U9.1. Stereo Vision

SJK002 Computer Vision

Master in Intelligent Systems

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- Introduction
  - What is stereo vision?
- Geometry of a binocular system
  - Projection geometry
  - Binocular geometry. Fundamental matrix
  - Rectification

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Introduction

- Projection of a scene (3D) in 2 o or more planes (2D)
  - Binocular system
  - Moving camera
  - System of a camera with mirrors
  - System of multiple cameras

Introduction

- Binocular system

http://users.rcn.com/mclaughl.dnai/products.htm

http://www.lucs.lu.se/Projects/Robots/Robots1990/StereoHead2.html

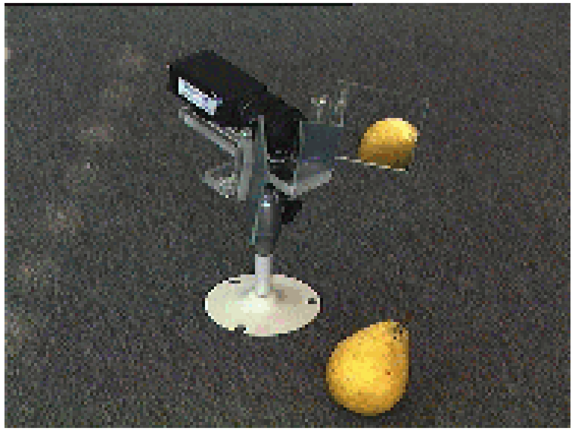
Introduction

- Moving camera

http://www.dis.uniroma1.it/~iocchi/stereo/stereo.html

Introduction

- System of a camer with mirrors

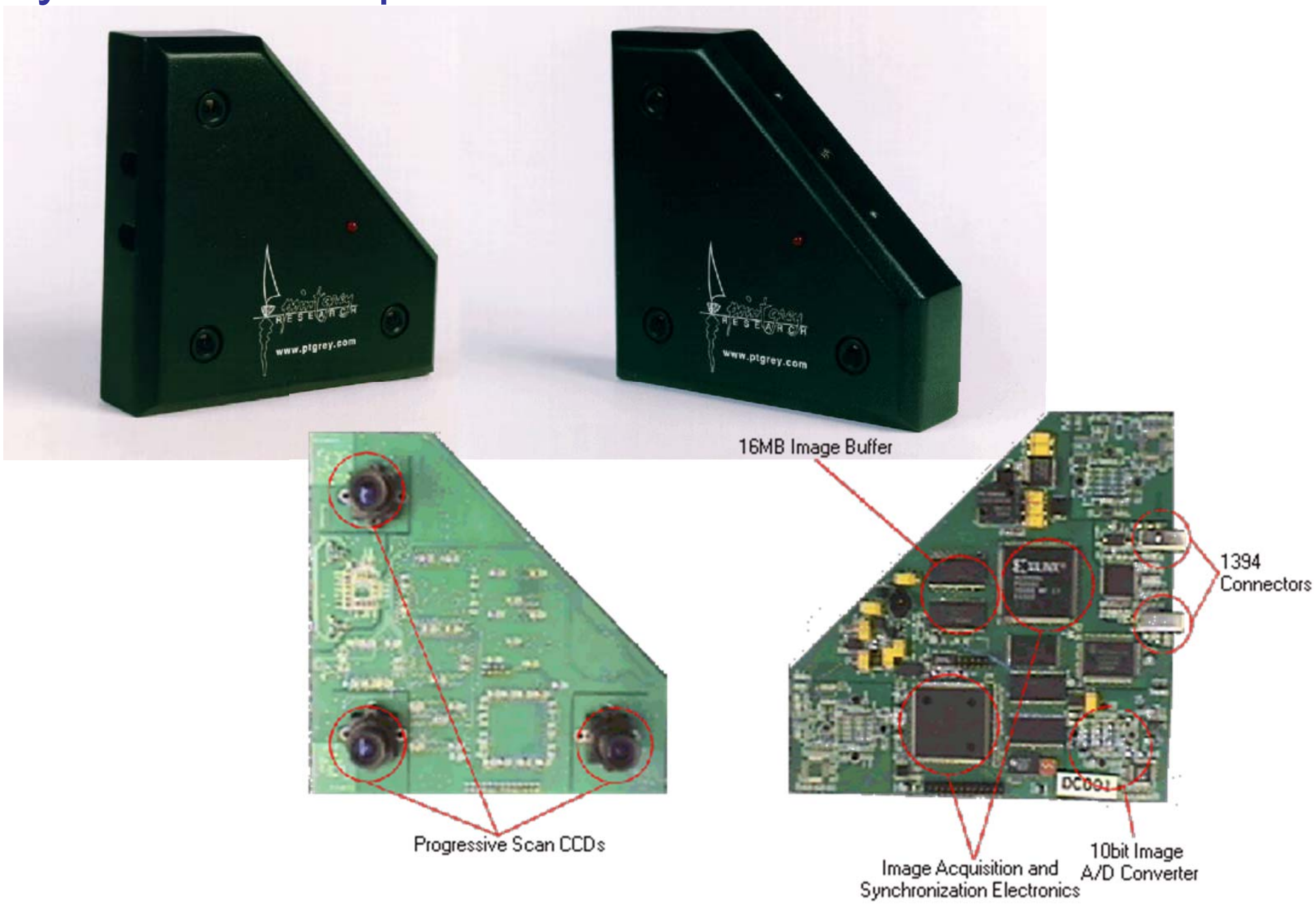


⇒

http://www-sop.inria.fr/robotvis/hardware/HardwarePictures.html

Introduction

- System of multiple cameras



http://www.ptgrey.com/products/digiclops

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What is stereo vision?

- Stereo vision:
  - 2 or more 2D images ⇒ 3D information



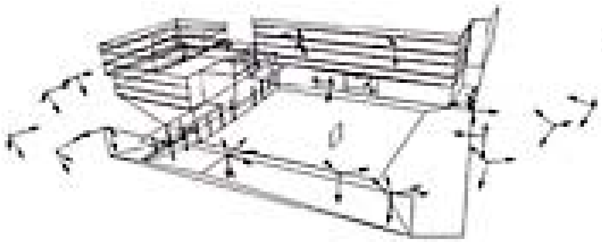


http://www.ai.sri.com/~konolige/svs/pictures.htm

What is stereo vision?

- Stereo vision:
  - 2 or more 2D images $\Rightarrow$ 3D information






http://www.inria.fr/

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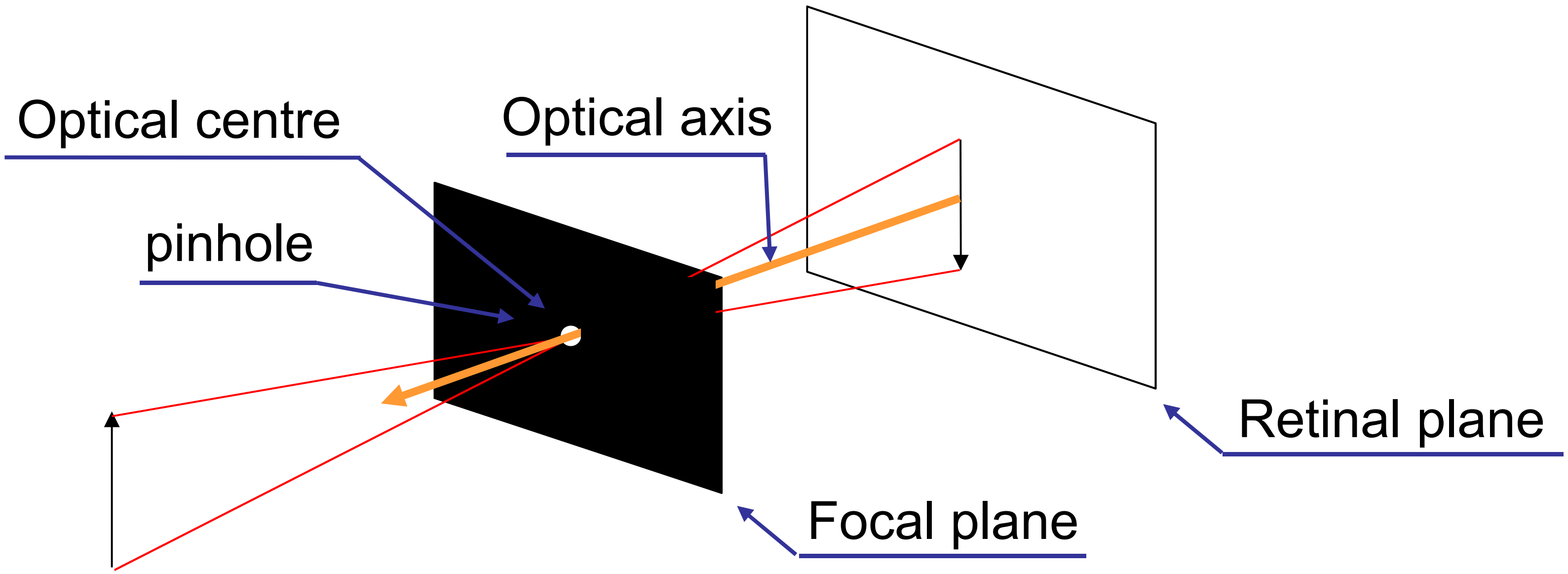
Projection geometry

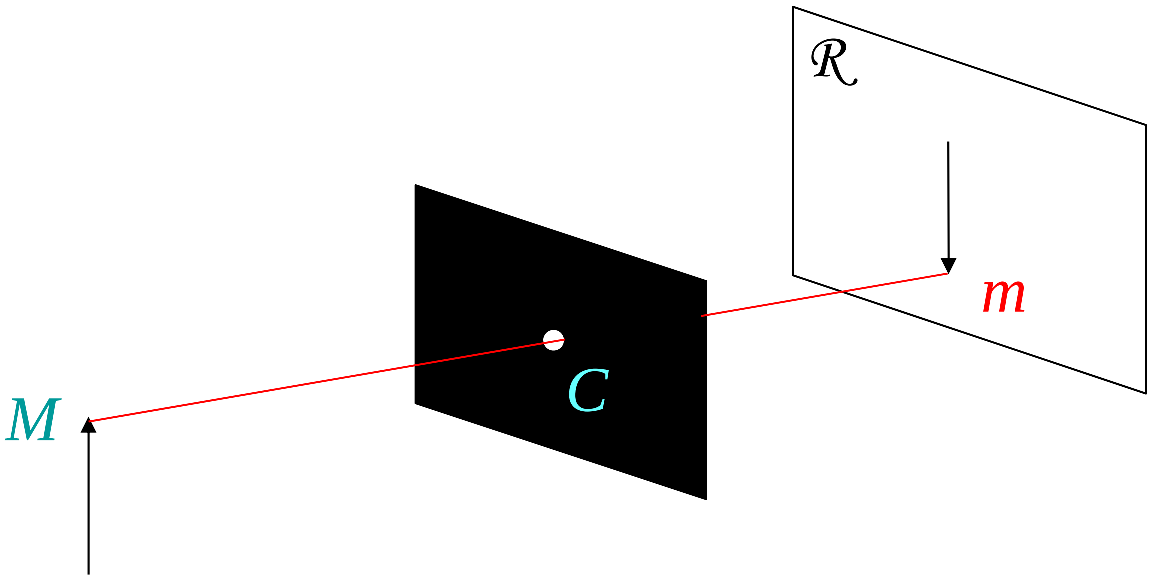
- Pinhole camera model



Perspective projection

Projection geometry

- Pinhole camera model



m = intersection of line CM with plane $\mathcal{R}$

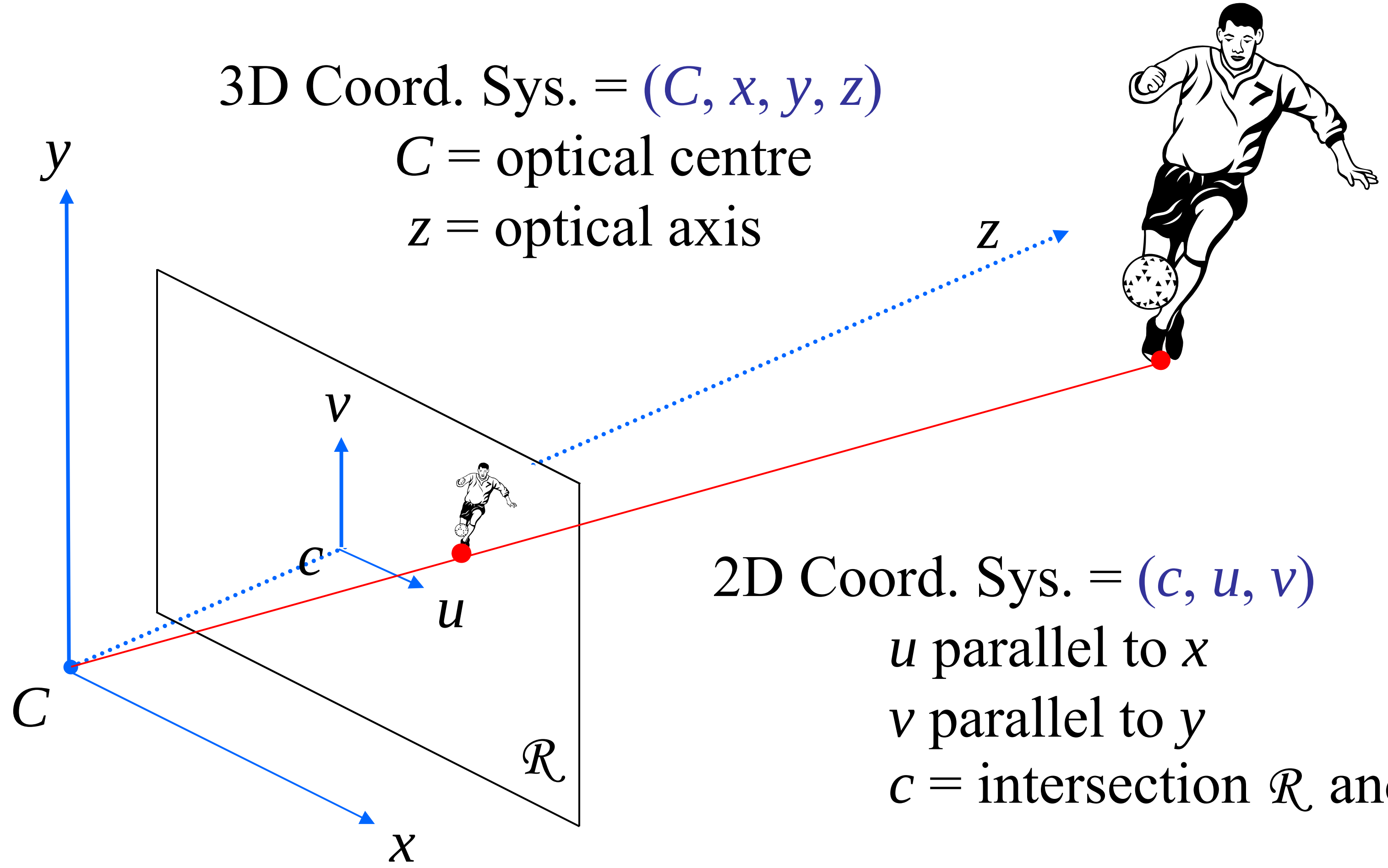
Projection geometry

- Coordinate systems

3D Coord. Sys. = (C, x, y, z)

C = optical centre

z = optical axis



2D Coord. Sys. = (c, u, v)

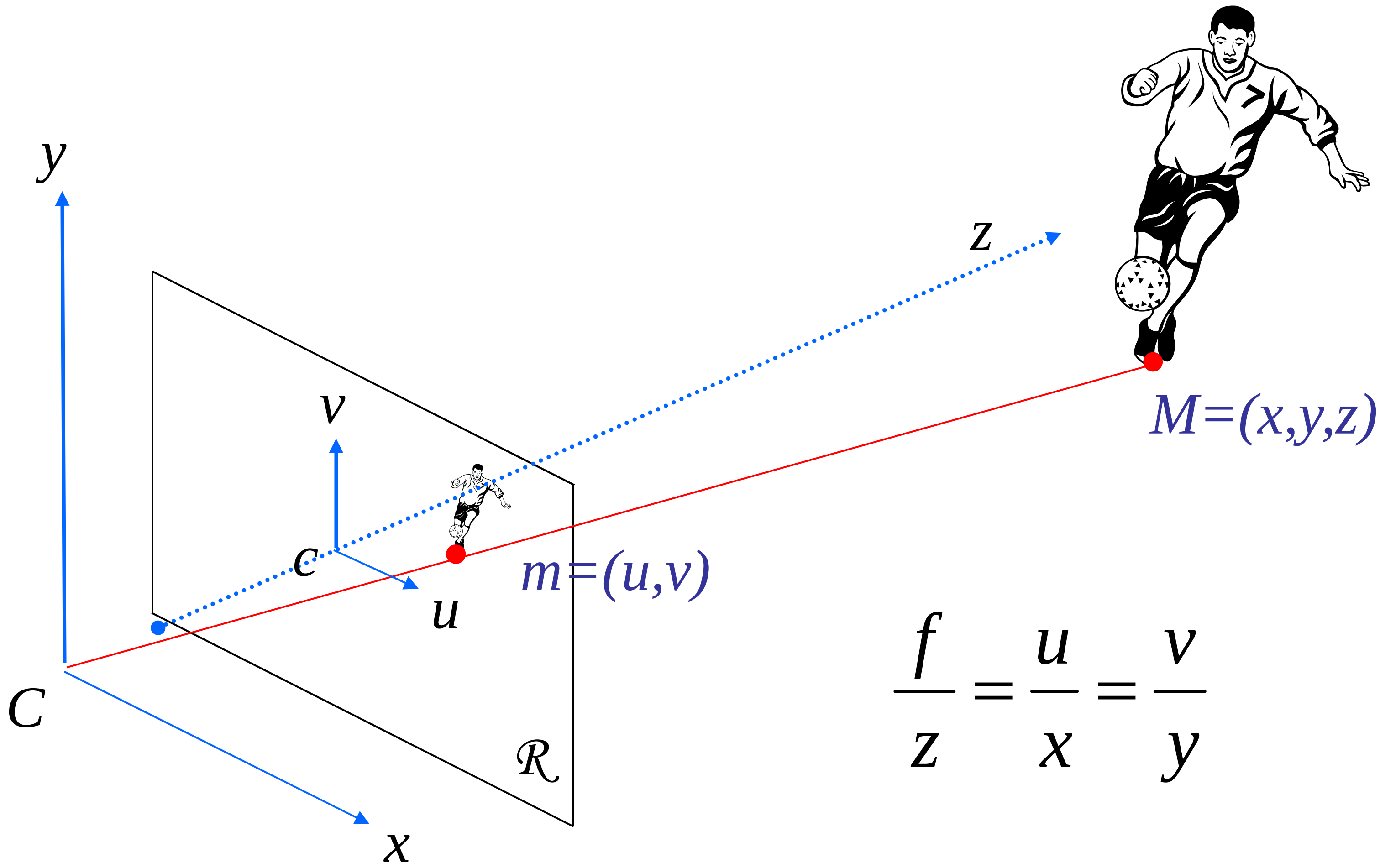
u parallel to x

v parallel to y

c = intersection $\mathcal{R}$ and z

Projection geometry

- From camera to image coordinates



$$\frac{f}{z} = \frac{u}{x} = \frac{v}{y}$$

Projection geometry

- Perspective projection matrix

2D

$$\frac{f}{z} = \frac{u}{x} = \frac{v}{y}$$

3D

2D 3D

$$\begin{bmatrix} U \\ V \\ S \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$u = U / S \quad v = V / S \quad si\ S \neq 0$$

Scale factor
S=z

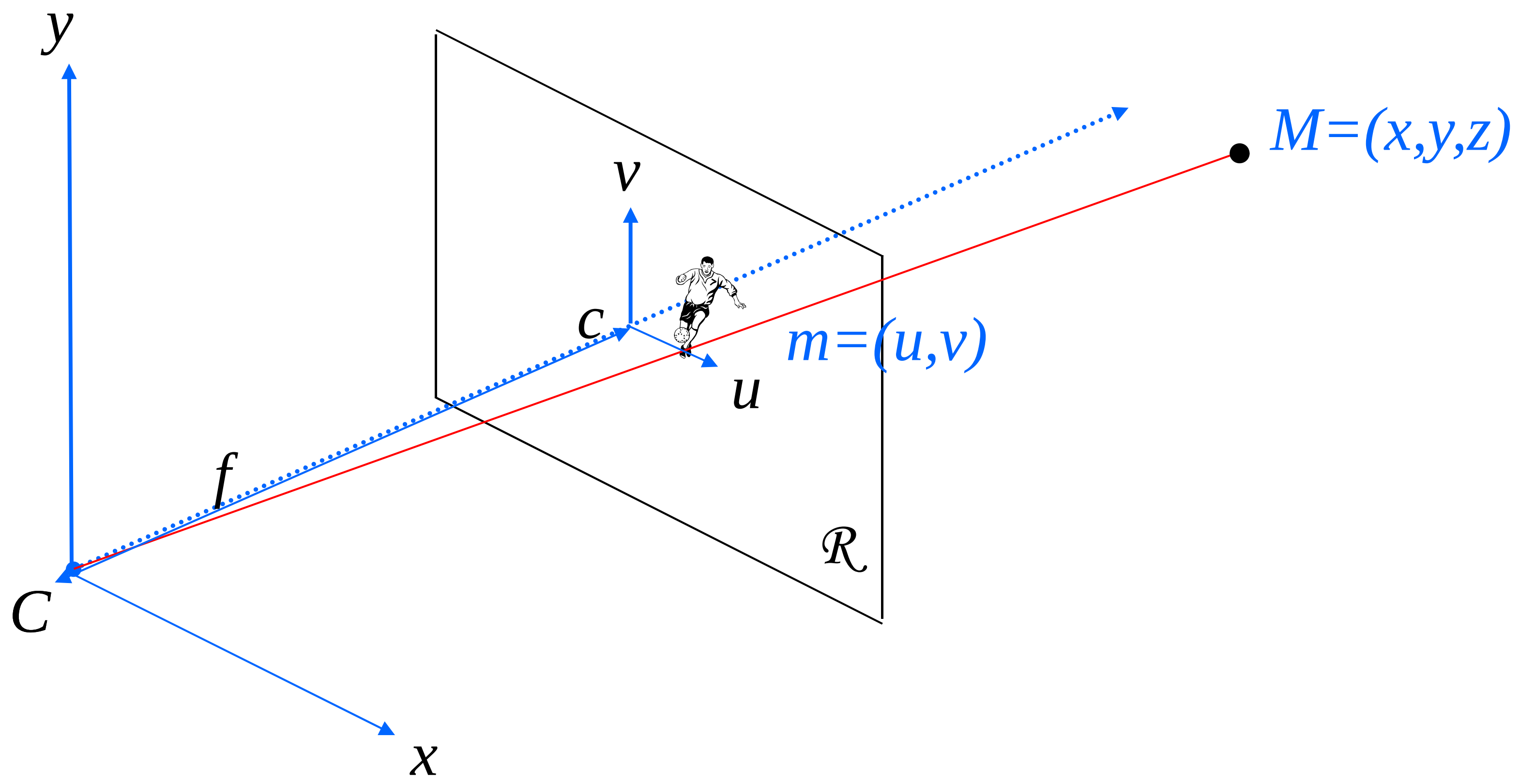
Homogeneous coordinates

P = projection matrix

$$\tilde{m} = P\tilde{M}$$

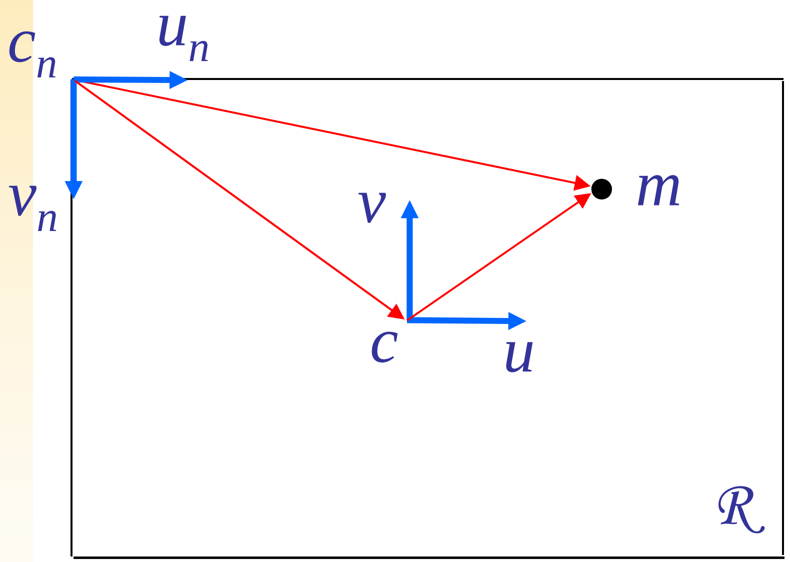
Projection geometry

- The objective is to represent m in normalized coordinates



Projection geometry

- From image coordinates to normalized coordinates (origin and pixel units)



Normalized image coordinates

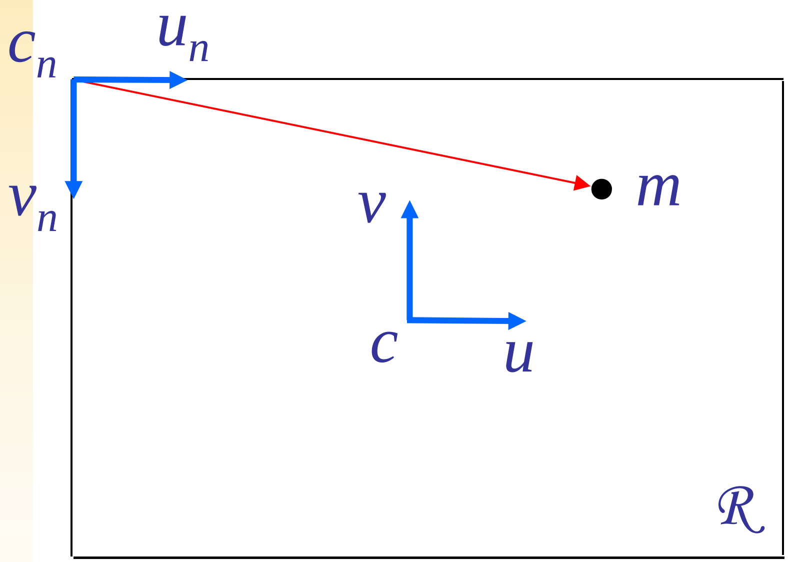
Coordinates change
de (c, u, v) a (c_n, u_n, v_n)

$$\overline{c_n m} = \overline{c_n c} + \overline{cm}$$

Add c position to m with respect to c_n :
(u_0, v_0)

Projection geometry

- From image coordinates to normalized coordinates



Normalized image coordinates

Units of u, v ⇨ *meters*

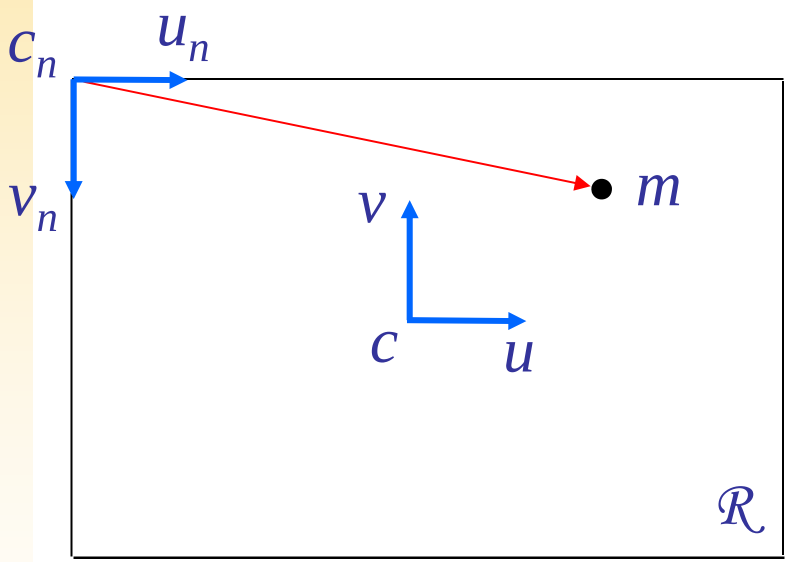
Units of u_n, v_n ⇨ *pixels*

k_u, k_v pixels/meter

In general $k_u \neq k_v$

Projection geometry

- From image coordinates to normalized coordinates



Change of coordinates origin

$$\begin{bmatrix} u_n \\ v_n \\ 1 \end{bmatrix} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

Normalized image coordinates

Projection geometry

- Perspective projection matrix

$$S\begin{bmatrix} u_n \\ v_n \\ 1 \end{bmatrix} = \begin{bmatrix} U \\ V \\ S \end{bmatrix} = \begin{bmatrix} fk_u & 0 & u_0 & 0 \\ 0 & fk_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$u_n = U/S \quad v_n = V/S \quad si\ S \neq 0$$

Intrinsic parameters: f, k_u, k_v, u_0, v_0

Projection geometry

- Perspective projection matrix

$$S\begin{bmatrix} u_n \\ v_n \\ 1 \end{bmatrix} = \begin{bmatrix} U \\ V \\ S \end{bmatrix} = \begin{bmatrix} \alpha_u & 0 & u_0 & 0 \\ 0 & \alpha_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$u_n = U / S \quad v_n = V / S \quad si\ S \neq 0$$

Focal length in units of u_n and v_n: α_u, α_v

Projection geometry

- Perspective projection matrix

$$S\begin{bmatrix} u_n \\ v_n \\ 1 \end{bmatrix} = \begin{bmatrix} U \\ V \\ S \end{bmatrix} = \begin{bmatrix} \alpha_u & -\alpha_u \cot\theta & u_0 & 0 \\ 0 & \alpha_v / \sin\theta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

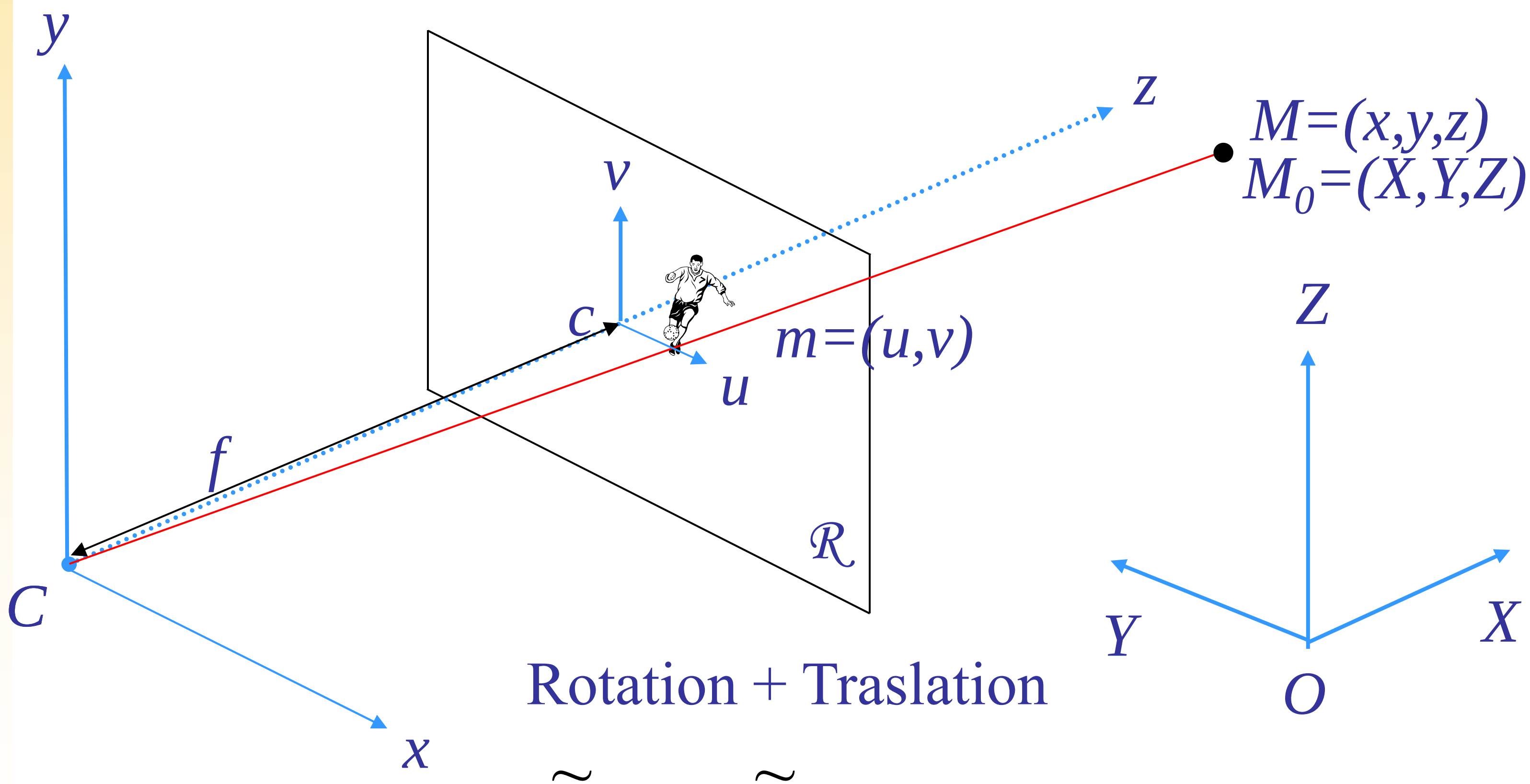
11 degrees of freedom

$$u_n = U / S \quad v_n = V / S \quad si\ S \neq 0$$

When there is a posible deviation of the optical axis

Projection geometry

- Extrinsic parameters:
  - from the world coordinates to camera coordinates



Rotation + Traslation

$$\tilde{M} = K\tilde{M}_o$$

Projection geometry

- Extrinsic parameters

Rotation + Traslation

$$\tilde{M} = K\tilde{M}_o$$

Rotation + Traslation

$$K = \begin{bmatrix} \boxed{\begin{matrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{matrix}} & \boxed{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix}$$

Projection geometry

- General form of the projection matrix

$$S\begin{bmatrix} u_n \\ v_n \\ 1 \end{bmatrix} = \begin{bmatrix} U \\ V \\ S \end{bmatrix} = P\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$u_n = U / S \quad v_n = V / S \quad si\ S \neq 0$$

Projection geometry

- General form of the projection matrix

$$P = \begin{bmatrix} fk_u & 0 & u_0 & 0 \\ 0 & fk_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Intrinsic parameters

3x4 matrix

Extrinsic parameters

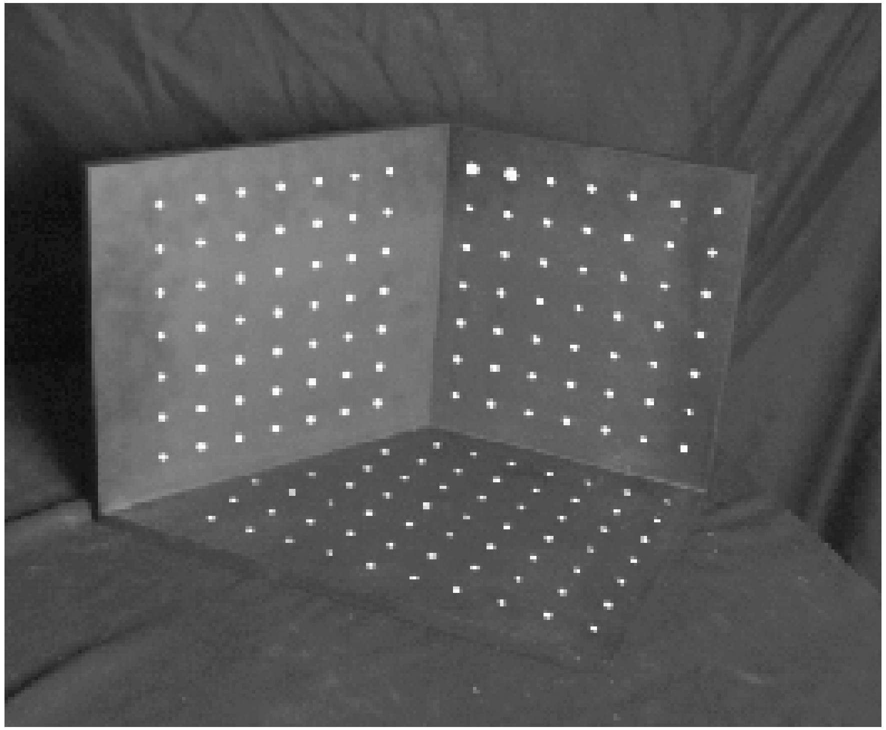
Projection geometry

- Calibration:
  - Step 1: Estimate matrix $\boldsymbol{P}$
  - Step 2: Estimate intrinsic and extrinsic parameters from $\boldsymbol{P}$

- For some applications (e.g. **stereo vision**) it is only necessary step 1.

Projection geometry

- Camera calibration
  - Linear regression (least squares).
  - Non linear optimization.
  - Multiple planar patterns.



Calibration: Linear regression

Intrinsic parameters

Extrinsic parameters

Projection matrix

$$\mathbf{x} \sim \mathbf{K}[\mathbf{R}|\mathbf{t}]\mathbf{X} = \mathbf{M}\mathbf{X}$$

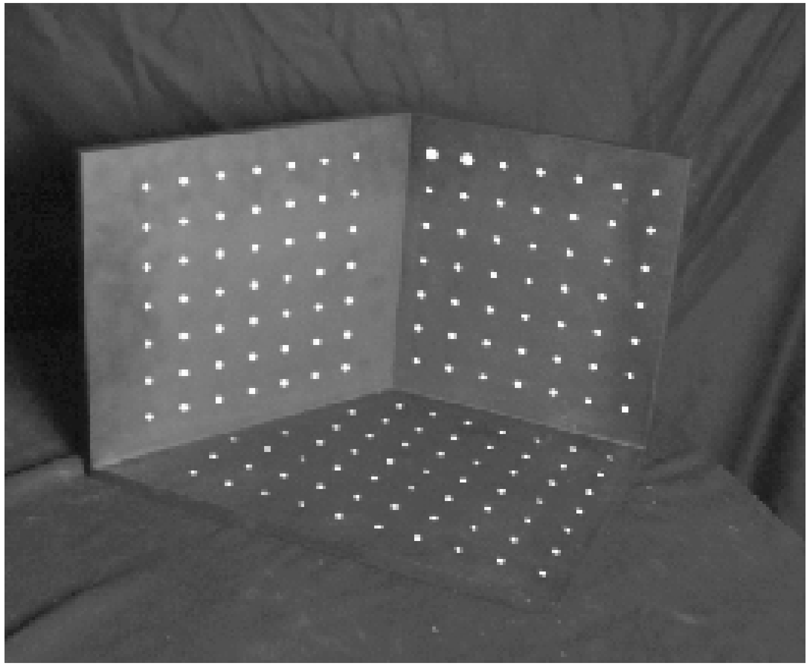
$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \sim \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Image projection of the 3D point

3D point

Linear regression

- Directly estimate the 11 unknown values of matrix M.
- Use a set of known 3D points (X_i,Y_i,Z_i) and measure the image position of the corresponding projected points (u_i,v_i)



Linear regression

$$u_i = \frac{m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1}$$

$$v_i = \frac{m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1}$$

$$u_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1) = m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}$$

$$v_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1) = m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}$$

Each point defines two equations

Solve for the projection matrix M using least squares

Linear regression

- Least squares:
  - p/2 points of the calibration grid (patterns)
  - n=11 unknown values of matrix M

Each row corresponds to the coefficients of one equation

$$\begin{bmatrix} a_{00} & a_{01} & \dots & a_{0n} \\ a_{10} & a_{11} & \dots & a_{1n} \\ \dots & \dots & \dots & \dots \\ a_{p0} & a_{p1} & \dots & a_{pn} \end{bmatrix} \begin{pmatrix} m_{00} \\ m_{01} \\ \dots \\ m_{22} \end{pmatrix} = \begin{pmatrix} b_0 \\ b_1 \\ \dots \\ b_n \end{pmatrix}$$

Independent terms vector

$$\mathbf{Am} = \mathbf{b}$$

Linear regression

- Least squares

$$\mathbf{Am = b}$$

Linear system with m equations and n unknowns

$$\mathbf{A^t A m = A^t b}$$

Normal equation which minimizes the sum of squared differences

$$\mathbf{m = (A^t A)^{-1} A^t b}$$

Solution vector m of n values to be estimated

Linear regression

- Advantages:
  - All camera parameters summarized in a single matrix.
  - Can predict how any 3D point is projected into the image plane.
- Disadvantages:
  - Does not provide specific information for each of the camera parameters.
  - Mixture of intrinsic and extrinsic parameters:
    - Dependent on camera position.
    - If camera moves, the estimated projection camera is not the same one.

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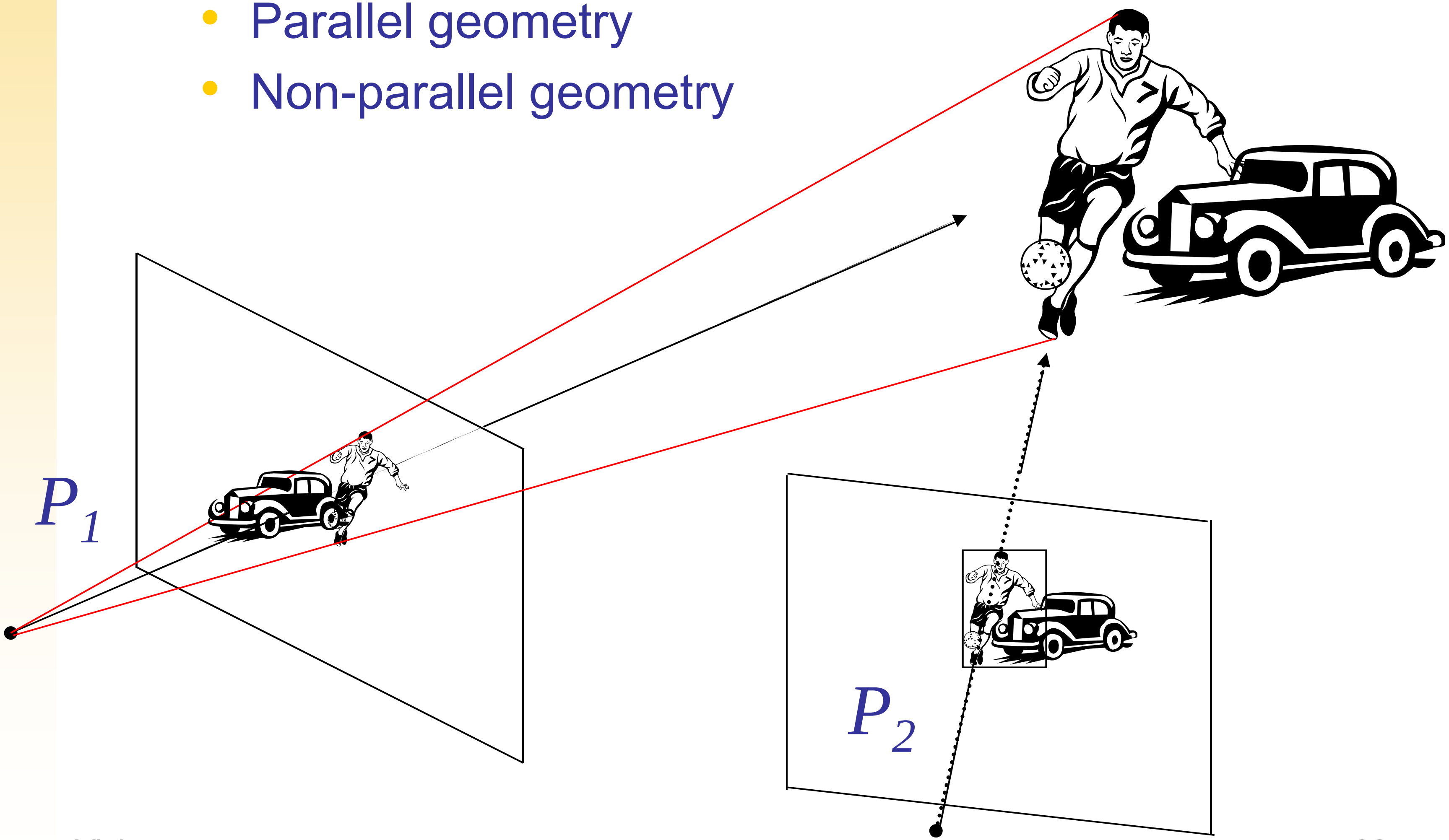
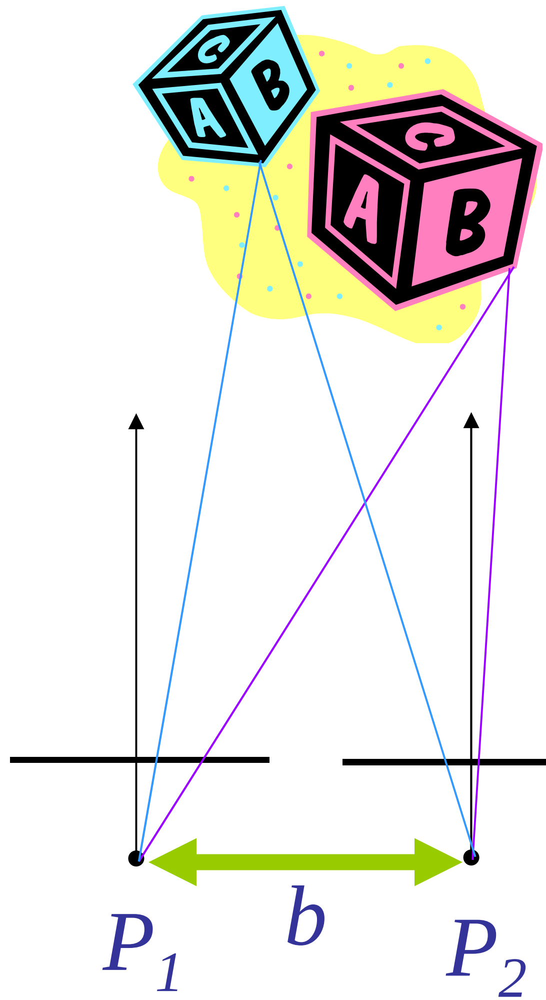
Binocular system geometry

- Geometry of 2 cameras:
  - Parallel geometry
  - Non-parallel geometry



Binocular system geometry

- The simplest case:
  - Parallel geometry



Binocular system geometry

- The simplest case:
  - Parallel geometry

$$P_1 = \begin{bmatrix} fk_u & 0 & u_0 & 0 \\ 0 & fk_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Change to the camera 1 coordinates system

$$P_2 = \begin{bmatrix} fk_u & 0 & u_0 & 0 \\ 0 & fk_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & b \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Binocular system geometry

- The simplest case:
  - Parallel geometry

$$P_1 = \begin{bmatrix} fk_u & 0 & u_0 & 0 \\ \boxed{0 \quad fk_v \quad v_0 \quad 0} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

They are the same!

$$P_2 = \begin{bmatrix} fk_u & 0 & u_0 & fk_u b \\ \boxed{0 \quad fk_v \quad v_0 \quad 0} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Binocular system geometry

- The simplest case:
  - Parallel geometry

$$\tilde{m}_1 = P_1 \tilde{M}, \quad m_1 = \begin{bmatrix} u_1 \\ v_1 \end{bmatrix}$$

$$\tilde{m}_2 = P_2 \tilde{M}, \quad m_2 = \begin{bmatrix} u_2 \\ v_2 \end{bmatrix}$$

$$\boxed{v_1 = v_2}$$

Binocular system geometry

- The simplest case:
  - Parallel geometry

$$u_1 = fk_u \frac{x}{z} + u_0$$

$$u_2 = fk_u \frac{x}{z} + u_0 + fk_u \frac{b}{z}$$

$$\boxed{u_2 = u_1 + fk_u \frac{b}{z}}$$

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Binocular system geometry

- The simplest case:
  - Parallel geometry

constant

$$u_2 - u_1 = \frac{f k_u b}{z}$$

$$u_2 = u_1 + f k_u \frac{b}{z}$$

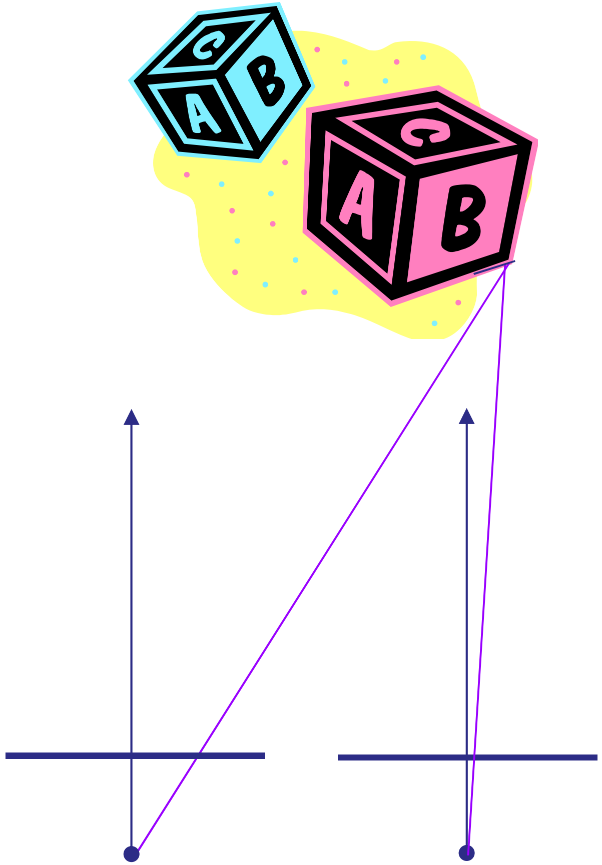
disparity

depth

Inversely proportional

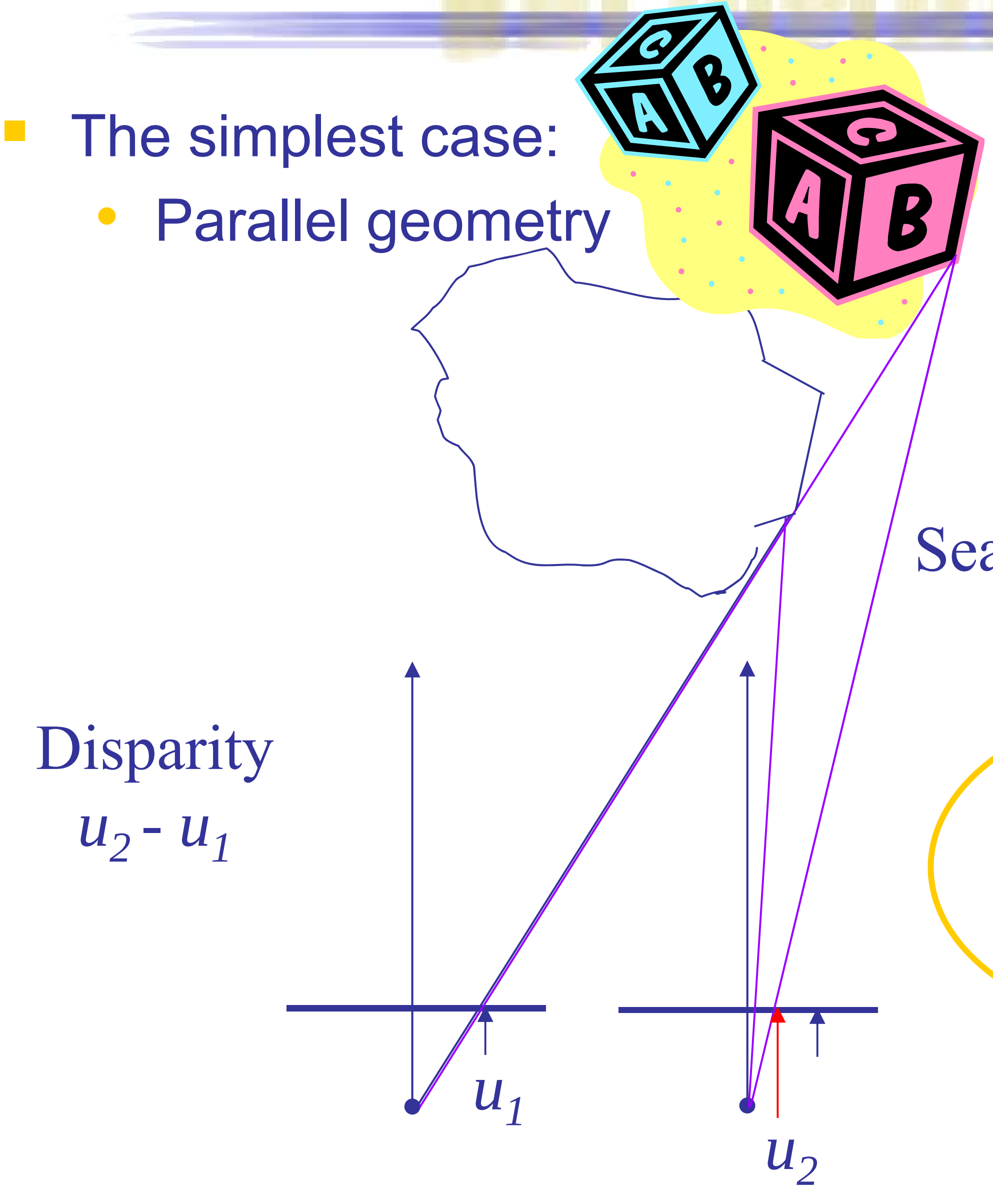
Binocular system geometry

- The simplest case:
  - Parallel geometry



Binocular system geometry

- The simplest case:
  - Parallel geometry

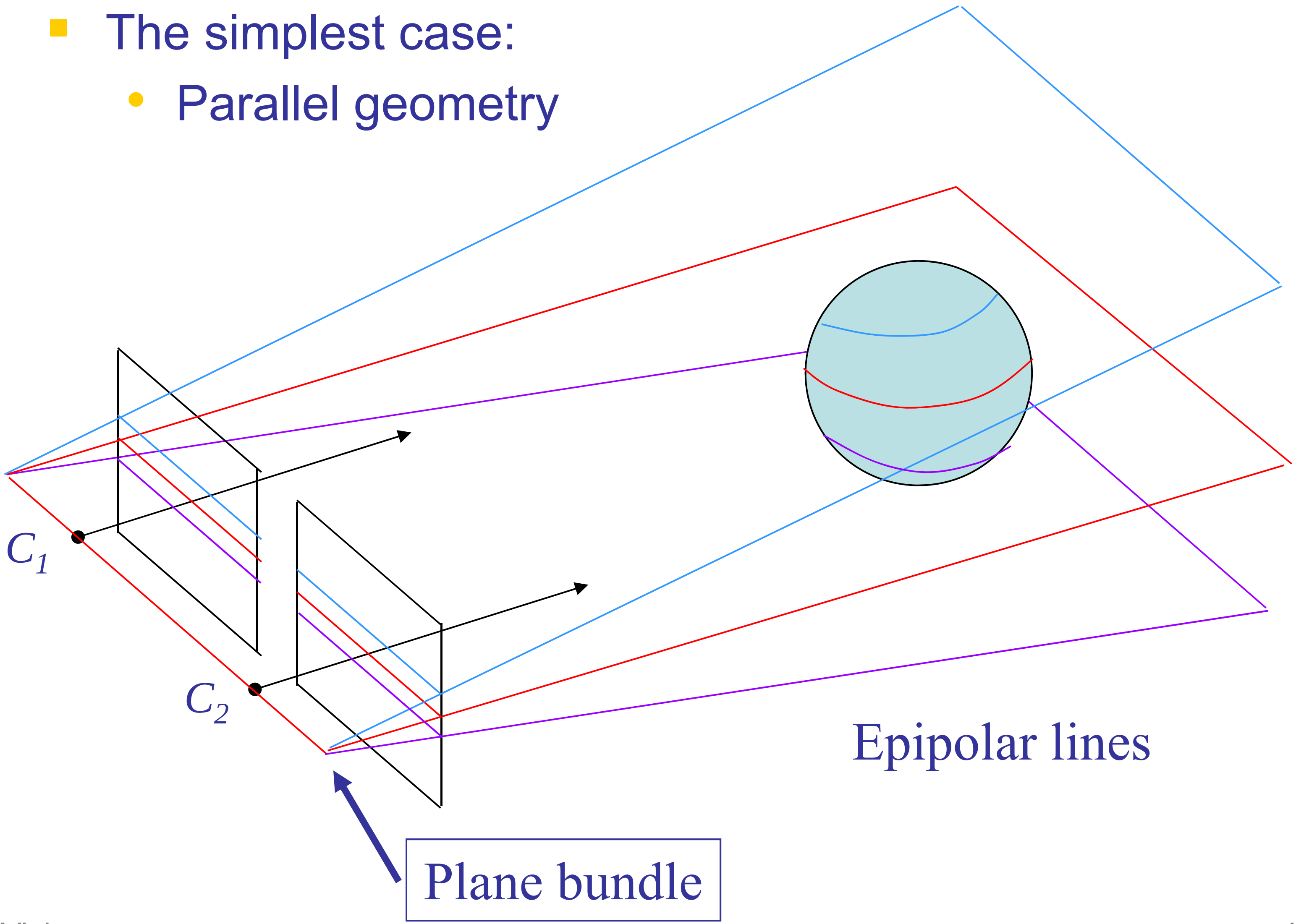


Search for homologous points:

The correspondence problem

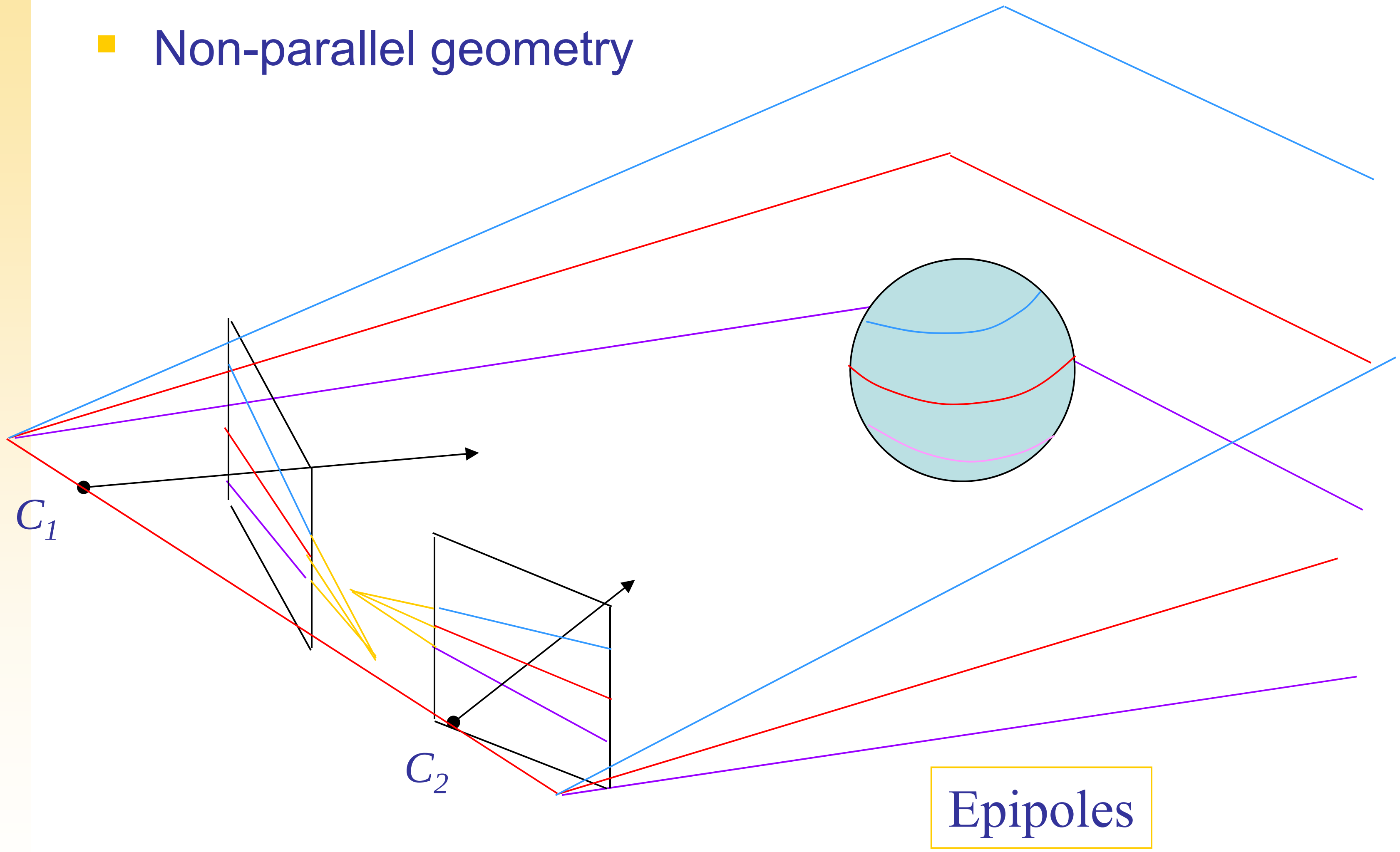
Binocular system geometry

- The simplest case:
  - Parallel geometry



Binocular system geometry

- Non-parallel geometry



Binocular system geometry

- Non-parallel geometry

$$P_1 = \begin{bmatrix} fk_u & 0 & u_0 & 0 \\ 0 & fk_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

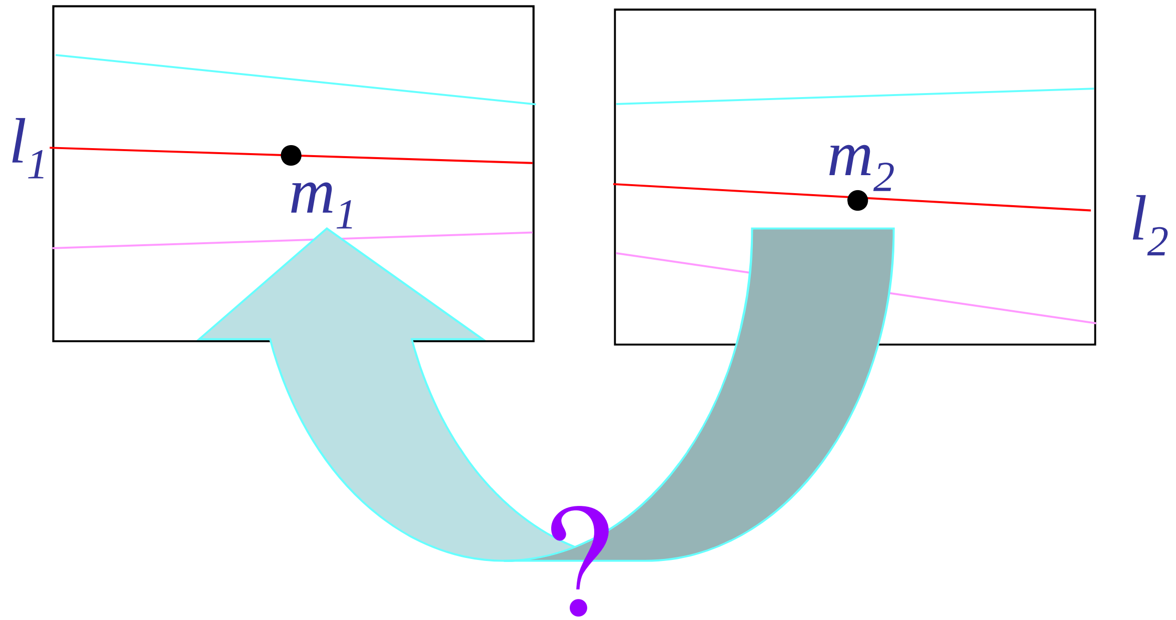
Assuming:

- World coordinates in (C_1, x_1, y_1, z_1)
- Both cameras are equal

$$P_2 = \begin{bmatrix} fk_u & 0 & u_0 & 0 \\ 0 & fk_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Binocular system geometry

- Fundamental matrix



Matriz 3x3

F= Fundamental matrix

$$F^{-1} = F^T$$

$$F\tilde{m}_1 = l_2$$

$$F^T\tilde{m}_2 = l_1$$

Homogeneous coordinates

l=(a,b,c)

l_2

$$\tilde{m}_2{}^T F\tilde{m}_1 = 0$$

$$\tilde{m}_1{}^T F^T\tilde{m}_2 = 0$$

l_1

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Binocular system geometry

- Calibration:
  - Obtain all calibration parameters (projection matrices $\boldsymbol{P_1}$ y $\boldsymbol{P_2}$)

$$s_1\tilde{m}_1 = P_1\tilde{M}$$
$$s_2\tilde{m}_2 = P_2\tilde{M}$$

$\boldsymbol{s_1, s_2}$: scale factors

  - Calculate $\boldsymbol{F}$ from $\boldsymbol{P_1}$ y $\boldsymbol{P_2}$

$$F = \left[q_2 - Q_2Q_1^{-1}q_1\right]_\times Q_2Q_1^{-1}$$

where $\boldsymbol{Q_i}$ are 3x3 matrices and $\boldsymbol{q_i}$ are 3x1 vectors,

$$P_i = [Q_i \quad q_i]$$

Factorization

3x3 matrix

3x1 matrix

Binocular system geometry

- **Weak** calibration:
  - Find pairs of homologous points in both images
  - Estimate F matrix from pairs of corresponding points
    - Method of 8-points
    - Linear methods
    - Non-linear methods

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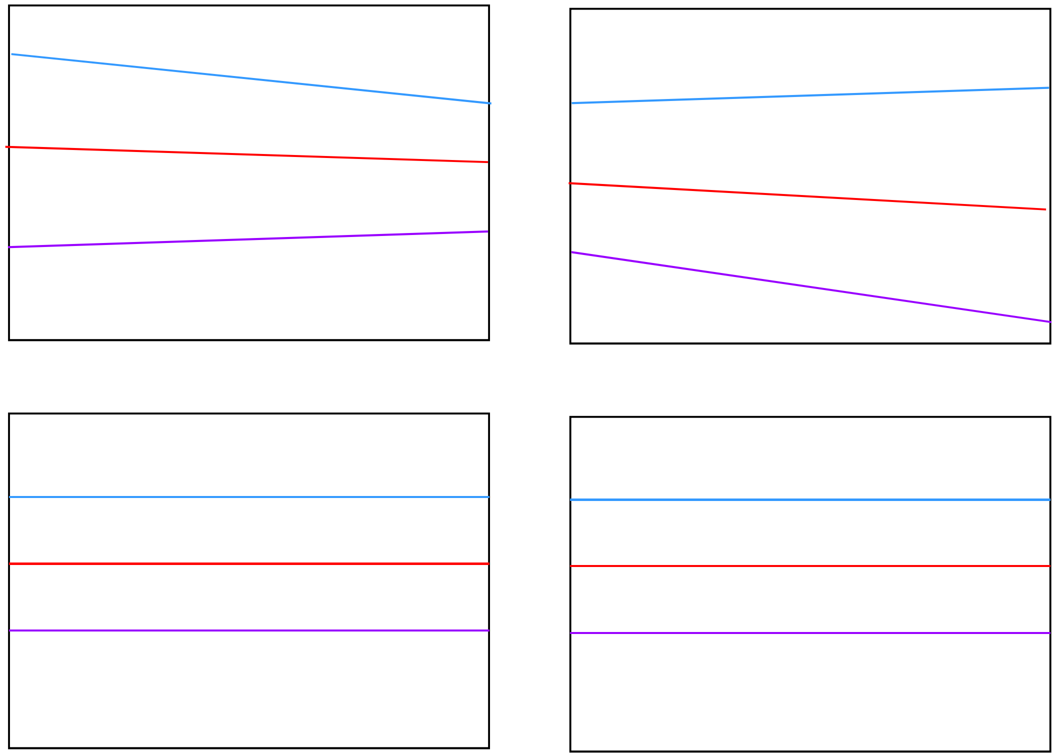
Binocular system geometry

- Rectification:
  - Make coincide epipolar lines with image rows



Binocular system geometry

- Rectification:
  - If the cameras have linear perspective
    - Rectify = project P_1 and P_2 in a rectification plane
  - If they have non-linear perspective
    - Must take into account distortion
  - If they do not have perspective projection
    - Very different methods

Binocular system geometry

- Rectification:
  - If the cameras have linear perspective
    - Rectify = project $\boldsymbol{P_1}$ and $\boldsymbol{P_2}$ in a rectification plane
    - Epipoles are sent to infinit
    - Rectified $\boldsymbol{F}$ is:

$$F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

Binocular system geometry

- Rectification:

$$F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$F\tilde{m}_1 = l_2$$

$$F \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ v_1 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = l_2$$

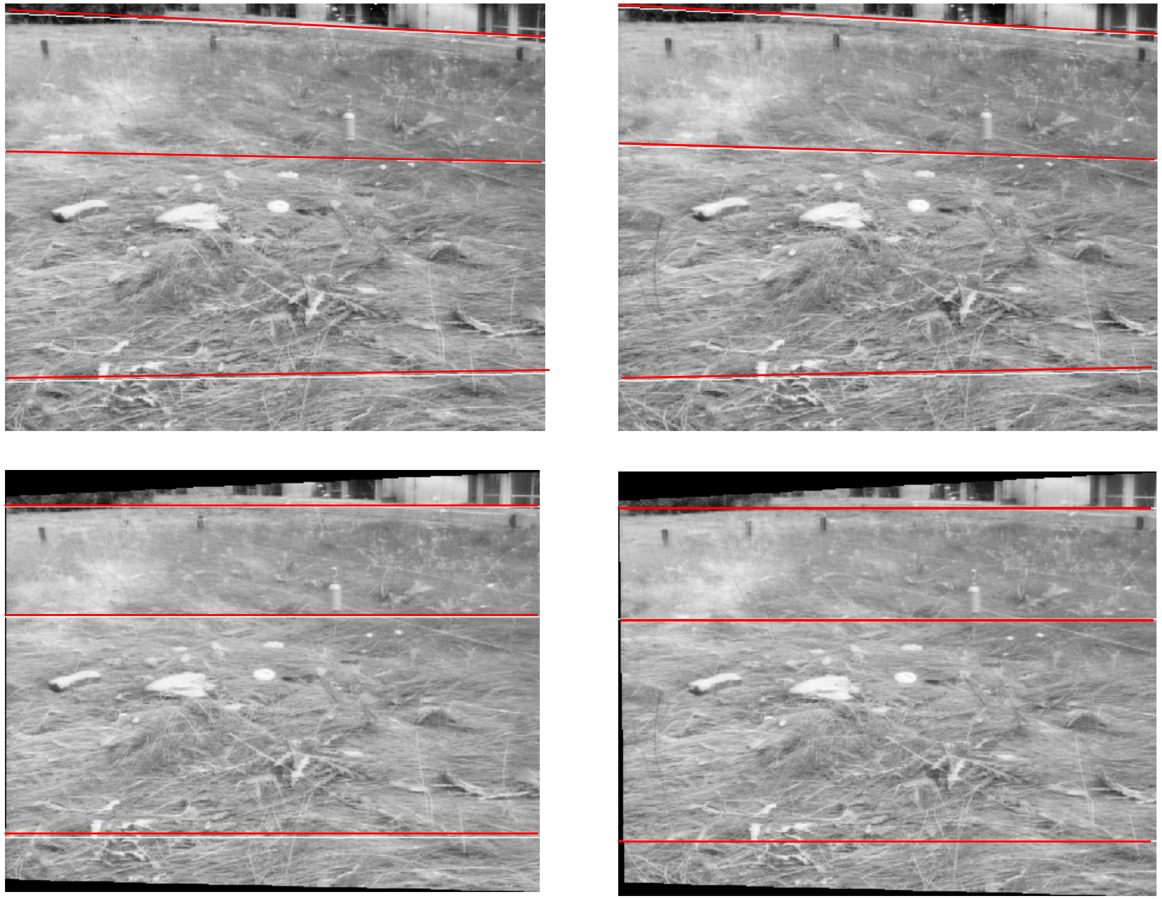
$$l_2 \tilde{m}_2 = 0$$

$$\begin{bmatrix} 0 & -1 & v_1 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = -v_2 + v_1 = 0$$

$$v_1 = v_2$$

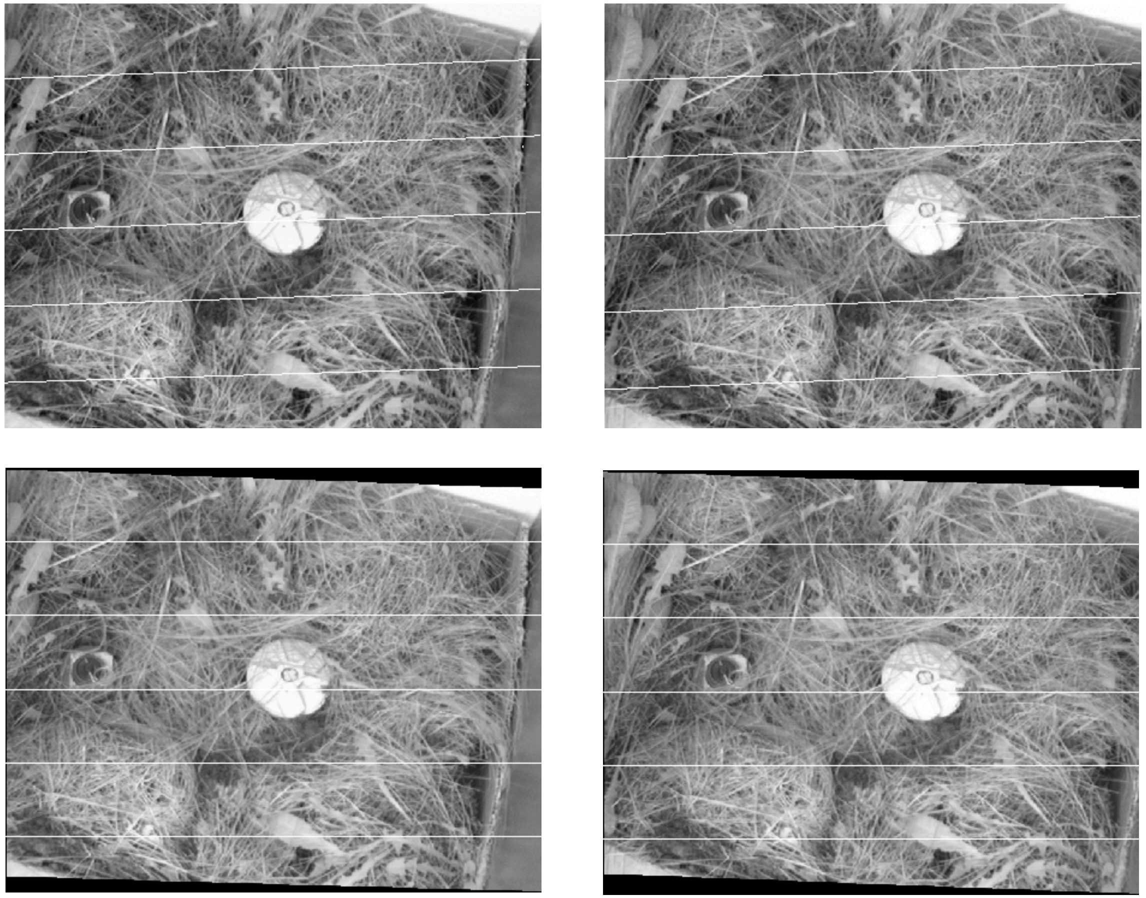
Binocular system geometry

- Rectification:



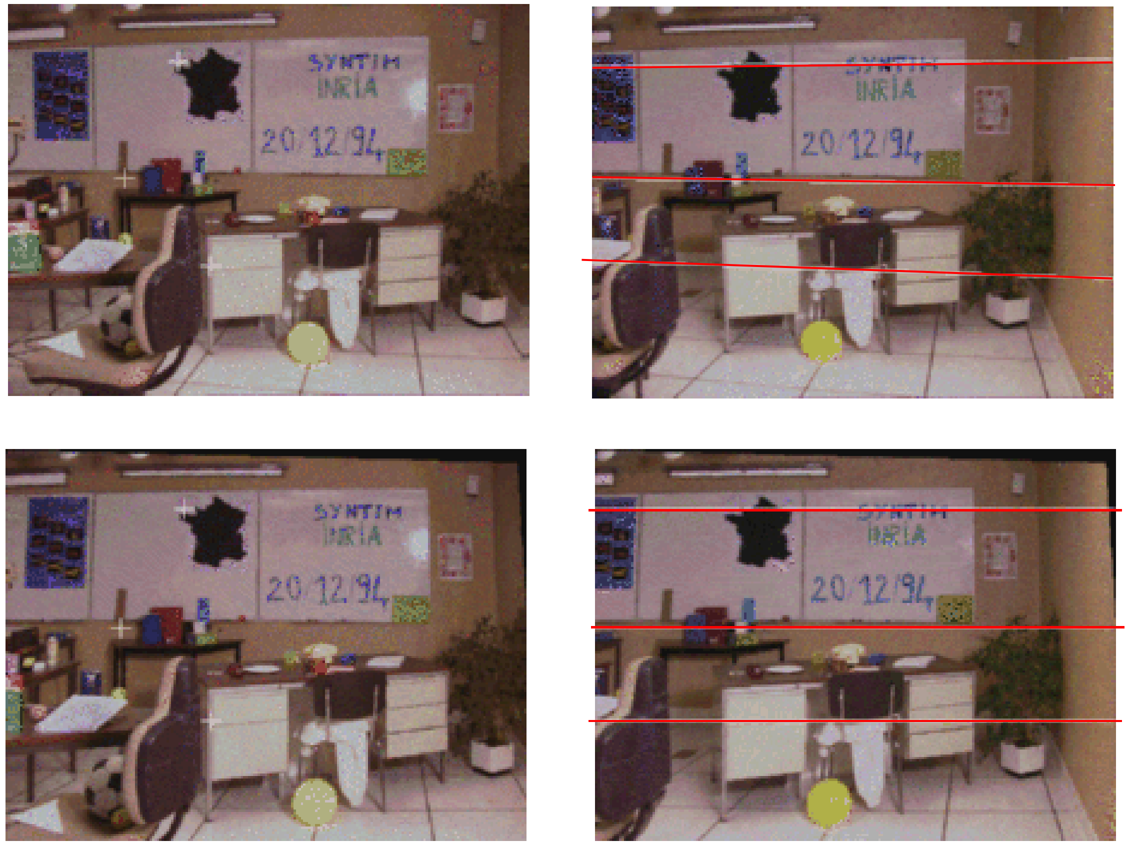
Binocular system geometry

- Rectification:



Binocular system geometry

- Rectification:



Binocular system geometry

- Rectification: comparison between linear and non-linear method

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