

1. The table below shows a data set \mathbf{Z} with 10 objects described by 4-dimensional feature vector.

Object	x_1	x_2	x_3	x_4	Label
1	5	2	1	4	3
2	6	1	4	0	2
3	-5	1	-4	7	2
4	0	1	3	2	2
5	6	2	1	-5	1
6	4	2	1	7	1
7	-6	3	3	1	2
8	0	2	1	8	3
9	-4	11	5	2	2
10	4	7	2	2	3

(a) What are C and N ?

(b) Consider a classifier G , which consists of the following set of discriminant functions:

$$g_1(\mathbf{x}) = x_1 + x_2$$

$$g_2(\mathbf{x}) = x_2 + 2x_3$$

$$g_3(\mathbf{x}) = 3x_4 - x_3$$

Run G on \mathbf{Z} and give the (guessed) labels of all the objects. For instance, the discriminant functions for object 1 are:

$$g_1(\mathbf{x}) = x_1 + x_2 = 5 + 2 = 7$$

$$g_2(\mathbf{x}) = x_2 + 2x_3 = 2 + 2 \cdot 1 = 4$$

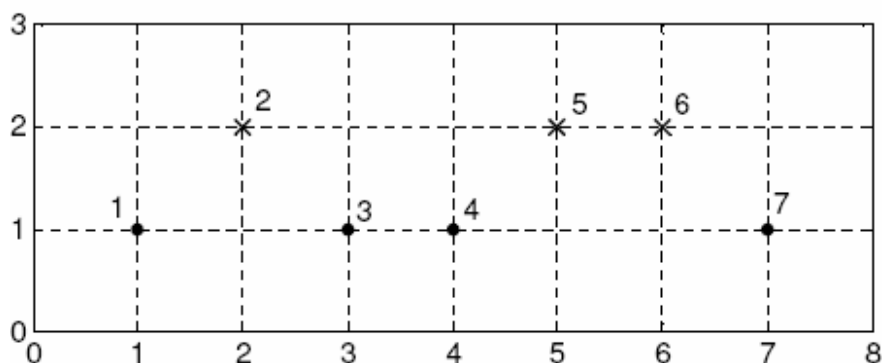
$$g_3(\mathbf{x}) = 3x_4 - x_3 = 3 \cdot 4 - 1 = 11$$

As $\max(g_1, g_2, g_3) = g_3$, then the class label predicted for object 1 is 3.

(c) Give the confusion matrix of G and calculate the error of the classifier G .

(d) Estimate the prior probabilities for the classes from \mathbf{Z} .

2. Consider the classification problem shown in the figure below. The data is depicted as points and the (true) class labels are shown by different markers: class 1 with dots and class 2 with crosses. The object numbers are given next to the markers. Your classifier operates in the following way: for a point \mathbf{x} that you want to classify, find the nearest point in \mathbf{Z} . Assign \mathbf{x} to the class of its nearest neighbour (this is called *the nearest neighbour classifier*). In the case of a tie, resolve in favour of the class with the lower number (class 1 here).



(a) Find the leave-one-out error of the nearest neighbour classifier.

(b) Consider the following experiment. Stage1: Objects 1,2,3 and 4 are used for “training” (they form the data set from which we select the nearest neighbour) and objects 5,6 and 7 are used for testing. Stage 2: Objects 1,3,5,7 are used for training and objects 2,4,6 are used for testing. Stage 3: objects 3,4,5 and 6 are used for training and objects 1,2, and 7 are used for testing. The errors on the three testing sets are averaged to obtain an estimate of the error of the classifier. Find this estimate.

(c) Which training-testing protocol have we used in **(b)**?