VPython: Modeling Orbits

Apparatus

VPython
computer

Goal

In this activity, you will write a VPython program that simulates Earth orbiting Sun. You will adjust the initial velocity in order to see how the initial velocity affects the orbit.

Introduction

The gravitational force law is:

$$\vec{F}_{grav\ on\ 2\ by\ 1} = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r} \tag{1}$$

where $\vec{r} = \vec{r}_2 - \vec{r}_1$. It might be easier to write this as:

$$|\vec{F}_{grav}| = G\frac{m_1 m_2}{|\vec{r}|^2} \tag{2}$$

$$\vec{F}_{grav} = -|\vec{F}_{grav}|\hat{r} \tag{3}$$

We want to simulate Earth's orbit around the Sun. You will need to determine an initial position and initial velocity for Earth. Let's pick the point on Earth's orbit that is shown below. Note: any point on the orbit could have been chosen.

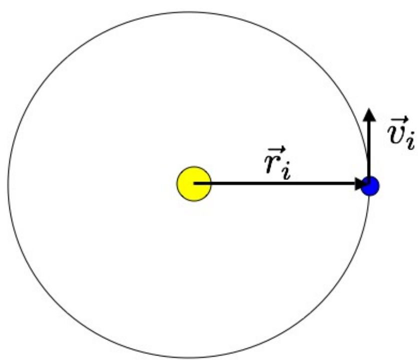

Figure 1: Earth's initial position and velocity in the simulation

Earth's average distance from the Sun is 1.496×10^9 m. What is the position vector shown in Figure 1? This will be the initial position of Earth in our simulation.

What is the time in seconds for Earth to complete one orbit? (Note: this is defined to be 1 year.)

What is the distance Earth travels in one complete orbit? (Note: the distance around a circle is called *circumference*.)

What is Earth's speed as it orbits Sun? (speed = distance / time)

What is the initial velocity vector in Figure 1 if Earth has the speed you just calculated in the previous question?

We will also need to know Sun's mass and Earth's mass so that we can calculate the gravitational force on Earth. Look up those values and record those values now.

Procedure

1. Start with a simulation that you have used before. A good one to start with is the projectile motion simulation. Open it and save it as a different name, such as *earth-orbit.py* for example.

2. Delete all of the graphs and plot statements since we won't need those right now.

3. Delete the floor or ground from the projectile motion simulation since we won't need that either. Basically, delete anything that you know will not be used in the simulation of Earth's orbit.

4. Change the ball to Earth and name it `earth`. Make its radius 6.4×10^9 which is 1000 times larger than Earth's actual radius. If we used its actual radius, we would not be able to see it in the simulation window. As its initial position, use the position you calculated in Fig. 1.

5. Change all references to the ball to `earth` in the program.

6. Create a sphere to represent Sun. Make its color white (or yellow is ok, but the yellow is due to atmospheric effects; Sun is actually white.) Make its position the origin.

7. Make the radius of Sun about 7×10^9 m (which is 10 times bigger than its actual radius). Again, this helps us see it clearly in the animation window.

8. Define the masses of Sun and Earth and name them `sun.m` and `earth.m` .

9. Define the universal gravitational constant.

   ```
   G=6.7e-11
   ```

10. Define the initial velocity of Earth to be what you calculated in Figure 1.

11. The time step should be small, relative to 1 year. Thus, 1 day is sufficiently small. Define *dt* to be 1 day in units of seconds.

12. Inside the `while` loop, you will need calculate the net force on Earth, use Newton's law of gravitation. The steps for calculating the gravitational force on Earth by the Sun are:

    ```
    r=earth.pos - sun.pos
    rmag=mag(r)
    rhat=r/rmag
    Fgrav=-(G*sun.m*earth.m/rmag**2)*rhat
    Fnet=Fgrav
    ```

13. Run your program, and you should see Earth travel in a circular orbit around Sun.

14. If there is dynamic autoscaling (where the camera zooms in and out), it can be annoying. Add the following line just above the while loop to turn off autoscaling.

    ```
    scene.autoscale=0
    ```

    You can turn this back on by setting `scene.autoscale=1` if you want the window to autoscale.

15. If it's not already in your program, add a trail so that you can see the path of Earth.

16. Run the program and verify that Earth travels in a circular orbit.

Application

1. Multiply the initial velocity of Earth by 1.1. What path do you get?
2. Multiply the initial velocity of Earth by 0.9. What path do you get?
3. Multiply the initial velocity of Earth by 1.5. What path do you get?
4. Multiply the initial velocity by sqrt(2). What path do you get?
5. Model the motion of Jupiter, assuming that its orbit is circular.