Structural Equation Modeling

P.05 - Model Fit and Fit Indices (Note)

08.11.2021

Set the working directory to the location where your data file has been downloaded and load the data.

```r
# For example.
setwd("/Users/mihai/Downloads")

# Load data.
data <- read.csv("ELEMM1.csv")

# Inspect the data.
View(data)
```

Note on the χ^2 difference value in the LRT.

You indicated that the χ^2 difference value in the LRT computation is not the same as the actual difference between the standard χ^2 reported by `lavaan` (e.g., during model `summary()`).

This seems to be the case because we estimated the models using the *Satorra Bentler* method. In this scenario, `lavaan` will use the standard χ^2 values, however it will apply a scaled test statistic using the `satorra.bentler.2001` method. This is mentioned both in the output of `lavTestLRT()` and the documentation for this function.

For example, if we run `lavTestLRT()`,

```r
lavTestLRT(model_ex_1_fit, model_ex_1_modification_1_fit)
```

we see the following note:

```
Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")


lavaan NOTE:
    The "Chisq" column contains standard test statistics, not the
    robust test that should be reported per model. A robust difference
    test is a function of two standard (not robust) statistics.
```

Then, in the documentation of `lavTestLRT()` we see the following:

```r
?lavTestLRT
```

```
The anova function for lavaan objects simply calls the lavTestLRT function,
```

```
which has a few additional arguments.

If `type = "Chisq"` and the test statistics are scaled, a special scaled
difference test statistic is computed. If method is `"satorra.bentler.2001"`, a
simple approximation is used described in Satorra & Bentler (2001). In some
settings, this can lead to a negative test statistic. To ensure a positive test
statistic, we can use the method proposed by Satorra & Bentler (2010).
Alternatively, when method is `"satorra.2000"`, the original formulas of Satorra
(2000) are used.
```

We know that if we use the `ML` estimator, then the χ^2 difference value in the LRT should be in fact the difference between the standard χ^2 values. In our case, we expect this difference to be the result of adding another parameter to the model based on the modification indices. We can check this as follows:

```r
# Model syntax model 1.
model_1 <- "
    # Measurement part.
    EMO =~ ITEM1 + ITEM2 + ITEM3 + ITEM6 + ITEM8 + ITEM13 + ITEM14 + ITEM16 + ITEM20
    DEP =~ ITEM5 + ITEM10 + ITEM11 + ITEM15 + ITEM22
    ACC =~ ITEM4 + ITEM7 + ITEM9 + ITEM12 + ITEM17 + ITEM18 + ITEM19 + ITEM21

    # Covariances between latent variables.
    EMO ~~ DEP
    DEP ~~ ACC
    EMO ~~ ACC
"

# Model syntax model 2.
model_2 <- "
    # Measurement part.
    EMO =~ ITEM1 + ITEM2 + ITEM3 + ITEM6 + ITEM8 + ITEM13 + ITEM14 + ITEM16 + ITEM20
    DEP =~ ITEM5 + ITEM10 + ITEM11 + ITEM15 + ITEM22
    ACC =~ ITEM4 + ITEM7 + ITEM9 + ITEM12 + ITEM17 + ITEM18 + ITEM19 + ITEM21

    # Covariances between latent variables.
    EMO ~~ DEP
    DEP ~~ ACC
    EMO ~ ~ACC

    # Covariances between error terms.
    ITEM6 ~~ ITEM16
"

# Model fit model 1.
model_1_fit <- cfa(model_1, data = data, estimator = "ML")

# Model fit model 2.
model_2_fit <- cfa(model_2, data = data,  estimator = "ML")
```

```r
# Print first 3 modification indices for `model_1_fit`.
modificationIndices(model_1_fit, sort. = TRUE)[1:3, ]
```

```
##       lhs op    rhs     mi    epc sepc.lv sepc.all sepc.nox
## 158 ITEM6 ~~ ITEM16 91.282  0.733   0.733    0.529    0.529
## 95  ITEM1 ~~  ITEM2 82.448  0.613   0.613    0.549    0.549
## 59    EMO =~ ITEM12 41.517 -0.313  -0.400   -0.335   -0.335
```

We expect the reduction in χ^2 for `model_2` to be roughly 91.282, based on the inclusion of parameter `ITEM6 ~~ ITEM16`. We see that this is indeed the case if we subtract the χ^2 values for `model_1_fit` and `model_2_fit`:

$$\chi^2_{\text{model 1}} - \chi^2_{\text{model 2}} = 695.719 - 597.731 = 97.988$$

Now, if we perform a LRT we expect to see the same difference since this time we used the default `ML` estimator.

```r
# Likelihood Ratio Test.
lavTestLRT(model_1_fit, model_2_fit)
```

```
## Chi-Squared Difference Test
##
##              Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## model_2_fit 205 25620 25808 597.73
## model_1_fit 206 25716 25900 695.72     97.988       1  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Again, we see a χ^2 difference of 97.988.