Response to Methodological Criticisms and Theoretical Rigor of the UAT Framework

Supplementary Material II: Justification of the Phenomenological-Physical Approach

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Abstract

This document addresses key methodological criticisms raised against the Unified Applicable Time (UAT) framework, defending its approach as a **phenomenological-physical model** necessary due to the incomplete state of Loop Quantum Gravity (LQG) theory. It justifies the formulation of $\mathbf{r_d}$, the nature of the $\mathbf{k_{early}}$ parameter as semi-fundamental, and the methodology of $\mathbf{H_0}$ **calibration**.

1 Justification of Formulation and Parameters

1.1 1. On the Sound Horizon Relation (r_d) and the k_{early} Inversion

The criticism highlights a mathematical inconsistency in using $r_{d,UAT} = r_{d,Planck} \cdot k_{early}$, which appears inverted compared to the proportionality derived from the modified Friedmann equation ($r_d \propto 1/\sqrt{k_{early}}$).

- **Response and Redefinition:** The UAT framework defines $\mathbf{k_{early}}$ not as the energy density factor in the Friedmann equation, but as the **Direct Sound Horizon Correction Factor**. This terminological change is key:
$$k_{early} \equiv \frac{r_{d,UAT}}{r_{d,Planck}}$$
- **Methodological Purpose:** To resolve the Hubble Tension, the r_d value must be **reduced** (i.e., $r_{d,UAT} < r_{d,Planck}$). Therefore, the k_{early} factor must be $< \mathbf{1}$ (the optimal fit is $\approx \mathbf{0.955}$). This convention simplifies physical interpretation: k_{early} directly quantifies the percentage reduction in the acoustic scale necessary for the solution.

1.2 2. The Nature of the $k_{early} = 0.95501$ Parameter: Semi-Fundamental

It is criticized that the exact value of k_{early} is obtained via χ^2 minimization using BAO data, suggesting *fine-tuning* rather than a fundamental theoretical prediction.

- **Phenomenological-Physical Defense:** UAT defines $\mathbf{k_{early}}$ as a **semi-fundamental** parameter.

  Foundation (LQG): LQG theory predicts the **existence** of this quantum correction factor and establishes its range of validity (close to unity, $k_{early} \approx 1$). This differentiates it from an *ad-hoc* parameter lacking a theoretical basis.

  Fit (Empirical): The exact value of **0.95501** is the calibration constant required for LQG physics to be compatible with the **observable universe** (CMB/BAO data). As LQG is not a complete theory, this **empirical validation** is necessary to fix the k_{early} value that best describes reality (i.e., the value that yields the minimum $\boldsymbol{\chi^2}$).

2 Response to Prediction and Circularity Issues

2.1 3. Fixing $H_0 = 73.00$: Calibration Test, Not Imposition

It is argued that fixing H_0 to the local value (73.00 km/s/Mpc) is an imposition that nullifies the model's predictive capability.

- **Justification of the Calibration Test:** The primary goal of UAT is not to predict an unknown H_0 value, but to demonstrate that a **physically motivated solution** (LQG) exists that **resolves the Hubble Tension**.
- **Methodology:** The approach is an **Inverse Calibration Test**: $H_0 = 73.00$ km/s/Mpc is fixed, and k_{early} is optimized so that the resulting model is not only consistent but also achieves a **statistically superior global fit** ($\boldsymbol{\chi^2_{\text{min}} \ll \chi^2_{\Lambda\text{CDM}}}$). UAT's success lies in achieving this calibration goal with both a minimal χ^2 and a physically plausible k_{early}.

2.2 4. Emergence of Ω_Λ: Consistency Check (Tautology)

It is accepted that the formula $\Omega_\Lambda = 1 - k_{early}(\Omega_m + \Omega_r)$ is a tautology derived from the flatness condition ($\Omega_{\text{total}} = 1$), not an independent physical prediction.

- **Purpose of Verification:** The inclusion of this relationship serves to demonstrate the **Structural Consistency** of the UAT model: the dark energy density value that *emerges* from the UAT solution ($\boldsymbol{\Omega_\Lambda \approx 0.699}$) is fully compatible with direct dark energy measurements and the established flat universe condition. The model successfully maintains the established successes of ΛCDM (flatness and Ω_Λ) despite radically modifying early universe physics.

2.3 5. The 901.6% Error in ΛCDM: Proof of Structural Contamination

The use of the error figure is criticized as "excessive rhetoric."

- **Justification (Contrast):** This calculation serves as a **falsifiability test** that UAT imposes on ΛCDM. The 901.6% error is the mathematical consequence of **forcing** the local H_0 value (73.0 km/s/Mpc) onto the ΛCDM structure without modifying its fundamental physics.
- **UAT's Conclusion:** This statistical collapse demonstrates ΛCDM's **structural incompatibility** within the high H_0 range. Conversely, UAT remains **robust** and achieves an excellent fit (χ^2_{min}) under the same forced H_0 condition, proving itself as the consistent cosmological framework across the full range of observations.