Re-evaluation of the Applicable Time Framework: A Unified Approach for Primordial Black Holes, Cosmology, and Quantum Gravity

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Abstract

We present a comprehensive numerical study on the evolution of primordial black holes (PBHs) with an initial mass of 10^{12} kg in the early universe, specifically during the recombination epoch ($z = 1089$). We introduce a novel concept: the "applicable time" ($t_{applied}$), and its formalization as the Unified Applicable Time (TAT) Framework. This framework adjusts the temporal scale of simulations to comprehensively incorporate cosmological conditions (redshift), relativistic effects (spacetime curvature near PBHs, based on the Schwarzschild metric), and quantum corrections (at Planck and Loop Quantum Gravity scales).

Our simulations detail key dynamic processes such as Hawking radiation, the accumulation of dark matter (PDM) and dark energy (PDE) in the PBH's vicinity, and their impact on the cosmic microwave background (CMB) and gravitational wave (GW) backgrounds. Results indicate that, under realistic density constraints ($f_{PBH} \leq 0.1$), PBHs have a negligible effect on the CMB, with a spectral distortion parameter $y \approx 1.09 \times 10^{-23}$ and a change in ionization fraction $\Delta x_e \approx 1.03 \times 10^{-23}$. These findings provide stringent upper limits for CMB distortions and offer a unified temporal framework that serves as a robust tool for modeling cosmological phenomena under extreme conditions. The utility of TAT for unifying general relativity, cosmology, and quantum mechanics is discussed, and its properties are compared with conventional temporal frameworks.

Primordial Black Holes, Applicable Time, Hawking Radiation, Cosmic Microwave Background, Gravitational Waves, Unification, General Relativity, Quantum Mechanics, Cosmology

1 Introduction

The study of phenomena in the early universe and near extreme gravitational objects, such as black holes, presents significant challenges for conventional temporal frameworks. These environments involve a complex interplay between cosmological expansion, intense gravitational fields, and quantum effects. Traditional time measures often fail to simultaneously account for these diverse physical influences, limiting their ability to model dynamic processes with the required precision in modern theoretical physics.[?, ?] For example, proper time diverges near singularities, while comoving time does not capture local gravitational dynamics.[?]

To overcome these limitations, the "Applicable Time" framework has been introduced as an innovative conceptual tool. This framework provides a constructed time scale that adjusts the duration of a local event as perceived by a distant observer, integrating the relevant physics from the event to the observation.[?, ?] This concept has evolved progressively, starting with basic considerations of light travel time and advancing towards comprehensive formulations that incorporate cosmological, relativistic, and quantum corrections.[?]

This report carries out a comprehensive re-evaluation of the Applicable Time framework. It will detail its conceptual evolution, its mathematical formulations, and its practical applications in numerical simulations of Primordial Black Holes (PBH) and in cosmological parameter studies. A critical comparison with traditional temporal concepts will be made, its robustness will be assessed, its predictive capabilities against observational limits will be discussed, and its broader implications for quantum gravity and the unification of fundamental theories will be synthesized.

2 The Applicable Time Framework: Evolution and Formulation

This section systematically details the conceptual evolution and mathematical formulations of the Applicable Time framework, highlighting the progressive integration of physical complexities, from simple light travel time to unified quantum-gravitational-cosmological effects.

2.1 Basic Applicable Time ($t_{applied}$)

The most elemental formulation of the Applicable Time framework focuses on adjusting the duration of an event by considering only the time it takes for light to travel from the event to the observer.[? ? ?] This variant represents the simplest form, operating in a flat, non-expanding spacetime, and taking into account exclusively the finite speed of light.[? ?]

The equation defining the Basic Applicable Time is:

$$t_{applied} = t_{event} + \frac{d}{c} \tag{1}$$

Where:

- t_{event}: Is the duration of the event in the local frame (in seconds).[? ?]
- d: Represents the distance to the observer (in meters).[? ?]
- c: Is the speed of light, whose value is $3 \times 10^8 m/s$.[? ?]

The physical purpose of this formulation is to synchronize the intrinsic duration of an event with its duration perceived by a distant observer, considering only the finite speed of information propagation.[?] For example, if an event has a duration of 1 second ($t_{event} = 1s$) and is located at a distance of 3×10^8 meters ($d = 3 \times 10^8 m$), the basic applicable time would be $t_{applied} = 1s + (3 \times 10^8 m/3 \times 10^8 m/s) = 1s + 1s = 2s$.[?]

The term d/c in this equation is fundamental, as it represents the time it takes for light to travel a given distance. In physics, the speed of light sets the maximum limit for information transfer, which in turn defines causality. Therefore, this initial formulation, despite its simplicity, directly addresses the causal link between an event and its observation. This principle of causality underlies all subsequent, more complex formulations, by ensuring that the observer's perception of "now" is causally connected to the event's "then." This highlights that the framework is built upon fundamental relativistic principles from its conception, even before explicitly introducing general relativity or cosmological factors.

2.2. Cosmic Applicable Time ($t_{applied,cosmic}$)

The next stage in the evolution of the Applicable Time framework introduces the effect of the expanding universe, incorporating redshift. This variant is crucial for cosmological observations, as it explains how time intervals are stretched due to the expansion of space.[? ? ?]

The equation for Cosmic Applicable Time is:

$$t_{applied,cosmic} = t_{event} \times (1 + z) + \frac{d_L}{c} \tag{2}$$

Where:

- z: Is the cosmological redshift.[? ?]
- d_L: Is the luminosity distance, defined as $d_L = (1 + z) \int_0^z \frac{c\ dz'}{H(z')}$.[?]
- $H(z)$: Represents the Hubble rate as a function of redshift.[?]

The physical purpose of this formulation is to account for cosmological time dilation (the stretching of time intervals) due to the expansion of space, ensuring that the perceived duration of distant events is accurately represented.[?] For example, for an event with $t_{event} = 1s$ occurring at a redshift $z = 1089$, and assuming a distance $d = 3 \times 10^8 m$ (where d/c simplifies to 1s), the cosmic applicable time would be $t_{applied} = 1s \times (1 + 1089) + 1s = 1091s$.[?]

The addition of the factor $(1 + z)$ to t_{event} directly scales the local event duration by the cosmological expansion. This is a fundamental concept in observational cosmology, where distant events are observed to unfold more slowly due to the universe's expansion. The term d_L/c, while still representing light travel time, uses luminosity distance, which is a cosmological distance measure. Thus, this variant explicitly bridges the local event frame (t_{event}) and the global expanding cosmological background, which is a crucial step for realistic astrophysical modeling. This formulation transcends the notion of a static spacetime, allowing the framework to describe phenomena within the dynamic and evolving universe, which is indispensable for studying events in the early universe.

2.3. Quantum Applicable Time ($t_{applied,quantum}$)

This version of the Applicable Time framework introduces gravitational and quantum corrections, essential for modeling processes occurring near singularities, such as those associated with PBHs. Its goal is to bridge general relativity and quantum mechanics in extreme gravitational environments.[? ? ?]

The equation for Quantum Applicable Time is:

$$t_{applied,quantum} = t_{event} \times (1+z) \times \sqrt{1 - \frac{r_s}{r}} \times \left(1 + \frac{l_{Planck}^2}{r^2}\right)^{-1} + \frac{d_L}{c} \quad (3)$$

Where:

- $r_s = \frac{2GM}{c^2}$: Is the Schwarzschild radius.[? ?]
- r: Represents the radial distance from the PBH (in meters).[? ?]
- $l_{Planck} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616 \times 10^{-35} m$: Is the Planck length.[? ?]
- G: Is the gravitational constant ($6.67430 \times 10^{-11} m^3 kg^{-1} s^{-2}$).[? ?]
- $\hbar$: Is the reduced Planck constant ($1.0545718 \times 10^{-34} J\ s$).[? ?]

The physical purpose of this formulation is to account for both gravitational time dilation near massive objects and quantum effects at extremely small (Planck) scales, providing a more complete description of time in extreme environments where classical relativity breaks down.[? ?] For example, for a PBH with $M = 10^{12} kg$, a $t_{event} = 1s$, $z = 1089$, $d = 3 \times 10^8 m$ and $r = 10 r_s$, the $t_{applied,quantum}$ is calculated at approximately $1035.083s$.[? ?]

The introduction of the term $\sqrt{1 - r_s/r}$ directly models gravitational time dilation, a central prediction of General Relativity. As r approaches r_s, this term tends to zero, implying infinite time dilation in a classical sense. However, the quantum correction $(1 + l_{Planck}^2/r^2)^{-1}$ becomes relevant at Planck scales. If r approaches l_{Planck}, this term modifies the behavior, preventing a classical divergence. This double-correction mechanism is crucial because it addresses the breakdown of classical General Relativity at singularities and attempts to incorporate quantum effects where spacetime curvature becomes extreme. This variant constitutes a direct attempt to build a phenomenological bridge between General Relativity and Quantum Mechanics, especially pertinent for understanding the interior of black holes and the very early universe, where both theories are essential. It suggests a possible resolution of classical singularities by introducing a quantum regularization.

2.4. Unified Applicable Time (TAT) ($t_{unified}$)

The most complete formulation of the framework, the Unified Applicable Time, dynamically integrates more refined cosmological, relativistic, and quantum effects, inspired by Loop Quantum Gravity (LQG). It provides a holistic view of temporal evolution under the combined influence of cosmic expansion, strong gravitational fields, and quantum corrections.[? ? ?]

The equation for Unified Applicable Time is:

$$t_{unified} = t_{event} \times \frac{1}{a(t)} \times \sqrt{\max\left(1 - \frac{2GM(t)}{c^2 r}, 10^{-10}\right)} \times \frac{1}{1 + \frac{A_{min}}{4\pi r_s(t)^2}} + \frac{d_{comoving}}{c} \tag{4}$$

Where:

- $a(t) = \frac{1}{1+z}$: Is the cosmological scale factor.[?,?]
- $M(t) = M_0(1 - \frac{t}{\tau})^{1/3}$ with $\tau = \frac{5120\pi G^2 M_0^3}{\hbar c^4}$: Represents the dynamic mass of the PBH.[?,?]
- $r_s(t) = \frac{2GM(t)}{c^2}$: Is the dynamic Schwarzschild radius.[?,?]
- $A_{min} = 4\sqrt{3}\pi\gamma l_{Planck}^2$, with $\gamma = 0.2375$: Is the minimum area from LQG.[?,?]
- $d_{comoving} = c\int_0^z \frac{dz'}{H(z')}$: Is the comoving distance.[?,?]

The physical purpose of this formulation is to provide a robust and dynamic framework for modeling processes under the combined and evolving influences of cosmic expansion, strong gravitational fields, and fundamental quantum gravity effects, which is particularly crucial for the long-term evolution of PBHs.[?,?] For example, for a PBH with $M_0 = 10^{12} kg$ at $z = 1089$, with $r = 10r_s$ and a $t_{event} = 1s$, the $t_{unified}$ is calculated at approximately $1.433 \times 10^{18} s$. This value is dominated by the comoving distance term.[?,?]

The calculation for $t_{unified}$ clearly reveals that the $d_{comoving}/c$ term overwhelmingly dominates the total value (approximately $10^{18} s$ versus approximately $10^3 s$ for the local event term). This implies that for events observed from a cosmologically distant observer (like us observing the recombination epoch), the time it takes for light to travel across the expanding universe is the primary factor determining the observed "applicable time." The local event duration, even with relativistic and quantum corrections, becomes a minor perturbation on this vast cosmological timescale. However, the inclusion of $M(t)$ and $r_s(t)$ gives the framework a dynamic character, allowing for the modeling of PBH evaporation over cosmic timescales. The A_{min} term, rooted in LQG, provides a more theoretically sound quantum correction compared to the phenomenological l_{Planck}^2/r^2 term of $t_{applied,quantum}$. The $\max(..., 10^{-10})$ function is a numerical stabilization, indicating that the model anticipates and elegantly handles regions where classical General Relativity would break down or lead to complex numbers. Consequently, while cosmological effects dominate the magnitude of unified time for distant observations, relativistic and quantum terms are critical for the physics of the local event, particularly for phenomena like PBH evaporation and singularity resolution. The framework thus provides a tool that bridges scales, where the large-scale cosmological context sets the observed temporal scale, but microphysical processes are precisely modeled by quantum gravity corrections. This also underscores the practical need for numerical stability when dealing with extreme physical regimes.

3 Comparative Analysis with Traditional Temporal Frameworks

This section critically compares the Applicable Time framework with conventional temporal concepts in astrophysics and relativity, highlighting the unique advantages and fundamental differences that justify its development.

3.1. Comoving Time ($t_{comoving}$)

Comoving time is conceptualized as the universe's "master clock," measured by observers moving with the Hubble flow, and is used to describe the global evolution of the cosmos.[?,?] Its definition is related to the scale factor $a(t)$ through the Friedmann equation: $dt_{comoving} = \frac{da}{H(a)a}$.[?]

Despite its utility for modeling large-scale cosmological evolution, comoving time presents significant limitations. It does not directly incorporate local effects, such as gravitational time dilation near massive objects, nor specific quantum corrections, nor the light travel time to a particular observer for a given event.[?,?]

In contrast, the Applicable Time framework (and its variants) modulates the duration of a local event with specific effects, such as cosmological expansion,

Table 1: Summary of Applicable Time Framework Variants

Time Framework Variant	**Equation**	**Key Components/-Factors**	**Primary Physical Phenomena Modeled**	**Typical Value (for** $t_{event} = 1s, z = 1089, d = 3 \times 10^8 m$**)**
Basic ($t_{applied}$)	$t_{event} + \frac{d}{c}$	t_{event}, d, c	Light travel time, Foundational Causality	$2s$ (for $d = 3 \times 10^8 m$)
Cosmic ($t_{applied,cosmic}$)	$t_{event} \times (1+z) + \frac{d_L}{c}$	$t_{event}, z, d_L, c, H(z)$	Cosmic expansion, Cosmological time dilation	$1091s$
Quantum ($t_{applied,quantum}$)	$t_{event} \times (1+z) \times \sqrt{1-\frac{r_s}{r}} \times (1 + \frac{l_{Planck}^2}{r^2})^{-1} + \frac{d_L}{c}$	$t_{event}, z, r_s, r, l_{Planck}, G, \hbar, d_L, c$	Gravitational time dilation, Planck-scale quantum effects, Singularity regularization	$1035.083s$
Unified (TAT) ($t_{unified}$)	$t_{event} \times \frac{1}{a(t)} \times \sqrt{\max\left(1-\frac{2GM(t)}{c^2 r}, 10^{-10}\right)} \times \frac{1}{1+\frac{A_{min}}{4\pi r_s(t)^2}} + \frac{d_{comoving}}{c}$	$t_{event}, a(t), M(t), \tau, r_s(t), A_{min}, \gamma, d_{comoving}, c$	Dynamic cosmic expansion, Strong gravity, Quantum gravity (LQG), Dynamic mass evolution	$1.433 \times 10^{18} s$ (dominated by $d_{comoving}/c$)

local gravity, quantum phenomena, and observer distance, aspects that comoving time does not encompass.[?] This fundamental difference lies in the fact that comoving time is inherently a global, large-scale cosmological coordinate. It is excellent for describing the expansion of the universe as a whole, but it lacks the granularity needed to capture specific phenomena of local gravitational wells or quantum interactions. The Applicable Time framework, by explicitly incorporating terms like r_s/r and l_{Planck}/r, operates at much smaller, localized scales. This highlights a fundamental difference in its intended domain of application: one for large-scale cosmic structure, the other for cosmic singularities. This mismatch of scales underscores the necessity of the Applicable Time framework to bridge macro-cosmology and micro-physics, a central theme in modern theoretical physics.

3.2. Proper Time (τ)

Proper time represents the time measured by a clock that moves along with a particle or observer, being a purely local measure.[? ?] Near a PBH, using the Schwarzschild metric, it is defined as $d\tau = \sqrt{1-\frac{r_s}{r}}dt$.[?]

Although proper time is ideal for local observers, it presents critical limitations. It is the "actual duration" of the event in its own frame, but it contains no information about how cosmic expansion or distance will affect the observation of that event. Furthermore, and crucially, proper time diverges near singularities ($r \to r_s$), where the factor $\sqrt{1-\frac{r_s}{r}}$ tends to zero, leading to extreme time dilation.[? ?]

In contrast, Applicable Time begins with proper time (as t_{event}) and "dresses" it with the effects that occur between the source and the observer, such as cosmological expansion and light travel time. Quantum Applicable Time, crucially, remains finite near singularities by incorporating quantum corrections, unlike proper time which diverges.[? ?] Proper time is an intrinsic, invariant measure of time for an object or observer in its own frame, independent of external observers. Applicable Time, conversely, is explicitly observer-dependent, incorporating factors such as redshift and observer distance. This highlights a shift in perspective: from an intrinsic property of the event itself to how that event is perceived and measured by a distant observer in a dynamic universe. The divergence of proper time at singularities, a classical problem, is directly addressed by quantum corrections in Applicable Time, suggesting a physical regularization. Therefore, the Applicable Time framework is inherently designed for "observational cosmology" from a theoretical standpoint, providing a tool to predict what a distant observer would measure, rather than what simply occurs locally. Its ability to handle singularities suggests a path for quantum gravity theories to resolve classical infinities.

3.3. Conformal Time (η)

Conformal time is a mathematical tool used in cosmology to simplify the FLRW metric, so that light cones appear as 45-degree lines.[?, ?] It is defined as $d\eta = \frac{dt}{a(t)}$, where $a(t)$ is the scale factor.[?]

Despite its utility for studying light propagation and causal structures in the early universe, conformal time is not designed to describe the "perceived duration" of an event. It does not incorporate local gravitational effects nor is it linked to a signal observed by a specific observer.[?, ?]

Unlike conformal time, which is a mathematical construct for global causal analysis, Applicable Time is a physical measure of perceived duration. It integrates local gravitational and quantum effects relevant to observation.[?] Conformal time is primarily a theoretical convenience for simplifying spacetime metrics and analyzing light cone structures; it is an abstract coordinate. Applicable Time, however, aspires to be a physically measurable quantity, directly linking theoretical constructs with possible observations. This distinction is crucial for bridging theoretical cosmology and empirical astrophysics. The Applicable Time framework aims to be a more direct tool for interpreting observational data, translating abstract theoretical models into concrete predictions of what instruments would detect.

4 Methodology: Simulation of Primordial Black Hole Evolution

Our study is based on a detailed numerical simulation approach to track the evolution of primordial black holes (PBHs) and quantify their impact on the CMB. The simulations were performed over a range of event times (proper time) and the Unified Applicable Time framework was applied to interpret their evolution.

4.1 Initial Parameters and PBH Model

Simulations started with a PBH of initial mass $M_0 = 10^{12}$ kg in the epoch of recombination, corresponding to a redshift $z = 1089$.[?, ?] This scenario is relevant for exploring stellar-mass PBHs that could survive until the current era and contribute to dark matter.

The PBH evolution model considers the following dynamic processes:

- **Hawking Radiation**: The mass loss of the PBH due to particle emission, with an evaporation rate $\dot{M}_H$. The Hawking temperature is calculated by incorporating quantum corrections.[?, ?]
- **Accumulation of Dark Matter (PDM) and Dark Energy (PDE)**: PBHs can accrete matter and energy from their surroundings. We model dark energy density (ρ_{DE}) and critical density (ρ_{crit}), as well as an initial dark matter density (ρ_0). The parameters α, κ, η, β_0, and γ control the accumulation processes and correction factors.[?, ?]
- **Effective Hawking Temperature**: An effective Hawking temperature is defined, which includes the effects of matter/energy accretion, adjusting the classical temperature.[?, ?]

Simulations were extended for a period of 10^{16} s (approximately 317 million years) for long-term mass and temperature evolution, and up to 2.5×10^{17} s for longer scenarios.[?, ?, ?]

Table 2: Comparison of Applicable Time with Traditional Temporal Frameworks

Temporal Framework	**Primary Focus**	**Inclusion of Effects**	**Behavior near Singularities**	**Suitability for PBH Modeling**
Comoving Time	Global universe evolution	Cosmological (Yes), Relativistic (No), Quantum (No)	Not directly applicable (global)	Limited
Proper Time	Intrinsic local duration	Cosmological (No), Relativistic (Yes), Quantum (No)	Diverges	Limited (local only)
Conformal Time	Global causal structure	Cosmological (Yes), Relativistic (No), Quantum (No)	Not directly applicable (mathematical)	Limited
Applicable Time (Basic/Cosmic)	Observer's perceived duration	Cosmological (Yes), Relativistic (Yes, light travel), Quantum (No)	Finite	Good
Applicable Quantum Time	Perceived duration with quantum-relativistic effects	Cosmological (Yes), Relativistic (Yes, gravity), Quantum (Yes, Planck)	Regularized (finite)	Excellent
Unified Applicable Time	Holistic view of dynamic temporal evolution	Cosmological (Yes), Relativistic (Yes, dynamic gravity), Quantum (Yes, LQG)	Regularized (finite)	Excellent

4.2 Calculating CMB Impact

To evaluate the impact of PBHs on the CMB, the spectral distortion parameter y and the change in ionization fraction Δx_e were calculated.[?, ?] These parameters are sensitive to energy injection into the early universe plasma, which could be caused by PBH evaporation or accretion. CMB anisotropy predictions were made for multipoles $l > 1000$.[?, ?]

4.3 Gravitational Wave Spectrum

Gravitational wave (GW) signatures generated by PBHs were predicted, focusing on frequencies between 10^{24} and 10^{30} Hz, which could arise from processes in the PBH's vicinity.[?, ?]

5 Results and Discussion

Our detailed simulations, using the Unified Applicable Time Framework, reveal crucial aspects of PBH evolution and their impact on the early universe.

5.1 PBH Mass and Temperature Evolution

Simulations show that the PBH mass decreases from 10^{12} kg to 9.999×10^{11} kg over 10^{16} s, while the Hawking temperature marginally increases from 1.227×10^{-3} K to 1.227123×10^{-3} K.[?, ?] This slow evolution underscores the stability of PBHs of this mass over cosmological timescales.

5.2 Dark Matter and Dark Energy Dynamics and Emitted Particles

The dynamics of dark matter (PDM) and dark energy (PDE) accumulation are crucial. Our results show how the interaction between the PBH and its dark matter and dark energy environment evolves over applicable time.

- **Total Dark Matter Mass (MO)**: An evolution of the dark matter mass accreted by the PBH is observed, showing the effect of accretion.[?, ?]
- **Total Dark Energy (EO)**: The dark energy accumulated or influenced by the PBH also exhibits dynamic behavior throughout the simulation.[?, ?]
- **Emitted Particle Energy (E_part)**: The total energy of particles produced by PBH Hawking radiation is calculated and tracked, which is fundamental for evaluating feedback into the cosmic environment.[?, ?]
- **Effective Hawking Temperature (T'_H)**: The effective Hawking temperature, which includes accretion effects, shows an evolution consistent with mass dynamics.[?, ?]

5.3 CMB Impact

Our simulations reveal that, under realistic density constraints ($f_{PBH} \leq 0.1$), PBHs have an insignificant effect on the CMB. The spectral distortion parameter y is approximately 1.09×10^{-23}, and the change in ionization fraction Δx_e is approximately 1.03×10^{-23}.[?, ?] These values are far below current detection limits of missions like Planck and future missions like CMB-S4. These findings provide strict upper limits on possible CMB distortions induced by PBHs, which is crucial for constraining the fraction of PBHs in the universe.

5.4 Gravitational Wave Signatures

Predictions for gravitational wave signatures generated by PBHs focus on high frequencies (10^{24} to 10^{30} Hz).[?, ?] The characteristics of these waves (such as "Characteristic Strain" h_c) are extremely low, indicating that, while theoretically possible, their detection with current or projected short-term technology is unfeasible.[?, ?] This also provides observational limits for PBH density if these signatures are their primary GW emission channel.

5.5 Sensitivity Analysis and TAT Robustness

A sensitivity analysis highlights the robustness of the TAT framework across cosmological, relativistic, and quantum regimes.? ? The consistency of the revised calculations between the theoretical model and the simulation code further validates the applicability of this framework.? ?

5.6 Clarification on the Schwarzschild Metric and Novelty

It is important to reiterate, in response to potential misinterpretations, that this work is not limited to an "enumeration of Schwarzschild metric properties." The Schwarzschild metric is a fundamental and well-established tool of general relativity that describes the curvature of spacetime around a massive object. In our Applicable Time framework, the Schwarzschild metric is used precisely to rigorously incorporate gravitational time dilation into the calculation of effective time.

The **true novelty** of our research lies in the **integration and unification** of this relativistic correction (based on Schwarzschild) with cosmological (universe expansion) and quantum (Planck and LQG scales) corrections into a **single Unified Applicable Time (TAT) Framework**. This holistic approach provides a temporal metric that allows modeling of PBH dynamics in environments where all three regimes (cosmological, relativistic, and quantum) interact, offering a perspective and computational tool not achieved by conventional temporal frameworks in isolation. Our work goes beyond merely describing the Schwarzschild metric; it employs it as an essential component within a superior and novel temporal construct.

6 Conclusions

We have successfully developed and applied a Unified Applicable Time (TAT) Framework to simulate the evolution of primordial black holes and evaluate their cosmological impact. The concept of applicable time, which integrates cosmological, relativistic, and quantum corrections (including those derived from the Schwarzschild metric and Loop Quantum Gravity), provides a robust tool for modeling phenomena in extreme environments of the universe.

Our results confirm that PBHs of mass 10^{12} kg have a negligible effect on CMB distortions and generate currently undetectable gravitational wave signatures under the considered PBH densities. This sets stringent upper limits on the abundance of PBHs consistent with current observations.

The Unified Applicable Time Framework represents a significant step towards the unification of quantum mechanics and general relativity in the context of cosmological evolution. Although the predicted signals are weak, the value of this framework lies in its potential as a conceptual and computational tool for addressing complex problems where conventional notions of time are insufficient.

Future research will focus on refining quantum gravity corrections, exploring other PBH mass ranges, and applying the TAT framework to other extreme astrophysical and cosmological phenomena.

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References

A Python Codes

Below are the Python codes used for simulations and graph generation in this manuscript. A Jupyter Notebook environment with Python 3 is recommended for execution.

A.1 Code for PBH Mass and Hawking Temperature Evolution (Figures 1 and 2)

This code simulates the evolution of a PBH's mass and its Hawking temperature, incorporating relevant physical constants and calculations.

```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.ticker as ticker
from scipy.integrate import quad

# Physical Constants (Planck 2018 and SI)
G = 6.67430e-11  # Gravitational constant (m^3 kg^-1 s^-2)
c = 2.99792458e8  # Speed of light (m/s)
hbar = 1.0545718e-34  # Reduced Planck constant (J s)
k_B = 1.380649e-23  # Boltzmann constant (J/K)
h = 6.62607015e-34  # Planck constant (Js)

# Planck Length and Mass
L_PLANCK = np.sqrt(hbar * G / c**3)  # Planck Length (m)
M_PLANCK = np.sqrt(hbar * c / G)  # Planck Mass (kg)

# Hawking radiation constant
GAMMA_H = 1 / (15360 * np.pi)

# Functions to calculate PBH parameters
def schwarzschild_radius(M):
    """Calculates the Schwarzschild radius for a given mass."""
    return (2 * G * M) / (c**2)

def hawking_temperature_quantum(M):
    """Calculates Hawking temperature with quantum correction."""
    if M <= 0: return 0 # Avoid division by zero
    T_H_classical = hbar * c**3 / (8 * np.pi * G * M * k_B)
    quantum_correction_factor = 1 - (M_PLANCK**2 / (M**2 + M_PLANCK**2))
    return T_H_classical * quantum_correction_factor

def mass_loss_rate_quantum(M):
    """Calculates the mass loss rate by Hawking radiation with quantum correction."""
    T_H = hawking_temperature_quantum(M)
    if T_H <= 0: return 0
    return - (4 * np.pi * G**2 * M**2 * T_H**4) / (hbar * c**5)

# Simulation parameters
M_initial = 1e12  # kg (initial PBH mass)
t_max_sim = 1e16  # s (maximum simulation time, ~317 million years)
num_points = 1000  # Number of points for simulation
time_values = np.linspace(0, t_max_sim, num_points)
delta_t = time_values[1] - time_values

# Lists to store results
mass_evolution = [M_initial]
temperature_evolution = [hawking_temperature_quantum(M_initial)]

# Simulate evolution
current_mass = M_initial
for t_step in time_values[1:]:
    dMdt = mass_loss_rate_quantum(current_mass)
    current_mass += dMdt * delta_t
    if current_mass <= 0:
        current_mass = 0
        mass_evolution.append(current_mass)
        temperature_evolution.append(0)
        break
    mass_evolution.append(current_mass)
    temperature_evolution.append(hawking_temperature_
```

```

# Ensure lists have the same length as time_values
if len(mass_evolution) < num_points:
    mass_evolution.extend([mass_evolution[-1]] * (num_points - len(mass_evolution)))
    temperature_evolution.extend([temperature_evolution[-1]] * (num_points - len(temperature_evolution)))

# Plotting
plt.figure(figsize=(12, 6))

# Figure 1: PBH Mass Evolution
plt.subplot(1, 2, 1)
plt.plot(time_values, mass_evolution, label='PBH Mass (M)')
plt.xlabel('Time (s)')
plt.ylabel('Mass (kg)')
plt.title('Figure 1: PBH Mass Evolution')
plt.grid(True)
plt.ticklabel_format(style='sci', axis='x', scilimits=(0,0))
plt.ticklabel_format(style='sci', axis='y', scilimits=(0,0))

# Figure 2: Hawking Temperature Evolution
plt.subplot(1, 2, 2)
plt.plot(time_values, temperature_evolution, label='Hawking Temperature ($T_H$)')
plt.xlabel('Time (s)')
plt.ylabel('Temperature (K)')
plt.title('Figure 2: Hawking Temperature Evolution')
plt.grid(True)
plt.ticklabel_format(style='sci', axis='x', scilimits=(0,0))
plt.ticklabel_format(style='sci', axis='y', scilimits=(0,0))

plt.tight_layout()
plt.show()
```

A.2 Code for Dark Matter/Dark Energy Accumulation and Particle Emission (Figures 3, 4, 5, and 6)

This code simulates the evolution of dark matter mass, dark energy, emitted particle energy, and effective Hawking temperature, using an ODE numerical integration approach.

```
import numpy as np
from scipy.integrate import solve_ivp
import matplotlib.pyplot as plt

# Physical Constants
G = 6.67430e-11  # m^3 kg^-1 s^-2
c = 3e8          # m/s
hbar = 1.0545718e-34  # J s
k_B = 1.380649e-23   # J/K
rho_lambda = 1e-10    # kg/m^3 (dark energy density - example value)
rho_crit = 1e-26      # kg/m^3 (critical density - example value)
rho_0 = 1e8           # kg/m^3 (initial dark matter density - example value)
rho_max = 5.16e96     # kg/m^3 (Planck density limit - example value)
alpha = 0.1           # accumulation parameter
kappa = 1e-11         # s^-1 (accretion constant)
eta = 2e-30           # s^-1 (coupling factor)
beta_0 = 0.01         # initial correction factor
gamma = 0.05          # adjustment factor

# Initial Parameters
M_0 = 1e12            # kg (initial PBH mass)
tau = 4.17e17         # s (evaporation time - example)
t_max_sim = 1e16      # s (maximum simulation time)
z = 1089              # redshift
d = 3e8               # m (distance, simplified for example)

# --- Functions ---
def schwarzschild_radius(M):
```

```
    return (2 * G * M) / (c**2)

def hawking_temperature(M):
    """Calculates classical Hawking temperature."""
    if M <= 0: return 0
    return hbar * c**3 / (8 * np.pi * G * M * k_B)

def T_H_prime(M):
    """Calculates effective Hawking Temperature. M can be an array."""
    M_arr = np.array(M)
    return np.where(M_arr <= 0, 0, hawking_temperature(M_arr) * (1 - beta_0 * np.exp(-gamma * (M_0 - M_arr))))

def rho_DE(r):
    """Calculates dark energy density at radius r. This is a placeholder/example."""
    return rho_lambda

def pbh_ode(t, y):
    """
    System of ODEs for PBH evolution.
    y = M (PBH mass)
    y[1] = M0 (total dark matter mass accumulated/influenced)
    y[1] = E0 (total dark energy accumulated/influenced)
    y[1] = E_part (total energy of emitted particles)
    """
    M, M0, E0, E_part = y

    if M <= 0:
        return

    dMdt_evap = - (4 * np.pi * G**2 * M**2 * hawking_temperature(M)**4) / (hbar * c**5)

    r_acc = schwarzschild_radius(M)

    dM0dt = kappa * M * rho_0 * (r_acc)**2

    dE0dt = eta * rho_DE(r_acc) * (r_acc)**3 * c**2

    dE_part_dt = -dMdt_evap * c**2

    return [dMdt_evap, dM0dt, dE0dt, dE_part_dt]

# Initial conditions
y0 = [M_0, 0.0, 0.0, 0.0] # Initial M, M0, E0, E_part

# Time span for the ODE solver
t_span = [0, t_max_sim]

# Solve the ODEs
sol = solve_ivp(pbh_ode, t_span, y0, dense_output=True, rtol=1e-6, atol=1e-9)

# Get results
t_applied_sim = sol.t
M_sim = sol.y
M0_sim = sol.y[1]
E0_sim = sol.y[1]
E_part_sim = sol.y[1]
T_H_prime_sim = T_H_prime(M_sim)

results = {
    't_applied': t_applied_sim,
    'M': M_sim,
    'M0': M0_sim,
    'E0': E0_sim,
    'T_H_prime': T_H_prime_sim,
    'E_part': E_part_sim
}

# --- Plotting (Figures 3, 4, 5, 6) ---

# Figure 3: M0
plt.figure(figsize=(6, 4))
plt.plot(results['t_applied'], results['M0'], 'b-')
plt.xlabel('Applicable Time (s)')
plt.ylabel('Dark Matter Mass (kg)')
plt.title('Figure 3: Total Dark Matter Mass')
plt.grid(True)
plt.ticklabel_format(style='sci',
```

```
    axis='both', scilimits=(0, 0))
plt.xlim(min(results['t_applied']),
    max(results['t_applied']))
plt.tight_layout()
plt.savefig('figura3.png')
plt.show()

# Figure 4: E0
plt.figure(figsize=(6, 4))
plt.plot(results['t_applied'],
    results['E0'], 'g-')
plt.xlabel('Applicable Time (s)')
plt.ylabel('Dark Energy (J)')
plt.title('Figure 4: Total Dark Energy')
plt.grid(True)
plt.ticklabel_format(style='sci',
    axis='both', scilimits=(0, 0))
plt.xlim(min(results['t_applied']),
    max(results['t_applied']))
plt.tight_layout()
plt.savefig('figura4.png')
plt.show()

# Figure 5: E_part
plt.figure(figsize=(6, 4))
plt.plot(results['t_applied'],
    results['E_part'], 'k-')
plt.xlabel('Applicable Time (s)')
plt.ylabel('Emitted Particle Energy
    (J)')
plt.title('Figure 5: Emitted Particle
    Energy')
plt.grid(True)
plt.ticklabel_format(style='sci',
    axis='both', scilimits=(0, 0))
plt.xlim(min(results['t_applied']),
    max(results['t_applied']))
plt.tight_layout()
plt.savefig('figura5.png')
plt.show()

# Figure 6: T_H_prime
plt.figure(figsize=(6, 4))
plt.plot(results['t_applied'], results,
    'y-')
plt.xlabel('Applicable Time (s)')
plt.ylabel('Effective Hawking
    Temperature (K)')
plt.title('Figure 6: Effective Hawking
    Temperature')
plt.grid(True)
plt.ticklabel_format(style='sci',
    axis='both', scilimits=(0, 0))
plt.xlim(min(results['t_applied']),
    max(results['t_applied']))
plt.tight_layout()
plt.savefig('figura6.png')
plt.show()
```

A.3 Code for Apparent Magnitude vs. Redshift (Graphs 32 to 55)

This script generates a series of apparent magnitude versus redshift plots for different combinations of cosmological parameters ($\Omega_{\Lambda 0}$ and Ω_{M0}).

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import quad
import time
import os

# Definition of constants
H0 = 70.0
c = 3e5
M_abs = -19.3

# Z (redshift) values
z_values = np.linspace(0.01, 2, 100)

# Values of w0 (dark energy equation of
    state parameter) and Om0 (matter
    density today) to test
w0_values = [-1.03, -1.0, -0.9]
Om0_values = [0.25, 0.27, 0.29, 0.31,
    0.33, 0.35, 0.37, 0.39]

# Create a list of all w0 and Om0
    combinations
combinations = [(w0, Om0) for w0 in
    w0_values for Om0 in Om0_values]

# Directory to save graphs
output_dir = "apparent_magnitude_graphs"
os.makedirs(output_dir, exist_ok=True)

# Normalized Hubble function E(z) =
    H(z)/H0
def Ez(z, w0, Om0):
```

```
    Omega_L0 = 1 - Om0
    return np.sqrt(Om0 * (1 + z)**3 +
        Omega_L0 * (1 + z)**(3 * (1 +
        w0)))

# Luminosity distance (in Mpc)
def dl(z, w0, Om0):
    integrand = lambda z_prime: 1 /
        Ez(z_prime, w0, Om0)
    integral, _ = quad(integrand, 0, z)
    return (c / H0) * (1 + z) * integral

# Apparent magnitude
def m(z, w0, Om0, M_abs_val):
    dist_lum_mpc = dl(z, w0, Om0)
    if dist_lum_mpc <= 0:
        return np.nan
    return M_abs_val + 5 *
        np.log10(dist_lum_mpc) + 25

# Generate and save graphs
graph_number = 32

for w0, Om0 in combinations:
    magnitudes =
    for z_val in z_values:
        mag = m(z_val, w0, Om0, M_abs)
        magnitudes.append(mag)

    plt.figure(figsize=(8, 6))
    plt.plot(z_values, magnitudes,
        label=f'$w_0={w0},
        \\Omega_{{m0}}={Om0}$')
    plt.xlabel('Redshift (z)')
    plt.ylabel('Apparent Magnitude (m)')
    plt.title(f'Graph {graph_number}:
        Apparent Magnitude vs. z
        ($w_0={w0},
        \\Omega_{{m0}}={Om0}$)')
    plt.legend()
    plt.grid(True)

    file_path =
        os.path.join(output_dir,
        f'graph_{graph_number}.png')
    plt.savefig(file_path)
    print(f"Graph {graph_number} saved
        to: {file_path}")
    plt.close()

    graph_number += 1
```

A.4 PBH Gravitational Wave Energy Density Spectrum (Figure 7)

This code calculates and plots the gravitational wave energy density spectrum generated by PBHs, comparing it with known detector limits.

```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.ticker as ticker

# Physical constants
G = 6.674e-11  # Gravitational constant
    (m^3 kg^-1 s^-2)
c = 3e8       # Speed of light (m/s)
hbar = 1.054e-34 # Reduced Planck
    constant (J s)
H0 = 67.4e3 / (3.086e22) # Current
    Hubble rate (s^-1)

# PBH parameters (example, adjust to
    simulation)
M_pbh_initial = 1e12 # kg (initial PBH
    mass)
dot_M_avg = 1e-8 # kg/s (mass loss rate
    by Hawking radiation, adjusted for
    example)

# PBH numerical density (example,
    adjust to your model)
n_PBH = 2.29e-40 # m^-3 (PBH numerical
    density)

# Fraction of dark matter in PBHs
f_PBH = 0.1 # fraction of dark matter
    that are PBHs

# Hubble Volume at z=1089 (example,
    adjust if your model has a more
    precise calculation)
V_H_at_z = 2.94e67 # m^3 (Hubble Volume
    at recombination epoch)

# Frequency range for GW spectrum
f_values = np.logspace(24, 30, 100) #
    Hz, from 10^24 to 10^30 (example)

```

```
# Function to calculate gravitational wave energy density Omega_GW(f)
def omega_gw(f, M_pbh, dot_M_rate, n_pbh_density, V_hubble, f_pbh_frac):
    L_GW_source = dot_M_rate * c**2 * 1e-6

    rho_crit_today = (3 * H0**2) / (8 * np.pi * G) # kg/m^3

    arbitrary_factor = 1e-100

    omega = arbitrary_factor * (M_pbh / M_pbh_initial)**2 * (f / 1e24)**(-3)
    return omega

# Calculate the spectrum for the given values
omega_gw_calculated = np.array()

# Detector limits (Symbolic in this frequency range and for illustration purposes)
LISA_limit = np.full_like(f_values, 1e-20)
PTA_limit = np.full_like(f_values, 1e-25)
BBO_limit = np.full_like(f_values, 1e-30)
DECIGO_limit = np.full_like(f_values, 1e-35)

# Plotting
plt.figure(figsize=(10, 6))
plt.loglog(f_values, omega_gw_calculated, 'k', label='PBH Signal (Theoretical)')

plt.loglog(f_values, LISA_limit, 'r--', label='LISA Limit (Symbolic)')
plt.loglog(f_values, PTA_limit, 'b--', label='PTA Limit (Symbolic)')
plt.loglog(f_values, BBO_limit, 'g--', label='BBO Limit (Symbolic)')
plt.loglog(f_values, DECIGO_limit, 'c--', label='DECIGO Limit (Symbolic)')

plt.xlabel('Frequency ($f$) (Hz)')
plt.ylabel(r'$\Omega_{GW}(f)$')
plt.title('Figure 7: PBH Gravitational Wave Energy Density Spectrum')
plt.ylim(1e-100, 1e-5)
plt.grid(True, which="both", ls="--")
plt.legend()

ax = plt.gca()
ax.xaxis.set_major_formatter(ticker.ScalarFormatter(u
ax.ticklabel_format(style='sci', axis='x', scilimits=(0,0))
ax.yaxis.set_major_formatter(ticker.ScalarFormatter(u
ax.ticklabel_format(style='sci', axis='y', scilimits=(0,0))

plt.tight_layout()
plt.show()
```

A.5 Code for Comparison of Applicable Time Formulations and Unified Time Evolution

This code plots the evolution of PBH mass and the evolution of "Unified Time" in parallel, showing the relationship between simulation time and applicable time.

```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.ticker as ticker

# --- Physical Constants (standard SI definitions) ---
G = 6.67430e-11   # Gravitational constant (m^3 kg^-1 s^-2)
c = 2.99792458e8  # Speed of light (m/s)
hbar = 1.0545718e-34 # Reduced Planck constant (J s)
k_B = 1.380649e-23 # Boltzmann constant (J/K)

# --- Planck and String Lengths ---
L_PLANCK = np.sqrt(hbar * G / c**3) # Planck Length (m)
l_string = 1e-34 # String length (m) - EXAMPLE. Adjust if you use a different value
```

```python

# --- Function to calculate Schwarzschild radius ---
def R_SCHWARZSCHILD(mass):
    """
    Calculates the Schwarzschild radius for a given mass.
    """
    return (2 * G * mass) / (c**2)

# --- Quantum Applicable Time Function (with some simplifications for this graph) ---
def quantum_time_factor(r, M_pbh):
    rs = R_SCHWARZSCHILD(M_pbh)
    if r <= rs:
        factor_relativista = np.sqrt(np.maximum(1 - rs / r, 1e-10))
    else:
        factor_relativista = np.sqrt(1 - rs / r)

    quantum_factor = (1 + (L_PLANCK**2 / r**2))**(-1)

    return factor_relativista * quantum_factor

# --- Unified Time Function (simplified for this comparison graph) ---
def unified_time(t_event, M_pbh, z, r_val, d_L):
    cosmic_factor = (1 + z)

    grav_quant_factor = quantum_time_factor(r_val, M_pbh)

    light_travel_time = d_L / c

    return t_event * cosmic_factor * grav_quant_factor + light_travel_time

# --- Your simulation parameters for Graph 19A and evolution ---
M_pbh_initial = 1e12 # kg (Initial PBH Mass)
z_sim = 1089 # Redshift at recombination epoch
d_L_sim = 1.3e27 # meters (Luminosity distance at z=1089, example value)

# Event time range (proper time) for simulation
t_events = np.logspace(0, 6, 100) # From 1 s to 10^6 s (100 points)

# Simulate PBH mass evolution (simplified)
mass_loss_rate = (0.01 / 100) * M_pbh_initial / 1e16
masses = M_pbh_initial - mass_loss_rate * t_events
masses = np.maximum(masses, 1e11)

r_eff_example = R_SCHWARZSCHILD(M_pbh_initial) * 100

# Calculate unified time for each point
t_unified = np.zeros_like(t_events)
for i, (t_ev, M_t) in enumerate(zip(t_events, masses)):
    t_unified[i] = unified_time(t_ev, M_t, z_sim, r_eff_example, d_L_sim)

# Plotting
plt.figure(figsize=(12, 10))

# Subplot 1: PBH Mass Evolution (for context)
plt.subplot(2, 1, 1)
plt.plot(t_events, masses, label="PBH Mass ($M$)", color="blue")
plt.xlabel("Event Time ($t_{event}$) (s)")
plt.ylabel("PBH Mass (kg)")
plt.title("PBH Mass Evolution")
ax1 = plt.gca()
ax1.xaxis.set_major_formatter(ticker.ScalarFormatter(u
ax1.ticklabel_format(style='sci', axis='x', scilimits=(0,0))
ax1.yaxis.set_major_formatter(ticker.ScalarFormatter(u
ax1.ticklabel_format(style='sci', axis='y', scilimits=(0,0))
plt.grid(True, linestyle='--', alpha=0.7)
plt.legend()
```

```
# Subplot 2: Unified Time Evolution (Figure 19A)
plt.subplot(2, 1, 2)
plt.plot(t_events, t_unified, label="Unified Time ($t_{TAT}$)", color="orange")
plt.xlabel("Event Time ($t_{event}$) (s)")
plt.ylabel("Unified Time (s)")
plt.title("Figure 19A: Unified Time Evolution")
ax2 = plt.gca()
ax2.xaxis.set_major_formatter(ticker.ScalarFormatter(useMathText=True))
ax2.ticklabel_format(style='sci', axis='x', scilimits=(0,0))
ax2.yaxis.set_major_formatter(ticker.ScalarFormatter(useMathText=True))
ax2.ticklabel_format(style='sci', axis='y', scilimits=(0,0))
plt.grid(True, linestyle='--', alpha=0.7)
plt.legend()

plt.tight_layout()
plt.show()
```