Cálculo I - Agr. 4 2020/21

1.º teste - turma TP4B-3

Resolução

1. $f(x)=\arccos(4x-4x^2)$.

(a) $D_f=\{x\in\mathbb{R} : 4x-4x^2\in \underbrace{D_{\arccos}}_{[-1,1]}\}$

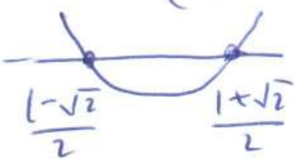
(50 pontos)

C.A.: $-1\leq 4x-4x^2\leq 1 \iff$

$\iff (4x-4x^2-1\leq 0 \wedge 4x-4x^2+1\geq 0)$

$\iff (4x^2-4x+1\geq 0 \wedge 4x^2-4x-1\leq 0)$

$\iff ((2x-1)^2\geq 0 \wedge 4x^2-4x-1\leq 0) \iff 4x^2-4x-1\leq 0$



$4x^2-4x-1=0 : x=\frac{4\pm\sqrt{16+16}}{8}=\frac{1\pm\sqrt{2}}{2}$.

$\therefore D_f=\left[\frac{1-\sqrt{2}}{2}, \frac{1+\sqrt{2}}{2}\right]$.

(b)

(120 pontos) $f'(x)=-\frac{(4x-4x^2)'}{\sqrt{1-(4x-4x^2)^2}}=-\frac{4-8x}{\sqrt{1-(4x-4x^2)^2}}$ em

$\left]\frac{1-\sqrt{2}}{2}, \frac{1+\sqrt{2}}{2}\right[\setminus\{\frac{1}{2}\}$.

$f'(x)>0 \iff 8x-4>0 \iff x>\frac{1}{2}$.

	$\frac{1-\sqrt{2}}{2}$		$1/2$		$\frac{1+\sqrt{2}}{2}$
f'		$-$		$+$	
f		$\searrow$		$\nearrow$	

f é contínua em $\left[\frac{1-\sqrt{2}}{2}, \frac{1+\sqrt{2}}{2}\right]$, estrit. decrescente em $\left[\frac{1-\sqrt{2}}{2}, \frac{1}{2}\right]$ e estrit. crescente em $\left[\frac{1}{2}, \frac{1+\sqrt{2}}{2}\right]$,

logo existe um mínimo (absoluto) no ponto $x=1/2$, $f(\frac{1}{2})=\arccos(1)=0$, e 2 máximos nos pontos $\frac{1-\sqrt{2}}{2}$ e $\frac{1+\sqrt{2}}{2}$. $f(\frac{1-\sqrt{2}}{2})=\arccos(4\frac{1-\sqrt{2}}{2}-4(\frac{1-\sqrt{2}}{2})^2)$

$=\arccos(-1)=\pi$, $f(\frac{1+\sqrt{2}}{2})=\arccos(4\frac{1+\sqrt{2}}{2}-4(\frac{1+\sqrt{2}}{2})^2)=\arccos(-1)=\pi$,

afinal nos pontos $\frac{1-\sqrt{2}}{2}$ e $\frac{1+\sqrt{2}}{2}$ o valor do máximo é comum, sendo absoluto.