CP - 1º teste - 2023/24 - LEI

1. $\underline{(u, y)} = \langle f, g \rangle$

$\equiv$ 6: Universal - x

$$\begin{cases} f = \pi_1 \cdot \underline{(u, y)} \\ g = \pi_2 \cdot \underline{(u, y)} \end{cases}$$

$\equiv$ 72: Ig. Ext., 73: Def-comp, 75: Def-const

$$\begin{cases} f\ u = \pi_1\ (u, y) \\ g\ y = \pi_2\ (u, y) \end{cases}$$

$\equiv$ 79: Def-proj

$$\begin{cases} f\ u = u \\ g\ y = y \end{cases}$$

$\equiv$ 75: Def-const, 72: Ig. Ext.

$$\begin{cases} f = \underline{u} \\ g = \underline{y} \end{cases}$$ //

2.

$\alpha = (id + [i_2, i_1]) \cdot [i_2, i_1]$

$coswap : A + B \longrightarrow B + A$

$$\begin{tikzcd}
A \arrow[r, "i_1"] \arrow[dr, "i_2"'] & A + B \arrow[d, "coswap"] & B \arrow[l, "i_2"'] \arrow[dl, "i_1"] \\
& B + A &
\end{tikzcd}$$

$$\begin{tikzcd}
A+(B+C) & A+(C+B) \arrow[l, "id + {[i_2, i_1]}"'] & (C+B)+A \arrow[l, "{[i_2, i_1]}"'] \arrow[ll, bend left, "\alpha"]
\end{tikzcd}$$

$\alpha : (C+B)+A \longrightarrow A+(B+C)$ //

$$\begin{tikzcd}
(C+B)+A \arrow[r, "\alpha"] \arrow[d, "(h+g)+f"] & A+(B+C) \arrow[d, "f+(g+h)"] \\
(C'+B')+A' \arrow[r, "\alpha"] & A'+(B'+C')
\end{tikzcd}$$

Prop. grátis de α:

$\alpha \cdot ((h+g)+f) = (f+(g+h)) \cdot \alpha$ //

3.

$[f \times h, g \times h] \cdot distl = [f, g] \times h$

$\equiv$ 33: Shunt-left, $distl^{\circ} = undistl = [i_1 \times id, i_2 \times id]$

$[f \times h, g \times h] = ([f, g] \times h) \cdot [i_1 \times id, i_2 \times id]$

$\equiv$ 20: Fusão-+

$[f \times h, g \times h] = [([f,g] \times h) \cdot (i_1 \times id), ([f,g] \times h) \cdot (i_2 \times id)]$

$\equiv$ 14: Functor-$\times$

$[f \times h, g \times h] = [([f,g] \cdot i_1) \times (h \cdot id), ([f,g] \cdot i_2) \times (h \cdot id)]$

$\equiv$ 18: Cancelamento-+, 1: Natural-id

$[f \times h, g \times h] = [f \times h, g \times h]$ //

4.

$\underline{Zero} : A \to T$

$One : Int \to T$

$Two : Int \times Int \to T$

$[\underline{Zero}, One] : A + Int \longrightarrow T$

$in : (A + Int) + (Int \times Int) \longrightarrow T$ //

$out \cdot [[\underline{Zero}, One], Two] = id$

$\equiv$ 20: Fusão-+, 17: Universal-+, 1: Natural-id

$$\begin{cases} out \cdot [\underline{Zero}, One] = i_1 \\ out \cdot Two = i_2 \end{cases}$$

$\equiv$ 20: Fusão-+, 17: Universal-+

$$\begin{cases} out \cdot \underline{Zero} = i_1 \cdot i_1 \\ out \cdot One = i_1 \cdot i_2 \\ out \cdot Two = i_2 \end{cases}$$

$\equiv$ 72: Ig. Ext., 75: Def-const, 73: Def-comp

$$\begin{cases} out\ (Zero) = i_1 \cdot i_1\ t \\ out\ (One\ t) = i_1 \cdot i_2\ t \\ out\ (Two\ t) = i_2\ t \end{cases}$$

//

5.

$\langle [id, id] \cdot (\pi_1 + \pi_1), [id, id] \cdot (\pi_2 + \pi_2) \rangle = [id, id]$

$\equiv$ 22: Absorção-+, 1: Natural-id

$\langle [\pi_1, \pi_1], [\pi_2, \pi_2] \rangle = [id, id]$

$\equiv$ 28: Lei da troca

$[\langle \pi_1, \pi_2 \rangle, \langle \pi_1, \pi_2 \rangle] = [id, id]$

$\equiv$ 8: Reflexão-×

$[id, id] = [id, id]$ //

6.

$(p \rightarrow g, h) \times f = p \cdot \pi_1 \rightarrow g \times f, h \times f$

$\equiv$ 10: Def-$\times$

$\langle (p \rightarrow g, h) \cdot \pi_1, f \cdot \pi_2 \rangle = p \cdot \pi_1 \rightarrow g \times f, h \times f$

$\equiv$ 31: 1ª lei de fusão cond.

$\langle ((p \cdot \pi_1) \rightarrow g \cdot \pi_1, h \cdot \pi_1), f \cdot \pi_2 \rangle = p \cdot \pi_1 \rightarrow g \times f, h \times f$

$\equiv$ 30: Def. cond. McCharty (x2)

$\langle [g \cdot \pi_1, h \cdot \pi_1] \cdot (p \cdot \pi_1)?, f \cdot \pi_2 \rangle = [g \times f, h \times f] \cdot (p \cdot \pi_1)?$

$\equiv$ 10: Def-$\times$

$\langle [g \cdot \pi_1, h \cdot \pi_1] \cdot (p \cdot \pi_1)?, f \cdot \pi_2 \rangle = [\langle g \cdot \pi_1, f \cdot \pi_2 \rangle, \langle h \cdot \pi_1, f \cdot \pi_2 \rangle] \cdot (p \cdot \pi_1)?$

$\equiv$ 28: Lei da troca, 9: Fusão-$\times$

$\langle [g \cdot \pi_1, h \cdot \pi_1] \cdot (p \cdot \pi_1)?, f \cdot \pi_2 \rangle = \langle [g \cdot \pi_1, h \cdot \pi_1] \cdot (p \cdot \pi_1)?, [f \cdot \pi_2, f \cdot \pi_2] \cdot (p \cdot \pi_1)? \rangle$

$\equiv$ 30: Def. cond. McCharty (3x), (E4)

$\langle ((p \cdot \pi_1) \rightarrow g \cdot \pi_1, h \cdot \pi_1), f \cdot \pi_2 \rangle = \langle ((p \cdot \pi_1) \rightarrow g \cdot \pi_1, h \cdot \pi_1), f \cdot \pi_2 \rangle$ //

7,

$invert \cdot in = [nil, rcons \cdot (id \times invert)]$

$\equiv$ Def. in, 20: Fusão-+, 17: Universal-+, 18: Cancelamento-+

$$\begin{cases} invert \cdot nil = nil \\ invert \cdot cons = rcons \cdot (id \times invert) \end{cases}$$

$\equiv$ 72: Ig. Ext, 73: Def-comp

$$\begin{cases} invert\ (nil\ \_) = nil\ \_ \\ invert\ (cons\ (a, u)) = rcons \cdot (id \times invert)\ (a, u) \end{cases}$$

$\equiv$ Def. nil, Def. cons, 78: Def-$\times$, 75: Def-id, Def. rcons

$$\begin{cases} invert\ [\,] = [\,] \\ invert\ (a : u) = invert\ u \ {+}{+}\ [a] \end{cases}$$

8.

Demonstrar que: $f = ap \cdot (\overline{f} \times id)$

Sim, prop. 36: Cancelamento-exp

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Demonstrar que: $(\overline{f} \times id) \cdot \langle \underline{a}, id \rangle = \langle \underline{\overline{f}\, a}, id \rangle$

$\equiv$ 11: Absorção-$\times$

$\langle \overline{f} \cdot \underline{a}, id \cdot id \rangle = \langle \overline{f} \cdot \underline{a}, id \rangle$

$\equiv$ 4: Absorção-const, 1: Natural-id

$\langle \underline{\overline{f}\, a}, id \rangle = \langle \underline{\overline{f}\, a}, id \rangle$ //

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$\overline{f}\, a = ap \cdot \langle \underline{\overline{f}\, a}, id \rangle$

$\equiv$ (E9: com $k = \overline{f}\, a$)

True //