Mathematics

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Preface

- Precalculus - Mathematics for Calculus by James Stewart

- Calculus by James Stewart

Part I

Precalculus

0.1 Logical Notation

for all $\forall$

there exists $\exists$

there does not exist $\nexists$

implies $\implies$

if and only if $\iff$

equivalent to $\equiv$

not equivalent to $\not\equiv$

0.2 Real Number Systems

Natural Numbers $\mathbb{N} = \{1, 2, 3, \dots\}$

Whole Numbers $\mathbb{W} = \{0, 1, 2, 3, \dots\}$

Integers $\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \dots\}$

Rational Numbers $\mathbb{Q} = \left\{ \frac{a}{b} : a, b \in \mathbb{Z},\ b \neq 0 \right\}$

Irrational Numbers $\mathbb{I} = \mathbb{R} \setminus \mathbb{Q}$

Real Numbers $\mathbb{R} = \mathbb{Q} \cup \mathbb{I}$

$$\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

0.3 Properties of Real Numbers

0.3.1 Commutative Property

addition $a + b = b + a$

multiplication $ab = ba$

0.3.2 Associative Property

addition $(a + b) + c = a + (b + c)$

multiplication $(ab)c = a(bc)$

0.3.3 Distributive Property

distributive $a(b + c) = ab + ac$

distributive $(b + c)a = ba + ca$

0.4 Properties of Negatives

1. $(-1)a = -a$

2. $-(-a) = a$

3. $(-a)b = a(-b) = -(ab)$

4. $(-a)(-b) = ab$

5. $-(a + b) = -a + -b$

6. $-(a - b) = -a + b$

0.5 Properties of Fractions

1. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

2. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$

3. $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

4. $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

5. $\frac{ac}{bc} = \frac{a}{b}$

6. If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$

0.6 The Real Line

The real number line is ordered. For any two real numbers a and b, one and only one of the following is true:

- $a < b$

- $a = b$

- $a > b$

0.7 Equality/Inequality Notation

less than $<$

greater than $>$

less than or equal to $\leq$

greater than or equal to $\geq$

equal to $=$

not equal to $\neq$

0.8   Set Notation

set of elements $\{\}$

such that $|$ or :

is an element of $\in$

is not an element of $\notin$

is a subset of $\subseteq$

is a proper subset of $\subset$

is a superset of $\supseteq$

is a proper superset of $\supset$

empty set $\emptyset$

universal set U

cardinality $|A|$

0.9   Set Operations

union $\cup$

intersection $\cap$

difference $\backslash$

complement A'

0.10   Interval Notation

open interval $(a, b) = \{x \in \mathbb{R} : a < x < b\}$

closed interval $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$

half-open interval $[a, b) = \{x \in \mathbb{R} : a \leq x < b\}$

half-open interval $(a, b] = \{x \in \mathbb{R} : a < x \leq b\}$

infinite interval $(a, \infty) = \{x \in \mathbb{R} : x > a\}$

infinite interval $[a, \infty) = \{x \in \mathbb{R} : x \geq a\}$

infinite interval $(-\infty, b) = \{x \in \mathbb{R} : x < b\}$

infinite interval $(-\infty, b] = \{x \in \mathbb{R} : x \leq b\}$

infinite interval $(-\infty, \infty) = \mathbb{R}$

0.11 Absolute Value and Distance

absolute value $|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$

distance between two points $d(a, b) = |a - b|$

0.11.1 Properties of Absolute Value

1. $|ab| = |a||b|$

2. **positive square root** $|a|^2 = a^2 \implies |a| = \sqrt{a^2}$

3. $|a^n| = |a|^n$, where $n \in \mathbb{Z}$ and $a \neq 0$ for $n < 0$

4. **triangle inequality** $|a + b| \leq |a| + |b|$

0.12 Questions

Precalculus - Mathematics for Calculus by James Stewart, Section 1.1.

0.13 Exponential Notation

$a^n = a \cdot a \cdot \ldots \cdot a$ (a multiplied n times)

0.13.1 Zero and Negative Exponents

$a^0 = 1$, where $a \neq 0$

$a^{-n} = \frac{1}{a^n}$,

0.13.2 Laws of Exponents

1. $a^m \cdot a^n = a^{m+n}$

2. $\frac{a^m}{a^n} = a^{m-n}$

3. $(a^m)^n = a^{mn}$

4. $(ab)^n = a^n b^n$

5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

6. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

7. $\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$

0.14 Scientific Notation

scientific notation $x = a \times 10^n$, where $1 \le |a| < 10$ and $n \in \mathbb{Z}$

0.15 Radicals

radical $\sqrt{a} = b \iff b^2 = a$ and $b \ge 0$

nth root $\sqrt[n]{a} = b \iff b^n = a$ and if n is even, then $a, b \ge 0$

0.15.1 Properties of nth Roots

1. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

2. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

3. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

4. $\sqrt[n]{a^n} = a$ if n is odd

5. $\sqrt[n]{a^n} = |a|$ if n is even

0.16 Rational Exponents

rational exponent $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

0.17 Questions

Precalculus - Mathematics for Calculus by James Stewart, Section 1.2.