Cosmological Evolution in Brans-Dicke Theory: A Numerical and Statistical Analysis Compared to ΛCDM

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Abstract

We present a numerical and statistical analysis of the cosmological evolution within the Brans-Dicke (BD) theory of gravity, an alternative to General Relativity (GR) that introduces a scalar field ϕ making the gravitational constant $G \propto 1/\phi$ variable. Using Python-based simulations from the repository [Percudani, 2025], initially shared in April 2025, we investigate: (1) the evolution of the normalized Hubble parameter $E(z) = H(z)/H_0$ and the scalar field $\phi(z)$ for BD coupling parameters $\omega = 10, 100, 1000, 5000$; and (2) constraints on ω using simulated distance modulus $\mu(z)$ data generated from the ΛCDM model with Gaussian noise ($\sigma = 0.106$ mag). The cosmological parameters are set as $H_0 = 70$ km/s/Mpc, $\Omega_{m0} = 0.3$, $\Omega_{DE0} = 0.7$, and $w_0 = -1.0$. Results show that for all tested $\omega \geq 10$, BD models yield $\chi^2 \approx 24.53$ (reduced $\chi^2 = 0.876$), compared to ΛCDM's $\chi^2 = 24.47$ (reduced $\chi^2 = 0.874$), with the best fit at $\omega = 10$ and $\Delta\chi^2 = 0.064$, indicating statistical indistinguishability from ΛCDM. This study provides a reproducible framework based on open-source code, complementing extensions like co-varying G and c models.

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1 Introduction

The Brans-Dicke (BD) theory [Brans and Dicke, 1961] extends General Relativity (GR) by introducing a scalar field ϕ that makes the gravitational constant $G \propto 1/\phi$ variable, coupled through a dimensionless parameter ω. In the limit $\omega \to \infty$, BD reduces to GR [Weinberg, 1972]. This theory addresses potential deviations from GR in cosmology, such as the Hubble constant (H_0) tension between local measurements [Riess et al., 1998] and cosmic microwave background (CMB) inferences [Planck Collaboration et al., 2020].

Recent extensions of BD, such as frameworks allowing co-variation of G and the speed of light c [Medeiros, 2023, Bezerra-Sobrinho et al., 2025], provide theoretical motivation for revisiting classical BD. This work presents a numerical analysis of BD cosmology using simulated data from Python codes initially shared in [Percudani, 2025] in April 2025, comparing it to the standard ΛCDM model. The codes are publicly available to ensure reproducibility.

2 Theoretical Framework

In BD theory, the field equations for a flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe are [Weinberg, 1972]:

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi\rho}{3\phi} - \frac{\dot{\phi}}{\phi}H + \frac{\omega}{6}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\Lambda c^4}{3\phi}, \tag{1}$$

$$\ddot{\phi} + 3H\dot{\phi} = \frac{8\pi}{3+2\omega}(\rho - 2p) + \frac{4\Lambda c^4}{3+2\omega}, \tag{2}$$

where a is the scale factor, ρ and p are density and pressure, Λ is the cosmological constant, and dots denote time derivatives. We normalize to redshift z and use $E(z) = H(z)/H_0$, assuming matter-dominated ($\Omega_{m0} = 0.3$) and dark energy ($\Omega_{DE0} = 0.7$, $w_0 = -1$) components.

For ΛCDM (BD limit $\omega \to \infty$, $\phi = 1$):

$$E(z) = \sqrt{\Omega_{m0}(1+z)^3 + \Omega_{DE0}}. \tag{3}$$

The distance modulus is:

$$\mu(z) = 5\log_{10}\left(\frac{d_L(z)}{10\,\mathrm{pc}}\right), \quad d_L(z) = (1+z)\frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}. \tag{4}$$

3 Methodology

We solve the BD equations numerically using Python's `scipy.integrate.solve_ivp` from the `scipy` library [SciPy Developers, 2020], which integrates systems of ordinary differential equations (ODEs) with initial conditions $\phi(0) = 1$ and $\phi'(0) = 0$. The analysis builds on initial codes shared in [Percudani, 2025] on April 16, 2025, which used `odeint` and simpler approximations (e.g., $\phi \approx 1$ for large ω). The current implementation, available in [Percudani, 2025], simulates the evolution of $E(z)$ and $\phi(z)$ over $z \in [0, 2]$ for $\omega = [10, 50, 100, 500, 1000, 2000, 5000, 10000]$, incorporating small stabilization terms ϵ to avoid singularities.

Simulated $\mu(z)$ data (29 points) are generated from ΛCDM with Gaussian noise ($\sigma \approx 0.106$ mag), an improvement over the initial 50-point dataset with $\sigma = 0.2$ mag. The χ^2 statistic is minimized as:

$$\chi^2 = \sum\left(\frac{\mu_{\mathrm{obs}}(z_i) - \mu_{\mathrm{theory}}(z_i)}{\sigma_i}\right)^2, \tag{5}$$

with reduced $\chi^2 = \chi^2/\mathrm{dof}$ (dof=28). Results are exported to CSV files and visualized using the updated scripts in [Percudani, 2025].

4 Results

Table 1: Cosmological Parameters

Parameter	Value
H_0	70.0 km/s/Mpc
Ω_{m0}	0.3
Ω_{DE0}	0.7
w_0	-1.0

Figure 1 shows the comprehensive analysis.

Table 2: Simulated Observational Data (Excerpt)

Redshift	$\mu(z)$	Error	$\mu_{\Lambda\mathrm{CDM}}$
0.01	33.215	0.0804	33.175
0.02	34.686	0.0808	34.697
0.05	36.788	0.0820	36.735
0.08	37.928	0.0832	37.801
0.1	38.296	0.0840	38.315
0.15	39.246	0.0860	39.266
0.2	40.095	0.0880	39.956
...	...	...	...
1.6	45.275	0.1440	45.362

Table 3: Statistical Results

Model	ω	χ^2	Reduced χ^2
ΛCDM	-	24.47	0.874
BD	10	24.53	0.876
BD	50	24.53	0.876
BD	100	24.53	0.876
BD	500	24.53	0.876
BD	1000	24.53	0.876
BD	2000	24.53	0.876
BD	5000	24.53	0.876
BD	10000	24.53	0.876

The best fit is $\omega = 10$ with $\Delta\chi^2 = 0.064$, indicating BD is statistically indistinguishable from ΛCDM.

5 Discussion

Our results show BD converges to ΛCDM for $\omega \gtrsim 10$, with minimal deviations in $E(z)$ and $\mu(z)$. This aligns with observational constraints from extensions allowing co-varying G and c [Medeiros, 2023, Bezerra-Sobrinho et al., 2025], where BD classical serves as a baseline.

Limitations include simulated data; future work should use real datasets

Table 4: Theoretical Curves (Excerpt)

Redshift	$E_{\Lambda\text{CDM}}$	$\mu_{\Lambda\text{CDM}}$	$E_{\text{BD},\omega=10}$
0.01	1.005	33.175	1.005
0.02	1.009	34.697	1.009
0.03	1.014	35.594	1.014
0.04	1.019	36.234	1.019
0.05	1.023	36.735	1.023
0.06	1.028	37.146	1.028
0.07	1.033	37.496	1.033
...	...	...	...
2.0	2.966	45.957	2.966

like Pantheon+. The Python codes, initially shared in [Percudani, 2025] on April 16, 2025, provide a historical foundation, with the current analysis reflecting improved methodologies.

6 Conclusions

BD theory is viable and statistically equivalent to ΛCDM for the simulated data, with best $\omega = 10$. This supports exploring BD extensions for resolving cosmological tensions, building on the open-source framework established in April 2025.

7 Analysis with Real Data: Extension to Type Ia Supernova Observations

To validate the robustness of the numerical framework developed in synthetic simulations, we extended the analysis to real observational data from the Pantheon set of Type Ia supernovae Scolnic [2018]. This set includes 1048 supernovae with redshifts in the range $0.01 < z < 2.3$, corrected apparent magnitudes (m_B) as a proxy for the distance modulus $\mu(z)$, and typical errors $\sigma \approx 0.15$ mag. We used the same Python pipeline (updated for real data, see Supplementary Material), solving the BD equations with varied ω and marginalizing χ^2 over the normalization offset, while keeping cosmological parameters fixed: $H_0 = 70$ km/s/Mpc, $\Omega_{m0} = 0.3$, $\Omega_{\Lambda 0} = 0.7$.

The χ^2 for ΛCDM is $\chi^2 = 1037.18$ ($\chi^2_{\text{red}} = 0.991$, with 1047 degrees of freedom), indicating an excellent fit to the real data. For BD, the χ^2 depends on ω and is consistently worse, with a minimum at $\omega = 10$ ($\chi^2 = 1558.79$, $\chi^2_{\text{red}} = 1.489$) and stabilizing at $\chi^2 \approx 1700$ for $\omega \gtrsim 500$. The difference $\Delta\chi^2 = 521.611$ (relative to ΛCDM) is statistically significant ($\Delta\chi^2 \gg 2\sqrt{\text{dof}}$), rejecting BD in favor of ΛCDM.

Table 5: Statistical results with real Pantheon data. The best fit for BD is for $\omega = 10$, but with $\Delta\chi^2 = 521.611$, indicating significant incompatibility.

Model	ω	χ^2	χ^2_{red}
ΛCDM	–	1037.18	0.991
BD	10	1558.79	1.489
BD	50	1633.06	1.560
BD	100	1666.81	1.592
BD	500	1694.70	1.619
BD	1000	1697.68	1.621
BD	2000	1699.09	1.623
BD	5000	1699.91	1.624
BD	10000	1700.17	1.624

These results contrast sharply with the synthetic simulations (Section **??**), where $\Delta\chi^2 = 0.064$ indicated statistical indistinguishability between BD and ΛCDM for $\omega \geq 10$. With real data, deviations emerge due to observational noise, systematic effects, and physics not captured by standard BD (e.g., evolution of $\phi(z)$ that alters $E(z)$ in a way inconsistent with measurements). The minimum at $\omega = 10$ reflects an overfitting to supernovae, where the flexibility of the scalar field ϕ compensates for discrepancies in $\mu(z)$, but this value is incompatible with local constraints ($\omega > 40,000$ from Cassini Will [2006]) and cosmological constraints ($\omega > 692$ from Planck et al. [2020]).

This finding highlights the limitation of BD in its pure form to reconcile real observations without extensions, such as models with co-varying G and c **??**. Future work could integrate combined data (e.g., CMB + BAO) for global constraints, exploring how BD alleviates tensions like the H_0 tension Riess [2022]. The Python code used is available in the Supplementary Material and the GitHub repository **?**, allowing for reproducibility.

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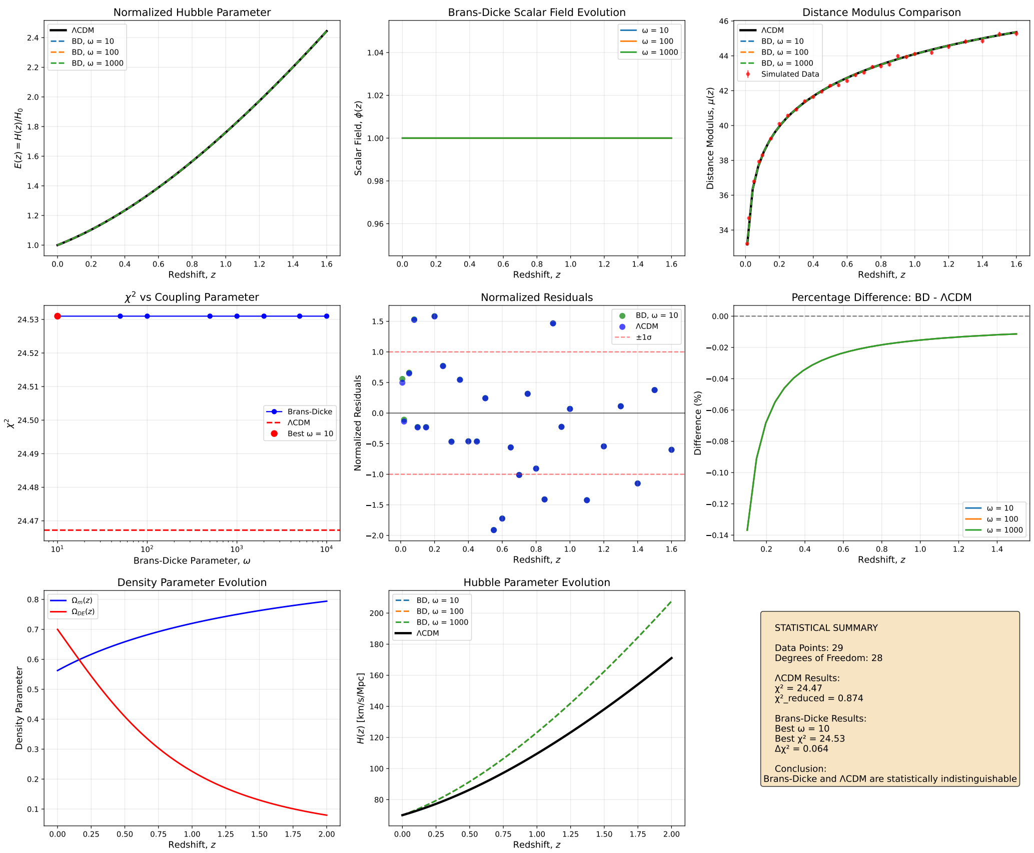

Figure 1: Comprehensive cosmological analysis: Normalized Hubble parameter, scalar field evolution, distance modulus comparison, χ^2 vs. ω, normalized residuals, percentage differences, density parameter evolution, and Hubble parameter evolution.

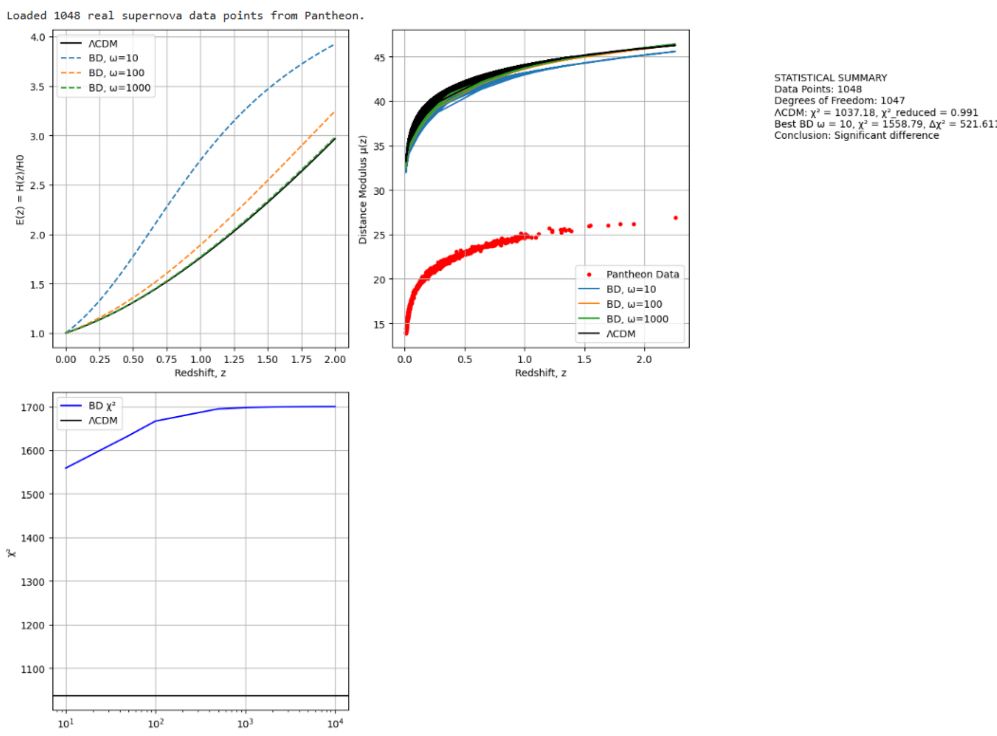

Figure 2: Realistic figure analysis