Lecture Notes Problems

January 22, 2018

1 All imports

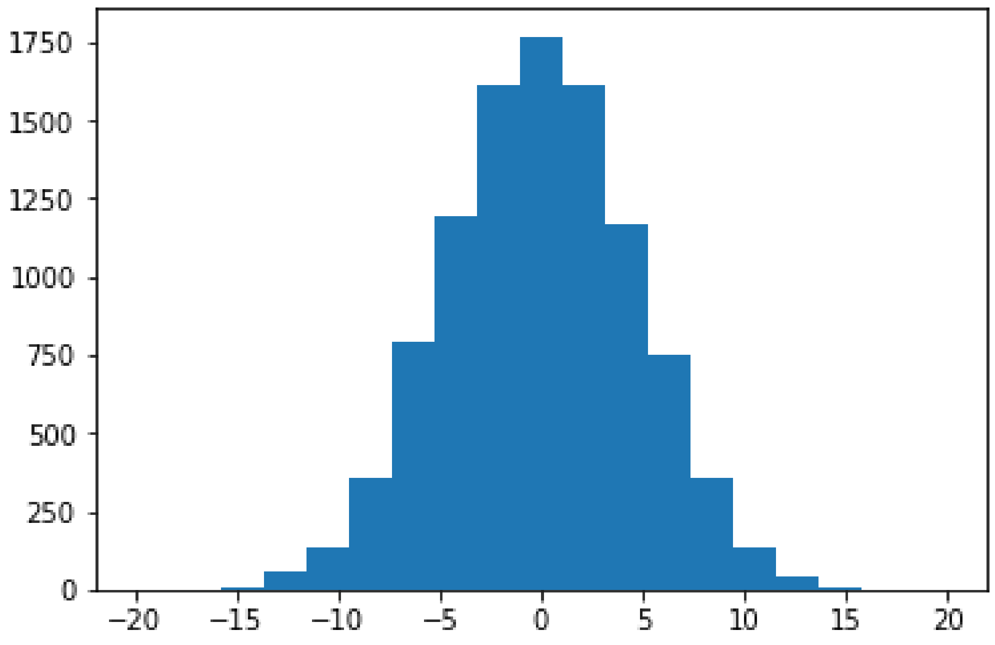
```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        import math
        from scipy.stats import norm # Gaussian i.e. Normal distribution
```

2 III.1

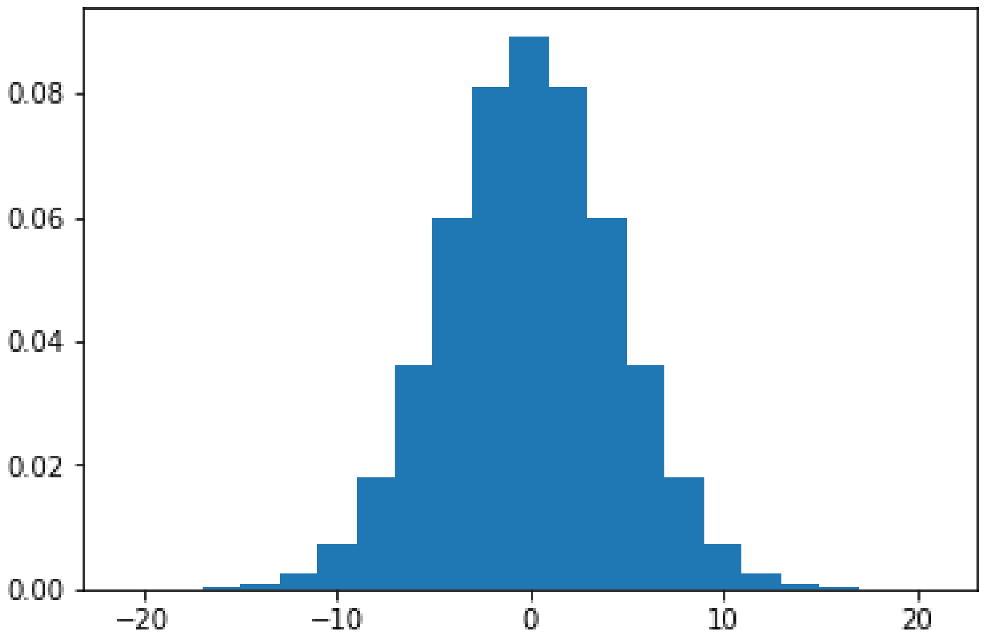
```
In [2]: def bernoulli_mean(p):
            return p

        def bernoulli_variance(p):
            return p*(1-p)
```

3 III.4

```
In [3]: def random_walk_sample(samples, walk_length):
            walks = np.random.randint(0,2,[samples,walk_length])*2-1 # value 1 is a right step,
            final_step = [sum(x) for x in walks]
            return final_step

In [4]: def plot_random_walk(samples, walk_length):
            final_step = random_walk_sample(samples=samples,walk_length=walk_length)
            plt.hist(final_step, bins=np.linspace(-walk_length,walk_length,walk_length))

In [5]: def plot_gaussian(walk_length):
            x = np.linspace(-walk_length,walk_length,walk_length+1)
            y = [norm.pdf(v, scale=math.sqrt(walk_length)) for v in x]
            plt.bar(x,y, width = 2)

In [6]: samples = 10000
        walk_length=20

In [7]: plot_random_walk(samples=samples,walk_length=walk_length)
```



```
In [8]: plot_gaussian(walk_length=walk_length)
```



3.0.1 Conclusions

The 2 graphs above are very similar, so indeed the Gaussian profile is a good approximation to a Random walk

for a large enough set of samples and random_walks.

Metric suggestion: Distance in L2 space between observations and the Guassian distribution.

4 III.7

```
In [9]: def max_eigenvalue_approximation(A,n):
            B = A
            for x in range(n): # In the end B = A^(32^n) normalized
                B = np.linalg.matrix_power(B,2**3) # B = A^32
                B = np.divide(B,np.linalg.norm(B)) # Normalizes B
            x = np.random.rand(len(A)) # Generates random A
            x = np.matmul(B,x) # Multiplies x by B, i.e. multiplies x by A 2**32 times
            x = np.divide(x,np.linalg.norm(x)) # Normalizes x
            x = np.matmul(A,x) # Calculates Ax
            eigenvalue = np.linalg.norm(x)
            print("Largest Eigenvalue: " + str(eigenvalue))
            return eigenvalue # This value approximates the max eigenvalue from below
```

```
In [10]: max_eigenvalue_approximation([[1,0],[1,2]],2)
```

```
Largest Eigenvalue: 2.0
```

```
Out[10]: 2.0
```

```
In [11]: np.matmul([[2,3],[11,13]],[1,2])
```

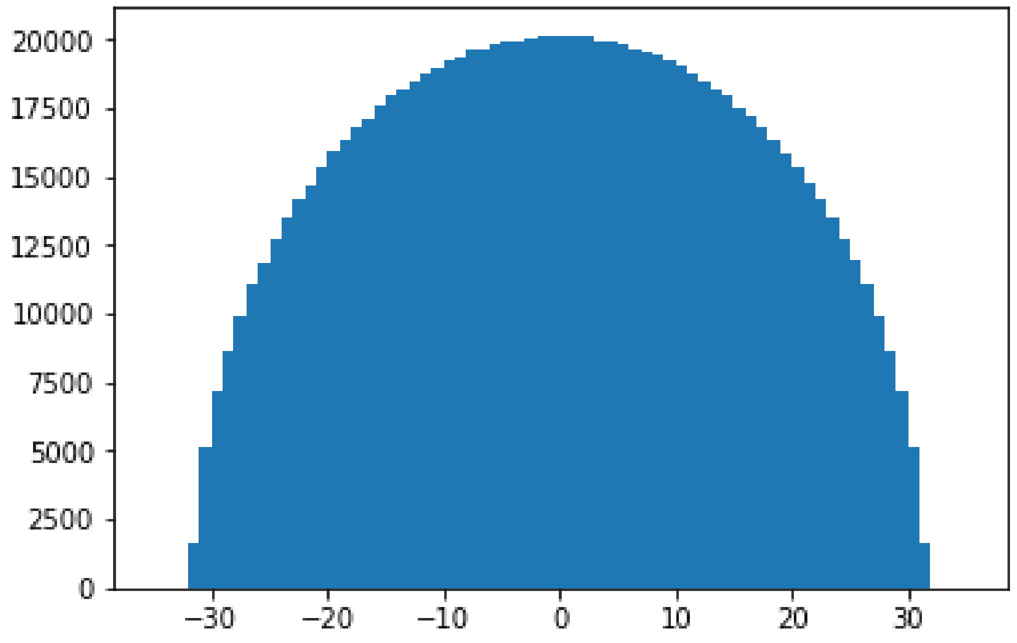
```
Out[11]: array([ 8, 37])
```

5 III.8

```
In [12]: def return_eigenvalues(A):
             return np.linalg.eigvals(A)
```

```
In [13]: def generate_random_symmetric_bernoulli_matrix(n):
             A = np.random.randint(0,2,[n,n]) # Generates a random (non symmetric) bernoulli mat
             for i in range(n):
                 for j in range(i):
                     value = A[i][j]^A[j][i] # Xors the 2 symetric entries so that value is unif
                     A[i][j] = value
                     A[j][i] = value
             return A # Returns the new
```

```
In [14]: def iii8_answer():
             A = generate_random_symmetric_bernoulli_matrix(1000)
             return return_eigenvalues(A)

In [15]: # n is the number of mattrices being run.
         # Higher n means waiting for longer, but with more statistical accuracy
         def eigenvalue_analysis(n):
             observed = [iii8_answer() for _ in range(n)]
             observed = np.concatenate(observed)
             plt.hist(observed,bins=np.linspace(-35,35,71))
             return observed

In [16]: observed = eigenvalue_analysis(1000)
```



The distribution seems to follow a half ellipse with x-radius of sqrt(1000)

The sqrt(1000) limit makes sense since that's the maximum possible eigenvalue for a 1000-sided matrix of zeroes and ones.