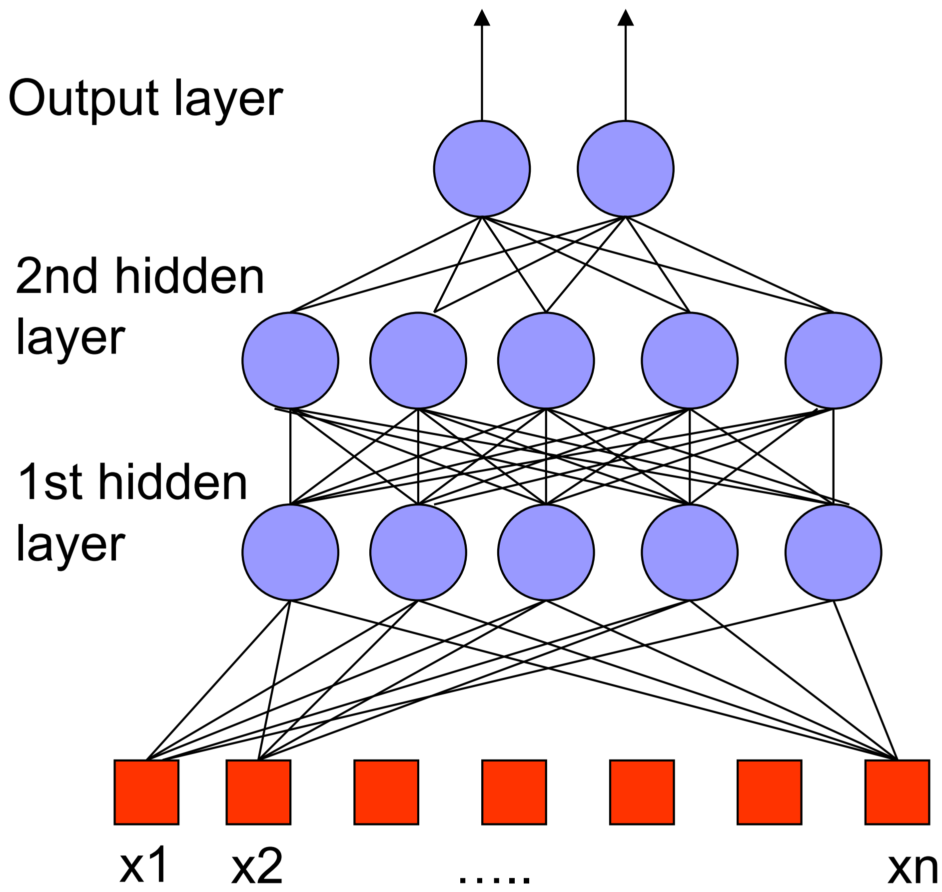
Introduction to Neural Networks

Neural Networks

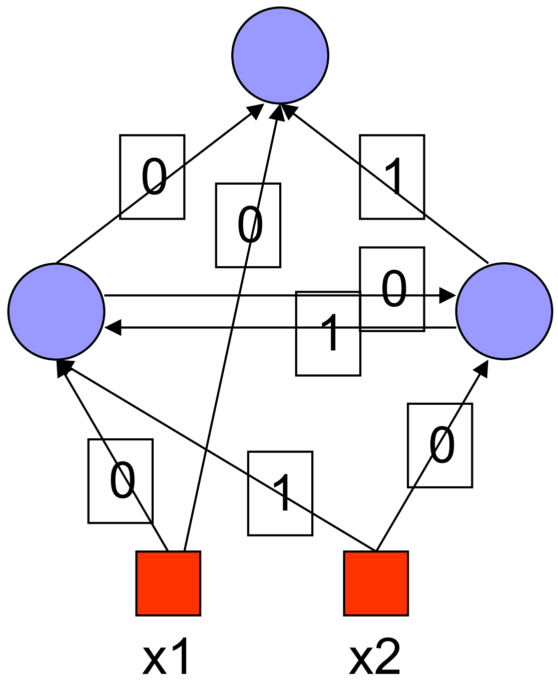
- A mathematical model to solve engineering problems
  - Group of connected neurons to realize compositions of non linear functions
- Tasks
  - Classification
  - Discrimination
  - Estimation
- 2 types of networks
  - Feed forward Neural Networks
  - Recurrent Neural Networks

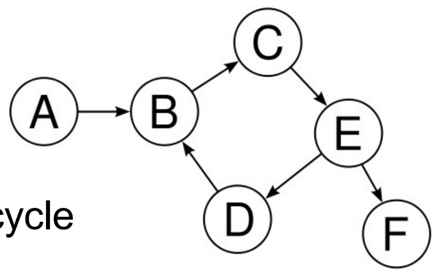
Feed Forward Neural Networks



- The information is propagated from the inputs to the outputs
- Computations of functions from n input variables by compositions of functions
- Time has no role (NO cycle between outputs and inputs)

Recurrent Neural Networks





A graph with a cycle

- Can have arbitrary topologies. i.e. connections between units form a cycle.
- Can model systems with internal states (dynamic ones)
- Delays are associated to a specific weight
- Training is more difficult
- Performance may be problematic
  - Stable Outputs may be more difficult to evaluate
  - Unexpected behavior (oscillation, chaos, …)

Properties of Neural Networks

- Supervised networks are universal approximators
- Theorem : Any limited function can be approximated by a neural network with a finite number of hidden neurons to an arbitrary precision

Supervised Learning

- The desired response of the neural network in function of particular inputs is well known.
- A “teacher” may provide examples and teach the neural network how to fulfill a certain task

Unsupervised learning

- Idea : group typical input data in function of resemblance criteria un-known a priori
- Data clustering
- No need of a "teacher"
  - The network finds itself the correlations between the data
  - Examples of such networks :
    - Self-Organizing Feature Maps (SOM)

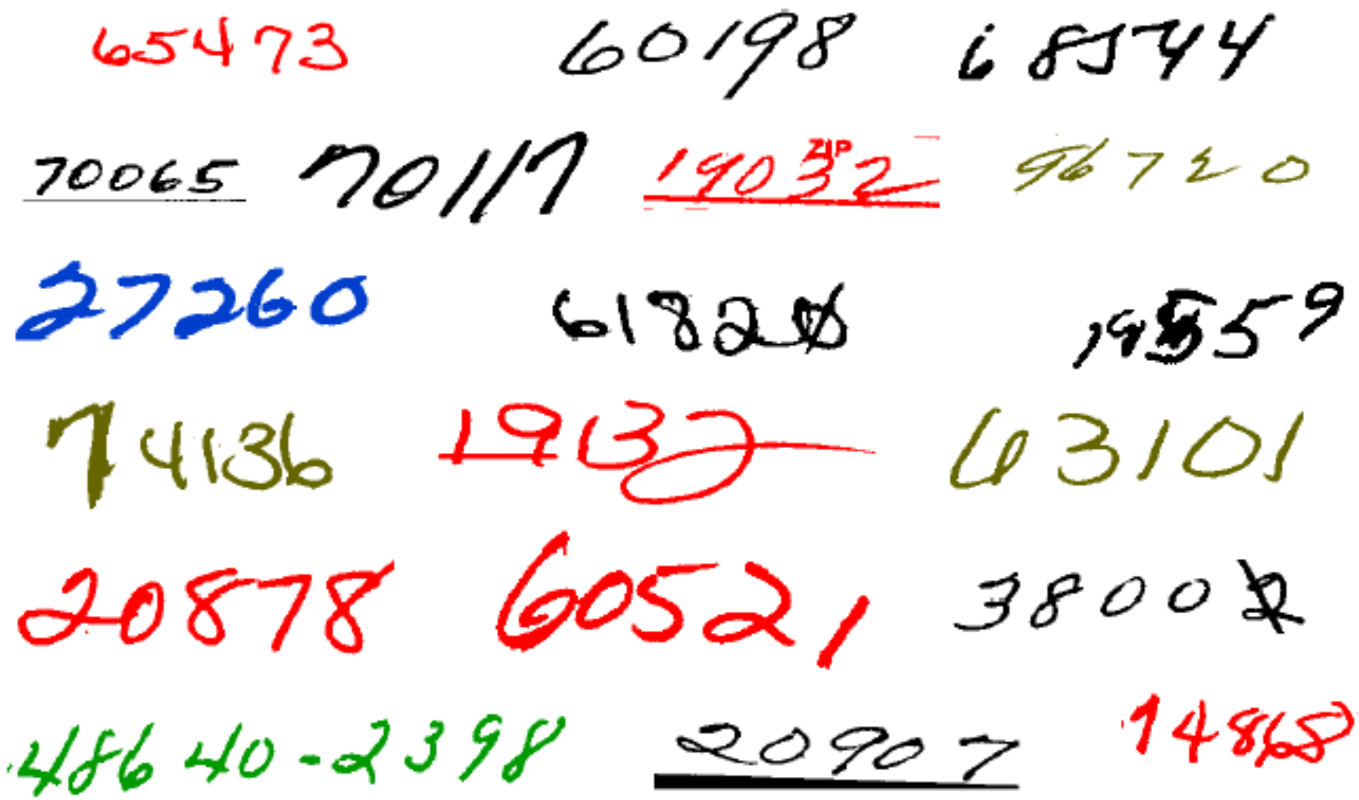
Semi-Supervised Learning

- Halfway between supervised and unsupervised
- The desired response of the neural network is known only for a subset of the inputs, together with unlabeled data.

Reinforcement Learning

- The desired response of the neural network in function of particular inputs is not known.
- Only a *reward* or *penalty* value is provided with the input examples
- *Q-learning:* delayed rewards

Example



Examples of handwritten postal codes
drawn from a database available from the US Postal service

What is needed to create a NN ?

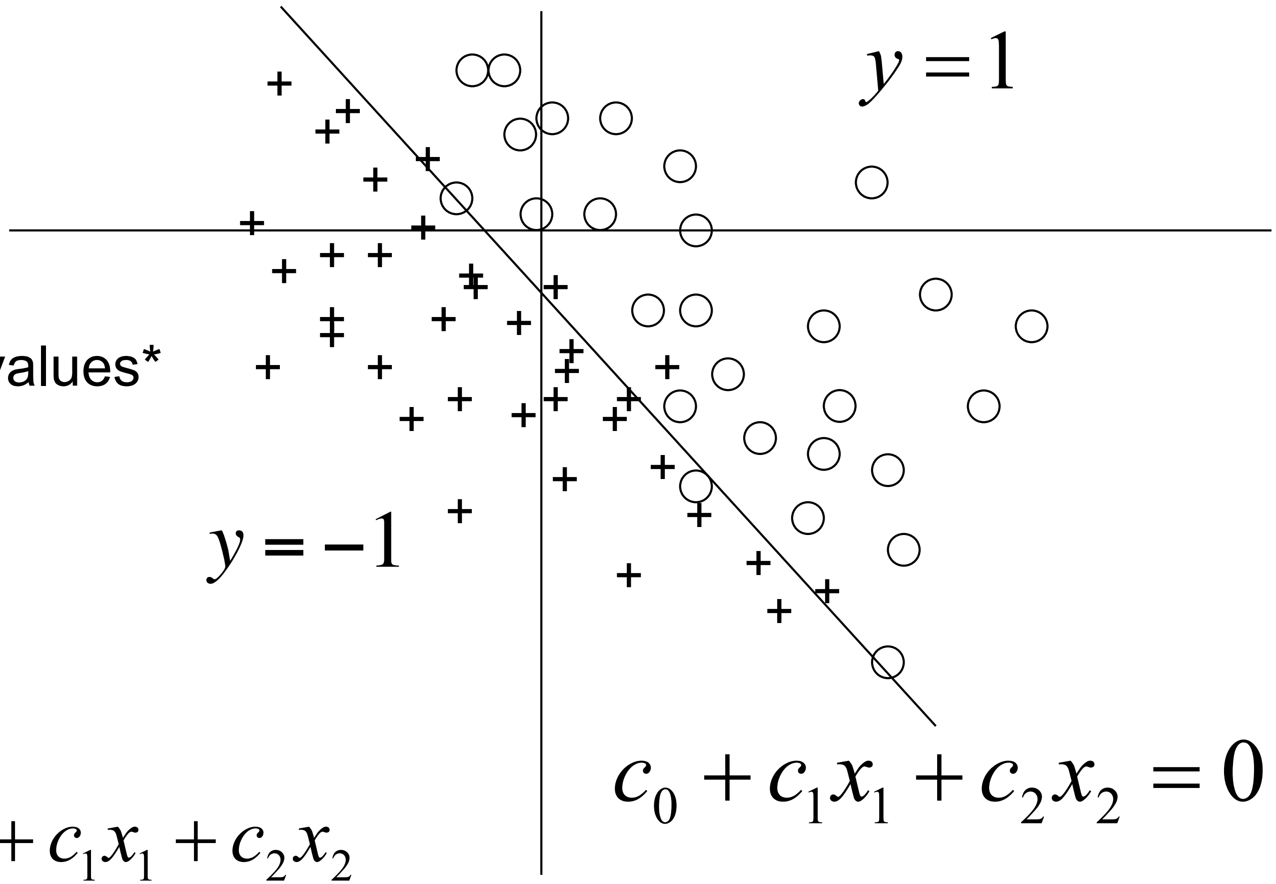
- Determination of relevant inputs
- Collection of data for the learning and testing phases of the neural network
- Finding the optimum number of hidden nodes
- Learning the parameters
- Evaluate the performances of the network
- If performances are not satisfactory then review all the precedent points

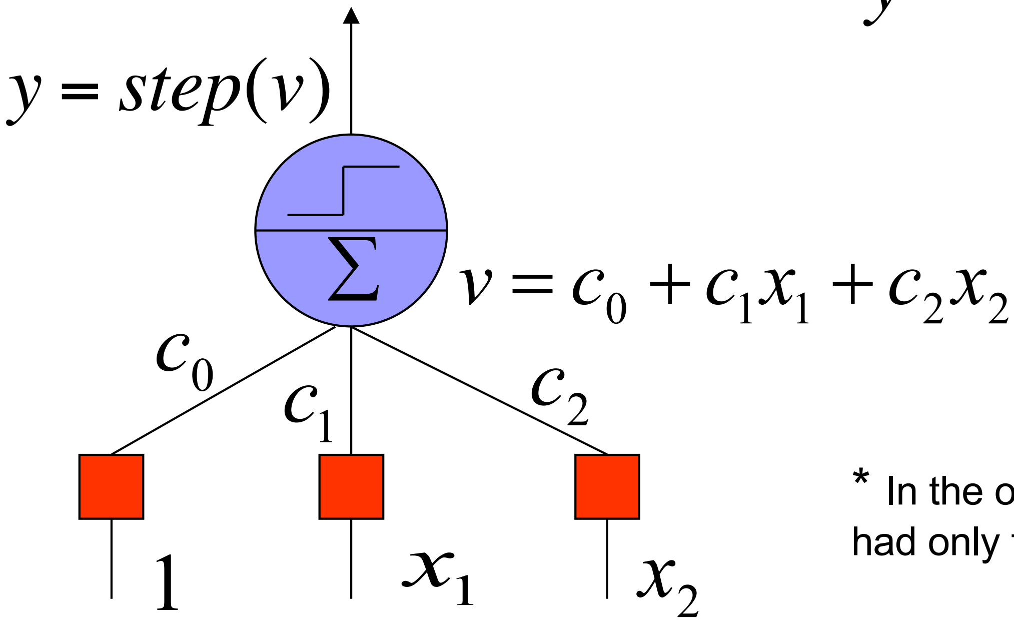
Popular neural architectures

- Perceptron
- Multi-Layer Perceptron (MLP)
- Radial Basis Function Network (RBFN)
- Self-Organizing Feature Maps (SOM)
- Other architectures

Perceptron

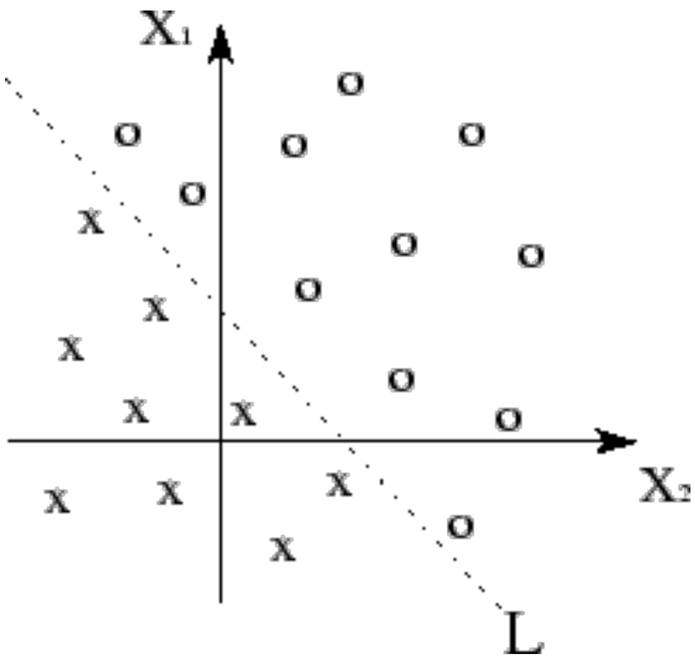
- Rosenblatt (1962)
- Linear separation
- Inputs :Vector of real values*
- Outputs :1 or -1





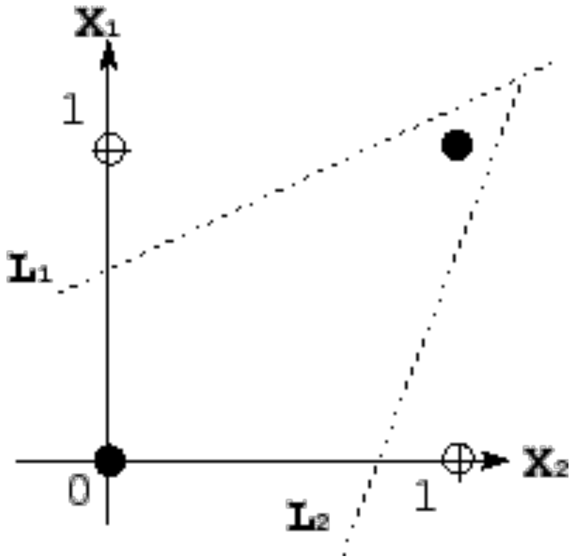
* In the original definition of perceptron inputs typically had only two states: ON and OFF

- The perceptron algorithm converges if examples are linearly separable

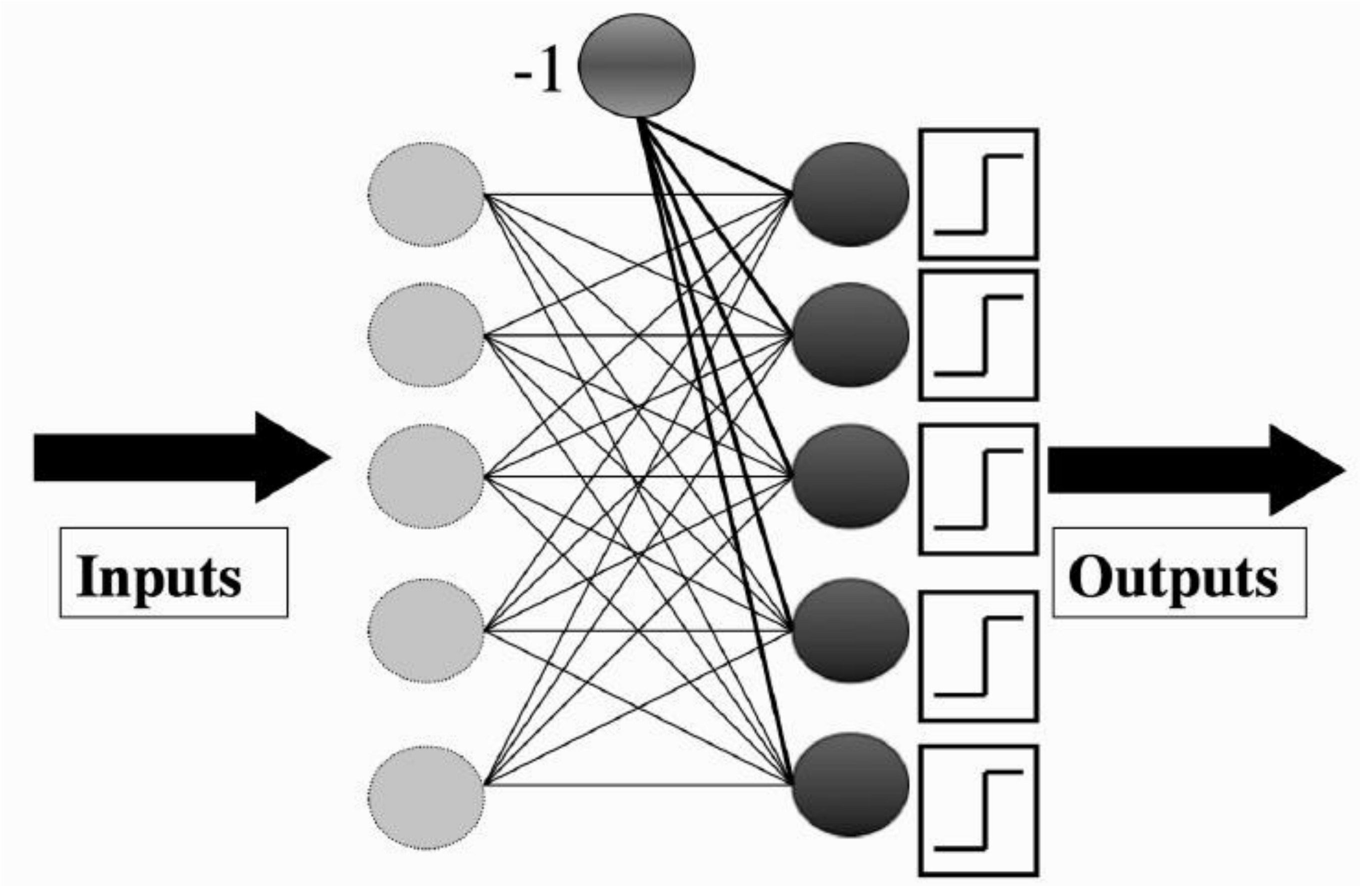


X_1	X_2	Y
0	0	0
0	1	1
1	0	1
1	1	0

$$Y = X_1 \oplus X_2$$

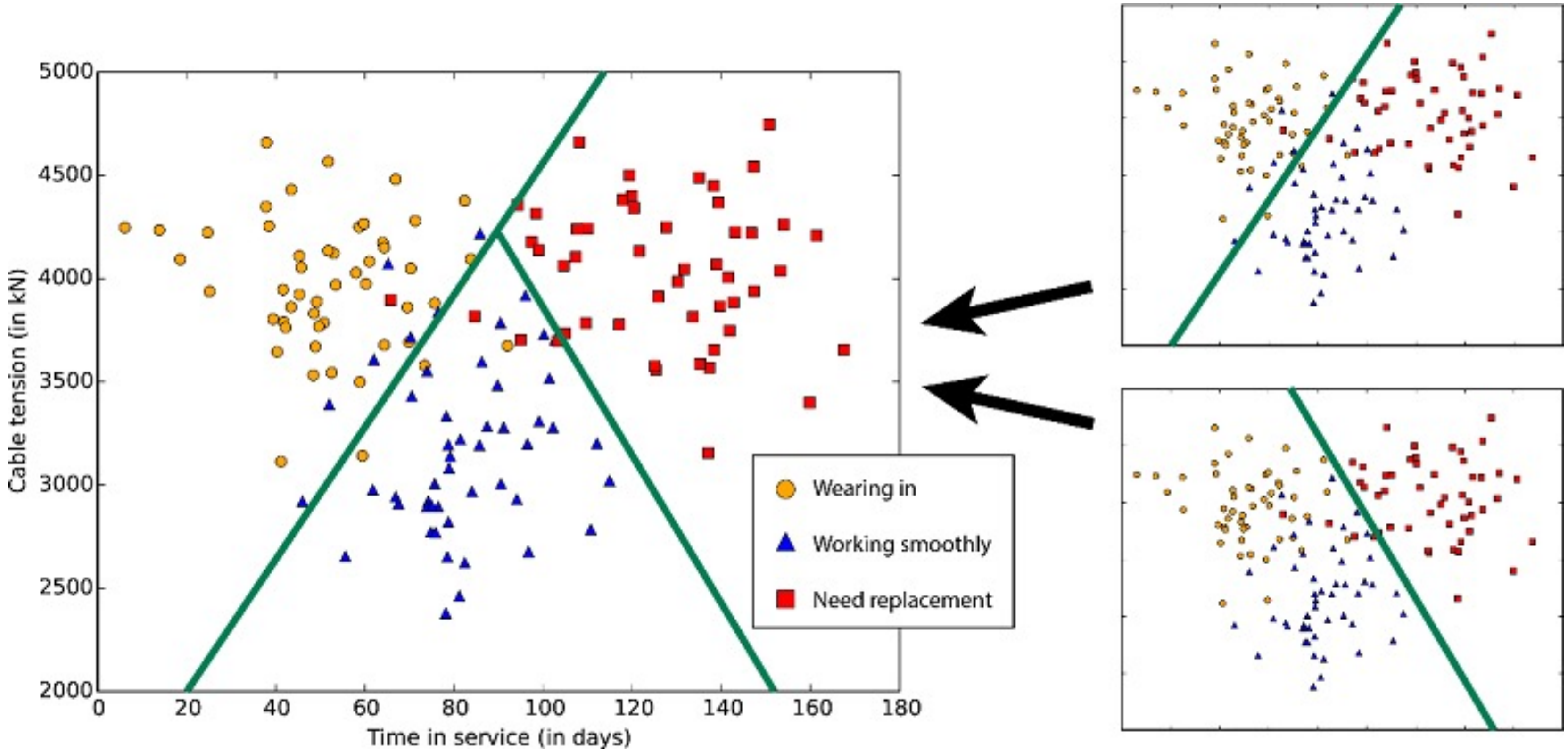


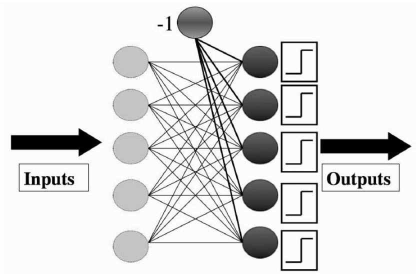
Perceptron with several outputs



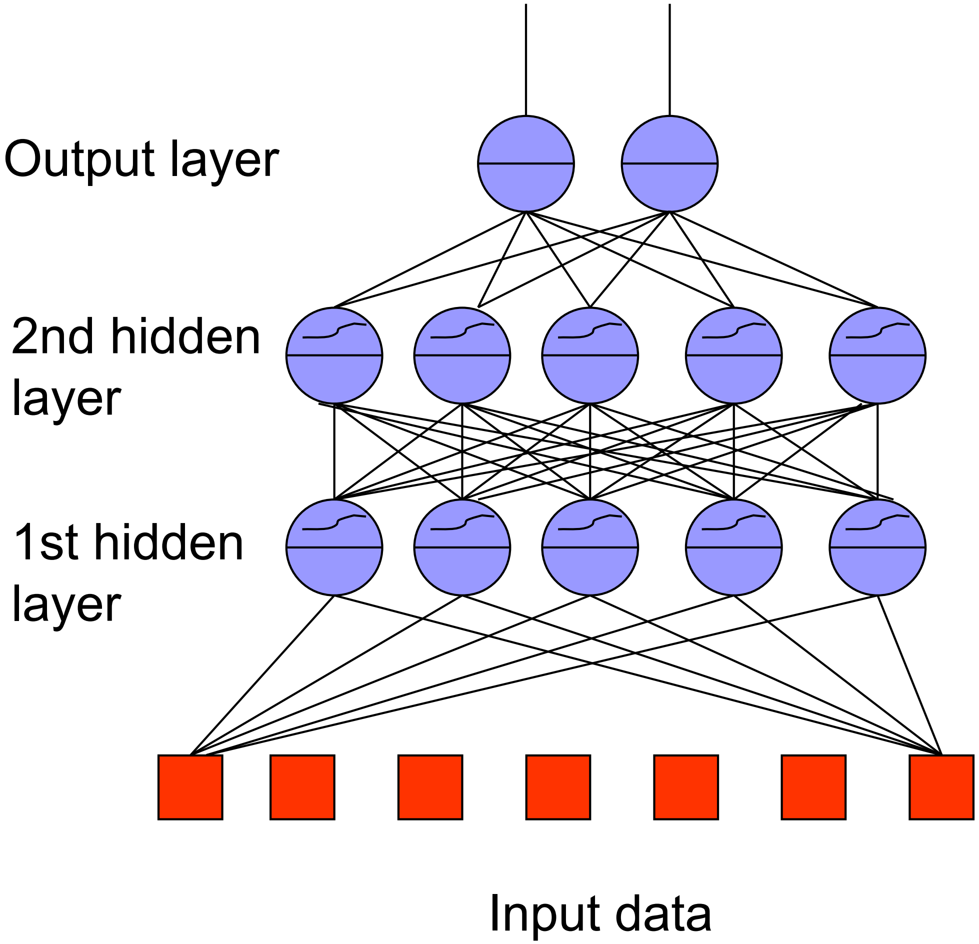
A perceptron network with several outputs (from Marsland Fig. 3.3)

Perceptron with several outputs

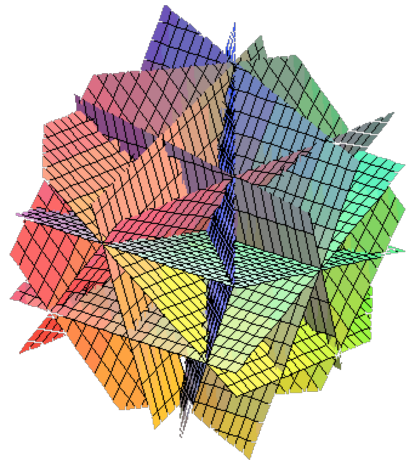




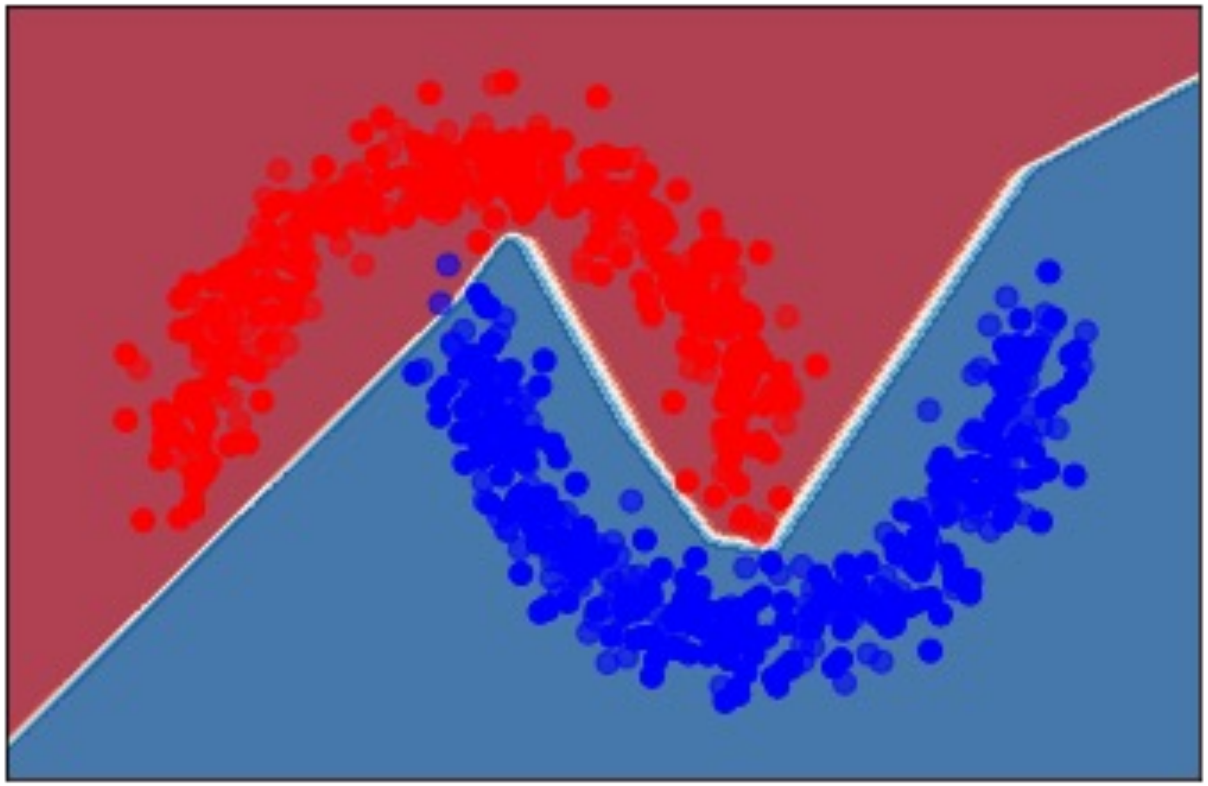
Multi-Layer Perceptron

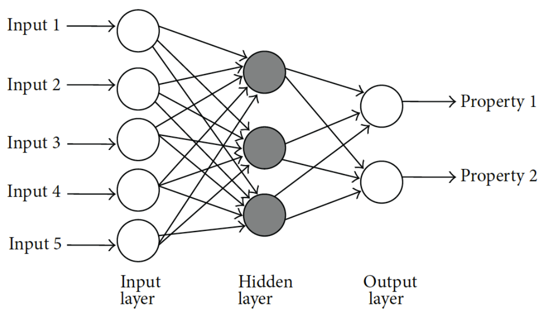


- One or more hidden layers



Multi-Layer Perceptron





Backpropagation

- Backpropagation is a process involved in training a neural network.
- It involves taking the error rate of a forward propagation and feeding this loss backward through the neural network layers to fine-tune the weights.
- Neural Networks online course

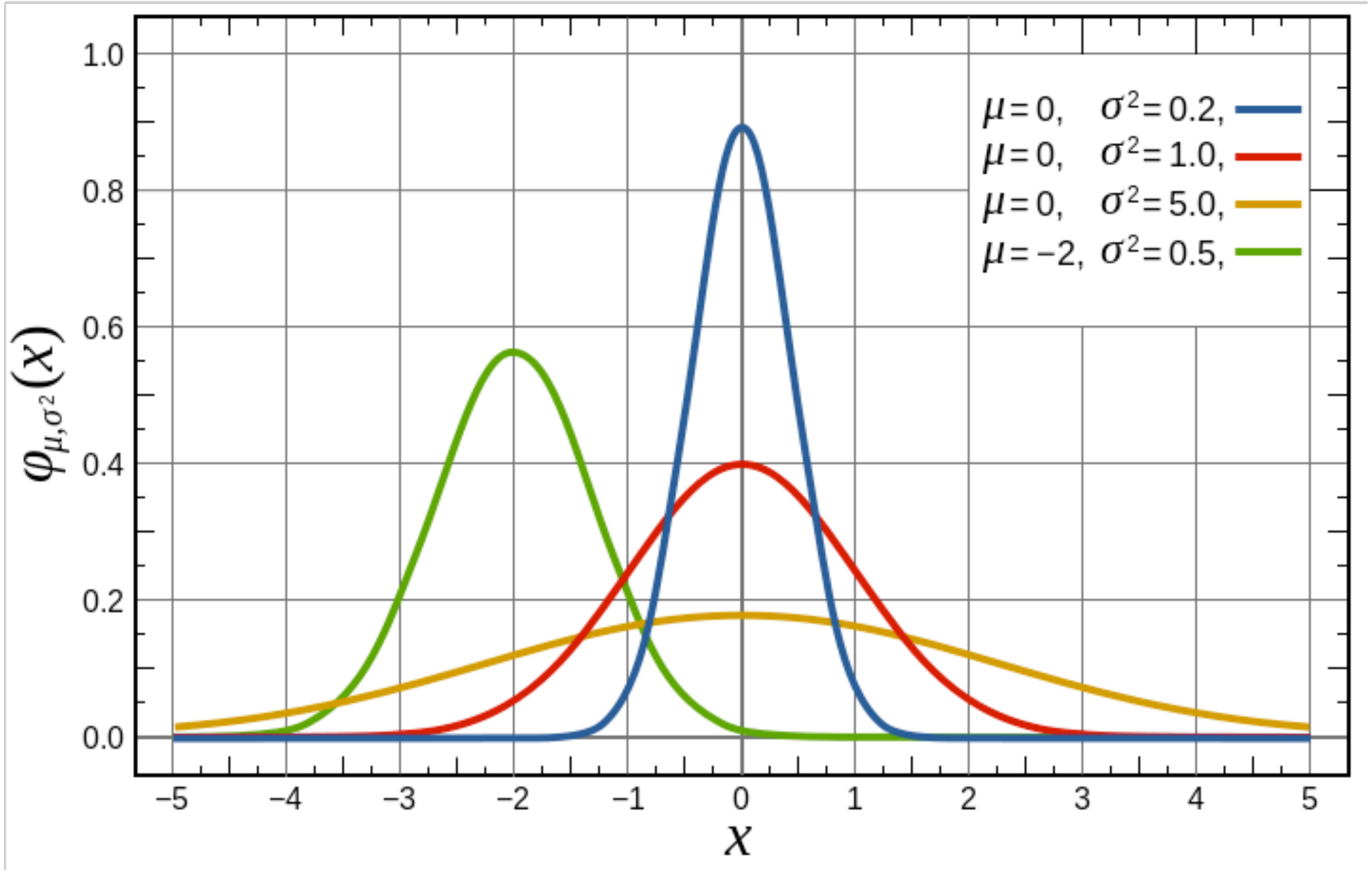
Radial Basis Functions

- A **radial basis function (RBF)** is a real-valued function whose value depends only on the distance from some other point *c*, called a *center*, $\varphi(\boldsymbol{x}) = f(||\boldsymbol{x}\text{-}\boldsymbol{c}||)$
- Any function φ that satisfies the property $\varphi(\boldsymbol{x}) = f(||\boldsymbol{x}\text{-}\boldsymbol{c}||)$ is a radial function.
- The distance is usually the Euclidean distance

$$||\, x - c \,||^2 = \sum_{i=1}^{N} \left(x_i - c_i \right)^2$$

Distance in 2D space:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$



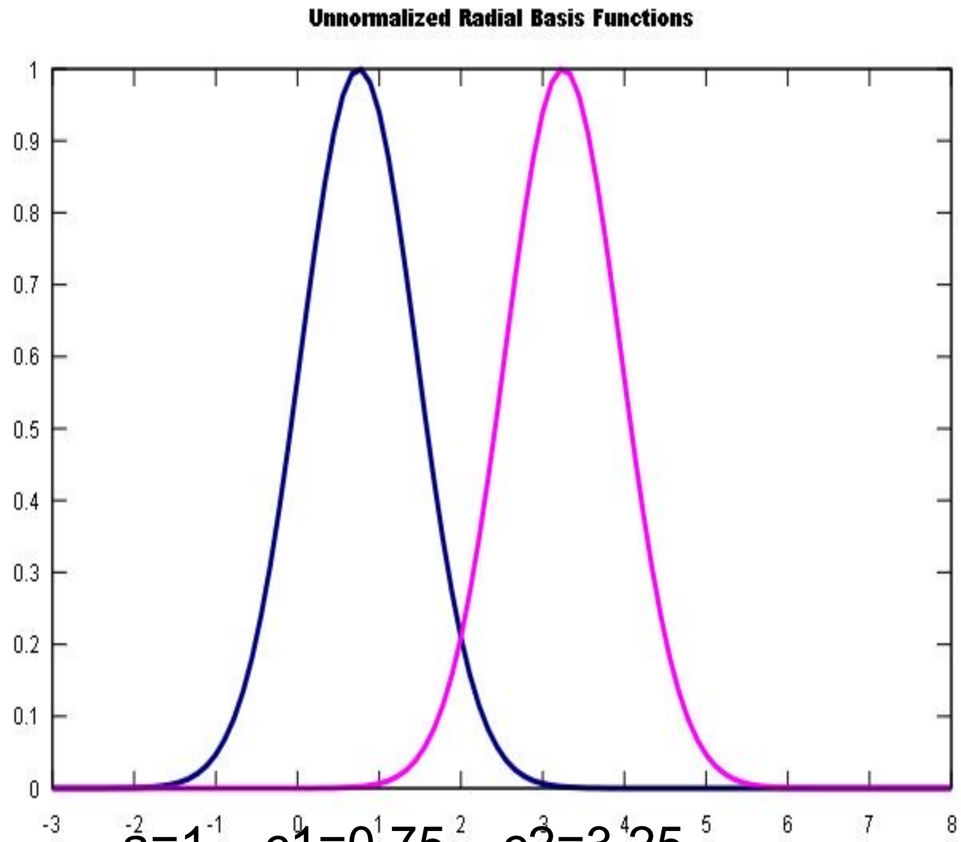
Normalized Gaussian curves with expected value μ and variance σ^2.
The corresponding parameters are $a = 1/(\sigma\sqrt{(2\pi)})$, $b = \mu$, $c = \sigma$

$$f(x) = ae^{-\frac{(x-b)^2}{2c^2}} \qquad g(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Radial Basis Functions

- The popular output of radial basis functions is the Gaussian function:

$$\Phi\left(\left\|x-c_j\right\|\right)=\exp\left(-a\left(\frac{\left\|x-c_j\right\|}{\sigma_j}\right)^2\right)$$



a=1, c1=0.75, c2=3.25

Radial Basis Functions

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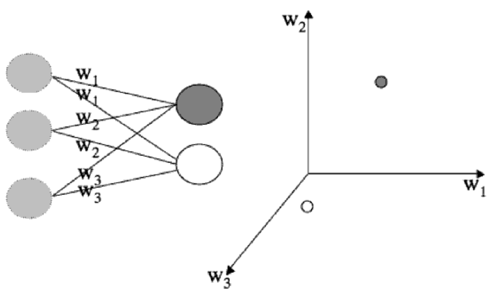
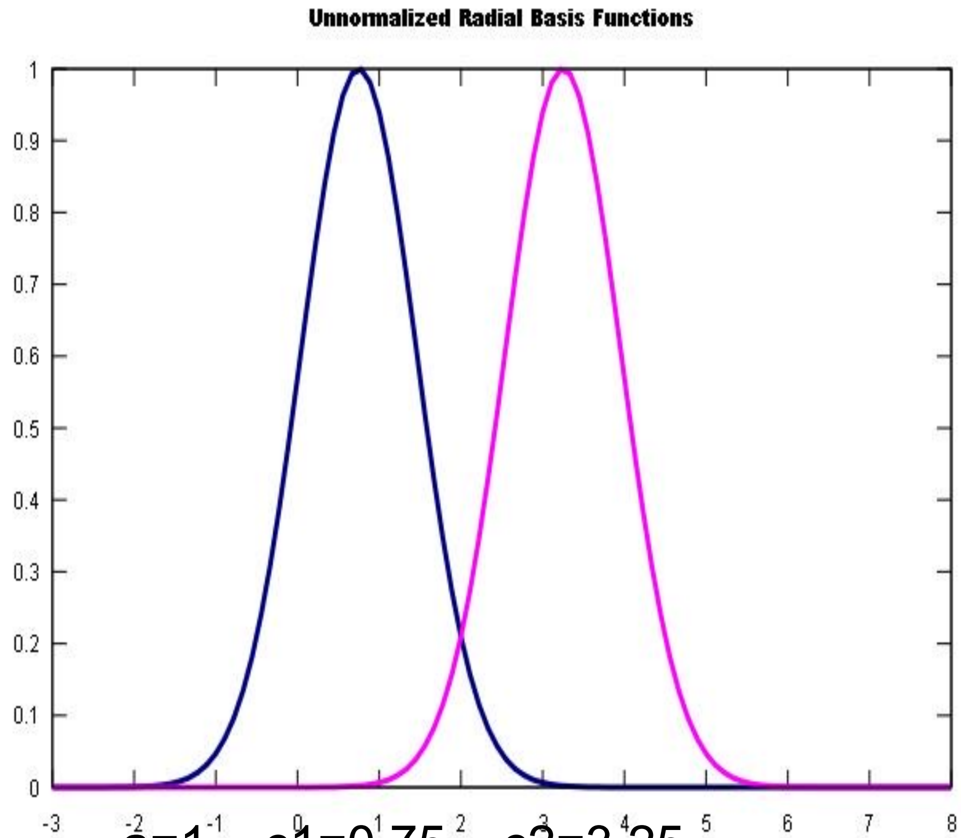
$$\Phi\left(\left\|x-c_j\right\|\right)=\exp\left(-a\left(\frac{\left\|x-c_j\right\|}{\sigma_j}\right)^2\right)$$

Sometimes the centers are called weights



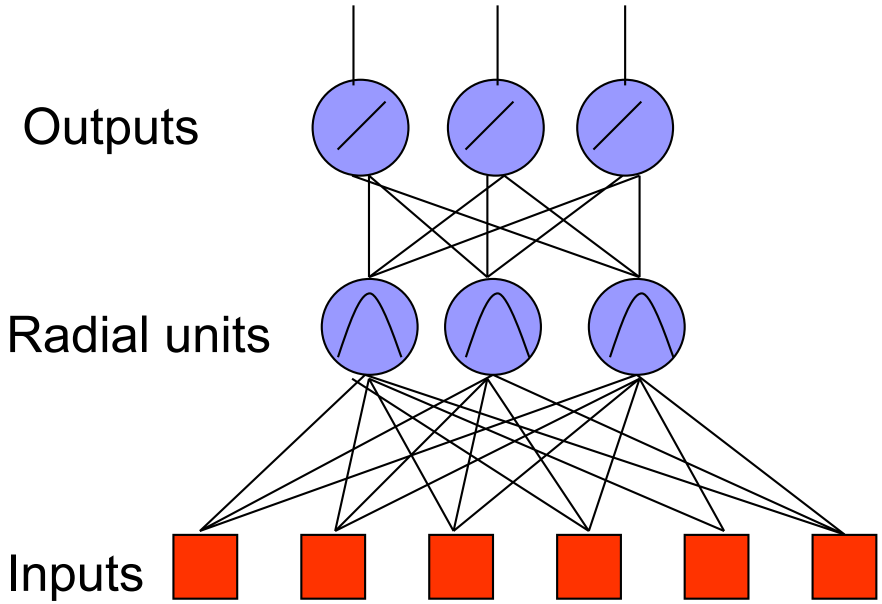
Weight Space = Input Space

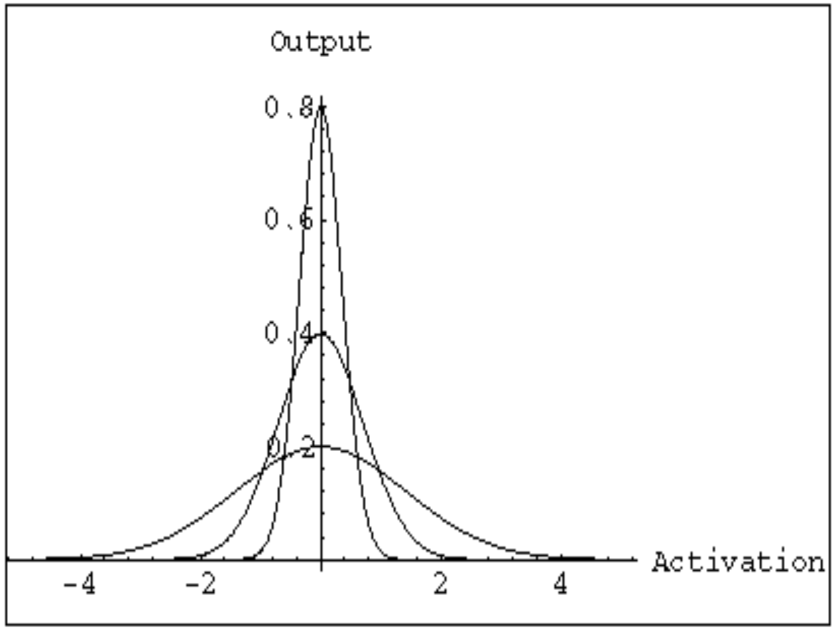


a=1, c1=0.75, c2=3.25

Radial Basis Functions Network (RBFN)

- Features
  - One hidden layer
  - The activation of a hidden unit is determined by a radial basis function



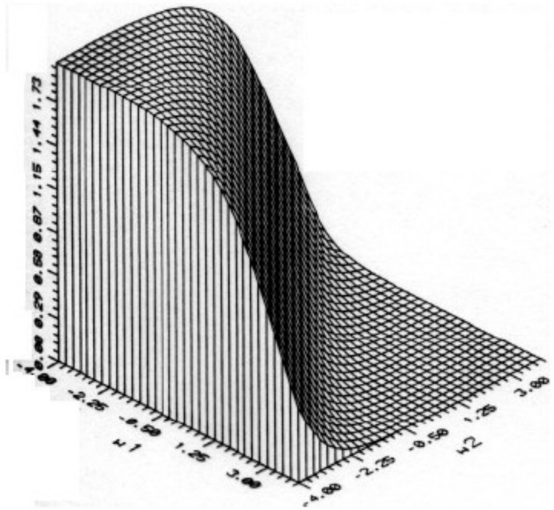


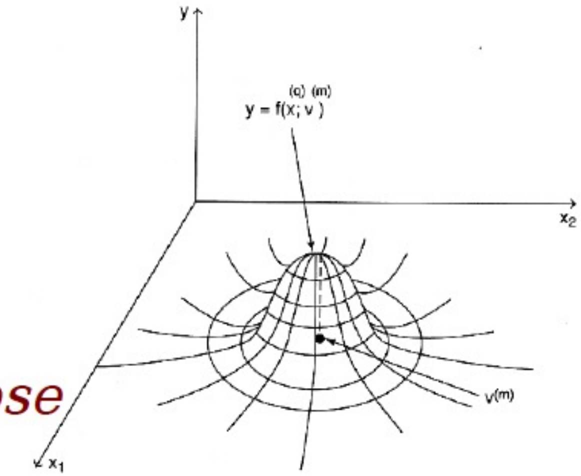
Sigmoidal vs. Gaussian Units

Sigmoidal unit:

$$y_j = \tanh\left(\sum_i w_{ji} x_i\right)$$

Decision boundary is a hyperplane





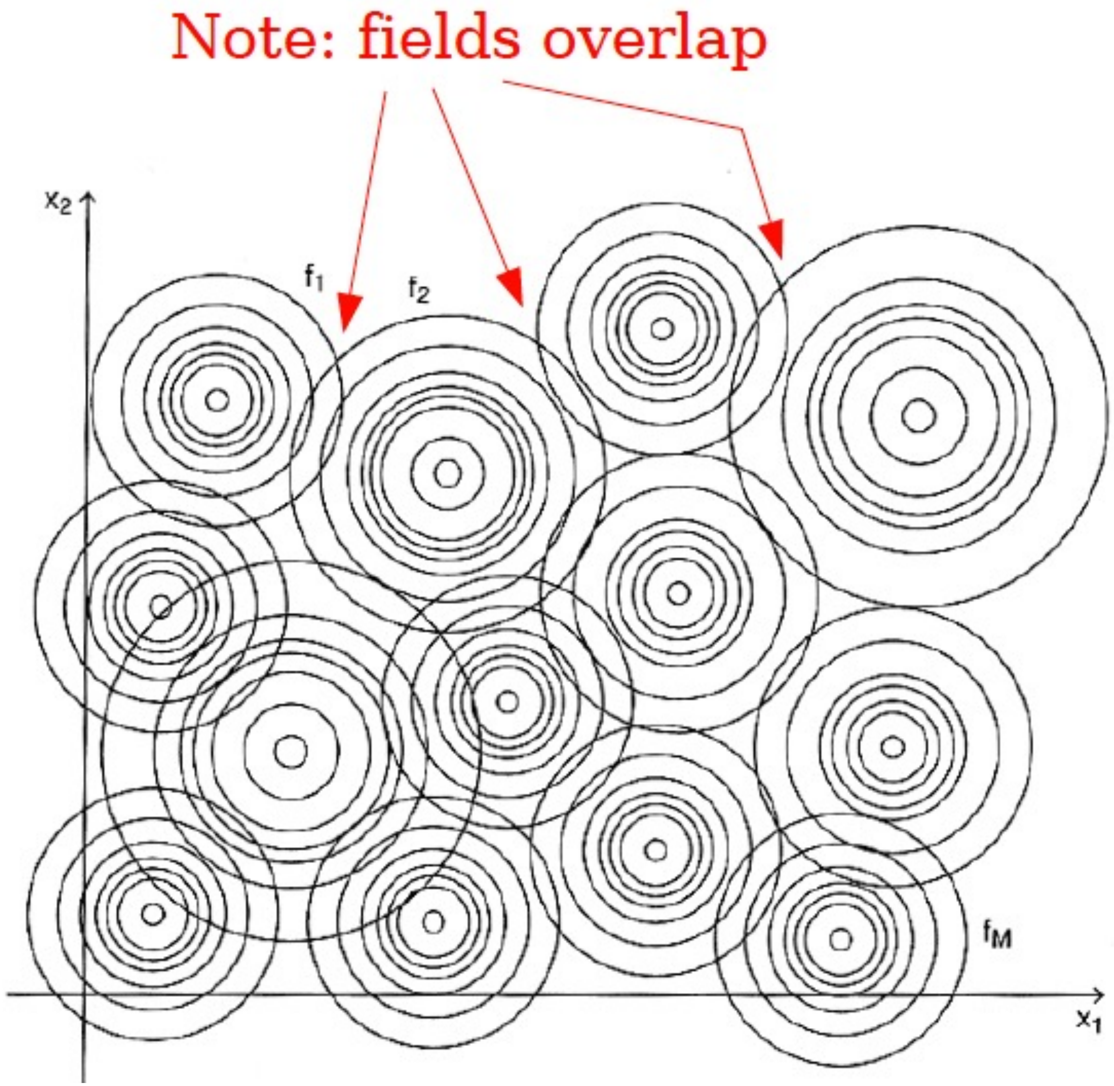
Gaussian unit:

$$y_j = \exp\left(\frac{-\|\vec{x}-\vec{\mu}_j\|^2}{\sigma_j^2}\right)$$

Decision boundary is a hyperellipse

Tiling the Input Space

Receptive fields in Neuroscience



- Generally, the hidden unit function is the Gaussian function
- The output Layer is linear:

$$s(x) = \sum_{j=1}^{K} W_j \Phi\left(\left\| x - c_j \right\|\right)$$

$$\Phi\left(\left\| x - c_j \right\|\right) = \exp(-w_j \left(\frac{\left\| x - c_j \right\|}{\sigma_j} \right)^2)$$

RBFN Learning

- The training is performed by deciding on
  - How many hidden nodes there should be
  - The centers and the sharpness of the Gaussians
- 2 steps (first unsupervised, second supervised)
  - In the 1st stage, the input data set is used to determine the parameters of the RBF
  - In the 2nd stage, RBFs are kept fixed while the second layer weights are learned (Simple BP algorithm like for MLPs or Perceptron)

Summary

- Neural networks are utilized as statistical tools
  - Adjust non linear functions to fulfill a task
  - Need of multiple and representative examples but fewer than in other methods
- Neural networks enable to model complex static phenomena (Feed-Forward) as well as dynamic ones (Recurent NN)
- NN are good classifiers BUT
  - Good representations of data have to be formulated
  - Training vectors must be statistically representative of the entire input space
  - Unsupervised techniques can help
- The use of NN needs a good comprehension of the problem

Self Organising Feature Maps (SOM)

- Used for Unsupervised Learning
- Weights in neurons must represent a class of pattern

one neuron, one class

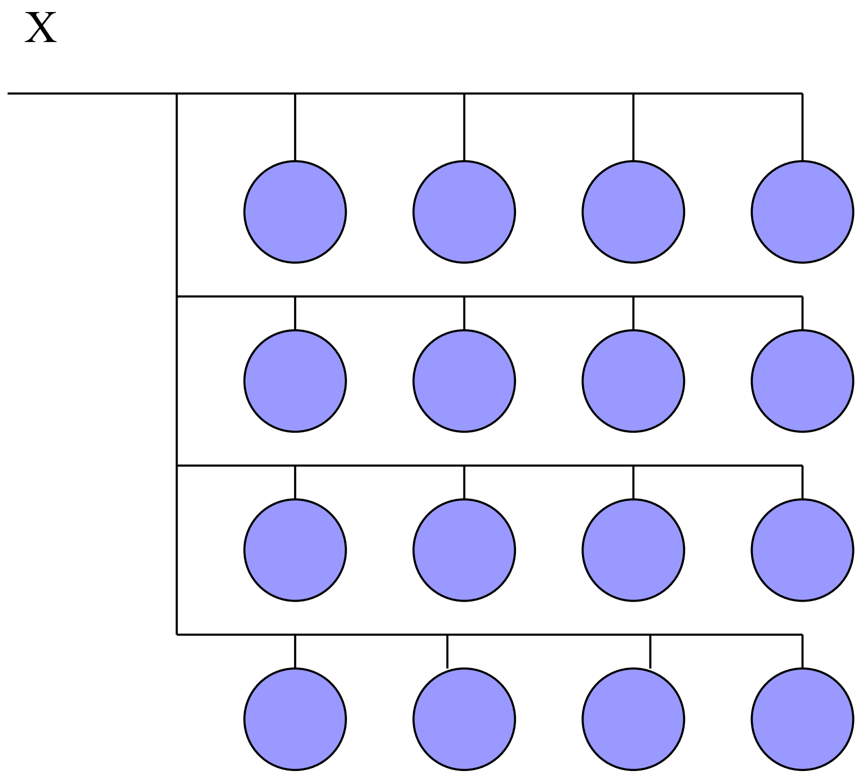
Four requirements for SOM

- Input pattern presented to all neurons and each produces an output.
- Output: measure of the match between input pattern and pattern stored by neuron.
- A competitive learning strategy selects neuron with largest response.
- A method of reinforcing the largest response

Architecture

- The Kohonen network (named after Teuvo Kohonen) is a self-organising network proposed in the 1980s
- Neurons are usually arranged on a 2-dimensional grid
- Inputs are sent to all neurons
- There are no connections between neurons

Architecture



Kohonen network

Output value

- The output of each neuron is the weighted sum
- There is no threshold or bias
- Input values and weights are normalized

“Winner takes all”

- Initially the weights in each neuron are random
- Input values are sent to all the neurons
- The outputs of each neuron are compared
- The “winner” is the neuron with the largest output value

Weight Space = Input Space

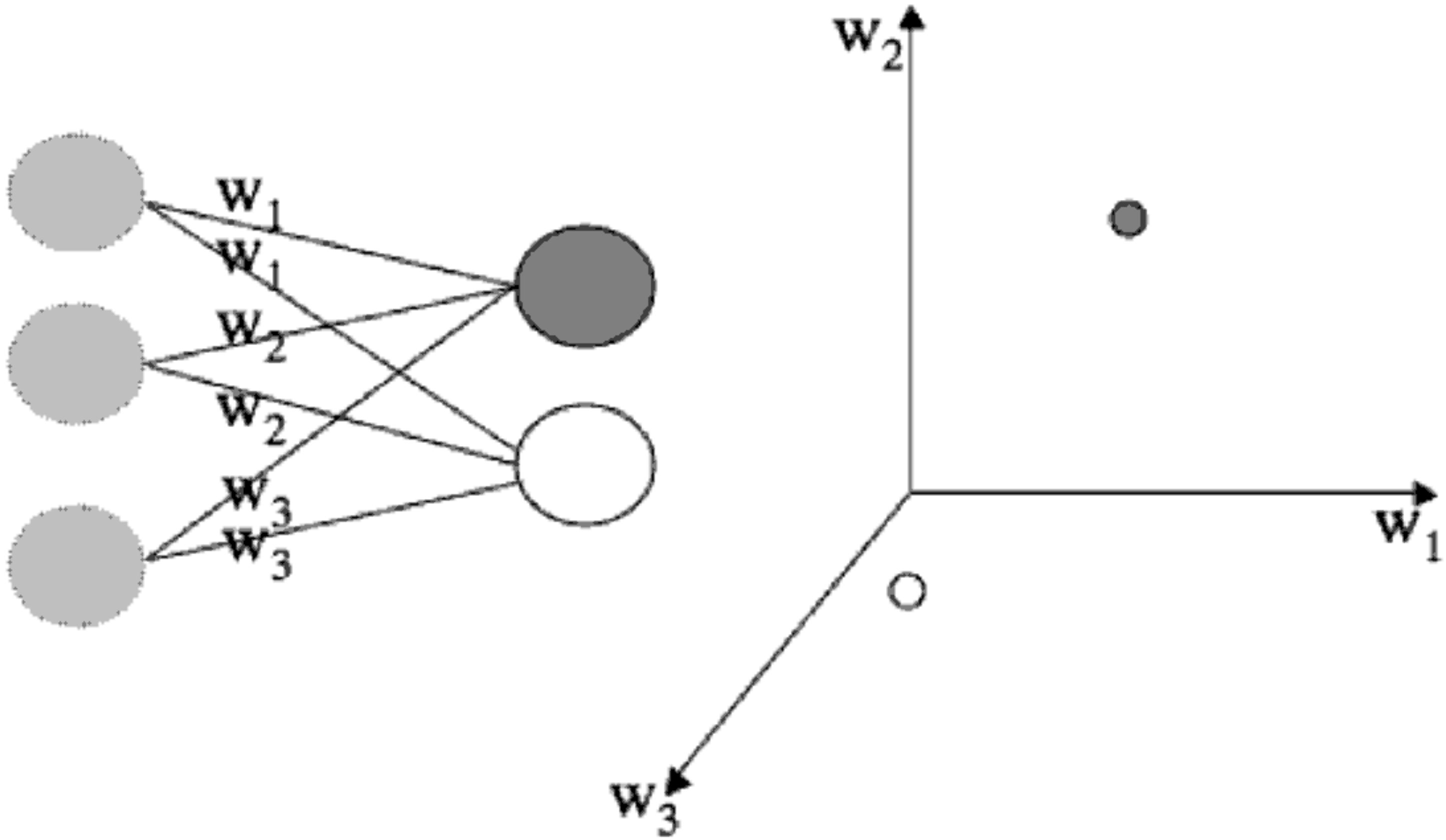
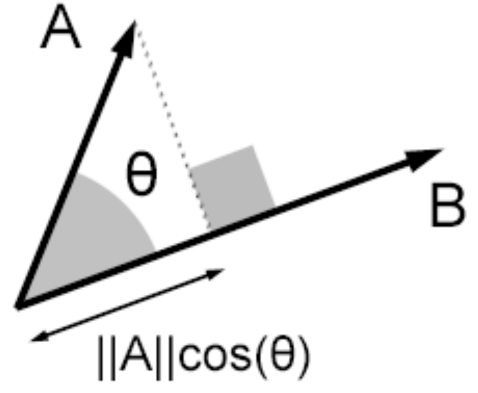


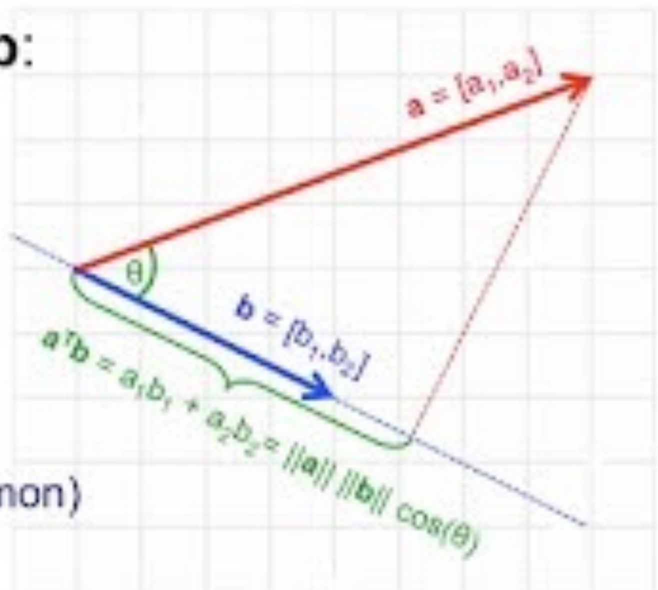
Fig. 2.1 Machine Learning, 2nd ed. By S. Marsland

The linear combination is a dot product

$$\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \bullet \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = a_x.b_x + a_y.b_y + a_z.b_z$$



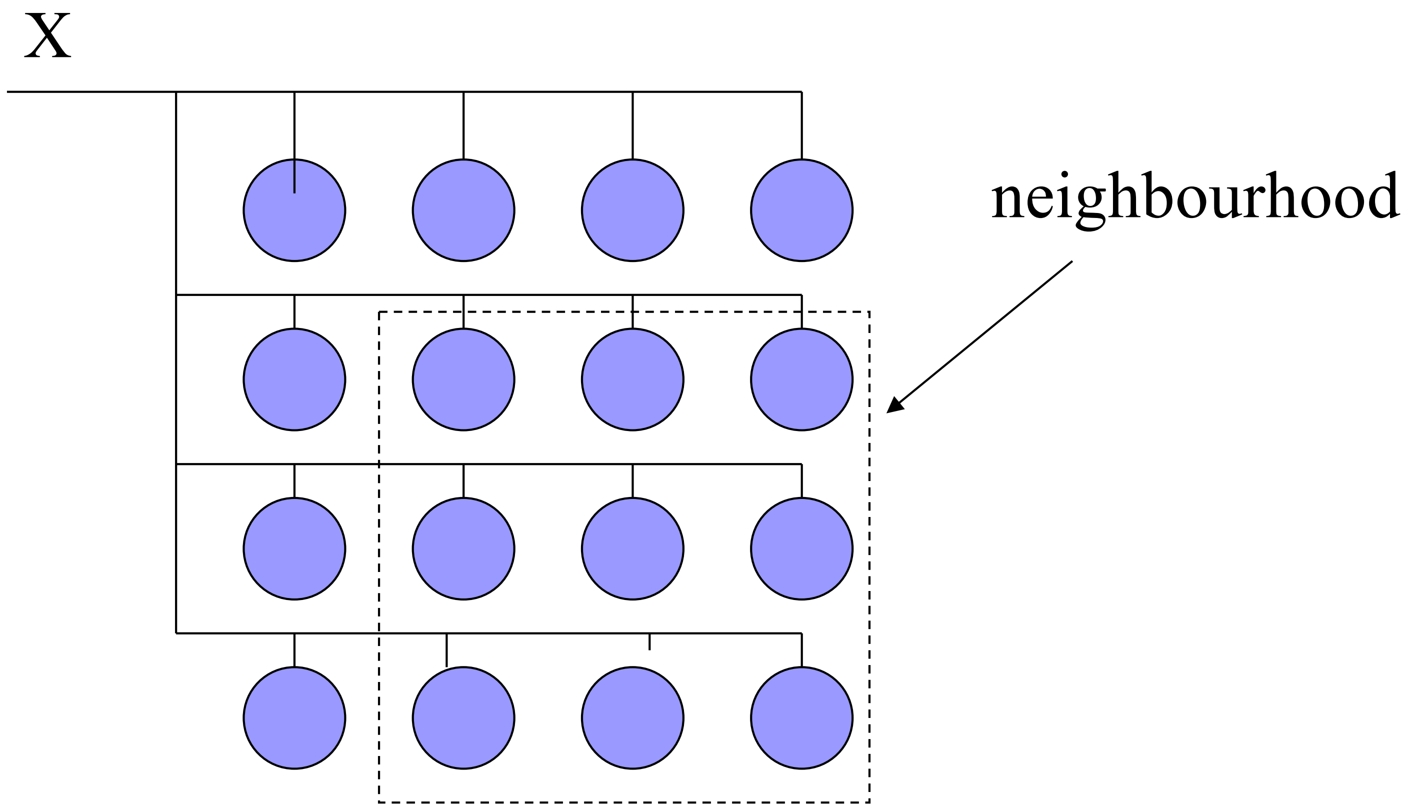
- Similarity of document vectors **a** and **b**:
  - **a** = [a_1 a_2 … a_d], **b** = [b_1 b_2 … b_d]
  - $\mathbf{a}^T\mathbf{b} = a_1b_1 + \ldots + a_db_d = \Sigma_i\, a_ib_i$
- Geometrically:
  - length of projection of **a** onto **b**
    - highest if **a**,**b** point in the same direction
    - zero if **a**,**b** are orthogonal (no words in common)
  - cosine of the angle between **a** and **b**



Training

- Having found the winner, the weights of the winning neuron are adjusted
- Weights of neurons in a surrounding neighbourhood are also adjusted

Neighbourhood

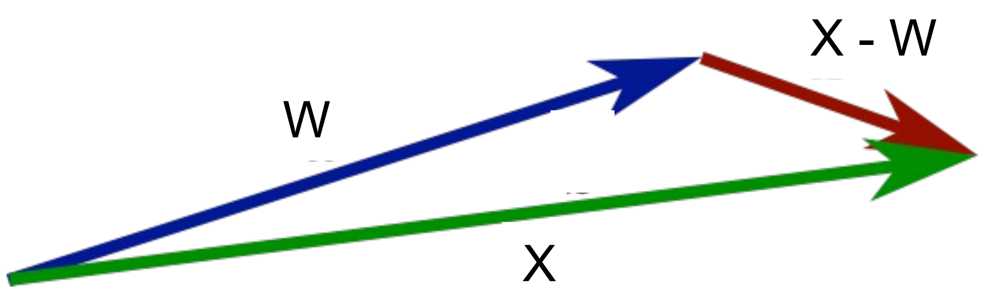


Kohonen network

Training

- As training progresses the neighbourhood gets smaller
- Weights are adjusted according to the following formula:

$$w' = w + \alpha(x - w)$$



Weight adjustment

- The learning coefficient (alpha) starts with a value of 1 and gradually reduces to 0
- This has the effect of making big changes to the weights initially, but no changes at the end
- The weights are adjusted so that they more closely resemble the input patterns

Weight adjustment details

$$\mathbf{W}_v(s + 1) = \mathbf{W}_v(s) + \Theta(u, v, s)\, \alpha(s)(\mathbf{D}(t) - \mathbf{W}_v(s))$$

Where:

- s is the step index,
- t an index into the training sample,
- u is the index of the best matching unit for D(t),
- α(s) is a monotonically decreasing learning coefficient
- D(t) is the input vector
- Θ(u, v, s) is the neighborhood function which gives the distance between the neuron u and the neuron v in step s

Example

- A Kohonen network receives the input pattern 0.6 0.6 0.6.
- Two neurons in the network have weights 0.5 0.3 0.8 and -0.6 –0.5 0.6.
- Which neuron will have its weights adjusted and what will the new values of the weights be if the learning coefficient is 0.4?

Answer

$$w' = w + \alpha(x - w)$$

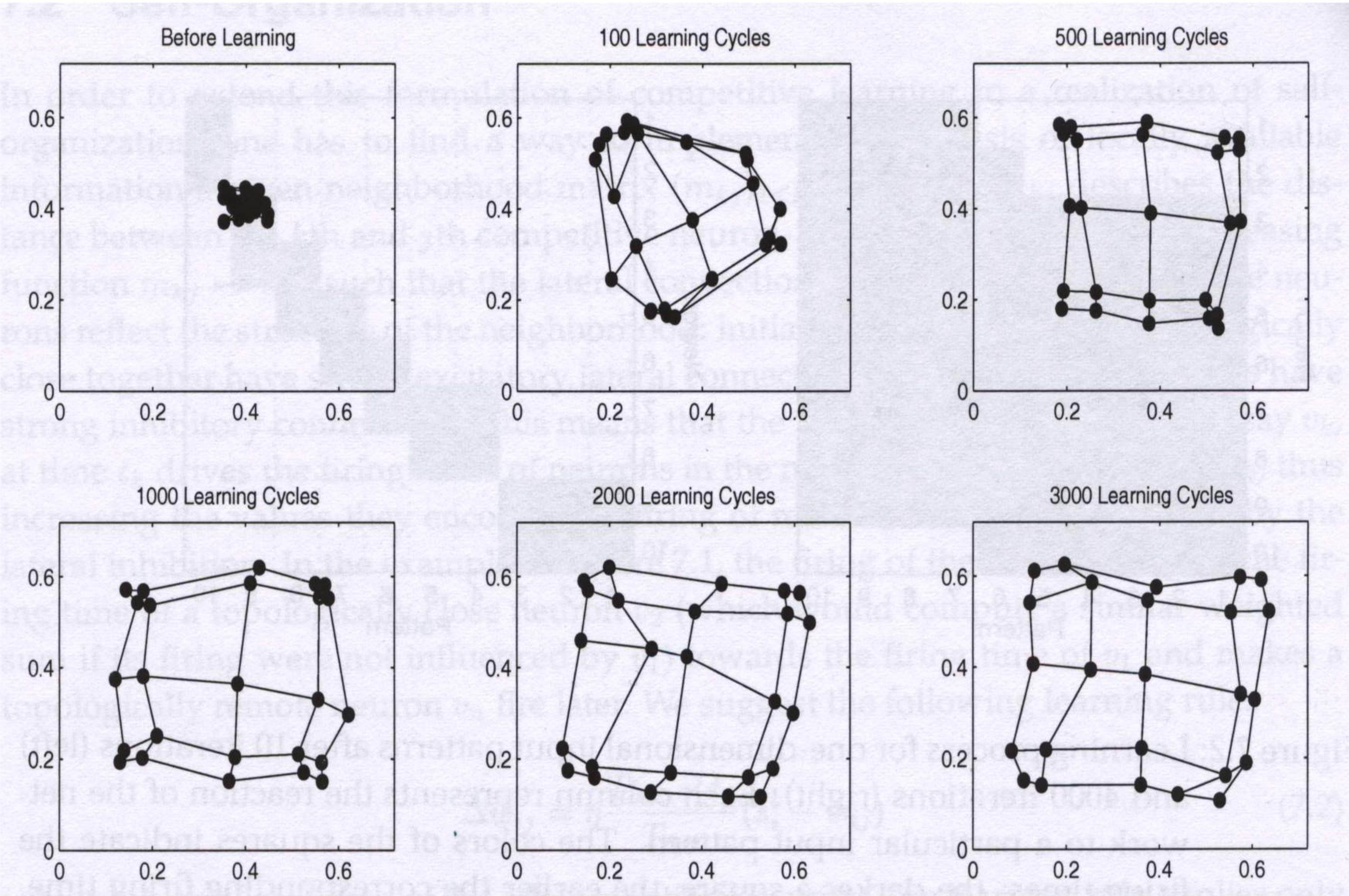
The weighted sums are 0.96 and –0.3 so the first neuron wins.
(0.6 X 0.3 + 0.6 X 0.5 + 0.6 X 0.8 = 0.18 + 0.30 + 0.48 = 0.96)
The weights become:

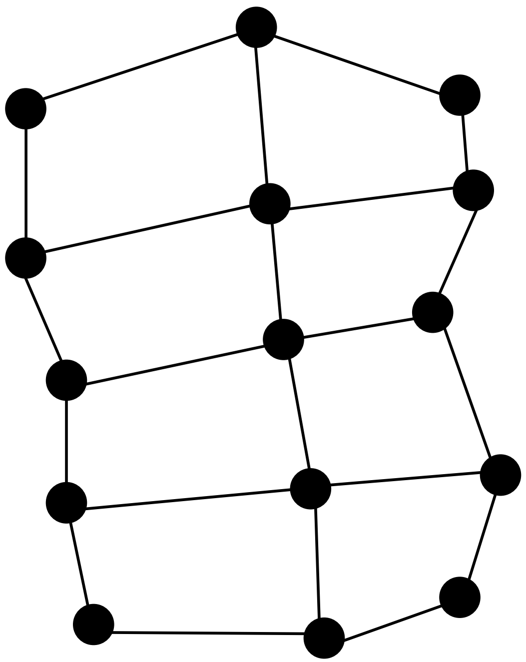
w1 = 0.5 + 0.4 *(0.6 – 0.5)
w1 = 0.5 + 0.4 * 0.1 = 0.5 + 0.04 = 0.54

w2 = 0.3 + 0.4 *(0.6 – 0.3)
w2 = 0.3 + 0.4 * 0.3 = 0.3 + 0.12 = 0.42

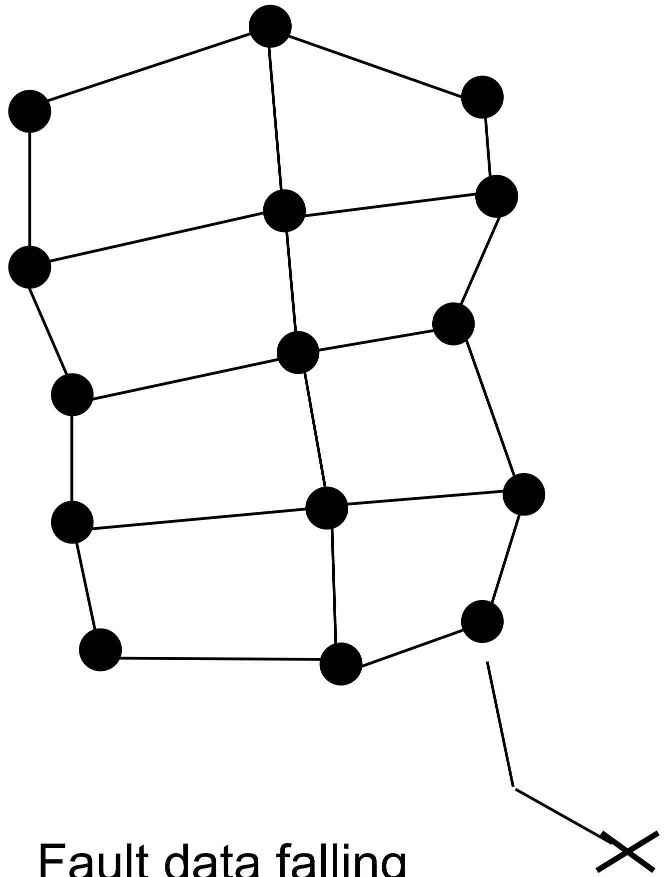
w3 = 0.8 + 0.4 *(0.6 – 0.8)
w3= 0.8 - 0.4 * 0.2 = 0.8 - 0.08 = 0.72

Visualizing a SOM





Kohonen network
representing
‘Normal’ space

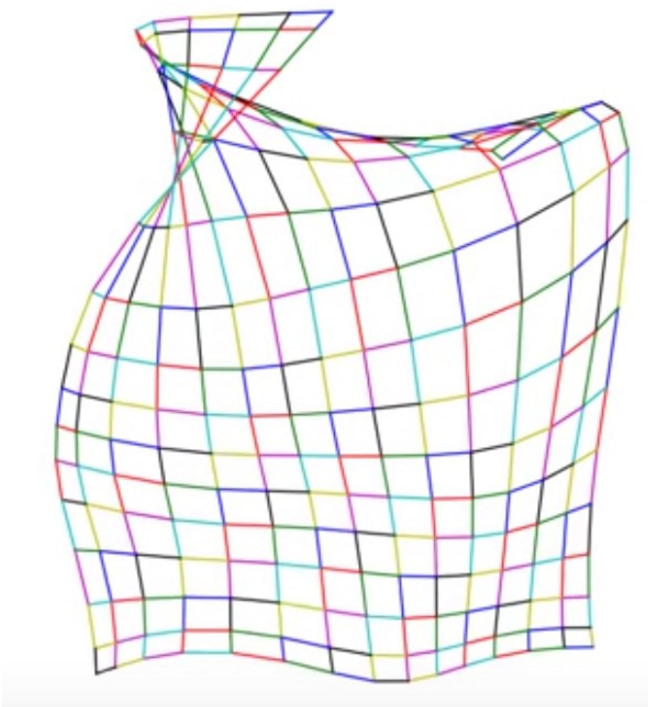


Fault data falling
outside
‘Normal’ space

Summary

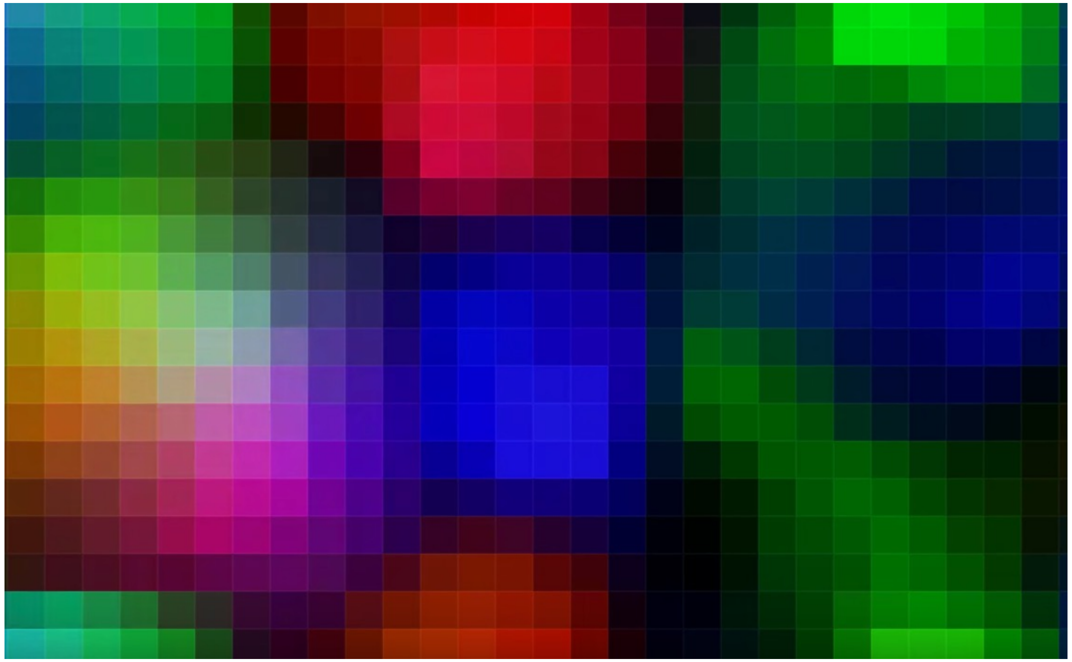
- The Kohonen network is self-organising
- It uses unsupervised training
- All the neurons are connected to the input
- A winner takes all mechanism determines which neuron gets its weights adjusted
- Neurons in a neighbourhood also get adjusted

Visualizing a SOM



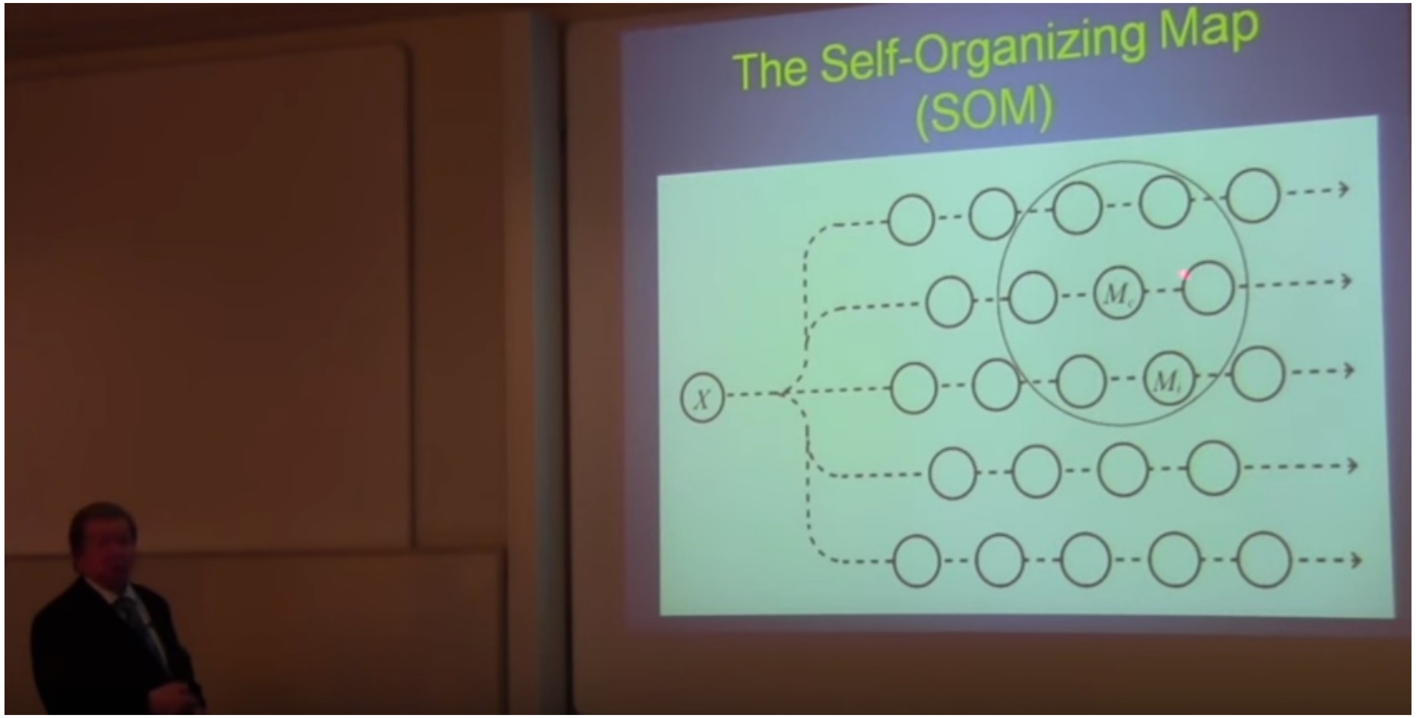
https://youtu.be/IixbH1gDhsg

Visualizing a SOM



https://youtu.be/dASyjPQtbS8

Prof. Kohonen explains SOMs



https://youtu.be/iWPhGKniTew

Prof. Teuvo Kohonen explains Self-Organizing Maps in May 2014

Watch from 12:05 to 16:05, he still uses his classical slides!