

## Session 5: Linear Time Calculus part 2

### 1 Garage door controller

*If you have not completed this exercise in the previous session, do it now. Otherwise move on to the next exercises.*

### 2 Nim

*If you have not completed this exercise in the previous session, do it now. Otherwise move on to the next exercises.*

### 3 Vending Machine

Starting from the VendingMachine.idp skeleton provided on Toledo, model a beverage vending machine using LTC. The aim is to model the vending machine from a user's standpoint, i.e. the actions that are available are actually actions performed by a user.

The users can pay one coin at a time (the coins are represented per 10 cents, e.g. 1 is a 10 cent coin), press a button to receive a certain beverage, ask for a refund or restock the machine. The machine keeps track of the current amount paid, as well as the stock of the different beverages. Each beverage has a certain price.

The actions work as follows:

**pay(c)** denotes a user inserting a coin of value  $c$ . The value of the coin should be added to the current paid amount. However the machine will not accept anymore coins when the current paid amount is already larger than or equal to the price of the most expensive beverage.

**pressButton(b)** means a users pressed the button for beverage  $b$ . Users can press a button at any time, but will only receive a beverage if they have paid at least the beverage's price. If the user receives the beverage, its price is deducted from the currently paid amount.

**askRefund** happens when the user asks to have his money returned. The currently paid amount is returned to the user. Obviously this is only possible if there is some of amount of currency in the machine.

**restock** allows the machine to be restocked. It is only meaningful to restock the machine when the stock of at least one of the beverages is below its maximum.

Only one action can happen at any one time.

The machine has an obvious short coming, i.e. when you select a beverage and paid more than the price, it does not automatically return the remaining money. To get the rest of the money back a user has to ask for a refund.

#### Tasks

1. Model this scenario by means of Linear Time Calculus.
2. Use IDP to simulate the workings of the vending machine.
3. Check that it is impossible to have an amount over 24 in the machine. Use the  $\text{isinvariant}(T, T', S)$  procedure for this. This returns true if  $T'$  can be proven to be an invariant of  $T$  in structure  $S$ .  $T'$  has to be a theory with a single sentence of the form  $\forall t[Time] : \phi[t]$ .
4. This only shows this is an invariant for the the given structure. Complete the single state and bstate invariants and the provided procedure to check whether it is actually an invariant of the theory.
5. If proof of invariant fails, try to find a counter-example structure  $S'$  and verify it by using  $\text{isinvariant}(T, T', S')$ .
6. Check that there can not be two restocks following each other. Why is this not really an invariant (consult the definition of an invariant)?