UNIVERSITATEA DIN BUCUREȘTI
FACULTATEA DE MATEMATICĂ ȘI INFORMATICĂ

TUTORIAT
ALGEBRĂ ȘI GEOMETRIE
II

STUDENȚI:

Radu Dimitrie Octavian

Țigănilă Ștefania

Bălan Mihai

BUCUREȘTI

2025

SLI. SG. Bază.

Exemplu 1:

Fie $S = \{(-1,3,4),(2,\ \alpha,-1),(1,2,2)\} \subset\ \mathbb{R}^3;\ \alpha \subset \mathbb{R}.\ \alpha = ?\, astfel\ incat\ S\ baza.$

Rezolvare:

$S\ baza\ \ \Leftrightarrow S\ SG\ si\ S\ SLI.$

$$\left.\begin{array}{r}|S| = 3\\ dim_{\mathbb{R}}\mathbb{R}^3 = 3\end{array}\right\} \Longrightarrow S\ este\ SG \qquad (1)$$

$Acum\ verificam\ daca\ S\ este\ SLI:$

$\forall\ a_1, a_2, a_3 \in\ \mathbb{R}\, astfel\ incat\ a_1\ (-1,3\ ,4) + a_2\ (2,\alpha,-1) + a_3(1,2,2) = 0_{\mathbb{R}}\ \Longrightarrow\ a_1 =$

$= \ a_2 = \ a_3 = 0_{\mathbb{R}}.$

$$\begin{cases}-a_1 + 2a_2 + a_3 = 0\\ 3a_1 + \alpha a_2 + 2a_3 = 0\\ 4a_1 - a_2 + 2a_3 = 0\end{cases} \quad A = \begin{pmatrix}-1 & 2 & 1\\ 3 & \alpha & 2\\ 4 & -1 & 2\end{pmatrix}$$

$S\ SLI \Leftrightarrow \det(A) \neq 0$ -------> criteriu LI

$$\det(A) = \begin{vmatrix}-1 & 2 & 1\\ 3 & \alpha & 2\\ 4 & -1 & 2\end{vmatrix} = \ldots = -6\alpha - 1.$$

$$\det(A) \neq 0 \Leftrightarrow -6\alpha - 1 \neq 0 \Leftrightarrow \alpha \neq -\frac{1}{6}.$$

$$Atunci\ S\ este\ SLI\ \forall\ \alpha \in \mathbb{R} \setminus \left\{-\frac{1}{6}\right\} \qquad (2)$$

$$Din\ relatiile\ (1)\ si\ (2) \Longrightarrow S\ este\ baza\ \forall\ \alpha \in \mathbb{R} \setminus \left\{-\frac{1}{6}\right\}.$$

Exemplu 2:

Fie $S' = \{(1,1,\beta),(-1,3,0),(-1,-1,2)\} \subset\ \mathbb{R}^3;\ \beta \subset \mathbb{R}.\ \beta = ?\, astfel\ incat\ S\ baza.$

--------------------------> $de\ facut$

Aflarea unui SLI maximal și extinderea la o bază a acestuia.

Exemplu 1:

$$(\mathbb{R}^3, +, \cdot)|_{\mathbb{R}}, S = \{u = (7\,,4\,,2), v = (2\,,1\,,4), w = (8\,,5\,,-4)\}$$

$$Sa\ se\ extraga\ SLI\ maximal\ S'\ si\ sa\ se\ extinda\ ala\ o\ baza\ in\ \mathbb{R}^3.$$

Rezolvare:

$$A = \begin{pmatrix} \mathbf{7} & \mathbf{2} & 8 \\ \mathbf{4} & \mathbf{1} & 5 \\ 2 & 4 & -8 \end{pmatrix}$$

$$\det(A) = \begin{vmatrix} 7 & 2 & 8 \\ 4 & 1 & 5 \\ 2 & 4 & -8 \end{vmatrix} = \cdots = 0.$$

$$Luam\ \begin{vmatrix} 7 & 2 \\ 4 & 1 \end{vmatrix}\ si\ verificam\ daca\ e\ \neq 0$$

$$\begin{vmatrix} 7 & 2 \\ 4 & 1 \end{vmatrix} = 7 - 8 = -1 \neq 0 \Longrightarrow S' = \{u, v\}\ SLI\ maximal\ in\ S.$$

$$Extindem\ S'\ la\ o\ baza\ in\ \mathbb{R}^3:$$

$$Alegem\ e_1 = (1\,,0\,,0).$$

$$rg\begin{pmatrix} 7 & 2 & 1 \\ 4 & 1 & 0 \\ 2 & 4 & 0 \end{pmatrix} =?$$

$$\begin{vmatrix} 7 & 2 & 1 \\ 4 & 1 & 0 \\ 2 & 4 & 0 \end{vmatrix} = \cdots = 14 \neq 0 \Longrightarrow rg\begin{pmatrix} 7 & 2 & 1 \\ 4 & 1 & 0 \\ 2 & 4 & 0 \end{pmatrix} = 3(maxim) \Longrightarrow S' \cup \{(1, 0, 0)\}\ baza.$$

$$Trivial\ am\ ales\ un\ vector\ din\ baza\ canonica, dar\ se\ poate\ alege\ orice\ vector\ astfel\ incat:$$

$$rg(S'|vector\ ales) = maxim$$

Metoda 2.

Observăm că unul dintre vectori se poate scrie sub forma unor combinații liniare a celorlalți vectori. (În cazul acesta $w = 2u - 3v$). Nu e trivial pentru acest caz 😊 dar o astfel de observație scutește calculul determinanților $\Longrightarrow$ $S' = \{u , v\}$. În cazul

$$S = \{u = (1, 2 ,3), v = (2 , 3, 4), w = \ (3, 5,7)\}\ se\ observa\ imediat\ ca\ u + v = w.$$

Exemplu 2:

$$(\mathbb{R}^3, +,\cdot)|_{\mathbb{R}}, S = \{u = (2 , 3 ,5), v = \left(2 , \frac{3}{2} , \frac{5}{2}\right), w = (4 ,6 ,10)\}$$

$$Sa\ se\ extraga\ SLI\ maximal\ S'\ si\ sa\ se\ extinda\ ala\ o\ baza\ in\ \mathbb{R}^3.$$

--------------------------> $de\ facut$

Subspații vectoriale. Bază într-un subspațiu vectorial.

Exemplu 1:

$$(\mathbb{R}^3, +,\cdot)|_{\mathbb{R}}, V' = \{(x, y, z) \in \mathbb{R}^3\ |7x - y + 3z = 0\}$$

$a)\ Este\ V'\ subspatiu\ vectorial\ =?\,?$

$b)\ Sa\ se\ precizeze\ o\ baza\ in\ V'.$

Rezolvare:

$$a) \left.\begin{matrix} \forall\ (x , y , z), (x', y', z') \in V' \\ \forall\ a, b \in \mathbb{R} \end{matrix}\right\} \Longrightarrow a(x , y , z) + b(x', y', z') \in V' \Leftrightarrow$$

$$\Leftrightarrow (ax + bx', ay + by', az + bz') \in V' \Leftrightarrow (x'', y'', z'') \in V'$$

$$7x'' - y'' + 2z'' = 7(ax + bx') - (ay + by') + 2(az + bz') = a(7x - y + 2z) +$$

$$+b(7x' - y' + 2z') = 0; dar \begin{cases} 7x - y + 2z = 0 \\ 7x' - y' + 2z' = 0 \end{cases} \Longrightarrow (x'', y'', z'') \in V'$$

$$b)\ (x , y , z) \in \Longrightarrow 7x - y + 3z = 0 \Longrightarrow y = 7x + 3z$$

$$(x\ ,y\ ,z\ ) = (x\ ,7x + 3z\ ,z\ ) = (x\ ,7x\ ,0) + (0\ ,3z\ ,z) = x(1\ ,7\ ,0) + z(0\ ,3\ ,1)$$

$$V' = \langle\{(1\ ,7\ ,0), (0\ ,3\ ,1)\}\rangle \Longrightarrow SG$$

$$rg\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{7} & \mathbf{3} \\ 0 & 1 \end{pmatrix} = 2(maxim) \Longrightarrow SLI$$

$$SG\ si\ SLI \Longrightarrow baza.$$

Exemplu 2:

$$(\mathbb{R}^3, +,\cdot)|_{\mathbb{R}}, V' = \{(x, y, z) \in \mathbb{R}^3 \mid \begin{cases} 3x - y = 0 \\ 2y + z = 0 \end{cases}\}$$

$a)\ Este\ V'\ subspatiu\ vectorial\ =??$

$b)\ Sa\ se\ precizeze\ o\ baza\ in\ V'.$

--------------------------> $de\ facut$

Repere. Coordonatele vectorilor în raport cu un reper.

Exemplu 1:

$$(\mathbb{R}^2, +,\cdot)|_{\mathbb{R}}, R_0 = \{e_1 = (1, 0), e_2 = (0, 1)\};\ R' = \{e_1' = (5, 2), e_2' = (3, 0)\}$$

$a)\ R'\ reper$

$b)\ R_0 \overset{A}{\rightarrow} R'\ ; R' \overset{B}{\rightarrow} R_0$

$A\ =?\ ; B\ =?\ ; Sunt\ la\ fel\ orientate?$

$c)\ x = (2, 5). Sa\ se\ precizeze\ coordonatele\ in\ raport\ cu\ R_0, R'\ =?$

Rezolvare:

$a)\ dim_{\mathbb{R}}\mathbb{R}^2 = 2$

$e_1' = 5e_1 + 2e_2$

$e_1' = 3e_1 + 0e_2$

$A = \begin{pmatrix} 5 & 3 \\ 2 & 0 \end{pmatrix}$;

$$\begin{cases} rg(A) = maxim \\ dim_{\mathbb{R}} \mathbb{R}^2 = 2 = |R'| \end{cases} \Longrightarrow reper$$

$b)\ R' \xrightarrow{A^{-1}} R_0;\ A^{-1} = ?;\ la\ fel\ orientate, invers\ orientate\ \Longrightarrow depinde\ de\ valoarea$

$determinantului.$

$c)\ Metoda\ "directa" : a(vector1\_reper) + b(vector2\_reper)\ =\ vector\_cautat;$

$Metoda\ 2 : Bordare\ reper\ cu\ vectorul\ cautat, forma\ esalon\ redusa.$