

Gravitational Wave and cosmological tests of Gravity and Dark Energy

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BERKELEY CENTER *for*
COSMOLOGICAL PHYSICS



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with JM Ezquiaga, (1710.05901 PRL)

D. Bettoni, JM Ezquiaga, K. Hinterbichler (1608.01982 PRD)

A. Barreira, F. Montanari, J. Renk (1707.02266 JCAP)

The case for modified gravity

- Alternatives to Λ

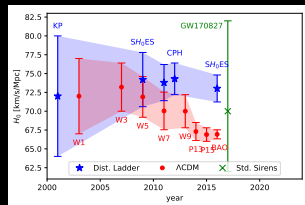
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- Λ CDM tensions \longrightarrow

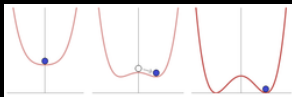
- Interesting theoretical questions

- *proxy for inflation/quantum gravity?*

- *cosmological constant problems?*



(Adapted from Fredman '17)



$\sim 36\%$ of unsolved problems in physics involve gravity

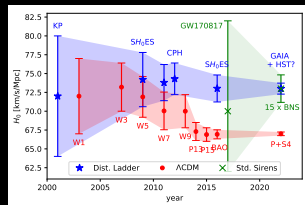
(see www.wikipedia.org/wiki/List_of_unsolved_problems_in_physics)

- Test gravity on all regimes

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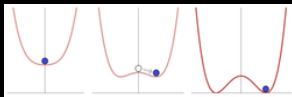
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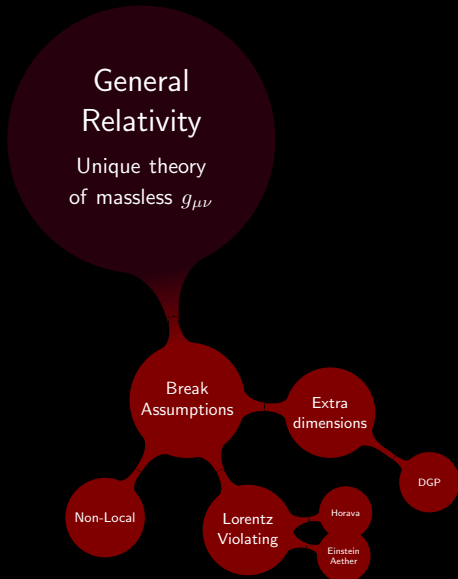


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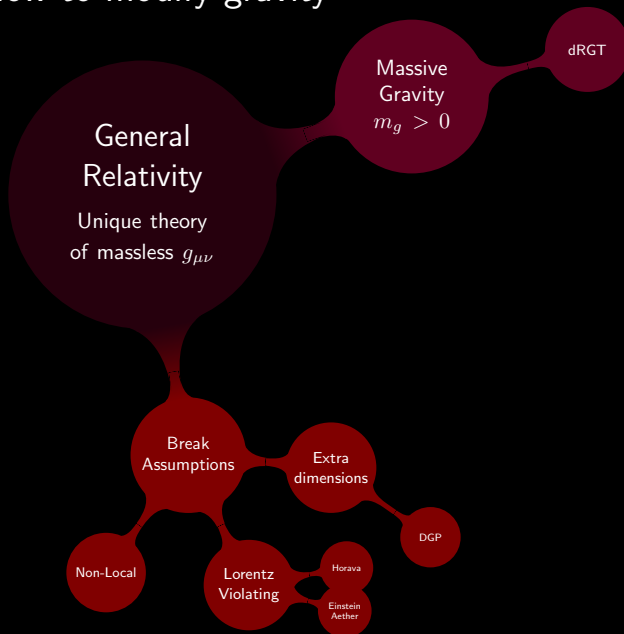
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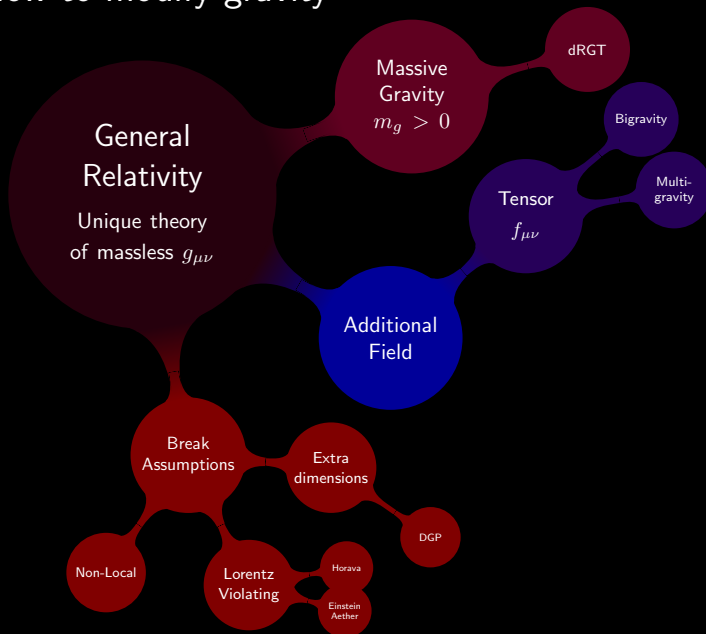
How to modify gravity



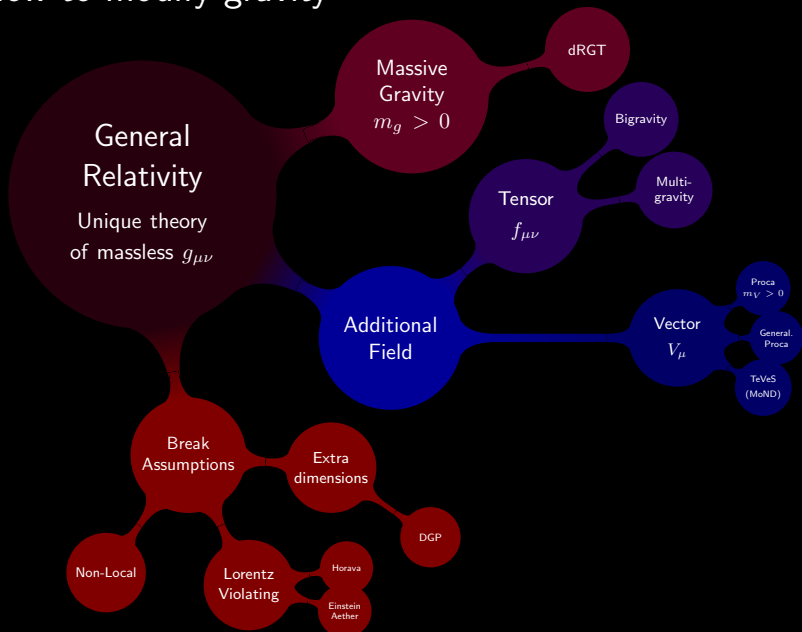
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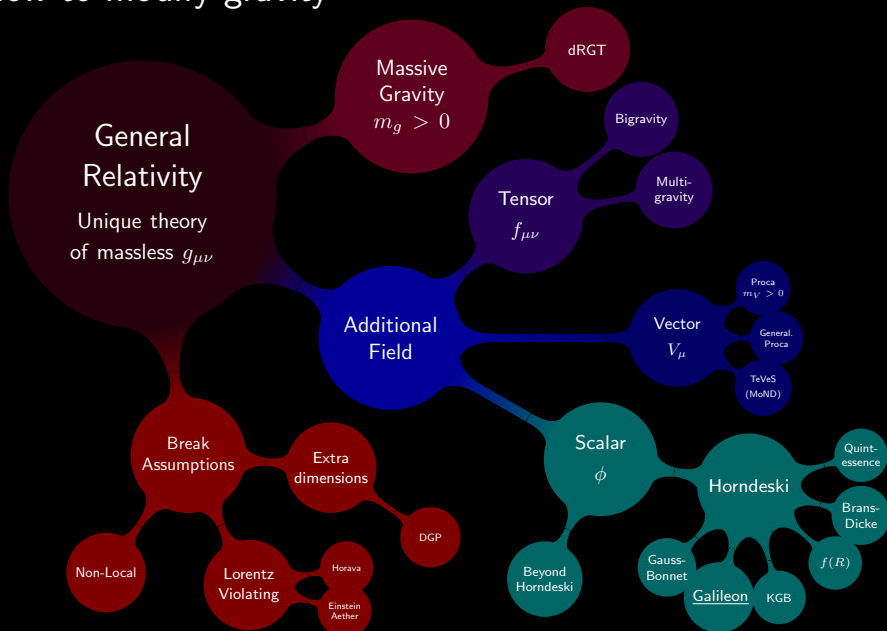
How to modify gravity



How to modify gravity



How to modify gravity



Test gravity: 3 approaches

1 - Model from Lagrangian

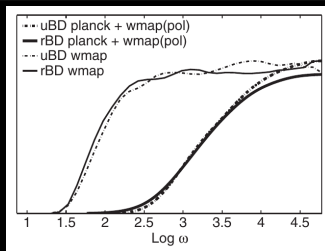
$$\underbrace{\sqrt{-g} \left\{ \frac{1}{16\pi G} R[g_{\mu\nu}] + \mathcal{L}_m \right\}}_{\text{Theory (Lagrangian)}} \rightarrow \underbrace{G_{\mu\nu} = 8\pi G T_{\mu\nu}}_{\text{Equations}} \rightarrow \underbrace{H(t), P(k, t)}_{\text{Solutions}}$$

- Specific, self-consistent
- Variable freedom: parameters + ICs \rightarrow several free functions
- Fully predictive: expansion history, N-body, GWs...

Example: Brans-Dicke

$$\mathcal{L} \propto \phi R - 2\Lambda - \frac{\omega}{\phi} (\partial\phi)^2$$

(Avilez & Skordis '14)



Test gravity: 3 approaches

- Model from Lagrangian

2 - Parameterize the solutions

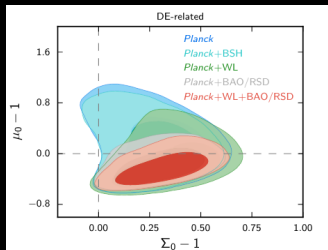
$$\nabla^2 \Psi = 4\pi G a^2 \mu(a, k) \rho \Delta, \quad \nabla^2 (\Phi + \Psi) = 8\pi G a^2 \Sigma(a, k) \rho \Delta$$

- Fully general
- Vast functional freedom: 2 functions of 2 variables
- Only linear regime, no expansion history

Example: (Planck '15 DE paper)

$$\mu = 1 + \mu_0 \Omega_{de}(a)$$

$$\Sigma = 1 + \Sigma_0 \Omega_{de}(a)$$



Test gravity: 3 approaches

- Model from Lagrangian
- Parameterize the solutions

3 - Effective theory approaches

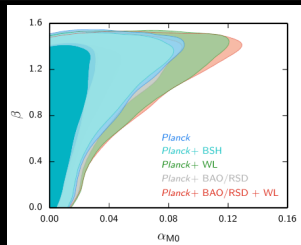
$$\mathcal{L} = \sum_i \alpha_i(t) \mathcal{O}_i$$

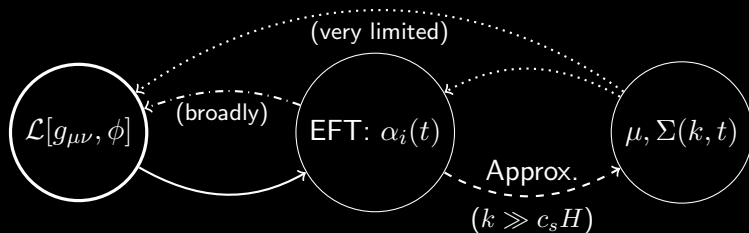
- Rather general: locality, covariance, d.o.f. # & type
- Large functional freedom $\mathcal{O}(\text{few})$ functions, 1 variable
- Limited info from other regimes GWs (no expansion history)

Example: (Planck '15 DE paper)

$$\alpha_M = \frac{a}{\Omega + 1} \frac{d\Omega}{da} = -\alpha_B$$

$$\Omega(a) = \exp \left[\frac{\alpha_{M,0}}{\beta} a^\beta \right] - 1$$





Outline:

- Tools for general theories (Horndeski in CLASS)
- EFT & Lagrangian models (Galileon)
- GW constraints on Lagrangians and EFT

$$\underbrace{\ddot{h}_{ij} + 3H(1 + \alpha_M)\dot{h}_{ij}}_{\delta(\sqrt{-g}M_*^2\dot{h}_{ij}^2)} + \underbrace{(1 + \alpha_T)k^2 h_{ij}}_{c_T^2, \text{ GW}} = 0 \quad (\text{tensors})$$

$$\underbrace{\alpha_K}_{\text{diagonal}} \delta\ddot{\phi} + 3H \underbrace{\alpha_B}_{\text{mixing}} \ddot{\Phi} + \underbrace{(\dots)}_{\alpha_K, \alpha_B, \alpha_M, \alpha_T} = 0 \quad (\text{scalar field})$$

Theory-specific relations:

$$G_2 - G_3\Box\phi + G_4R + G_{4,X}[\nabla\nabla\phi]^2 + G_5G_{\mu\nu}\phi^{;\mu\nu} - \frac{G_{5,X}}{6}[\nabla\nabla\phi]^3$$

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Kineticity: α_K

Standard kinetic term $\rightarrow c_S^2$

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M_p running: α_M

Variation rate of effective M_p

Horndeski in EFT language

(Bellini & Sawicki '14)

$$\underbrace{\ddot{h}_{ij} + 3H(1 + \alpha_M)\dot{h}_{ij}}_{\delta(\sqrt{-g}M_*^2\dot{h}_{ij}^2)} + \underbrace{(1 + \alpha_T)k^2 h_{ij}}_{c_T^2, \text{ GW}} = 0 \quad (\text{tensors})$$

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Variation rate of effective M_p

Tensor speed excess: α_T

GW at $c_g^2 = 1 + \alpha_T$

Horndeski in the Cosmic Linear Anisotropy Solving System

Goals: $\left\{ \begin{array}{l} \star \text{ DE/MG predictions in as much detail as } \Lambda\text{CDM} \\ \star \text{ public tool, valid for a large class of theories} \end{array} \right.$

hi_class

- Flexibility:
 - ★ New models trivially added
 - ★ Compatible massive ν 's...
- Accuracy:
 - ★ Full linear dynamics + ICs
 - ★ Tested to $\mathcal{O}(0.1\%)$ (Bellini+ '17)
- Speed: MCMC-able

www.hiclass-code.net (MZ, Bellini, Sawicki, Lesgourgues, Ferreira '16)

hi_class in practice

$$\left. \begin{array}{l} G_2, G_3, G_4, G_5 \\ \phi(t_0), \dot{\phi}(t_0) \end{array} \right\} \longrightarrow \left\{ \begin{array}{l} \text{Kineticity } \alpha_K \\ \text{Braiding } \alpha_B \\ M_p \text{ running } \alpha_M \\ \text{Tensor speed } \alpha_T \end{array} \right\} \longrightarrow \left\{ \begin{array}{l} D_A(z) \\ C_\ell \\ P(k) \\ \dots \end{array} \right.$$

a) Full theory + IC

b) or Parameterize $w(z), \alpha_i(z)$

Analogy with Quintessence

$$\underline{V(\phi), \phi_0, \dot{\phi}_0} \quad \text{vs} \quad \underline{w(z)}$$

EFT tests

(Alonso, Bellini, Ferreira, MZ '16)

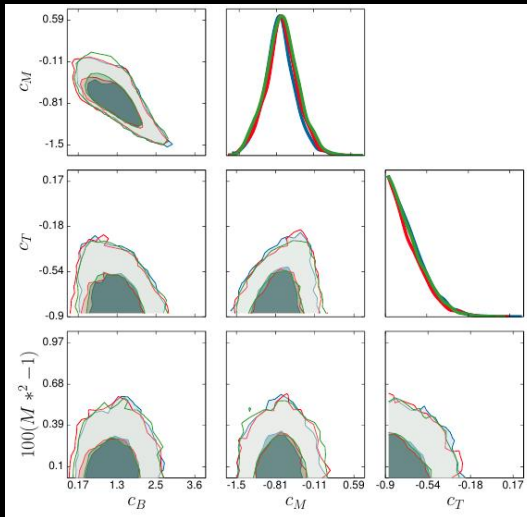
Horndeski parameterization

- $\alpha_i = c_i \cdot \Omega_{de}(z)$

- Current: (Bellini+ '15)

$$\Delta c_i \sim 1$$

CMB+BAO+RSD+PK



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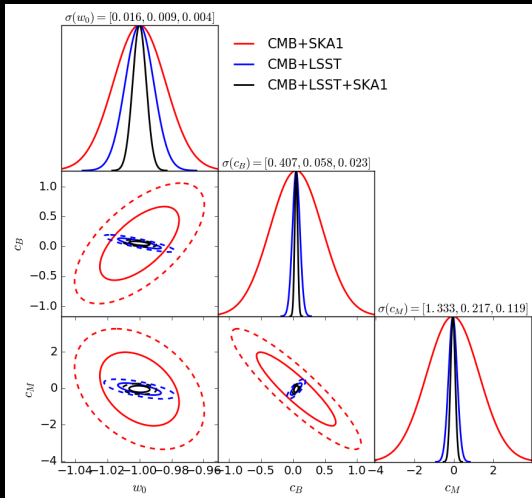
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- Future:

$$\Delta c_i \sim 0.1 - 0.01$$

- CMB S4 + SKA
- + LSST
- + DESI-like



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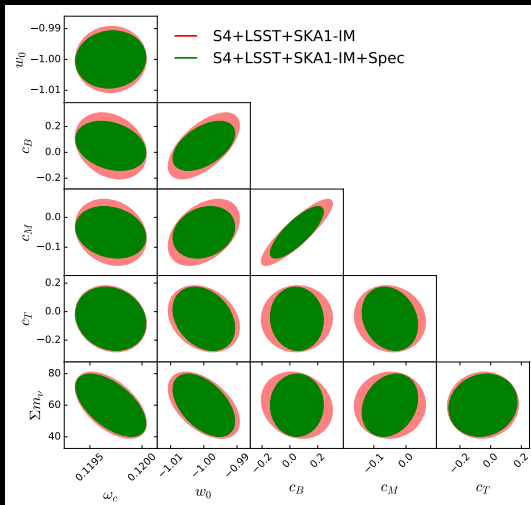
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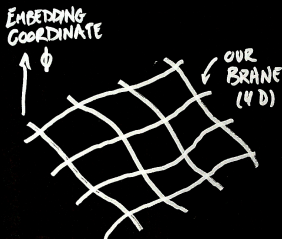
Horndeski's Theory, with

$\boxed{\Lambda = 0}$ + simple choice of $G_i(X, \phi)$: (recall: $X \equiv -(\partial\phi)^2/2$)

$$\begin{aligned} & \frac{M_p^2}{2} R - \textcolor{brown}{X} - c_3 \frac{X}{M^3} \nabla^2 \phi && \rightarrow \text{Gal3 (cubic): 0 extra params} \\ & + c_4 \frac{X^2}{M^6} \left(\frac{M_p^2}{2} R + \frac{2}{X} [\nabla \nabla \phi]^2 \right) && \rightarrow \text{Gal4 (quartic): 1 extra params} \\ & + c_5 \frac{X^2}{M^9} \left(G_{\mu\nu} \phi^{;\mu\nu} - \frac{1}{3X} [\nabla \nabla \phi]^3 \right) && \rightarrow \text{Gal5 (quintic): 2 extra params} \end{aligned}$$

- Related to
 - ★ Massive Gravity: $\phi \rightarrow$ helicity 0
 - ★ DGP/extra dim: $\phi \leftrightarrow x^5$ coord.
- Screening: $\Rightarrow \sim$ GR on small scales
- Self-accelerating solutions ($\Lambda = 0$)

(de Felice Tsujikawa '10, Barreira+ '14)



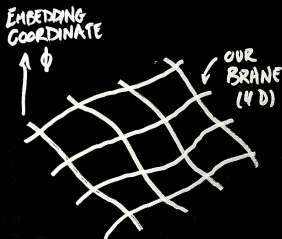
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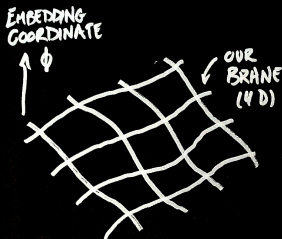
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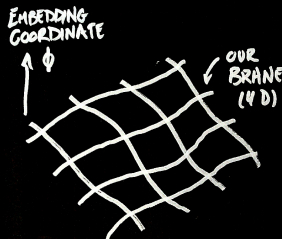
→ Gal4 (quartic): 1 extra params

$$+ c_5 \frac{X^2}{M^9} \left(G_{\mu\nu} \phi^{;\mu\nu} - \frac{1}{3X} [\nabla \nabla \phi]^3 \right)$$

→ Gal5 (quintic): 2 extra params

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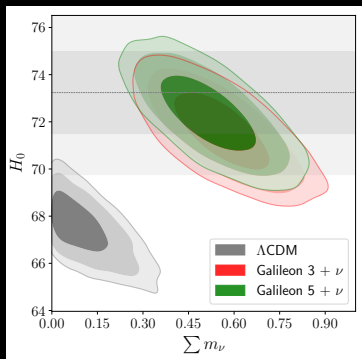
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$\Lambda = 0$ Galileon Gravity

(Barreira+ '14, Renk+ 17')

Planck+BAO:

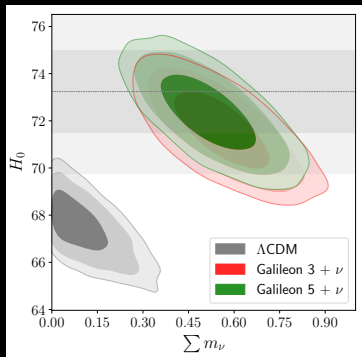


- H_0 compatible (Λ CDM 3.4σ !)
- if $\Sigma m_\nu \approx 0.6$ eV
- slight tension with other data

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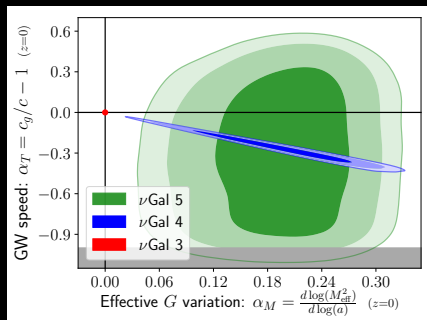
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→ modifies GW propagation

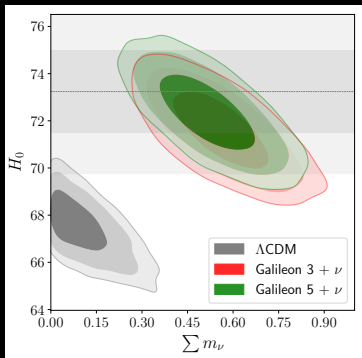
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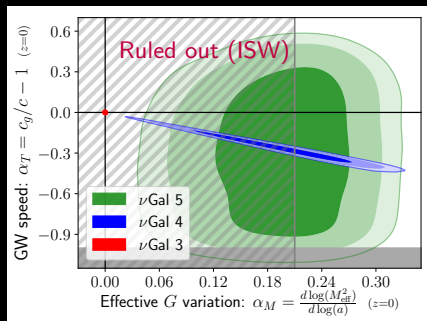
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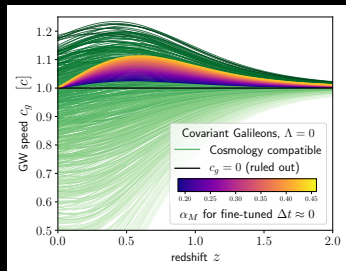
- ISW effect (from Planck \times WISE):
→ kills ν Gal3 (8.2σ)
→ non-standard GW propagation

No way out from GWs

GWs on FRW (Bellini+Sawicki, Gleyzes+ '14)

$$\ddot{h}_{ij} + \underbrace{(1 + \alpha_T)}_{c_g^2 \neq c} k^2 h_{ij} + \dots = 0$$

- Can't fine tune $\alpha_T(z) = 0$
- Galileons with $\alpha_T(z) = 0$
 \Rightarrow ruled out by ISW (8.2 σ)



No waveform distortion \Rightarrow need EM counterpart

$$\omega^2 = (1 + \alpha_T) k^2$$

Massive gravity tested with GW alone: $m_g < 7.7 \cdot 10^{-23} \text{ eV}$ (GW170104)

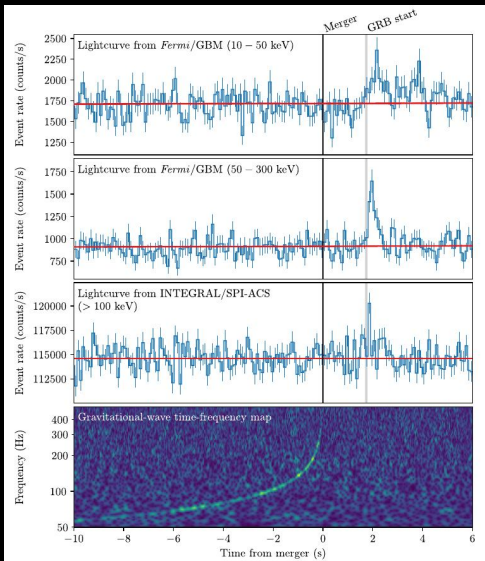
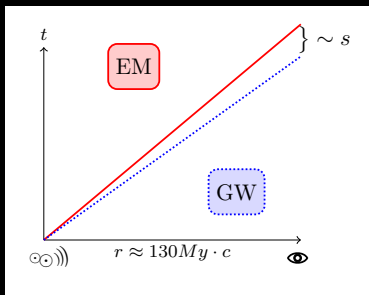
$$\ddot{h}_{ij} + k^2 h_{ij} + m_g^2 h_{ij} = 0 \quad \Rightarrow \quad \omega \approx k + \frac{m_g^2}{2k^2}$$

GW 170817 + GRB 170817A

(LIGO+Fermi+... 1710.05834)

Very precise bound :

$$-3 \cdot 10^{-15} \leq \alpha_T \leq 6 \cdot 10^{-16}$$



Conditions for variable c_g

(Bettoni, Ezquiaga, Hinterbichler & MZ '16)

Operationally: $\ddot{h}_{ij} + c_g^2 \vec{\nabla}^2 h_{ij} + \dots = 0$

GW effective metric - any background, $k^2 \gg |R_{\mu\nu}|$

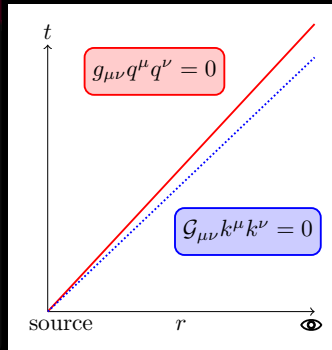
$$\text{GW eq} \propto \underbrace{(C\Box + \mathcal{D}_{\mu\nu}\partial^\mu\partial^\nu)}_{\mathcal{G}_{\mu\nu}\partial^\mu\partial^\nu} h_{ij}$$

1) Non-trivial $\phi(x)$ configuration (any)

Dark Energy $\rightarrow \dot{\phi} \sim H_0$

2) ϕ -derivatives coupled to curvature

Modified Gravity: $\mathcal{D}_{\mu\nu} \propto \partial_\mu\phi\partial_\nu\phi \dots$

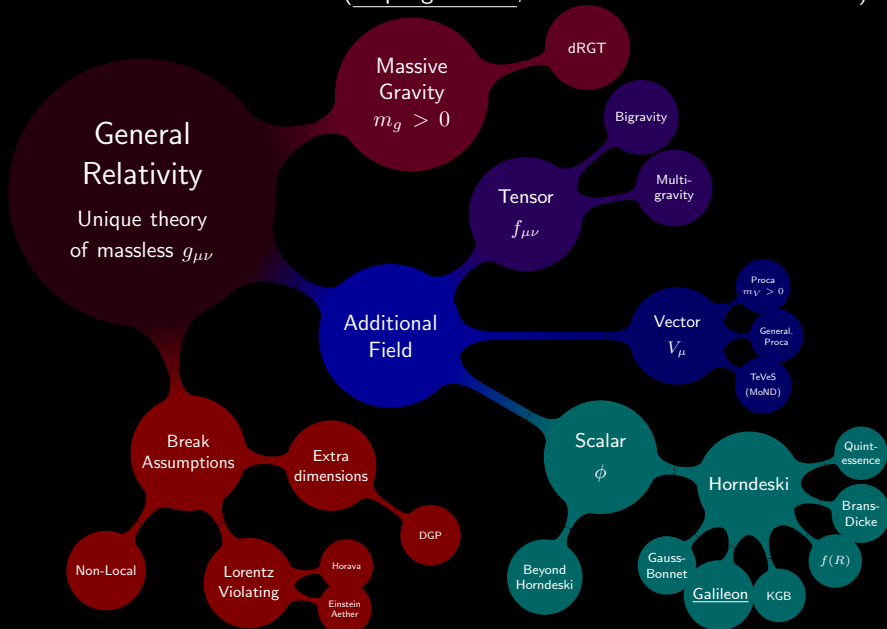


(1,2) $\Rightarrow \phi$ changes the effective medium in which GWs propagate.

(2) \Rightarrow binary classification of theories

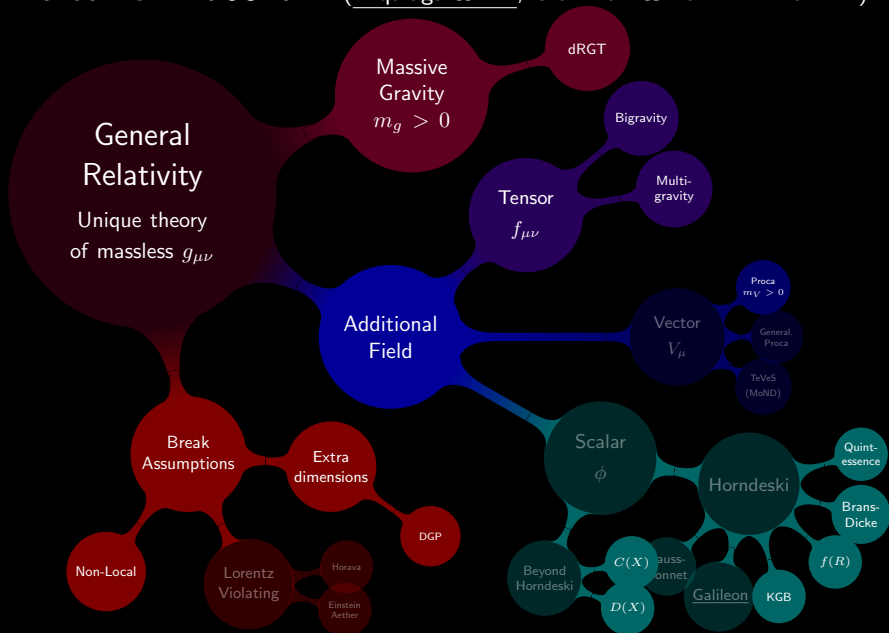
DE after GW170817

(Ezquiaga & MZ, Creminelli & Vernizzi - '17 PRL)

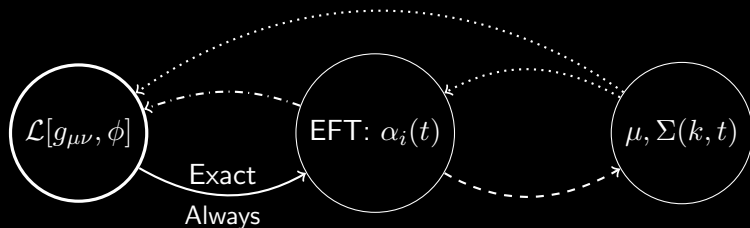


DE after GW170817

(Ezquiaga & MZ, Creminelli & Vernizzi - '17 PRL)



What's left after GW170817?



Lagrangian

- All Simple Ths.
- 2 special Ths.
- massive GR (?)

Effective Theory

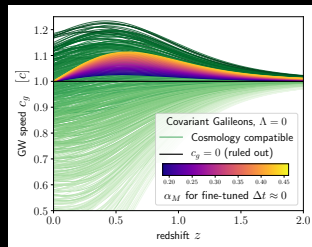
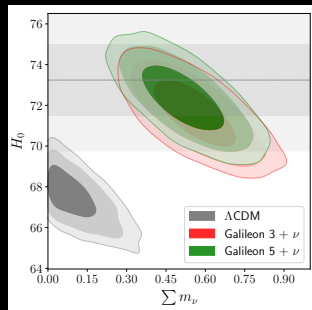
- $\alpha_T = 0$
- all other α 's free

Parameterization

- Everything goes!

Conclusions

- \exists Interesting Dark Energy models
 - ★ Λ CDM tensions?
 - ★ Very predictive!
- GW propagation \rightarrow critical test of gravity
 - ★ either $c_g = c$ or not
 - ★ Dead ends after GW170817
- Complementarity is essential:
 - ★ Cosmology
 - Cross-correlations (e.g. ISW)
 - ★ Gravitational Waves
- Need to go back to our blackboards...



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hi_class

[illegible]

Miguel Zumalacárregui (Berkeley)

GW & cosmo tests of Gravity and Dark Energy

Scalar-Tensor gravity

- ★ Old-School: $\frac{f(\phi)R}{16\pi G} - \frac{1}{2}(\partial\phi)^2 - V(\phi) \supset$ Quintessence/Inflation,
 \supset Brans-Dicke, $f(R)$ (Jordan '59, Brans & Dicke '61)

★ Horndeski's Theory (1974)

$g_{\mu\nu} + \boxed{\phi} + \text{Local} + 4\text{-D} + \text{Lorentz theory with } \boxed{2^{nd} \text{ order Eqs.}}$

4× functions $G_i(X, \phi)$ of ϕ , $X \equiv -(\partial\phi)^2/2$

$$\mathcal{L}_H = G_2 - G_3 \nabla^2 \phi + G_4 R + G_{4,X} [\nabla \nabla \phi]^2 + G_5 G_{\mu\nu} \phi^{;\mu\nu} - \frac{G_{5,X}}{6} [\nabla \nabla \phi]^3$$

- \supset GR, quint/k-essence, Brans-Dicke, $f(R)$, chameleons...
kinetic gravity braiding, covariant Galileon, Gauss-Bonnet...

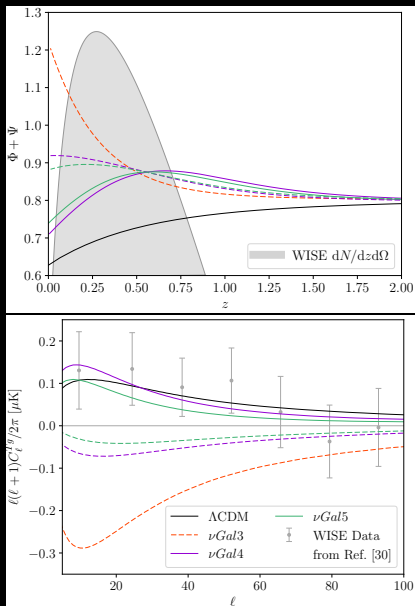
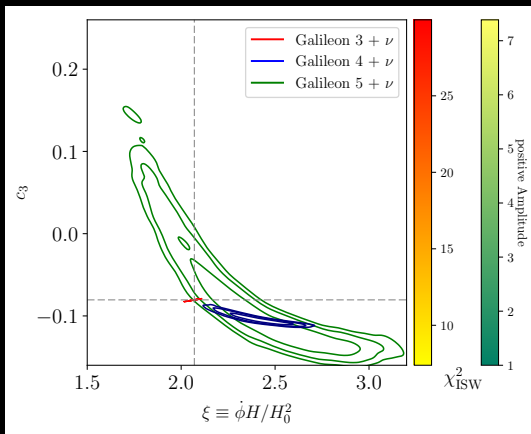
★ Beyond Horndeski \rightarrow *discovered by accident!*

(MZ & Garcia-Bellido '13, Gleyzes et al. '14, Langlois & Noui '15)

Galileon and Integrated Sachs-Wolfe effect

(Renk+ '17)

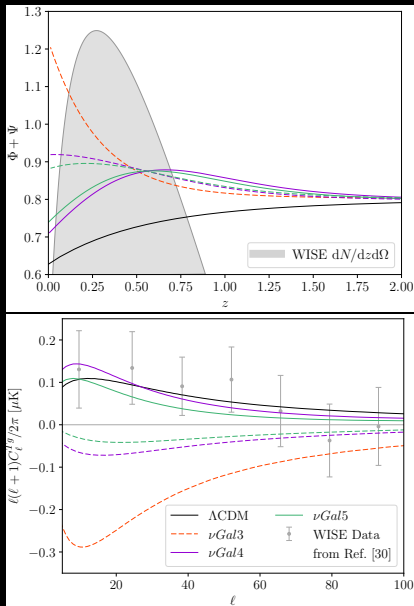
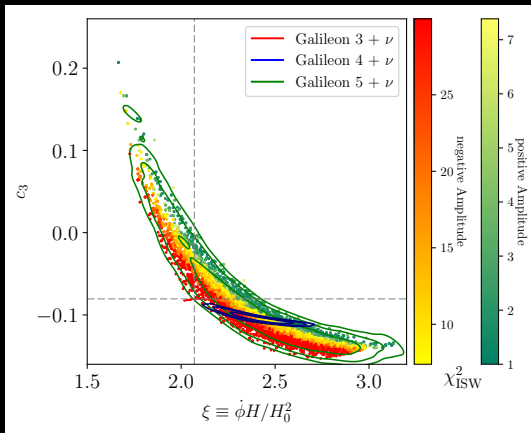
$$\Delta_{\ell}^{\text{ISW}} = \int_{\tau_*}^{\tau_0} d\tau (\Phi' + \Psi') j_{\ell}$$



Galileon and Integrated Sachs-Wolfe effect

(Renk+ '17)

$$\Delta_{\ell}^{\text{ISW}} = \int_{\tau_*}^{\tau_0} d\tau (\Phi' + \Psi') j_{\ell}$$

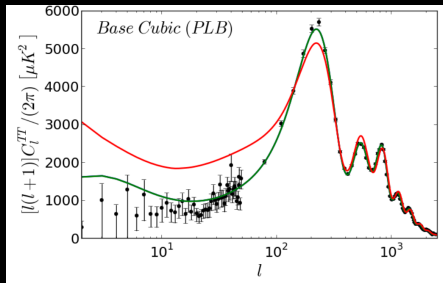
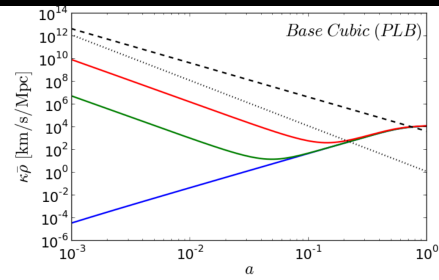


Galileon: Tracker solution

(Barreira+ '14)

$$\text{Symmetry } \phi \rightarrow \phi + C \quad \Rightarrow \text{conserved } \mathcal{J}^\mu \quad \Rightarrow \mathcal{J}^0 \propto a^{-3} \rightarrow 0$$

$$\dot{\phi}(t)H(t) = \xi \cdot H_0^2 M_P = \text{constant}$$



- Evolution to tracker: no fine tuning
- Tracker by $z_T \sim \infty$, $z_T \approx 6$,
 $z_T \approx 2.5$ (Ω_{de} small but relevant)
- Inviability if out of tracker late (i.e. while Ω_{de} significant)
- Indistinguishable if reached earlier