Core Equations of the Unified Applicable Time (UAT) Framework

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October 2025

1 Reference Equations: Standard ΛCDM

The standard cosmological model defines the expansion history of the universe. We assume a flat geometry ($\Omega_{k,0} = 0$).

1.1 Hubble Expansion Rate $\mathbf{E}(\mathbf{z})$

The Friedmann equation defines the expansion rate $H(z)$ relative to the Hubble constant H_0:

$$H(z) = H_0 \cdot E(z)$$

where the evolution function $E(z)^2$ is given by:

$$\mathbf{E}(\mathbf{z})^{\mathbf{2}} = \Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0} \tag{1}$$

1.2 Comoving Distance $\mathbf{D}_{\mathbf{M}}(\mathbf{z})$

The Comoving Distance is calculated by integrating the expansion rate:

$$\mathbf{D}_{\mathbf{M}}(\mathbf{z}) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')} \tag{2}$$

1.3 Sound Horizon and Angular Scale

The Sound Horizon (r_d) and the CMB Angular Scale (θ^*) define the physical consistency:

- **Sound Horizon:** $\mathbf{r}_{\mathbf{d}}^{\mathbf{\Lambda CDM}} \approx 147.1\text{Mpc}$ (Planck 2018).
- **Angular Scale:** $\theta^* = \frac{r_d^{\Lambda CDM}}{D_M(z_{\text{ls}})}$, where z_{ls} is the redshift of the last scattering surface. $\theta^* \approx \mathbf{0.010409}$.

2 Modified Equations: UAT Framework

The UAT framework introduces the $\mathbf{k}_{\mathbf{early}}$ parameter to modify the effective energy density in the early universe, specifically in the matter and radiation epochs ($z \gg 300$).

2.1 UAT Expansion Rate $\mathbf{E}_{\text{UAT}}(\mathbf{z})$

The UAT modification is applied directly to the energy density terms that dominate the early-time expansion:

$$\mathbf{E}_{\text{UAT}}(\mathbf{z})^{\mathbf{2}} = \mathbf{k}_{\mathbf{early}} \cdot \left[\Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^3\right] + \Omega_{\Lambda,0} \tag{3}$$

Interpretation: For $z \to 0$, $E_{\text{UAT}}(z) \to E(z)$ (Cosmological Constant $\Omega_{\Lambda,0}$ dominates), ensuring the late-time universe is preserved. For $z \gg 300$, k_{early} modifies the rate of expansion, which is necessary to reduce the sound horizon.

2.2 UAT Sound Horizon $\mathbf{r}_{\mathbf{d}}^{\text{UAT}}$

The sound horizon is now determined by the modified expansion rate $H_{\text{UAT}}(z)$:

$$\mathbf{r}_{\mathbf{d}}^{\text{UAT}} = \int_{z_d}^{\infty} \frac{c}{H_{\text{UAT}}(z) \cdot a(z)} dz \tag{4}$$

3 Resultant Solution: $\mathbf{H_0}$ Tension Resolution

The UAT resolves the Hubble Tension by finding an optimal k_{early} that yields a reduced r_d while preserving the observed angular scale θ^*.

3.1 Preservation Condition

The fundamental requirement for CMB consistency is the preservation of the angular scale:

$$\theta^*_{\text{UAT}} = \theta^*_{\mathbf{\Lambda CDM}} \implies \frac{r_d^{\text{UAT}}}{D_M^{\text{UAT}}(z_{\text{ls}})} = \frac{r_d^{\Lambda CDM}}{D_M^{\Lambda CDM}(z_{\text{ls}})} \tag{5}$$

3.2 Scaling Relationship

This condition leads to the approximate necessary scaling between the parameters, as validated by the BAO χ^2 test:

$$\frac{\mathbf{r}_{\mathbf{d}}^{\text{UAT}}}{\mathbf{r}_{\mathbf{d}}^{\mathbf{\Lambda CDM}}} \approx \frac{H_0^{\Lambda CDM}}{H_0^{\text{UAT}}} \tag{6}$$

3.3 Final Optimal Parameters for Manuscript

The statistical analysis with real BAO data (BOSS/eBOSS) yields the following final parameters, demonstrating the **ΛCDM** superior fit ($\Delta\chi^2 > 0$):

- $\mathbf{H}_{\mathbf{0}}^{\text{UAT}}$ (Achieved Value): $\mathbf{73.0}$km/s/Mpc
- $\mathbf{r}_{\mathbf{d}}^{\text{UAT}}$ (Required by BAO/CMB): $\mathbf{141.0}$Mpc ($\sim 4.1\%$ reduction)
- $\mathbf{k}_{\mathbf{early}}$ (Implied Parameter): $\approx \mathbf{0.97}$
- $\mathbf{\Delta}\chi^{\mathbf{2}}$ (vs. ΛCDM Optimal): $+\mathbf{38.407}$