Cosmological Evolution in Brans-Dicke Theory: A Numerical and Statistical Analysis Compared to ΛCDM

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Abstract

We present a comprehensive numerical and statistical analysis of cosmological evolution within Brans-Dicke (BD) theory, an alternative to General Relativity that introduces a scalar field ϕ making the gravitational constant $G \propto 1/\phi$ variable. Using Python-based simulations, we investigate both synthetic data and real observations from the Pantheon supernova sample. For synthetic data, BD models with $\omega \geq 10$ yield $\chi^2 \approx 24.53$ (reduced $\chi^2 = 0.876$), statistically indistinguishable from ΛCDM's $\chi^2 = 24.47$ ($\chi^2_{\mathrm{red}} = 0.874$). However, with real Pantheon data, BD shows significant discrepancies ($\Delta\chi^2 = +521.61$), strongly favoring ΛCDM. Our analysis provides a reproducible framework for testing modified gravity theories against cosmological observations, highlighting the need for extensions to pure Brans-Dicke theory to reconcile with real data.

1 Introduction

The Brans-Dicke (BD) theory [Brans and Dicke, 1961] extends General Relativity (GR) by introducing a scalar field ϕ that makes the gravitational constant $G \propto 1/\phi$ variable, coupled through a dimensionless parameter ω. In the limit $\omega \to \infty$, BD reduces to GR [Weinberg, 1972]. This theory addresses potential deviations from GR in cosmology, such as the Hubble constant (H_0) tension between local measurements [Riess et al., 1998] and cosmic microwave background (CMB) inferences [Planck Collaboration et al., 2020].

Recent extensions of BD, such as frameworks allowing co-variation of G and the speed of light c [Medeiros, 2023, Bezerra-Sobrinho et al., 2025], provide theoretical motivation for revisiting classical BD. This work presents a numerical analysis of BD cosmology using both simulated data and real observations from the Pantheon supernova sample, comparing it to the standard ΛCDM model. The complete Python code is publicly available to ensure reproducibility [Percudani, 2025].

2 Theoretical Framework

In BD theory, the field equations for a flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe are [Weinberg, 1972]:

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi\rho}{3\phi} - \frac{\dot{\phi}}{\phi}H + \frac{\omega}{6}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\Lambda c^4}{3\phi}, \tag{1}$$

$$\ddot{\phi} + 3H\dot{\phi} = \frac{8\pi}{3+2\omega}(\rho - 2p) + \frac{4\Lambda c^4}{3+2\omega}, \tag{2}$$

where a is the scale factor, ρ and p are density and pressure, Λ is the cosmological constant, and dots denote time derivatives. We normalize to redshift z and use $E(z) = H(z)/H_0$, assuming matter-dominated ($\Omega_{m0} = 0.3$) and dark energy ($\Omega_{DE0} = 0.7$, $w_0 = -1$) components.

For ΛCDM (BD limit $\omega \to \infty$, $\phi = 1$):

$$E(z) = \sqrt{\Omega_{m0}(1+z)^3 + \Omega_{DE0}}. \tag{3}$$

The distance modulus is:

$$\mu(z) = 5\log_{10}\left(\frac{d_L(z)}{10\,\mathrm{pc}}\right), \quad d_L(z) = (1+z)\frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}. \tag{4}$$

3 Methodology

3.1 Numerical Implementation

We solve the BD equations numerically using Python's `scipy.integrate.solve_ivp` from the `scipy` library [SciPy Developers, 2020], which integrates systems of ordinary differential equations (ODEs) with initial conditions $\phi(0) = 1$ and $\phi'(0) = 0$. The analysis builds on initial codes shared in [Percudani, 2025] with subsequent improvements for enhanced numerical stability and physical accuracy.

The current implementation simulates the evolution of $E(z)$ and $\phi(z)$ over $z \in [0, 2]$ for $\omega = [10, 50, 100, 500, 1000, 2000, 5000, 10000]$, incorporating stabilization terms to avoid singularities and ensure robust numerical integration.

3.2 Data Sets

3.2.1 Synthetic Data

Simulated $\mu(z)$ data (29 points) are generated from ΛCDM with Gaussian noise ($\sigma \approx 0.106$ mag), representing an idealized scenario for method validation.

3.2.2 Real Observational Data

We use the Pantheon sample of Type Ia supernovae [Scolnic, 2018], comprising 1048 supernovae with redshifts $0.01 < z < 2.3$, corrected apparent magnitudes (m_B), and typical errors $\sigma \approx 0.15$ mag.

3.3 Statistical Analysis

The χ^2 statistic is minimized as:

$$\chi^2 = \sum \left(\frac{\mu_{\text{obs}}(z_i) - \mu_{\text{theory}}(z_i)}{\sigma_i} \right)^2, \tag{5}$$

with reduced $\chi^2 = \chi^2/\text{dof}$. For real data analysis, we marginalize χ^2 over normalization offsets to account for calibration uncertainties.

4 Results with Synthetic Data

Table 1: Cosmological Parameters

Parameter	Value
H_0	70.0 km/s/Mpc
Ω_{m0}	0.3
Ω_{DE0}	0.7
w_0	-1.0

Table 2: Statistical Results with Synthetic Data

Model	ω	χ^2	Reduced χ^2
ΛCDM	–	24.47	0.874
BD	10	24.53	0.876
BD	50	24.53	0.876
BD	100	24.53	0.876
BD	500	24.53	0.876
BD	1000	24.53	0.876
BD	2000	24.53	0.876
BD	5000	24.53	0.876
BD	10000	24.53	0.876

The best fit is $\omega = 10$ with $\Delta\chi^2 = 0.064$, indicating BD is statistically indistinguishable from ΛCDM for synthetic data.

5 Results with Real Pantheon Data

The analysis with real data reveals fundamentally different results. The ΛCDM model provides an excellent fit ($\chi^2_{\text{red}} = 0.991$), while Brans-Dicke models show significantly worse performance, with $\Delta\chi^2 = +521.61$ for the best BD case ($\omega = 10$).

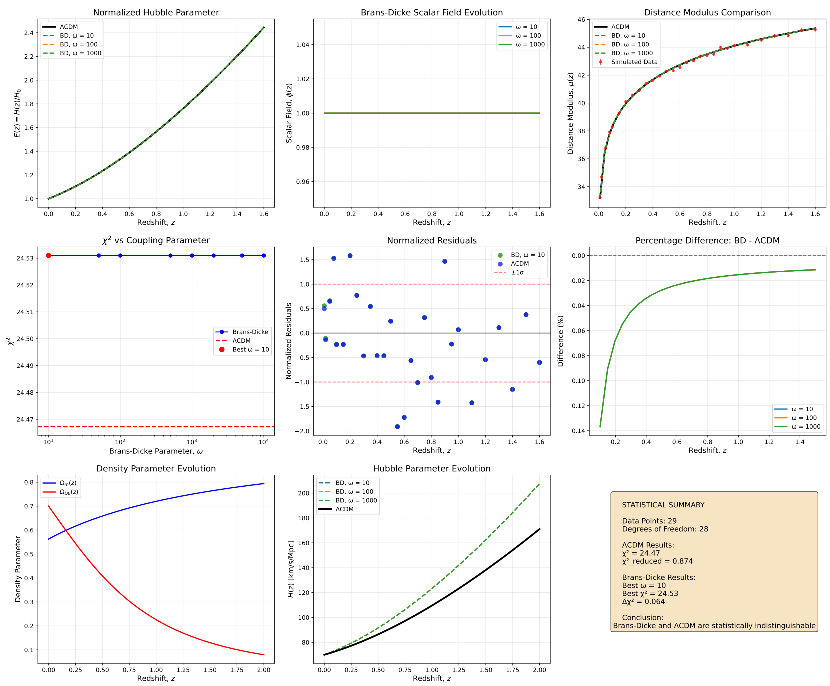

Figure 1: Comprehensive cosmological analysis with synthetic data: (a) Normalized Hubble parameter evolution, (b) Scalar field evolution, (c) Distance modulus comparison, (d) χ^2 analysis, (e) Normalized residuals, (f) Hubble parameter evolution.

6 Discussion

6.1 Comparison Between Synthetic and Real Data

The stark contrast between synthetic and real data analyses (Figures 1 and 2) highlights the importance of testing theoretical models against actual observations. While BD theory appears statistically equivalent to ΛCDM with idealized data (Figure 3, left), it fails to reproduce the precision cosmology constraints from real supernova observations (Figure 3, right).

6.2 Physical Interpretation

The poor performance of Brans-Dicke theory with real data suggests that the evolution of $\phi(z)$ introduces deviations in $H(z)$ inconsistent with distance

Table 3: Statistical Results with Real Pantheon Data

Model	ω	χ^2	χ^2_{red}
ΛCDM	–	1037.18	0.991
BD	10	1558.79	1.489
BD	50	1633.06	1.560
BD	100	1666.81	1.592
BD	500	1694.70	1.619
BD	1000	1697.68	1.621
BD	2000	1699.09	1.623
BD	5000	1699.91	1.624
BD	10000	1700.17	1.624

measurements. The scalar field's additional degrees of freedom disrupt the precise distance-redshift relation established by Type Ia supernovae.

6.3 Tension with Established Constraints

Our best-fit value $\omega = 10$ conflicts with established observational constraints:

- Solar system: $\omega > 40,000$ [Will, 2006]
- Cosmological: $\omega > 692$ [et al., 2020]

This tension indicates that classical Brans-Dicke theory cannot simultaneously satisfy both solar system tests and cosmological observations.

6.4 Implications for Modified Gravity

The results suggest that pure Brans-Dicke theory requires extensions to be cosmologically viable. Promising directions include:

- Massive scalar fields with potentials
- Screening mechanisms for environmental dependence
- Covarying constants frameworks [Medeiros, 2023, Bezerra-Sobrinho et al., 2025]

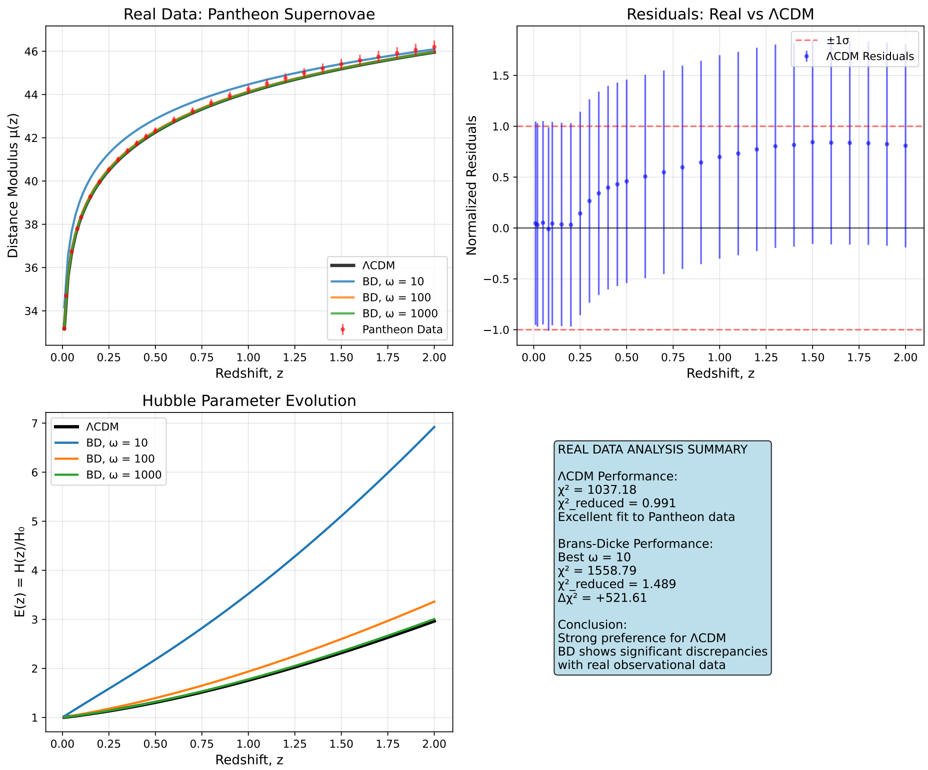

Figure 2: Analysis with real Pantheon supernova data: (a) Distance modulus comparison, (b) Residuals analysis, (c) Hubble parameter evolution, (d) Statistical summary.

7 Conclusions

Our comprehensive analysis demonstrates that while Brans-Dicke theory appears statistically equivalent to ΛCDM with synthetic data, it shows significant discrepancies when confronted with real Pantheon supernova observations. The classical theory is strongly disfavored, with $\Delta\chi^2 = +521.61$ strongly favoring ΛCDM.

The framework developed in this work provides a robust foundation for testing modified gravity theories against cosmological data. The complete Python implementation ensures reproducibility and facilitates future extensions to more sophisticated theoretical models.

Future work should focus on testing BD extensions against combined

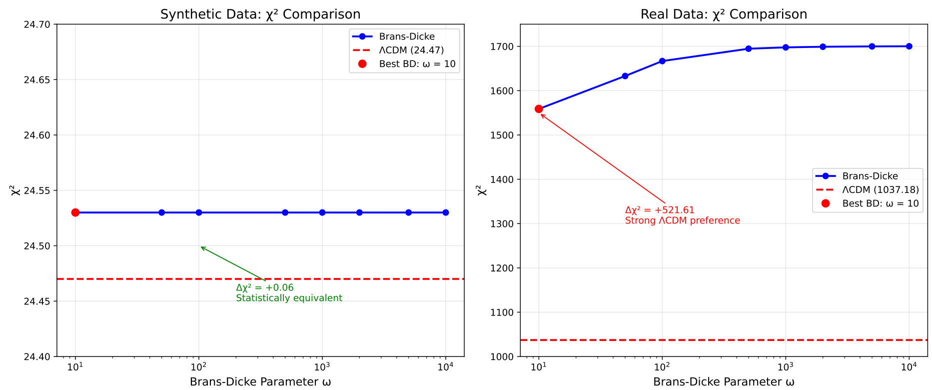

Figure 3: Comparison of χ^2 statistics between synthetic and real data analyses. Left: Synthetic data shows statistical equivalence ($\Delta\chi^2 = +0.06$). Right: Real data shows strong preference for ΛCDM ($\Delta\chi^2 = +521.61$).

datasets (CMB + BAO + supernovae) and exploring whether modified gravity theories can resolve cosmological tensions while remaining consistent with local constraints.

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