In [140]:

```
reset()
x=var('x')
y=function('y')(x)
eqd=diff(y,x)==2*y
show(eqd)
desolve(eqd,y)
show(desolve(eqd,y,show_method=True))
```

$$\frac{\partial}{\partial x}y(x) = 2\,y(x)$$

$$\left[Ce^{(2\,x)}, \mathtt{linear}\right]$$

In [49]:

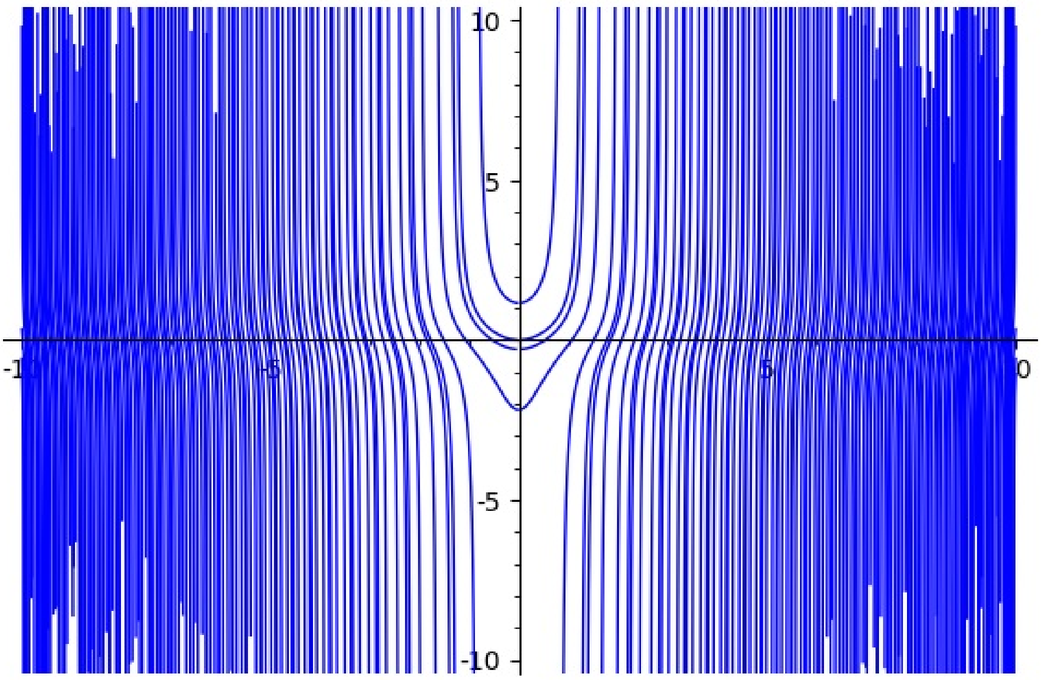
```
x=var('x')
y=function('y')(x)
eqd=diff(y,x)==2*x*(1+y^2)
show(eqd)
s=desolve(eqd,y)
show(s)
ans1=solve(s,y(x))
show(ans1)
ysol(x,_C)=ans1[0].rhs()
show(ysol)
g=plot(ysol(x,0),x,-10,10,detect_poles='True',ymin=-10,ymax=10)
for i in [1..3]:
    g=g+plot(ysol(x,i),x,-10,10,detect_poles='True',ymin=-10,ymax=10)
show(g)
```

$$\frac{\partial}{\partial x}y(x) = 2\left(y(x)^2 + 1\right)x$$

$$\frac{1}{2}\arctan(y(x)) = \frac{1}{2}x^2 + C$$

$$\left[y(x) = \tan\left(x^2 + 2\,C\right)\right]$$

$(x, C) \mapsto$



In [89]:

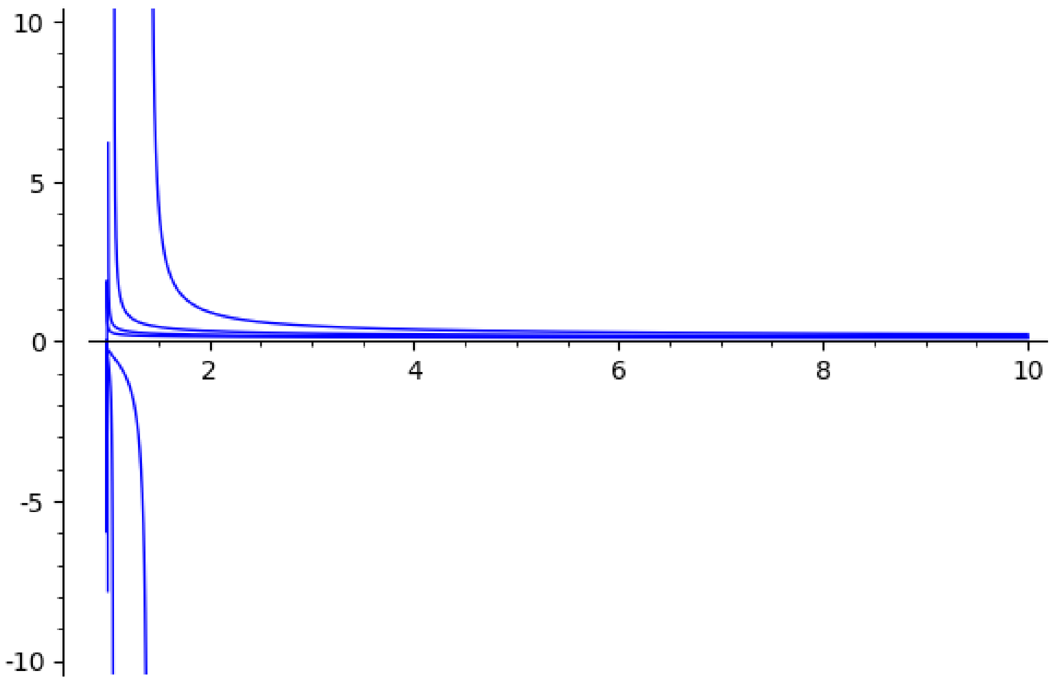
```
x=var('x')
y=function('y')(x)
eqd=(x^2-1)*diff(y,x)+2*x*y^2 == 0
show(eqd)
s=desolve(eqd,y)
show(s)
ans1=solve(s,y(x))
show(ans1)
sol(x,_C)=ans1[0].rhs()
show(sol)
g=plot(sol(x,0),x,1,10,detect_poles='True',ymin=-10,ymax=10)
for i in [1..3]:
    g=g+plot(sol(x,i),x,1,10,detect_poles='True',ymin=-10,ymax=10)
show(g)
```

$$2\,xy(x)^2 + \left(x^2 - 1\right)\frac{\partial}{\partial x}y\left(x\right) = 0$$

$$\frac{1}{2\,y\left(x\right)} = C + \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1)$$

$$\left[y\left(x\right) = \frac{1}{2\,C + \log(x+1) + \log(x-1)}\right]$$

$(x, C) \qquad \mapsto$



In [93]:

```
x=var('x')
y=function('y')(x)
eqd=2*x^2*diff(y,x)==x^2+y^2
show(eqd)
s=desolve(eqd,y)
show(s)
ans1=solve(s,y(x))
show(ans1)
sol1(x,_C)=ans1[1].rhs()
show(sol1)
g=plot(sol1(x,1),x,1,10,detect_poles='True',ymin=-10,ymax=10)
for i in [2..10]:
    g=g+plot(sol1(x,i),x,1,10,detect_poles='True',ymin=-10,ymax=10)
show(g)
```

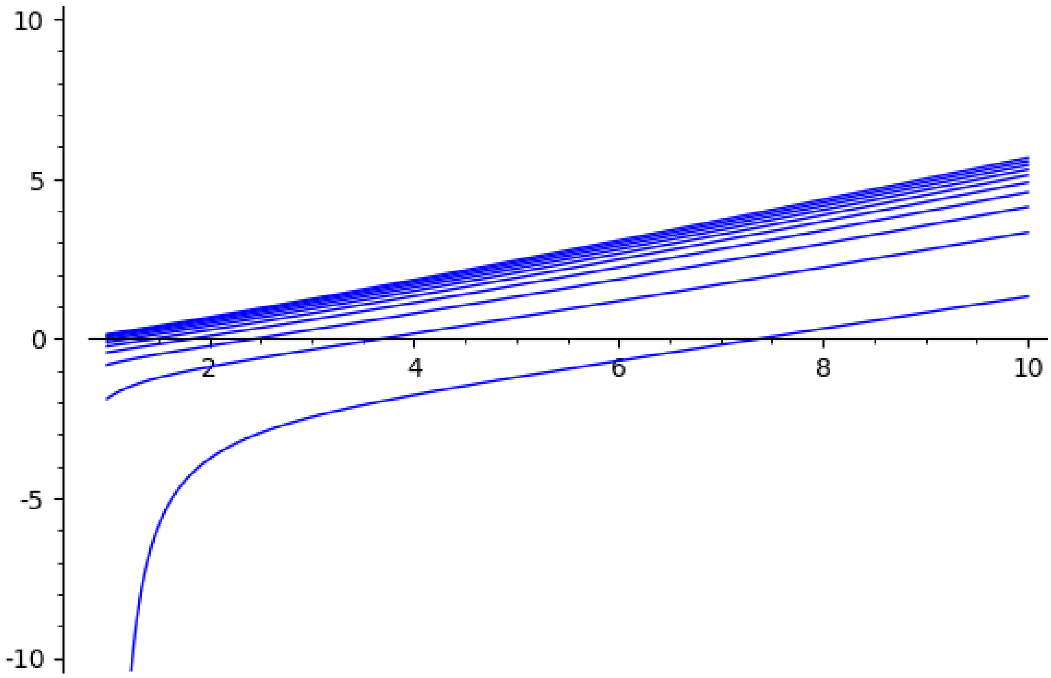
$$2\,x^2\frac{\partial}{\partial x}y\left(x\right) = x^2 + y\left(x\right)^2$$

$$2\,x\frac{\partial}{\partial x}y(x) = x^{2} + y(x)^{2}$$

$$Cx = e^{\left(\frac{2\,x}{x-y(x)}\right)}$$

$$\left[y(x) = \frac{x\log\left(-\frac{1}{\sqrt{Cx}}\right) + x}{\log\left(-\frac{1}{\sqrt{Cx}}\right)}, y(x) = \frac{x\log(Cx) - 2\,x}{\log(Cx)}\right]$$

$(x, C) \quad \mapsto$



In [110]:

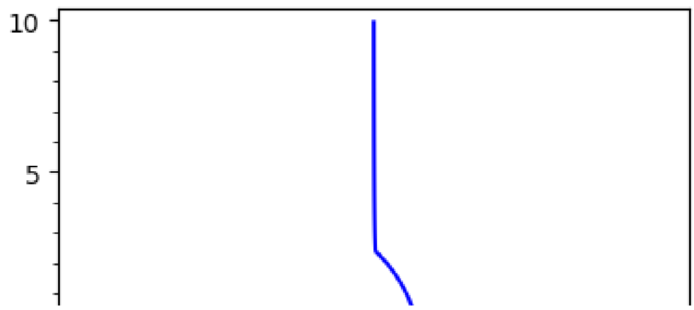
```
x=var('x')
y=function('y')(x)
eqd=diff(y,x)==-(x+y)/y
show(eqd)
s=desolve(eqd,y)
show(s)
ans1=solve(s,y(x))
show(ans1)
yy=var('yy')
f(x,yy,_C)=s.substitute(y(x)==yy)
implicit_plot(f(x,yy,1),(x,-10,10),(yy,-10,10))
```

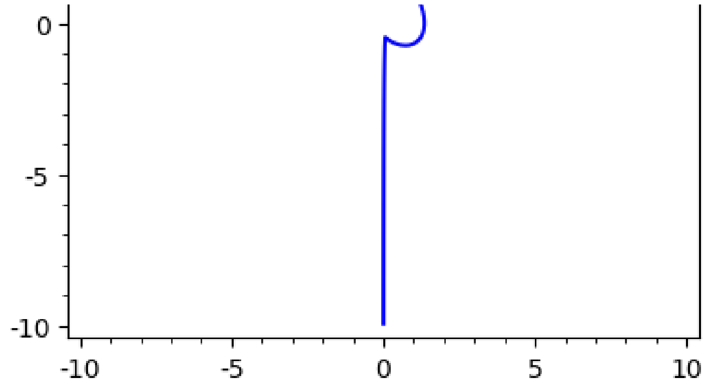
$$\frac{\partial}{\partial x}y(x) = -\frac{x + y(x)}{y(x)}$$

$$Cx = e^{\left(-\frac{1}{6}\sqrt{3}\left(\sqrt{3}\log\left(\frac{x^2+xy(x)+y(x)^2}{x^2}\right) - 2\arctan\left(\frac{\sqrt{3}(x+2\,y(x))}{3\,x}\right)\right)\right)}$$

$[]$

Out[110]:





In [129]:

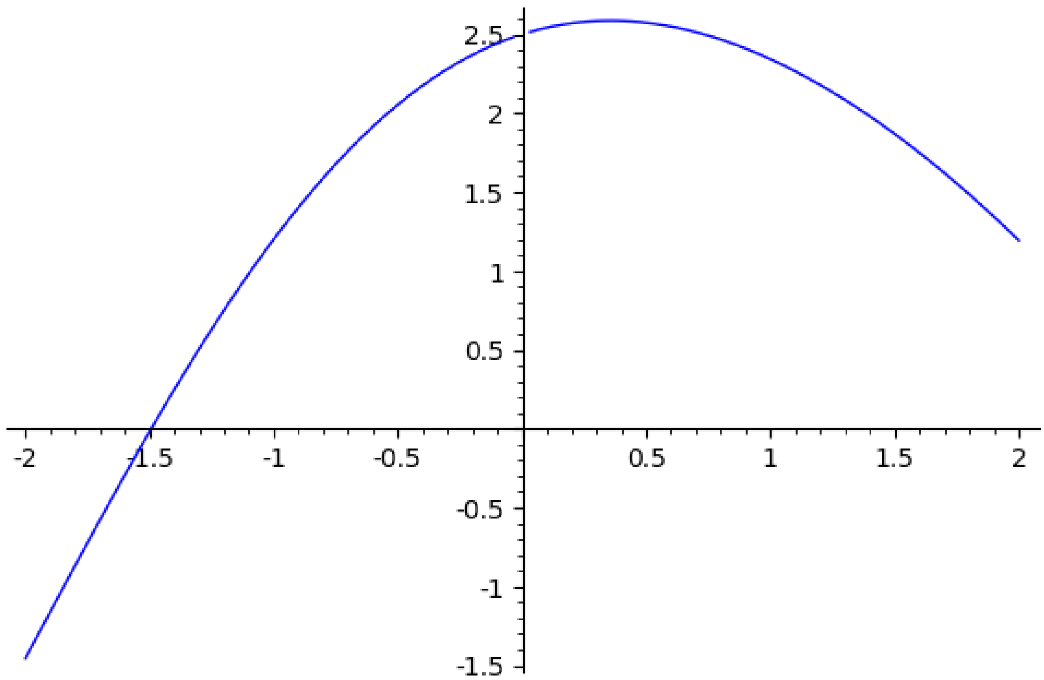
```
x=var('x')
y=function('y')(x)
eqd=diff(y,x,2)+y==sin(x)+cos(x)
show(eqd)
s=desolve(eqd,y)
show(s)
ans1=solve(x,y(x))
_K1,_K2=var('_K1,_K2')
sol1=s.substitute(_K1==1,_K2==2)
show(sol1)
plot(sol1,x,-2,2,detect_poles='True')
```

$$y\left(x\right) + \frac{\partial^{2}}{(\partial x)^{2}}y\left(x\right) = \cos\left(x\right) + \sin\left(x\right)$$

$$K_{2} \cos\left(x\right) - \frac{1}{2} \left(x - 1\right) \cos\left(x\right) + K_{1} \sin\left(x\right) + \frac{1}{2} x \sin\left(x\right)$$

$$-\frac{1}{2} \left(x - 1\right) \cos\left(x\right) + \frac{1}{2} x \sin\left(x\right) + 2 \cos\left(x\right) + \sin\left(x\right)$$

Out[129]:



In [138]:

```
x=var('x')
y=function('y')(x)
eqd=diff(y,x,2)-y==e^(2*x)
show(eqd)
s=desolve(eqd,y)
show(s)
```

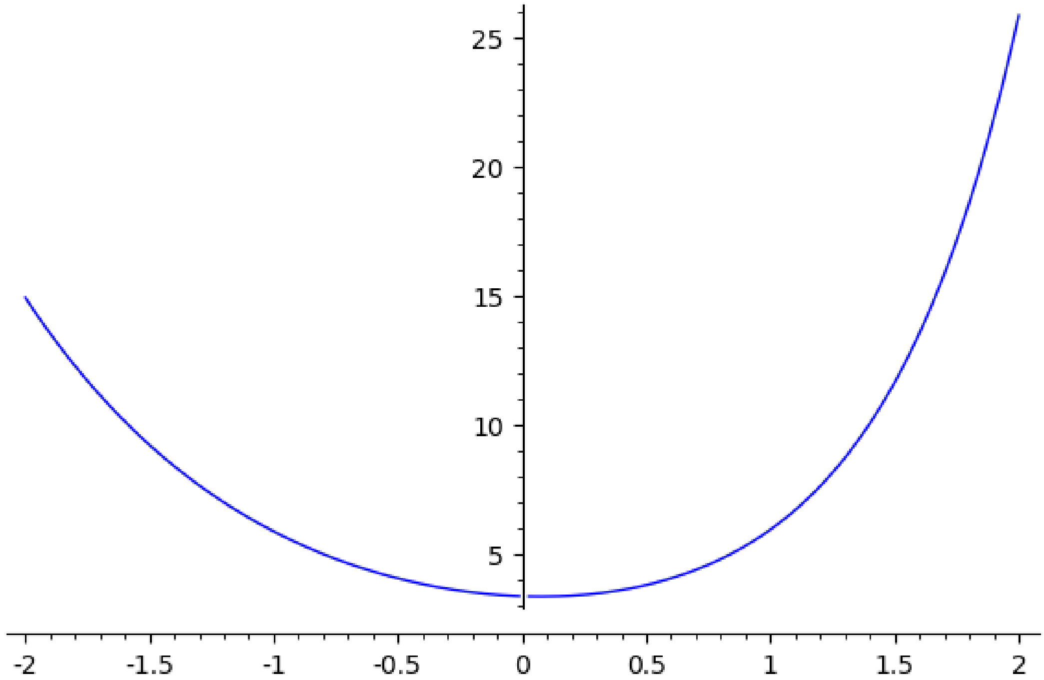
```
ans1=solve(x,y(x))
_K1,_K2=var('_K1,_K2')
sol1=s.substitute(_K1==1,_K2==2)
show(sol1)
plot(sol1,x,-2,2,detect_poles='True')
```

$$-y(x) + \frac{\partial^2}{(\partial x)^2}y(x) = e^{(2x)}$$

$$K_2 e^{(-x)} + K_1 e^x + \frac{1}{3}e^{(2x)}$$

$$\frac{1}{3}e^{(2x)} + 2e^{(-x)} + e^x$$

Out[138]:



In [141]:

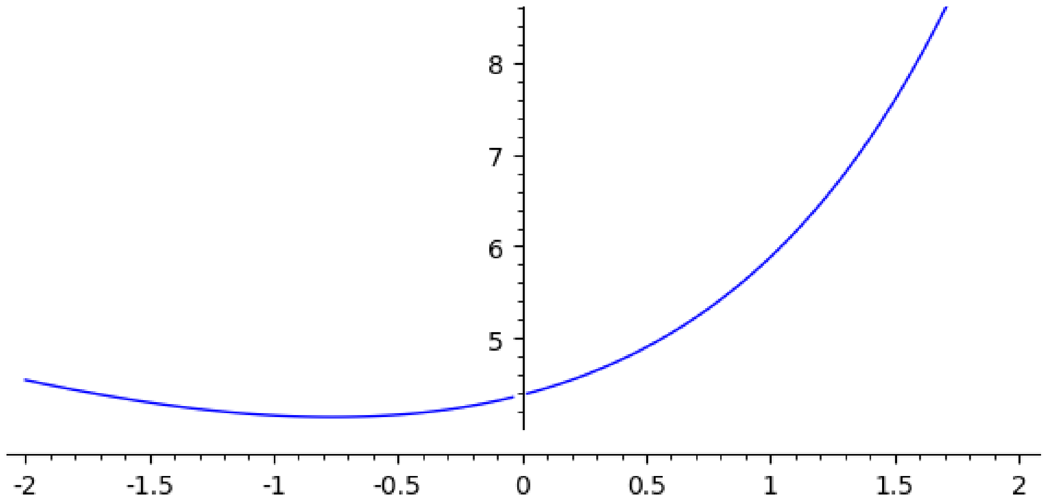
```
x=var('x')
y=function('y')(x)
eqd=diff(y,x,2)-diff(y,x)==1/(1+e^x)
show(eqd)
s=desolve(eqd,y)
show(s)
ans1=solve(x,y(x))
_K1,_K2=var('_K1,_K2')
sol1=s.substitute(_K1==1,_K2==2)
show(sol1)
plot(sol1,x,-2,2,detect_poles='True')
```

$$-\frac{\partial}{\partial x}y(x) + \frac{\partial^2}{(\partial x)^2}y(x) = \frac{1}{e^x + 1}$$

$$K_1 e^x + e^x \log\left((e^x + 1)e^{(-x)}\right) + K_2 - x + \log(e^x + 1)$$

$$e^x \log\left((e^x + 1)e^{(-x)}\right) - x + e^x + \log(e^x + 1) + 2$$

Out[141]:



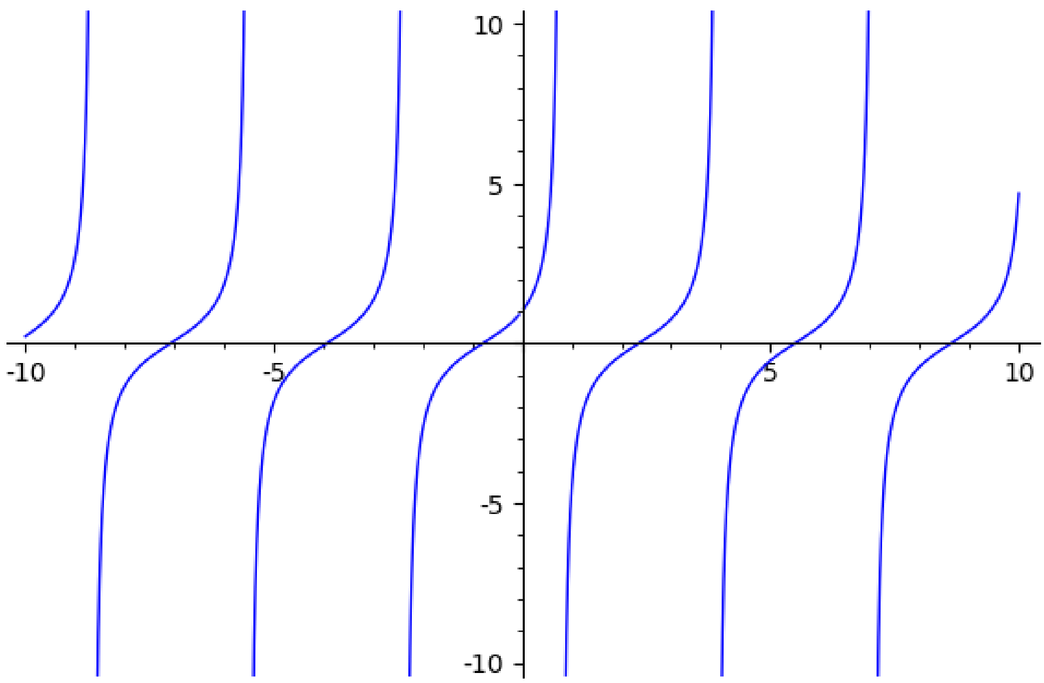
In [150]:

```
x=var('x')
y=function('y')(x)
eqd=diff(y,x)==1+y^2
s=desolve(eqd,y,ics=[0,1])
show(s)
sol=solve(s,y(x))
show(sol)
ans1=sol[0].rhs()
plot(ans1,x,-10,10,detect_poles='True',ymin=-10,ymax=10)
```

$$\arctan(y(x)) = \frac{1}{4}\pi + x$$

$$\left[y\left(x\right) = \tan\left(\frac{1}{4}\pi + x\right)\right]$$

Out[150]:



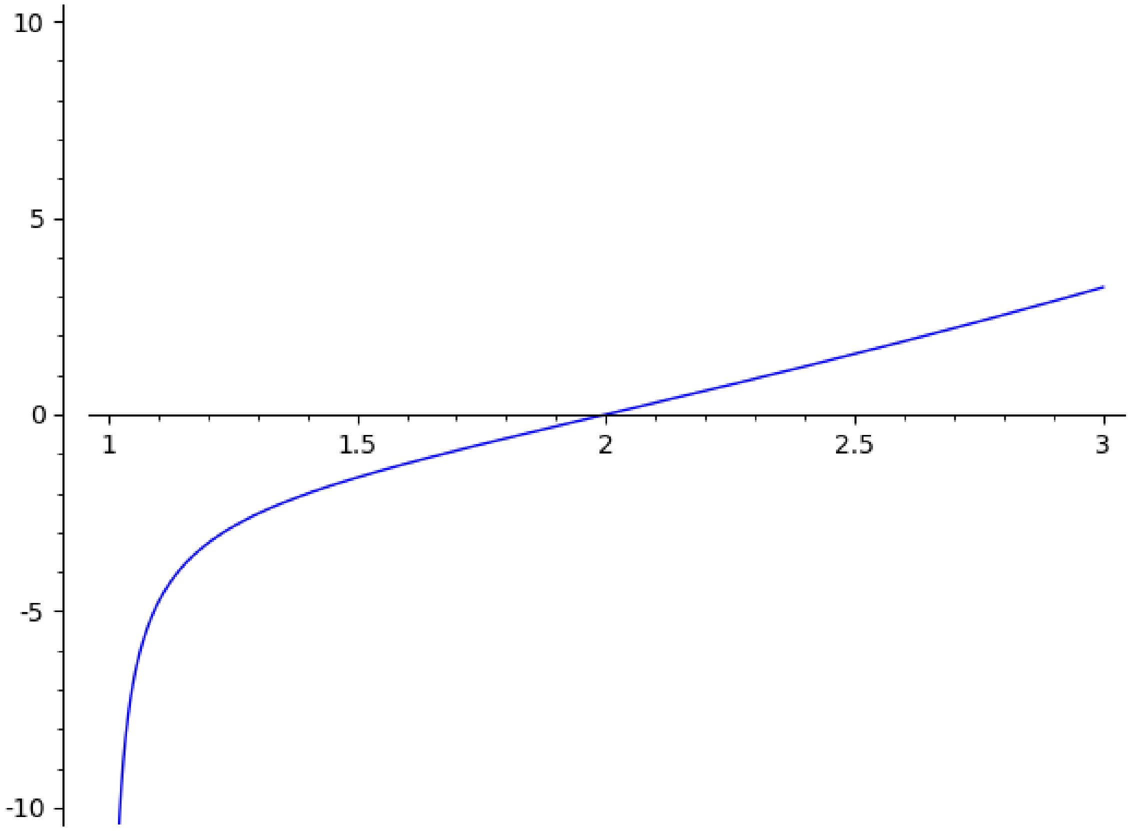
In [1]:

```
x=var('x')
y=function('y')(x)
eqd=diff(y,x)==1/(1-x^2)*y+1+x
s=desolve(eqd,y,ics=[2,0])
show(s)
plot(s,x,1,3,detect_poles='True',ymin=-10,ymax=10)
```

$$\sqrt{x^2-1}\sqrt{x+1}\,x - \sqrt{x+1}\left(2\sqrt{3} - \log\left(2\sqrt{3}+4\right)\right) \quad - \sqrt{x+1}\,\log\left(2\,x + 2\sqrt{x^2-1}\right)$$

$$\overline{2\sqrt{x-1}}$$

Out[1]:



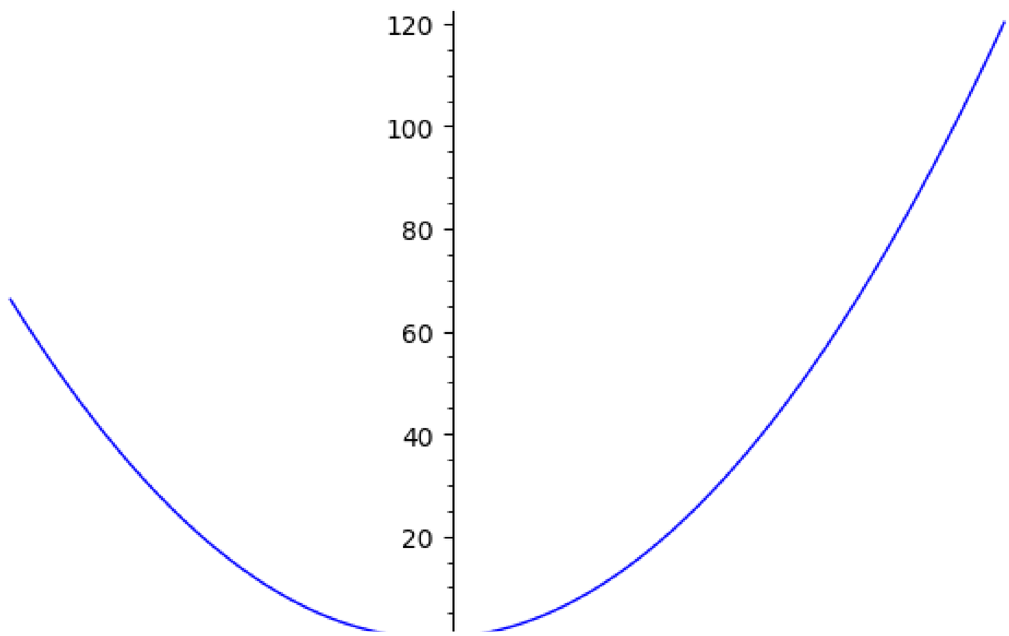
In [11]:

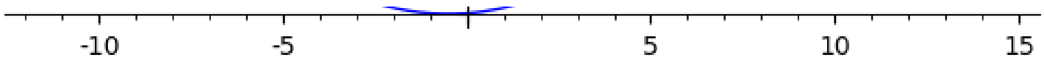
```
x=var('x')
y=function('y')(x)
eq1=diff(y,x)-2*y==-x^2
show(eq1)
sol=desolve(eq1,y,ics=[0,1/4])
show(sol)
plot(sol,x,-12,15)
```

$$-2\,y\left(x\right)+\frac{\partial}{\partial x}y\left(x\right)=-x^{2}$$

$$\frac{1}{2}\,x^{2}+\frac{1}{2}\,x+\frac{1}{4}$$

Out[11]:





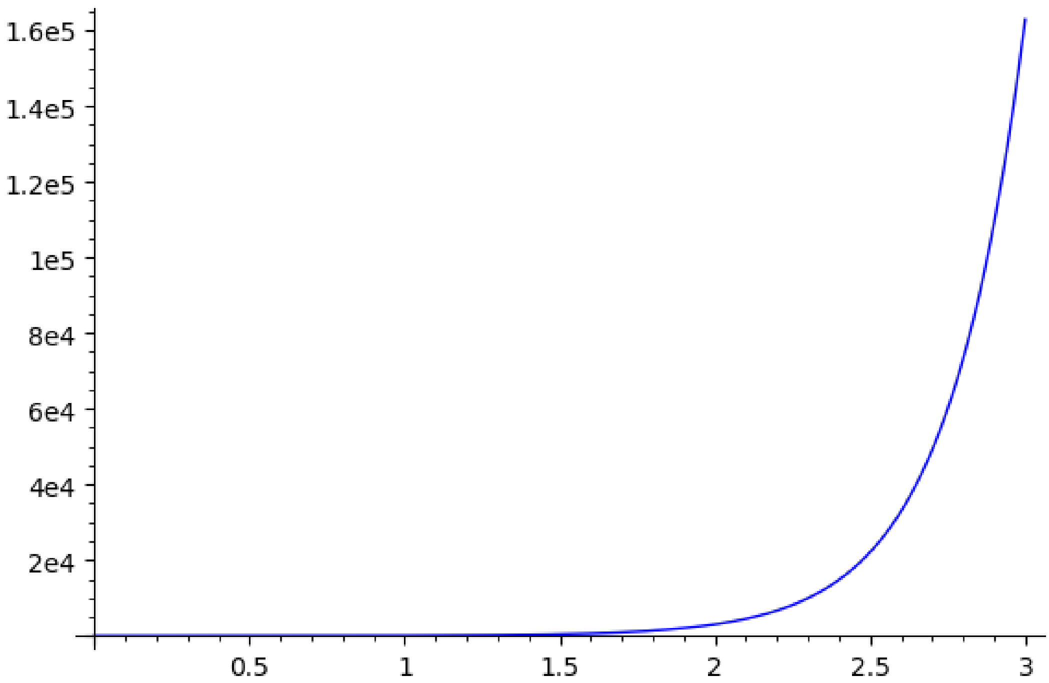
In [23]:

```
x=var('x')
y=function('y')(x)
equ=diff(y,x,2)-5*diff(y,x)+4*y==0
show(equ)
sol=desolve(equ,y,ics=[0, 5, 8])
show(sol)
plot(sol,x,0,3)
```

$$4\,y\left(x\right) - 5\,\frac{\partial}{\partial x}y\left(x\right) + \frac{\partial^{2}}{(\partial x)^{2}}y\left(x\right) = 0$$

$$e^{\left(4\,x\right)} + 4\,e^{x}$$

Out[23]:



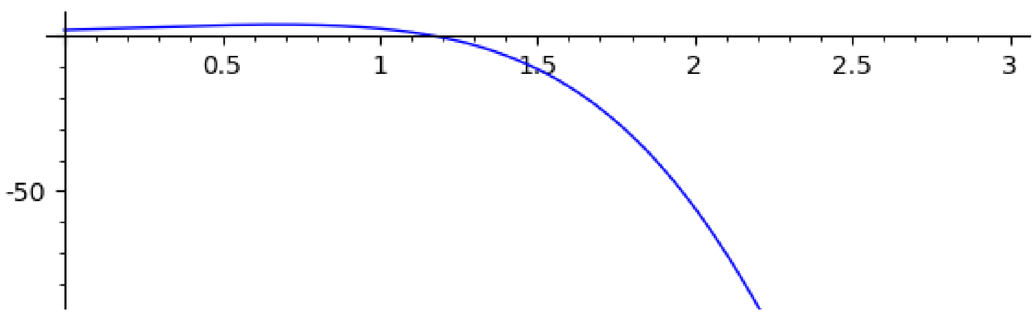
In [24]:

```
x=var('x')
y=function('y')(x)
equ=diff(y,x,2)-4*diff(y,x)+5*y==2*x^2*e^x
show(equ)
sol=desolve(equ,y,ics=[0, 2, 3])
show(sol)
plot(sol,x,0,3)
```

$$5\,y\left(x\right) - 4\,\frac{\partial}{\partial x}y\left(x\right) + \frac{\partial^{2}}{(\partial x)^{2}}y\left(x\right) = 2\,x^{2}e^{x}$$

$$\left(\cos\left(x\right) - 2\,\sin\left(x\right)\right)\,e^{\left(2\,x\right)} + \left(x^{2} + 2\,x + 1\right)\,e^{x}$$

Out[24]:



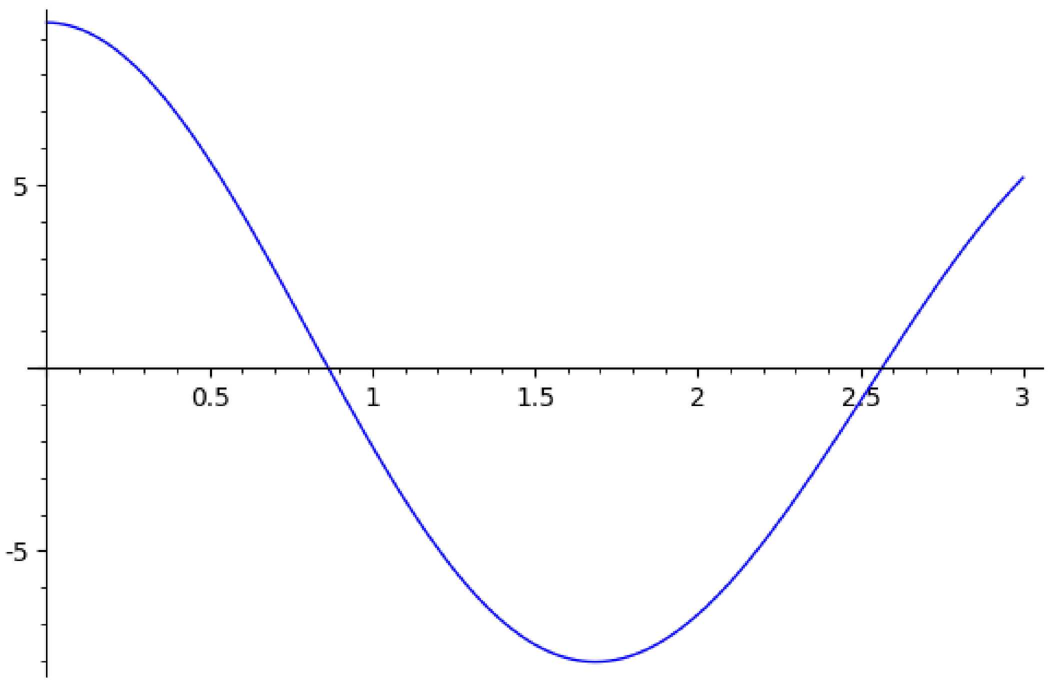
In [28]:

```
x=var('x')
y=function('y')(x)
equ=diff(y,x,2)+4*y==4*(sin(2*x)+cos(2*x))
show(equ)
sol=desolve(equ,y,ics=[pi, 2*pi, 2*pi])
show(sol)
plot(sol,x,0,3)
```

$$4\,y\left(x\right) + \frac{\partial^{2}}{(\partial x)^{2}}y\left(x\right) = 4\,\cos\left(2\,x\right) + 4\,\sin\left(2\,x\right)$$

$$\frac{1}{2}\,\left(6\,\pi - 1\right)\,\cos\left(2\,x\right) - \frac{1}{2}\,\left(2\,x - 1\right)\,\cos\left(2\,x\right) + x\sin\left(2\,x\right) + \frac{1}{2}\,\sin\left(2\,x\right)$$

Out[28]:



In [69]:

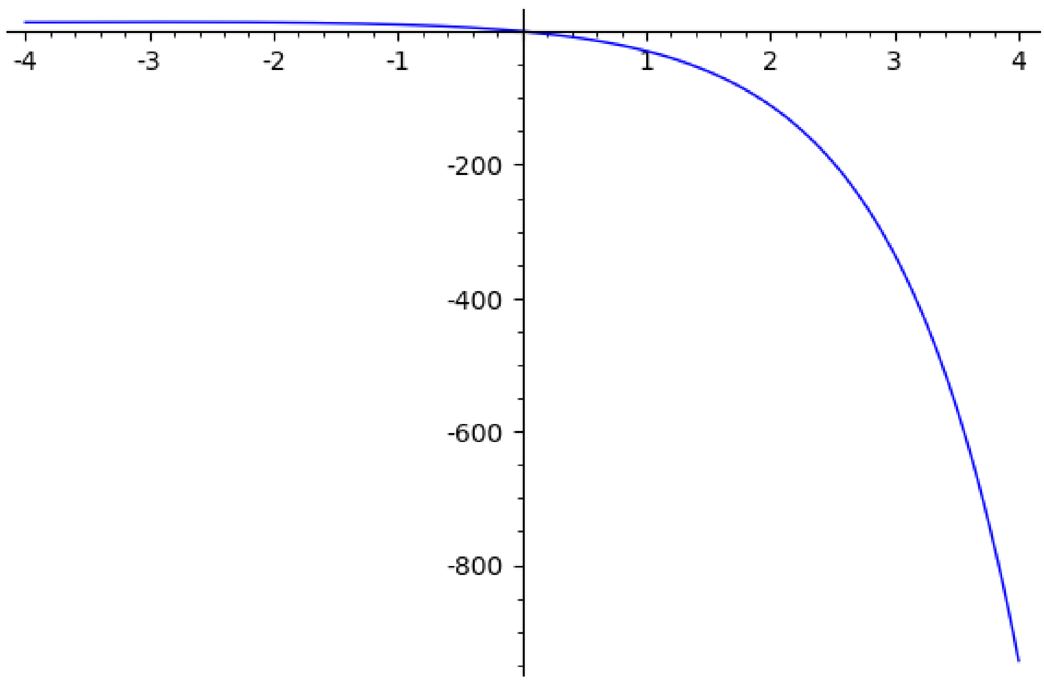
```
x,a,b=var('x,a,b')
y=function('y')(x)
equ=diff(y,x)==y+a*x+b
show(equ)
sol(a,b)=desolve(equ,y,ics=[0,1],ivar=x)
show(sol)
res=sol(1,-5+4*e)==sol(2,-7+4*e^2)
show(solve(res,x))
plot(res,x,-4,4)
```

$$\frac{\partial}{\partial x}y\left(x\right) = ax + b + y\left(x\right)$$

$\left(a, b\right) \ \mapsto$

$$\left[x = \left(4\,e^{2} - 4\,e - 1\right)\,e^{x} - 4\,e^{2} + 4\,e + 1\right]$$

Out[69]:



In [128]:

```
reset()
x,a=var('x,a')
y=function('y')(x)
equ=diff(y,x,2)-diff(y,x)-2*y==0
show(equ)
s=desolve(equ,y,ics=[0,a,2])
show(s)
sol=solve(s,x)
show(sol[2])
func=sol[2].rhs()
show(func)
eqd=limit(func,x=infinity)==0
show(solve(eqd,a))
```

$$-2\,y\left(x\right) - \frac{\partial}{\partial x}y\left(x\right) + \frac{\partial^{2}}{(\partial x)^{2}}y\left(x\right) = 0$$

$$\frac{1}{3}\,(a+2)\,e^{(2\,x)} + \frac{2}{3}\,(a-1)\,e^{(-x)}$$

$$x = \frac{1}{3}\log\left(-\frac{2\,a}{a+2} + \frac{2}{a+2}\right)$$

$$\frac{1}{3}\log\left(-\frac{2\,a}{a+2} + \frac{2}{a+2}\right)$$

$$[a = 0]$$