Law of large numbers

Mihai Cîra

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Abstract

The Law of Large Numbers is a statistical principle that states that as the number of trials in a random process increases, the average of the results will approach the expected value. This paper provides a comprehensive overview of the Law of Large Numbers, including a clear explanation of the concept, a demonstration of its application through a programmed example, and an examination of the law under different parameters. The program presents a simulation of the law in action, allowing readers to visualize the convergence of the average towards the expected value as the number of trials increases. The discussion of the law under different parameters explores the impact of varying sample sizes and distributions on the rate of convergence. This paper provides a valuable resource for anyone looking to deepen their understanding of the Law of Large Numbers and its applications in the field of statistics.

Contents

1 The theorem

The law of large numbers is a fundamental concept in probability theory that states that as the number of trials or independent events increases, the average of the results obtained from those trials will converge to the expected value or the true mean of the underlying population.

In other words, as the sample size increases, the average of the sample values becomes more and more accurate in reflecting the population mean. This result can be expressed mathematically as follows: let $X_1, X_2, ..., X_n$ be a sequence of independent and identically distributed (i.i.d) random variables with expected value μ. Then, as n approaches infinity, the sample mean (the average of the random variables), converges to μ in probability.

In practical applications, the law of large numbers is often used to make inferences about population parameters based on a sample of data. For example, in a large enough sample, the average income of a group of people will approximate the population mean income, even if there is significant variation in the individual incomes in the sample.

2 The program

2.1 The code

Here's a simple implementation of the law of large numbers, in Python (version 3.10.7 used):

```
import random

def law_of_large_numbers(n):
    sum = 0
    for i in range(n):
        sum += random.random()
    avg = sum / n
    return avg

# Run the simulation with a large sample size

result = law_of_large_numbers(1000000)
print(result)
```

The law of large numbers states that as the sample size increases, the average of the sample values approaches the expected value of the population. In this case, the expected value of the uniform distribution (which generates random numbers between 0 and 1) is 0.5. Running the simulation with a large sample size (e.g. 1 million) should produce an average value close to 0.5.

2.2 Analysis of the program

The previous program run time was **0.061017 seconds**, and the printed result of the program was 0.500324. The environment that the program has been run on is a Windows device with a AMD Ryzen 7 5700U 1.80GHz processor, that runs on a 64-bit operating system.

2.3 Other parameters

The previous example of code has been run with the n parameter having the value of **1 000 000**. As stated above, the result of that program was a value (0.500324) that's very close to the expected value (0.50). Here are some other examples with other parameters, both smaller and larger than the previous example:

- Parameter n having the value of **10**, the result is: **0.4255**;
- Parameter n having the value of **50**, the result is: **0.5479**;
- Parameter n having the value of **500**, the result is: **0.4985**;
- Parameter n having the value of **5 000**, the result is: **0.5051**;
- Parameter n having the value of **10 000**, the result is: **0.5036**;
- Parameter n having the value of **10 000 000**, the result is: **0.5001**;

As stated in the first section (The theorem), you can observe that as much as the n value increases, the result gets closer to the desired μ value.

3 A real-life example of the theorem

A real-life example of this can be seen in the insurance industry. Insurance companies use the law of large numbers to determine the expected number of claims they will receive from their policyholders. They gather data on past claims, and use that information to estimate the average number of claims they can expect to receive in the future.

For example, let's say an insurance company has data on 100,000 car insurance policyholders. They know that, on average, 10% of these policyholders will file a claim in a given year. Based on this information, they can expect to receive 10,000 claims in a year. However, if they only have data on 10 policyholders, they can't be as confident in their estimate.

As the number of policyholders increases, the estimate of the average number of claims will become more accurate, approaching the expected value. This represents an example of the law of large numbers in action.

4 Conclusion

In conclusion, the Law of Large Numbers is a fundamental principle in statistics that has a wide range of applications in fields such as finance, insurance, and engineering. The demonstration of the law through a programmed example, as well as the examination of its behavior under different parameters, highlights the importance of understanding this concept in order to make informed decisions based on probabilities. This paper serves as a comprehensive resource for those looking to gain a deeper understanding of the Law of Large Numbers and its implications. Further research could be conducted to explore the limitations and assumptions of the law and to examine its application in real-world scenarios. Ultimately, the Law of Large Numbers is a powerful tool that can provide insight into the behavior of random processes and support informed decision-making in a variety of fields.