Supplementary Information 3

Theoretical Justification for the k_{early} Approximation

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1. Purpose and Computational Constraint

The Unified Applicable Time (UAT) Framework is fundamentally based on a complex microphysical hypothesis (detailed in Supplementary Information 1, Eq. 1), which unifies the cosmological scale factor with corrections derived from Loop Quantum Gravity (LQG) and General Relativity.

The direct integration of this complex formulation into standard MCMC cosmological solvers (e.g., Cobaya, CosmoMC) is computationally challenging, as these solvers are optimized for the canonical form of the **Friedmann Equation**, which defines the expansion rate $E(z)^2$. To bridge the gap between the foundational theory (Eq. 1, SI 1) and observational validation via MCMC, a **phenomenological simplification** was required.

2. The Phenomenological Bridge: k_{early}

We introduced the free parameter $\mathbf{k}_{\text{early}}$ as a computational bridge to approximate the net effect of the UAT/LQG corrections on the universe's expansion history. This approximation modifies the standard expansion rate $E(z)^2$ (see Supplementary Information 2, Eq. 1) into the UAT-modified rate ($E_{\text{UAT}}(z)^2$):

$$E_{\text{UAT}}(z, k_{\text{early}})^2 = k_{\text{early}} \cdot \Omega_{r,0}(1+z)^4 + k_{\text{early}} \cdot \Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0} \tag{1}$$

3. Theoretical Justification for Parameter Placement

The crucial physical decision in Equation 1 is that $\mathbf{k}_{\text{early}}$ multiplies the density terms for **radiation (Ω_r) and matter (Ω_m), but explicitly leaves the vacuum energy term ($\Omega_{\Lambda,0}$) untouched. This placement is theoretically mandated by the UAT hypothesis and serves two critical goals:

3.1. Targeting the Early Universe (High Density)

The physical effects derived from the LQG foundation (as referenced in the original UAT equation) are associated with **quantum gravity corrections**, which are expected to be significant only in the **high-density, early universe** ($z \gg 300$). The energy densities of matter and radiation, $\Omega_m(z)$ and $\Omega_r(z)$, dominate the total energy budget during these early epochs. Therefore, multiplying these terms with k_{early} ensures that the new physics acts precisely where the quantum corrections are physically relevant.

3.2. Preserving Late-Time Consistency

The Λ term, representing vacuum energy, dominates the expansion rate at **low redshift** (late times, $z \to 0$). The UAT Framework aims to resolve the Hubble Tension by **reducing the sound horizon (r_d)** (an early-time effect) while **preserving the late-time distance scale consistency** (a late-time effect). By leaving $\mathbf{\Omega_{\Lambda,0}}$ unmultiplied by k_{early}, the model ensures:

- The modification rapidly vanishes at low z.
- The late-time expansion history is preserved, maintaining consistency with local distance ladder measurements, thereby providing a robust solution to the Tension.

In summary, the parameter $\mathbf{k}_{\text{early}}$ acts as a **necessary effective density correction factor** at high z, translating the complexity of the microphysical UAT equation into a functional form that can be rigorously tested against global cosmological data.