https://github.com/migueserna/ModeladoySimulacion/blob/main/Simulaciones/Ejercicios_capitulo_3_MASG.ipynb

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taller#2 Bifurcaciones

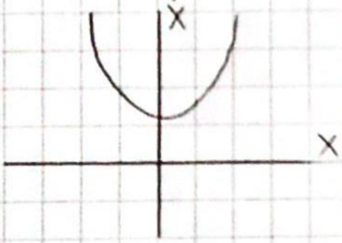
3.1 saddle-node-Bifurcation

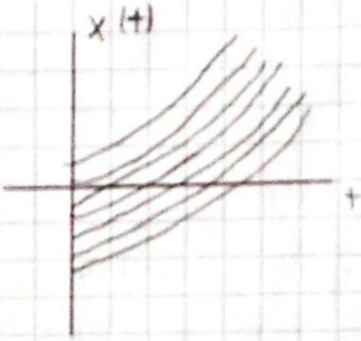
Para cada uno de los siguientes ejercicios, esboza todos los campos vectoriales cualitativamente diferentes que se producen al variar r. Demuestre que se produce valor crítico de r, a determinar por ultimo diagrama de bifurcacion de puntos fijos x^* frente a r

3.1.1 $\dot{x} = 1 + rx + x^2$

para $r = 0$:

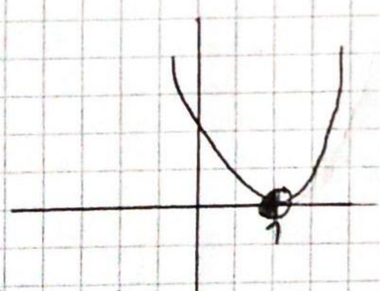
$\dot{x} = 1 + x^2$ → No hay equilibrio

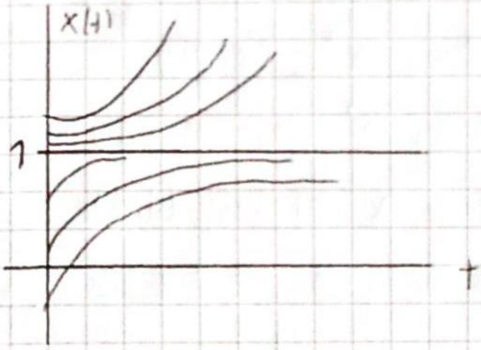




Para $r = -2$

$\dot{x} = x^2 - 2x + 1$ – Pto, semiestable

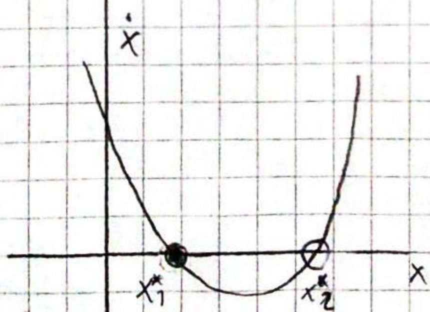


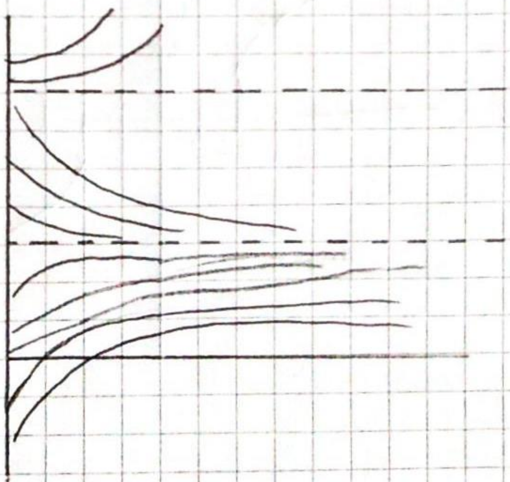


Para $r < -2$

$\dot{x} = x^2 + rx + 1$ → $x^2 + rx + 1 = 0$

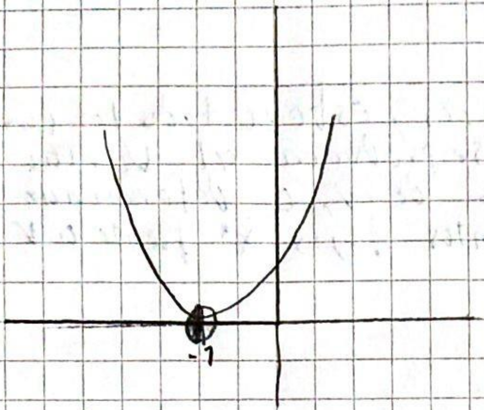
$x^* = -\frac{r}{2} \pm \frac{1}{2}\sqrt{r^2 - 4}$ → Punto estable
Punto inestable

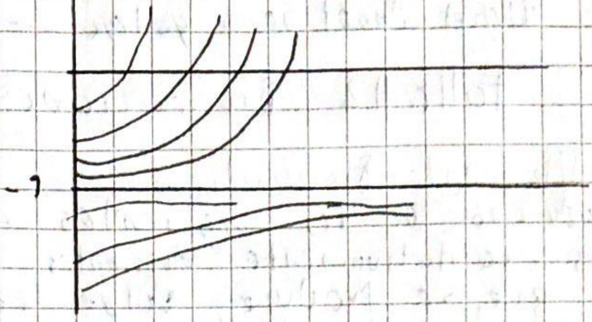




Para $v = 2$

$x^* = 1 \rightarrow$ semiestable





en resumen

$v = \pm 2 \rightarrow$ semi estable

$|v| > 2 \rightarrow$ uno estable y uno inestable

$|v| < 2 \rightarrow$ no hay punto de equilibrio

$\dot{x} = x^2 + vx + 1 \rightarrow x^* = -\frac{v}{2} \pm \frac{1}{2}\sqrt{v^2 - 4}$

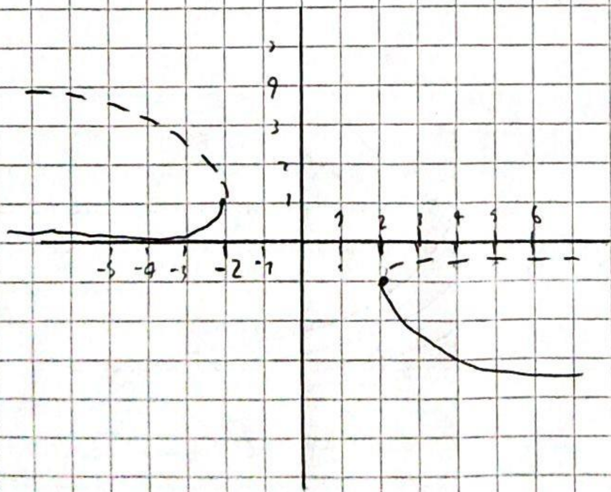
Si $|v| < 2$, $x^* =$ complejo

Si $|v| = 2$, x^* es una solución real única

Si $|v| > 2$, x^* es real doble

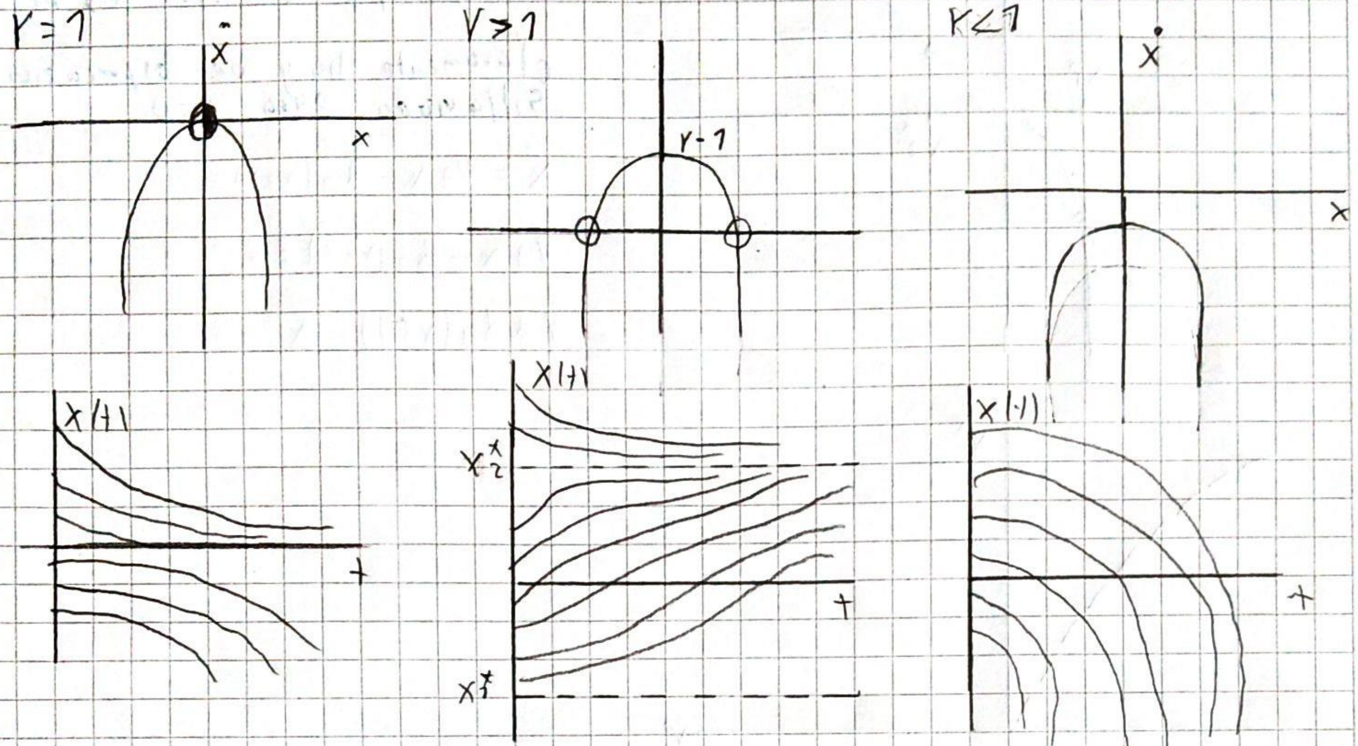
Hay bifurcación en $v = \pm 2$

$x^* = -\frac{1}{2} \pm \frac{1}{2}\sqrt{v^2 - 4}$



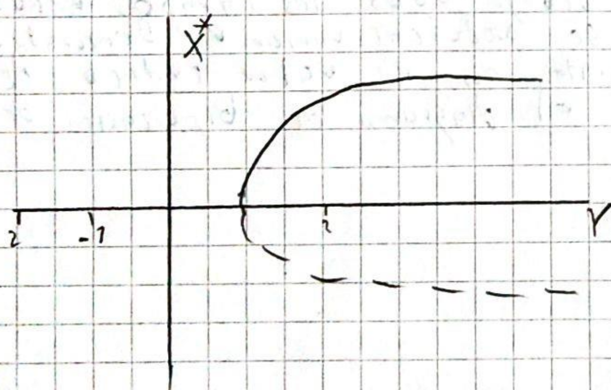
3.7.2

$\dot{x} = r - \cosh x$



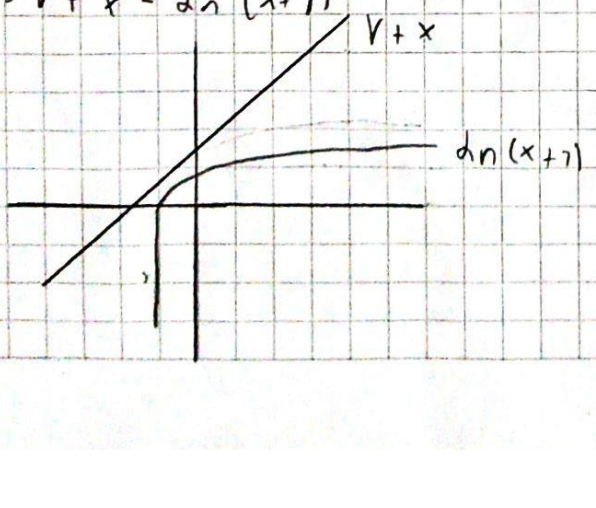
en r=1 ocurre una bifurcacion silla nudo

$r - \cosh(x) = 0 \Rightarrow r = \cosh x$

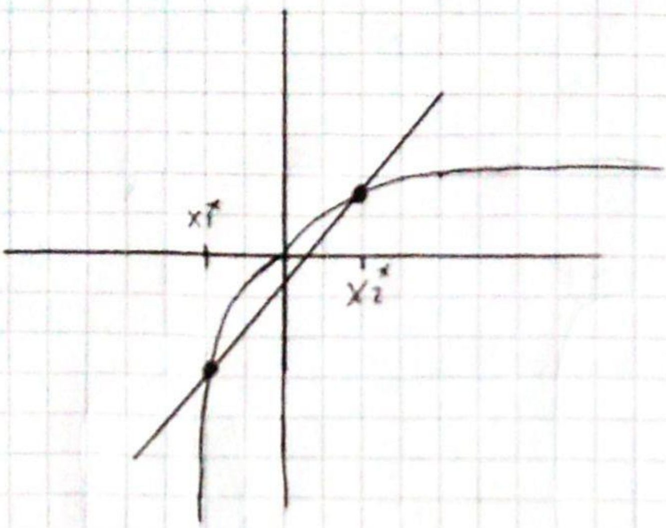


3.7.3

$\dot{x} = r + x - \ln(x+1)$



Puede verse que para $r>0$ no hay ningun punto de equilibrio, cuando $r=0$ hay un unico punto de equilibrio en $x=0$ y cuando $r<0$ existen 2 puntos de equilibrio



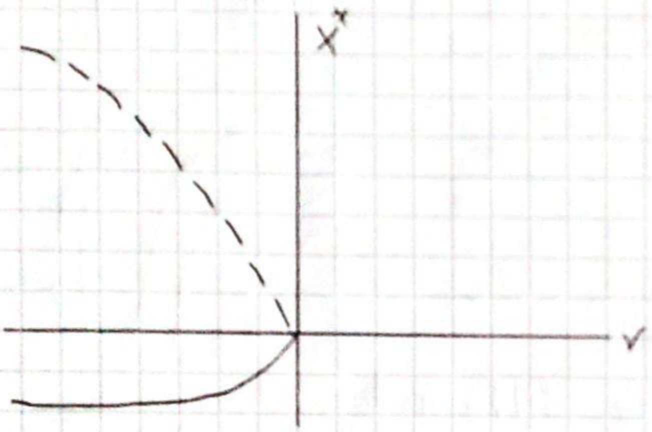
el punto de equilibrio menor a cero x_1^* es estable en cambio que el otro es inestable

claramente hay una bifurcacion sillanodo para $v=0$

$\dot{x} = v + x - \ln(x+1)$

$v + x - \ln(x+1) = 0$

$v = \ln(x+1) - x$



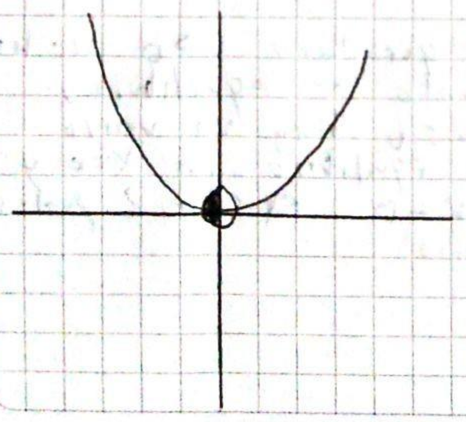
3.2. Bifurcacion transcritica

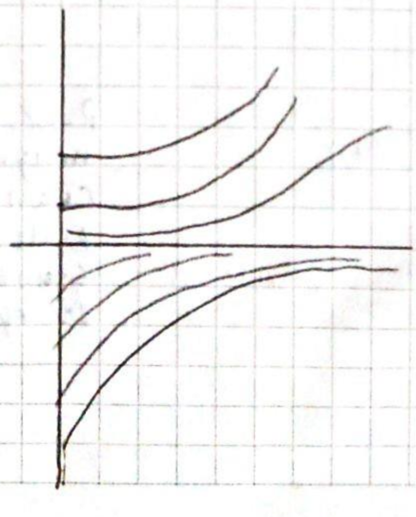
Para cada uno de los siguientes dibuje todos los campos vectoriales cualitativamente diferente que se producen al variar v, Demuestre que se produce una bifurcacion transcrita en un valor critico de v a determinar por ultimo dibuje el diagrama de bifurcacion de puntos fijos x^* frente a v

3.2.1

$\dot{x} = vx + x^2$

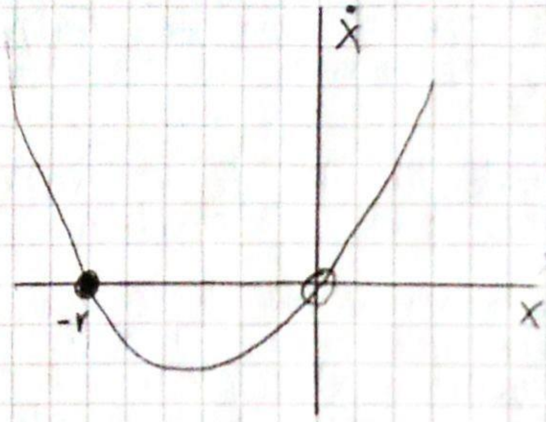
$v = 0 \quad \dot{x} = x^2$

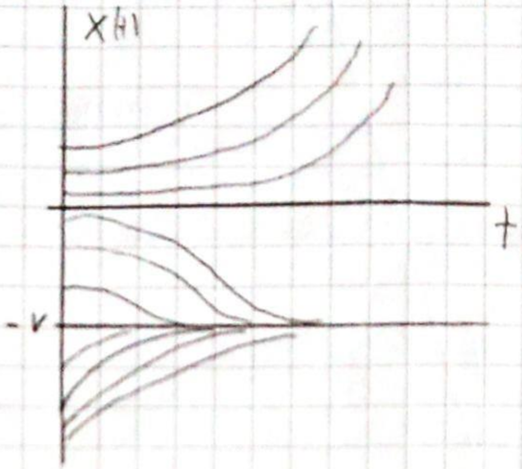




$r > 0$

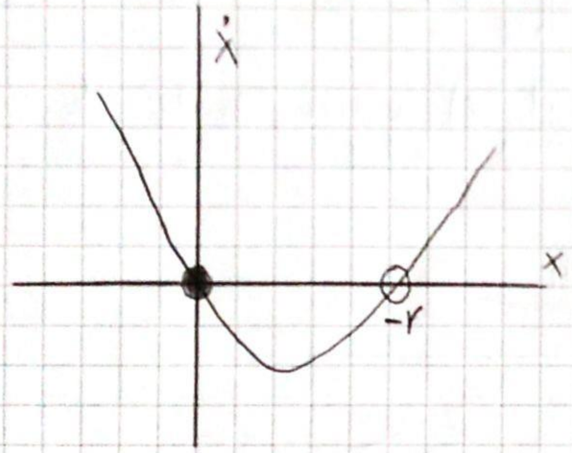
$\dot{x} = rx + x^2 \rightarrow rx + x^2 = 0 \rightarrow x(r+x) = 0 \quad x_1^* = 0 \;,\; x_2^* = -r$

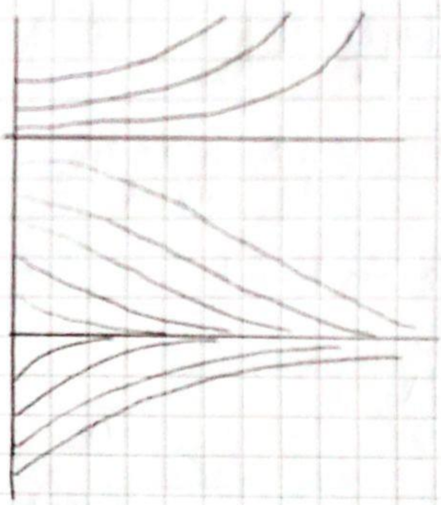




$r < 0$

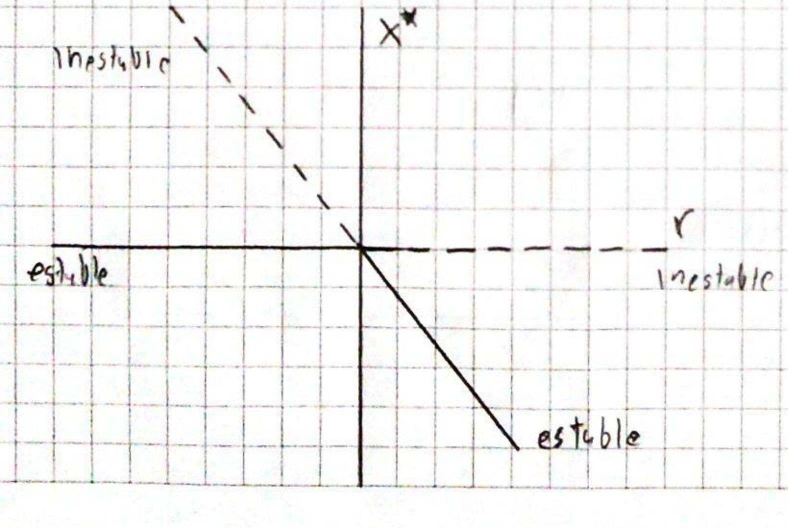
$x_1^* = 0 \;,\; x_2^* = -r \;,\; r < 0$





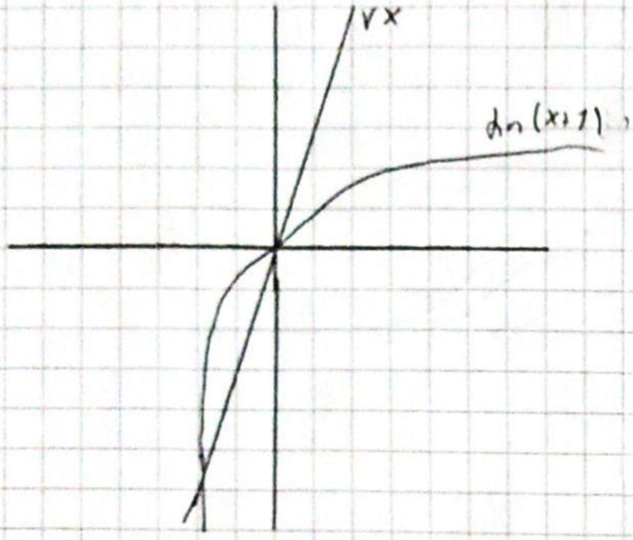
$x^* = 0$ es un punto de equilibrio para todos los valores a r. Sin embargo su estabilidad cambia:

$r < 0 \rightarrow x^* = 0$ estable, $r = 0 \rightarrow x^* = 0$ semiestable, $r > 0 \rightarrow x^* = 0$ inestable



3.2.2

$\dot{x} = rx - \ln(x+1)$



es evidente que para cualquier r hay un punto de equilibrio $x^* = 0$

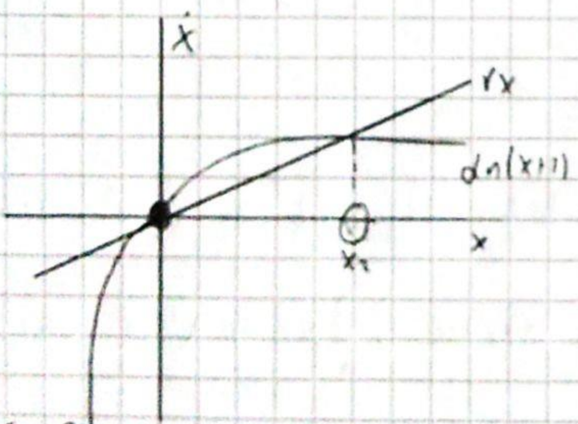
Depende del valor de r existe otro punto de equilibrio

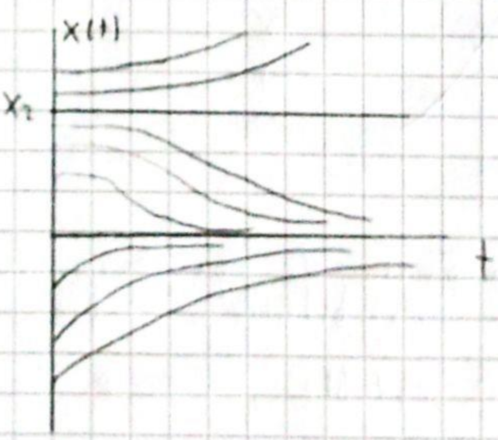
existe un valor de r que hace que exista el punto de equilibrio $x^* = 0$, este se da cuenta la recta rx es tangente a la curva ln(x+1)

$r_{cr} = \frac{d(\ln(x+1))}{dx}\Big|_{x=0} \to R_{cr} = \frac{1}{x+1}\Big|_{x=0} = 1 \to R_{cr} = 1$

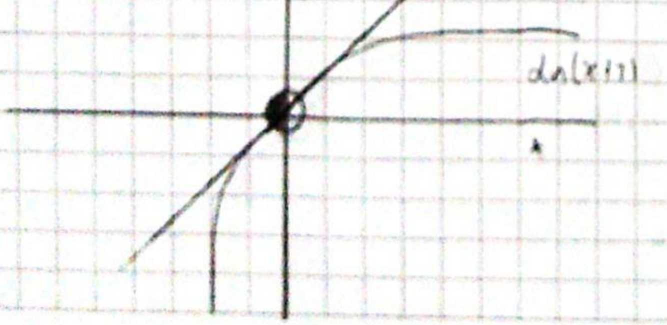
existe otro $r < 1$ a partir del cual el unico punto de equilibrio es $x = 0$ este queria por determina r ?

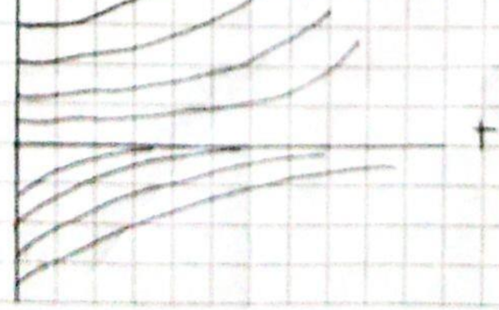
Para $r_{cr} < r < 1$





$r = 1$

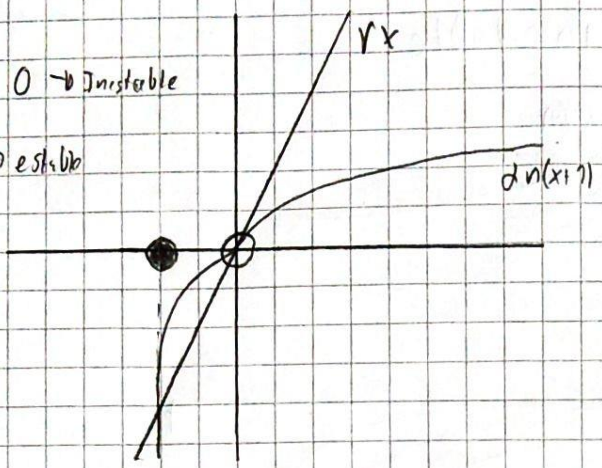


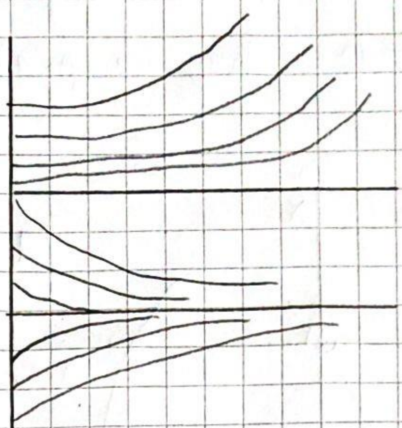


$r > 1$

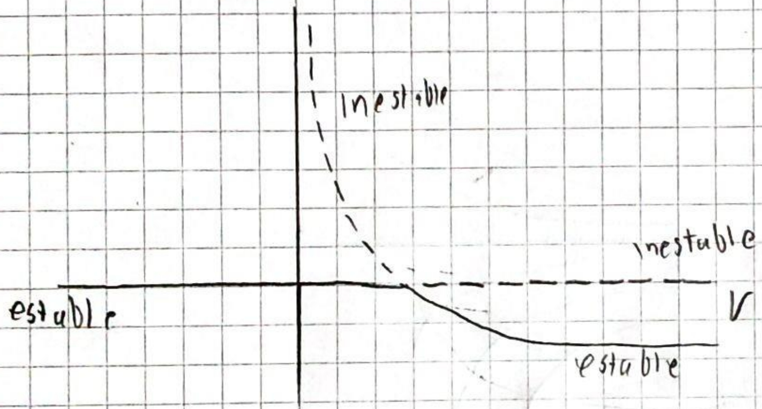
$x_1^* = 0 \rightarrow$ Inestable

$x_2^* \rightarrow$ estable





$x^* = 0$ es : inestable $r > 1$, Semiestable $r = 1$, estable $r < 1$

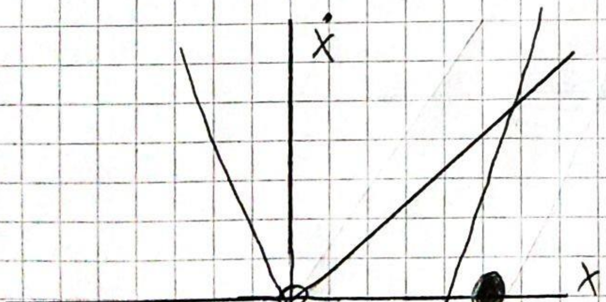


$rx = \ln(x+1)$

$r = \frac{\ln(x+1)}{x}$

3.2.3

$\dot{x} = x - rx(1-x)$



$x^* = 0$ siempre es un punto de equilibrio para cualquier r

$\dot{x} = x - rx(1-x)$

$\dot{x} = x[1 - r(1-x)]$

$x[1 - r(1-x)] = 0 \rightarrow x^* = 0$

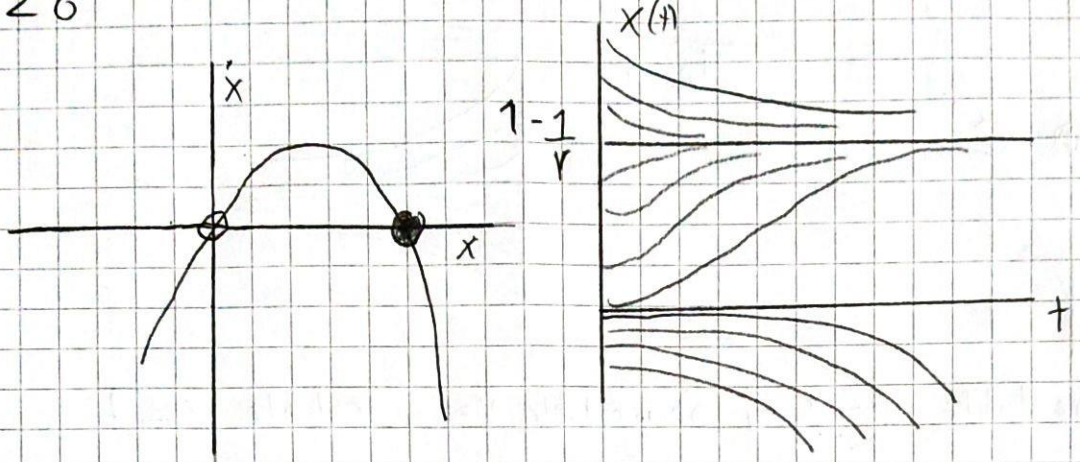
$1 - r(1-x) = 0 \rightarrow r(1-x) = 1 \rightarrow 1 - x = \frac{1}{r}$

$x^* = 1 - \frac{1}{r}$ → cuando $r = 1$ este punto de equilibrio se encuentra con el otro

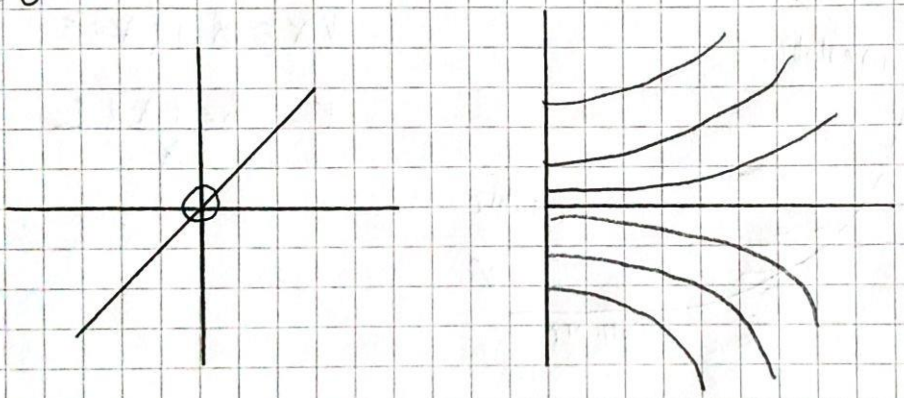
Para $r=1$ solo hay un punto de equilibrio Semiestable

Para $r=0$ hay un punto inestable

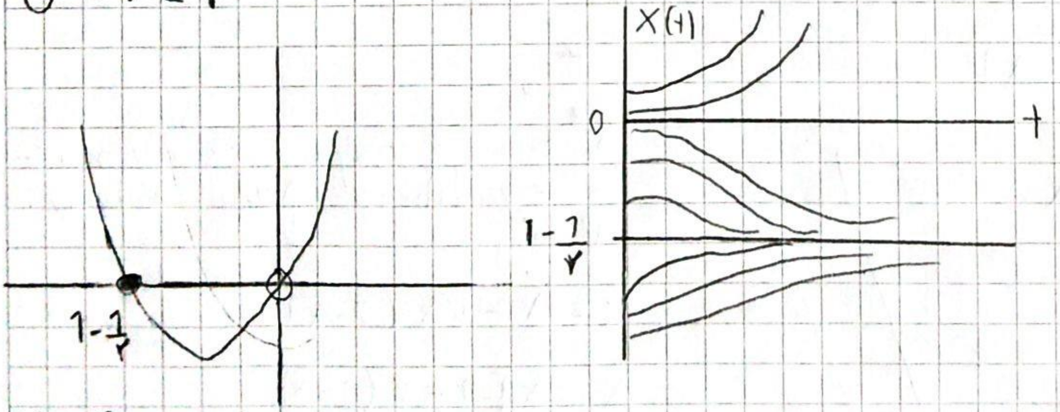
$r<0$



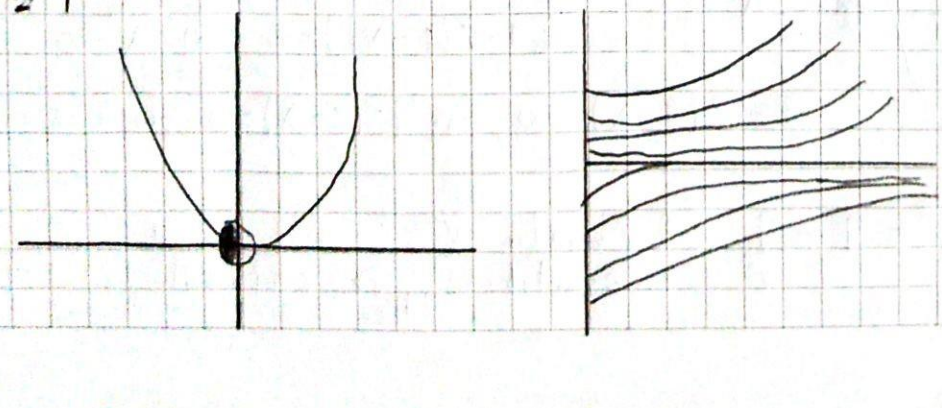
$r=0$



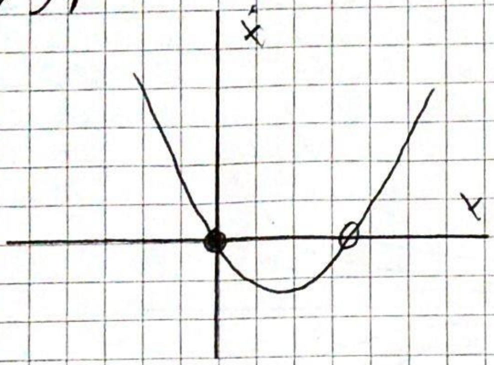
$0<r<1$

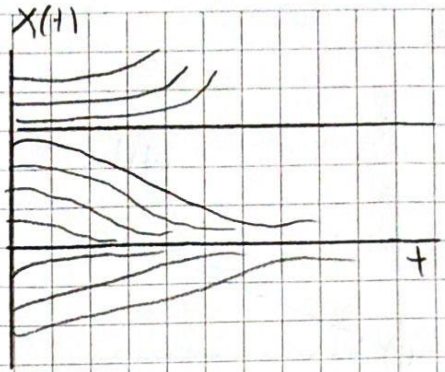


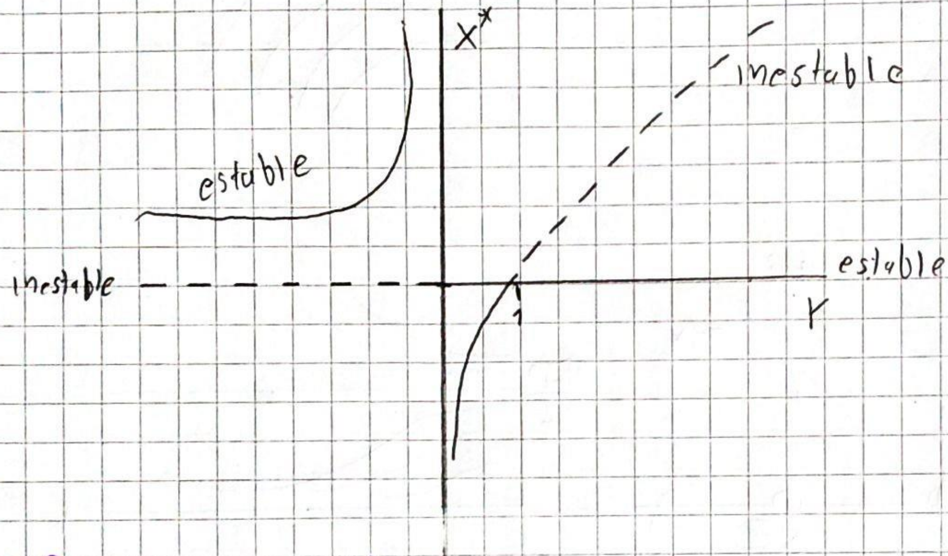
$r=1$



$v > 1$







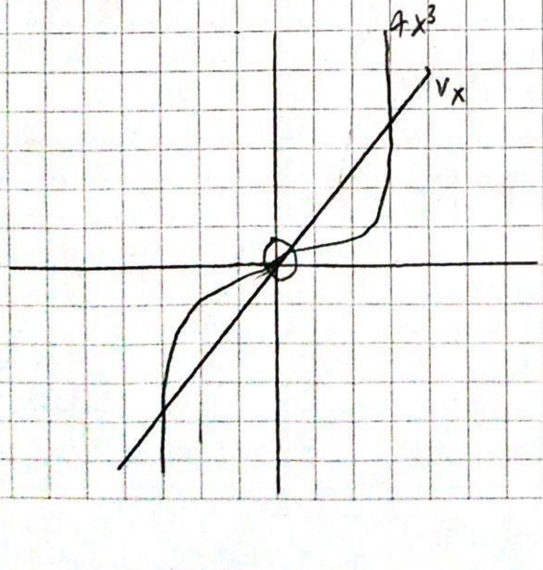
$x^* = 1 - \frac{1}{v}$

3.4 Bifurcacion en horquilla

In the following ejercicios, esboza todos los campos vectoriale cualitativamente diferente que se produce al variar v Demuestre que se produce una bifurcacion en horquilla en un valor critico de v (a determinar) y clasifique la bifurcacion como supercritica y subcritica. Por ultimo dibuje el diagrama de biparcacion de x^* frente a v

3.4.1

$\dot{x} = vx + 4x^3$



$x^* = 0 \to vx + 4x^3 = 0$

$x(v + 4x^2) = 0 \to x_1^* = 0$

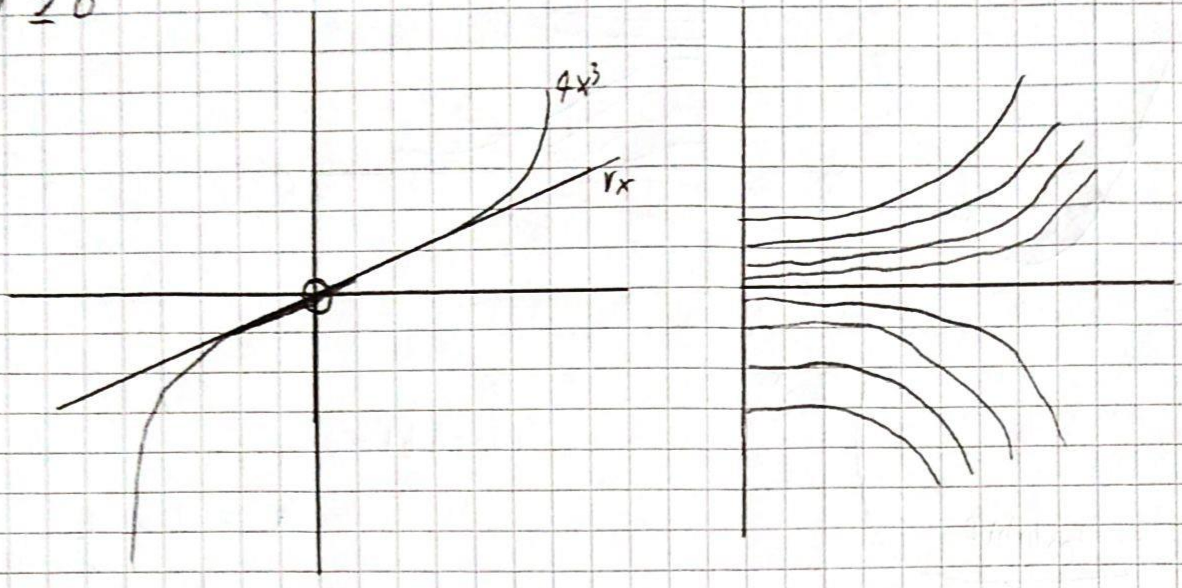
$v + 4x^2 = 0 \to 4x^2 = -v$

$x^2 = -\frac{v}{4}$ Para $v > 0$ no existen puntos de equilibrio

Si $v < 0$

$x^2 = -\frac{v}{4}$ $x = \frac{\sqrt{-v}}{4}$ $v < 0$

$r \geq 0$



$r < 0$

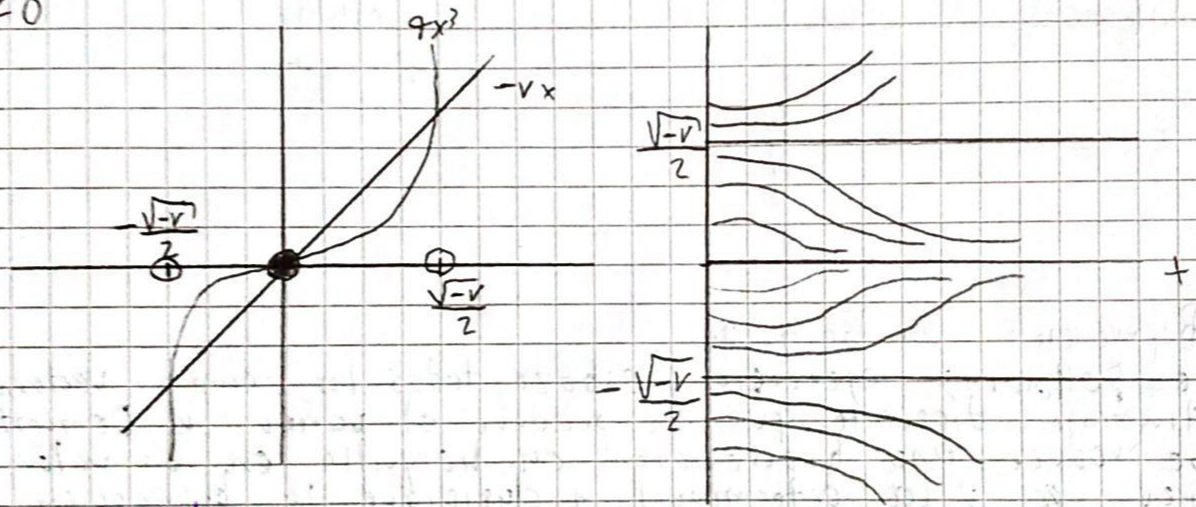
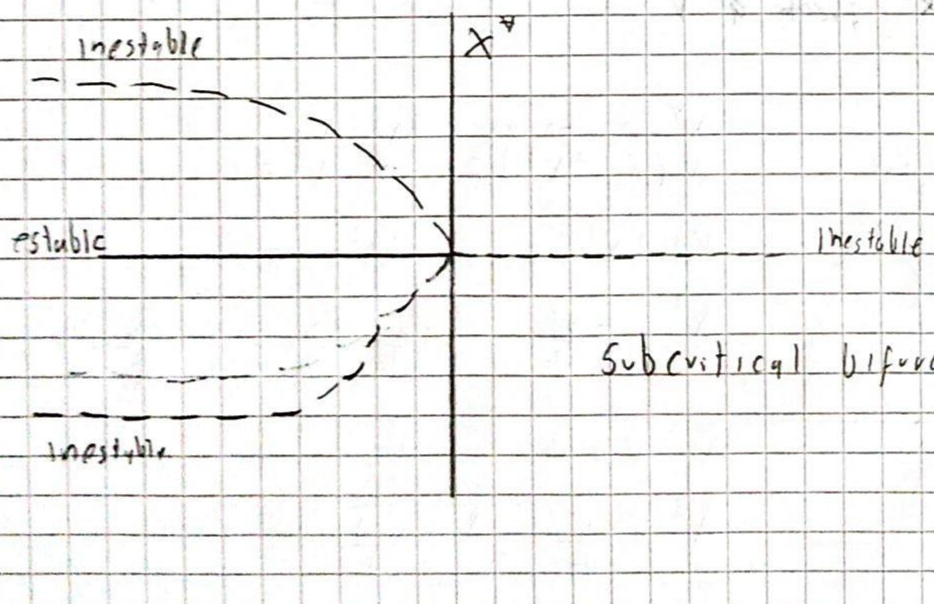


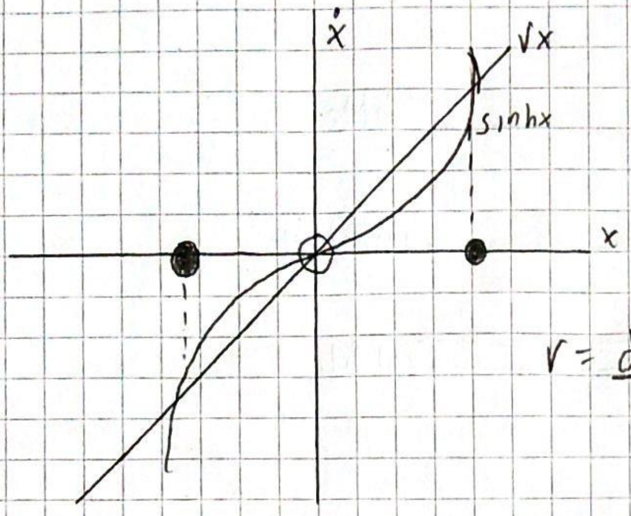
Diagrama de bifurcacion



Subcritical bifurcation

3.4.2

$\dot{x} = rx - \sinh x \to rx - \sinh x = 0 \to rx = \sinh x \to r = \frac{\sinh x}{x}$



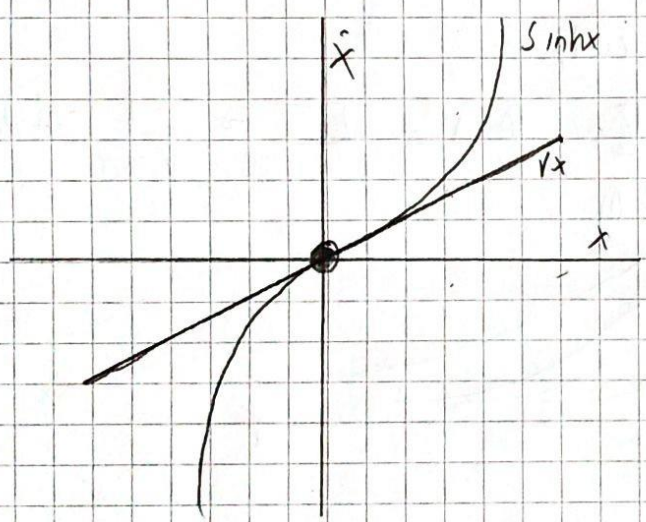
$x^* = 0 \to$ Punto de equilibrio para cualquier r su estabilidad varia

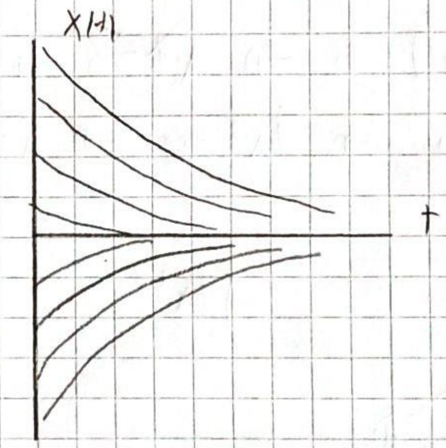
Cuando rx es tangente a sinh x en el origen es el punto critico en el cual aparecen los nuevos puntos de equilibrio

$r = \frac{d\sinh x}{dx}\Big|_{x=0} \to r = \cosh x\Big|_{x=0}$

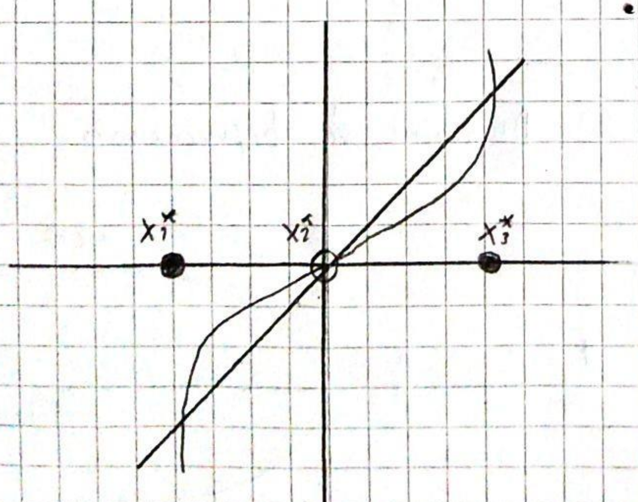
$r = 1$

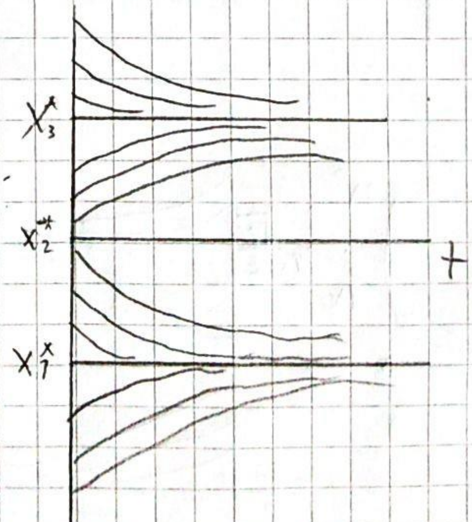
$r \leq 1$



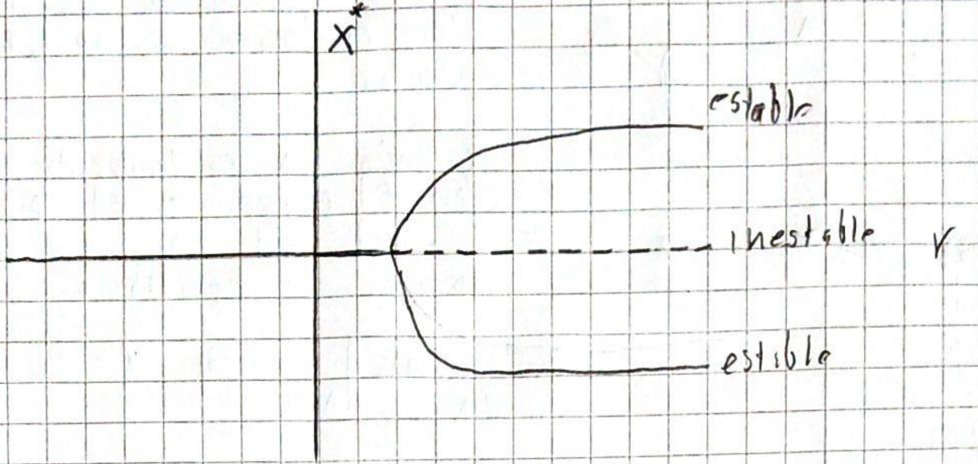


$r > 1$





$rx - \sinh x = 0 \rightarrow rx = \sinh x \rightarrow r = \frac{\sinh x}{x^*} \rightarrow$ Puntos de equilibrio implícita



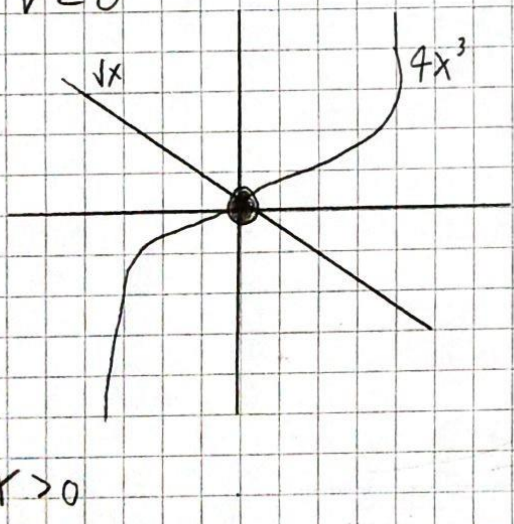
3.4.3

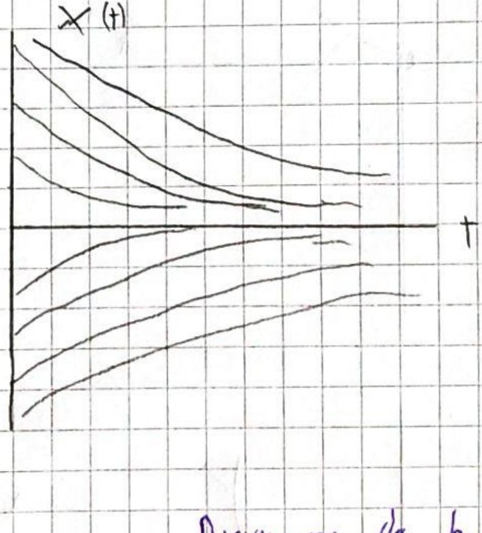
$\dot{x} = rx - 4x^3 \rightarrow \dot{x} = x(r - 4x^2)$

$x(r - 4x^2) = 0 \rightarrow x^* = 0 \rightarrow$ Para cualquier r

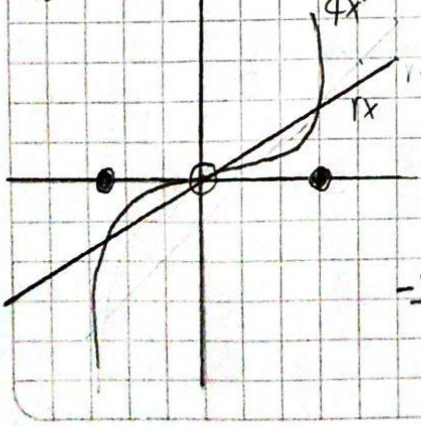
$r - x^2 = 0 \rightarrow 4x^2 = r \rightarrow x^2 = \frac{r}{4} \rightarrow x^* = \frac{\sqrt{r}}{2} \rightarrow$ solo existe $r > 0$

$r < 0$





$r > 0$



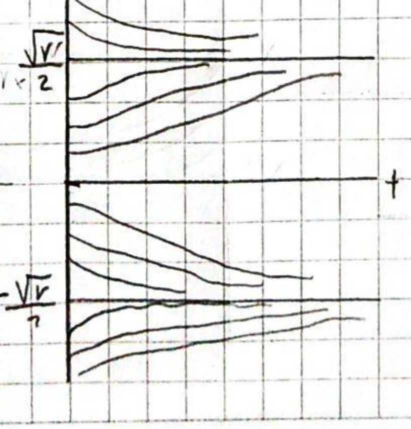


Diagrama de bifurcaciones