a) Conmutatividad

Sea $|a\rangle = a^{\alpha}|q_{\alpha}\rangle$, $|m\rangle = m^{\alpha}|q_{\alpha}\rangle$ dos cuaterniones con $a^{\alpha}, m^{\alpha} \in \mathbb{R}$

$$|a\rangle + |m\rangle = |m\rangle + |a\rangle$$

Con la cerradura bajo la suma se probo $|a\rangle + |m\rangle$ por lo tanto hace falta probar $|m\rangle + |a\rangle$

Para $|m\rangle + |a\rangle$

$$|m\rangle + |a\rangle = (m^0|q_0\rangle + m^1|q_1\rangle + m^2|q_2\rangle + m^3|q_3\rangle) + (a^0|q_0\rangle + a^1|q_1\rangle + a^2|q_2\rangle + a^3|q_3\rangle)$$

$$= (m^0 + a^0)|q_0\rangle + (m^1 + a^1)|q_1\rangle + (m^2 + a^2)|q_2\rangle + (m^3 + a^3)|q_3\rangle$$

$$= c^{\alpha}|q_{\alpha}\rangle \qquad m^{\alpha} + a^{\alpha} = c^{\alpha}$$

a) Asociatividad

Sea $|a\rangle = a^{\alpha}|q_{\alpha}\rangle$, $|m\rangle = m^{\alpha}|q_{\alpha}\rangle$, $|Y\rangle = Y^{\alpha}|q_{\alpha}\rangle$ con $a^{\alpha}, m^{\alpha}, Y^{\alpha} \in \mathbb{R}$

$$(|a\rangle + |m\rangle) + |Y\rangle = [(a_0 + m_0)|q_0\rangle + (a^1 + m^1)|q_1\rangle + (a^2 + m^2)|q_2\rangle + (a^3 + m^3)|q_3\rangle] + Y^0|q_0\rangle + Y^1|q_1\rangle + Y^2|q_2\rangle + Y^3|q_3\rangle$$

$$= (a_0 + m_0 + Y_0)|q_0\rangle + (a^1 + m^1 + Y^1)|q_1\rangle + (a^2 + m^2 + Y^2)|q_2\rangle + (a^3 + m^3 + Y^3)|q_3\rangle$$

Con la cerradura bajo la suma se probó $|a\rangle + |m\rangle$ por lo tanto hace falta probar $|m\rangle + |a\rangle$

Para $|m\rangle + |a\rangle$

$$|m\rangle + |a\rangle = (m^0|q_0\rangle + m^1|q_1\rangle + m^2|q_2\rangle + m^3|q_3\rangle) + (a^0|q_0\rangle + a^1|q_1\rangle + a^2|q_2\rangle + a^3|q_3\rangle)$$

$$= (m^0 + a^0)|q_0\rangle + (m^1 + a^1)|q_1\rangle + (m^2 + a^2)|q_2\rangle + (m^3 + a^3)|q_3\rangle$$

$$= C^\alpha |q_\alpha\rangle \qquad m^\alpha + a^\alpha = C^\alpha$$

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a) Asociatividad

Sea $|a\rangle = a^\alpha |q_\alpha\rangle$, $|m\rangle = m^\alpha |q_\alpha\rangle$, $|Y\rangle = Y^\alpha |q_\alpha\rangle$ con $a^\alpha, m^\alpha, Y^\alpha \in \mathbb{R}$

$$(|a\rangle + |m\rangle) + |Y\rangle = [(a_0 + m_0)|q_0\rangle + (a^1 + m^1)|q_1\rangle + (a^2 + m^2)|q_2\rangle + (a^3 + m^3)|q_3\rangle] + Y^0|q_0\rangle + Y^1|q_1\rangle + Y^2|q_2\rangle + Y^3|q_3\rangle$$

$$= (a_0 + m_0 + Y_0)|q_0\rangle + (a^1 + m^1 + Y^1)|q_1\rangle + (a^2 + m^2 + Y^2)|q_2\rangle + (a^3 + m^3 + Y^3)|q_3\rangle$$

$$= i^\alpha q_\alpha$$

$$(|Y\rangle + |a\rangle) + |m\rangle = [(Y^0 + a^0)|q_0\rangle + (Y^1 + a^1)|q_1\rangle + (Y^2 + a^2)|q_2\rangle + (Y^3 + a^3)|q_3\rangle] + m^0|q_0\rangle + m^1|q_1\rangle + m^2|q_2\rangle + m^3|q_3\rangle$$

$$= (Y^0 + a^0 + m^0)|q_0\rangle + (Y^1 + a^1 + m^1)|q_1\rangle + (Y^2 + a^2 + m^2)|q_2\rangle + (Y^3 + a^3 + m^3)|q_3\rangle$$

$$= i^\alpha q_\alpha$$

• a) existencia del neutro aditivo

Sea $|a\rangle = a^{\alpha}|q_{\alpha}\rangle$, $|m\rangle = m^{\alpha}|q_{\alpha}\rangle$ donde $a^{\alpha}, m^{\alpha} \in \mathbb{R}$ y $m^0 = m^1 = m^2 = m^3 = 0$

$$|a\rangle + |m\rangle = (a^0 + m^0)|q_0\rangle + (a^1 + m^1)|q_1\rangle + (a^2 + m^2)|q_2\rangle + (a^3 + m^3)|q_3\rangle$$

$$= (a^0 + 0)|q_0\rangle + (a^1 + 0)|q_1\rangle + (a^2 + 0)|q_2\rangle + (a^3 + 0)|q_3\rangle$$

$$= a^0|q_0\rangle + a^1|q_1\rangle + a^2|q_2\rangle + a^3|q_3\rangle$$

$$= a^{\alpha}|q_{\alpha}\rangle$$

---

• a) existencia de inverso aditivo

Sea $|a\rangle = a^{\alpha}|q_{\alpha}\rangle$, $|-a\rangle = -a^{\alpha}|q_{\alpha}\rangle$

$$|a\rangle + (|-a\rangle) = (a^0 - a^0)|q_0\rangle + (a^1 - a^1)|q_1\rangle + (a^2 - a^2)|q_2\rangle + (a^3 - a^3)|q_3\rangle$$

$$= 0|q_0\rangle + 0|q_1\rangle + 0|q_2\rangle + 0|q_3\rangle$$

---

• a) Cerradura respecto a la multiplicación por escalar

Sea $|a\rangle = a^{\alpha}|q_{\alpha}\rangle$ y $a^{\alpha}, \beta \in \mathbb{R}$

$$\beta|a\rangle = \beta(a^{\alpha}|q_{\alpha}\rangle) = \beta(a^0|q_0\rangle + a^1|q_1\rangle + a^2|q_2\rangle + a^3|q_3\rangle)$$

$$= a^0|a_0\rangle + a^1|a_1\rangle + a^2|a_2\rangle + a^3|a_3\rangle$$

$$= a^\alpha|a_\alpha\rangle$$

---

• a) existencia de inverso aditivo

sea $|a\rangle = a^\alpha|a_\alpha\rangle$, $|-a\rangle = -a^\alpha|a_\alpha\rangle$

$$|a\rangle + |-a\rangle = (a^0-a^0)|a_0\rangle + (a^1-a^1)|a_1\rangle + (a^2-a^2)|a_2\rangle + (a^3-a^3)|a_3\rangle$$

$$= 0|a_0\rangle + 0|a_1\rangle + 0|a_2\rangle + 0|a_3\rangle$$

---

• a) Cerradura respecto a la multiplicación por escalar

sea $|a\rangle = a^\alpha|a_\alpha\rangle$ y $a^\alpha, \beta \in \mathbb{R}$

$$\beta|a\rangle = \beta(a^\alpha|a_\alpha\rangle) = \beta(a^0|a_0\rangle + a^1|a_1\rangle + a^2|a_2\rangle + a^3|a_3\rangle)$$

$$= \beta a^0|a_0\rangle + \beta a^1|a_1\rangle + \beta a^2|a_2\rangle + \beta a^3|a_3\rangle \quad \boxed{\beta a^i = K^i}$$

$$= K^0|a_0\rangle + K^1|a_1\rangle + K^2|a_2\rangle + K^3|a_3\rangle$$

---

• a) asociatividad de la multiplicación escalar

$$\gamma(\beta|a\rangle) = (\gamma|a\rangle)\beta$$

$$= [\gamma(a^0|a_0\rangle + a^1|a_1\rangle + a^2|a_2\rangle + a^3|a_3\rangle)]\beta$$

$$= [\gamma a^0|a_0\rangle + \gamma a^1|a_1\rangle + \gamma a^2|a_2\rangle + \gamma a^3|a_3\rangle]\beta$$

$$= \beta\gamma a^0|a_0\rangle + \beta\gamma a^1|a_1\rangle + \beta\gamma a^2|a_2\rangle + \beta\gamma a^3|a_3\rangle$$

$$= \gamma(\beta a^0|a_0\rangle + \beta a^1|a_1\rangle + \beta a^2|a_2\rangle + \beta a^3|a_3\rangle$$

$$= \gamma(\beta|a\rangle)$$

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a) elemento neutro del cuerpo

$$1 \odot |a\rangle = 1 \odot (a^0 + a^1|q_1\rangle + a^2|q_2\rangle + a^3|q_3\rangle)$$
$$= 1 \cdot a^0 + 1 \cdot a^1|q_1\rangle + 1 \cdot a^2|q_2\rangle + 1 \cdot a^3|q_3\rangle$$
$$= a^0 + a^1|q_1\rangle + a^2|q_2\rangle + a^3|q_3\rangle$$
$$= |a\rangle$$

a) distributividad respecto a la suma de vectores

$$\beta(|a\rangle + |m\rangle) = \beta[(a_0 + a^1|q_1\rangle + a^2|q_1\rangle + a^3|q_3\rangle) + (m_0 + m^1|q_1\rangle + m^2|q_2\rangle + m^3|q_3\rangle)]$$
$$= \beta[(a_0 + m_0) + (a^1 + m^1)|q_1\rangle + (a^2 + m^2)|q_2\rangle + (a^3 + m^3)|q_3\rangle]$$
$$= \beta(a_0 + m_0) + \beta(a^1 + m^1)|q_1\rangle + \beta(a^2 + m^2)|q_2\rangle + (a^3 + m^3)|q_3\rangle$$
$$= \beta a_0 + \beta m_0 + \beta a^1|q_1\rangle + \beta m^1|q_1\rangle + \beta a^2|q_2\rangle + \beta m^2|q_2\rangle + \beta a^3|q_3\rangle + \beta m^3|q_3\rangle$$
$$= \beta a_0 + \beta a^1|q_1\rangle + \beta a^2|q_2\rangle + \beta a^3|q_3\rangle + \beta m^0 + \beta m^1|q_1\rangle + \beta m^2|q_2\rangle + \beta m^3|q_3\rangle$$
$$= \beta(a_0 + a^1|q_1\rangle + a^2|q_2\rangle + a^3|q_3\rangle) + \beta(m^0 + m^1|q_1\rangle + m^2|q_2\rangle + m^3|q_3\rangle)$$
$$= \beta|a\rangle + \beta|m\rangle$$

a) Distributividad respecto a la suma de escalares

$= a^0 + a^1|q_1\rangle + a^2|q_2\rangle + a^3|q_3\rangle$

$= |q\rangle$

---

a) distributividad respecto a la suma de vectores

$\beta(|q\rangle + |m\rangle) = \beta[(a_0 + a^1|q_1\rangle + a^2|q_2\rangle + a^3|q_3\rangle) + (m_0 + m^1|q_1\rangle + m^2|q_2\rangle + m^3|q_3\rangle)]$

$= \beta[(a_0 + m_0) + (a^1 + m^1)|q_1\rangle + (a^2 + m^2)|q_2\rangle + (a^3 + m^3)|q_3\rangle]$

$= \beta(a_0 + m_0) + \beta(a^1 + m^1)|q_1\rangle + \beta(a^2 + m^2)|q_2\rangle + (a^3 + m^3)|q_3\rangle$

$= \beta a_0 + \beta m_0 + \beta a^1|q_1\rangle + \beta m^1|q_1\rangle + \beta a^2|q_2\rangle + \beta m^2|q_2\rangle + \beta a^3|q_3\rangle + \beta m^3|q_3\rangle$

$= \beta a_0 + \beta a^1|q_1\rangle + \beta a^2|q_2\rangle + \beta a^3|q_3\rangle + \beta m^0 + \beta m^1|q_1\rangle + \beta m^2|q_2\rangle + \beta m^3|q_3\rangle$

$= \beta(a_0 + a^1|q_1\rangle + a^2|q_2\rangle + a^3|q_3\rangle) + \beta(m^0 + m^1|q_1\rangle + m^2|q_2\rangle + m^3|q_3\rangle)$

$= \beta|a\rangle + \beta|m\rangle$

---

a) Distributividad respecto a la suma de escalares

$(\beta + \gamma)|a\rangle = (\beta + \gamma)(a^0 + a^1|q_1\rangle + a^2|q_2\rangle + a^3|q_3\rangle)$

$= (\beta + \gamma)a^0 + (\beta + \gamma)a^1|q_1\rangle + (\beta + \gamma)a^2|q_2\rangle + (\beta + \gamma)a^3|q_3\rangle$

$= \beta a^0 + \gamma a_0 + \beta a^1|q_1\rangle + \gamma a^1|q_1\rangle + \beta a^2|q_2\rangle + \gamma a^2|q_2\rangle + \beta a^3|q_3\rangle + \gamma a^3|q_3\rangle$

$= \beta a^0 + \beta a^1|q_1\rangle + \beta a^2|q_2\rangle + \beta a^3|q_3\rangle + \gamma a^0 + \gamma a^1|q_1\rangle + \gamma a^2|q_2\rangle + \gamma a^3|q_3\rangle$

$= \beta(a^0 + a^1|q_1\rangle + a^2|q_2\rangle + a^3|q_3\rangle) + \gamma(a^0 + a^1|q_1\rangle + a^2|q_2\rangle + a^3|q_3\rangle)$

$= \beta|q\rangle + \gamma|q\rangle$

b) $|b\rangle \equiv (b^0, \mathbf{b})$ y $|r\rangle \equiv (r^0, \mathbf{r})$

$$|b\rangle \odot |r\rangle = (b^0 + \mathbf{b}\cdot \mathbf{q})(r^0 + \mathbf{r}\cdot\mathbf{q})$$

$$= b^0 r^0 + b^0 \mathbf{r}\cdot\mathbf{q} + r^0 \mathbf{b}\cdot\mathbf{q} + (\mathbf{b}\cdot\mathbf{q})(\mathbf{r}\cdot\mathbf{q})$$

$$= b^0 r^0 + b^0 \sum_j r^j q_j + r^0 \sum_i b^i q_i + \sum_j \sum_i r^j b^i q_j q_i$$

donde se puede expresar de la forma

$$= b^0 r^0 + b^0 r^j q_j + r^0 b^i q_i + r^j b^i (-\delta_{ij} 1 + \varepsilon_{ijk} q_k)$$

$$= b^0 r^0 + b^0 r^j q_j + r^0 b^i q_i + r^j b^i (-\delta_{ij} 1) + r^j b^i \varepsilon_{ijk} q_k$$

donde se puede identificar las partes vectorial y escalar

$$= b^0 r^0 - r^j b^i \delta_{ij} 1 + b^0 r^j q_j + r^0 b^i q_i + r^j b^i \varepsilon_{ijk} q_k$$

$$= b^0 r^0 - \mathbf{b}\cdot\mathbf{r} + r^0 \mathbf{b} + b^0 \mathbf{r} + \mathbf{b}\times\mathbf{r}$$

---

c) $|d\rangle = |b\rangle \odot |r\rangle = b^\alpha |q_\alpha\rangle \odot r^\alpha |q_\alpha\rangle$

$$= b^\alpha r^\beta (|q_\alpha\rangle |q_\beta\rangle)$$

$$= b^0 r^0 |q_0 q_0\rangle + b^0 r^j |q_0 q_j\rangle + r^0 b^i |q_i q_0\rangle + b^i r^j |q_i q_j\rangle$$

donde se puede expresar de la forma

$$= b^0 r^0 + b^0 r^j q_j + r^0 b^i q_i + r^j b^i (-\delta_{ij} 1 + \varepsilon_{ijk} q_k)$$

$$= b^0 r^0 + b^0 r^j q_j + r^0 b^i q_i + r^j b^i (-\delta_{ij} 1) + r^j b^i \varepsilon_{ijk} q_k$$

donde se puede identificar las partes vectorial y escalar

$$= b^0 r^0 - r^j b^i \delta_{ij} 1 + b^0 r^j q_j + r^0 b^i q_i + r^j b^i \varepsilon_{ijk} q_k$$

$$= b^0 r^0 - b \cdot r + r^0 b + b^0 r + b \times r$$

---

c) $|d\rangle = |b\rangle \odot |r\rangle = b^\alpha |q_\alpha\rangle \odot r^\alpha |q_\alpha\rangle$

$$= b^\alpha r^\beta (|q_\alpha\rangle |q_\beta\rangle)$$

$$= b^0 r^0 |q_0 q_0\rangle + b^0 r^j |q_0 q_j\rangle + r^0 b^i |q_i q_0\rangle + b^i r^j |q_i q_j\rangle$$

$$= b^0 r^0 |q_0\rangle + b^0 r^j |q_j\rangle + b^i r^0 |q_i\rangle + b^i r^j (-\delta_{ij} |q_0\rangle + \varepsilon_{ijk} |q_k\rangle)$$

donde al agrupar se obtiene

$$a = b^0 r^0 - b^j r_j \quad ; \quad d^i = b^0 r^i + r^0 b^i + \varepsilon^i{}_{jk} b^j r^k$$

- se toma la parte simetrica y antisimetrica de $d^i$ y la reescribimos

$$S^{(0j)} = b^0 r^j + r^0 b^j \quad ; \quad A^{[jk]i} b_j r_k = \varepsilon^i{}_{jk} b^j r^k$$

- de forma que ahora se agrupa la parte escalar y vectorial

$$|d\rangle = a|q_0\rangle + S^{(\alpha j)} \delta^0_\alpha |q_j\rangle + A^{[jk]i} b_j r_k |q_i\rangle$$

d) $a = b^0 r^0 - b^j r_j \rightarrow$ escalar

$S^{(ij)} = b^0 r^j + r^0 b^j \rightarrow$ Parte simétrica del producto de las componentes vectoriales

$A^{(jki)} = \varepsilon^i{}_{jk} b^j r^k \rightarrow$ Parte antisimétrica del producto vectorial

• Se obtiene un pseudovector por el comportamiento antisimétrico del producto de dos cuaterniones

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e) Para que sean base se necesita independencia lineal, por lo tanto

• $\alpha\sigma_1 + \beta\sigma_2 + \gamma\sigma_3 + \lambda\sigma_4 = 0$

$$\begin{pmatrix} 0 & \alpha \\ \alpha & 0 \end{pmatrix} + \begin{pmatrix} 0 & -\beta i \\ \beta i & 0 \end{pmatrix} + \begin{pmatrix} \gamma & 0 \\ 0 & -\gamma \end{pmatrix} + \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = 0$$

$$\begin{pmatrix} \gamma+\lambda & \alpha-\beta i \\ \alpha+\beta i & \lambda-\gamma \end{pmatrix} = 0 \Rightarrow \begin{array}{l} \lambda+\gamma=0 \Rightarrow \lambda=-\gamma \\ \alpha+\beta i=0 \Rightarrow \alpha=-\beta i \\ \alpha-\beta i=0 \Rightarrow \alpha=\beta i \\ \lambda-\gamma=0 = \lambda=\gamma \end{array}$$

Por lo tanto $\alpha=\beta=\gamma=\lambda=0$

• Para la matriz 2x2 se toma la relación

$$|b\rangle = \begin{pmatrix} z & w \\ -w^* & z^* \end{pmatrix} = \alpha\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \beta\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + \gamma\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} z & w \\ -w^* & z^* \end{pmatrix} = \begin{pmatrix} \gamma+\lambda & \alpha-\beta i \\ \alpha+\beta i & \lambda-\gamma \end{pmatrix}$$

$z^* = \lambda-(z-\lambda)$ • $-w^* = \alpha+i(\alpha-w)$

• Se obtiene un pseudovector por el comportamiento antisimétrico del producto de dos cuaterniones

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e) Para que sean base se necesita independencia lineal, por lo tanto

• $\alpha\sigma_1 + \beta\sigma_2 + \gamma\sigma_3 + \lambda\sigma_4 = 0$

$$\begin{pmatrix} 0 & \alpha \\ \alpha & 0 \end{pmatrix} + \begin{pmatrix} 0 & -\beta i \\ \beta i & 0 \end{pmatrix} + \begin{pmatrix} \gamma & 0 \\ 0 & -\gamma \end{pmatrix} + \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = 0$$

$$\begin{pmatrix} \gamma+\lambda & \alpha-\beta i \\ \alpha+\beta i & \lambda-\gamma \end{pmatrix} = 0 \Rightarrow \begin{array}{l} \lambda+\gamma=0 \Rightarrow \lambda=-\gamma \\ \alpha+\beta i=0 \Rightarrow \alpha=-\beta i \\ \alpha-\beta i=0 \Rightarrow \alpha=\beta i \\ \lambda-\gamma=0 = \lambda=\gamma \end{array}$$

Por lo tanto $\alpha=\beta=\gamma=\lambda=0$

• Para la matriz 2X2 se toma la relación

$$|b\rangle = \begin{pmatrix} z & w \\ -w^* & z^* \end{pmatrix} = \alpha\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \beta\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + \gamma\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} z & w \\ -w^* & z^* \end{pmatrix} = \begin{pmatrix} \gamma+\lambda & \alpha-\beta i \\ \alpha+\beta i & \lambda-\gamma \end{pmatrix}$$

$z = \gamma+\lambda$

$w = \alpha - \beta i$

$-w^* = \alpha + \beta i$

$z^* = \lambda - \gamma$

• $z^* = \lambda - (z-\lambda)$

$\lambda = \dfrac{z^*+z}{2}$

• $z = \gamma + (z^* + \gamma)$

$\gamma = \dfrac{z - z^*}{2}$

• $-w^* = \alpha + (\alpha - w)$

$\alpha = \dfrac{w - w^*}{2}$

• $w = (-w^* - \beta i) - \beta i$

$\beta = \dfrac{i(w+w^*)}{2}$

f) Para mostrar que es una posible representación para la base de cuaterniones analizamos la independencia lineal

$$\alpha|q_1\rangle + \beta|q_2\rangle + \gamma|q_3\rangle + \lambda|q_4\rangle = 0$$

$$\alpha\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} + \beta\begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} + \gamma\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} + \lambda\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} \lambda & \alpha & -\gamma & -\beta \\ -\alpha & \lambda & -\beta & \gamma \\ \gamma & \beta & \lambda & \alpha \\ \beta & -\gamma & -\alpha & \lambda \end{bmatrix} = 0 \Rightarrow$$ donde se puede notar que:

$\lambda = 0$, $\alpha = 0$ $\beta = 0$, $\gamma = 0$

---

g) $\langle a|b\rangle = |a\rangle^{*} \odot |b\rangle$

1. $\langle a|a\rangle = |a\rangle^{*} \odot |a\rangle$

$$= (a^0 - a^i|q_i\rangle)(a^0 + a^j|q_j\rangle$$
$$= a^0a^0|q_0\rangle|q_0\rangle + a^0a^j|q_0\rangle|q_j\rangle - a^0a^i|q_i\rangle|q_j\rangle - a^ja^i|q_j\rangle|q_i\rangle$$
$$= a^0a^0 1 - a^ia^i(-1)$$
$$= a^0a^0 + a^ia^i$$

2. $\langle a|b\rangle = \langle b|a\rangle^{*}$

$$\langle a|b\rangle = |a\rangle^{*} \odot |b\rangle = (a^0|q_0\rangle - a^i|q_i\rangle)(b^0|q_0\rangle + b^j|q_j\rangle)$$
$$= a^0b^0|q_0\rangle|q_0\rangle + a^0b^j|q_0\rangle|q_j\rangle - b^0a^i|q_0\rangle|q_i\rangle - a^ib^j|q_i\rangle|q_j\rangle$$
$$= a^0b^0 1 + a^0b^j|q_j\rangle - b^0a^i|q_i\rangle - a^ib^j|q_i\rangle|q_j\rangle$$

$$\langle b|a\rangle^{*} = (|b\rangle^{*} \odot |a\rangle)^{*} = |b\rangle \odot |a\rangle^{*}$$
$$= (b^0|q_0\rangle + b^j|q_j\rangle)(a^0|q_0\rangle + a^i|q_i\rangle)$$
$$= b^0a^0|q_0\rangle|q_0\rangle - b^0a^i|q_0\rangle|q_i\rangle + a^0b^j|q_j\rangle|q_0\rangle - a^ib^j|q_j\rangle|q_i\rangle$$

$$\begin{bmatrix} \lambda & \alpha & -\gamma & \beta \\ -\alpha & \lambda & -\beta & \gamma \\ \gamma & \beta & \lambda & \alpha \\ \beta & -\gamma & -\alpha & \lambda \end{bmatrix} = 0 \Rightarrow$$

donde se puede notar que:

$\lambda = 0, \quad \alpha = 0 \quad \beta = 0, \quad \gamma = 0$

g) $\langle a | b \rangle = |a\rangle^* \odot |b\rangle$

1. $\langle a | a \rangle = |a\rangle^* \odot |a\rangle$

$$= (a^0 - a^i |q_i\rangle)(a^0 + a^j |q_j\rangle)$$
$$= a^0 a^0 |q_0\rangle|q_0\rangle + a^0 a^j |q_0\rangle|q_j\rangle - a^0 a^i |q_i\rangle|q_j\rangle - a^j a^i |q_j\rangle|q_i\rangle$$
$$= a^0 a^0 1 - a^i a^i (-1)$$
$$= a^0 a^0 + a^i a^i$$

2. $\langle a | b \rangle = \langle b | a \rangle^*$

$$\langle a | b \rangle = |a\rangle^* \odot |b\rangle = (a^0 |q_0\rangle - a^i |q_i\rangle)(b^0 |q_0\rangle + b^j |q_j\rangle)$$
$$= a^0 b^0 |q_0\rangle|q_0\rangle + a^0 b^j |q_0\rangle|q_j\rangle - b^0 a^i |q_0\rangle|q_i\rangle - a^i b^j |q_i\rangle|q_j\rangle$$
$$= a^0 b^0 1 + a^0 b^j |q_j\rangle - b^0 a^i |q_i\rangle - a^i b^j |q_i\rangle|q_j\rangle$$

$$\langle b | a \rangle^* = (|b\rangle^* \odot |a\rangle)^* = |b\rangle \odot |a\rangle^*$$
$$= (b^0 |q_0\rangle + b^j |q_j\rangle)(a^0 |q_0\rangle + a^i |q_i\rangle)$$
$$= b^0 a^0 |q_0\rangle|q_0\rangle - b^0 a^i |q_0\rangle|q_i\rangle + a^0 b^j |q_0\rangle|q_j\rangle - a^i b^j |q_i\rangle|q_j\rangle$$

3. $\langle a | \alpha b + \beta c \rangle = \alpha \langle a | b \rangle + \beta \langle a | c \rangle$

$$= |a\rangle^* \odot |\alpha b + \beta c\rangle = (a^0 - a^i |q_i\rangle)(\alpha b + \beta c)$$

$$= \alpha a^0 (b^0 + b^j |q_j\rangle) + \beta a^0 (c^0 + c^m |q_m\rangle - \alpha a^i |q_i\rangle (b^0 + b^j |q_j\rangle) - \beta a^i |q_i\rangle (c^0 + c^m |q_m\rangle)$$

$$= \alpha a^0 b^0 + \alpha a^0 b^j |q_j\rangle + \beta a^0 c^0 + \beta a^0 c^m |q_m\rangle - \alpha b^0 a^i |q_i\rangle - \alpha a^i b^j |q_i\rangle|q_j\rangle - \beta c^0 a^i |q_i\rangle - \beta a^i c^m |q_i\rangle|q_m\rangle$$

$$= (\alpha a^0 b^0 + \alpha a^0 b^j |q_j\rangle - \alpha b^0 a^i |q_i\rangle - \alpha a^i b^j |q_i\rangle|q_j\rangle) + (\beta a^0 c^0 + \beta a^0 c^m |q_m\rangle - \beta c^0 a^i |q_i\rangle - \beta a^i c^m |q_i\rangle|q_m\rangle)$$

$$= \alpha (a^0 b^0 + a^0 b^j |q_j\rangle - b^0 a^i |q_i\rangle - a^i b^j |q_i\rangle|q_j\rangle) + \beta (a^0 c^0 + a^0 c^m |q_m\rangle - c^0 a^i |q_i\rangle - a^i c^m |q_i\rangle|q_m\rangle)$$

$$= \alpha \langle a | b \rangle + \beta \langle a | c \rangle$$

4. $\langle \alpha a + \beta b | c \rangle = \alpha^* \langle a | c \rangle + \beta^* \langle b | c \rangle$

$= \langle \alpha a + \beta b | c \rangle = |\alpha a + \beta b\rangle^* \odot |c\rangle$

$= \left(\alpha^*(a^0 - a^i |q_i\rangle) + (\beta^*(b^0 - b^j |q_j\rangle)\right)(c^0 + c^m |q_m\rangle)$

$= \alpha^* a^0 c^0 - \alpha^* c^0 a^i |q_i\rangle + \beta^* c^0 b^0 - \beta^* c^0 b^j |q_j\rangle + \alpha^* a^0 c^m |q_m\rangle - \alpha^* a^i c^m |q_i\rangle|q_m\rangle + \beta^* b^0 c^m |q_m\rangle - \beta^* b^j c^m |q_j\rangle|q_m\rangle$

$= (\alpha^* a^0 c^0 - \alpha^* c^0 a^i |q_i\rangle + \alpha^* a^0 c^m |q_m\rangle - \alpha^* a^i c^m |q_i\rangle|q_m\rangle) + (\beta^* c^0 b^0 - \beta^* c^0 b^j |q_j\rangle + \beta^* b^0 c^m |q_m\rangle - \beta^* b^j c^m |q_j\rangle|q_m\rangle)$

$= \alpha^*(a^0 c^0 - c^0 a^i |q_i\rangle + a^0 c^m |q_m\rangle - a^i c^m |q_i\rangle|q_m\rangle) + \beta^*(c^0 b^0 - c^0 b^j |q_j\rangle + b^0 c^m |q_m\rangle - b^j c^m |q_j\rangle|q_m\rangle)$

$= \alpha^* \langle a | c \rangle + \beta^* \langle b | c \rangle$

5. $\langle a | 0 \rangle = \langle 0 | a \rangle = 0$

$\langle a | 0 \rangle = |a\rangle^* \odot |0\rangle$

$= (a^0 - a^i |q_i\rangle)(0)$

$= 0a^0 - 0a^i |q_i\rangle$

$= 0 + 0$

$= 0$

$\langle 0 | a \rangle = |0\rangle^* \odot |a\rangle$

$= (0)(a^0 + a^i |q_i\rangle)$

$= 0a^0 + 0a^i |q_i\rangle$

$= 0 + 0$

$= 0$

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h) $\langle a | b \rangle = \frac{1}{2}\left[ \langle \widetilde{a} | b \rangle - |q_i\rangle \odot \langle \widetilde{a} | b \rangle \odot |q_i\rangle \right]$

$= (\alpha^* a^0 c^0 - \alpha^* c^0 a^i |q_i\rangle + \alpha^* a^0 c^m |q_m\rangle - \alpha^* a^i c^m |q_i\rangle|q_m\rangle) + (\beta^* c^0 b^0 + \beta^* c^0 b^j |q_j\rangle + \beta^* b^0 c^m |q_m\rangle - \beta^* b^j c^m |q_j\rangle|q_m\rangle)$

$= \alpha^* (a^0 c^0 - c^0 a^i |q_i\rangle + a^0 c^m |q_m\rangle - a^i c^m |q_i\rangle|q_m\rangle) + \beta^* (c^0 b^0 - c^0 b^j |q_j\rangle + b^0 c^m |q_m\rangle - b^j c^m |q_j\rangle|q_m\rangle)$

$= \alpha^* \langle a|c\rangle + \beta^* \langle b|c\rangle$

5. $\langle a|0\rangle = \langle 0|a\rangle = 0$

$$\begin{aligned}
\langle a|0\rangle &= |a\rangle^* \odot |0\rangle \\
&= (a^0 - a^i|q_i\rangle)(0) \\
&= 0a^0 - 0a^i|q_i\rangle \\
&= 0 + 0 \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\langle 0|a\rangle &= |0\rangle^* \odot |a\rangle \\
&= (0)(a^0 + a^i|q_i\rangle) \\
&= 0a^0 + 0a^i|q_i\rangle \\
&= 0 + 0 \\
&= 0
\end{aligned}$$

---

h) $\langle a|b\rangle = \frac{1}{2}\left[\langle a|b\rangle - |q_1\rangle \odot \langle a|b\rangle \odot |q_1\rangle\right]$

$= \frac{1}{2}\left[|a\rangle^* \odot |b\rangle - |q_1\rangle \odot |a\rangle^* \odot |b\rangle \odot |q_1\rangle\right]$

$= \frac{1}{2}\left[(a^0 - a^1|q_1\rangle)(b^0 + b^2|q_2\rangle) - |q_1\rangle \odot (a^0 - a^1|q_1\rangle)(b^0 + b^2|q_2\rangle) \odot |q_1\rangle\right]$

$= \frac{1}{2}\left[a^0b^0 + a^0b^2|q_2\rangle - b^0a^1|q_1\rangle - a^1b^2|q_1\rangle|q_2\rangle - |q_1\rangle(a^0b^0 + a^0b^2|q_2\rangle - b^0a^1|q_1\rangle - a^1b^2|q_1\rangle|q_2\rangle)|q_1\rangle\right]$

$= \frac{1}{2}\left[a^0b^0 + a^0b^2|q_2\rangle - b^0a^1|q_1\rangle - a^1b^2|q_1\rangle|q_2\rangle + a^0b^0 + a^0b^2|q_2\rangle - b^0a^1|q_1\rangle - a^1b^2|q_1\rangle|q_2\rangle\right.$

$= \frac{1}{2}\left[2a^0b^0 + 2a^0b^2|q_2\rangle - 2b^0a^1|q_1\rangle - 2a^1b^2|q_1\rangle|q_2\rangle\right]$

$= \frac{1}{\cancel{2}} \cdot \cancel{2}\left[a^0b^0 + a^0b^2|q_2\rangle - b^0a^1|q_1\rangle - a^1b^2|q_1\rangle|q_2\rangle\right]$

$= a^0b^0 + a^0b^2|q_2\rangle - b^0a^1|q_1\rangle - a^1b^2|q_1\rangle|q_2\rangle$

h.1) $\langle a|a\rangle = \||a\rangle\|^2$

$$\cdot\ \langle a|a\rangle = \frac{1}{2}\left[\widehat{\langle a|a\rangle} - |q_1\rangle \odot \widehat{\langle a|a\rangle} \odot |q_1\rangle\right]$$

$$= \frac{1}{2}\left[|a\rangle^{*} \odot |a\rangle - |q_1\rangle \odot (|a\rangle^{*} \odot |a\rangle) \odot |q_1\rangle\right]$$

$$= \frac{1}{2}\left[(a^0 - a^1|q_1\rangle)(a^0 + a^1|q_1\rangle) - |q_1\rangle \odot (a^0 - a^1|q_1\rangle)(a^0 + a^1|q_1\rangle) \odot |q_1\rangle\right]$$

$$= \frac{1}{2}\left[a^0a^0 + \cancel{a^0a^1|q_1\rangle} - \cancel{a^0a^1|q_1\rangle} - a^1a^1|q_1\rangle|q_1\rangle - |q_1\rangle \odot (a^0a^0 + \cancel{a^0a^1|q_1\rangle} - \cancel{a^0a^1|q_1\rangle} - a^1a^1|q_1\rangle|q_1\rangle) \odot |q_1\rangle\right]$$

$$= \frac{1}{2}\left[a^0a^0 + a^1a^1 - |q_1\rangle \odot (a^0a^0 + a^1a^1) \odot |q_1\rangle\right]$$

$$= \frac{1}{2}\left[a^0a^0 + a^1a^1 + a^0a^0 + a^1a^1\right]$$

$$= \frac{1}{2}\left[2a^0a^0 + 2a^1a^1\right]$$

$$= \frac{1}{\cancel{2}}\cancel{2}\left[(a^0)^2 + (a^1)^2\right] = (a^0)^2 + (a^1)^2$$

h.2) $\langle a|b\rangle = \langle b|a\rangle^{*}$

Ya se probo $\langle a|b\rangle$ por lo tanto se mostrará $\langle b|a\rangle^{*}$

$$\langle b|a\rangle^{*} = \frac{1}{2}\left[(\widehat{\langle b|a\rangle})^{*} - |q_1\rangle \odot (\widehat{\langle b|a\rangle})^{*} \odot |q_1\rangle\right]$$

$$= \frac{1}{2}\left[(|b\rangle^{*} \odot |a\rangle)^{*} - |q_1\rangle \odot ((|b\rangle^{*} \odot |a\rangle)^{*} \odot |q_1\rangle\right]$$

$$= \frac{1}{2}\left[(b^0 + b^1|q_1\rangle)(a^0 - a^2|q_2\rangle) - |q_1\rangle \odot (b^0 + b^1|q_1\rangle)(a^0 - a^2|q_2\rangle) \odot |q_1\rangle\right]$$

$$= \frac{1}{2}\left[a^0a^0 + a^1a^1 - |q_1\rangle \odot (a^0a^0 + a^1a^1) \odot |q_1\rangle\right]$$

$$= \frac{1}{2}\left[a^0a^0 + a^1a^1 + a^0a^0 + a^1a^1\right]$$

$$= \frac{1}{2}\left[2a^0a^0 + 2a^1a^1\right]$$

$$= \frac{1}{\cancel{2}}\cancel{2}\left[(a^0)^2 + (a^1)^2\right] = (a^0)^2 + (a^1)^2$$

h. 2) $\langle a|b\rangle = \langle b|a\rangle^*$

Ya se probo $\langle a|b\rangle$ por lo tanto se mostrará $\langle b|a\rangle^*$

$$\langle b|a\rangle^* = \frac{1}{2}\left[(\widetilde{\langle b|a\rangle})^* - |q_1\rangle \odot (\widetilde{\langle b|a\rangle})^* \odot |q_1\rangle\right]$$

$$= \frac{1}{2}\left[(|b\rangle^* \odot |a\rangle)^* - |q_1\rangle \odot (|b\rangle^* \odot |a\rangle)^* \odot |q_1\rangle\right]$$

$$= \frac{1}{2}\left[(b^0 + b^1|q_1\rangle)(a^0 - a^2|q_2\rangle) - |q_1\rangle \odot (b^0 + b^1|q_1\rangle)(a^0 - a^2|q_2\rangle) \odot |q_1\rangle\right]$$

$$= \frac{1}{2}\left[b^0a^0 - b^0a^2|q_2\rangle + a^0b^1|q_1\rangle - a^2b^1|q_2\rangle|q_1\rangle - |q_1\rangle \odot (b^0a^0 - b^0a^2|q_2\rangle + a^0b^1|q_1\rangle - a^2b^1|q_2\rangle|q_1\rangle) \odot |q_1\rangle\right]$$

$$= \frac{1}{2}\left[b^0a^0 - b^0a^2|q_2\rangle + a^0b^1|q_1\rangle - a^2b^1|q_2\rangle|q_1\rangle + b^0a^0 - b^0a^2|q_2\rangle + a^0b^1|q_1\rangle - a^2b^1|q_2\rangle|q_1\rangle\right]$$

$$= \frac{1}{2}\left[2(b^0a^0) - 2(b^0a^2|q_2\rangle) + 2(a^0b^1|q_1\rangle) - 2(a^2b^1|q_2\rangle|q_1\rangle)\right]$$

$$= \frac{1}{\cancel{2}}\cancel{2}\left[b^0a^0 - b^0a^2|q_2\rangle + a^0b^1|q_1\rangle - a^2b^1|q_2\rangle|q_1\rangle\right]$$

$$= b^0a^0 - b^0a^2|q_2\rangle + a^0b^1|q_1\rangle - a^2b^1|q_2\rangle|q_1\rangle$$

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$\cdot\ \langle a|b\rangle = \frac{1}{2}\left[\widetilde{\langle a|b\rangle} - |q_1\rangle \odot \widetilde{\langle a|b\rangle} \odot |q_1\rangle\right]$

$3.\ \langle a|\alpha b+\beta c\rangle =$

$= \frac{1}{2}\left[\widetilde{\langle a|\alpha b+\beta c\rangle} - |q_1\rangle \odot \widetilde{\langle a|\alpha b+\beta c\rangle} \odot |q_1\rangle\right]$

$= \frac{1}{2}\left[|a\rangle^{*} \odot |\alpha b+\beta c\rangle - |q_1\rangle \odot |a\rangle^{*} \odot |\alpha b+\beta c\rangle \odot |q_1\rangle\right]$

$= \frac{1}{2}\left[(a^0 - a^1|q_1\rangle)(\alpha b+\beta c) - |q_1\rangle \odot (a^0 - a^1|q_1\rangle)(\alpha b+\beta c) \odot |q_1\rangle\right]$

$= \frac{1}{2}\Big[(a^0 - a^1|q_1\rangle)(\alpha b^0 + \alpha b^2|q_2\rangle) + (a^0 - a^1|q_1\rangle)(\beta c^0 + \beta c^3|q_3\rangle) - |q_1\rangle \odot (a^0 - a^1|q_1\rangle)(\alpha b^0 + \alpha b^2|q_2\rangle) + \ldots$

$\ldots (a^0 - a^1|q_1\rangle)(\beta c^0 + \beta c^3|q_3\rangle) \odot |q_1\rangle\Big]$

$= \frac{1}{2}\Big[(\alpha a^0 b^0 + \alpha a^0 b^2|q_2\rangle - \alpha b^0 a^1|q_1\rangle - \alpha b^2 a^1|q_2\rangle|q_1\rangle) + (\beta a^0 c^0 + \beta a^0 c^3|q_3\rangle - \beta c^0 a^1|q_1\rangle - \beta a^1 c^3|q_1\rangle|q_3\rangle \ldots$

$- |q_1\rangle \odot \big[(\alpha a^0 b^0 + \alpha a^0 b^2|q_2\rangle - \alpha b^0 a^1|q_1\rangle - \alpha b^2 a^1|q_2\rangle|q_1\rangle + \beta a^0 c^0 + \beta a^0 c^3|q_3\rangle - \beta c^0 a^1|q_1\rangle - \beta a^1 c^3|q_1\rangle|q_3\rangle) \odot |q_1\rangle\Big]$

$= \frac{1}{2}\Big[(\alpha a^0 b^0 + \alpha a^0 b^2|q_2\rangle - \alpha b^0 a^1|q_1\rangle - \alpha b^2 a^1|q_2\rangle|q_1\rangle + \beta a^0 c^0 + \beta a^0 c^3|q_3\rangle - \beta c^0 a^1|q_1\rangle - \beta a^1 c^3|q_1\rangle|q_3\rangle \ldots$

$\ldots \alpha a^0 b^0 + \alpha a^0 b^2|q_2\rangle - \alpha b^0 a^1|q_1\rangle - \alpha b^2 a^1|q_2\rangle|q_1\rangle + \beta a^0 c^0 + \beta a^0 c^3|q_3\rangle - \beta c^0 a^1|q_1\rangle - \beta a^1 c^3|q_1\rangle|q_3\rangle\Big]$

$= \frac{1}{2}\Big[(\alpha a^0 b^0 + \alpha a^0 b^2|q_2\rangle - \alpha b^0 a^1|q_1\rangle - \alpha b^2 a^1|q_2\rangle|q_1\rangle + \alpha a^0 b^0 + \alpha a^0 b^2|q_2\rangle - \alpha b^0 a^1|q_1\rangle - \alpha b^2 a^1|q_2\rangle|q_1\rangle) + \ldots$

$\ldots + (\beta a^0 c^0 + \beta a^0 c^3|q_3\rangle - \beta c^0 a^1|q_1\rangle - \beta a^1 c^3|q_1\rangle|q_3\rangle + \beta a^0 c^0 + \beta a^0 c^3|q_3\rangle - \beta c^0 a^1|q_1\rangle - \beta a^1 c^3|q_1\rangle|q_3\rangle)\Big]$

$= \frac{1}{2}\Big[(2\alpha a^0 b^0 + 2a^0 b^2|q_2\rangle - 2\alpha b^0 a^1|q_1\rangle - 2\alpha b^2 a^1|q_2\rangle|q_1\rangle) + (2\beta a^0 c^0 + 2\beta a^0 c^3|q_3\rangle - 2\beta c^0 a^1|q_1\rangle - 2\beta a^1 c^3|q_1\rangle|q_3\rangle)\Big]$

$= \frac{1}{\cancel{2}} \cdot \cancel{2}\alpha(a^0 b^0 + 2a^0 b^2|q_2\rangle - b^0 a^1|q_1\rangle - b^2 a^1|q_2\rangle|q_1\rangle) + \frac{1}{\cancel{2}} \cdot \cancel{2}\beta(a^0 c^0 + a^0 c^3|q_3\rangle - c^0 a^1|q_1\rangle - a^1 c^3|q_1\rangle|q_3\rangle)$

$= \alpha\langle a|b\rangle + \beta\langle a|c\rangle$

4. $\langle \alpha a + \beta b | c \rangle = \alpha^* \langle a | c \rangle + \beta^* \langle b | c \rangle$

$$= \frac{1}{2}\left[ \langle \alpha a + \beta b | c \rangle - |q_1\rangle \odot \langle \alpha a + \beta b | c \rangle \odot |q_1\rangle \right]$$

$$= \frac{1}{2}\left[ |\alpha a + \beta b\rangle^* \odot |c\rangle - |q_1\rangle \odot |\alpha a + \beta b\rangle^* \odot |c\rangle \odot |q_1\rangle \right]$$

$$= \frac{1}{2}\left[ ((\alpha a)^* + (\beta b)^*)(c^0 + c^1 |q_1\rangle) - |q_1\rangle \odot ((\alpha a)^* + (\beta b)^*)(c^0 + c^1 |q_1\rangle) \odot |q_1\rangle \right]$$

$$= \frac{1}{2}\left[ (\alpha^* a^0 - \alpha^* a^2 |q_2\rangle)(c^0 + c^1 |q_1\rangle) + (\beta^* b^0 - \beta^* b^3 |q_3\rangle)(c^0 + c^1 |q_1\rangle) - |q_1\rangle \odot (\alpha^* a^0 - \alpha^* a^2 |q_2\rangle)(c^0 + c^1 |q_1\rangle) + (\beta^* b^0 - \beta^* b^3 |q_3\rangle)(c^0 + c^1 |q_1\rangle) \odot |q_1\rangle \right]$$

$$= \frac{1}{2}\Big[ \alpha^* a^0 c^0 + \alpha^* a^0 c^1 |q_1\rangle - \alpha^* c^0 a^2 |q_2\rangle - \alpha^* a^2 c^1 |q_2\rangle|q_1\rangle + \beta^* b^0 c^0 + \beta^* b^0 c^1 |q_1\rangle - \beta^* c^0 b^3 |q_3\rangle - \beta^* b^3 c^1 |q_3\rangle|q_1\rangle - |q_1\rangle \odot \ldots$$
$$\ldots (\alpha^* a^0 c^0 + \alpha^* a^0 c^1 |q_1\rangle - \alpha^* c^0 a^2 |q_2\rangle - \alpha^* a^2 c^1 |q_2\rangle|q_1\rangle + \beta^* b^0 c^0 + \beta^* b^0 c^1 |q_1\rangle - \beta^* c^0 b^3 |q_3\rangle - \beta^* b^3 c^1 |q_3\rangle|q_1\rangle) \odot |q_1\rangle \Big]$$

$$= \frac{1}{2}\Big[ \alpha^* a^0 c^0 + \alpha^* a^0 c^1 |q_1\rangle - \alpha^* c^0 a^2 |q_2\rangle - \alpha^* a^2 c^1 |q_2\rangle|q_1\rangle + \beta^* b^0 c^0 + \beta^* b^0 c^1 |q_1\rangle - \beta^* c^0 b^3 |q_3\rangle - \beta^* b^3 c^1 |q_3\rangle|q_1\rangle + \ldots$$
$$\ldots \alpha^* a^0 c^0 + \alpha^* a^0 c^1 |q_1\rangle - \alpha^* c^0 a^2 |q_2\rangle - \alpha^* a^2 c^1 |q_2\rangle|q_1\rangle + \beta^* b^0 c^0 + \beta^* b^0 c^1 |q_1\rangle - \beta^* c^0 b^3 |q_3\rangle - \beta^* b^3 c^1 |q_3\rangle|q_1\rangle \Big]$$

$$= \frac{1}{2}\left[ (2\alpha^* a^0 c^0 + 2\alpha^* a^0 c^1 |q_1\rangle - 2\alpha^* c^0 a^2 |q_2\rangle - 2\alpha^* a^2 c^1 |q_2\rangle|q_1\rangle) + (2\beta^* b^0 c^0 + 2\beta^* b^0 c^1 |q_1\rangle - 2\beta^* c^0 b^3 |q_3\rangle - 2\beta^* b^3 c^1 |q_3\rangle|q_1\rangle) \right]$$

$$= \frac{1}{\cancel{2}} \cdot \cancel{2}\alpha^* (a^0 c^0 + a^0 c^1 |q_1\rangle - c^0 a^2 |q_2\rangle - a^2 c^1 |q_2\rangle|q_1\rangle) + \frac{1}{2} \cdot 2\beta^* (b^0 c^0 + b^0 c^1 |q_1\rangle - c^0 b^3 |q_3\rangle - b^3 c^1 |q_3\rangle|q_1\rangle)$$

$$= \alpha^* \langle a | c \rangle + \beta^* \langle b | c \rangle$$

5. Sean $|a\rangle, |b\rangle \in \mathbb{H}$ se tiene

$$= \frac{1}{2}\Big[(\alpha^* a^0 - \alpha^* a^2|q_2\rangle)(c^0 + c^1|q_1\rangle) + (\beta^* b^0 - \beta^* b^3|q_3\rangle)(c^0 + c^1|q_1\rangle) - |q_1\rangle \odot (\alpha^* a^0 - \alpha^* a^2|q_2\rangle)(c^0 + c^1|q_1\rangle) + (\beta^* b^0 - \beta^* b^3|q_3\rangle)(c^0 + c^1|q_1\rangle)$$

$$\odot|q_1\rangle\Big]$$

$$= \frac{1}{2}\Big[\alpha^* a^0 c^0 + \alpha^* a^0 c^1|q_1\rangle - \alpha^* c^0 a^2|q_2\rangle - \alpha^* a^2 c^1|q_2\rangle|q_1\rangle + \beta^* b^0 c^0 + \beta^* b^0 c^1|q_1\rangle - \beta^* c^0 b^3|q_3\rangle - \beta^* b^3 c^1|q_3\rangle|q_1\rangle - |q_1\rangle \odot \ldots$$

$$\ldots(\alpha^* a^0 c^0 + \alpha^* a^0 c^1|q_1\rangle - \alpha^* c^0 a^2|q_2\rangle - \alpha^* a^2 c^1|q_2\rangle|q_1\rangle + \beta^* b^0 c^0 + \beta^* b^0 c^1|q_1\rangle - \beta^* c^0 b^3|q_3\rangle - \beta^* b^3 c^1|q_3\rangle|q_1\rangle) \odot |q_1\rangle\Big]$$

$$= \frac{1}{2}\Big[\alpha^* a^0 c^0 + \alpha^* a^0 c^1|q_1\rangle - \alpha^* c^0 a^2|q_2\rangle - \alpha^* a^2 c^1|q_2\rangle|q_1\rangle + \beta^* b^0 c^0 + \beta^* b^0 c^1|q_1\rangle - \beta^* c^0 b^3|q_3\rangle - \beta^* b^3 c^1|q_3\rangle|q_1\rangle + \ldots$$

$$\ldots \alpha^* a^0 c^0 + \alpha^* a^0 c^1|q_1\rangle - \alpha^* c^0 a^2|q_2\rangle - \alpha^* a^2 c^1|q_2\rangle|q_1\rangle + \beta^* b^0 c^0 + \beta^* b^0 c^1|q_1\rangle - \beta^* c^0 b^3|q_3\rangle - \beta^* b^3 c^1|q_3\rangle|q_1\rangle\Big]$$

$$= \frac{1}{2}\Big[(2\alpha^* a^0 c^0 + 2\alpha^* a^0 c^1|q_1\rangle - 2\alpha^* c^0 a^2|q_2\rangle - 2\alpha^* a^2 c^1|q_2\rangle|q_1\rangle) + (2\beta^* b^0 c^0 + 2\beta^* b^0 c^1|q_1\rangle - 2\beta^* c^0 b^3|q_3\rangle - 2\beta^* b^3 c^1|q_3\rangle|q_1\rangle\Big]$$

$$= \frac{1}{\cancel{2}} \cdot \cancel{2}\alpha^*(a^0 c^0 + a^0 c^1|q_1\rangle - c^0 a^2|q_2\rangle - a^2 c^1|q_2\rangle|q_1\rangle) + \frac{1}{2} \cdot 2\beta^*(b^0 c^0 + b^0 c^1|q_1\rangle - c^0 b^3|q_3\rangle - b^3 c^1|q_3\rangle|q_1\rangle)$$

$$= \alpha^* \langle a|c\rangle + \beta^* \langle b|c\rangle$$

5. Sean $|a\rangle, |0\rangle \in \mathbb{H}$ se tiene

$$\langle a|0\rangle = \frac{1}{2}\Big[\widetilde{\langle a|0\rangle} - |q_1\rangle \odot \widetilde{\langle a|0\rangle} \odot |q_1\rangle\Big]$$

$$= \frac{1}{2}\big[a^* \odot 0 - |q_1\rangle \odot (a^* \odot 0) \odot |q_1\rangle\big]$$

$$= \frac{1}{2}\big[0 - |q_1\rangle \odot 0 \odot |q_1\rangle\big]$$

$$= 0$$

$$\langle 0|a\rangle = \frac{1}{2}\Big[\widetilde{\langle 0|a\rangle} - |q_1\rangle \odot \widetilde{\langle 0|a\rangle} \odot |q_1\rangle\Big] = \frac{1}{2}\big[0^* \odot |a\rangle - |q_1\rangle \odot (0^* \odot |a\rangle) \odot |q_1\rangle\big]$$

$$= 0$$

$$\langle 0|a\rangle = \langle a|0\rangle = 0$$

i) $n(|b\rangle) = \| |a\rangle \| = \sqrt{\langle a|a\rangle} = \sqrt{|a\rangle^{*} \odot |a\rangle}$

$= \sqrt{(a^0 - a^i|q_i\rangle)(a^0 + a^j|q_j\rangle)}$

$= \sqrt{a^0a^0 + \cancel{a^0a^j|q_j\rangle} - \cancel{a^0a^i|q_i\rangle} - a^ia^j|q_i\rangle|q_j\rangle}$

$= \sqrt{a^0a^0 + a^ia^i}$

$= \sqrt{(a^0)^2 + (a^i)^2}$

note que se cumple con:

- $\| |a\rangle \| \geq 0$
- $\sqrt{(a^0)^2 + (a^i)^2} = K$ ; $K \in \mathbb{R}$
- Homogeneidad $\|\lambda a\| = \sqrt{(\lambda a^0)^2 + (\lambda a^i)^2} = |\lambda| \, \|a\|$
- Desigualdad triangular al ser norma euclidea

// // //

j) $\overline{|a\rangle} = \dfrac{|a\rangle^{*}}{\| |a\rangle \|^2}$

$|a\rangle \odot \overline{|a\rangle} = (a^0 + a^i|q_i\rangle)\left(\dfrac{a^0 - a^j|q_j\rangle}{(a^0)^2 + (a^i)^2}\right)$

$= \dfrac{a^0a^0}{(a^0)^2 + (a^i)^2} - \cancel{\dfrac{a^0a^j|q_j\rangle}{(a^0)^2 + (a^i)^2}} + \cancel{\dfrac{a^0a^i|q_i\rangle}{(a^0)^2 + (a^i)^2}} - \dfrac{a^ia^j|q_i\rangle|q_j\rangle}{(a^0)^2 + (a^i)^2}$

$= \dfrac{(a^0)^2}{(a^0)^2 + (a^i)^2} + \dfrac{(a^i)^2}{(a^0)^2 + (a^i)^2} = \dfrac{(a^0)^2 + (a^i)^2}{(a^0)^2 + (a^i)^2} = 1$

K) $|a\rangle = a_0 + a \qquad |b\rangle = b_0 + b$

$|a\rangle \odot |b\rangle = (a_0 b_0 - a \cdot b,\ a^0 b + b^0 a + a \times b) = (c_0, c) = |c\rangle \in \mathbb{H}$

$= |a\rangle = \begin{bmatrix} a_0 + a_1 i & a_2 + a_3 i \\ -a_2 + a_3 i & a_0 - a_1 i \end{bmatrix} = M_1 \qquad |b\rangle = \begin{bmatrix} b_0 + b_1 i & b_2 + b_3 i \\ -b_2 + b_3 i & b_0 - b_1 i \end{bmatrix} = M_2$

$|d\rangle = \begin{bmatrix} d_0 + d_1 i & d_2 + d_3 i \\ -d_2 + d_3 i & d_0 - d_1 i \end{bmatrix} = M_3$

$|a\rangle \odot (|b\rangle \odot |d\rangle) = (|a\rangle \odot |b\rangle \odot |d\rangle)$

$M_1 (M_2 M_3) = (M_1 M_2) M_3$ note que de antemano se sabe que la multiplicación de matrices es asociativa por lo que la de los cuaterniones tambien

—— // —— // —— // —— // ——

I) $|v'\rangle = \overline{|a\rangle} \odot |v\rangle \odot |a\rangle$

$= \frac{|a\rangle^{*}}{\| |a\rangle \|^2} \odot |v\rangle \odot |a\rangle$

$= \left( \frac{(a^0 - a^i |q_i\rangle)}{(a^0)^2 + (a^i)^2} \right) (v^j |q_j\rangle) \odot |a\rangle$

$$\begin{bmatrix} d_0 + d_1 i & d_2 + d_3 i \\ -d_2 + d_3 i & d_0 - d_1 i \end{bmatrix}$$

$$|a\rangle \odot (|b\rangle \odot |d\rangle) = (|a\rangle \odot |b\rangle \odot |d\rangle)$$

$$M_1 (M_2 M_3) = (M_1 M_2) M_3$$

note que de antemano se sabe que la multiplicación de matrices es asociativa por lo que la de los cuaterniones también

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I) $|V'\rangle = \overline{|a\rangle} \odot |V\rangle \odot |a\rangle$

$$= \frac{|a\rangle^*}{\| |a\rangle \|^2} \odot |V\rangle \odot |a\rangle$$

$$= \left( \frac{(a^0 - a^i |q_i\rangle)}{(a^0)^2 + (a^i)^2} \right) (V^j |q_j\rangle) \odot |a\rangle$$

$$= \left( \frac{a^0 V^j |q_j\rangle - a^i V^j |q_i\rangle |q_j\rangle}{(a^0)^2 + (a^i)^2} \right) (a^0 + a^k |q_k\rangle)$$

$$= \frac{a^0 a^0 V^j |q_j\rangle}{(a^0)^2 + (a^i)^2} - \frac{a^0 a^i V^j \cancel{|q_i\rangle} |q_j\rangle}{\cancel{(a^0)^2} + (a^i)^2} + \frac{a^0 a^k V^j \cancel{|q_j\rangle |q_k\rangle}}{\cancel{(a^0)^2} + (a^i)^2} - \frac{a^i V^j a^k |q_i\rangle |q_j\rangle |q_k\rangle}{(a^0)^2 + (a^i)^2}$$

$$= \frac{(a^0)^2 V^j |q_j\rangle}{(a^0)^2 + (a^i)^2} + \frac{(a^i)^2 V^j |q_j\rangle}{(a^0)^2 + (a^i)^2}$$

$$= \frac{((a^0)^2 + (a^i)^2)}{(a^0)^2 + (a^i)^2} (V^j |q_j\rangle) = V^j |q_j\rangle$$

$$\|V'\|^2 = ((V^1)^2 + (V^2)^2 + (V^3)^2) \equiv ((V^1)^2 + (V^2)^2 + (V^3)^2) = \|V\|^2$$

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