In [13]:

```
t,r,x0,r0,K=var('t,r,x0,r0,K')
x=function('x')(t)
print("a:")
eqMalthaus=diff(x,t)==r*x
solMalthaus(x0,r,t)=desolve(eqMalthaus,x,ics=[0,x0],ivar=t)
show('Solutia generala pentru modelul lui Malthaus:\t', solMalthaus)
print("b:")
eq=solMalthaus(1000,r,10)==30000
show(eq)
r1=solve(eq,r)
show(r1)
#alegem doar raspunsurile reale
r1=r1[9]
show(r1)
show(numerical_approx(r1.rhs()))
```

a:

Solutia generala pentru modelul lui Malthaus: (x_0, r, t)

b:

$$1000\, e^{(10\, r)} = 30000$$

$$\left[r = \log\left(\frac{1}{4} \cdot 30^{\frac{1}{10}} \left(\sqrt{5} + i\, \sqrt{-2\,\sqrt{5} + 10} + 1\right)\right), r = \log\left(\frac{1}{4} \cdot 30^{\frac{1}{10}} \left(\sqrt{5} + i\, \sqrt{2\,\sqrt{5} + 10} - 1\right)\right), r = \log\left(\right.\right.$$

$$r = \frac{1}{10} \log(30)$$

$$0.340119738166216$$

In [34]:

```
reset()
t,C1,C2=var('t,C1,C2')
x=function('x')(t)
y=function('y')(t)
eq1=diff(x,t)==2*x-y
eq2=diff(y,t)==3*x-2*y
print("a:")
sol=desolve_system([eq1,eq2],[x,y],[0,C1,C2])
show(sol[0])
show(sol[1])
print("b:")
sol=desolve_system([eq1,eq2],[x,y],[0,2,4])
show(sol[0])
show(sol[1])
reprx=plot(sol[0].rhs()(t), t, -3, 3, color='red')
repry=plot(sol[1].rhs()(t), t, -3, 3, color='blue')
show(reprx+repry)
```

a:

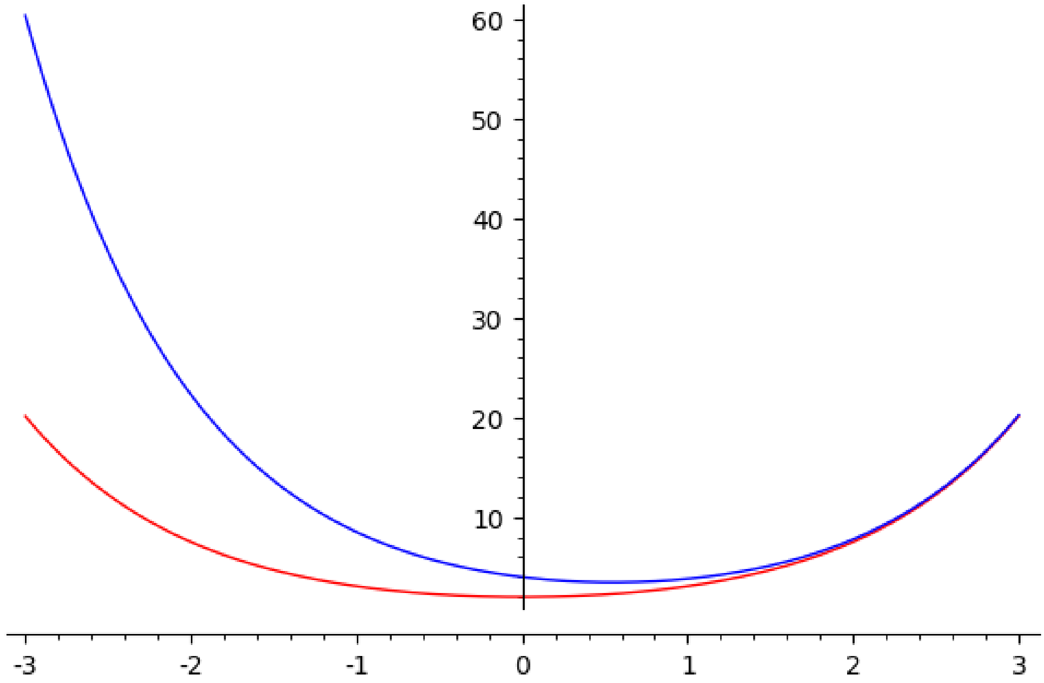
$$x(t) = -\frac{1}{2}\left(C_1 - C_2\right) e^{(-t)} + \frac{1}{2}\left(3\, C_1 - C_2\right) e^{t}$$

$$y(t) = -\frac{3}{2}\left(C_1 - C_2\right) e^{(-t)} + \frac{1}{2}\left(3\, C_1 - C_2\right) e^{t}$$

b:

$$x(t) = e^{(-t)} + e^{t}$$

$$y(t) = 3\, e^{(-t)} + e^{t}$$



In [1]:

```
reset()
t,v=var('t,v')
x=function('x')(t)
x(t)=t*(9-t^2)
print("a:")
eqa=solve(x(t)==0, t)
show(eqa)
t1=eqa[0].rhs()
t2=eqa[1].rhs()
t3=eqa[2].rhs()
show(diff(x,t)(t1) < 0)
print("\t=> t1 este local asimptotic stabil")
show(diff(x,t)(t2) < 0)
print("\t=> t2 este local asimptotic stabil")
show(diff(x,t)(t3) > 0)
print("\t=> t3 este instabil")
print("b:")
y=function('y')(t)
deq=diff(y,t)==y*(9-y^2)
show(deq)
repr1=desolve_rk4(deq, y, [0,0], step=0.01, end_points=[0,2], output='plot')
repr2=desolve_rk4(deq, y, [0,2], step=0.01, end_points=[0,2], output='plot')
repr3=desolve_rk4(deq, y, [0,3], step=0.01, end_points=[0,2], output='plot')
repr4=desolve_rk4(deq, y, [0,4], step=0.01, end_points=[0,2], output='plot')
repr5=desolve_rk4(deq, y, [0,-1], step=0.01, end_points=[0,2], output='plot')
repr6=desolve_rk4(deq, y, [0,-3], step=0.01, end_points=[0,2], output='plot')
repr7=desolve_rk4(deq, y, [0,-4], step=0.01, end_points=[0,2], output='plot')
show(repr1+repr2+repr3+repr4+repr5+repr6+repr7)
```

a:

$$[t = (-3), t = 3, t = 0]$$

$$(-18) < 0$$

=> t1 este local asimptotic stabil

$$(-18) < 0$$

=> t2 este local asimptotic stabil

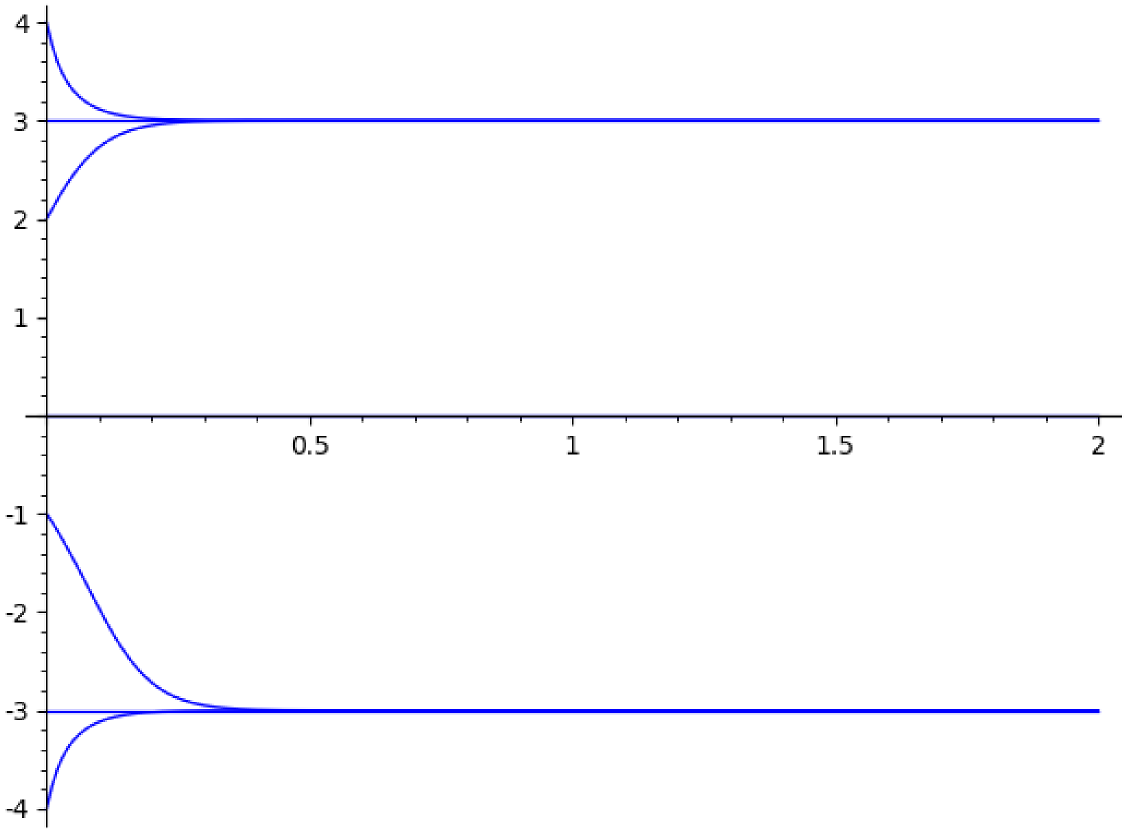
$$9 > 0$$

=> t3 este instabil

b:

$$\frac{\partial}{\partial t} y(t) = -\left(y(t)^2 - 9\right) y(t)$$

$$\frac{\partial}{\partial t} y(t) = -\left(y(t)^2 - 9\right) y(t)$$



In [84]:

```
reset()
x,y,t=var('x,y,t')
f1(x,y)=4*x-x*y^2
f2(x,y)=x-y
print("a:")
sol=solve([f1(x,y)==0,f2(x,y)==0],x,y)
show(sol)
print("b:")
J=jacobian((f1(x,y),f2(x,y)),(x,y))
show(J)
show(J(x=0,y=0).eigenvalues())
show(J(x=-2,y=-2).eigenvalues())
show(J(x=2,y=2).eigenvalues())
```

a:

$$\left[\left[x = 0, y = 0\right], \left[x = \left(-2\right), y = \left(-2\right)\right], \left[x = 2, y = 2\right]\right]$$

b:

$$\begin{pmatrix} -y^2 + 4 & -2\,xy \\ 1 & -1 \end{pmatrix}$$

$$\left[-1, 4\right]$$

$$\left[-\frac{1}{2} i \sqrt{31} - \frac{1}{2}, \frac{1}{2} i \sqrt{31} - \frac{1}{2}\right]$$

$$\left[-\frac{1}{2} i \sqrt{31} - \frac{1}{2}, \frac{1}{2} i \sqrt{31} - \frac{1}{2}\right]$$