

$$1. a) f(x) = \frac{2}{\sqrt{x}}, \quad \mathcal{D}_f = \mathbb{R}^+$$

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$$g(x) = \sqrt{5-x}, \quad \mathcal{D}_g =]-\infty, 5]$$

Os gráficos de f e g só se podem interceptar em $I = \mathcal{D}_f \cap \mathcal{D}_g =]0, 5]$. Para $x \in I$:

$$\frac{2}{\sqrt{x}} = \sqrt{5-x} \Leftrightarrow \sqrt{x} \sqrt{5-x} = 2 \Leftrightarrow \begin{matrix} \uparrow \\ x \in I \end{matrix}$$

$$x(5-x) = 4 \Leftrightarrow 5x - x^2 = 4 \Leftrightarrow$$

$$x^2 - 5x + 4 = 0 \Leftrightarrow x = \frac{5 \pm \sqrt{25 - 4(4)}}{2} \Leftrightarrow$$

$$x = \frac{5 \pm 3}{2} \Leftrightarrow x = 1 \vee x = 4$$

Os pontos de interseção são $P = (1, \frac{2}{\sqrt{1}})$
e $Q = (4, \frac{2}{\sqrt{4}})$. $P = (1, 2)$ e $Q = (4, 1)$

b) f e g são de classe \mathcal{C}^∞ em $]0, 5[$ e

$$f'(x) = [2x^{-1/2}]' = 2(-\frac{1}{2})x^{-3/2} = -x^{-3/2} < 0$$

$$f''(x) = -(-\frac{3}{2})x^{-5/2} = \frac{3}{2}x^{-5/2} > 0$$

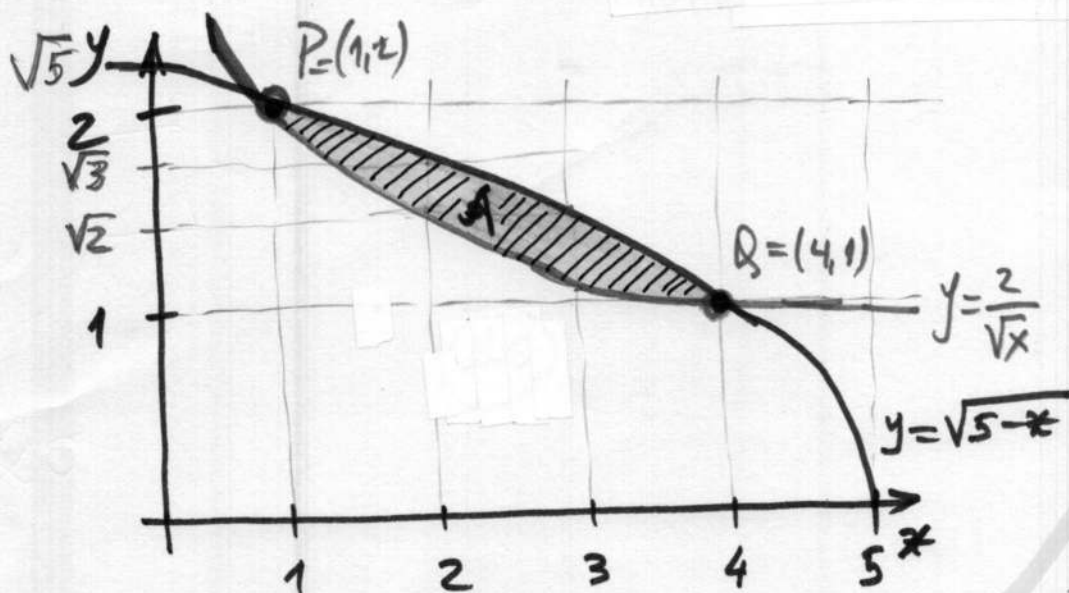
f é estritamente decrescente e convexa em I

$$g'(x) = [(5-x)^{1/2}]' = \frac{1}{2}(5-x)^{-1/2}(-1) = -\frac{1}{2}(5-x)^{-1/2} < 0$$

$$g''(x) = -\frac{1}{2}(-\frac{1}{2})(5-x)^{-3/2}(-1) = -\frac{1}{4}(5-x)^{-3/2} > 0$$

g é estritamente decrescente e côncava em I

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$$\sqrt{5} \approx 2.2$$

$$\sqrt{2} \approx 1.4$$

$$\sqrt{3} \approx 1.7$$

x	$\frac{2}{\sqrt{x}}$	$\sqrt{5-x}$
0	1.0	$\sqrt{5}$
1	2	2
2	$\frac{2}{\sqrt{2}} = \sqrt{2}$	$\sqrt{3}$
3	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$
4	1	1

$$c) A = \int_1^4 \left| \frac{2}{\sqrt{x}} - \sqrt{5-x} \right| dx = \int_1^4 \sqrt{5-x} - \frac{2}{\sqrt{x}} dx$$

$$A = \int_1^4 (5-x)^{1/2} - 2x^{-1/2} dx \quad \begin{array}{l} \text{Regra Barrow} \\ \text{f e g contínuas} \end{array}$$

$$A = \left[\frac{-(5-x)^{1/2+1}}{+1/2+1} - \frac{2x^{-1/2+1}}{-1/2+1} \right]_1^4 = \left[-\frac{2}{3} (5-x)^{3/2} - 4x^{1/2} \right]_1^4$$

$$A = \left[-\frac{2}{3} (5-4)^{3/2} - 4(4)^{1/2} \right] - \left[-\frac{2}{3} (5-1)^{3/2} - 4(1)^{1/2} \right]$$

$$A = -\frac{2}{3} 1^{3/2} - 8 + \frac{2}{3} (4)^{3/2} + 4 = -\frac{2}{3} (1) - 4 + \frac{2}{3} 4\sqrt{4}$$

$$A = -\frac{2}{3} (1) - 4 + \frac{8}{3} (2) = \frac{-2-12+16}{3} = \frac{2}{3} > 0$$