1. $\{P_n\} \rightleftarrows P_{(x)} = a_0 + a_1 X + a_2 X^2 \ldots + a_{n-1} X^{n-1}$

$$= \sum_{i=0}^{n-1} a_i X^i \quad \forall a \in \mathbb{R}$$

asumimos que es un espacio vectorial por lo tanto se toma otro vector perteneciente

$m_{(x)} = d_0 + d_1 X + d_2 X^2 + \ldots + d_{n-1} X^{n-1}$

$$\sum_{i=0}^{n-1} d_i X^i \quad \forall d \in \mathbb{R}$$

• Cerradura y conmutatividad

$P_{(x)} + m_{(x)} = (a_0 + d_0) + (a_1 + d_1) X + (a_2 + d_2) X^2 + \ldots + (a_{n-1} + d_{n-1}) X^{n-1}$

$$= \sum_{i=0}^{n-1} (a+d)_i X^i \quad a, d \in \mathbb{R}$$

$m(X) + P(x) = (d_0 + a_0) + (d_1 + a_1) X + (d_2 + a_2) X^2 + \ldots + (d_{n-1} + a_{n-1}) X^{n-1}$

$$= \sum_{i=0}^{n-1} (d+a)_i X^i$$

• Asociatividad

$(m(x) + Y(x)) + P(x) = [(d_0 + C_0) + (d_1 + C_1) X + \ldots \; (_{n-1}) X^{n-1}] + P(x)$

$$m(x) = d_0 + d_1x + \dots + d_{n-1}x^{n-1}$$
$$= \sum_{i=0}^{n-1} d_i x^i \quad \forall i \in \mathbb{R}$$

• [illegible] conmutatividad

$$P(x) + m(x) = (a_0 + d_0) + (a_1 + d_1)x + (a_2 + d_2)x^2 + \dots + (a_{n-1} + d_{n-1})x^{n-1}$$
$$= \sum_{i=0}^{n-1} (a+d)_i x^i \quad a, d \in \mathbb{R}$$

$$m(x) + P(x) = (d_0 + a_0) + (d_1 + a_1)x + (d_2 + a_2)x^2 + \dots + (d_{n-1} + a_{n-1})x^{n-1}$$
$$= \sum_{i=0}^{n-1} (d+a)_i x^i$$

• Asociatividad

$$(m(x) + Y(x)) + P(x) = [(d_0 + C_0) + (d_1 + C_1)x + \dots + (d_{n-1} + C_{n-1})x^{n-1}] + P(x)$$
$$= (d_0 + C_0 + a_0) + (d_1 + C_1 + a_1)x + \dots + (d_{n-1} + C_{n-1} + a_{n-1})x^{n-1}$$
$$= \sum_{i=0}^{n-1} (d+C+a)_i x^i$$

$$(P(x) + m(x)) + (Y(x)) = [(a_0 + d_0) + (a_1 + d_1)x + \dots + (a_{n-1} + d_{n-1})x^{n-1}] + Y(x)$$
$$= [(a_0 + d_0 + C_0) + (a_1 + d_1 + C_1)x + \dots + (a_{n-1} + d_{n-1} + C_{n-1})x^{n-1}$$
$$= \sum_{i=0}^{n-1} (a+d+c)_i x^i$$

• Existencia del neutro aditivo

- Se analiza el caso

$$a_0 = a_1 = a_2 = \dots = a_{n-1} = 0$$

- Por lo tanto nuestro vector $P(x)$ es

$$P(x) = 0 + 0x + 0x^2 + \dots + 0x^{n-1}$$

• Existencia de inversos aditivos

- al estar trabajando con $a \in \mathbb{R}$ se toma

$$-P(x) = -[a_0 + a_1x + a_2x^2 + \dots + a_{n-1}x^{n-1}]$$

$$= -a_0 - a_1x - a_2x^2 - \dots - a_{n-1}x^{n-1}$$

• Cerradura multiplicación por escalar

$$\alpha P(x) = \alpha[a_0 + a_1x + a_2x^2 + \dots + a_{n-1}x^{n-1}]$$

$$= \alpha a_0 + \alpha a_1x + \alpha a_2x^2 + \dots + \alpha a_{n-1}x^{n-1}$$

$$= \sum_{i=0}^{n-1} \alpha a_i x^i = \alpha \sum_{i=0}^{n-1} a_i x^i$$

• Sean $\alpha, \beta \in \mathbb{R}$

$$(\alpha P(x))\beta = [\alpha a_0 + \alpha a_1 x + \alpha \dots$$

$P(x) = 0 + 0x + 0x^2 + \ldots + 0x^{n-1}$

- Existencia de inversos aditivos

- al estar trabajando con $a_i \in R$ [illegible]

$-P(x) = -[a_0 + a_1 x + a_2 x^2 + \ldots + a_{n-1} x^{n-1}]$

$= -a_0 - a_1 x - a_2 x^2 - \ldots - a_{n-1} x^{n-1}$

- Cerradura multiplicación por escalar

$\alpha P(x) = \alpha [a_0 + a_1 x + a_2 x^2 + \ldots + a_{n-1} x^{n-1}]$

$= \alpha a_0 + \alpha a_1 x + \alpha a_2 x^2 + \ldots + \alpha a_{n-1} x^{n-1}$

$= \sum_{i=0}^{n-1} \alpha a_i x^i = \alpha \sum_{i=0}^{n-1} a_i x^i$

- Sean $\alpha, \beta \in \mathbb{R}$

$(\alpha P(x))\beta = [\alpha a_0 + \alpha a_1 x + \alpha a_2 x^2 + \ldots + \alpha a_{n-1} x^{n-1}]\beta$

$= \beta\alpha a_0 + \beta\alpha a_1 x + \beta\alpha a_2 x^2 + \ldots + \beta\alpha a_{n-1} x^{n-1}$

$= \beta\alpha \sum_{i=0}^{n-1} a_i x^i$

$(\beta P(x))\alpha = [\beta a_0 + \beta a_1 x + \beta a_2 x^2 + \ldots + \beta a_{n-1} x^{n-1}]\alpha$

$= \alpha\beta a_0 + \alpha\beta a_1 x^1 + \alpha\beta a_2 x^2 + \ldots + \alpha\beta a_{n-1} x^{n-1}$

$= \alpha\beta \sum_{i=0}^{n-1} a_i x^i$

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$= -a_0 - a_1 X - a_2 X^2 - \dots - a_{n-1} X^{n-1}$

· Cerradura multiplicación por escalar

$$\alpha P(x) = \alpha [a_0 + a_1 X + a_2 X^2 + \dots + a_{n-1} X^{n-1}]$$
$$= \alpha a_0 + \alpha a_1 X + \alpha a_2 X^2 + \dots + \alpha a_{n-1} X^{n-1}$$
$$= \sum_{i=0}^{n-1} \alpha a_i X^i = \alpha \sum_{i=0}^{n-1} a_i X^i$$

· Sean $\alpha, \beta \in \mathbb{R}$

$$(\alpha P(x))\beta = [\alpha a_0 + \alpha a_1 X + \alpha a_2 X^2 + \dots + \alpha a_{n-1} X^{n-1}]\beta$$
$$= \beta\alpha a_0 + \beta\alpha a_1 X + \beta\alpha a_2 X^2 + \dots + \beta\alpha a_{n-1} X^{n-1}$$
$$= \beta\alpha \sum_{i=0}^{n-1} a_i X^i$$

$$(\beta P(x))\alpha = [\beta a_0 + \beta a_1 X + \beta a_2 X^2 + \dots + \beta a_{n-1} X^{n-1}]\alpha$$
$$= \alpha\beta a_0 + \alpha\beta a_1 X^1 + \alpha\beta a_2 X^2 + \dots + \alpha\beta a_{n-1} X^{n-1}$$
$$= \alpha\beta \sum_{i=0}^{n-1} a_i X^i$$

· Elemento neutro del producto

$1 \cdot P(x) = \sum_{i=0}^{n-1} (1 \cdot a_i) x^i = \sum a_i x^i = P(x)$

· Distributividad respecto a la suma de vectores

$\alpha(P(x) + m(x)) = \alpha[(a_0 + d_0) + (a_1 + d_1)x + \ldots + (a_{n-1} + d_{n-1})x^{n-1}]$

$= \alpha(a_0 + d_0) + \alpha(a_1 + d_1)x + \ldots + \alpha(a_{n-1} + d_{n-1})x^{n-1}]$

$= \alpha a_0 + \alpha d_0 + (\alpha a_1 + \alpha d_1)x + \ldots + (\alpha a_{n-1} + \alpha d_{n-1})x^{n-1}$

$= \alpha a_0 + \alpha d_0 + \alpha a_1 x + \alpha d_1 x + \ldots + \alpha a_{n-1} x^{n-1} + \alpha d_{n-1} x^{n-1}$

$= (\alpha a_0 + \alpha a_1 x + \ldots + \alpha a_{n-1} x^{n-1}) + (\alpha d_0 + \alpha d_1 x + \ldots + \alpha d_{n-1} x^{n-1})$

$= \sum_{i=0}^{n-1} \alpha a_i x^i + \sum_{i=0}^{n-1} \alpha d_i x^i$

$= \alpha \sum_{i=0}^{n-1} a_i x^i + \alpha \sum_{i=0}^{n-1} d_i x^i = \alpha P(x) + \alpha m(x)$

· distributividad respe[illegible] a la suma de escalares

$(\alpha + \beta)P(x) = (\alpha + \beta)$ [illegible] $a_1 x + a_2 x^2 + \ldots + a$ [illegible]

$= \alpha a_0 + \alpha d_0 + \alpha(a_1 + d_1)x + \dots + \alpha(a_{n-1} + d_{n-1})x^{n-1}$

$= \alpha a_0 + \alpha d_0 + \alpha a_1 x + \alpha d_1 x + \dots + \alpha a_{n-1} x^{n-1} + \alpha d_{n-1} x^{n-1}$

$= (\alpha a_0 + \alpha a_1 x + \dots + \alpha a_{n-1} x^{n-1}) + (\alpha d_0 + \alpha d_1 x + \dots + \alpha d_{n-1} x^{n-1})$

$= \sum_{i=0}^{n-1} \alpha a_i x^i + \sum_{i=0}^{n-1} \alpha d_i x^i$

$= \alpha \sum_{i=0}^{n-1} a_i x^i + \alpha \sum_{i=0}^{n-1} d_i x^i = \alpha P(x) + \alpha M(x)$

• distributividad respecto a la suma de escalares

$(\alpha+\beta)P(x) = (\alpha+\beta)(a_0 + a_1 x + a_2 x^2 + \dots + a_{n-1} x^{n-1})$

$= (\alpha a_0 + \alpha a_1 x + \alpha a_2 x^2 + \dots + \alpha a_{n-1} x^{n-1}) + (\beta a_0 + \beta a_1 x + \beta a_2 x^2 + \dots + \beta a_{n-1} x^{n-1})$

$= \alpha(a_0 + a_1 x + a_2 x^2 + \dots + a_{n-1} x^{n-1}) + \beta(a_0 + a_1 x + a_2 x^2 + \dots + a_{n-1} x^{n-1})$

$= \alpha \sum_{i=0}^{n-1} a_i x^i + \beta \sum_{i=0}^{n-1} a_i x^i$

$= \alpha P(x) + \beta P(x)$

---

b. Para $a_i \in \mathbb{Z}$

• no sera un espacio vectorial porque no podrá cumplir con el elemento neutro del cuerpo pues si se quiere tomar el elemento neutro de un coeficiente o escalar de la forma $2k+1$ no se podria realizar por la falta de los racionales

6. I)

- Contiene al polinomio cero
- Cerradura bajo la suma

$$p(x)+q(x)=(a_0+b_0)+(a_1+b_1)X+(a_2+b_2)X^2+\dots+(a_{n-1}+b_{n-1})X^{n-1}$$

donde $a, b \in \mathbb{R}$ por lo tanto su suma está definida dentro de los reales

- Cerradura bajo multiplicación por escalar

$$\alpha p(x) = \alpha a_0 + \alpha a_1 X + \alpha a_2 X^2 + \dots + \alpha a_{n-1} X^{n-1}$$
$$= \alpha (a_0 + a_1 X + a_2 X^2 + \dots + a_{n-1} X^{n-1})$$

II)

- Contiene al polinomio cero
- Cerradura bajo la suma

$$p(x)+q(x)=(a_2+b_2)X^2+(a_4+b_4)X^4+\dots+(a_{2n}+b_{2n})X^{2n}$$

donde $a, b \in \mathbb{R}$ y su suma esta definida sobre los reales

- Cerradura bajo escalar

$\alpha P(x) = \alpha a_0 + \alpha a_1 x + \alpha a_2 x^2 + \dots + \alpha a_n x^n$ [illegible]

$= \alpha(a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n)$ [illegible]

II)

- Contiene al Polinomio cero

- Cerradura bajo la suma

$P(x) + g(x) = (a_2 + b_2)x^2 + (a_4 + b_4)x^4 + \dots + (a_{2n} + b_{2n})x^{2n}$

donde $a, b \in \mathbb{R}$ y su suma esta definida sobre los reales

- Cerradura bajo escalar

$\alpha P(x) = \alpha(a_2 x^2 + a_4 x^4 + \dots + a_{2n} x^{2n})$

$= \alpha a_2 x^2 + \alpha a_4 x^4 + \dots + \alpha a_{2n} x^{2n}$

III)

- Contiene al Polinomio cero para $a_2 + a_3 + a_4 + \dots + a_{n-1} = 0$

- Cerradura bajo la suma

$P(x) + g(x) = (a_2 + b_2)x^2 + (a_3 + b_3)x^3 + \dots + (a_{n-1} + b_{n-1})x^{n-1}$

donde $a, b \in \mathbb{R}$ Por lo tanto su suma está definida sobre los reales

- Cerradura bajo escalar

$\alpha P(x) = \alpha(a_2 x^2 + a_3 x^3 + \dots + a_{n-1} x^{n-1})$

$= \alpha a_2 x^2 + \alpha a_3 x^3 + \dots + \alpha a_{n-1} x^{n-1})$

I

• el polinomio cero está incluido $P_{(x)} = (X-1)r = 0$

• Cerradura bajo la suma

$P_{(x)} + P_{(x)} = (X-1)r + (X-1)s = (X-1)(r+s)$

• Cerradura bajo escalar

$\alpha P_{(x)} = \alpha (X-1) r$

$= (X-1) \alpha r$

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