ERGOG

The Initial Value Problem

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IFEG CONICET UNC

UNC FAMAF

IVP:Given an equation system, determine whether there exists data such that:

- Determines a unique solution (existence and uniqueness).
- Those solutions depend continuously on the data (depends on the assumed topology [Sobolev spaces vs. Gevrey spaces]).

Outline

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- The Initial Value Problem:
  - Old stuff < 1950. Cauchy-Kowalevski Theorem.
  - The question of well posedness and predictability.
  - Symmetric and Strongly hyperbolic systems.
  - Examples: Wave equation, and Electromagnetism.
  - Open problems.

Socio/psycho-logy page

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- Why is this important?
- Today we have computers and can find general solutions.
  - Theory fully developed when simulations started.
  - Entered very late in GR
- Most textbooks in physics are older than the advances in PDE's.
  - Are based in a handful of exact solutions (very important also, but stay in the realm of Cauchy-Kowalevski, convergence of series??)
  - Landau / Eckart Theory of relativistic dissipative fluids.

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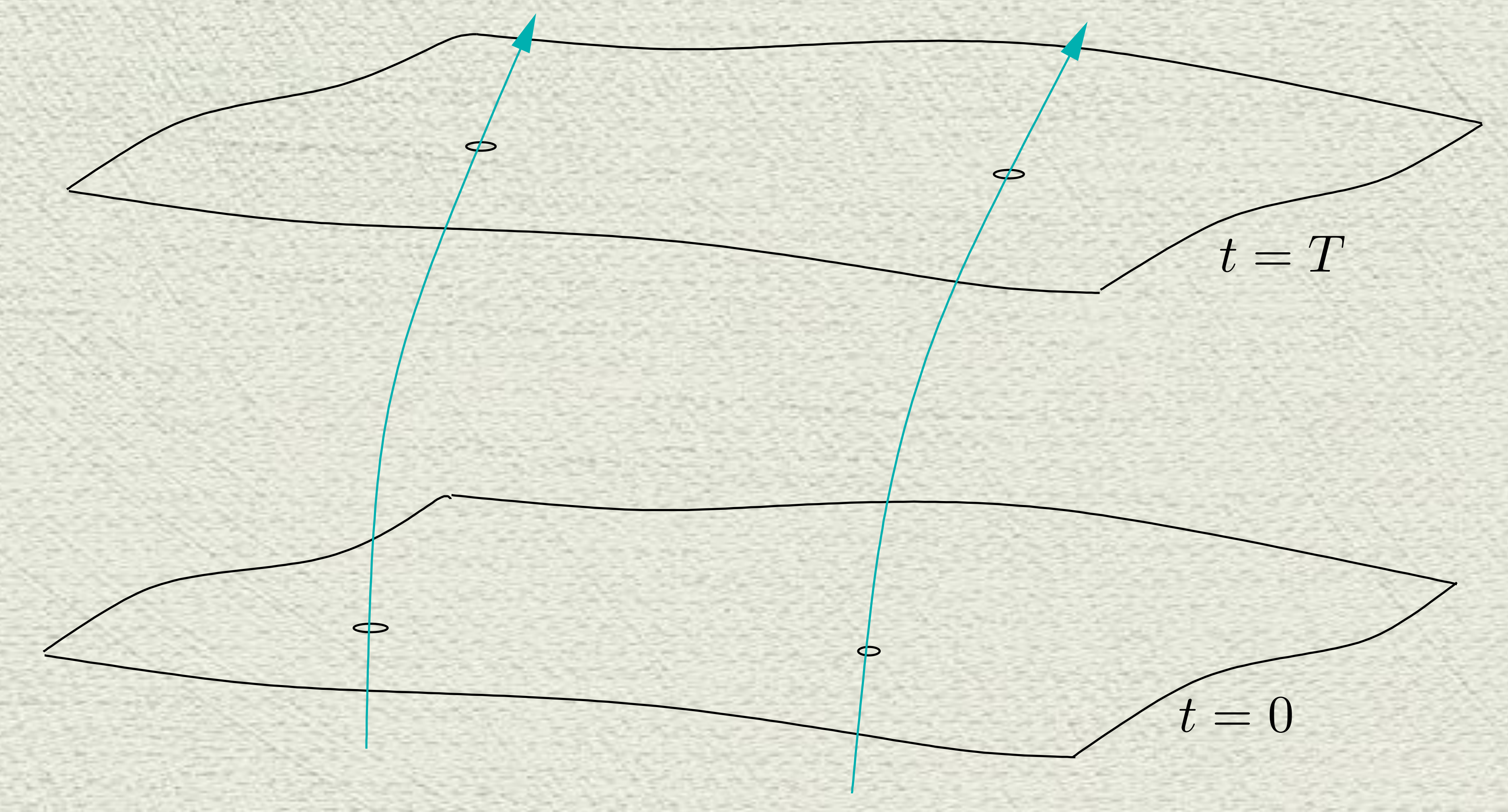

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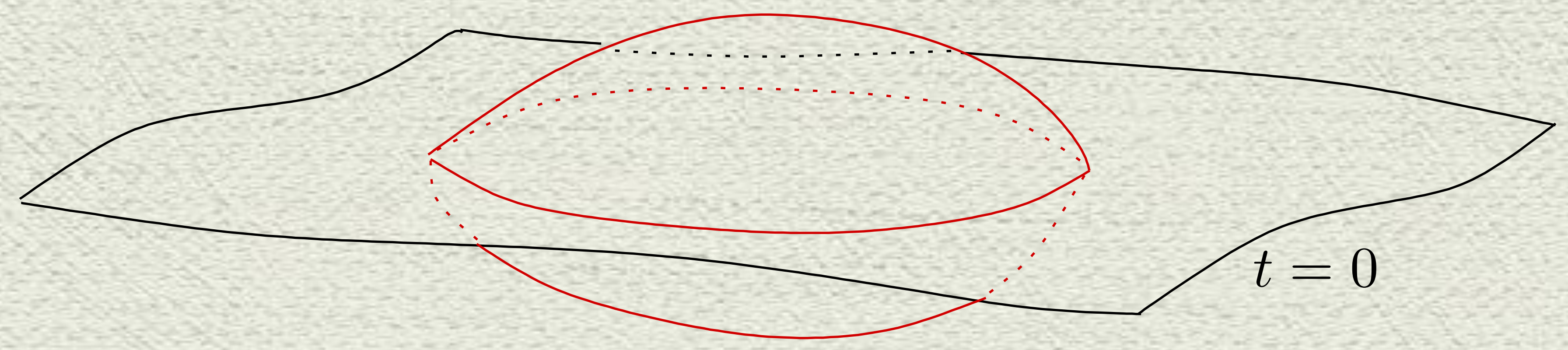

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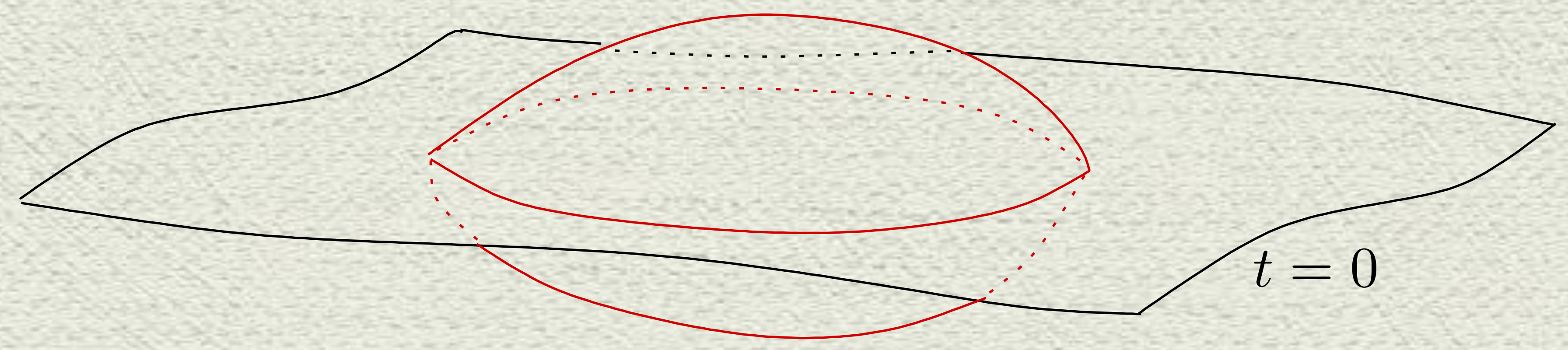

$$u^\alpha\big|_{t=0} = f^\alpha$$

$$\begin{aligned} u^\alpha(x,\delta t) &= u^\alpha(x) + \delta t\partial_t u^\alpha(x,0) \\ &= f^\alpha(x) + \delta t[A^{\alpha i}{}_\beta(f,x,0)\nabla_i f^\beta + B^\alpha(f,x,0)] \end{aligned}$$

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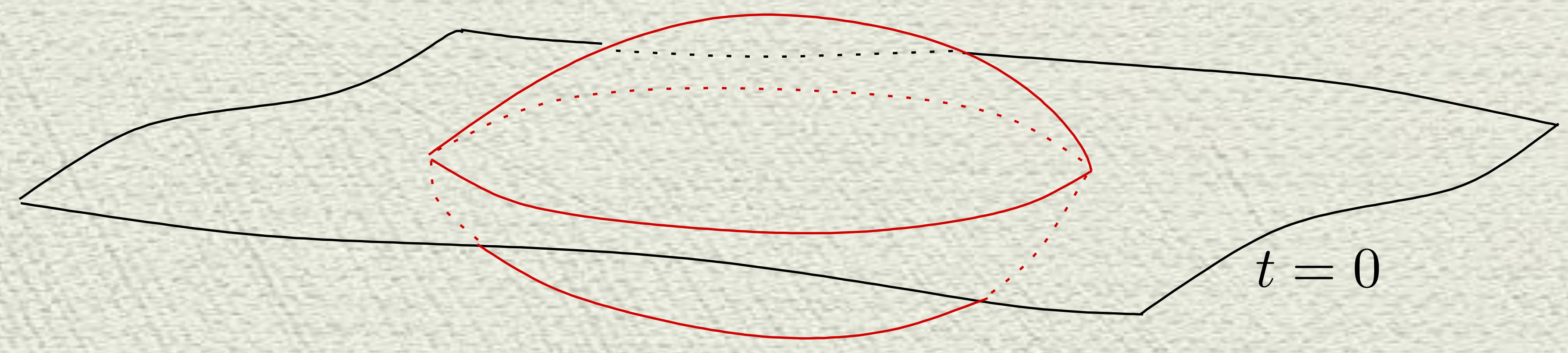

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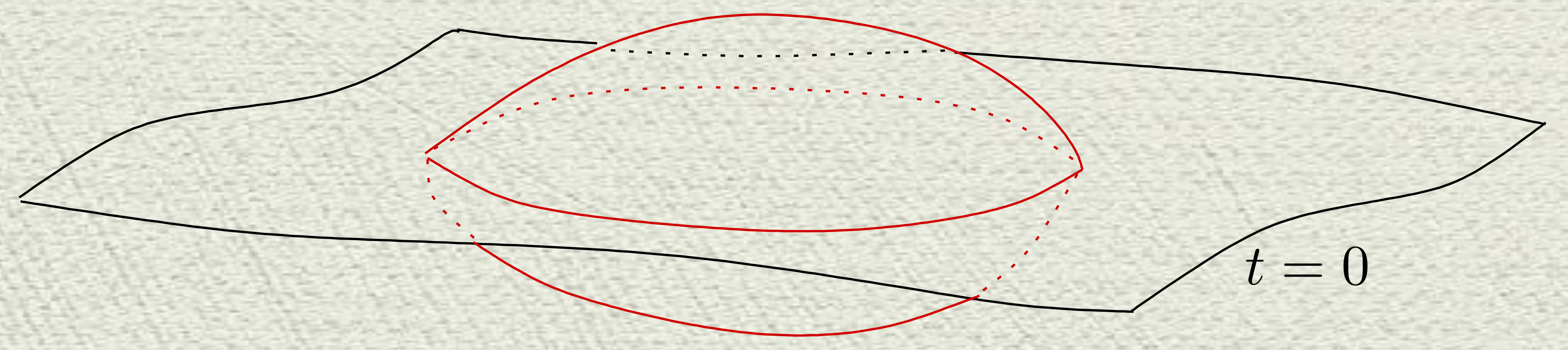

Analytic

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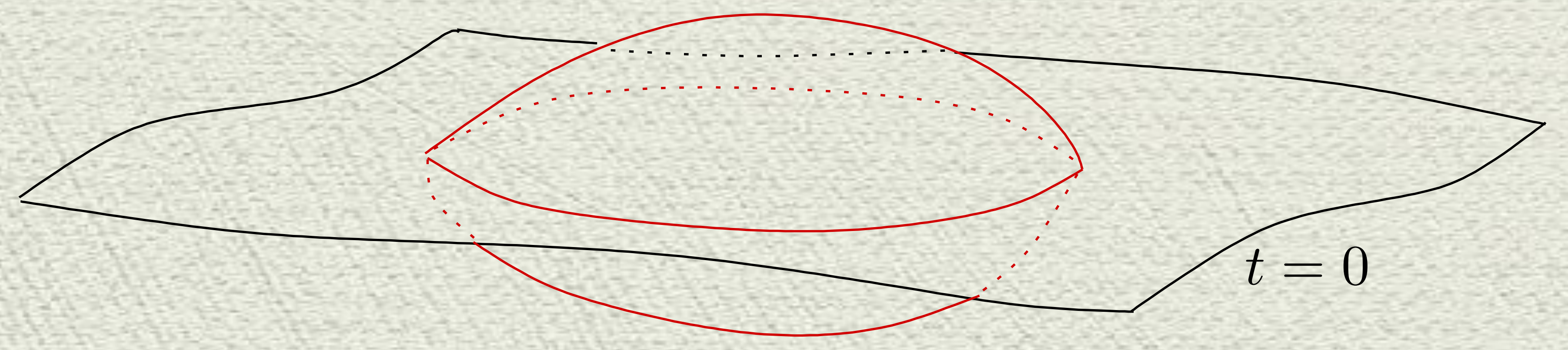


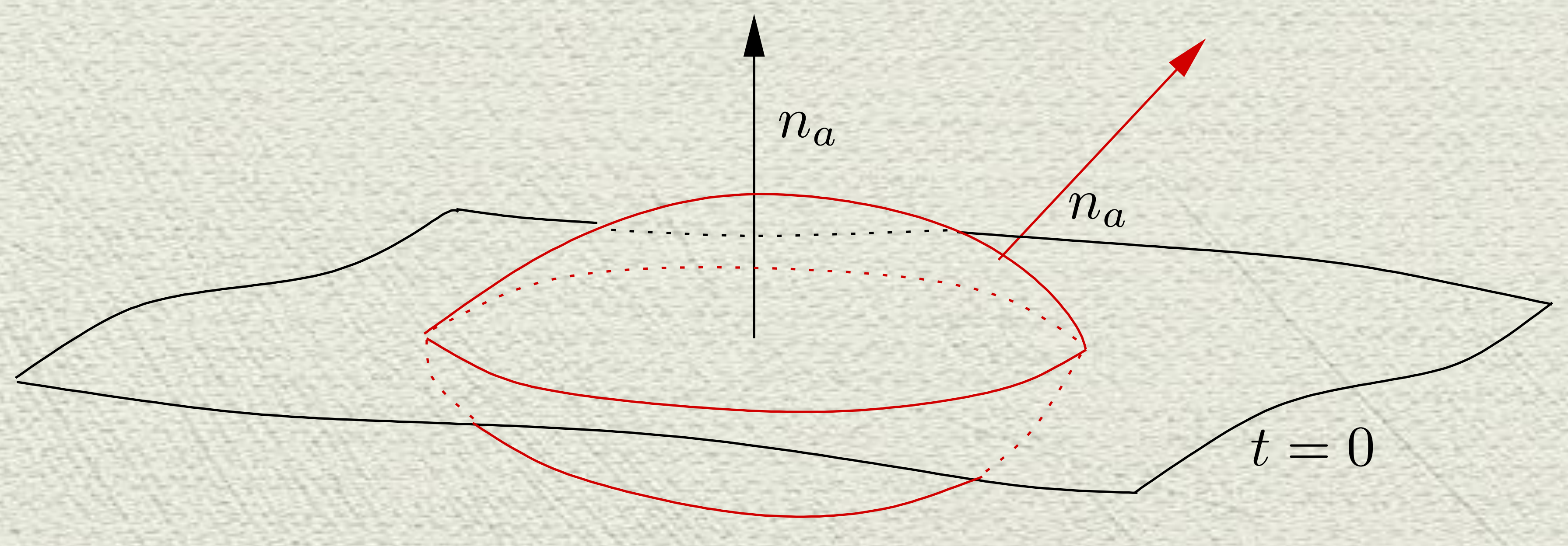
Then there exists a unique, analytic solution in a nbh of t=0

Cauchy problem

- Write f(x) in Taylor series, around x_0.
- Write u(t,x) in Taylor series around $(t=0,x_0)$.
- Get all coefficients for the u(t,x) series from the f(x) series using the equations.
- Show that convergence of Taylor series for f(x) implies convergence of Taylor series for u(t,x).

Cauchy Problem

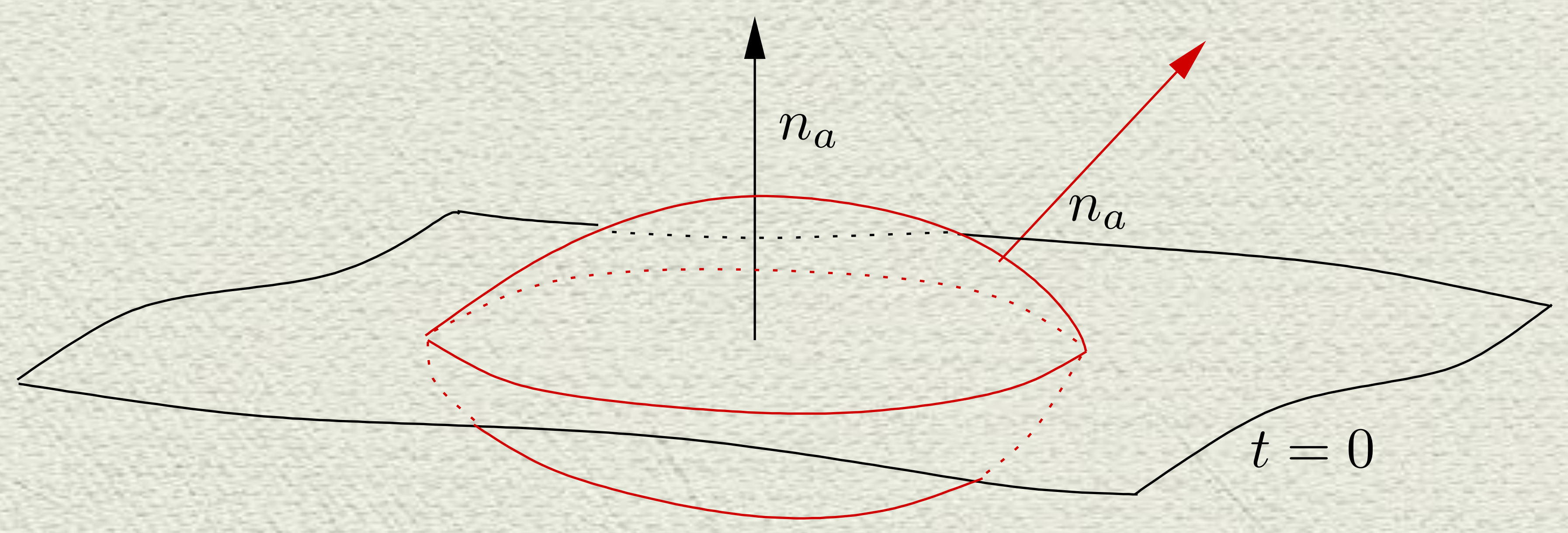
- Cauchy–Kowalevski theorem: If ${A^{\alpha i}}_{\beta}$ and B^{α} are analytic, then for each analytic initial data f^{α} there exists a nbh. around Σ_0 and an analytic, unique, solution satisfying the initial conditions, $u|_{\Sigma_0} = f$.
- Holmgrem's theorem: Even when f is not analytic, if a solution exists, it is unique in a region bounded by characteristic surfaces.
- Everything seemed to be OK



Cauchy Problem

Definition: Given a manifold M, and a set of fields on it, u^α, a characteristic surface of a quasi-linear equation $A^{\alpha a}{}_\beta(u)\nabla_a u^\beta = F^\alpha(u)$ is a hypersurface such that its normal, n_a at every point satisfies,

$$\det(A^{\alpha a}{}_\beta n_a) = 0.$$



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“Modern” stuff

- Cauchy-Kowalevski gives only existence for analytic initial data.
- There are equations which only have solutions if the data is analytic.
- In many cases if a solution exists it is not a continuos function with respect to the initial data. (Not well posed.)
- No good for physics! There is no predictability.

Example: Laplacian

$$\phi_{,11} + \phi_{,22} = f$$

$$\begin{pmatrix} u \\ v \end{pmatrix}_{,1} + \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}_{,2} = \begin{pmatrix} f \\ 0 \end{pmatrix} \qquad u := \phi_{,1} \quad v := \phi_{,2}$$

- In this case $0 = det(A^{\mu} n_{\mu}) = n_1^2 + n_2^2 \Longrightarrow n_{\mu} = 0$ so there is no characteristic surfaces.
- Elliptic equations, if you know the solution on a nbh. of a point you know it everywhere.
- Something funny...

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Eigenvectors of

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

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$$u(t, x) = c^+(t, x) v_+ + c^-(t, x) v_-$$

$$\partial_t c^\pm = \pm i \partial_x c^\pm$$

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$$u(t,x) = c^+(t,x)v_+ + c^-(t,x)v_- \qquad c^\pm(t,x) = C^\pm(k)e^{\omega t \pm ikx}$$

$$\partial_t c^\pm = \pm i \partial_x c^\pm \qquad\qquad \omega = \pm k$$

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$$c^+(0, x) = e^{ix} + \frac{1}{10000} e^{i1000x}$$

$$c^+(t, x) = e^t e^{ix} + \frac{e^{1000t}}{10000} e^{i1000x}$$

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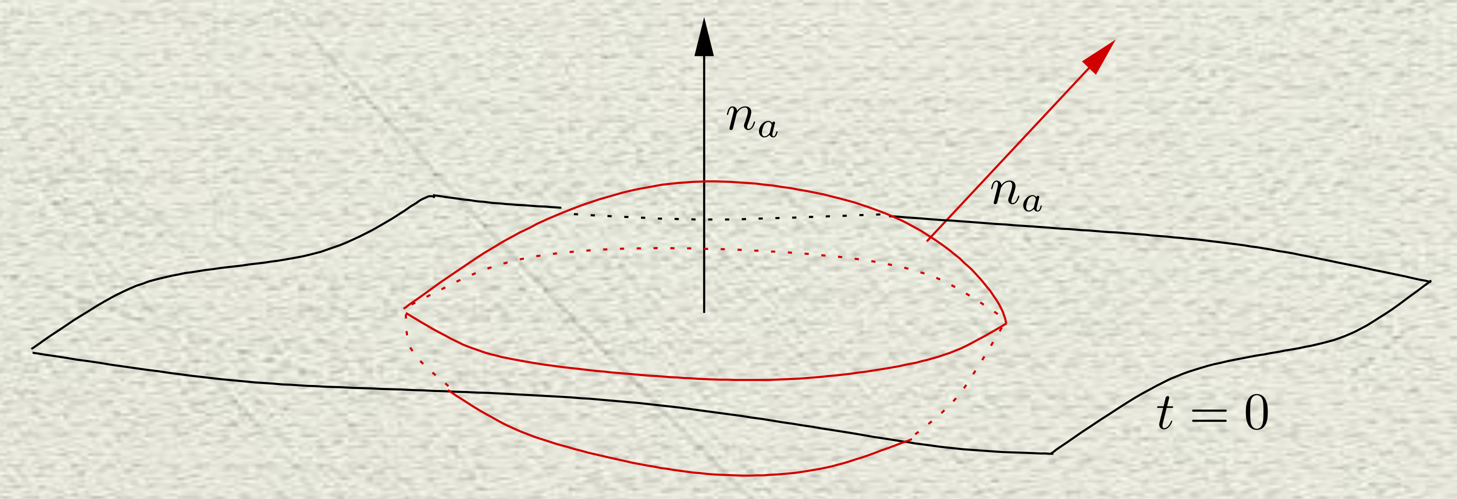
- In this case $0 = det(A^{\mu} n_{\mu}) = n_1^2 - n_2^2 \implies n_{\mu} = (1, \pm 1)$ so there are characteristic surfaces.
- Hyperbolic equations, characteristics are maximum propagation speeds.
- Concept of domain of dependence.



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Example: Heat equation

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- In this case $0 = det(A^{\mu} n_{\mu}) = n_1^2 \implies n_{\mu} = (0,1)$ so there is only a characteristic surface.
- Parabolic equations, if you know the solution on a nbh. of a point you know it everywhere.
- Something funny...

Constant coefficient system (on a torus)

$$u_t(t,x) = P(\partial)u(t,x) \qquad P(\partial_j) = \sum_{|\alpha|\leq m} A^\alpha D_\alpha$$

non-characteristic surface

$$u(t,x) = \sum_k e^{ix\cdot k}\hat{u}(t,k) \qquad \hat{u}(t,k) := \int e^{-ix\cdot k}u(t,x)$$

$$\hat{u}_t = P(ik)\hat{u} \quad \Longrightarrow \quad \hat{u}(t,k) = e^{P(ik)t}u(0,k)$$

$$f(x) = \sum_{|k|<r} e^{ix\cdot k}\hat{f}(k) \quad \Longrightarrow \quad u(t,x) = \sum_{|k|<r} e^{ix\cdot k}e^{P(ik)t}u(0,k)\hat{f}(k)$$

- Good for finite series (Cauchy-Kowalevski)

Constant coefficient system

- When do solutions at a given time have a continuous dependence with respect to their initial data? *Hadamard*

- This happens if and only if $|e^{P(ik)}| < C$ independent of k. (When considering the same norm for initial data and solution at the given time).

- **IF:** $||u(t,\cdot)||^2 = ||\hat{u}(t,\cdot)||^2 \leq C||\hat{f}(\cdot)||^2 = C||f(\cdot)||^2$

- **ONLY IF:** Otherwise there exists a sequence $\{(k_l, \hat{f}_l)\}$ with $||\hat{f}_l|| = 1$ and $|e^{P(ik_l)}\hat{f}_l| = l$

Constant coefficient system

Constant coefficient system

- For Laplacian equation it is not bounded, grows exponentially with $|k|$.
- For Wave equation it is bounded by one, $P(ik)$ is diagonalizable and has imaginary eigenvalues.
- For Heat equation it is not bounded, either $|e^{P(ik)}|$ or $|e^{-P(ik)}|$ grows exponentially with $|k|^2$.

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Without stability we can not compute!

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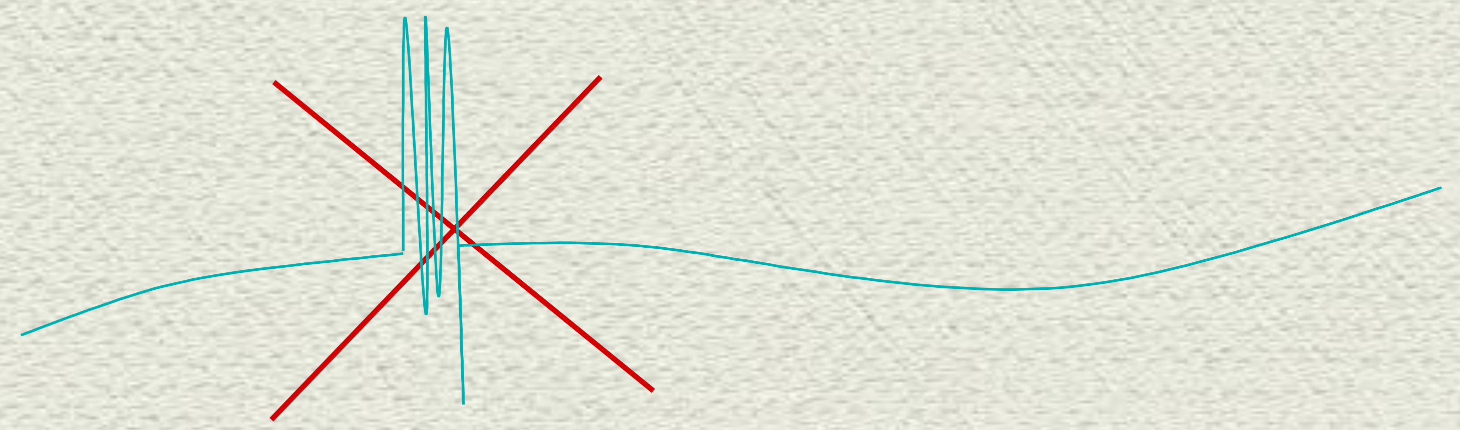
- Alternatively: It is SH if there exist $H = H(k)$, positive definite bilinear form such that $H(k)A^a k_a$ is symmetric.
- **Symmetric hyperbolicity:** If H does not depend on k_a. Most physical systems can be cast in Symmetric Hyp. form.

Strong hyperbolicity

- High frequency limit.



- Strong-Hyperbolicity: $\longrightarrow$ Waves can be decomposed into a complete set of plane waves, each one of them with purely imaginary frequency.

Strong hyperbolicity

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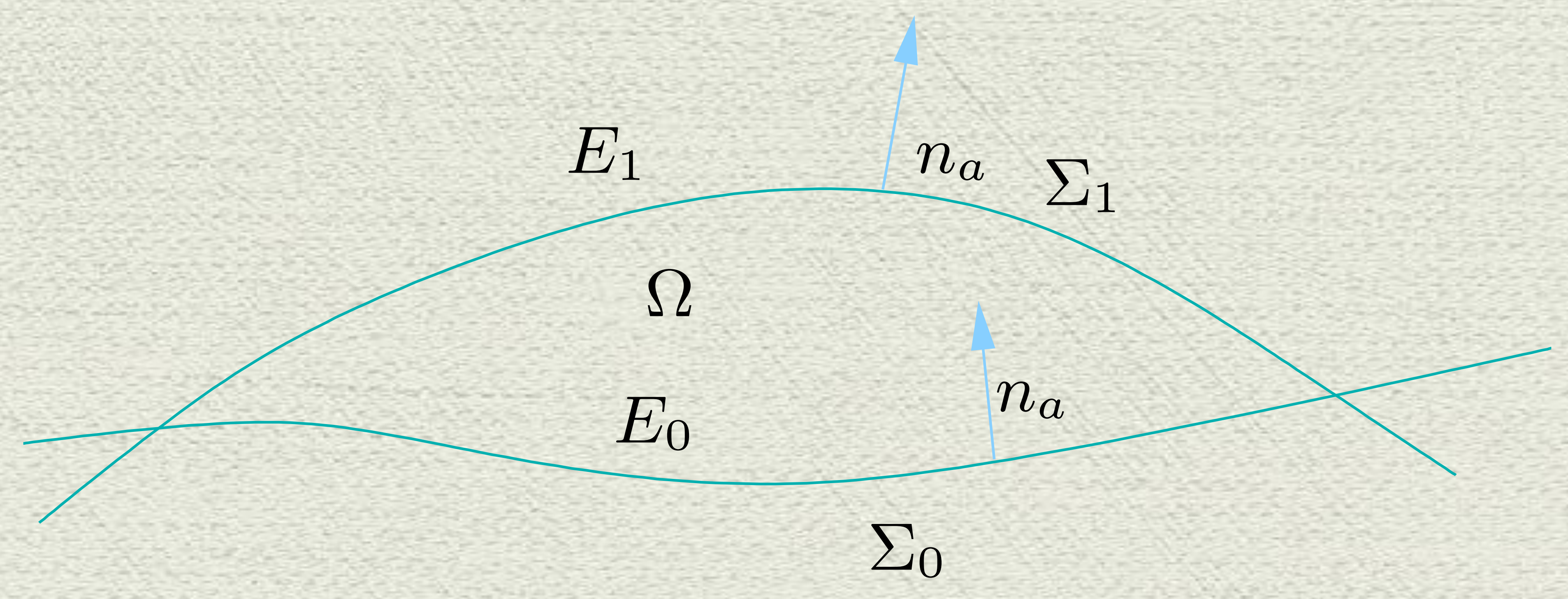
- Allows energy estimates:



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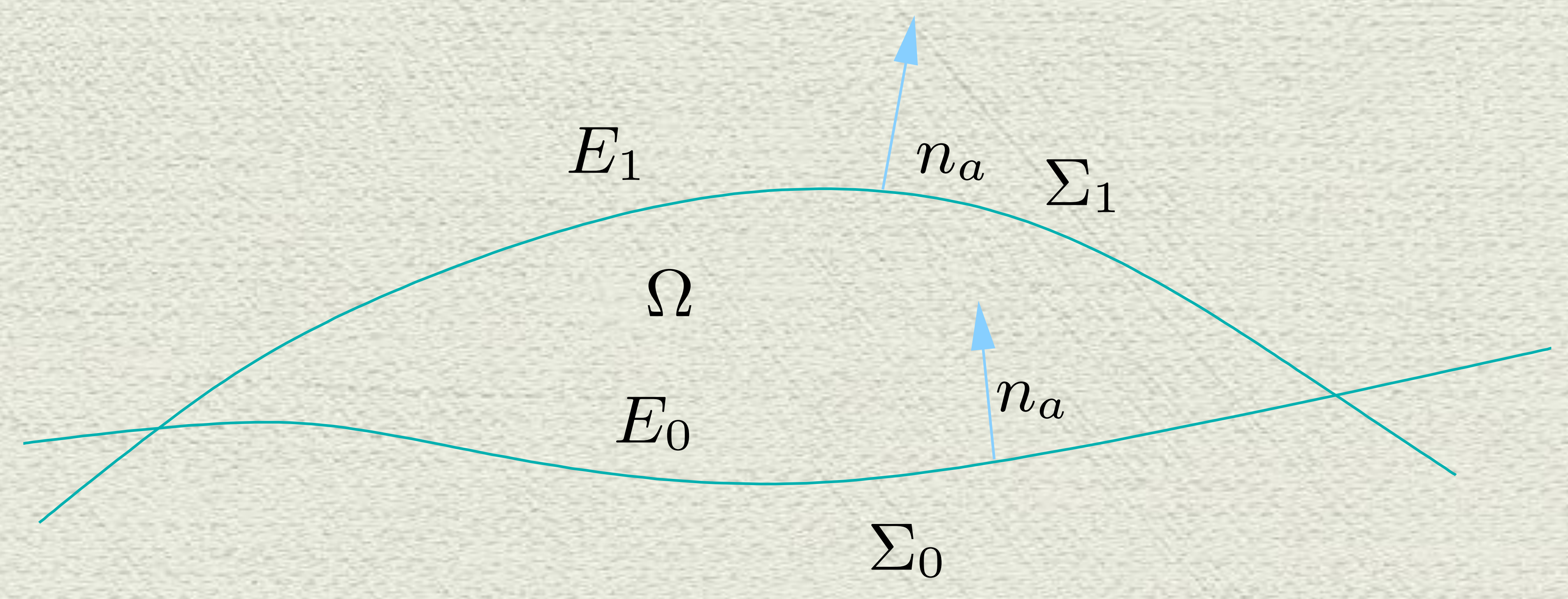

- Symmetric Hyperbolic: Usual energies

- Strongly Hyperbolic: Pseudodifferential Calculus

Quasilinear equations

General quasilinear equation:

$$A^{a}{}_{A\alpha}(u)\nabla_a u^{\alpha} = F_A(u)$$

$$u^{\alpha} = \{set\ \ of\ \ tensors\ \ in\ \ some\ \ manifold\ \ M\}$$

Fiber bundle over M of dimension m

$$F_A(u) = \{set\ \ of\ \ tensors\ \ also\ \ in\ \ M\}$$

Fiber bundle over M of dimension q

They can be also maps from manifolds into manifolds. (The differential among these bundles is always well defined)

Quasilinear systems of equations

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Electromagnetism

$$\nabla_{[a}F_{bc]} = 0 \qquad 4$$

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There are constraints.

$$\varepsilon^{tabc}\nabla_a F_{bc} = 0 \qquad \nabla^a F_{at} = 0 \qquad 2$$

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There are constraints.

$$\nabla_i \phi = l_i \quad \epsilon^{tijk}\nabla_j l_k = 0 \qquad 6$$

Symmetric hyperbolic systems (a la Geroch)

- Geroch paper on SH systems

$$A^a{}_{A\alpha}(u)\nabla_a u^\alpha = F_A(u)$$

Is symmetric hyperbolic or admits a hyperbolization if there exists $H_\beta{}^A$ such that:

$$H_\beta{}^A A^a_{A\alpha} = H_{(\beta}{}^A A^a_{|A|\alpha)}$$

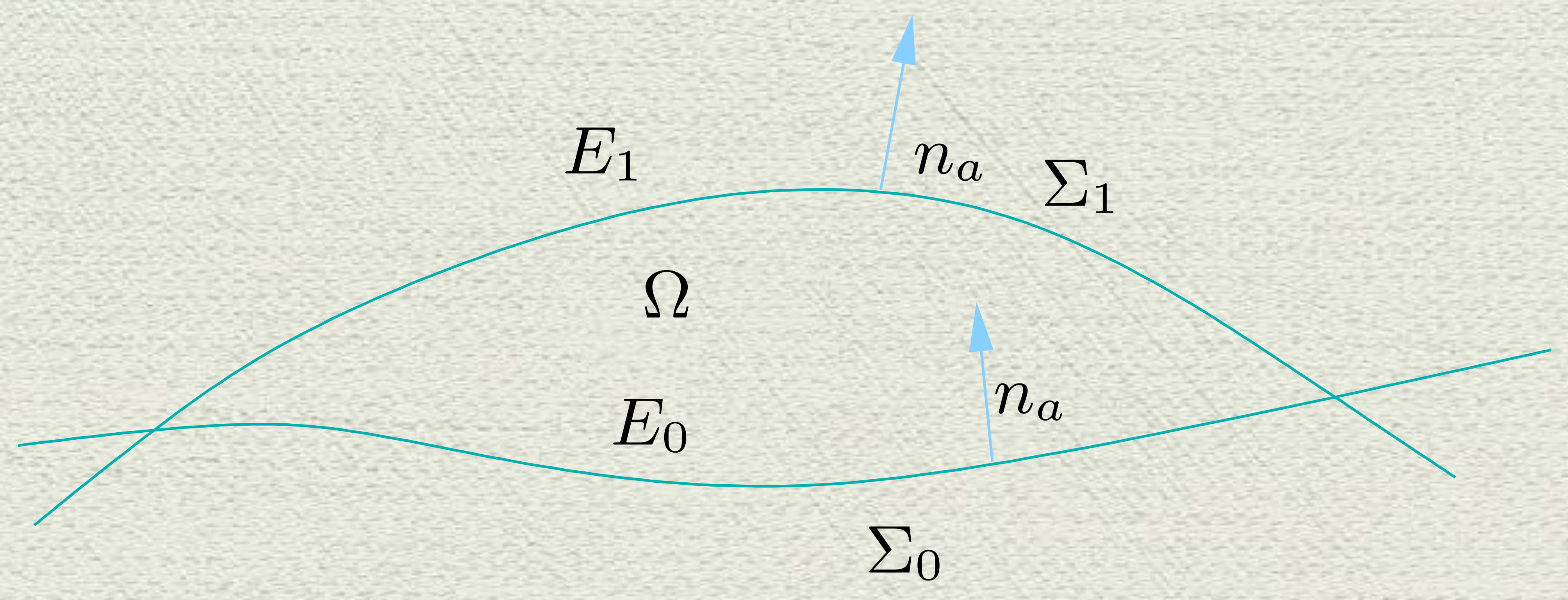
is symmetric and there exists n_a such that

$$H_\beta{}^A A^a_{A\alpha} n_a$$

is positive definite.

Strong hyperbolicity

- Allows energy estimates:



Symmetric hyperbolic systems

$$H_\beta{}^A A^a_{A\alpha} n_a$$

$$J^a := H_\beta{}^A A^a{}_{A\alpha} u^\beta u^\alpha \qquad \nabla_a J^a := H_\beta{}^A F_A u^\beta$$

A mathematical energy is defined by,

$$\mathcal{E}(t) = \int_{\Sigma_t} H_\beta{}^A A^a_{A\alpha} u^\beta u^\alpha n_a \, dV$$

$$\tilde{\mathcal{E}}(t) \leq \mathcal{F}(\tilde{\mathcal{E}}(0))$$

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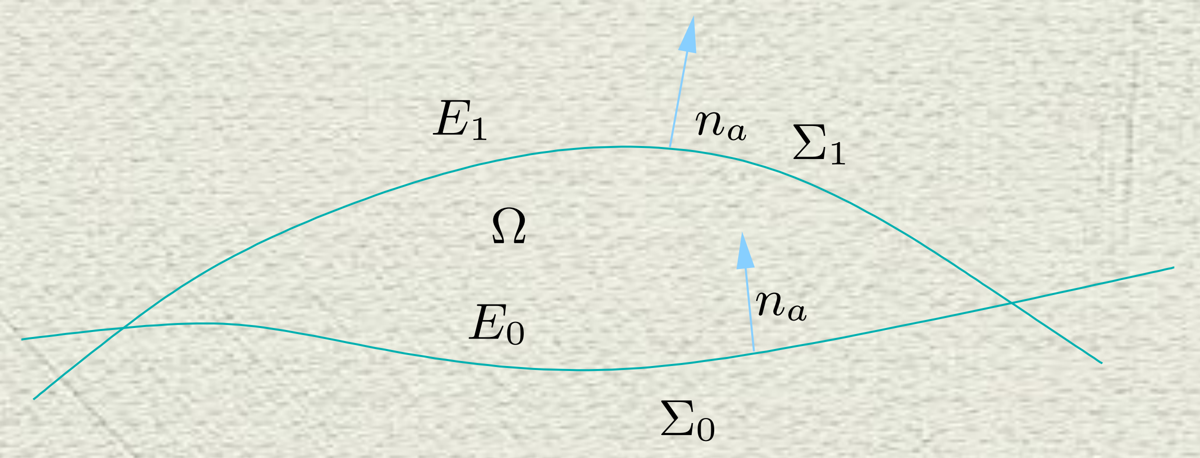

Symmetric hyperbolic systems

$$H_\beta{}^A A^a{}_{A\alpha} n_a > 0 \qquad\qquad H_\beta{}^A A^a{}_{A\alpha} \hat{n}_a > 0$$

$$H_\beta{}^A A^a{}_{A\alpha}(x n_a + y \hat{n}_a) > 0 \;\; \forall \;\; x, y > 0$$

The set of all co-vectors for which the above expression is positive is a cone. The cone of Cauchy planes. Its boundary is called the characteristic cone.

Causality is derived from this cone structure.

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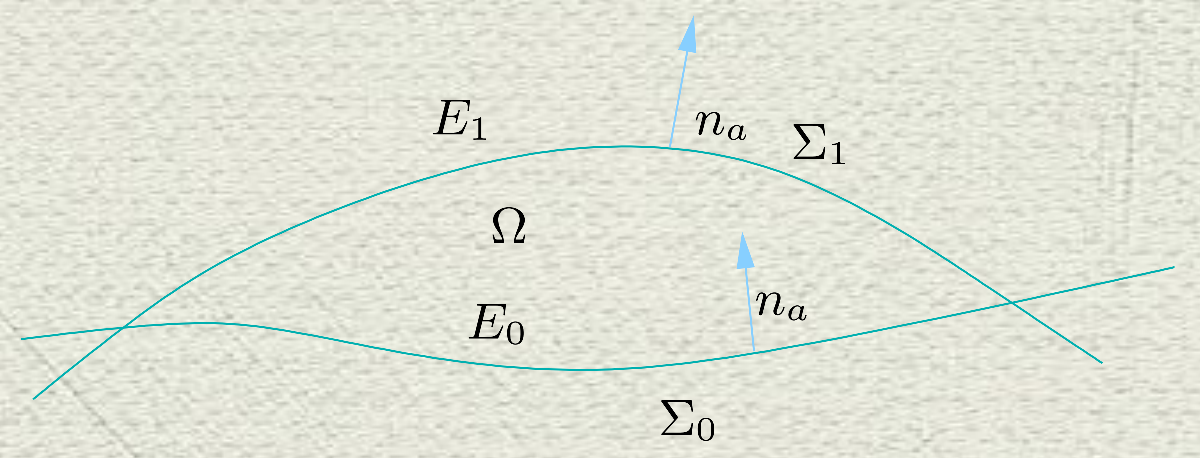

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Symmetric hyperbolic systems

Example: Wave equation

$$\nabla^a \nabla_a \phi = J$$

$$\begin{aligned} \nabla_a \phi &= l_a \\ \nabla_{[a} l_{b]} &= 0 \\ \nabla^a l_a &= J \end{aligned}$$

Symmetrizer: map from the space of equations to the space of unknowns.

Symmetric hyperbolic systems

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$$A^n{}_a = \delta^n{}_a, \quad A_{ab}{}^{nc} = \delta_{[a}{}^n \delta_{b]}{}^c, \quad A^{na} = g^{na}$$

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$$H_\alpha{}^\Gamma A^n_{\Gamma\beta}u_1^\alpha u_2^\beta = -v^n\phi_1\phi_2 - u^n g^{ab}l_{1a}l_{2b} + u^c l_{1c}l_2^n + u^c l_1^n l_{2c}$$

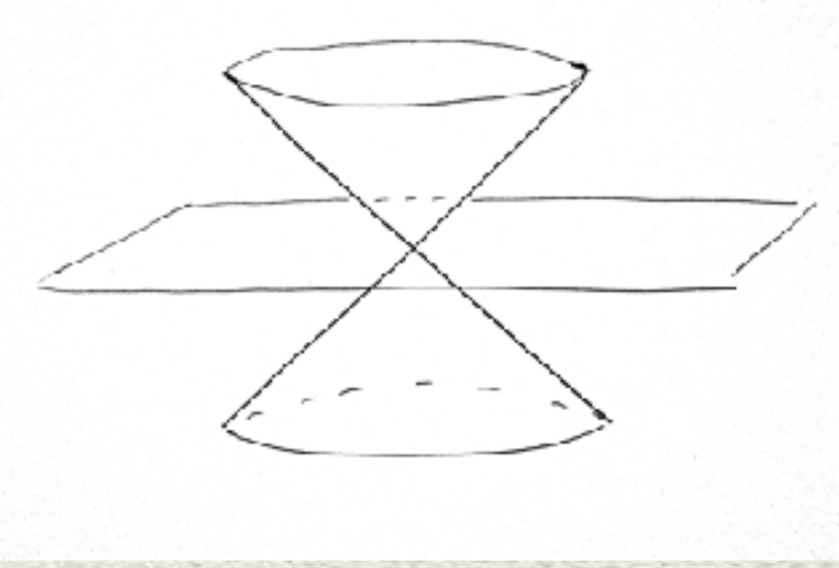
Pseudo-differential way

Wave Equation

$$\frac{\partial^2 \hat{\phi}}{\partial t^2} = -k^2 \hat{\phi} \qquad v := \partial_t \hat{\phi}, \qquad u := |k| \hat{\phi}$$

$$\frac{\partial}{\partial t} \begin{pmatrix} v \\ u \end{pmatrix} = |k| \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} v \\ u \end{pmatrix}$$

$$i\omega = \pm i|k|, \qquad \begin{pmatrix} 1 \\ \pm i \end{pmatrix}$$



System strongly hyperbolic!

Pseudo-differential way

Force-Free in Euler Potentials

$$\begin{aligned} F_{cb} \nabla_a F^{ab} &= 0 \\ \nabla_{[a} F_{bc]} &= 0 \end{aligned}$$

$$F_{ab} = \varepsilon^{ij} \nabla_a \phi_i \nabla_b \phi_j \ , \qquad A_a = \frac{1}{2} \varepsilon^{ij} \phi_i \nabla_a \phi_j$$

$$\varepsilon^{ij} \nabla_a \phi_k \nabla_c (\nabla^a \phi_i \nabla^c \phi_j) = 0, \qquad k = 1, 2,$$

Pseudo-differential way

Force-Free in Euler Potentials

$$\partial_t U = \mathbb{A}\, U, \qquad U = \begin{pmatrix} \partial_t \hat{\phi}_i \\ |k| \hat{\phi}_i \end{pmatrix}, \quad i = 1, 2.$$

$$\mathbb{A} = \begin{pmatrix} 0 & 0 & \frac{-k_1^2 - k_3^2}{|\vec{k}|} & -\frac{(\vec{\ell}_1 \cdot \vec{k})(\vec{\ell}_2 \cdot \vec{k})}{|\vec{k}| G_{22}} \\ 0 & 0 & -\frac{(\vec{\ell}_1 \cdot \vec{k})(\vec{\ell}_2 \cdot \vec{k})}{|\vec{k}| G_{11}} & \frac{-k_2^2 - k_3^2}{|\vec{k}|} \\ |\vec{k}| & 0 & 0 & 0 \\ 0 & |\vec{k}| & 0 & 0 \end{pmatrix}, \qquad \begin{aligned} \ell_{ai} &:= \nabla_a \bar{\phi}_i, \\ G_{ij} &:= \ell_{ai} g^{ab} \ell_{bj} \end{aligned}$$

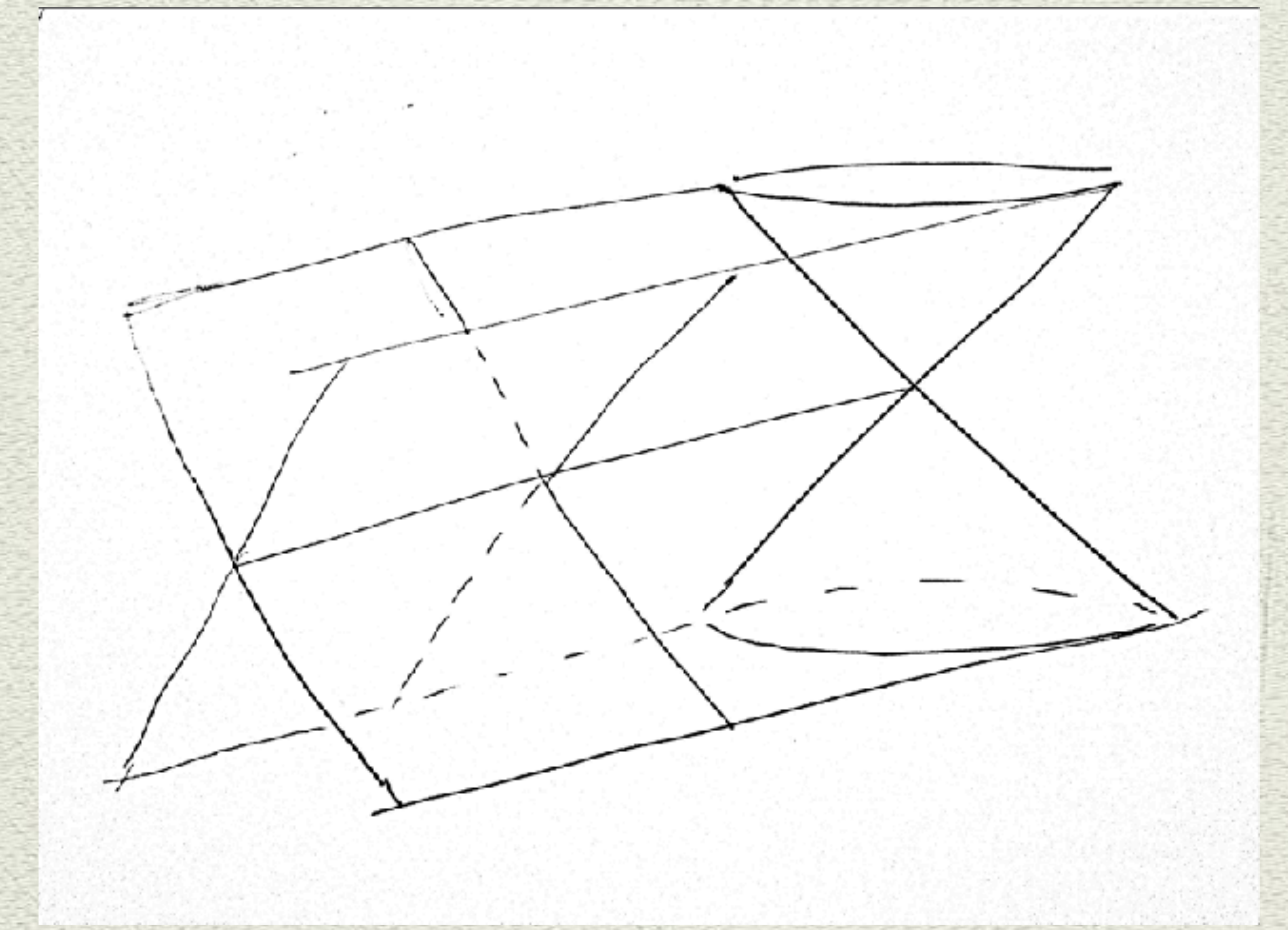
System is Ill Posed! $\quad k_0 = k_3 = 0$

Pseudo-differential way

Force-Free in Euler Potentials

$$(g^{ab}k_a k_b)\left(\left[g^{cd} - G^{-1ij}\ell_i^c \ell_j^d\right] k_c k_d\right) = 0$$

$$k_0 = k_3 = 0$$



System is Ill Posed!

OR, and Marcelo E. Rubio

Gracias por su atención!

Grav17

April 3rd-7th, 2017

La Falda, Córdoba, Argentina

Organized by:

General Relativity and Gravitation Group
FaMAF - Universidad Nacional de Córdoba

Organizing committee: *Federico Carrasco, María E. Gabach, Emanuel Gallo, Omar E. Ortiz, Oscar Reula*

The conference Grav17 will be held in La Falda, Córdoba (ARGENTINA), from April 3rd through April 7th, 2017. The venue of the conference is the "Hotel del Lago".

List of confirmed invited speakers

- Abhay Ashtekar, Penn State University, USA.
- Florian Beyer, University of Otago, New Zeland.
- Andrea Costa, Instituto de Astronomía Teórica y Experimental, Argentina.
- Jőrg Frauendiener, University of Otago, New Zeland.
- Helmut Friedrich, Max Planck Institute for Gravitational Physics, Germany.
- Gabriela González, Louisiana State University and LIGO, USA.
- Philippe Le Floch, Université Pierre et Marie Curie (UPMC, Paris 6), France.
- Luis Lehner, Perimeter Institute for Theoretical Physics, Canada.
- Alejandro Pérez, Université de Marseille, France.
- Jorge Pullin, Lousiana State University, USA.
- Martín Reiris, Universidad de Montevideo, Uruguay.
- Olivier Sarbach, Instituto de Física y Matemáticas Universidad Michoacana de San Nicolás de Hidalgo, México.
- Manuel Tiglio, San Diego Supercomputer Center, USA.
- Gabriela Vila, Instituto Argentino de Radioastronomía, Argentina.
- Robert Wald, The University of Chicago, USA.