Cálculo I - Agrupamento 4

Exame Final - 1.ª Parte

<u>Resolução</u>

1) $A = \{(x,y) \in \mathbb{R}^2 : 2x \leq y \leq -x^2+5x\}$

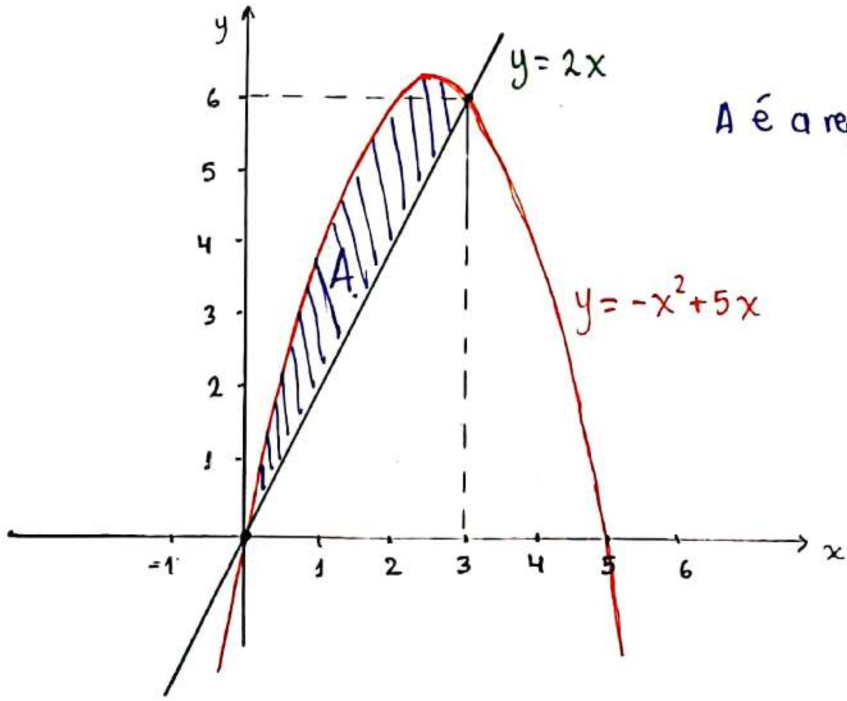
(a) $\begin{cases} y = 2x \\ y = -x^2+5x \end{cases} \Leftrightarrow \begin{cases} 2x = -x^2+5x \\ \underline{} \end{cases} \Leftrightarrow \begin{cases} x^2 - 5x + 2x = 0 \\ \underline{} \end{cases}$

$\Leftrightarrow \begin{cases} x^2 - 3x = 0 \\ \underline{} \end{cases} \Leftrightarrow \begin{cases} x(x-3) = 0 \\ \underline{} \end{cases} \Leftrightarrow \begin{cases} x=0 \vee x-3=0 \\ \underline{} \end{cases}$

$\Leftrightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \vee \begin{cases} x = 3 \\ y = 6 \end{cases}$

Os pontos de interseção pedidos são : $(0,0)$ e $(3,6)$

(b)



A é a região sombreada.

(c) $\text{Área de } A = \int_0^3 -x^2+5x-2x \, dx = \int_0^3 -x^2+3x \, dx$

$= \left[-\frac{x^3}{3} + \frac{3x^2}{2} \right]_0^3 = -9 + \frac{27}{2} = \frac{9}{2}$ //

Cálculo I - agr. 4 — Resolução do exame final

2ª parte — 18/02/2021

1. (a) $\begin{cases} -1 \leq \dfrac{1}{\sqrt{2-2x^2}} \leq 1 \\ 2-2x^2 > 0 \end{cases} \Leftrightarrow \begin{cases} \dfrac{1}{\sqrt{2-2x^2}} \leq 1 \\ 2-2x^2 > 0 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 2-2x^2 \geq 1 \\ 2-2x^2 > 0 \end{cases} \Leftrightarrow 2-2x^2 \geq 1 \Leftrightarrow x^2 \leq \frac{1}{2} \Leftrightarrow x \in \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$

Resposta: $D_f = \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$.

(b) f é contínua em $\left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$ e diferenciável em $\left]-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right[$.

$$f'(x) = \frac{1}{\sqrt{1-\frac{1}{2-2x^2}}} \cdot \left(-\frac{1}{2}\right)(2-2x^2)^{-3/2} \cdot (-4x)$$

$$= \frac{2x}{\sqrt{1-\frac{1}{2-2x^2}} \cdot (2-2x^2)^{3/2}}.$$

Se $x \in \left]-\frac{\sqrt{2}}{2}, 0\right[$, $f'(x) < 0 \Rightarrow f$ é monótona decrescente.

Se $x \in \left]0, \frac{\sqrt{2}}{2}\right[$, $f'(x) > 0 \Rightarrow f$ é monótona crescente.

Logo f tem 2 máximos nos pontos $x = -\frac{\sqrt{2}}{2}$ e $x = \frac{\sqrt{2}}{2}$.

$f\left(-\frac{\sqrt{2}}{2}\right) = f\left(\frac{\sqrt{2}}{2}\right) = \arcsin\left(\frac{1}{\sqrt{2-1}}\right) = \arcsin 1 = \frac{\pi}{2}$,

logo os máximos são globais.

f tem 1 mínimo global no ponto $x = 0$,

$f(0) = \arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$.

2. (a) $\dfrac{2x^2+7x+4}{x(x^2+2x+2)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+2x+2}$

$$\frac{2x^2+7x+4}{x(x^2+2x+2)} = \frac{A(x^2+2x+2)+x(Bx+C)}{x(x^2+2x+2)}$$

$$2x^2+7x+4 = Ax^2+2Ax+2A+Bx^2+Cx$$

$$\begin{cases} 2=A+B \\ 7=2A+C \\ 4=2A \end{cases} \Leftrightarrow \begin{cases} 2=2+B \\ 7=4+C \\ A=2 \end{cases} \Leftrightarrow \begin{cases} B=0 \\ C=3 \\ A=2 \end{cases} \Rightarrow \frac{2x^2+7x+4}{x(x^2+2x+2)} = \frac{2}{x} + \frac{3}{x^2+2x+2}$$

$$\int \frac{2x^2+7x+4}{x(x^2+2x+2)}\,dx = \int \frac{2}{x} + \frac{3}{x^2+2x+2}\,dx = 2\ln|x| + 3\int \frac{1}{(x+1)^2+1}\,dx$$

$= 2\ln|x| + 3\operatorname{arctg}(x+1) + C$, C constante em intervalos.

(b) $x = \frac{1}{3}\operatorname{tg} t$, $t \in]-\frac{\pi}{2}, \frac{\pi}{2}[\Rightarrow \frac{dx}{dt} = \frac{1}{3}\frac{1}{\cos^2 t}$, $t = \operatorname{arctg}(3x)$.

$$\int \frac{1}{(9x^2+1)^{3/2}}\,dx = \int \frac{1}{(\operatorname{tg}^2 t+1)^{3/2}}\,\frac{1}{3}\frac{1}{\cos^2 t}\,dt = \frac{1}{3}\int \frac{1}{\left(\frac{1}{\cos^2 t}\right)^{3/2}} \cdot \frac{1}{\cos^2 t}\,dt$$

$= \frac{1}{3}\int |\cos t|^3 \cdot \frac{1}{\cos^2 t}\,dt = \frac{1}{3}\int \cos t\,dt$

$= \frac{1}{3}\sin t + C = \frac{1}{3}\sin(\operatorname{arctg}(3x)) + C$

C.A.: $t \in]-\frac{\pi}{2}, \frac{\pi}{2}[$, logo $\cos t > 0 \Rightarrow |\cos t| = \cos t$

$= \pm\frac{1}{3}\sqrt{\frac{(3x)^2}{1+(3x)^2}} + C = \frac{1}{\cancel{3}}\frac{\cancel{3}x}{\sqrt{1+9x^2}} + C$

$= \dfrac{x}{\sqrt{1+9x^2}} + C$, $C \in \mathbb{R}$.

C.A.: $x = \frac{1}{3}\frac{\sin t}{\cos t}$, $t \in]-\frac{\pi}{2}, \frac{\pi}{2}[$

$\Rightarrow x$ tem o mesmo sinal que $\sin t$