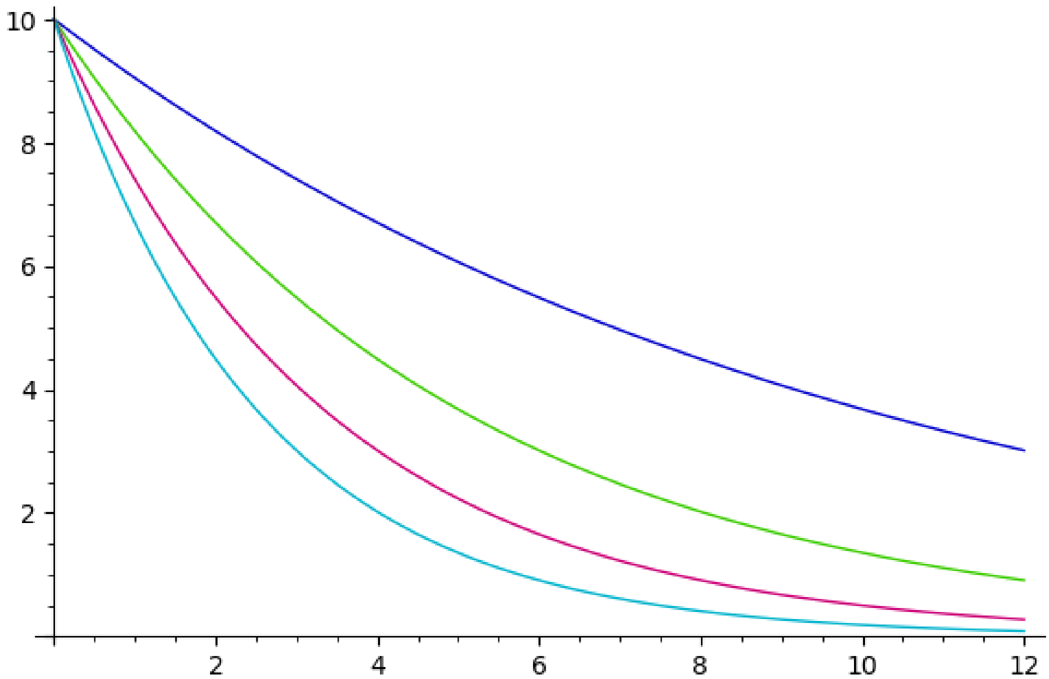
In [162]:

```
t,x0,k=var('t,x0,k')
x=function('x')(t)
print("A:")
equ=diff(x, t)==-k*x(t)
sol(t,k,x0)=desolve(equ, [x, t], [0, x0])
show(sol)
print("B:")
show(plot([sol(t,0.1,10),sol(t,0.2,10),sol(t,0.3,10),sol(t,0.4,10)],t,0,12))
print("C:")
T12=var('T12')
carbon=sol(T12,k,x0)==x0/2
show(carbon)
carbonsol=solve(carbon,k)
show(carbonsol)
k1(T12)=carbonsol[0].rhs()
show(k1)
kC14=k1(5730)
show(kC14)
print("D:")
T=var('T')
equ2=sol(T,kC14,1)==0.2
show(numerical_approx(solve(equ2,T)[0].rhs()))
print("E:")
an1=sol(T,kC14,100)==93.021
an2=sol(T,kC14,100)==91.57
print("%f - %f" % (1988-numerical_approx(solve(an2,T)[0].rhs()), (1988-numerical_approx(
solve(an1,T)[0].rhs()))))
```

A:

$$\left(t, k, x_0\right) \mapsto x_0 e^{(-kt)}$$

B:



C:

$$x_0 e^{\left(-T_{12}k\right)} = \frac{1}{2} x_0$$

$$\left[k = \frac{\log(2)}{T_{12}}\right]$$

$$T_{12} \mapsto \frac{\log(2)}{T_{12}}$$

$$\frac{1}{5730}\log(2)$$

D:

$$13304.6479837046$$

E:
1259.985896 - 1389.950471

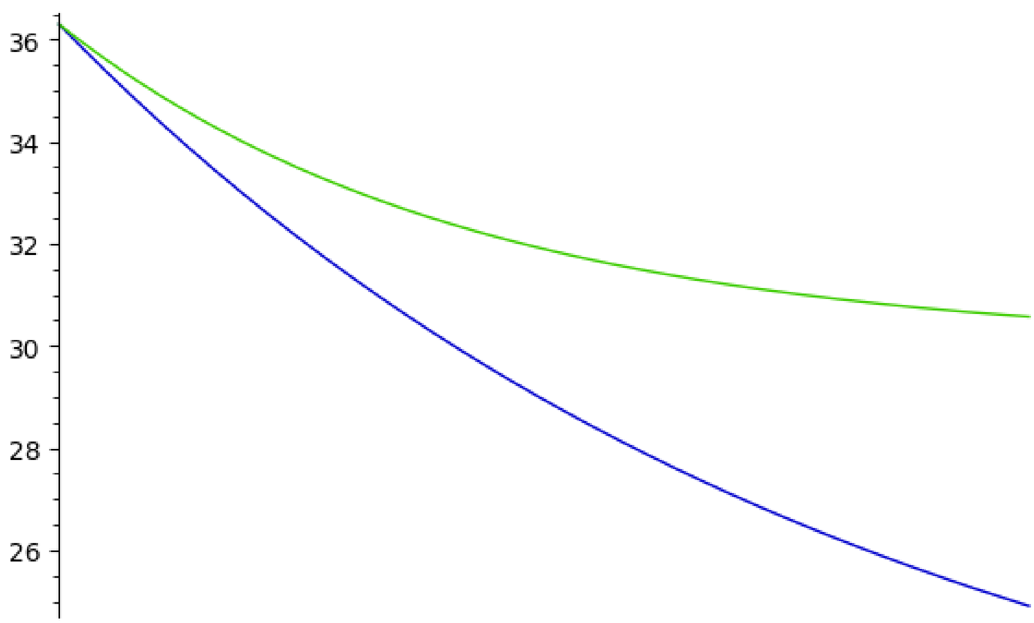
In [10]:

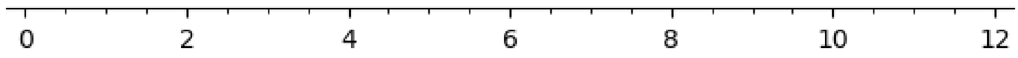
```
t,T0,k,Tm=var('t,T0,k,Tm')
T=function('T')(t)
print("A:")
equ=diff(T, t)==-k*(T(t)-Tm)
sol(t,k,T0,Tm)=desolve(equ, [T, t], [0, T0])
show(sol)
print("B:")
show(plot([sol(t,0.1,36.3,20),sol(t,0.2,36.3,30)],t,0,12))
print("C:")
eq1=sol(1,k,34.22,21)==34.11
ans1=solve(eq1,k)
k1=ans1[0].rhs()
show(k1)
Td=var('Td')
eq2=sol(Td,k1,36.3,21)==34.22
show(find_root(eq2,0,24))
```

A:

$$\left(t, k, T_0, \mathrm{Tm}\right) \mapsto \left(\mathrm{Tm}e^{(kt)} + T_0 - \mathrm{Tm}\right)e^{(-kt)}$$

B:





C:

$$\log\left(\frac{1322}{1311}\right)$$

$$17.488044171407882$$

In [79]:

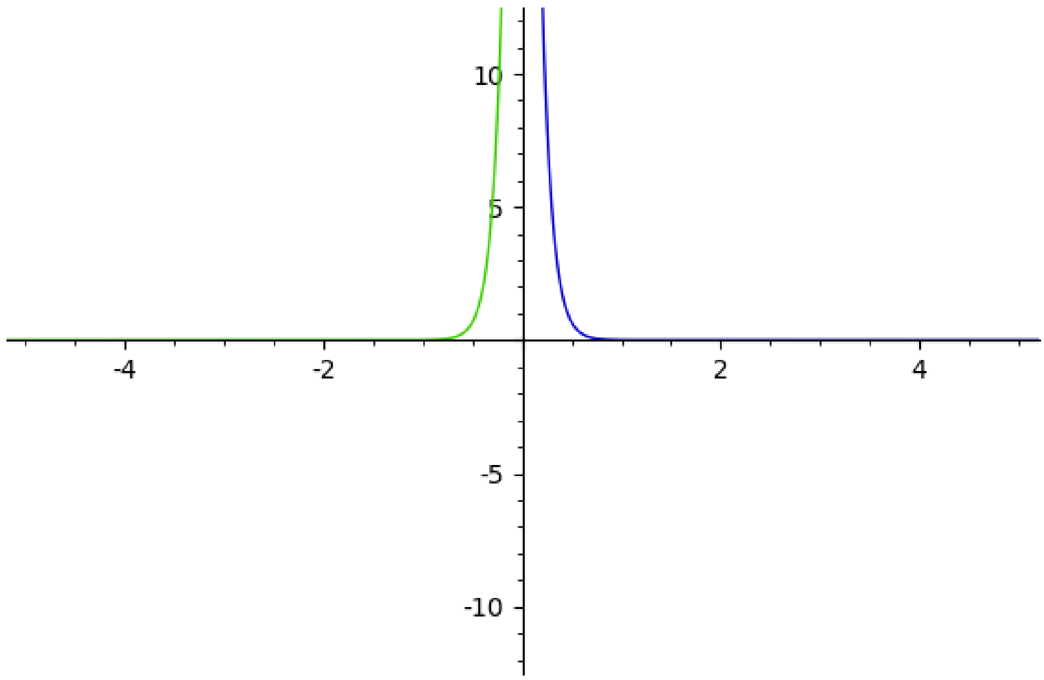
```
t,r,x0,r0,K=var('t,r,x0,r0,K')
x=function('x')(t)
print("A:")
eqMalthaus=diff(x,t)==r*x
solMalthaus(x0,r,t)=desolve(eqMalthaus,x,ics=[0,x0],ivar=t)
show('Solutia generala pentru modelul lui Malthaus:\t', solMalthaus)
eqVerhulst=diff(x,t)==r0*x*floor(1-x/K)
solVerhulst(x0,r0,K,t)=K/(1+(K/r0-1)*e^(-r0*t))
show('Solutia generala pentru modelul lui Verhulst:\t', solVerhulst)
print("B:")
show(plot([solMalthaus(100,-10,t), solMalthaus(100,10,t)],t,-50,50),xmin=-5,xmax=5,ymin=
-12,ymax=12)
print("C:")
eq=solMalthaus(25000,r,2)==30000
show(eq)
r1=solve(eq,r)[1].rhs()
show(r1)
eq2=solMalthaus(25000,r1,5)
show('Populatia estimata in 5 ani cu modelul lui Malthaus:\t', numerical_approx(eq2))
print("D:")
eq2=solVerhulst(20000,r0,K,2)==40000
show(eq2)
eq3=solVerhulst(20000,r0,K,3)==50000
show(eq3)
K1=(solve(eq2,K))[0].rhs()
show(K1)
K2=(solve(eq3,K))[0].rhs()
show(K2)
show(K1==K2)
show(solve(K1==K2,r0))
r0=solve(K1==K2,r0)[1]
show(r0)
eqFin=r0.rhs()-r0
r0=find_root(eqFin,-1000000,1000000)
show('r0:\t', r0)
K3=solVerhulst(20000,r0,K,2)==40000
K3=numerical_approx(solve(K3,K)[0].rhs())
show('K:\t', K3)
eqFin=solVerhulst(20000,r0,K3,7)
show('Populatia estimata in 7 ani cu modelul lui Verhulst:\t', eqFin)
```

A:

$$\text{Solutia generala pentru modelul lui Malthaus:}\left(x_0, r, t\right) \mapsto x_0 e^{(rt)}$$

$$\text{Solutia generala pentru modelul lui Verhulst:}\left(x_0, r_0, K, t\right) \mapsto \frac{K}{\left(\frac{K}{r_0} - 1\right)e^{\left(-r_0 t\right)} + 1}$$

B:



C:

$$25000\, e^{(2r)} = 30000$$

$$\log\left(\frac{1}{5}\sqrt{6}\sqrt{5}\right)$$

Populatia estimata in 5 ani cu modelul lui Malthaus:39436.0241403720

D:

$$\frac{K}{\left(\frac{K}{r_0} - 1\right)e^{\left(-2\, r_0\right)} + 1} = 40000$$

$$\frac{K}{\left(\frac{K}{r_0} - 1\right)e^{\left(-3\, r_0\right)} + 1} = 50000$$

$$\frac{40000\left(r_0 e^{\left(2\, r_0\right)} - r_0\right)}{r_0 e^{\left(2\, r_0\right)} - 40000}$$

$$\frac{50000\left(r_0 e^{\left(3\, r_0\right)} - r_0\right)}{r_0 e^{\left(3\, r_0\right)} - 50000}$$

$$40000\left(r_0 e^{\left(2\, r_0\right)} - r_0\right) \quad 50000\left(r_0 e^{\left(3\, r_0\right)} - r_0\right)$$

$$\frac{\cdots}{r_0 e^{(2 r_0)} - 40000} = \frac{\cdots}{r_0 e^{(3 r_0)} - 50000}$$

$$\left[r_0 = 0, r_0 = \frac{200000}{e^{(2 r_0)} + e^{r_0} + 5} \right]$$

$$r_0 = \frac{200000}{e^{(2 r_0)} + e^{r_0} + 5}$$

r0:5.269441607527001

K:50064.7475719972

Populatia estimata in 7 ani cu modelul lui Verhulst:50064.7475719517

In [ ]: