RĂDULESCU MIHAI-ALEXANDRU

GRUPA 234

EXAMEN PBS

ID = 123

Ex 1: Notăm: $D+$ = are boala

$D-$ = nu are boala

$T+$ = testat pozitiv

$T-$ = testat negativ

$P(D+) = \frac{1}{1000} \Rightarrow P(D-) = 1 - \frac{1}{1000}$

"fals pozitiv": $P(T+|D-) = 0,03 \Rightarrow P(T-|D-) = 0,97$

"fals negativ": $P(T-|D+) = 0,03 \Rightarrow P(T+|D+) = 0,97$

Probabilitatea de a avea boala este

$$P(D+|T+) = \frac{P(T+|D+) * P(D+)}{P(T+)}$$

$P(D+) = \frac{1}{1000} = 0,123$

$P(T+|D+) = 0,97$

$P(T+) = 0,03 * \left(1000 - \frac{123}{1000}\right) + 0,97 * \frac{123}{1000}$

$= 0,14562$

$\Rightarrow P(D+|T+) = \frac{0,97 \cdot 0,123}{0,14562} = 0,8793 = 81,93\%$

Ex 2: Putem obține rezultatul folosind funcția RV din limbajul R

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Cod  i ← 123
     X ← RV(outcomes = 1:3, probs = c((1/1000), (1/500),
((1000 - 3*i)/1000)))
     E(X^2)
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$E(X^2) = 6,786$

Ex 4: Pentru $i = 123$:)

:) $P(\text{123 aversuri} \mid p) = C_{200}^{123} p^{123}(1-p)^{77} \Rightarrow$

:) $\frac{d}{dp} P(\text{date} \mid p) = C_{200}^{123}\left[123p^{122}(1-p)^{77} - 77p^{123}(1-p)^{76}\right] = 0$

:) $123p^{122}(1-p)^{77} - 77p(1-p)^{76} = 0$

$123p^{122}(1-p)^{77} = 77p(1-p)^{76} \mid \Rightarrow$

$p \in (0,1)$

:) $123(1-p) = 77p$

$123 = 200p$

$p = \frac{123}{200} = 0,61\overline{5}$

Ex 5 : A) $P(\text{avers}) = P(A \mid \text{avers la prima}) + P(B \mid \text{avers la prima})$

$P(A \mid \text{avers la prima}) = P(\text{avers} \mid A) * P(A) / P(\text{avers la prima})$

$= 0,5 * (i/200)$

$P(B \mid \text{avers la prima}) = P(\text{avers} \mid B) * P(B) / P(\text{avers la tirada})$

$= 0,8 * (200 - i)/200$

$P(\text{avers}) = 0,5 \cdot i/200 + 0,8 * (200 - i)/200 \Big| =)$

$i = 123$

$=) P(\text{avers}) = 0,615$

B) $P(\text{avers la segona}) = P(A \mid \text{avers la prima}) * P(\text{avers} \mid A) + P(B \mid \text{avers la prima}) * P(\text{avers} \mid B)$

$P(A \mid \text{avers la prima}) = P(\text{avers} \mid A) * P(A) / P(\text{avers}) \text{ la} \Big| =)$

$P(B \mid \text{avers la prima}) = P(\text{avers} \mid B) * P(B) / P(\text{avers})$

$=) P(\text{avers la segona}) = P(\text{avers} \mid A)^2 * P(A) / P(\text{avers}) + P(\text{avers} \mid B)^2 * P(B) / P(\text{avers})$

$= \left[0,25 * \frac{i}{200} + 0,64 * \left(\frac{200 - i}{200}\right)\right] / 0,615$

$= 0,650$

Ex 3: $i = 123 \Rightarrow X \sim (0, 123)$, $Y = \sqrt{X}$

$$\left.\begin{array}{l} Y = \sqrt{X} \\ X \in [0, 123] \end{array}\right| \Rightarrow Y \in [0, \sqrt{123}] \Rightarrow Y \in [0, 11,09]$$

Pot. afla cdf din definiția

$$F_Y(y) = P(Y \leq y) = P(\sqrt{X} \leq y) = P(X \leq y^2) = F_X(y^2) \Big| \Rightarrow$$

$F_X(x) = (x-a)/(b-a) = x/123$

$\Rightarrow cdf = F_X(y^2) = \frac{y^2}{123}$

Pentru a afla pdf derivez cdf:

$$f_Y(y) = F_Y'(y) = \left(\frac{y^2}{123}\right)' = \frac{2y}{123}$$

Rezultate:

$y \in [0, \sqrt{123}]$

$F(y) = \frac{y^2}{123}$

$f(y) = \frac{2y}{123}$

Ex 6: Intervalul ~~normal~~ de încredere $(1-\alpha)$ normal corespunzător lui θ este dat de formula $\bar{x} \pm z_{\alpha/2} \cdot \frac{1}{2\sqrt{n}}$

$\bar{x}$ = media datelor = 101/1600

Intervalul de 95% încredere $\Rightarrow (1-\alpha) = 0,95 \Rightarrow \alpha = 0,05$

Pentru $\alpha = 0,05 \Rightarrow z_{\alpha/2} = z_{0,05} = 1,96 \Rightarrow$

$$\Rightarrow \theta = \frac{101}{1600} \pm \frac{1,96}{2\sqrt{1600}} \Rightarrow$$

$i = 123$

$$\Rightarrow \theta = \frac{10 \cdot 123}{1600} \pm \frac{1,96}{2 \cdot 40}$$

$$= \frac{123}{160} \pm 0,0245$$

$$= 0,7687 \pm 0,0245$$

$$= [0,7442 ; 0,7932]$$