LEIC-T 2023/2024
Aprendizagem - Machine Learning

Homework IV - Group 007

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HM 4

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a) $x_1 = \begin{pmatrix} 2{,}5 \\ 2{,}5 \end{pmatrix}$, $x_2 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, $x_3 = \begin{pmatrix} 0{,}5 \\ 0{,}55 \end{pmatrix}$

$\pi_1 = 0{,}6$, $\pi_2 = 0{,}4$

$C_1\left(u_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \Sigma_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\right)$, $C_2\left(u_2 = \begin{pmatrix} 0{,}5 \\ 0{,}5 \end{pmatrix}, \Sigma_2 \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\right)$

For $x^{(1)}$

- For cluster $C=1$
  - Prior $p(C=1) = 0{,}6$
  - Likelihood: $p(x^{(1)} \mid C=1) = \frac{1}{2\pi} \frac{1}{\det(\Sigma_1)} \exp\left(-\frac{1}{2}(x_1 - u_1)^T (\Sigma_1)^{-1} (x_1 - u_1)\right) =$

$= \frac{1}{2\pi} \times \frac{1}{1} \exp\left(-\frac{1}{2}\left(\begin{pmatrix} 2{,}5 \\ 2{,}5 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}\right)^T \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \left(\begin{pmatrix} 2{,}5 \\ 2{,}5 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}\right)\right) =$

$= \frac{1}{2\pi} \times \exp\left(-\frac{1}{2}(0{,}5, 0{,}5) \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0{,}5 \\ 0{,}5 \end{pmatrix}\right) = \frac{1}{2\pi} \times \exp\left(-\frac{1}{2}(0{,}5, 0{,}5) \begin{pmatrix} 0 \\ 0{,}5 \end{pmatrix}\right) =$

$= \frac{1}{2\pi} \times \exp(-0{,}125) = 0{,}14$

  - Joint Probability: $p(C=1)\, p(x_1 \mid C=1) = 0{,}6 \times 0{,}14 = 0{,}084$

- For Cluster $C=2$
  - Prior $p(C=2) = 0{,}4$
  - Likelihood : $p(x^1|C=1) = \frac{1}{2\pi}\frac{1}{\det(\Sigma_2)} \exp\left(-\frac{1}{2}(x_1-\mu_2)(\Sigma_2)^{-1}(x_1-\mu_2)\right) =$

$= \frac{1}{2\pi} \times \frac{1}{1} \exp\left(-\frac{1}{2}\left(\begin{pmatrix}2{,}5\\2{,}5\end{pmatrix} - \begin{pmatrix}0{,}5\\0{,}5\end{pmatrix}\right)^T \begin{pmatrix}1&0\\-1&1\end{pmatrix}\left(\begin{pmatrix}2{,}5\\2{,}5\end{pmatrix} - \begin{pmatrix}0{,}5\\0{,}5\end{pmatrix}\right)\right) =$

$= \frac{1}{2\pi} \times \exp\left(-\frac{1}{2}(2,2)\begin{pmatrix}1&0\\-1&1\end{pmatrix}\begin{pmatrix}2\\2\end{pmatrix}\right) = \frac{1}{2\pi} \times \exp\left(-\frac{1}{2}(2\ 2)\begin{pmatrix}2\\0\end{pmatrix}\right) = \frac{1}{2\pi} \times \exp(-2$

$= 0{,}022$

  - Joint Probability : $P(C=2)\,P(x_1|C=2) = 0{,}4 \times 0{,}022 = 0{,}0088$

$P(C=1|x_1) = \frac{P(C=1, x_1)}{P(C=1, x_1) + P(C=2, x_1)} = \frac{0{,}084}{0{,}084 + 0{,}0088} = 0{,}905$

$P(C=2|x_1) = \frac{P(C=2, x_1)}{P(C=1, x_1) + P(C=2, x_1)} = \frac{0{,}0088}{0{,}084 + 0{,}0088} = 0{,}095$

For x^2

- For Cluster $C=1$
  - Prior $p(C=1) = 0{,}6$
  - Likelihood : $p(x_2|C=1) = \frac{1}{2\pi}\frac{1}{\det(\Sigma_1)} \times \exp\left(-\frac{1}{2} \times \left(\begin{pmatrix}2\\2\end{pmatrix} - \begin{pmatrix}2\\2\end{pmatrix}\right)^T \times \begin{pmatrix}1&-1\\0&1\end{pmatrix} \times \left(\begin{pmatrix}2\\2\end{pmatrix} - \begin{pmatrix}2\\2\end{pmatrix}\right)\right)$

$= \frac{1}{2\pi} \times \exp\left(-\frac{1}{2}(0\,0)\begin{pmatrix}1&-1\\0&1\end{pmatrix}\begin{pmatrix}0\\0\end{pmatrix}\right) = \frac{1}{2\pi} \times \exp\left(-\frac{1}{2} \times 0\right) = \frac{1}{2\pi} \times 1 = \frac{1}{2\pi}$

  - Joint Probability : $P(C=1)\,P(x_2|C=1) = 0{,}6 \times \frac{1}{2\pi} = 0{,}095$

- For Cluster $C=2$
  - Prior $p(C=2) = 0{,}4$
  - Likelihood : $P(x_2|C=2) = \frac{1}{2\pi} \times \frac{1}{\det(\Sigma_2)} \times \exp\left(-\frac{1}{2} \times (x_2-\mu_2)^T (\Sigma_2)^{-1}(x_2-\mu_2)\right) =$

$= \frac{1}{2\pi} \times \exp\left(-\frac{1}{2} \times \left(\begin{pmatrix}2\\2\end{pmatrix} - \begin{pmatrix}0{,}5\\0{,}5\end{pmatrix}\right)^T \begin{pmatrix}1&0\\-1&1\end{pmatrix}\begin{pmatrix}1{,}5\\1{,}5\end{pmatrix}\right) = \frac{1}{2\pi} \times \exp\left(-\frac{1}{2} \times (1{,}5\ \ 1{,}5)\begin{pmatrix}1{,}5\\0\end{pmatrix}\right) =$

$= \frac{1}{2\pi} \exp\left(-\frac{1}{2} \times 2{,}25\right) = \frac{1}{2\pi} \times \exp(-1{,}125) = 0{,}0517$

  - Joint Probability : $P(C=1)\,P(x_2|C=2) = 0{,}4 \times 0{,}0517 = 0{,}02$

$P(C=1|x_2) = \frac{P(C_1, x_2)}{P(C_1, x_2) + P(C_2, x_2)} = \frac{0{,}095}{0{,}095 + 0{,}02} = 0{,}83$

$P(C=2|x_2) = \frac{P(C_2, x_2)}{P(C_1, x_2) + P(C_2, x_2)} = \frac{0{,}02}{0{,}095 + 0{,}02} = 0{,}17$

For x^3

- For Cluster $C=1$
  - Prior $p(C=1)=0{,}6$
  - Likelihood: $p(x_3|C=1)=\frac{1}{2\pi}\times\frac{1}{\det(\Sigma_1)}\times\exp\left(-\frac{1}{2}\times(x_3-u_1)^T(\Sigma_1)^{-1}(x_3-u_1)\right)=$

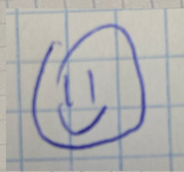
$=\frac{1}{2\pi}\times\exp\left(-\frac{1}{2}\times\left(\begin{pmatrix}0{,}5\\0{,}55\end{pmatrix}-\begin{pmatrix}2\\2\end{pmatrix}\right)^T\times\begin{pmatrix}1&-1\\0&1\end{pmatrix}\times\left(\begin{pmatrix}0{,}5\\0{,}55\end{pmatrix}-\begin{pmatrix}2\\2\end{pmatrix}\right)\right)=$

$=\frac{1}{2\pi}\times\exp\left(-\frac{1}{2}\times\begin{pmatrix}-1{,}5\\-1{,}45\end{pmatrix}^T\times\begin{pmatrix}1&-1\\0&1\end{pmatrix}\times\begin{pmatrix}-1{,}5\\-1{,}45\end{pmatrix}\right)=\frac{1}{2\pi}\times\exp\left(-\frac{1}{2}\times(-1{,}5\ \ -1{,}45)\times\begin{pmatrix}-0{,}05\\-1{,}45\end{pmatrix}\right)$

$=\frac{1}{2\pi}\times\exp\left(-\frac{1}{2}\times2{,}1775\right)=\frac{1}{2\pi}\exp(-1{,}089)=0{,}0536$

Joint Probability

$\frac{P(C=1,x_3)}{P(C=1,x_3)+P(C_2,x_3)}$



$P(C=1)\,P(x_3|C=1)=0{,}6\times0{,}0536=0{,}0322$

- For $C=2$
  - Prior $p(C=2)=0{,}4$
  - Likelihood $p(x^3|C=2)=\frac{1}{2\pi}\times\frac{1}{\det(\Sigma_2)}\times\exp\left(-\frac{1}{2}(x_3-u_2)^T(\Sigma_2)^{-1}(x_3-u_2)\right)=$

$=\frac{1}{2\pi}\times\exp\left(-\frac{1}{2}\left(\begin{pmatrix}0{,}5\\0{,}55\end{pmatrix}-\begin{pmatrix}0{,}5\\0{,}5\end{pmatrix}\right)^T\begin{pmatrix}1&0\\-1&1\end{pmatrix}\left(\begin{pmatrix}0{,}5\\0{,}55\end{pmatrix}-\begin{pmatrix}0{,}5\\0{,}5\end{pmatrix}\right)\right)=$

$=\frac{1}{2\pi}\times\exp\left(-\frac{1}{2}(0\ \ 0{,}05)\begin{pmatrix}1&0\\-1&1\end{pmatrix}\begin{pmatrix}0\\0{,}05\end{pmatrix}\right)=\frac{1}{2\pi}\times\exp\left(-\frac{1}{2}(0;\ 0{,}05)\begin{pmatrix}0\\0{,}05\end{pmatrix}\right)=$

$=\frac{1}{2\pi}\times\exp\left(-\frac{1}{2}\times0{,}0025\right)=\frac{1}{2\pi}\times\exp(-0{,}00125)=0{,}159$

Joint Probability

$P(C=2)\,P(x_3|C=2)=0{,}4\times0{,}159=0{,}0636$

$P(C=1|x_3)=\frac{P(C=1,x_3)}{P(C_1,x_3)+P(C_2,x_3)}=\frac{0{,}0322}{0{,}0322+0{,}0636}=0{,}34$

$P(C=2|x_3)=\frac{P(C=2;x_3)}{P(C_2;x_3)+P(C_1,x_3)}=\frac{0{,}0636}{0{,}0636+0{,}0322}=0{,}66$

M-STEP

• For C=1

• For likelihood

• $\mu^1 = \frac{0{,}905\begin{pmatrix}2{,}5\\2{,}5\end{pmatrix} + 0{,}83\begin{pmatrix}2\\2\end{pmatrix} + 0{,}34\begin{pmatrix}0{,}5\\0{,}55\end{pmatrix}}{0{,}905 + 0{,}83 + 0{,}34} =$

$$= \frac{\begin{pmatrix}2{,}2625\\2{,}2625\end{pmatrix} + \begin{pmatrix}1{,}66\\1{,}66\end{pmatrix} + \begin{pmatrix}0{,}17\\0{,}187\end{pmatrix}}{2{,}075} = \frac{\begin{pmatrix}4{,}09\\4{,}11\end{pmatrix}}{2{,}075} = \begin{pmatrix}1{,}97\\1{,}98\end{pmatrix}$$

$$\Sigma^1_{11} = \frac{0{,}905(2{,}5-1{,}97)(2{,}5-1{,}97) + 0{,}83(2-1{,}97)(2-1{,}97) + 0{,}34(\cancel{2}\,0{,}5-\cancel{2}\,1{,}97)(\cancel{2}\,0{,}5-1{,}97)}{2{,}075} =$$

$$= \frac{0{,}254 + 0{,}000747 + 0{,}739}{2{,}075} = 0{,}48$$

$$\Sigma^1_{12} = \Sigma^1_{21} = \frac{0{,}905(2{,}5-1{,}97)(2{,}5-1{,}98) + 0{,}83(2-1{,}97)(2-1{,}98) + 0{,}34(0{,}5-1{,}97)(0{,}55-1{,}98)}{2{,}075} =$$

$$= \frac{0{,}249 + 0{,}000498 + 0{,}715}{2{,}075} = 0{,}465$$

$$\Sigma_{22} = \frac{0{,}905(2{,}5-1{,}98)(2{,}5-1{,}98) + 0{,}83(2-1{,}98)(2-1{,}98) + 0{,}34(0{,}55-1{,}98)(0{,}55-1{,}98)}{2{,}075} =$$

$$= \frac{0{,}245 + 0{,}000332 + 0{,}695}{2{,}075} = 0{,}453$$

$$P(x|C=1) = N\left(\mu = \begin{pmatrix}1{,}97\\1{,}98\end{pmatrix}, \begin{bmatrix}0{,}48 & 0{,}465\\ 0{,}465 & 0{,}453\end{bmatrix} = \Sigma_1\right)$$

$$P(C=1) = \frac{0{,}905 + 0{,}83 + 0{,}34}{0{,}905 + 0{,}83 + 0{,}34 + 0{,}06 + 0{,}17 + 0{,}095} = 0{,}692$$

• For $c=2$

$$\mu_2 = \frac{0{,}095\begin{pmatrix}2{,}5\\2{,}5\end{pmatrix} + 0{,}17\begin{pmatrix}2\\2\end{pmatrix} + 0{,}66\begin{pmatrix}0{,}5\\0{,}55\end{pmatrix}}{0{,}095 + 0{,}17 + 0{,}66} = \frac{\begin{pmatrix}0{,}2375\\0{,}2375\end{pmatrix} + \begin{pmatrix}0{,}34\\0{,}34\end{pmatrix} + \begin{pmatrix}0{,}33\\0{,}363\end{pmatrix}}{0{,}925}$$

$$= \frac{\begin{pmatrix}0{,}9075\\0{,}9405\end{pmatrix}}{0{,}925} = \begin{pmatrix}0{,}98\\1{,}02\end{pmatrix}$$

$$\Sigma_{11} = \frac{0{,}095 \times (2{,}5 - 0{,}98)(2{,}5 - 0{,}98) + 0{,}17(2 - 0{,}98)(2 - 0{,}98) + 0{,}66(0{,}5 - 0{,}98)(0{,}5 - 0{,}98)}{0{,}425}$$

$$= \frac{0{,}219 + 0{,}177 + 0{,}192}{0{,}925} = 0{,}592$$

$$\Sigma^2_{12} = \Sigma^2_{21} = \frac{0{,}095(2{,}5 - 0{,}98)(2{,}5 - 1{,}02) + 0{,}17(2 - 0{,}98)(2 - 1{,}02) + 0{,}66(0{,}5 - 0{,}98)(0{,}55 - 1{,}02)}{0{,}925} =$$

$$= \frac{0{,}214 + 0{,}17 + 0{,}149}{0{,}925} = 0{,}576$$

$$\Sigma^2_{22} = \frac{0{,}095(2{,}5 - 1{,}02)(2{,}5 - 1{,}02) + 0{,}17(2 - 1{,}02)(2 - 1{,}02) + 0{,}66(0{,}55 - 1{,}02)(0{,}55 - 1{,}02)}{0{,}425} =$$

$$= \frac{0{,}208 + 0{,}163 + 0{,}146}{0{,}925} = 0{,}559$$

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$$p(x \mid c=2) = N\left(\mu^2 = \begin{pmatrix}0{,}98\\1{,}02\end{pmatrix}, \Sigma^2 = \begin{pmatrix}0{,}592 & 0{,}576\\0{,}576 & 0{,}559\end{pmatrix}\right)$$

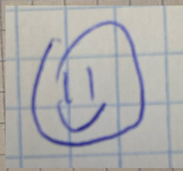
$$P(c=2) = \frac{0{,}925}{3} = 0{,}308$$

b)

$$\|x_1 - c_1\|_2^2 = \left\|\begin{pmatrix}2,5\\2,5\end{pmatrix} - \begin{pmatrix}2\\2\end{pmatrix}\right\|_2^2 = \left\|\begin{pmatrix}0,5\\0,5\end{pmatrix}\right\|_2^2 = \frac{\sqrt{2}}{2}$$

$$\|x_1 - c_2\|_2^2 = \left\|\begin{pmatrix}2,5-0,5\\2,5-0,5\end{pmatrix}\right\|_2^2 = \left\|\begin{pmatrix}2\\2\end{pmatrix}\right\|_2^2 = 2,828$$

x_1 pertence a C_1



$$\|x_2 - c_1\|_2^2 = \left\|\begin{pmatrix}2\\2\end{pmatrix} - \begin{pmatrix}2\\2\end{pmatrix}\right\|_2^2 = 0$$

$$\|x_2 - c_2\|_2^2 = \left(\left\|\begin{pmatrix}2-0,5\\2-0,5\end{pmatrix}\right\|^2\right) = 2,12$$

x_2 pertence a C_1

$$\|x_3 - c_1\|_2^2 = \left(\left\|\begin{pmatrix}0,5\\0,55\end{pmatrix} - \begin{pmatrix}2\\2\end{pmatrix}\right\|_2\right) = \begin{pmatrix}1,5\\1,45\end{pmatrix} = 2,09$$

$$\|x_3 - c_2\|_2^2 = \left(\left\|\begin{pmatrix}0,5-0,5\\0,55-0,5\end{pmatrix}\right\|\right) = \begin{pmatrix}0\\0,05\end{pmatrix}^{(1)} = 0,05$$

x_3 pertence a C_2

$$s(x_1) = 1 - \frac{\|x_1 - x_2\|}{\|x_1 - x_3\|} = \frac{1 - \left(\begin{pmatrix}2,5\\2,5\end{pmatrix} - \begin{pmatrix}2\\2\end{pmatrix}\right)}{\left\|\begin{pmatrix}2,5\\2,5\end{pmatrix} - \begin{pmatrix}0,5\\0,55\end{pmatrix}\right\|} = 1 - \frac{\left\|\begin{pmatrix}0,5\\0,5\end{pmatrix}\right\|_2^2}{\left\|\begin{pmatrix}2\\1,95\end{pmatrix}\right\|_2^2} =$$

$$1 - \frac{\frac{\sqrt{2}}{2}}{2,828} = \cancel{0,4945}\ 0,747$$

$$s(x_2) = 1 - \frac{\|x_2 - x_1\|}{\|x_2 - x_3\|} = 1 - \frac{\frac{\sqrt{2}}{2}}{\left\|\begin{pmatrix}1,5\\1,45\end{pmatrix}\right\|} = 1 - \frac{\frac{\sqrt{2}}{2}}{2,09} =$$

$$= \cancel{0,323}\ 0,662$$

~~s(x3) = 1 - ||~~

$$s(C_1) = \frac{s(x_1) + s(x_2)}{2} = \frac{\overset{0,747 + 0,662}{\cancel{0,495 + 0,323}}}{2} = \cancel{0,409}\ 0,705$$

II Software Experiments

(a)

```python
import matplotlib.pyplot as plt
from sklearn import metrics, datasets, cluster, mixture
from sklearn.decomposition import PCA

# Load the wine dataset
data = datasets.load_wine()
X, y = data.data, data.target

# Initialize a list to store silhouette scores for k-means and EM
kmeans_silhouettes = []
em_silhouettes = []

# Try different values of k (number of clusters)
k_values = [2, 3, 4]

for k in k_values:
    # K-means clustering
    kmeans_algo = cluster.KMeans(n_clusters=k, algorithm='elkan', n_init=10)
    kmeans_model = kmeans_algo.fit(X)
    kmeans_labels = kmeans_model.labels_
    kmeans_silhouette = metrics.silhouette_score(X, kmeans_labels, metric='euclidean')
    kmeans_silhouettes.append(kmeans_silhouette)

    # EM clustering
    em_algo = mixture.GaussianMixture(n_components=k, covariance_type='full', n_init=10)
    em_model = em_algo.fit(X)
    em_labels = em_model.predict(X)
    em_silhouette = metrics.silhouette_score(X, em_labels, metric='euclidean')
    em_silhouettes.append(em_silhouette)

# Display the silhouette scores for k-means and EM with different k values
for i, k in enumerate(k_values):
    print(f'K-Means with {k} clusters - Silhouette: {kmeans_silhouettes[i]}')
    print(f'EM with {k} clusters - Silhouette: {em_silhouettes[i]}')

# Now, perform PCA with two components and repeat the clustering experiments
pca = PCA(n_components=2)
X_pca = pca.fit(X).transform(X)

# Initialize new lists for silhouette scores with PCA
kmeans_silhouettes_pca = []
em_silhouettes_pca = []
```

```python
for k in k_values:
    # K-means clustering with PCA
    kmeans_algo_pca = cluster.KMeans(n_clusters=k, algorithm='elkan', n_init=10)
    kmeans_model_pca = kmeans_algo_pca.fit(X_pca)
    kmeans_labels_pca = kmeans_model_pca.labels_
    kmeans_silhouette_pca = metrics.silhouette_score(X_pca, kmeans_labels_pca, metric='euclidean')
    kmeans_silhouettes_pca.append(kmeans_silhouette_pca)

    # EM clustering with PCA
    em_algo_pca = mixture.GaussianMixture(n_components=k, covariance_type='full', n_init=10)
    em_model_pca = em_algo_pca.fit(X_pca)
    em_labels_pca = em_model_pca.predict(X_pca)
    em_silhouette_pca = metrics.silhouette_score(X_pca, em_labels_pca, metric='euclidean')
    em_silhouettes_pca.append(em_silhouette_pca)

# Display the silhouette scores for k-means and EM with PCA and different k values
for i, k in enumerate(k_values):
    print(f'K-Means with PCA and {k} clusters - Silhouette: {kmeans_silhouettes_pca[i]}')
    print(f'EM with PCA and {k} clusters - Silhouette: {em_silhouettes_pca[i]}')
```

```
K-Means with 2 clusters - Silhouette: 0.6568536504294317
EM with 2 clusters - Silhouette: 0.5510515269549274

K-Means with 3 clusters - Silhouette: 0.5711381937868838
EM with 3 clusters - Silhouette: 0.34726590057721557

K-Means with 4 clusters - Silhouette: 0.5620323449580341
EM with 4 clusters - Silhouette: 0.32712767339444565
```

```
K-Means with PCA and 2 clusters - Silhouette: 0.6572176888364498
EM with PCA and 2 clusters - Silhouette: 0.6419971749179373

K-Means with PCA and 3 clusters - Silhouette: 0.5722554756855063
EM with PCA and 3 clusters - Silhouette: 0.2623333079949891

K-Means with PCA and 4 clusters - Silhouette: 0.5633930017441461
EM with PCA and 4 clusters - Silhouette: 0.16208459426663982
```

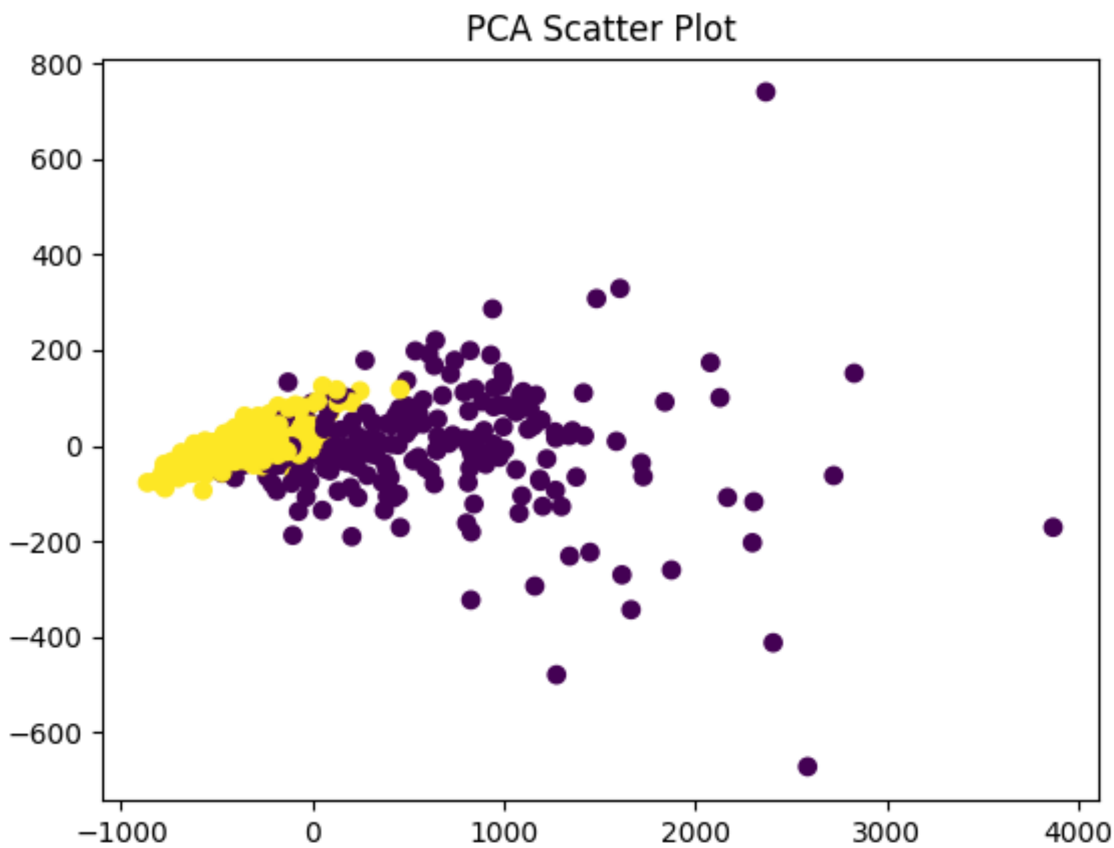
The ideal k-values for K-Means and EM clustering are as follows:
Without PCA:
- For K-Means, the ideal k-value is 2, with a silhouette score of 0.657.
- For EM, the ideal k-value is 2, with a silhouette score of 0.551.

With PCA (2 components):
- For K-Means with PCA, the ideal k-value is 2, with a silhouette score of 0.657.
- For EM with PCA, the ideal k-value is 2, with a silhouette score of 0.642.

The ideal k-values for K-Means and EM are the same when using PCA and when not using PCA in this specific case. However, it's important to note that the ideal k-value can vary depending on the dataset and the specific problem we are trying to solve. In this case, k=2 seems to provide the best clustering results based on silhouette scores.

(b)

```
import matplotlib.pyplot as plt
from sklearn import metrics, datasets, cluster, mixture
from sklearn.decomposition import PCA

# Load the breast_cancer dataset
data = datasets.load_breast_cancer()
X, y = data.data, data.target

# K-means clustering with 2 clusters
kmeans_algo = cluster.KMeans(n_clusters=2, algorithm='elkan', n_init=10)
kmeans_model = kmeans_algo.fit(X)
kmeans_labels = kmeans_model.labels_
kmeans_silhouette = metrics.silhouette_score(X, kmeans_labels, metric='euclidean')

# EM clustering with 2 clusters
em_algo = mixture.GaussianMixture(n_components=2, covariance_type='full', n_init=10)
em_model = em_algo.fit(X)
em_labels = em_model.predict(X)
em_silhouette = metrics.silhouette_score(X, em_labels, metric='euclidean')

# Perform PCA with two components
pca = PCA(n_components=2)
X_pca = pca.fit(X).transform(X)

# Scatter plot of PCA mapped data
plt.scatter(X_pca[:, 0], X_pca[:, 1], c=y)
plt.title("PCA Scatter Plot")
plt.show()

print("K-Means Silhouette Score: ", kmeans_silhouette)
print("EM Silhouette Score: ", em_silhouette)
```



K−Means Silhouette Score: 0.6972646156059464
EM Silhouette Score: 0.5315172918032405

Based on the silhouette values and the scatter plot of the PCA−mapped data, K−Means clustering with 2 clusters appears to be better, as it has a higher silhouette score (0.697) compared to EM clustering (0.532).