EDO's de Bernoulli

$$y' + a(x)\,y = b(x)\,y^{\alpha} \qquad \begin{cases} \text{se } \alpha = 0 \Rightarrow \text{Linear de 1ª ordem} \\ \text{se } \alpha \neq 0 \Rightarrow \text{mudança de variável } z = y^{1-\alpha} \end{cases}$$

$\boxed{\alpha \neq 0}$

$$y' + a(x)\,y = b(x)\,y^{\alpha}, \quad z = y^{1-\alpha} \qquad z' = (1-\alpha)\,y^{-\alpha}\,y'$$

$$\times y^{-\alpha} \rightarrow \quad y^{-\alpha} y' + a(x)\,y\,y^{-\alpha} = b(x)\,y^{\alpha}\,y^{-\alpha}$$

simplificam

$$\times(1-\alpha) \quad \boxed{(1-\alpha)\,y^{-\alpha}\,y'} + a(x)\,y\,(1-\alpha)\,\boxed{y^{-\alpha}} = b(x)\,\cancel{y^{\alpha}}\,(1-\alpha)\,\cancel{y^{-\alpha}}$$

$$\boxed{z'} + (1-\alpha)\,a(x)\,\boxed{z} = (1-\alpha)\,b(x)$$ esta já é uma EDO de 1ª Ordem

exemplo:

$$x y' + y = y^2 \ln(x), \quad x > 0$$

$$\times y^{-2} \quad x y^{-2} y' + y^{-2} y = y^2 y^{-2} \ln(x)$$

$$x y^{-2} y' + y^{-1} = \ln(x)$$

$$\times(-1) \quad -x y^{-2} y' - y^{-1} = -\ln(x)$$

$$x z' - z = -\ln(x)$$ EDO linear

$$z' - \tfrac{1}{x} z = -\tfrac{\ln(x)}{x}$$

$$\tfrac{1}{x} z' - \tfrac{1}{x^2} z = -\frac{\ln(x)}{x^2}$$

$$\left(\frac{1}{x} z\right)' = -\frac{\ln(x)}{x^2}$$

$$\frac{z}{x} = -\int \frac{\ln(x)}{x^2}\,dx$$

Fator integrante

$$\mu(x) = e^{\int -1/x\,dx} = e^{-\int 1/x\,dx} = e^{-\ln(x)}$$

$$= \cancel{e}^{\ln(1/x)} = \frac{1}{x}$$

$\alpha = 2$ Bernoulli

$z = y^{-1}$

$z' = -1\,y^{-2}\,y'$

$$\frac{z}{x} = -\int \underbrace{x^{-2}}_{f'}\,\underbrace{\ln(x)}_{g}\,dx$$

$$\frac{z}{x} = \left[-\frac{1}{x}\ln(x) + \int \frac{1}{x^2}\,dx\right]$$

$$\frac{z}{x} = \frac{1}{x}\ln(x) - \frac{1}{x} + C, \quad C \in \mathbb{R}$$

$$z = \ln(x) - 1 + xC, \quad C \in \mathbb{R}$$

integral geral da EDO de primeira ordem

$$y^{-1} = \ln(x) - 1 + xC \rightarrow y = \frac{1}{\ln(x) - 1 + xC}, \quad C \in \mathbb{R}$$

$\alpha = 5$

$z = y^{-4}$

$z' = -4y^{-5}y'$

$$y' - \tfrac{1}{2x}\,y = 5x^2 y^5, \quad x \neq 0$$

$$\times y^{-5} \quad y^{-5} y' - \frac{1}{2x}\,y^1 y^{-5} = 5x^2 y^{-5} y^5$$

$$y^{-5} y' - \frac{1}{2x}\,y^{-4} = 5x^2 \xrightarrow{\times(-4)} -4y^{-5}y' - \frac{1}{2x}(-4)\,y^{-4} = -20x^2$$

$$z' + \frac{2}{x} z = -20x^2$$ EDO de 1ª ordem $\xrightarrow{\times x^2}$ $x^2 z' + 2x z = -20x^4$

$$(x^2 z)' = -20x^4 \rightarrow x^2 z = -\int 20x^4\,dx \rightarrow x^2 z = -20\left(\tfrac{x^5}{5} + C\right)$$

Int. geral da EDO linear

$$\boxed{z = -4x^3 - \frac{20C}{x^2}, \quad C \in \mathbb{R}} \rightarrow \boxed{y^{-4} = -4x^3 - \frac{20C}{x^2}, \quad C \in \mathbb{R}}$$

integral geral da EDO Bernoulli na forma implícita folha 4 - exercício 1

1. Verifique se as seguintes funções são solução (em $\mathbb{R}$) das equações diferenciais dadas:

(a) $y = \operatorname{sen} x - 1 + e^{-\operatorname{sen} x}$ $\qquad \frac{dy}{dx} + y\cos x = \frac{1}{2}\operatorname{sen}(2x);$

(b) $z = \cos x$ $\qquad z'' + z = 0;$

(c) $y = \cos^2 x$ $\qquad y'' + y = 0;$

(d) $y = Cx - C^2 \quad (C \in \mathbb{R})$ $\qquad (y')^2 - xy' + y = 0.$

a) $y = \operatorname{sen} x - 1 + e^{-\operatorname{sen} x}$

$y' = (\operatorname{sen} x - 1 + e^{-\operatorname{sen} x})' = \cos x + (-\operatorname{sen} x)' e^{-\operatorname{sen} x} = \cos(x) - \cos(x) e^{-\operatorname{sen}(x)}$

$\frac{dy}{dx} + y\cos(x) = \frac{1}{2}\operatorname{sen}(2x)$

$y' + y\cos(x) = \frac{1}{2}\operatorname{sen}(2x)$

$\cancel{\cos(x)} - \cancel{\cos(x) e^{-\operatorname{sen}(x)}} + \operatorname{sen}(x) \times \cos(x) - \cancel{\cos(x)} + \cancel{\cos(x) e^{-\operatorname{sen}(x)}} = 1/2 \operatorname{sen}(2x)$

$\operatorname{sen}(x) \times \cos(x) = \frac{1}{2}\operatorname{sen}(2x)$

$\operatorname{sen}(x) \times \cos(x) = \frac{1}{\cancel{2}} \times \cancel{2}\operatorname{sen}(x) \times \cos(x) \longrightarrow \operatorname{sen}(x) \times \cos(x) = \operatorname{sen}(x) \times \cos(x)$ // c.q.p.

b)

$z = \cos(x)$; $z' = -\operatorname{sen}(x)$; $z'' = -\cos(x)$

$z'' + z = 0$

$-\cos(x) + \cos(x) = 0$ // c.q.p.

folha 4 - exercício 5

b) $y' - \sqrt{1-x^2} = 0$

$y = \int \sqrt{1-x^2}\, dx$

$x = \operatorname{sen} t$

$\frac{dx}{dt} = \cos t \iff dx = \cos t \, dt$

$y = \int \sqrt{1 - \operatorname{sen}^2 t} \cos t \, dt$

$y = \int \cos^2 t \, dt \longrightarrow y = \int \frac{1+\cos(2t)}{2} dt \longrightarrow y = \int \frac{1}{2} dt + \int \frac{1}{2}\cos(2t)\, dt$

$y = \frac{1}{2}t + \frac{1}{4}\operatorname{sen}(2t) + c, \quad c \in \mathbb{R}$

$y = \frac{1}{2}\operatorname{arcsen}(x) + \frac{1}{4} 2\operatorname{sen}(\operatorname{arcsen}(x)) \cdot \cos(\operatorname{arcsen}(x))$

$y = \frac{1}{2}\operatorname{arcsen}(x) + \frac{1}{2} x \cdot \sqrt{1-x^2}$

Nota:

$\sqrt{a^2 - x^2} \rightarrow x = a\operatorname{sen} t$

$\sqrt{x^2 - a^2} \rightarrow x = a \sec t$

$\sqrt{x^2 + a^2} \rightarrow x = a \tan t$

Nota:

$\cos^2 x + \operatorname{sen}^2 x = 1$

$\cos x = \sqrt{1 - \operatorname{sen}^2 x}$

$\cos x = \sqrt{1 - \operatorname{sen}^2(\operatorname{arcsen}(x))}$

$\cos x = \sqrt{1 - x^2}$