Math Guide

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Preface

This is a math guide for The University of Queensland's MATH1051, MATH1052, MATH2001, MATH1061, and MATH2302 courses. It is intended to be a quick reference for students studying these subjects.

Part I

MATH1051 Calculus & Linear Algebra I

Chapter 1

Eigenvalues and Eigenvectors

Consider a linear transformation represented by a square matrix, denoted A. When a vector is transformed by the matrix, its direction and magnitude can change. **Eigenvectors**, denoted $\underline{x}$, are special vectors that only change in magnitude (not direction) under this transformation. The magnitude by which the eigenvector is scaled is called the **eigenvalue**, denoted λ.

The relationship between a matrix A, its eigenvalues λ, and eigenvectors $\underline{x}$ is given by the equation:

$$A\underline{x} = \lambda\underline{x}$$

This is known as the **definition of eigenvalues and eigenvectors**.

1.1 Eigenvalue Equation

To find the eigenvalues λ of a matrix A, we solve the equation:

$$|A - \lambda I| = 0$$

1.2 Eigenvector Equation

To find the eigenvectors $\underline{x}$ corresponding to a specific eigenvalue λ, we solve the equation:

$$(A - \lambda I)\underline{x} = \underline{0}$$

1.3 Derivation of Eigenvalue and Eigenvector Equations

To find the eigenvector equation, we can rearrange the definition of eigenvalues and eigenvectors.

$$\begin{aligned} A\underline{x} &= \lambda\underline{x} \\ \iff A\underline{x} &= \lambda I\underline{x} \\ \iff A\underline{x} - \lambda\underline{x} &= \underline{0} \\ \iff (A - \lambda I)\underline{x} &= \underline{0} \end{aligned}$$

To find the eigenvalue equation, note that eigenvectors are non-zero vectors. Therefore, the equation $(A - \lambda I)\underline{x} = \underline{0}$ must have non-trivial solutions. This occurs when the matrix $(A - \lambda I)$ is singular, when its determinant is zero:

$$|A - \lambda I| = 0$$

Therefore, we have derived both the eigenvalue and eigenvector equations from the definition of eigenvalues and eigenvectors.

Part II

MATH1052 Multivariable Calculus & Linear Algebra I

Part III

MATH2001 Calculus & Linear Algebra II

Part IV

MATH1061 Discrete Mathematics I

Part V

MATH2302 Discrete Mathematics II