Trabalho de férias
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1- Mostra que:

a) $\frac{(3-\sqrt{12})^2}{(2+3\sqrt{3})} = \frac{87\sqrt{3}-150}{23}$

b) $\frac{(1+\sqrt{27})^2}{(1+5\sqrt{3})} = -\frac{59+73\sqrt{3}}{37}$

2- Determina J e M de modo a que obtenhas as raízes assinaladas e descobre a restante.

a) $x^3 + Jx^2 + Mx + 56$, raízes: 2 e 4

b) $x^3 - 7x^2 - Jx - M$, raízes: 5 e -1

3- Determina as 3 raízes dos seguintes polinómios:

a) $x^3 - x$

b) $x^3 - x^2$

c) $x^3 - 2x^2 + x$

4- Resolve em ordem a x

a) $-2 < 5x + 3 < 8$

b) $2x - \frac{4-x}{3} \leq 5x$

5- Representa em potência de base 5

$$\frac{5^{\frac{1}{6}} \times 5^{\frac{1}{2}}}{(\sqrt[3]{5})^{-2}}$$

1-

a) $\frac{(3-\sqrt{12})^2}{(2+3\sqrt{3})} = \frac{(3-\sqrt{12})(3-\sqrt{12})}{(2+3\sqrt{3})} = \frac{(9-6\sqrt{12}+12)}{(2+3\sqrt{3})} = \frac{(21-6\sqrt{12})}{(2+3\sqrt{3})} = \frac{(21-6\sqrt{12})(2-3\sqrt{3})}{(2+3\sqrt{3})(2-3\sqrt{3})} =$
$= \frac{42-63\sqrt{3}-12\sqrt{12}+18\sqrt{36}}{4-9\times3} = \frac{42-63\sqrt{3}-12\sqrt{4\times3}+18\times6}{4-27} = \frac{42+108-63\sqrt{3}-12\times2\sqrt{3}}{-23} =$
$= \frac{150-87\sqrt{3}}{-23} = \frac{87\sqrt{3}-150}{23}$

b) $\frac{(1+\sqrt{27})^2}{(1+5\sqrt{3})} = \frac{1+2\sqrt{27}+27}{(1+5\sqrt{3})} = \frac{(28+2\sqrt{27})}{(1+5\sqrt{3})} = \frac{(28+2\sqrt{27})(1-5\sqrt{3})}{(1+5\sqrt{3})(1-5\sqrt{3})} = \frac{28-140\sqrt{3}+2\sqrt{27}-10\sqrt{81}}{1-25\times3} =$
$= \frac{28-140\sqrt{3}+2\sqrt{9\times3}-10\times9}{1-75} = \frac{28-90-140\sqrt{3}+2\times3\sqrt{3}}{-74} = \frac{-62-134\sqrt{3}}{-74} = \frac{31+67\sqrt{3}}{37}$

2-
a)

$$\begin{array}{c|cccc}
 & 1 & J & M & 56 \\
2 & & 2 & 4+2J & 8+4J+2M \\
\hline
 & 1 & 2+J & 4+2J+M & \boxed{64+4J+2M=0} \\
4 & & 4 & 24+4J & \\
\hline
 & 1 & 6+J & \boxed{28+6J+M=0} &
\end{array}$$

$$\begin{cases} 64+4J+2M=0 \\ 28+6J+M=0 \end{cases} (=) \begin{cases} -------- \\ M=-28-6J \end{cases} (=) \begin{cases} 64+4J-56-12J=0 \\ ------------ \end{cases}$$

$$(=) \begin{cases} 8=8J \\ ---- \end{cases} \quad (=) \begin{cases} J=1 \\ 28+6+M=0 \end{cases} \quad (=) \begin{cases} J=1 \\ M=-34 \end{cases}$$

Substituindo J no ultimo resultado do Ruffini vem: $x+(6+1)=0 \quad (=)$ $x=-7$ e obtém-se a ultima raiz.

b)

$$\begin{array}{c|cccc}
 & 1 & -7 & -J & -M \\
5 & & 5 & -10 & -5J-50 \\
\hline
 & 1 & -2 & -J-10 & \boxed{-M-5J-50=0} \\
-1 & & -1 & 4 & \\
\hline
 & 1 & -3 & \boxed{-J-7=0} &
\end{array}$$

$$\left\{\begin{array}{l} -M-5J-50=0 \\ -J-7=0 \end{array}\right. (=) \left\{\begin{array}{l} -------- \\ J=-7 \end{array}\right. (=) \left\{\begin{array}{l} -M+35-50 \\ ------------- \end{array}\right.$$

$$(=) \left\{\begin{array}{l} -M-15=0 \\ ------- \end{array}\right. \quad (=) \left\{\begin{array}{l} M=-15 \\ J=-7 \end{array}\right.$$

A partir do último resultado do Ruffini, aparece: $x-3=0 \quad (=) \quad x=3$

3-

a) $x^3-x=0(=)x(x^2-1)=0(=)x=0 \vee x^2-1=0(=)x=0 \vee x^2=1(=)$
$(=)x=0 \vee x=\pm\sqrt{1}(=)x=0 \vee x=-1 \vee x=1$

b) $x^3-x^2=0(=)x \times x \times (x-1)=0(=)x=0 \vee x=0 \vee x-1=0(=)$
$(=)x=0 \vee x=0 \vee x=1$

c) $x^3-2x^2+x=0(=)x(x^2-2x+1)=0(=)x=0 \vee x^2-2x+1=0(=)$
$(=)x=0 \vee x=1 \vee x=1$

4-

a) $-2<5x+3<8(=)-5<5x<5(=)-1<x<1$

b) $2x-\frac{4-x}{3} \leq 5x(=)6x-4+x \leq 15x(=)6x-15x+x \leq 4(=)-8x \leq 4(=)$
$(=)8x \geq -4(=)x \geq -\frac{1}{2}$

5-

$\frac{5^{\frac{1}{6}} \times 5^{\frac{1}{2}}}{(\sqrt[3]{5})^{-2}} = \frac{5^{\frac{4}{6}}}{5^{\frac{-2}{3}}} = 5^{(\frac{4}{6}-\frac{-2}{3})} = 5^{(\frac{4}{6}+\frac{4}{6})} = 5^{\frac{8}{6}} = 5^{\frac{4}{3}}$