```
In [19]:
```

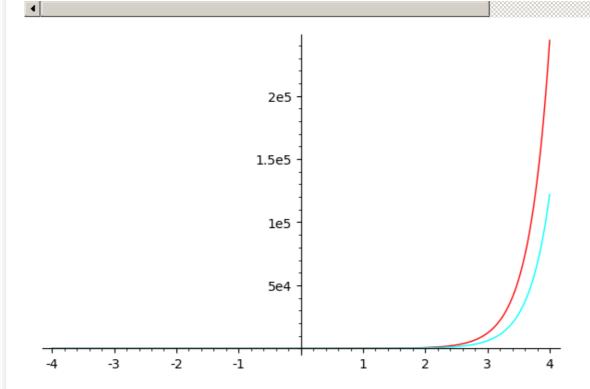
```
t=var('t')
x=function('x')(t)
y=function('y')(t)
eq1=diff(x,t) == x+4*y
eq2=diff(y,t)==x+y
syst=[eq1, eq2]
C1, C2=var('C1, C2')
sol=desolve system(syst, [x, y], [0, C1, C2])
show(sol)
sol x(t,C1,C2)=sol[0].rhs()
show(sol x)
reprx=plot(sol x(t, 1, 1), t, -4, 4, color='red')
sol_y(t,C1,C2) = sol[1].rhs()
show(sol_y)
repry=plot(sol_y(t, 1, 1), t, -4, 4, color='cyan')
show(reprx+repry)
```

$$\left[ x\left( t 
ight) = rac{1}{2} \left( C_1 + 2\,C_2 
ight) \,e^{(3\,t)} \, + rac{1}{2} \left( C_1 - 2\,C_2 
ight) e^{(-t)}, y\left( t 
ight) = rac{1}{4} \left( C_1 + 2\,C_2 
ight) e^{(3\,t)} \, - rac{1}{4} \left( C_1 - 2\,C_2 
ight) e^{(-t)} 
ight]$$

 $(t,C_1,C_2) \\ \mapsto$ 

<del>. , , .</del>

 $(t,C_1,C_2) \qquad \qquad \mapsto \qquad$ 



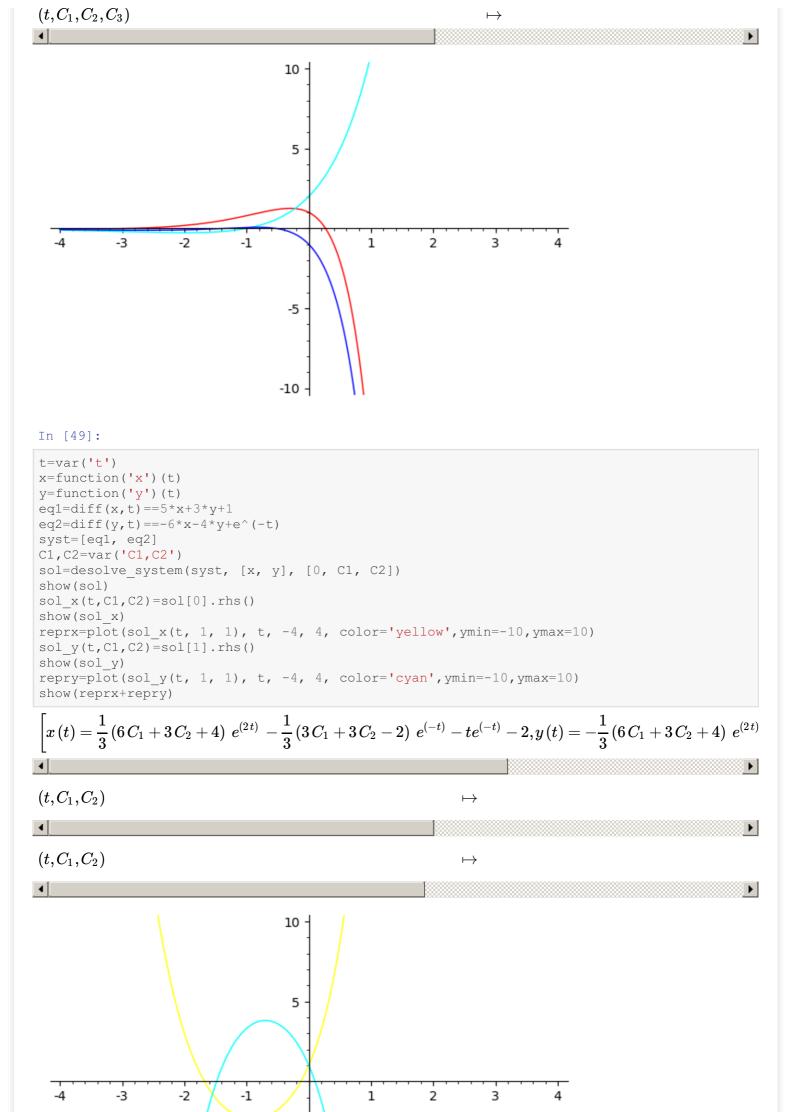
## In [48]:

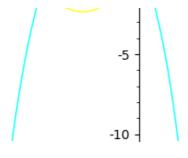
```
t=var('t')
x=function('x')(t)
y=function('y')(t)
eq1=diff(x,t)==2*x-y
eq2=diff(y,t)==x+2*y
syst=[eq1, eq2]
C1,C2=var('C1,C2')
sol=desolve_system(syst, [x, y], [0, C1, C2])
show(sol)
sol_x(t,C1,C2)=sol[0].rhs()
show(sol_x)
reprx=plot(sol_x(t, 1, 1), t, -4, 4, color='yellow')
```

```
show(sol_y)
repry=plot(sol_y(t, 1, 1), t, -4, 4, color='cyan')
show(reprx+repry)
                   \left|x\left(t
ight)=\left(C_{1}\cos(t)-C_{2}\sin(t)
ight)\,e^{\left(2\,t
ight)}\,,y\left(t
ight)=\left(C_{2}\cos(t)+C_{1}\sin(t)
ight)\,e^{\left(2\,t
ight)}\,
ight|
(t, C_1, C_2)
(t, C_1, C_2)
                                                                            \mapsto
             -3
                         -2
                                    -1
                                                          1
                                        -1000
                                        -2000
                                        -3000
                                        -4000
In [40]:
t=var('t')
x = function('x')(t)
y=function('y')(t)
z=function('z')(t)
eq1=diff(x,t)==x-y+z
eq2=diff(y,t)==x+y-z
eq3=diff(z,t)==-y+2*z
syst=[eq1, eq2, eq3]
C1, C2, C3=var('C1, C2, C3')
sol=desolve system(syst, [x, y, z], [0, C1, C2, C3])
show(sol)
sol x(t,C1,C2,C3) = sol[0].rhs()
show(sol x)
reprx=plot(sol x(t, 1, 2, -1), t, -4, 4, color='red', ymin=-10, ymax=10)
sol_y(t,C1,C2,C3) = sol[1].rhs()
show(sol_y)
repry=plot(sol_y(t, 1, 2, -1), t, -4, 4, color='cyan',ymin=-10,ymax=10)
sol z(t,C1,C2,C3) = sol[2].rhs()
show(sol z)
reprz=plot(sol_z(t, 1, 2, -1), t, -4, 4, color='blue', ymin=-10, ymax=10)
show(reprx+repry+reprz)
\left| x\left( t 
ight) = {C_1}t{e^t} - {C_3}t{e^t} - \left( {{C_1} + {C_2} - 2\,{C_3}} 
ight)\,\,{e^{\left( {2\,t} 
ight)}} \,+ \left( {2\,{C_1} + {C_2} - 2\,{C_3}} 
ight)\,\,{e^t},y\left( t 
ight) = {C_1}t{e^t} - {C_3}t{e^t} + {C_2}{e^t},z\left( t 
ight) = {C_1}t{e^t} + {C_3}t{e^t} + {C_2}t{e^t}
(t, C_1, C_2, C_3)
                                                                                \mapsto
```

 $sol_y(t,C1,C2) = sol[1].rhs()$ 

 $(t, C_1, C_2, C_3)$ 





### In [47]:

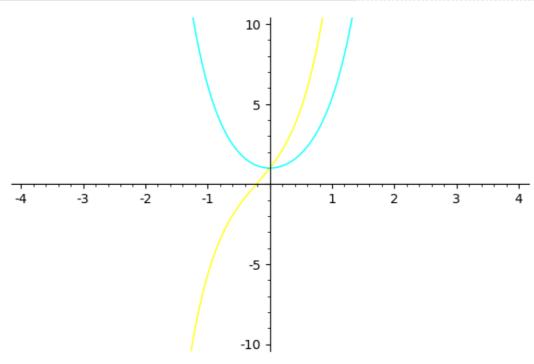
```
t=var('t')
x=function('x')(t)
y=function('y')(t)
eq1=diff(x,t) == x+3*y+cos(t)
eq2=diff(y,t)==x-y+2*t
syst=[eq1, eq2]
C1, C2=var('C1, C2')
sol=desolve_system(syst, [x, y], [0, C1, C2])
show(sol)
sol x(t,C1,C2)=sol[0].rhs()
show(sol x)
reprx=plot(sol_x(t, 1, 1), t, -4, 4, color='yellow',ymin=-10,ymax=10)
sol_y(t,C1,C2) = sol[1].rhs()
show(sol y)
repry=plot(sol_y(t, 1, 1), t, -4, 4, color='cyan',ymin=-10,ymax=10)
show(reprx+repry)
```

$$\left[x\left(t\right) = \frac{3}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right. \\ \left. + \frac{1}{40}\left(10\,C_1 - 30\,C_2 - 19\right)\,e^{(-2\,t)} \right. \\ \left. - \frac{3}{2}\,t - \frac{1}{5}\cos(t) + \frac{1}{5}\sin(t), y\left(t\right) = \frac{3}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 - 30\,C_2 - 19\right)\,e^{(-2\,t)} \right] \\ \left. + \frac{1}{5}\cos(t) + \frac{1}{5}\sin(t), y\left(t\right) = \frac{3}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 - 30\,C_2 - 19\right)\,e^{(-2\,t)} \right] \\ \left. + \frac{1}{5}\cos(t) + \frac{1}{5}\sin(t), y\left(t\right) = \frac{3}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \right] \\ \left. + \frac{1}{40}\left(10\,C_1 + 10\,C_2 + 9\right)\,e^{(2\,t)} \\ \left. + \frac$$

 $\blacksquare$ 

 $(t,C_1,C_2)$  $\mapsto$ 

 $(t,C_1,C_2)$  $\mapsto$ 



## In [46]:

```
t=var('t')
x=function('x')(t)
y=function('y')(t)
z=function('z')(t)
```

```
eq1=diff(x,t) == x-2*y-2*z+e^(-t)
eq2=diff(y,t)==-2*x+y+2*z
eq3=diff(z,t) == 2*x-y-3*z+e^(-t)
syst=[eq1, eq2, eq3]
C1, C2, C3=var('C1, C2, C3')
sol=desolve_system(syst, [x, y, z], [0, C1, C2, C3])
sol x(t,C1,C2,C3) = sol[0].rhs()
show(sol x)
reprx=plot(sol x(t, 1, 2, -1), t, -4, 4, color='red', ymin=-10, ymax=10)
sol_y(t,C1,C2,C3) = sol[1].rhs()
show(sol y)
repry=plot(sol y(t, 1, 2, -1), t, -4, 4, color='cyan', ymin=-10, ymax=10)
sol_z(t,C1,C2,C3) = sol[2].rhs()
show(sol z)
reprz=plot(sol z(t, 1, 2, -1), t, -4, 4, color='blue', ymin=-10, ymax=10)
show(reprx+repry+reprz)
\int x(t) = rac{1}{3} \sqrt{3} (2\,C_1 - C_2 - 2\,C_3) \ \sinh(\sqrt{3}t) - C_2 \cosh(\sqrt{3}t) + (C_1 + C_2) \, e^{(-t)} + t e^{(-t)}, y(t) = -rac{1}{3} \sqrt{3} (2\,C_1) + t e^{(-t)}
(t, C_1, C_2, C_3)
                                                                     \mapsto
(t,C_1,C_2,C_3)
                                                                     \mapsto
(t, C_1, C_2, C_3)
                                                                     \mapsto
                                      10
                                       5
            -3
                     -2
                                                                      3
                                                            2
```

#### In [55]:

```
t=var('t')
x=function('x')(t)
y=function('y')(t)
eq1=diff(x,t)==x+4*y
eq2=diff(y,t)==x+y
syst=[eq1, eq2]
sol=desolve_system(syst, [x, y], [0, 1, 2])
show(sol)
sol_x(t)=sol[0].rhs()
show(sol_x)
reprx=plot(sol_x(t), t, -4, 4, color='red', ymin=-10, ymax=10)
sol_y(t)=sol[1].rhs()
```

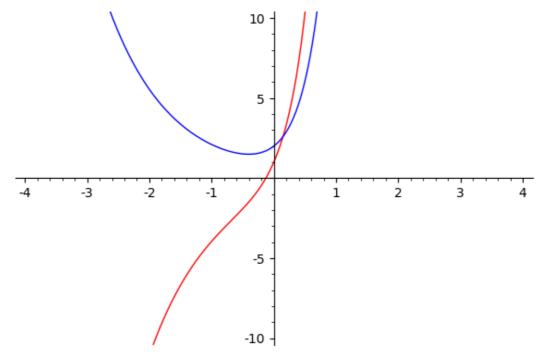
show(sol\_y)
repry=plot(sol\_y(t), t, -4, 4, color='blue',ymin=-10,ymax=10)
show(reprx+repry)

$$\left[ x\left( t
ight) =rac{5}{2}e^{\left( 3\,t
ight) }\,\,-rac{3}{2}e^{\left( -t
ight) },y\left( t
ight) =rac{5}{4}e^{\left( 3\,t
ight) }\,\,+rac{3}{4}e^{\left( -t
ight) }
ight]$$

 $t \mapsto$ 

1

 $t \mapsto$ 



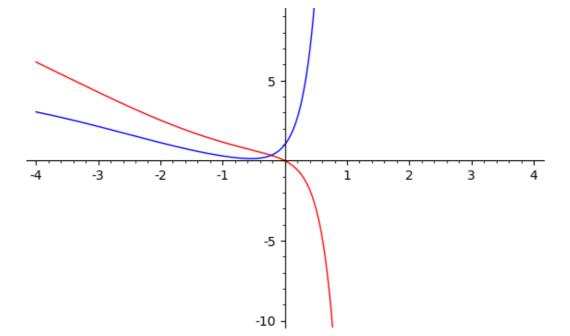
# In [57]:

```
t=var('t')
x=function('x')(t)
y=function('y')(t)
eq1=diff(x,t)==x-y+t-1
eq2=diff(y,t)==-2*x+4*y+cos(t)
syst=[eq1, eq2]
sol=desolve_system(syst, [x, y], [0, 0, 1])
show(sol)
sol_x(t)=sol[0].rhs()
show(sol_x)
reprx=plot(sol_x(t), t, -4, 4, color='red', ymin=-10, ymax=10)
sol_y(t)=sol[1].rhs()
show(sol_y)
repry=plot(sol_y(t), t, -4, 4, color='blue', ymin=-10, ymax=10)
show(reprx+repry)
```

$$\left[x\left(t\right)=-\frac{1}{13}\left(10\sqrt{17}\sinh\left(\frac{1}{2}\sqrt{17}t\right)-33\cosh\left(\frac{1}{2}\sqrt{17}t\right)\right)\ e^{\left(\frac{5}{2}t\right)}\ -2\,t-\frac{1}{26}\cos(t)+\frac{5}{26}\sin(t)-\frac{5}{2},y\left(\frac{5}{2}t\right)\right]$$

1

 $t \mapsto$ 



# In [64]:

```
t=var('t')
x=function('x')(t)
y=function('y')(t)
eq1=diff(x,t)==x+2*y+e^(-t)
eq2=diff(y,t)==-2*x+y+1
syst=[eq1, eq2]
sol=desolve_system(syst, [x, y], [0, 0, 1])
show(sol)
sol_x(t)=sol[0].rhs()
show(sol_x)
reprx=plot(sol_x(t), t, -4, 4, color='red', ymin=-30, ymax=30)
sol_y(t)=sol[1].rhs()
show(sol_y)
repry=plot(sol_y(t), t, -4, 4, color='blue', ymin=-30, ymax=40)
show(reprx+repry)
```

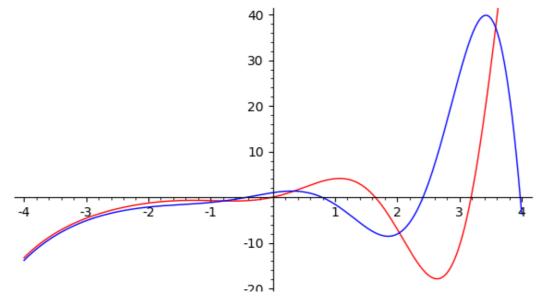
$$\left[x\left(t\right) = -\frac{1}{20}\left(3\cos(2\,t) - 29\sin(2\,t)\right)\ e^t - \frac{1}{4}\,e^{\left(-\,t\right)} + \frac{2}{5}, y\left(t\right) = \frac{1}{20}\left(29\cos(2\,t) + 3\sin(2\,t)\right)\ e^t - \frac{1}{4}\,e^{\left(-\,t\right)} - \frac{1}{5}\cos(2\,t)\right]\right] + \frac{1}{20}\left(29\cos(2\,t) + 3\sin(2\,t)\right) \left(-\frac{1}{4}\,e^{\left(-\,t\right)} - \frac{1}{5}\cos(2\,t)\right) \left(-\frac{1}{20}\cos(2\,t) + \frac{1}{20}\cos(2\,t)\right) \left(-\frac{1}{4}\,e^{\left(-\,t\right)} - \frac{1}{5}\cos(2\,t)\right) \left(-\frac{1}{4}\,e^{\left(-\,t\right)} - \frac{1}{5}\cos(2\,t)\right) \left(-\frac{1}{20}\cos(2\,t) + \frac{1}{20}\cos(2\,t)\right) \left(-\frac{1}{4}\,e^{\left(-\,t\right)} - \frac{1}{5}\cos(2\,t)\right) \left(-\frac{1}{20}\cos(2\,t) + \frac{1}{20}\cos(2\,t)\right) \left(-\frac{1}{20}\cos(2\,t)\right) \left(-\frac{1$$

t

 $\mapsto$ 

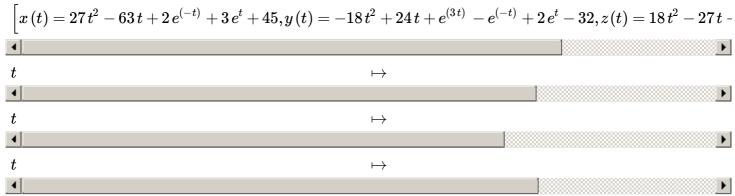
1

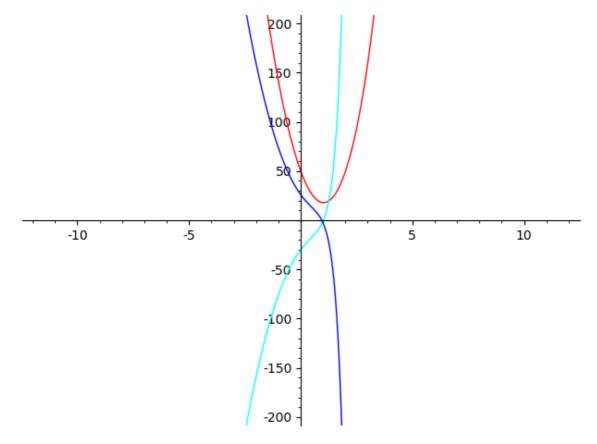
 $\mapsto$ 



#### In [1]:

```
t=var('t')
x = function('x')(t)
y=function('y')(t)
z=function('z')(t)
eq1=diff(x,t) ==-x+3*y+3*z+27*t^2
eq2=diff(y,t) == 2 \times x - 2 \times y - 5 \times z + 3 \times t
eq3=diff(z,t) == -2 \times x + 3 \times y + 6 \times z + 3
syst=[eq1, eq2, eq3]
sol=desolve\_system(syst, [x, y, z], [0, 50, -30, 26])
show(sol)
sol x(t) = sol[0].rhs()
show(sol x)
reprx=plot(sol x(t), t, -12, 12, color='red', ymin=-200, ymax=200)
sol y(t) = sol[1].rhs()
show(sol_y)
repry=plot(sol_y(t), t, -12, 12, color='cyan', ymin=-200, ymax=200)
sol_z(t) = sol[2].rhs()
show(sol z)
reprz=plot(sol z(t), t, -12, 12, color='blue', ymin=-200, ymax=200)
show(reprx+repry+reprz)
```





```
t=var('t')
x=function('x')(t)
y=function('y')(t)
eql=diff(x,t)==x+y
eq2=diff(y,t)==-2*x+4*y
sol1=desolve_system([eq1, eq2], [x,y], [0,3,0])
show(sol1)
show(limit(sol1[0].rhs(),t=infinity), limit(sol1[1].rhs(),t=infinity))
sol2=desolve_system([eq1, eq2], [x,y], [0,0,3])
show(sol2)
show(limit(sol2[0].rhs(),t=infinity), limit(sol2[1].rhs(),t=infinity))
sol3=desolve_system([eq1, eq2], [x,y], [0,-3,0])
show(sol3)
show(limit(sol3[0].rhs(),t=infinity), limit(sol3[1].rhs(),t=infinity))
sol4=desolve_system([eq1, eq2], [x,y], [0,0,-3])
show(sol4)
show(limit(sol4[0].rhs(),t=infinity), limit(sol4[1].rhs(),t=infinity))
```

$$egin{align} \left[x\left(t
ight) = -3\,e^{(3\,t)} \,+ 6\,e^{(2\,t)}\,, y\left(t
ight) = -6\,e^{(3\,t)} \,+ 6\,e^{(2\,t)}\,
ight] \ &-\infty - \infty \ \left[x\left(t
ight) = 3\,e^{(3\,t)} \,- 3\,e^{(2\,t)}\,, y\left(t
ight) = 6\,e^{(3\,t)} \,- 3\,e^{(2\,t)}\,
ight] \ &+\infty + \infty \ \left[x\left(t
ight) = 3\,e^{(3\,t)} \,- 6\,e^{(2\,t)}\,, y\left(t
ight) = 6\,e^{(3\,t)} \,- 6\,e^{(2\,t)}\,
ight] \ &+\infty + \infty \ \left[x\left(t
ight) = -3\,e^{(3\,t)} \,+ 3\,e^{(2\,t)}\,, y\left(t
ight) = -6\,e^{(3\,t)} \,+ 3\,e^{(2\,t)}\,
ight] \ &-\infty - \infty \ \end{pmatrix}$$

## In [17]:

```
t,C1,C2=var('t,C1,C2')
x=function('x')(t)
y=function('y')(t)
eq1=diff(x,t)==y
eq2=diff(y,t)==-x-2*y
sol=desolve_system([eq1, eq2], [x,y],[0,C1,C2])
show(sol)
show(limit(sol[0].rhs(),t=infinity))
show(limit(sol[1].rhs(),t=infinity))
```

$$\left[ x\left( t 
ight) = {C_1}t{e^{\left( - t 
ight)}} + {C_2}t{e^{\left( - t 
ight)}} + {C_1}{e^{\left( - t 
ight)}}, y\left( t 
ight) = - {C_1}t{e^{\left( - t 
ight)}} - {C_2}t{e^{\left( - t 
ight)}} + {C_2}{e^{\left( - t 
ight)}} 
ight]$$