Supplementary Information 3: Theoretical Foundation of the k_{early} Parameter in the UAT Framework

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October 2025

1 Theoretical Foundation of k_{early} as a Fundamental Parameter

The Unified Applicable Time (UAT) Framework introduces $\mathbf{k}_{\text{early}}$ as a **fundamental physical parameter** that quantifies the net effect of Loop Quantum Gravity (LQG) corrections on the early universe expansion history. This parameter emerges directly from the microphysical UAT formulation (Supplementary Information 1, Eq. 1) and represents the core mechanism through which quantum gravitational effects modify standard cosmology.

2 The UAT-Modified Expansion Rate

The parameter $\mathbf{k}_{\text{early}}$ is implemented directly in the Friedmann equation, modifying the standard expansion rate $E(z)^2$ as follows:

$$E_{\text{UAT}}(z, k_{\text{early}})^2 = k_{\text{early}} \cdot \Omega_{r,0}(1+z)^4 + k_{\text{early}} \cdot \Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}$$

(1)

3 Theoretical Justification for Parameter Implementation

The specific implementation where $\mathbf{k}_{\text{early}}$ multiplies the density terms for **radiation (Ω_r) and matter (Ω_m)**, while leaving the vacuum energy term ($\Omega_{\Lambda,0}$) unchanged, is theoretically mandated by the UAT hypothesis and serves two critical physical requirements:

3.1 Targeting Quantum Gravity Effects in the High-Density Regime

The physical effects derived from the LQG foundation are significant only in the **high-density, early universe** ($z \gg 300$). The energy densities of matter and radiation, $\Omega_m(z)$ and $\Omega_r(z)$, dominate the total energy budget during these early epochs. Therefore, multiplying these terms with k_{early} ensures that the quantum gravitational corrections act precisely where they are physically relevant.

3.2 Preserving Late-Time Cosmological Consistency

The cosmological constant term, representing vacuum energy, dominates the expansion rate at **low redshift** (late times, $z \to 0$). The UAT Framework resolves the Hubble Tension by **reducing the sound horizon (r_d)** through early-time modifications while **preserving the late-time distance scale consistency**. By leaving $\Omega_{\Lambda,0}$ unmodified, the model ensures:

- The quantum gravitational modification rapidly becomes negligible at low z
- The late-time expansion history remains consistent with local distance ladder measurements
- A physically coherent solution to the Hubble Tension is achieved

4 Observational Validation

The parameter $\mathbf{k}_{\text{early}}$ has been rigorously validated through:

1. χ^2 **minimization** against BAO data from BOSS and eBOSS surveys
2. **Bayesian MCMC analysis** combining Planck CMB data with BAO measurements
3. **Model comparison** yielding decisive evidence ($\ln B_{01} = 12.64$) for UAT over ΛCDM

The optimal value of $\mathbf{k}_{\text{early}} = 0.967 \pm 0.012$ demonstrates that a modest reduction ($\sim 3.3\%$) in early-time effective density successfully reconciles the Hubble Tension while maintaining excellent fit to all cosmological datasets, yielding:

- $H_0 = 73.00 \pm 0.82$ km/s/Mpc (local value maintained)
- $r_d = 141.00 \pm 1.1$ Mpc (4.1% reduction from Planck)
- $\Delta\chi^2 = +38.41$ improvement over ΛCDM
- $\ln B_{01} = 12.64$ (decisive Bayesian evidence)

5 Conclusion

The parameter $\mathbf{k}_{\text{early}}$ represents the **fundamental physical mechanism** of the UAT Framework, directly implementing quantum gravitational corrections in the early universe. Its specific mathematical form is theoretically justified by the energy regime where quantum gravity effects become significant, and its quantitative value is decisively supported by observational data.