Aprendizagem 2023

Lab 10: Dimensionality Reduction

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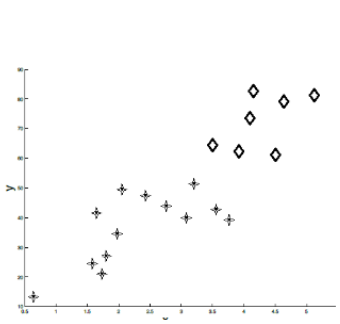
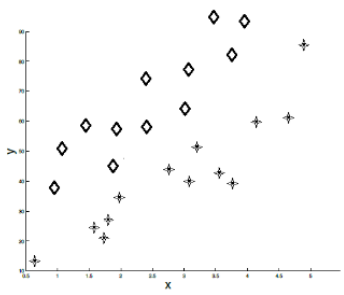
Practical exercises

1. Given the following datasets where observations are in $\mathbb{R}^2$ and belong to one of two classes:

	y_1	y_2
$\mathbf{x}_1$	0	0
$\mathbf{x}_2$	4	0
$\mathbf{x}_3$	2	1
$\mathbf{x}_4$	6	3





Which principal components can accurately discriminate the class per dataset?

2. The following top-7 eigenvalues explain 90% of the variation of dataset X:

λ_1=20, λ_2=10, λ_3=5, λ_4=4, λ_5=3, λ_6=2, λ_7=1

What is the most accurate information regarding X:

i. X has less than 7 attributes
ii. X has 7 attributes
iii. X has more than 7 attributes
iv. X has more than 11 attributes

3. Given a set of data points in $\mathbb{R}^3$, the following covariance matrix was obtained:

$$\begin{bmatrix} 91.43 & 171.92 & 297.99 \\ & 373.92 & 545.21 \\ & & 1297.26 \end{bmatrix}$$

as well as the following eigenvectors retrieved:

$$\boldsymbol{u}_1 = \begin{pmatrix} 0.2179 \\ 0.4145 \\ 0.8836 \end{pmatrix}, \boldsymbol{u}_2 = \begin{pmatrix} -0.2466 \\ -0.8525 \\ 0.4608 \end{pmatrix}, \boldsymbol{u}_3 = \begin{pmatrix} 0.9443 \\ -0.3183 \\ -0.0836 \end{pmatrix}$$

Please select the more complete answer:

i. eigenvalue λ1 is approximately 1626
ii. eigenvalue λ2 is approximately 129
iii. eigenvalues λ1 and λ2 explain >99% of the variation in data
iv. all of the above

4. Given the following dataset:

	y_1	y_2
$\mathbf{x}_1$	1	-1
$\mathbf{x}_2$	0	1
$\mathbf{x}_3$	-1	0

and the corresponding eigenvectors and eigenvalues:

λ_1=3/2 and λ_2=1/2

$$\boldsymbol{u}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \boldsymbol{u}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

a) Transform the input data using PCA

b) [*optional*] Assess the recovery error when considering the most informative component only