21 Fev 2022

Q2.b) do 2.º Teste em Recurso

$$\sum_{n=0}^{+\infty}\left(\frac{99}{10^{2n+2}}-\frac{3/2}{(n+1)(n+2)}+\frac{2^{n-1}}{3^n}\right)=S$$

- A série $\displaystyle\sum_{n=0}^{+\infty}\frac{99}{10^{2n+2}}=\sum_{n=0}^{+\infty}\frac{99}{100^{n+1}}=99\sum_{n=0}^{+\infty}\left(\frac{1}{100}\right)^{n+1}$

é geométrica de razão $\frac{1}{100}\in\,]-1,1[$ logo convergente.

$$\sum_{n=0}^{+\infty}\left[99\left(\frac{1}{100}\right)^{n+1}\right]=\frac{1.^{\circ}\text{ termo}}{1-\frac{1}{100}}=\frac{99/100}{1-1/100}=1=S_1$$

- A série $\displaystyle\sum_{n=0}^{+\infty}\frac{3/2}{(n+1)(n+2)}=\frac{3}{2}\sum_{n=0}^{+\infty}\left(\frac{A}{n+1}+\frac{B}{n+2}\right)$ para

alguns $A,B\in\mathbb{R}$ a calcular pelo método dos coef. indeterminados:

$$\frac{1}{(n+1)(n+2)}=\frac{A}{n+1}+\frac{B}{n+2}=\frac{A(n+2)+B(n+1)}{(n+1)(n+2)}$$

$$\frac{(A+B)n+(2A+B)}{(n+1)(n+2)}=\frac{1}{(n+1)(n+2)}\Rightarrow\begin{cases}A+B=0\\2A+B=1\end{cases}\quad\begin{cases}A=1\\B=-1\end{cases}$$

$$A=1$$

Então

$$\sum_{n=0}^{+\infty}\frac{3/2}{(n+1)(n+2)}=\frac{3}{2}\underbrace{\sum_{n=0}^{+\infty}(u_n-u_{n+1})}_{\text{Série de Mengoli}},\ \text{com } u_n=\frac{1}{n+1}$$

$$\sum_{n=1}^{+\infty}\frac{3/2}{(n+1)(n+2)}=\frac{3}{2}\left(u_0-\lim_{n\to+\infty}\frac{1}{n+1}\right)=\frac{3}{2}\left(\frac{1}{1}\right)=\frac{3}{2}=S_2$$

- A série $\displaystyle\sum_{n=0}^{+\infty}\frac{2^{n-1}}{3^n}=\sum_{n=0}^{+\infty}\frac{1}{2}\left(\frac{2}{3}\right)^n$ é geométrica

de razão $\frac{2}{3}\in\,]-1,1[$ logo convergente e tem soma igual a $\dfrac{1/2}{1-\frac{2}{3}}=\dfrac{1/2}{1/3}=\dfrac{3}{2}=S_3$

Concluo: $S=S_1-S_2+S_3=1-\frac{3}{2}+\frac{3}{2}=1.$ //