LEIC-T 2023/2024
Aprendizagem - Machine Learning
Homework 2
Deadline 9/10/2024 20:00
Submit on Fenix as pdf

I) Bayesian Classifier (8 pts)

Given a data set describing a sample

x_1	x_2	Class
0.5	0.5	A
1	1.5	A
1.5	0.8	A
2	1.8	A
2	0	B
2	1	B
3	0	B
5	1.2	B

And the query vector $x = (1,2)^T$

a) (3pts) Compute the most probable class for the query vector, under the Naive Bayes assumption, using 1-dimensional Gaussians to model the likelihoods. (Hint, the likelihood is described of each class is described by two Gaussians (Normal Distributions, each distribution is defined by a mean value and standard deviation..)

Solution

Gaussian distribution or normal is defined by *the relative probability*

$$p(x|\mu,\sigma^2) = \mathcal{N}(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\cdot\pi}\cdot\sigma}\cdot\exp\left(-\frac{1}{2\cdot\sigma^2}\cdot(x-\mu)^2\right)$$

We have two classes and two independent variables. Each likelihood of class is specified by two Normal distributions. (Naïve assumption)

$\square_{A1}$=$0.5 + 1 + 1.5 + 2)/4$=1.25

$\square_{A2}$=$0.5 + 1.5 + 0.8 + 1.8)/4 = 1.15$

σ_{A1}=$(((0.5-1.25)^2 + (1-1.25)^2 + (1.5-1.25)^2 + (2-1.25)^2)/3)^{1/2} = 0.645497$

σ_{A2}=$(((0.5-1.15)^2 + (1.5-1.15)^2 + (0.8-1.15)^2 + (1.8-1.15)^2)/3)^{1/2} = 0.60277$

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□$_{B1}$=$(2+2+3+5)/4$=3

□$_{B2}$=$(0+1+0+1.2)/4=0.55$

σ$_{B1}$=$(((0.5-1.25)^2+(1-1.25)^2+(1.5-1.25)^2+(2-1.25)^2)/3)^{1/2}=0.645497$

σ$_{B2}$=$((0.5-1.15)$^2 $+(1.5-1.15)$^2 $+(0.8-1.15)$^2 $+(1.8-1.15)^2)/3)$^$(1/2)$= 0.6027

p(A)=0.5, p(B)=0.5

p(A, xquery) = p(xquery,1|A1) · p(xquery,2|A2) · p(A)= 0.57338* 0.2448799*0.5=

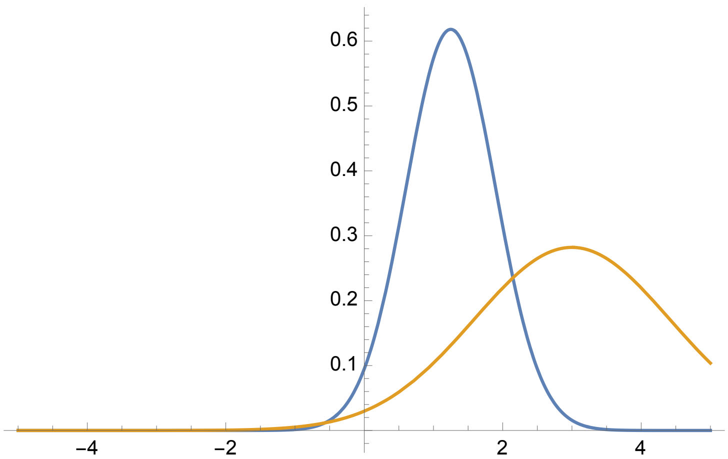
= 0.070204

p(B, xquery) = p(xquery,1|B1) · p(xquery,2|B2) · p(B) = $\frac{1}{2e\sqrt{\pi}}$* 0.04797*0.5=

= 0.0024891

p(A|xquery) = p(A, xquery)/ (p(A,xquery) +p(B,xquery))= 0.96575880048

p(B|xquery) = p(A, xquery)/ (p(A,xquery) +p(B,xquery))= 0.0342411995



First dimension (x1) of Normal distribution for class A and B

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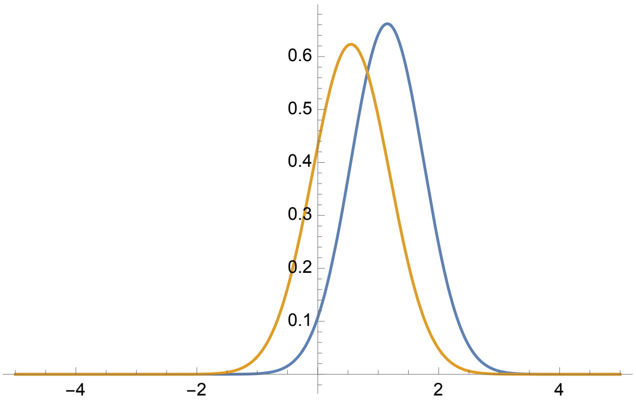


Second dimension (x2) of Normal distribution for class A and B

b) (3 pts) Compute the most probable class for the query vector assuming that the likelihoods are 2-dimensional Gaussians.

Over D dimensional space

$$p(\mathbf{x}|\boldsymbol{\mu},\Sigma) = \mathcal{N}(\mathbf{x}|\boldsymbol{\mu},\Sigma) = \frac{1}{(2\cdot\pi)^{D/2}}\cdot\frac{1}{|\Sigma|^{1/2}}\cdot\exp\left(-\frac{1}{2}\cdot(\mathbf{x}-\boldsymbol{\mu})^T\Sigma^{-1}\cdot(\mathbf{x}-\boldsymbol{\mu})\right)$$

where

- $\boldsymbol{\mu}$ is the D dimensional mean vector
- Σ is a $D\times D$ covariance matrix
- $|\Sigma|$ is the determinant of Σ

$\mu_A = (1.25, 1.15)^T$

$$ca11 = (((0.5-1.25)\hat{}2 + (1-1.25)\hat{}2 + (1.5-1.25)\hat{}2 + (2-1.25)\hat{}2))\ /\ 3 = 0.416666$$

$$ca22 = (((0.5-1.15)\hat{}2 + (1.5-1.15)\hat{}2 + (0.8-1.15)\hat{}2 + (1.8-1.15)\hat{}2\ ))\ /\ 3 = 0.36333$$

$$ca12 = \begin{matrix}((0.5-1.25)*(0.5-1.15)+(1-1.25)*(1.5-1.15)+(1.5-1.25)*(0.8-1.15)\\ +(2-1.25)*(1.8-1.15))/3 = 0.26666\end{matrix}$$

ΣA=

$$\begin{pmatrix} 0.416667 & 0.266667 \\ 0.266667 & 0.363333 \end{pmatrix}$$

Σ^{-1} A=

$$\begin{pmatrix} 4.52595 & -3.3218 \\ -3.3218 & 5.19031 \end{pmatrix}$$

Det(A)=0.080277

μB=(3, 0.55)^T

$$cb11 = (((2-3)\text{^}2 + (2-3)\text{^}2 + (3-3)\text{^}2 + (5-3)\text{^}2)) \,/\, 3$$

$$cb22 = (((0-0.55)\text{^}2 + (1-0.55)\text{^}2 + (0-0.55)\text{^}2 + (1.2-0.55)\text{^}2\,)) \,/\, 3 = 0.41$$

$cb12 = (((2-3)*(0-0.55) + (2-3)*(1-0.55) + (3-3)*(0-0.55) + (5-3)*(1.2-0.55))) \,/\, 3$=0.466667

ΣB=

$$\begin{pmatrix} 2 & 0.466667 \\ 0.466667 & 0.41 \end{pmatrix}$$

Σ^{-1} B=

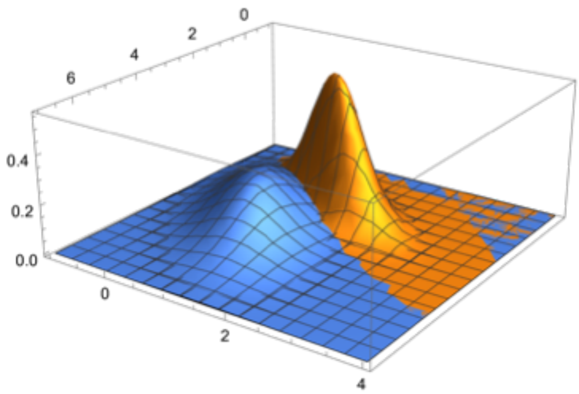
$$\begin{pmatrix} 0.680812 & -0.774908 \\ -0.774908 & 3.32103 \end{pmatrix}$$

Det(B)=0.6022

p(A, xquery) = p(xquery,1|A) · p(A) = 0.03691769*0.5= 0.0184588496

p(B,xquery)=p(xquery,1|B)·p(B) = 0.0001692*0.5= 0.0000846

p(A|xquery) = p(A, xquery)/ (p(A,xquery) +p(B,xquery))= 0.9954376



c) (1 pts)

x_3	Class
0	A
1	A
1	A
0	A
1	B
1	B
0	B
1	B

And the query vector *x3 =True=1*

Compute the most probable class, with x3 being a categorial class 1=True, 0=False.

Solution:

p(A|1)=card(A.1)/card(1)=2/5

P(B|1)= card(B.1)/card(1)=3/5

Most probable class is B

d) (1pts) Given a data set describing a sample combining the data set before

x_1	x_2	x_3	Class
0.5	0.5	0	A
1	1.5	1	A
1.5	0.8	1	A
2	1.8	0	A
2	0	1	B
2	1	1	B
3	0	0	B
5	1.2	1	B

x1 and x2 are dependable and x3 is independent of x1 and x2. x3 is a categorial class.
And the query vector $x = (1,2,1)^T$ Compute the most probable class and indicate the estimated relative probability.

Hint,

p(A, xquery) = p((1,2)|A) ·P(1|A) ·p(A)

p(B, xquery) = p((1,2)|B) ·P(1|B) ·p(B)

you have already computed the values in b) and in c)

P(1|A) =card(A.1)/card(A)=2/4
P(1|B) =card(A.1)/card(B)=3/4

Solution:

p(A, xquery) = p(xquery,1|A) · p(1|A) ·p(A) = 0.03691769*2/4*0.5= 0.0092294

p(B, xquery) = p(xquery,1|B) · p(1|B) ·p(B) = 0.0001692*3/4*0.5= 0.00006345

$p(A|x_{query})$ = p(A, xquery)/ (p(A,xquery) +p(B,xquery))= 0.993172

$p(B|x_{query})$ =1- $p(A|x_{query})$=0.006