III.Mathematical Toolbox

February 10, 2018

1 All imports

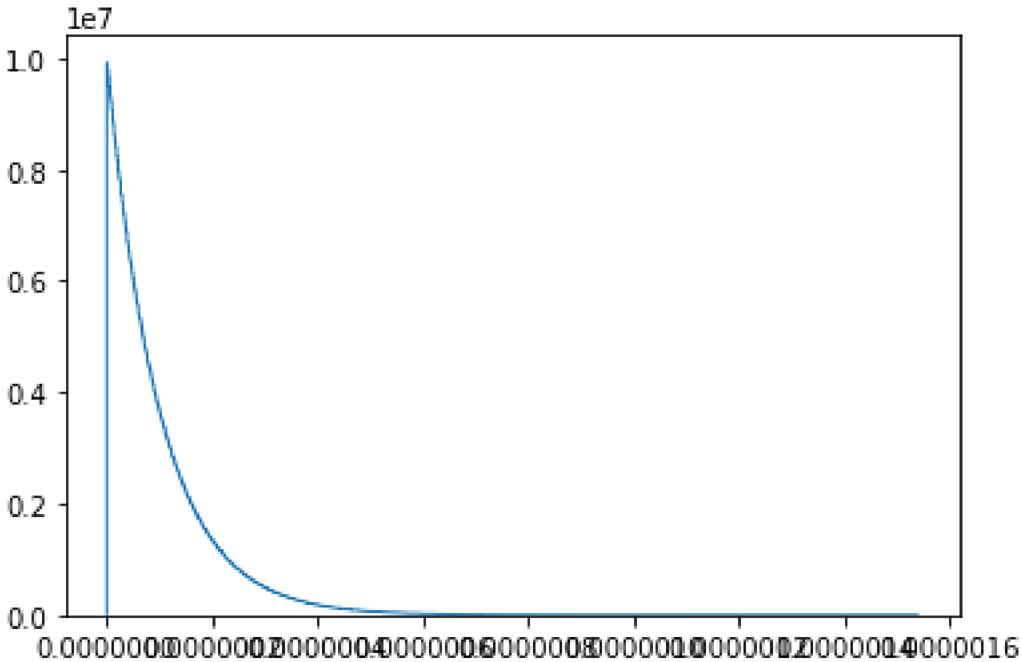
```
In [1]: from collections import Counter
        import pandas as pd
        import numpy as np
        import matplotlib.pyplot as plt
        import math
        from scipy.stats import norm # Gaussian i.e. Normal distribution
```

2 III.1

```
In [2]: def bernoulli_mean(p):
            return p

        def bernoulli_variance(p):
            return p*(1-p)
```

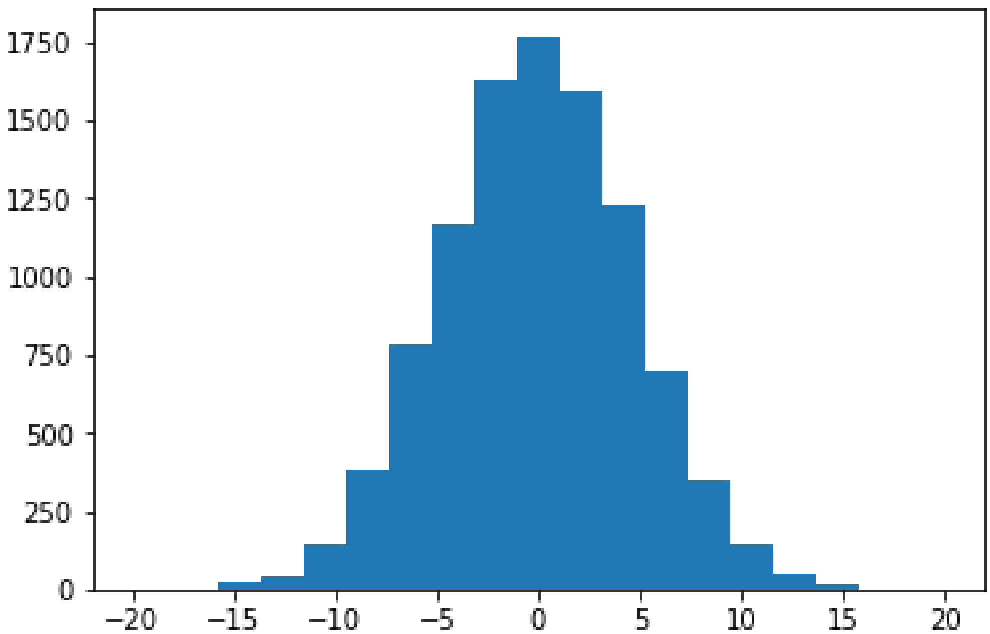
3 III.3

```
In [3]: def inter_interval_distribution(n):
            x = sorted(np.random.rand(n))
            y = list(zip(x[1:],x[:-1]))
            z = [t[0]-t[1] for t in y]
            plt.hist(z, bins=int(math.sqrt(n/10)), normed=1, histtype='step')
            return z

In [4]: z = inter_interval_distribution(10000000)
```

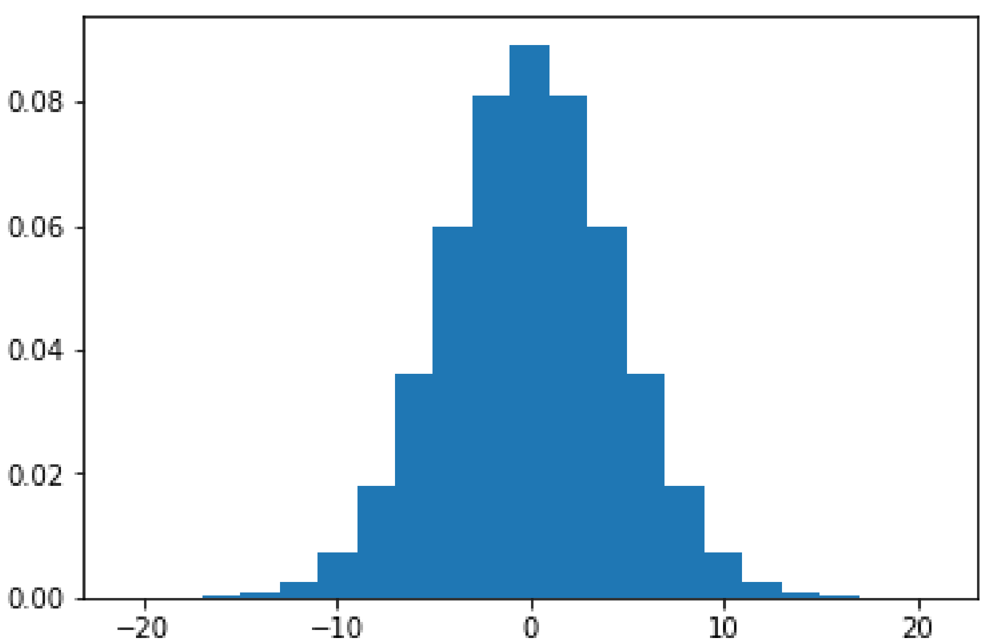


4 III.4

```
In [5]: def random_walk_sample(samples, walk_length):
            walks = np.random.randint(0,2,[samples,walk_length])*2-1 # value 1 is a right step,
            final_step = [sum(x) for x in walks]
            return final_step

In [6]: def plot_random_walk(samples, walk_length):
            final_step = random_walk_sample(samples=samples,walk_length=walk_length)
            plt.hist(final_step, bins=np.linspace(-walk_length,walk_length,walk_length))

In [7]: def plot_gaussian(walk_length):
            x = np.linspace(-walk_length,walk_length,walk_length+1)
            y = [norm.pdf(v, scale=math.sqrt(walk_length)) for v in x]
            plt.bar(x,y, width = 2)

In [8]: samples = 10000
        walk_length = 20

In [9]: plot_random_walk(samples=samples,walk_length=walk_length)
```




```
In [10]: plot_gaussian(walk_length=walk_length)
```



4.0.1 Conclusions

The 2 graphs above are very similar, so indeed the Gaussian profile is a good approximation to a Random walk. For a large enough set of samples and random_walks.

Metric suggestion: L2 distance between observations' cumulative distribution function and the Guassian distribution.

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Importing the text as a single string

```
In [11]: # You can change quran.txt to any txt file you want to try, and then run all the cells
         f = open('datasets/quran.txt', 'r')
         text = ''.join(f.readlines())
```

Making a word count:

```
In [12]: text = text.lower()
         for stringy in ["\n",",",".","'",":","?",";","!"]:
             text = text.replace(stringy, "")
         words = text.split(" ")
         frequency_count = Counter(words)
         d = dict(frequency_count)
```

```
In [13]: table = pd.DataFrame(list(d.items()))
         table.columns=(["Word","Quran_Count"])
         table.sort_values(by="Quran_Count",inplace=True,ascending=False)
         table.reset_index(drop=True,inplace=True)
         table.head(5)
```

Out[13]:

	Word	Quran_Count
0	the	8725
1	and	7667
2	of	4466
3	to	3641
4	you	3350

Getting the frequency of words in the English dictionary:

```
In [14]: f = open('datasets/english_word_count.txt', 'r')
         text = f.readlines()
         def process_line(line):
             line = line.replace("\n", "")
             return line.split('\t')
         frequencies = [process_line(line) for line in text]
         frequencies = {x[0]:int(x[1]) for x in frequencies}
```

Merging the tables:

```
In [15]: table['English_word'] = table["Word"].map(frequencies)
```

```
In [16]: table.head(5)

Out[16]:   Word  Quran_Count  English_word
         0  the         8725  2.313585e+10
         1  and         7667  1.299764e+10
         2   of         4466  1.315194e+10
         3   to         3641  1.213698e+10
         4  you         3350  2.996181e+09
```

Sorting the table by the ratio of frequencies, where ratio is defined as (up to a scalar): **Ratio:** Frequency of a word in the text / Frequency of word in common English

```
In [17]: table['Ratio'] = np.divide(table['Quran_Count'],table['English_word'])*(10**6)
         table.sort_values(by="Ratio",inplace=True,ascending=False)
         table.reset_index(drop=True,inplace=True)

In [18]: table[['Word','Ratio']].head(20).T

Out[18]:                   0          1         2            3        4        5  \
         Word   chastisement  evildoers    tiding  disbelieves    haply  gehenna
         Ratio        2275.3    1313.31   1286.81      1177.39  1100.15  962.464

                          6        7            8           9         10  \
         Word   unbelievers    whoso  recompensed  unthankful  similitudes
         Ratio      923.551  884.406       874.92     769.769      757.732

                          11       12       13         14          15         16  \
         Word   disbelieved   abased  tarried  niggardly  disbelieve  idolaters
         Ratio      715.237  662.663  616.118    595.593     576.764     548.08

                      17          18      19
         Word   couldst  whensoever  smites
         Ratio  536.813     499.322  459.01
```

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```
In [19]: def max_eigenvalue_approximation(A,n):
             B = A
             for x in range(n): # In the end B = A^(32^n) normalized
                 B = np.linalg.matrix_power(B,2**3) # B = A^32
                 B = np.divide(B,np.linalg.norm(B)) # Normalizes B
             x = np.random.rand(len(A)) # Generates random A
             x = np.matmul(B,x) # Multiplies x by B, i.e. multiplies x by A 2**32 times
             x = np.divide(x,np.linalg.norm(x)) # Normalizes x
             x = np.matmul(A,x) # Calculates Ax
             eigenvalue = np.linalg.norm(x)
             print("Largest Eigenvalue: " + str(eigenvalue))
             return eigenvalue # This value approximates the max eigenvalue from below
```

```
In [20]: max_eigenvalue_approximation([[1,0],[1,2]],2)
```

```
Largest Eigenvalue: 2.0
```

```
Out[20]: 2.0
```

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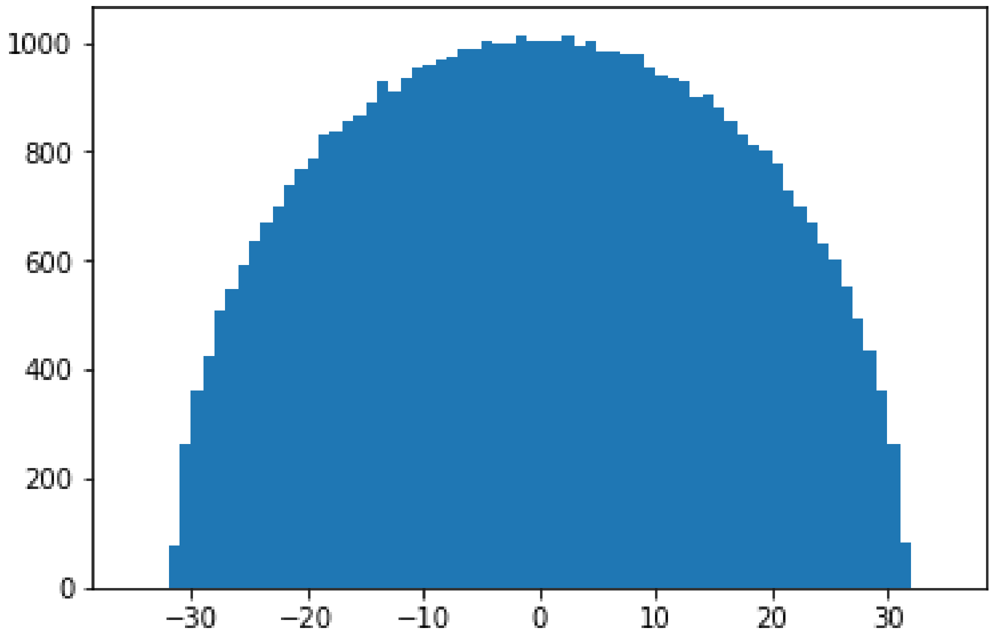
```
In [21]: def return_eigenvalues(A):
             return np.linalg.eigvals(A)
```

```
In [22]: def generate_random_symmetric_bernoulli_matrix(n):
             A = np.random.randint(0,2,[n,n]) # Generates a random (non symmetric) bernoulli mat
             for i in range(n):
                 for j in range(i):
                     value = A[i][j]^A[j][i] # Xors the 2 symetric entries so that value is unif
                     A[i][j] = value
                     A[j][i] = value
             return A # Returns the new
```

```
In [23]: def iii8_answer():
             A = generate_random_symmetric_bernoulli_matrix(1000)
             return return_eigenvalues(A)
```

```
In [24]: # n is the number of mattrices being run.
         # Higher n means waiting for longer, but with more statistical accuracy
         def eigenvalue_analysis(n):
             observed = [iii8_answer() for _ in range(n)]
             observed = np.concatenate(observed)
             plt.hist(observed,bins=np.linspace(-35,35,71))
             return observed
```

```
In [25]: # This line takes a long time to run. Can you improve it?
         observed = eigenvalue_analysis(50)
```



The distribution seems to follow a half ellipse with x-radius of sqrt(1000). The sqrt(1000) limit makes sense since that's the maximum possible eigenvalue for a 1000-sided matrix of zeroes and ones.