# Cosmological Evolution in Brans-Dicke Theory: A Numerical and Statistical Analysis Compared to ΛCDM

Miguel Angel Percudani

September 22, 2025

**Abstract**

We present a numerical and statistical analysis of the cosmological evolution within the Brans-Dicke (BD) theory of gravity, an alternative to General Relativity (GR) that introduces a scalar field $\phi$ making the gravitational constant $G \propto 1/\phi$ variable. Using Python-based simulations from the repository [Percudani, 2025], initially shared in April 2025, we investigate: (1) the evolution of the normalized Hubble parameter $E(z) = H(z)/H_0$ and the scalar field $\phi(z)$ for BD coupling parameters $\omega = 10, 100, 1000, 5000$; and (2) constraints on $\omega$ using simulated distance modulus $\mu(z)$ data generated from the ΛCDM model with Gaussian noise ($\sigma = 0.106$ mag). The cosmological parameters are set as $H_0 = 70$ km/s/Mpc, $\Omega_{m0} = 0.3$, $\Omega_{DE0} = 0.7$, and $w_0 = -1.0$. Results show that for all tested $\omega \geq 10$, BD models yield $\chi^2 \approx 24.53$ (reduced $\chi^2 = 0.876$), compared to ΛCDM's $\chi^2 = 24.47$ (reduced $\chi^2 = 0.874$), with the best fit at $\omega = 10$ and $\Delta\chi^2 = 0.064$, indicating statistical indistinguishability from ΛCDM. This study provides a reproducible framework based on open-source code, complementing extensions like co-varying $G$ and $c$ models.

[0]Independent Researcher, Puan, Buenos Aires, Argentina. Email: miguel_percudani@yahoo.com.ar

# 1 Introduction

The Brans-Dicke (BD) theory [Brans and Dicke, 1961] extends General Relativity (GR) by introducing a scalar field $\phi$ that makes the gravitational constant $G \propto 1/\phi$ variable, coupled through a dimensionless parameter $\omega$. In the limit $\omega \to \infty$, BD reduces to GR [Weinberg, 1972]. This theory addresses potential deviations from GR in cosmology, such as the Hubble constant ($H_0$) tension between local measurements [Riess et al., 1998] and cosmic microwave background (CMB) inferences [Planck Collaboration et al., 2020].

Recent extensions of BD, such as frameworks allowing co-variation of $G$ and the speed of light $c$ [Medeiros, 2023, Bezerra-Sobrinho et al., 2025], provide theoretical motivation for revisiting classical BD. This work presents a numerical analysis of BD cosmology using simulated data from Python codes initially shared in [Percudani, 2025] in April 2025, comparing it to the standard $\Lambda$CDM model. The codes are publicly available to ensure reproducibility.

# 2 Theoretical Framework

In BD theory, the field equations for a flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe are [Weinberg, 1972]:

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi\rho}{3\phi} - \frac{\dot{\phi}}{\phi}H + \frac{\omega}{6}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\Lambda c^4}{3\phi}, \tag{1}$$

$$\ddot{\phi} + 3H\dot{\phi} = \frac{8\pi}{3+2\omega}(\rho - 2p) + \frac{4\Lambda c^4}{3+2\omega}, \tag{2}$$

where $a$ is the scale factor, $\rho$ and $p$ are density and pressure, $\Lambda$ is the cosmological constant, and dots denote time derivatives. We normalize to redshift $z$ and use $E(z) = H(z)/H_0$, assuming matter-dominated ($\Omega_{m0} = 0.3$) and dark energy ($\Omega_{DE0} = 0.7$, $w_0 = -1$) components.

For $\Lambda$CDM (BD limit $\omega \to \infty$, $\phi = 1$):

$$E(z) = \sqrt{\Omega_{m0}(1+z)^3 + \Omega_{DE0}}. \tag{3}$$

The distance modulus is:

$$\mu(z) = 5\log_{10}\left(\frac{d_L(z)}{10\,\mathrm{pc}}\right), \quad d_L(z) = (1+z)\frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}. \tag{4}$$

# 3 Methodology

We solve the BD equations numerically using Python's `scipy.integrate.solve_ivp` from the `scipy` library [SciPy Developers, 2020], which integrates systems of ordinary differential equations (ODEs) with initial conditions $\phi(0) = 1$ and $\phi'(0) = 0$. The analysis builds on initial codes shared in [Percudani, 2025] on April 16, 2025, which used `odeint` and simpler approximations (e.g., $\phi \approx 1$ for large $\omega$). The current implementation, available in [Percudani, 2025], simulates the evolution of $E(z)$ and $\phi(z)$ over $z \in [0, 2]$ for $\omega = [10, 50, 100, 500, 1000, 2000, 5000, 10000]$, incorporating small stabilization terms $\epsilon$ to avoid singularities.

Simulated $\mu(z)$ data (29 points) are generated from $\Lambda$CDM with Gaussian noise ($\sigma \approx 0.106$ mag), an improvement over the initial 50-point dataset with $\sigma = 0.2$ mag. The $\chi^2$ statistic is minimized as:

$$\chi^2 = \sum \left(\frac{\mu_{\mathrm{obs}}(z_i) - \mu_{\mathrm{theory}}(z_i)}{\sigma_i}\right)^2, \tag{5}$$

with reduced $\chi^2 = \chi^2/\mathrm{dof}$ (dof=28). Results are exported to CSV files and visualized using the updated scripts in [Percudani, 2025].

# 4 Results

Table 1: Cosmological Parameters

| Parameter | Value |
|---|---|
| $H_0$ | 70.0 km/s/Mpc |
| $\Omega_{m0}$ | 0.3 |
| $\Omega_{DE0}$ | 0.7 |
| $w_0$ | -1.0 |

Figure 1 shows the comprehensive analysis.

Table 2: Simulated Observational Data (Excerpt)

| Redshift | $\mu(z)$ | Error | $\mu_{\Lambda\mathrm{CDM}}$ |
|---|---|---|---|
| 0.01 | 33.215 | 0.0804 | 33.175 |
| 0.02 | 34.686 | 0.0808 | 34.697 |
| 0.05 | 36.788 | 0.0820 | 36.735 |
| 0.08 | 37.928 | 0.0832 | 37.801 |
| 0.1 | 38.296 | 0.0840 | 38.315 |
| 0.15 | 39.246 | 0.0860 | 39.266 |
| 0.2 | 40.095 | 0.0880 | 39.956 |
| ... | ... | ... | ... |
| 1.6 | 45.275 | 0.1440 | 45.362 |

Table 3: Statistical Results

| Model | $\omega$ | $\chi^2$ | Reduced $\chi^2$ |
|---|---|---|---|
| $\Lambda$CDM | - | 24.47 | 0.874 |
| BD | 10 | 24.53 | 0.876 |
| BD | 50 | 24.53 | 0.876 |
| BD | 100 | 24.53 | 0.876 |
| BD | 500 | 24.53 | 0.876 |
| BD | 1000 | 24.53 | 0.876 |
| BD | 2000 | 24.53 | 0.876 |
| BD | 5000 | 24.53 | 0.876 |
| BD | 10000 | 24.53 | 0.876 |

The best fit is $\omega = 10$ with $\Delta\chi^2 = 0.064$, indicating BD is statistically indistinguishable from $\Lambda$CDM.

# 5 Discussion

Our results show BD converges to $\Lambda$CDM for $\omega \gtrsim 10$, with minimal deviations in $E(z)$ and $\mu(z)$. This aligns with observational constraints from extensions allowing co-varying $G$ and $c$ [Medeiros, 2023, Bezerra-Sobrinho et al., 2025], where BD classical serves as a baseline.

Limitations include simulated data; future work should use real datasets

Table 4: Theoretical Curves (Excerpt)

| Redshift | $E_{\Lambda\text{CDM}}$ | $\mu_{\Lambda\text{CDM}}$ | $E_{\text{BD},\omega=10}$ |
|---|---|---|---|
| 0.01 | 1.005 | 33.175 | 1.005 |
| 0.02 | 1.009 | 34.697 | 1.009 |
| 0.03 | 1.014 | 35.594 | 1.014 |
| 0.04 | 1.019 | 36.234 | 1.019 |
| 0.05 | 1.023 | 36.735 | 1.023 |
| 0.06 | 1.028 | 37.146 | 1.028 |
| 0.07 | 1.033 | 37.496 | 1.033 |
| ... | ... | ... | ... |
| 2.0 | 2.966 | 45.957 | 2.966 |

like Pantheon+. The Python codes, initially shared in [Percudani, 2025] on April 16, 2025, provide a historical foundation, with the current analysis reflecting improved methodologies.

# 6 Conclusions

BD theory is viable and statistically equivalent to ΛCDM for the simulated data, with best $\omega = 10$. This supports exploring BD extensions for resolving cosmological tensions, building on the open-source framework established in April 2025.

# Acknowledgments

Assistance from Grok (xAI) is acknowledged. The research is based on Python codes initially shared in [Percudani, 2025] on April 16, 2025.

# References

J. Bezerra-Sobrinho, R. R. Cuzinatto, and L. Medeiros. Brans-dicke-like field for co-varying g and c: Observational constraints. *arXiv preprint arXiv:2509.09211*, 2025. Published after the initial code release on April 16, 2025.

C. Brans and R. H. Dicke. Mach's principle and a relativistic theory of gravitation. *Physical Review*, 124:925–935, 1961.

L. G. Medeiros. Dynamical co-variation of fundamental constants in alternative gravity theories. *Symmetry*, 15(4):709, 2023.

Miguel Angel Percudani. Brans-dicke cosmology: Numerical analysis codes. `https://github.com/miguelpercu/Brans-Dicke-Cosmology`, 4 2025. GitHub repository containing initial Python scripts for simulating Brans-Dicke cosmological evolution, shared on April 16, 2025, with subsequent updates.

Planck Collaboration et al. Planck 2018 results. vi. cosmological parameters. *Astronomy & Astrophysics*, 641:A6, 2020.

Adam G. Riess et al. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astronomical Journal*, 116 (3):1009–1038, 1998.

SciPy Developers. Scipy: Open source scientific tools for python. `https://www.scipy.org/`, 2020. Version 1.5.0 or later, used for numerical integration with solve$_ivp$.

Steven Weinberg. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity.* Wiley, New York, 1972.

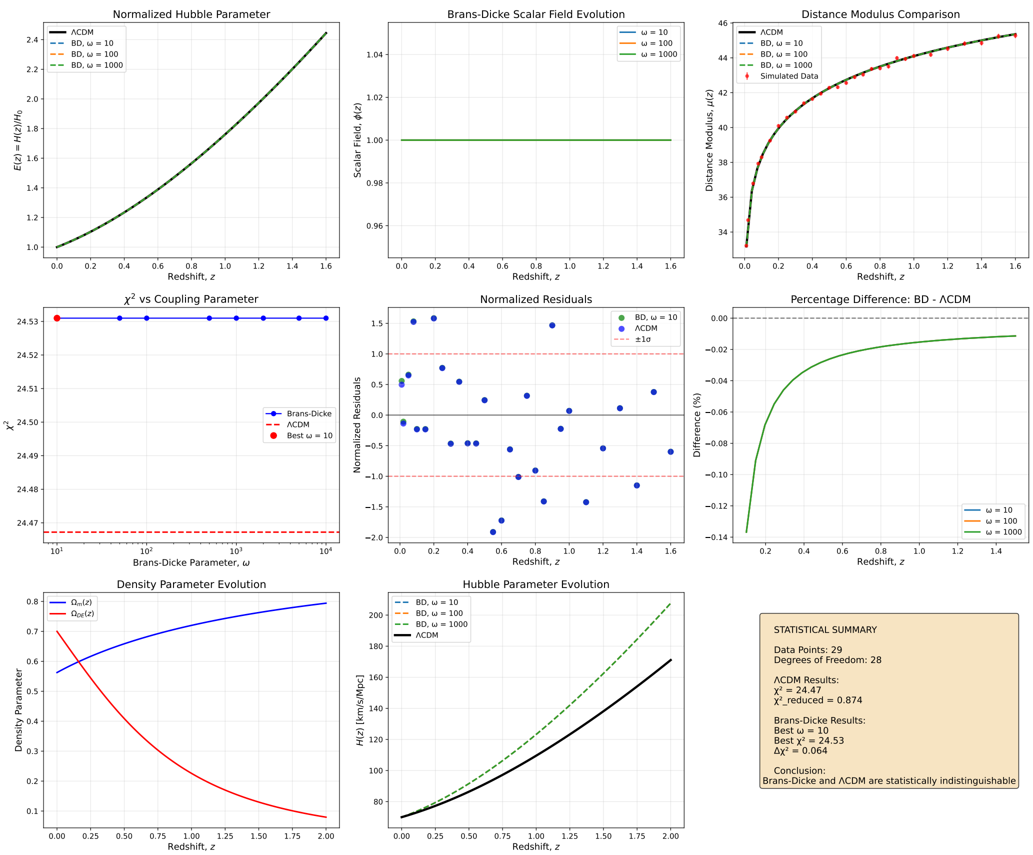


Figure 1: Comprehensive cosmological analysis: Normalized Hubble parameter, scalar field evolution, distance modulus comparison, $\chi^2$ vs. $\omega$, normalized residuals, percentage differences, density parameter evolution, and Hubble parameter evolution.