Exercício 2a).

Primitivação por partes: $\boxed{\int \psi'\varphi = \psi\varphi - \int \psi\varphi'}$

<u>1ª Alternativa:</u>

Tomar $f'(x) = e^{6x}$, $g(x) = \cos(2x)$ e obter $f(x) = \dfrac{e^{6x}}{6}$, $g'(x) = -2\sin(2x)$. *(2.5+2.5)*

$$\int \underbrace{e^{6x}}_{f'(x)}\underbrace{\cos(2x)}_{g(x)}\,\mathrm{d}x = \frac{e^{6x}}{6}\cos(2x) - \int \frac{e^{6x}}{6}(-2\sin(2x))\,\mathrm{d}x \qquad (5)$$

Identificar $i'(x) = e^{6x}$, $h(x) = \sin(2x)$ e obter $i(x) = \dfrac{e^{6x}}{6}$, $h'(x) = 2\cos(2x)$. *(2.5+2.5)*

$$= \frac{e^{6x}}{6}\cos(2x) + \frac{1}{3}\int \underbrace{e^{6x}}_{i'(x)}\underbrace{\sin(2x)}_{h(x)}\,\mathrm{d}x$$

$$= \frac{e^{6x}}{6}\cos(2x) + \frac{1}{3}\left(\frac{e^{6x}}{6}\sin(2x) - \int \frac{e^{6x}}{6}(2\cos(2x))\,\mathrm{d}x\right) \qquad (5)$$

$$= \frac{e^{6x}}{6}\cos(2x) + \frac{e^{6x}}{18}\sin(2x) - \frac{1}{9}\int e^{6x}\cos(2x)\,\mathrm{d}x$$

Resolução correcta da equação relativa ao problema *(10)*

$$\int e^{6x}\cos(2x)\,\mathrm{d}x = \frac{e^{6x}}{6}\cos(2x) + \frac{e^{6x}}{18}\sin(2x) - \frac{1}{9}\int e^{6x}\cos(2x)\,\mathrm{d}x$$

$$\Leftrightarrow$$

$$\frac{10}{9}\int e^{6x}\cos(2x)\,\mathrm{d}x = \frac{e^{6x}}{6}\cos(2x) + \frac{e^{6x}}{18}\sin(2x) + c_1,\ c_1 \in \mathbb{R}$$

$$\Leftrightarrow$$

$$\int e^{6x}\cos(2x)\,\mathrm{d}x = \frac{9}{10}\left(\frac{e^{6x}}{6}\cos(2x) + \frac{e^{6x}}{18}\sin(2x)\right) + c_2,\ c_2 \in \mathbb{R}$$

$$= \frac{e^{6x}(3\cos(2x) + \sin(2x))}{20} + c_2,\ c_2 \in \mathbb{R}.$$

$2^{\underline{a}}$ Alternativa:

Tomar $f(x) = e^{6x}$, $g'(x) = \cos(2x)$ e obter $f'(x) = 6e^{6x}$, $g(x) = \dfrac{\sin(2x)}{2}$. *(2.5+2.5)*

$$\int \underbrace{e^{6x}}_{f(x)} \underbrace{\cos(2x)}_{g'(x)}\ \mathrm{d}x = e^{6x}\frac{\sin(2x)}{2} - \int 6e^{6x}\frac{\sin(2x)}{2}\ \mathrm{d}x \qquad (5)$$

$$= e^{6x}\frac{\sin(2x)}{2} - 3\int e^{6x}\sin(2x)\ \mathrm{d}x$$

Identificar $i(x) = e^{6x}$, $h'(x) = \sin(2x)$ e obter $i'(x) = 6e^{6x}$, $h(x) = -\dfrac{\cos(2x)}{2}$. *(2.5+2.5)*

$$= e^{6x}\frac{\sin(2x)}{2} - 3\int \underbrace{e^{6x}}_{i(x)} \underbrace{\sin(2x)}_{h'(x)}\ \mathrm{d}x$$

$$= e^{6x}\frac{\sin(2x)}{2} - 3\left(e^{6x}\left(-\frac{\cos(2x)}{2}\right) - \int 6e^{6x}\left(-\frac{\cos(2x)}{2}\right)\ \mathrm{d}x\right) \quad (5)$$

$$= e^{6x}\frac{\sin(2x)}{2} + 3e^{6x}\frac{\cos(2x)}{2} - 9\int e^{6x}\cos(2x)\ \mathrm{d}x$$

Resolução correcta da equação relativa ao problema *(10)*

$$\int e^{6x}\cos(2x)\ \mathrm{d}x = e^{6x}\frac{\sin(2x)}{2} + 3e^{6x}\frac{\cos(2x)}{2} - 9\int e^{6x}\cos(2x)\ \mathrm{d}x$$

$$\Leftrightarrow$$

$$10\int e^{6x}\cos(2x)\ \mathrm{d}x = e^{6x}\frac{\sin(2x)}{2} + 3e^{6x}\frac{\cos(2x)}{2} + c_1,\ c_1 \in \mathbb{R}$$

$$\Leftrightarrow$$

$$\int e^{6x}\cos(2x)\ \mathrm{d}x = \frac{1}{10}\left(e^{6x}\frac{\sin(2x)}{2} + 3e^{6x}\frac{\cos(2x)}{2}\right) + c_2,\ c_2 \in \mathbb{R}$$

$$= \frac{e^{6x}(3\cos(2x) + \sin(2x))}{20} + c_2,\ c_2 \in \mathbb{R}.$$