ProblemSheet2

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Class: C5.4 Networks
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1 All imports

```
In [1]: import numpy as np
        import networkx as nx
        import math
        import itertools
        import matplotlib.pyplot as plt
        import pandas as pd
        import seaborn as sns
```

2 Utils

```
In [2]: def draw(G,**kwargs):
            if len(G)<20:
                nx.draw_spring(G,
                               node_size=400,
                               with_labels=True)
            else:
                nx.draw_spring(G,
                               node_size=.5,
                               with_labels=False)
```

```
In [3]: def create_undirected_graph(edges):
            G=nx.Graph()
            G.add_edges_from(edges)
            return G
```

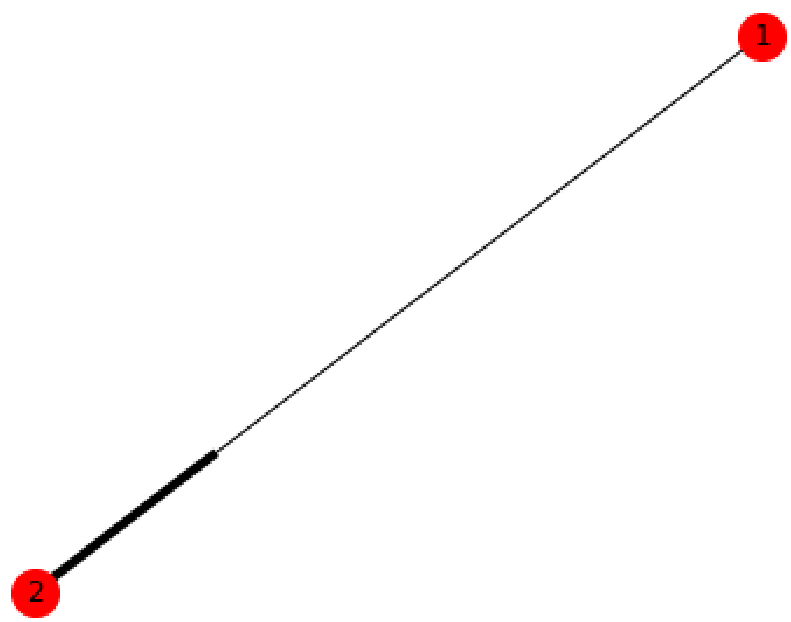
```
In [4]: def create_directed_graph(edges):
            DG=nx.DiGraph()
            DG.add_edges_from(edges)
            return DG
```

```
In [5]: def load_graph_from_tsv(file):
            f = open(file,"r")
            text = f.readlines()
            clean = lambda x:x.strip("\n").split(" ")
            node_pairs = list(map(clean,text[2:]))
            node_pairs = [(int(x[0]),int(x[1])) for x in node_pairs]
            node_pairs[:4]
            G = nx.Graph()
            G.add_edges_from(node_pairs)
            return G
```

3 (Q2) Connectedness

a) Draw a graph that is weakly connected but not strongly connected. Write down the adjacency matrix of this graph. What can happen to a random walk in such a graph, and what implication does this have for the asymptotic density of walkers on the nodes?

```
In [6]: DG = create_directed_graph([(1,2)])
        draw(DG)
```



Adjacency matrix $M = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Given the simplicity of the drawn graph, the only thing that can happen is for the walk to end on the 2nd node and get stuck there. For a general weakly connected graph then we can write a partial ordering on the strongly connected components. From that partial ordering we can make a tree, and a random walk will inevitably get stuck on one of the leaves.

b) Consider the adjacency-matrix representation of a graph. What is the difference between the spectrum of directed networks versus that of undirected networks? (Recall that the set of eigenvalues of a matrix is called the spectrum of that matrix.)

Answer: Since undirected matrices are represented by symmetric matrices, its eigenvalues are always real, whereas directed graphs are represented by more general matrices which might have non-real eigenvalues. Hence undirected graphs have matrices whose spectrum is a subset of the reals, whereas in directed graphs that might not be true.

4 (Q3) Clustering coefficients

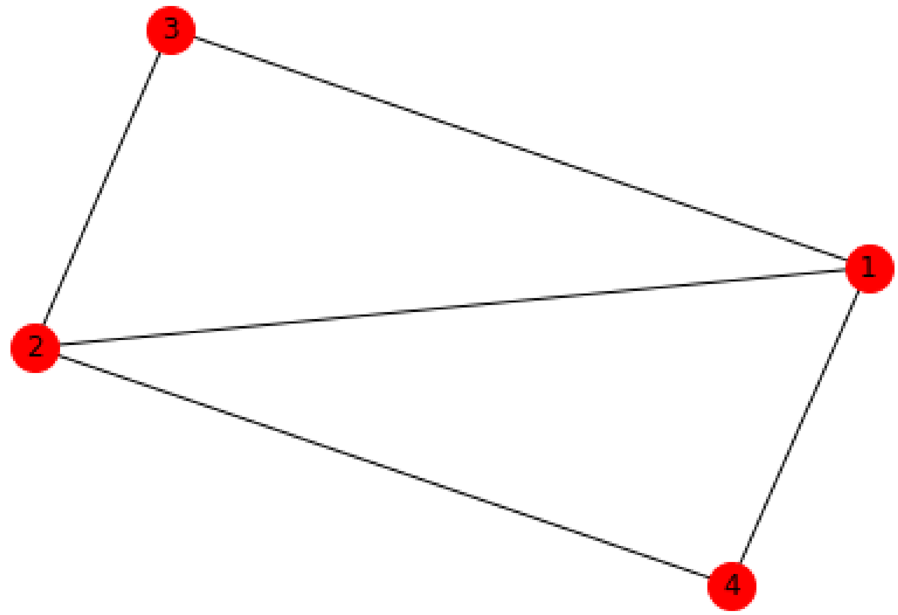
Draw a very small network in which the global clustering coefficient and mean local clustering coefficient have different values. Write down the adjacency matrix for this network.

Answer: Letting $c_k :=$ Number of triangles including the kth node $* \frac{2}{(\deg(k)-1)\deg(k)}$, then the mean local clustering coefficient is given by $\sum_k c_k$ where the sum goes through all nodes k. The global clustering coefficient (whose definition can be found on wikipedia but not on the lecture notes) is given by $3 * \frac{number.of.connected.triangles}{number.of.connected.triples}$. In order to compute the global clustering coefficient we use **nx.transitivity** and for the mean local clustering coefficient we use **nx.average_clustering**.

```
In [7]: def clustering_analysis(G):
            print("Global clustering coefficient: " + str(nx.transitivity(G)))
            print("Mean Local coefficient: " + str(nx.average_clustering(G)))
            draw(G)
```

```
In [8]: G = create_undirected_graph([(1,2),(2,3),(1,3),(1,4),(2,4)])
        clustering_analysis(G)
```

```
Global clustering coefficient: 0.75
Mean Local coefficient: 0.8333333333333333
```



This graph has adjacency Matrix $M = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$. In other words, $(1,4)$ is the only missing edge

5 (Q4) Small-world

Ex.IV.4: Generate graphs of the model c from Figure (13) and calculate the dependence of the network diameter and clustering coefficient on the number of shortcuts. Is this model, very similar to the so-called Watts-Strogatz model, a good model for a social network? Why or why not?

Answer: In order to compute the diameter we will use **nx.diameter** and for the clustering coefficient we will use **nx.transitivity** (i.e. the global clustering coefficient). For the number of shortcuts ...

(unfinished since I could not find model c from Figure (13)

6 (Q5) Centrality measures

Ex.IV.5: Take an undirected network and measure the correlation between different centrality measures. The correlation can either be estimated with the centrality values (Spearman) or with their associated ranking (Kendall). Construct an example of a graph where one node has a small degree centrality but a high betweenness centrality.

Answer: Given 2 random variables (or sets of observation) X and Y, we have

Pearson Correlation: $\frac{\text{Cov}(X,Y)}{\sigma(X)\sigma(Y)}$

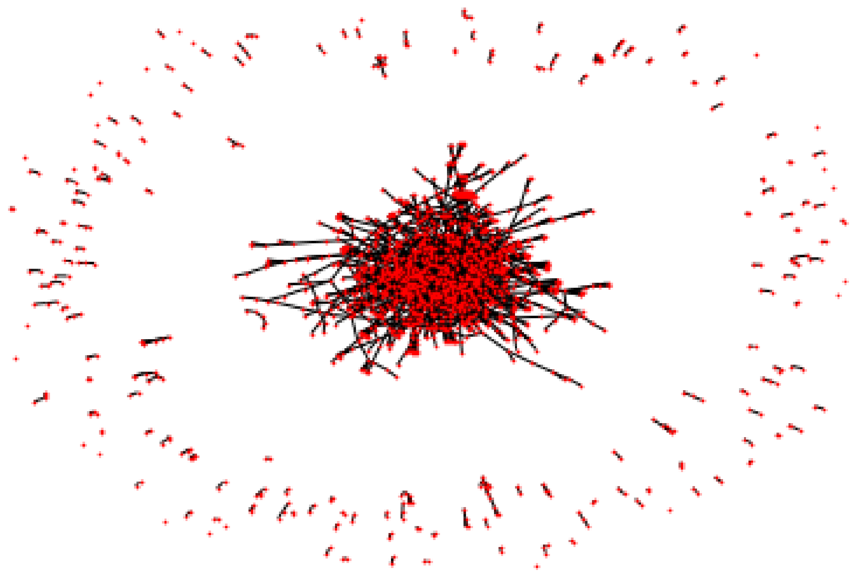
Kendall Correlation: After ordering the observation pairs, use $\frac{(number.of.concordant.pairs)-(number.of.discordant.pairs)}{\frac{1}{2}n(n-1)}$

Spearman Correlation: Pearson correlation after mapping the observations X_i, Y_i to their ranks.

In [9]:
```
def build_centrality_measures_dataframe(G):

    # Builds dictionaries with the different metrics
    betweenness = nx.betweenness_centrality(G)
    degree = dict(nx.degree(G))
    closeness = nx.closeness_centrality(G)
    katz = nx.katz_centrality(G)
    pagerank = nx.pagerank(G,alpha=0.5) # The $ \alpha=.85 $ was chosen randomly. Is the

    # Builds a dataframe with the measures as columns and the nodes as rows
    df = pd.DataFrame({'betweenness':betweenness,
                       'degree':degree,
                       'closeness':closeness,
                       'katz':katz,
                       'pagerank':pagerank})
    return df

# Requires the centrality_measures_dataframe as input
def correlation_of_centrality_metrics(df):
    # Builds correlation matrices for the different metrics
    for metric in ['pearson', 'kendall', 'spearman']:
        print("\n\n" + str(metric.capitalize()) + ' correlation:')
        print(df.corr(metric))
```

Dataset used: The propo dataset used consists of nodes representing proteins and edges representing pairs of interacting proteins.

In [10]:
```
G = load_graph_from_tsv("datasets/propo.tsv")
draw(G)
```



```
In [11]: df = build_centrality_measures_dataframe(G)
         df.describe()
```

Out[11]:

	betweenness	closeness	degree	katz	pagerank
count	1870.000000	1870.000000	1870.000000	1870.000000	1870.000000
mean	0.001891	0.091932	2.435294	0.020112	0.000535
std	0.006035	0.051226	3.164618	0.011416	0.000374
min	0.000000	0.000000	1.000000	0.013533	0.000313
25%	0.000000	0.080883	1.000000	0.014203	0.000362
50%	0.000000	0.112335	1.000000	0.016208	0.000446
75%	0.001219	0.126392	3.000000	0.021312	0.000548
max	0.129420	0.183020	56.000000	0.200559	0.009283

```
In [12]: correlation_of_centrality_metrics(df)
```

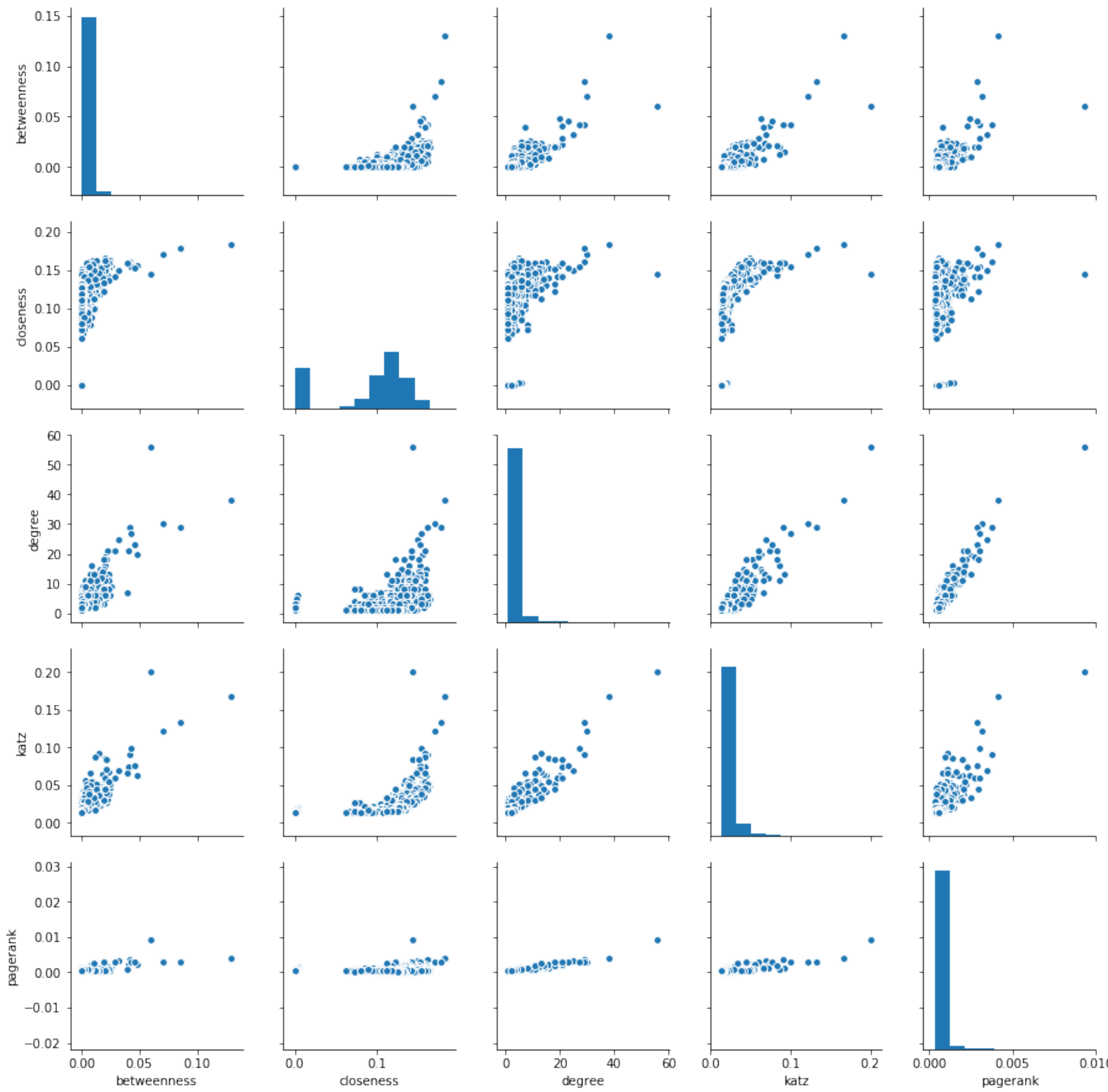
Pearson correlation:

	betweenness	closeness	degree	katz	pagerank
betweenness	1.000000	0.297399	0.837694	0.818457	0.739478
closeness	0.297399	1.000000	0.302620	0.456009	0.090823
degree	0.837694	0.302620	1.000000	0.868335	0.929309
katz	0.818457	0.456009	0.868335	1.000000	0.726429
pagerank	0.739478	0.090823	0.929309	0.726429	1.000000

```
Kendall correlation:
             betweenness  closeness    degree      katz  pagerank
betweenness     1.000000   0.409058  0.813356  0.548139  0.548832
closeness       0.409058   1.000000  0.359753  0.713957 -0.081977
degree          0.813356   0.359753  1.000000  0.569681  0.638604
katz            0.548139   0.713957  0.569681  1.000000  0.064198
pagerank        0.548832  -0.081977  0.638604  0.064198  1.000000


Spearman correlation:
             betweenness  closeness    degree      katz  pagerank
betweenness     1.000000   0.523995  0.897253  0.676930  0.699047
closeness       0.523995   1.000000  0.463058  0.880813 -0.064068
degree          0.897253   0.463058  1.000000  0.687259  0.764166
katz            0.676930   0.880813  0.687259  1.000000  0.176845
pagerank        0.699047  -0.064068  0.764166  0.176845  1.000000
```

```
In [13]: sns.pairplot(df)
```

```
Out[13]: <seaborn.axisgrid.PairGrid at 0x7f8e25f33eb8>
```



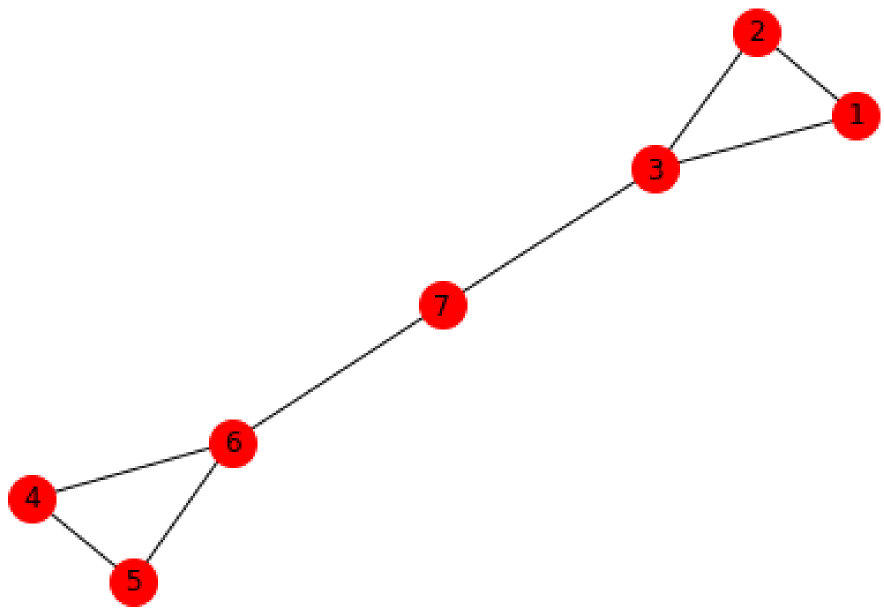
b) Construct an example of a graph where one node has a small degree centrality but a high betweenness centrality.

Answer: One possile approach is to take 2 complete graphs K_n, pick one node from each and connect them both to a $(2n+1)th$ node. That node will have deg 2 but betweenness $1 - \frac{2\binom{n}{2}}{\binom{2n+1}{2}} = 1 - 2\frac{n(n-1)}{2n(2n-1)} = 1 - \frac{n-1}{2n-1} = \frac{n}{2n-1}$, which converges to $\frac{1}{2}$ as $n \to \infty$. On the other hand the average degree of a node in this graph goes to infinity as $n \to \infty$.

```
In [14]: G = create_undirected_graph([(1,2),(2,3),(1,3),(4,5),(4,6),(5,6),(3,7),(6,7)])
         draw(G)
         comparison = pd.DataFrame({'degree':dict(nx.degree(G)),
                                    'betweenness':nx.betweenness_centrality(G)})
         comparison.index.name='node'
         comparison.sort_values(by=['degree','betweenness']).T
```

Out[14]:

node	1	2	4	5	7	3	6
betweenness	0.0	0.0	0.0	0.0	0.6	0.533333	0.533333
degree	2.0	2.0	2.0	2.0	2.0	3.000000	3.000000



Node 7 has deg 2 but betweeness $\frac{3}{2(3)-1} = \frac{3}{5}$, whereas all other nodes have at least deg 2 and at most betweenness $\frac{3}{5}$