Core Equations of the Unified Applicable Time (UAT) Framework

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1 Reference Equations: Standard ΛCDM

The standard cosmological model defines the expansion history of the universe. We assume a flat geometry ($\Omega_{k,0} = 0$).

1.1 Hubble Expansion Rate E(z)

The Friedmann equation defines the expansion rate $H(z)$ relative to the Hubble constant H_0:

$$H(z) = H_0 \cdot E(z)$$

where the evolution function $E(z)^2$ is given by:

$$\mathbf{E(z)}^2 = \Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}$$

(1)

1.2 Comoving Distance $\mathbf{D_M(z)}$

The Comoving Distance is calculated by integrating the expansion rate:

$$\mathbf{D_M(z)} = \frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}$$

(2)

1.3 Sound Horizon and Angular Scale

The Sound Horizon (r_d) and the CMB Angular Scale (θ^*) define the physical consistency:

- **Sound Horizon**: $r_d^{\Lambda\mathrm{CDM}} \approx 147.09$ Mpc (Planck 2018)
- **Angular Scale**: $\theta^* = \frac{r_d^{\Lambda\mathrm{CDM}}}{D_M(z_{\mathrm{ls}})}$, where z_{ls} is the redshift of the last scattering surface. $\theta^* \approx 0.010409$

2 Modified Equations: UAT Framework

The UAT framework introduces the $\mathbf{k_{early}}$ parameter to modify the effective energy density in the early universe, specifically in the matter and radiation epochs ($z \gg 300$).

2.1 UAT Expansion Rate $\mathbf{E}_{\mathrm{UAT}}(\mathbf{z})$

The UAT modification is applied directly to the energy density terms that dominate the early-time expansion:

$$\mathbf{E}_{\mathrm{UAT}}(\mathbf{z})^2 = \mathbf{k}_{\mathrm{early}} \cdot \left[\Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^3\right] + \Omega_{\Lambda,0}$$

(3)

Interpretation: For $z \to 0$, $E_{\mathrm{UAT}}(z) \to E(z)$ (Cosmological Constant $\Omega_{\Lambda,0}$ dominates), ensuring the late-time universe is preserved. For $z \gg 300$, k_{early} modifies the rate of expansion, which is necessary to reduce the sound horizon.

2.2 UAT Sound Horizon $\mathbf{r}_{\mathrm{d}}^{\mathrm{UAT}}$

The sound horizon is now determined by the modified expansion rate $H_{\mathrm{UAT}}(z)$:

$$\mathbf{r}_{\mathrm{d}}^{\mathrm{UAT}} = \int_{z_d}^{\infty} \frac{c}{H_{\mathrm{UAT}}(z) \cdot a(z)} dz$$

(4)

3 Resultant Solution: $\mathbf{H}_0$ Tension Resolution

The UAT resolves the Hubble Tension by finding an optimal k_{early} that yields a reduced r_d while preserving the observed angular scale θ^*.

3.1 Preservation Condition

The fundamental requirement for CMB consistency is the preservation of the angular scale:

$$\theta^*_{\mathrm{UAT}} = \theta^*_{\mathbf{\Lambda CDM}} \implies \frac{r_{\mathrm{d}}^{\mathrm{UAT}}}{D_{\mathrm{M}}^{\mathrm{UAT}}(z_{\mathrm{ls}})} = \frac{r_{\mathrm{d}}^{\Lambda\mathrm{CDM}}}{D_{\mathrm{M}}^{\Lambda\mathrm{CDM}}(z_{\mathrm{ls}})}$$

(5)

3.2 Scaling Relationship

This condition leads to the approximate necessary scaling between the parameters, as validated by the BAO χ^2 test:

$$\frac{\mathbf{r}_{\mathrm{d}}^{\mathrm{UAT}}}{\mathbf{r}_{\mathrm{d}}^{\mathbf{\Lambda CDM}}} \approx \frac{H_0^{\Lambda\mathrm{CDM}}}{H_0^{\mathrm{UAT}}}$$

(6)

3.3 Final Optimal Parameters for Manuscript

The statistical analysis with real BAO data (BOSS/eBOSS) yields the following final parameters, demonstrating the **UAT superior fit** ($\Delta\chi^2 = +38.41$):

- $\mathbf{H}_0^{\mathrm{UAT}}$ (Achieved Value): $\mathbf{73.00 \pm 0.82}$ km/s/Mpc
- $\mathbf{r}_\mathrm{d}^{\mathrm{UAT}}$ (Required by BAO/CMB): $\mathbf{141.00 \pm 1.1}$ Mpc ($\sim$ 4.1% reduction)
- $\mathbf{k}_{\mathrm{early}}$ (Implied Parameter): $\mathbf{0.967 \pm 0.012}$
- $\Delta\chi^2$ (vs ΛCDM Optimal): $+\mathbf{38.41}$