Cálculo I - agr. 4 2020/21

2º teste - turma TP4A-6

Resolução

(30pts) 1. (a) $\int \ln(x^2+1)\,dx = x\ln(x^2+1) - \int x \frac{2x}{x^2+1}\,dx$

$= x\ln(x^2+1) - \int \left(2 - \frac{2}{x^2+1}\right) dx$

$= x\ln(x^2+1) - 2x + 2\,\mathrm{arctg}\,x + c, \quad c \in \mathbb{R}$

C.A.: $\frac{2x^2}{x^2+1} = \frac{2x^2+2-2}{x^2+1}$

$= 2 - \frac{2}{x^2+1}$

(b)
(30pts.) $\int \frac{x-4}{x^2+x-2}\,dx$

$= \int \frac{2}{x+2} - \frac{1}{x-1}\,dx$

$= 2\ln|x+2| - \ln|x-1| + c,$

c constante em intervalos.

C.A. $x^2+x-2 = (x+2)(x-1)$

$\frac{x-4}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1} \Rightarrow$

$x - 4 = A(x-1) + B(x+2) \Leftrightarrow$

$\Leftrightarrow x - 4 = Ax - A + Bx + 2B \Leftrightarrow$

$\begin{cases} 1 = A+B \\ -4 = -A+2B \end{cases} \Leftrightarrow \begin{cases} A = 2 \\ B = -1 \end{cases}$

(c) $\int \frac{x^2}{\sqrt{4-x^2}}\,dx$
(40pts.)

$= \int \frac{(2\sin t)^2}{\cancel{2\cos t}} \cdot \cancel{2\cos t}\,dt$

$= \int 4\sin^2 t\,dt$

$= \int 2(1-\cos 2t)\,dt$

$= 2t - \sin 2t + c = 2t - 2\sin t \cdot \cos t + c$

$= 2\arcsin\frac{x}{2} - x \cdot \frac{\sqrt{4-x^2}}{2} + c, \quad c \in \mathbb{R}.$

C.A. $x = 2\sin t, \; t \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$

$\sqrt{4-x^2} = \sqrt{4 - 4\sin^2 t} = \sqrt{4\cos^2 t}$

$= 2|\cos t| = 2\cos t; \quad dx = 2\cos t\,dt$

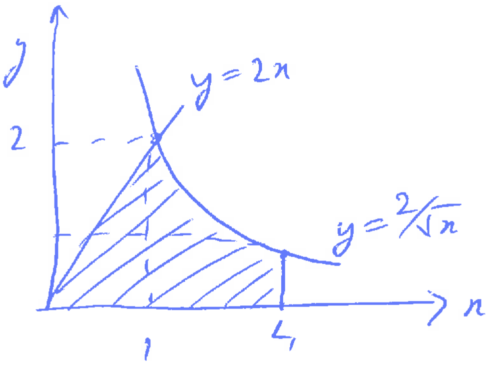
$t = \arcsin\frac{x}{2}$

2. $A = \{(x,y) \in \mathbb{R}^2 : y \geq 0,\ y \leq 2x,\ y \leq \frac{2}{\sqrt{x}},\ 0 \leq x \leq 4\}$

(a) (10 pts.) $\begin{cases} y = 2x \\ y = \frac{2}{\sqrt{x}} \end{cases} \Longleftrightarrow \begin{cases} — \\ x = \frac{1}{\sqrt{x}} \end{cases} \Longleftrightarrow \begin{cases} — \\ x\sqrt{x} = 1 \end{cases} \Longleftrightarrow \begin{cases} y = 2 \\ x = 1 \end{cases}$

O ponto de interseção é $(1,2)$.

(b) (30 pts.)



(c) (30 pts.) $\text{Área}(A) = \int_0^1 2x\,dx + \int_1^4 \frac{2}{\sqrt{x}}\,dx = [x^2]_0^1 + \left[2\,\frac{x^{1/2}}{1/2}\right]_1^4$

$= 1 + [4\sqrt{x}]_1^4 = 1 + 8 - 4 = 5.$

3.(a) (10 pts.) $S(f, P_n, C_n) = \sum_{i=1}^{n} f(\xi_i)\cdot(x_i - x_{i-1})$, sendo

$P_n = \{a = x_0 < x_1 < \dots < x_{n-1} < x_n = b\}$, $C_n = \{\xi_1, \dots, \xi_n\}$.

(b) (20 pts) $\underline{1.^{\text{o}}\text{ Método}}$. Sejam $P_n = \{0 < \frac{1}{n} < \dots < \frac{n-1}{n} < 1\}$,

$C_n = \{\frac{1}{n}, \dots, \frac{n-1}{n}, 1\}$; então $S(f, P_n, C_n)$

$= \sum_{i=1}^{n} f(\frac{i}{n})\cdot\frac{1}{n} = \sum_{i=1}^{n} \frac{n}{i}\cdot\frac{1}{n} = \sum_{i=1}^{n}\frac{1}{i}$ e $\lim_{n\to\infty} S(f, P_n, C_n) = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{i} = \infty$.

$\underline{2.^{\text{o}}\text{ Método}}$ Sejam $P_n = \{0 < \frac{1}{n} < \dots < \frac{n-1}{n} < 1\}$ e $C_n = \{\xi_1, \dots, \xi_n\}$,

sendo $\xi_1 = \frac{1}{n^2} \in [0, \frac{1}{n}]$. Então $S(f, P_n, C_n) = \sum_{i=1}^{n} f(\xi_i)\cdot\frac{1}{n}$

$> f(\xi_1)\cdot\frac{1}{n} = f(\frac{1}{n^2})\cdot\frac{1}{n} = n^2\cdot\frac{1}{n} = n \xrightarrow[(n\to\infty)]{} \infty$. Em ambos os

casos, a sequência de somas de Riemann diverge para o infinito.