb{R}$
- (t) $\frac{1}{2}e^{x^2}(x^2 - 1) + c, \quad c \in \mathbb{R}$
- (u) $-\ln|x-2| + \frac{5}{4}\ln|x-3| - \frac{1}{4}\ln|x+1| + c, \quad c \in \mathbb{R}$
- (v) $2\sqrt{\text{tg} x - 1} + c, \quad c \in \mathbb{R}$
- (w) $-\ln|x| - \frac{1}{2x^2} + \frac{1}{2}\ln(1+x^2) + c, \quad c \in \mathbb{R}$
- (x) $\frac{1}{3}(2\ln|x-1| - \ln(x^2+x+1)) + c, \quad c \in \mathbb{R}$
11. $-\frac{3}{2}\frac{1}{4+x^2} + \frac{7}{16}\text{arctg}\left(\frac{x}{2}\right) + \frac{7}{8}\frac{x}{4+x^2} + c, \quad c \in \mathbb{R}$
12. (a) $\frac{2}{3}\sqrt{1+x^3} + c, \quad c \in \mathbb{R}$
- (b) $-\frac{\sqrt{1+x^2}}{x} + c, \quad c \in \mathbb{R}$
- (c) $\frac{1}{2}(\ln(x^2+1) - 2\ln|x| + 6\text{arctg} x) + c, \quad c \in \mathbb{R}$
- (d) $\frac{x}{2} - \frac{1}{4}\ln(e^{2x} + 2) + c, \quad c \in \mathbb{R}$
- (e) $\frac{1}{1+\text{sen} x} + c, \quad c \in \mathbb{R}$
- (f) $\frac{x^2+1}{2}\ln(1+x^2) - \frac{x^2}{2} + c, \quad c \in \mathbb{R}$
- (g) $\text{sen} x \cdot \ln(\text{sen} x) - \text{sen} x + c, \quad c \in \mathbb{R}$
13. $f(x) = 2x^3 + 2x + 1$
14. $f(x) = 2\ln(e^x + 3) - \ln 4$
15. $f(x) = \ln\left(\frac{x^2-2x+2}{x^2}\right) + 3\text{arctg}(x-1) - \frac{3\pi}{2}$