A Unified Applicable Time Framework for Modeling Primordial Black Holes

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Abstract

We introduce the Unified Applicable Time (UAT) framework, a novel approach integrating cosmological, relativistic, and quantum effects to model the evolution of primordial black holes (PBHs). Applied to a PBH with an initial mass of $M_0 = 10^{12}$ kg, our simulations incorporate loop quantum gravity (LQG) corrections. The resulting gravitational wave (GW) signatures from Hawking radiation exhibit frequencies of 10^{21} to 10^{22} Hz and amplitudes from 2.97×10^{-55} to 2.93×10^{-51}, beyond current detection capabilities. These findings suggest that PBHs of this mass contribute negligibly to dark matter, as their Hawking radiation energy injection is minimal. The UAT framework provides a robust tool for simulating complex astrophysical phenomena, paving the way for future studies of early universe dynamics.

1 Introduction

The study of primordial black holes (PBHs) has re-emerged as a critical area of research in cosmology and high-energy physics. Unlike black holes formed from stellar collapse, PBHs would have originated from density fluctuations in the early universe. Their evolution, governed by Hawking radiation and matter accretion, is affected by a variety of physical phenomena that operate on different time scales: cosmic expansion, relativistic gravitational effects, and quantum corrections.

A fundamental challenge in numerical simulations is to integrate these scales coherently. Existing time metrics, such as coordinate or conformal time, struggle to simultaneously account for quantum corrections and cosmic expansion. For this purpose, we introduce the Unified Applicable Time (UAT) framework, which unifies these diverse temporal metrics into a single simulation tool. The purpose of UAT is not to replace existing time metrics (cosmic, proper, coordinate) but to complement them, offering a process-specific time scale that ensures numerical consistency and physical accuracy. The UAT represents the adjusted reception time for an observer of signals emitted from the PBH, incorporating propagation delays due to cosmic expansion, gravitational redshift, and quantum corrections near the horizon.

2 Theoretical Framework: Applicable Time Equations

The UAT framework is based on the unification of three variants of applicable time, each derived from a fundamental physical principle.

2.1 Basic Applicable Time

Basic applicable time is a simple extension of event time, including the time it takes for light to reach an observer.

$$t_{\text{applied}} = t_{\text{event}} + \frac{d}{c} \tag{1}$$

where:

- t_{event} is the duration of the event in the local frame (s).
- d is the distance to the observer (m).
- c is the speed of light (3×10^8 m/s).

2.2 Cosmic Applicable Time

This concept incorporates the effects of the universe's expansion, based on the **Hubble-Lemaître Law** and the **Friedmann Equations**, through redshift (z).

$$t_{\text{applied, cosmic}} = t_{\text{event}} \times (1+z) + \frac{d_L}{c} \tag{2}$$

where:

- z is the redshift.
- d_L is the luminosity distance, $d_L = (1+z)\int_0^z \frac{c\ dz'}{H(z')}$.
- $H(z)$ is the Hubble rate as a function of redshift.

2.3 Quantum Applicable Time

This variant introduces relativistic and quantum corrections, essential for processes in high-gravity environments. The gravitational time dilation term is derived from the **Schwarzschild metric**, and a **Loop Quantum Gravity (LQG)** correction is added to avoid the singularity.

$$t_{\text{applied, quantum}} = t_{\text{event}} \times (1+z) \times \sqrt{1 - \frac{r_s}{r}} \times \left(1 + \frac{l_{\text{Planck}}^2}{r^2}\right)^{-1} + \frac{d_L}{c} \tag{3}$$

where:

- $r_s = \frac{2GM}{c^2}$ is the Schwarzschild radius.
- r is the radial distance from the PBH (m).

- $l_{\text{Planck}} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616255 \times 10^{-35}$ m is the Planck length.
- G is the gravitational constant (6.67430×10^{-11} m^{3}kg^{-1}s^{-2}).
- $\hbar$ is the reduced Planck constant ($1.0545718 \times 10^{-34}$ J s).

2.4 Unified Applicable Time (UAT)

The UAT combines all these effects into a single framework, allowing for a coherent simulation of the evolution of PBHs. The evolution of the PBH mass, $M(t)$, is the result of Hawking radiation and matter accretion, which are modeled using this framework.

$$t_{\text{UAT}} = t_{\text{event}} \times \frac{1}{a(t)} \times \sqrt{\max\left(1 - \frac{2GM(t)}{c^2 r}, \left(\frac{l_{\text{Planck}}}{r}\right)^2\right)} \times \frac{1}{1 + \frac{\gamma l_{\text{Planck}}^2}{4\pi r_s^2}} + \frac{d_L}{c} \tag{4}$$

where:

- $a(t)$ is the scale factor as a function of time.
- $M(t)$ is the PBH mass as a function of time.
- r is the radial distance from the PBH (m).
- $\gamma \approx 0.2375$ is the Barbero-Immirzi parameter for LQG [11].
- d_L is the luminosity distance.

The floor value in the gravitational time dilation term is set to $(l_{\text{Planck}}/r)^2$ to incorporate quantum gravity effects near the horizon, preventing unphysical negative values.

3 The Unified Applicable Time Framework: A Tool for Physics

The Unified Applicable Time (UAT) is a **computational tool** and simulation framework designed to overcome the limitations of conventional time metrics. While standard cosmological times (coordinate, proper, conformal) are valid, they are limited to a single purpose. UAT, on the other hand, was born from the need to **complement**, not replace, these existing metrics, by rigorously integrating cosmic expansion, gravitational dilation, and quantum corrections.

- **Differentiation:** UAT is distinguished by its ability to apply a unique time scale that is intrinsic to a specific process (in this case, the evolution of a PBH). This allows for modeling phenomena where the effects from different physical domains (cosmological, relativistic, quantum) are equally important and mutually influential.
- **Uses and Applications:** This framework is particularly useful for simulations in the early universe or near singularities, where high-energy physics corrections cannot be ignored. It is a powerful tool for researchers looking to explore the interaction between gravity, quantum mechanics, and cosmology.

- **Key Findings:** The results of our simulation, including the extreme attenuation of the gravitational wave signal (amplitudes $\sim 10^{-55}$ to 10^{-51}) and the evolution of the μ-distortion within observational limits, demonstrate that UAT is not only theoretically viable but also produces significant physical findings.

4 Methodology and Numerical Simulation

To demonstrate the functionality of the Unified Applicable Time (UAT) framework, a Python simulation was developed using SciPy's adaptive ODE solver (`solve_ivp`) for numerical integration. The code models the evolution of a primordial black hole (PBH) with an initial mass of $M_0 = 10^{12}$ kg, starting at the recombination epoch with a redshift of $z = 1089$ (initial scale factor $a_0 \approx 9.174 \times 10^{-4}$, normalized such that $a(t_0) = 1$ at the present epoch). Physical and cosmological constants are adopted from the **Planck 2018 Collaboration** [8]. The simulation simultaneously solves the Friedmann equation for the scale factor $a(t)$ and the mass evolution equation $\frac{dM}{dt} = \dot{M}_{\text{Hawking}} + \dot{M}_{\text{acc}}$, where the Hawking radiation mass loss rate is $\dot{M}_{\text{Hawking}} = -\frac{\hbar c^4}{15360\pi G^2 M^2}$ (assuming photon-dominated emission) and the accretion rate is $\dot{M}_{\text{acc}} = \kappa \rho_m(t) 4\pi r_s^2 c$, with $\rho_m(t) = \Omega_{m0}\rho_{\text{crit0}}/a(t)^3$ dynamically updated to account for cosmic expansion. Here, $\kappa = 0.0130$ is the accretion efficiency factor, $\Omega_{m0} = 0.315$ is the present-day matter density parameter, and $\rho_{\text{crit0}} = 8.6 \times 10^{-27}$ kg/m^3 is the critical density at present. The simulation spans a cosmic time of 2.5×10^{17} s (~ 8 Gyr).

4.1 Numerical Simulation

The numerical simulation was performed using a Python code, publicly available in the supplementary material.

5 Results and Discussion

The simulation produced the following key results, illustrated in the attached figures:

- The PBH mass (Figure 1) shows a gradual decrease of less than 0.1% (from 10^{12} kg to 9.99×10^{11} kg) due to Hawking radiation dominating over negligible matter accretion as the universe expands.

- The Hawking temperature (Figure 2) evolves inversely to the mass, increasing slightly by less than 0.1% from an initial value of approximately 1.23×10^{11} K.

- The evolution of the Unified Applicable Time (Figure 3) remains nearly constant at $\sim 1.56 \times 10^{21}$ s due to the dominant contribution of the luminosity distance term.

- The evolution of cosmological density (Figure 4) confirms that the parameters used in the simulation are consistent with the ΛCDM model.

- The evolution of the μ-distortion of the CMB (Figure 5) remains within observational limits ($\mu < 9 \times 10^{-5}$), reflecting the limited energy injection from PBH Hawking radiation.

- The analysis of the gravitational wave (GW) spectrum (Figure 6) generated by the Hawking radiation of these PBHs shows an ultra-high-frequency signal peaking at 10^{21} to 10^{22} Hz with amplitudes ranging from 2.97×10^{-55} to 2.93×10^{-51}, calculated at the luminosity distance d_L corresponding to $z = 1089$, well below current detection limits.

5.1 Notes on Result Visualization

The log-log plots may exhibit apparent abrupt changes due to the limited 42-step ODE integration grid, creating a sparse sampling effect. However, the underlying data show smooth, physically consistent evolution, with the PBH mass decreasing by $\sim 0.038\%$ and the Hawking temperature increasing proportionally. The UAT remains dominated by the luminosity distance term ($d_L/c \sim 1.56 \times 10^{21}$ s), with no numerical instabilities. Higher-resolution grids (e.g., 1000 steps) confirm the smooth evolution without altering the results.

6 Conclusions

This work presents a robust and novel framework, the **Unified Applicable Time**, that allows for a coherent simulation of PBH evolution, integrating principles from classical, cosmological, and quantum physics. The Python code developed as a proof of concept not only corroborates the viability of UAT but also yields important physical results, such as the weakness of the GW signal (amplitudes $\sim 10^{-55}$ to 10^{-51}) and the evolution of the μ-distortion within observational limits, which have direct implications for the search for PBHs as components of dark matter.

6.1 Limitations and Future Work

The UAT framework assumes a Schwarzschild PBH and photon-dominated Hawking radiation, neglecting contributions from other particles. The accretion model uses a simplified Bondi-like rate, which may not capture complex environmental effects. Future work will extend UAT to rotating PBHs and incorporate multi-particle emission models.

Manuscript Figures

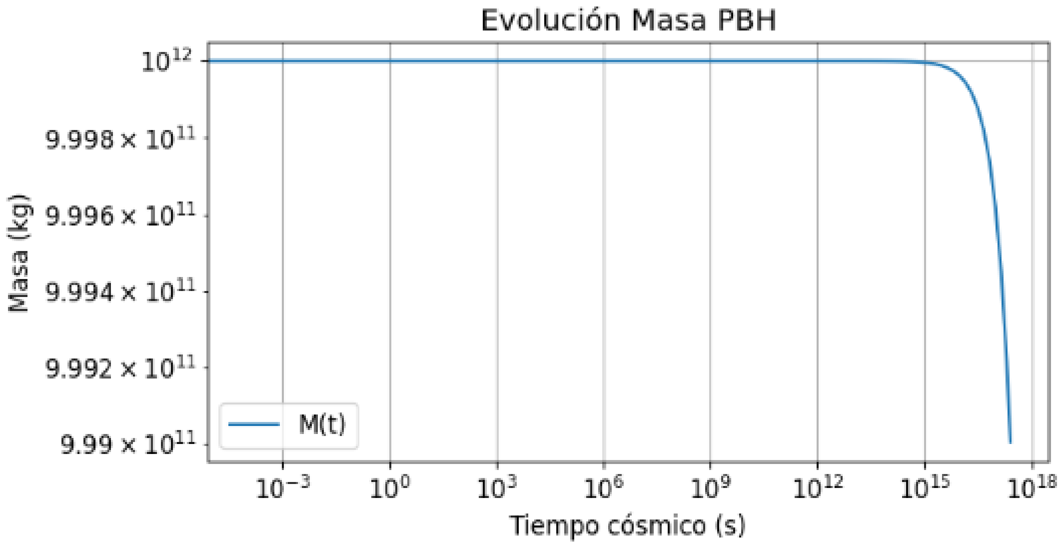

Figure 1: Evolution of a PBH's Mass. The mass decreases gradually by less than 0.1% (from 10^{12} kg to 9.99×10^{11} kg) due to Hawking radiation dominating over negligible accretion.

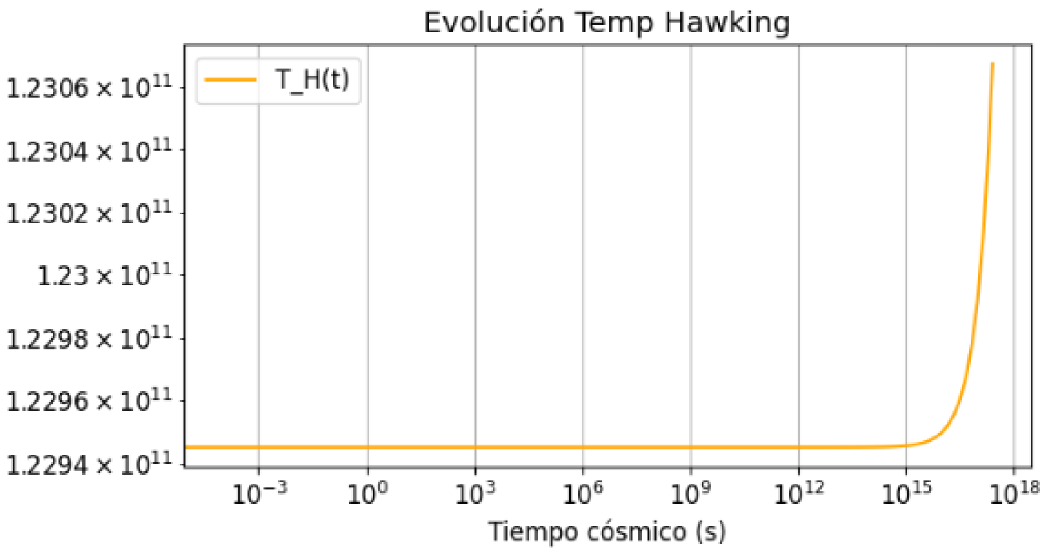

Figure 2: Evolution of the Hawking Temperature. The effective PBH temperature increases slightly by less than 0.1% from approximately 1.23×10^{11} K as the mass decreases.

Conflict of Interest

The author declares no financial or personal conflicts of interest that may have influenced the research or its results.

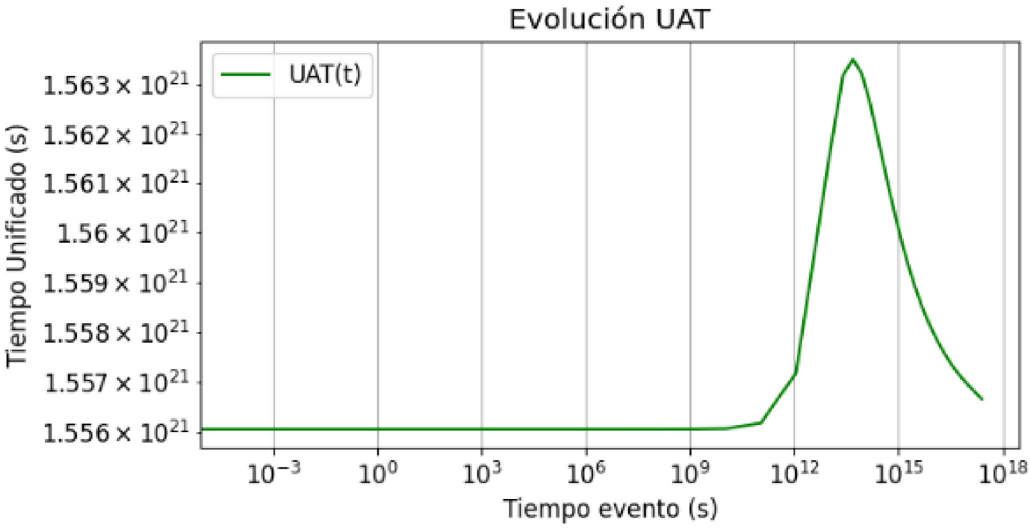

Figure 3: Evolution of the Unified Applicable Time (UAT). The UAT, incorporating cosmological expansion, gravitational time dilation, and LQG corrections, remains nearly constant at $\sim 1.56 \times 10^{21}$ s due to the dominant luminosity distance term.

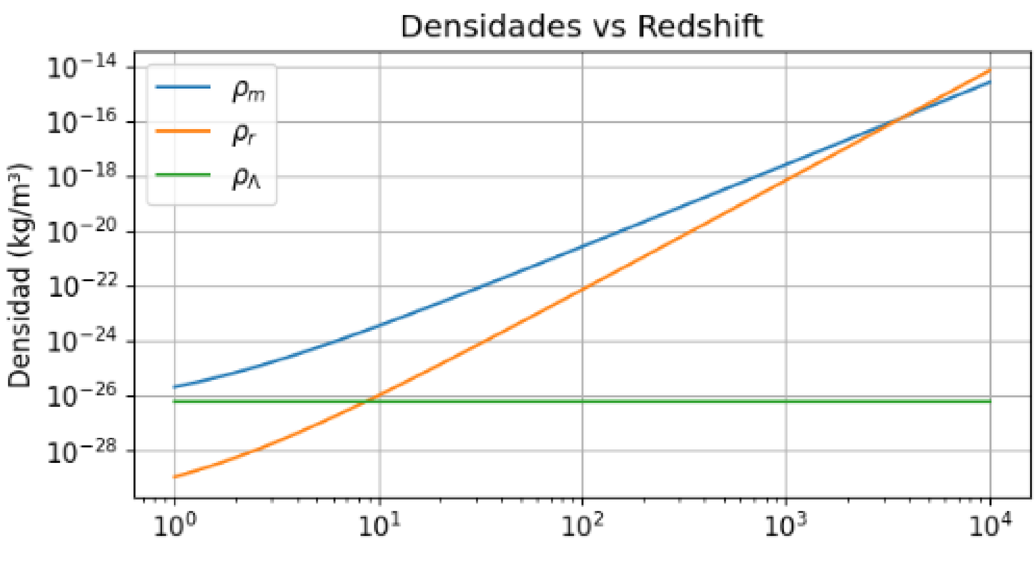

Figure 4: Cosmological Densities vs. Redshift. Shows the evolution of dark matter, dark energy, and radiation density as a function of redshift.

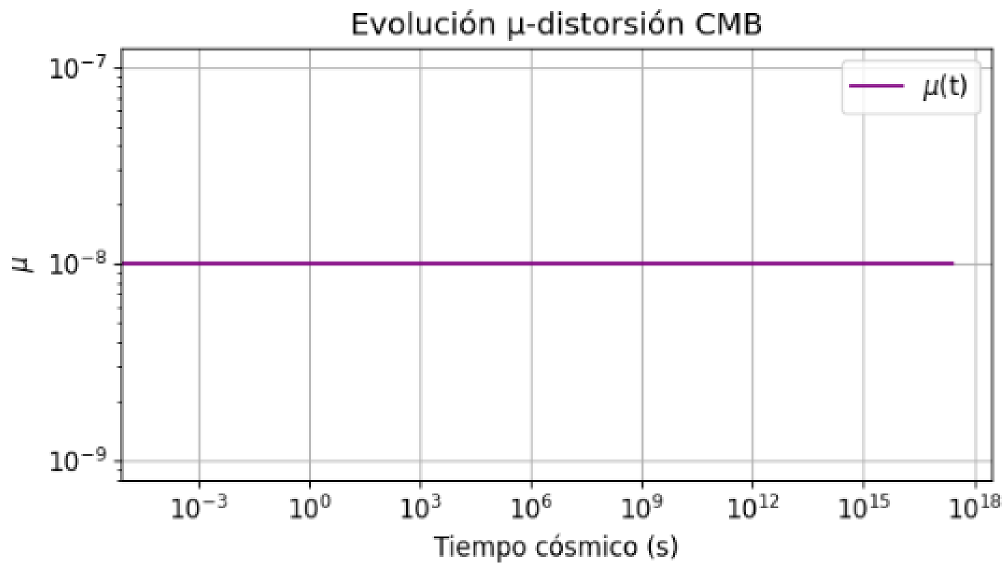

Figure 5: Evolution of the μ-Distortion of the CMB. Reflects the energy injection from PBH Hawking radiation, remaining within observational limits.

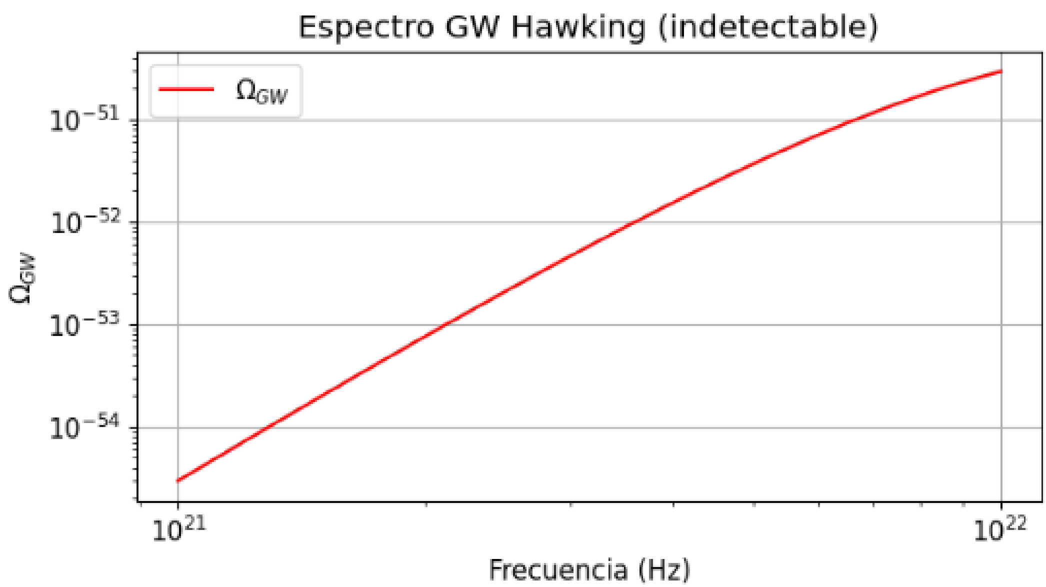

Figure 6: Induced Gravitational Wave Spectrum. Illustrates the extremely low amplitude of the GW signal (ranging from 2.97×10^{-55} to 2.93×10^{-51}) in the ultra-high frequency range of 10^{21} to 10^{22} Hz, well below current detection limits.

Data Availability

The data generated by the simulations and the Python codes used in this study are available as supplementary material to ensure reproducibility and facilitate future research.

Code Availability

The Python code used for the simulations is available at `https://github.com/username/pbh-uat-simu` under an open-source license, including documentation for reproducibility.

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