P.07 - Measurement Invariance

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Lab Description

During this practical you will learn how to test for various types of measurement invariance when dealing with both cross-sectional (i.e., *Exercise 1*) and longitudinal data (i.e., *Exercise 2*).

For this practical you will need the following packages:

- `lavaan` for structural equation modeling
- `semPlot` for visualizing structural equation models
- `mvtnorm` for generating multivariate normal data
- `GGally` for visualizing multivariate normal data

You can install and load these packages using the following code:

```
# Install packages.
install.packages(c("lavaan", "semPlot", "mvtnorm", "GGally"))

# Load the packages.
library(lavaan)
library(semPlot)
library(mvtnorm)
library(GGally)
```

Exercise 1

This exercise consists of two parts. In the first part, we prepare the data (i.e., in the form of means and covariances). In the second part, you are asked to perform the measurement invariance tests discussed during the lecture. Note that instead of using the entire data for our measurement invariance investigation, we will instead use mean vectors and covariance matrices (i.e., see the *Multiple groups* section from the `lavaan` tutorial).

Part 1. Preparing the data.

In Table 4.3 of Beaujean (2014, p. 66) (i.e., depicted in *Figure 1*) we are presented with the means and covariances for a set of eight random variables. Each random variables is assumed to be normally distributed

and represents an item on the Wechsler Intelligence Scale for Children-Third Edition.

Table 4.3 Covariances and Means for Wechsler Intelligence Scale for Children-Third Edition Subtests.

	Info	Sim	Vocab	Comp	PicComp	PicArr	BlkDsgn	ObjAsmb
Information	9.364	7.777	6.422	5.669	3.048	3.505	3.690	3.640
Similarities	7.777	12.461	8.756	7.445	4.922	4.880	5.440	4.641
Vocabulary	6.422	8.756	10.112	6.797	4.513	4.899	5.220	4.877
Comprehension	5.669	7.445	6.797	8.123	4.116	5.178	3.151	3.568
Picture Completion	3.048	4.922	4.513	4.116	6.200	5.114	3.587	3.819
Picture Arrangement	3.505	4.880	4.899	5.178	5.114	15.603	6.219	5.811
Block Design	3.690	5.440	5.220	3.151	3.587	6.219	11.223	6.501
Object Assembly	3.640	4.641	4.877	3.568	3.819	5.811	6.501	9.797
Subtest Mean	10.090	12.070	10.250	9.960	10.900	11.240	10.300	10.440

(a) Youth with Manic Symptoms ($n = 81$). Data taken from Beaujean et al. (2012, p. 5).

	Info	Sim	Vocab	Comp	PicComp	PicArr	BlkDsgn	ObjAsmb
Information	9.610	5.844	6.324	4.405	4.464	3.478	5.270	4.297
Similarities	5.844	8.410	6.264	4.457	4.547	2.967	4.930	4.594
Vocabulary	6.324	6.264	9.000	5.046	4.512	2.970	4.080	4.356
Comprehension	4.405	4.457	5.046	8.410	3.712	2.871	3.254	3.158
Picture Completion	4.464	4.547	4.512	3.712	10.240	3.802	5.222	4.963
Picture Arrangement	3.478	2.967	2.970	2.871	3.802	10.890	3.590	3.594
Block Design	5.270	4.930	4.080	3.254	5.222	3.590	11.560	6.620
Object Assembly	4.297	4.594	4.356	3.158	4.963	3.594	6.620	10.890
Subtest Mean	10.100	10.300	9.800	10.100	10.100	10.100	9.900	10.200

(b) WISC-III Norming Sample ($n = 200$). Data taken from Wechsler (1991).

Figure 1: Table 4.3 presented in Beaujean (2014).

Together, these items are assumed to measure two latent construct, namely the Verbal-Comprehension (VC) and Visual-Spatial (VS) components of intelligence, as depicted in the model in *Figure 2*.

In our case, we have two groups, (1) one containing youth with manic symptoms (i.e., $N = 81$), and (2) one representing the norming sample (i.e., $N = 200$). To keep things short and readable, we will use the following abbreviations for the variable names:

- `inf` = Information
- `sim` = Similarities
- `voc` = Vocabulary
- `com` = Comprehension
- `p_com` = Picture Completion
- `p_arr` = Picture Arrangement
- `b_des` = Block Design
- `o_ass` = Object Assembly

We start by storing the variable names, means, and covariances.

```
# Variable names.
var_names <- c("inf", "sim", "voc", "com", "p_com", "p_arr", "b_des", "o_ass")

# Means and covariances for group with manic symptoms (i.e., group 1).
```

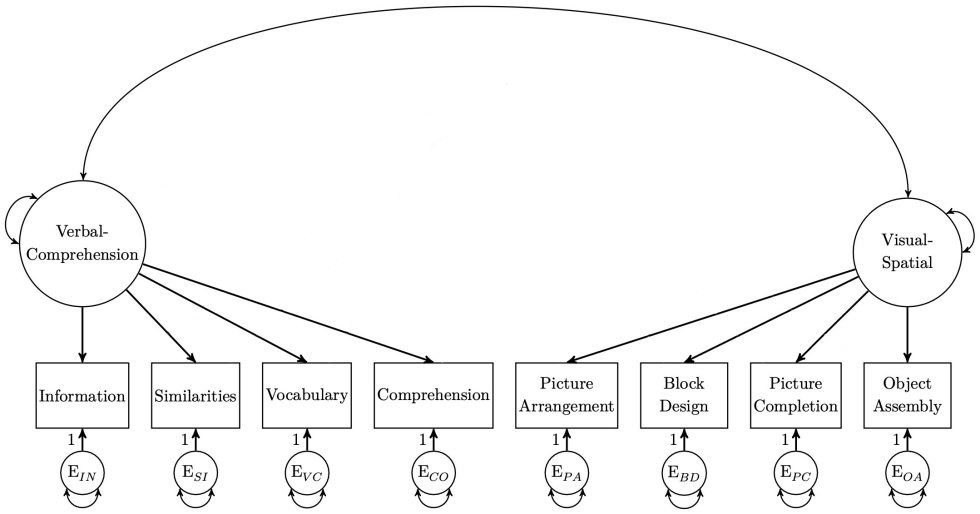


Figure 2: Adaptation of *Figure 4.3* from Beaujean (2014).

```
# Means for group 1.
group_1_means <- c(10.09, 12.07, 10.25, 9.96, 10.90, 11.24, 10.30, 10.44)

# Lower triangle of the covariance matrix for group 1.
group_1_cov_lower <- c(
    9.364, 7.777, 12.461, 6.422, 8.756, 10.112, 5.669,
    7.445, 6.797, 8.123, 3.048, 4.922, 4.513, 4.116,
    6.200, 3.505, 4.880, 4.899, 5.178, 5.114, 15.603,
    3.690, 5.440, 5.220, 3.151, 3.587, 6.219, 11.223,
    3.640, 4.641, 4.877, 3.568, 3.819, 5.811, 6.501, 9.797
)

# Create the entire covariance matrix for group 1.
group_1_cov <- lav_matrix_lower2full(group_1_cov_lower)

# Means and covariances for norming group (i.e., group 2).
# Means for group 2.
group_2_means <- c(10.10, 10.30, 9.80, 10.10, 10.10, 10.10, 9.90, 10.20)

# Lower triangle of the covariance matrix for group 2.
group_2_cov_lower <- c(
    9.610, 5.844, 8.410, 6.324, 6.264, 9.000, 4.405,
    4.457, 5.046, 8.410, 4.464, 4.547, 4.512, 3.712,
    10.240, 3.478, 2.967, 2.970, 2.871, 3.802, 10.890,
    5.270, 4.930, 4.080, 3.254, 5.222, 3.590, 11.560,
    4.297, 4.594, 4.356, 3.158, 4.963, 3.594, 6.620, 10.890
)

# Create the entire covariance matrix for group 2.
group_2_cov <- lav_matrix_lower2full(group_2_cov_lower)

# Add the variable names to the means and covariances for both groups.
# Add the names for the means.
names(group_1_means) <- var_names
names(group_2_means) <- var_names

# Add the names for the covariances.
```

```r
rownames(group_1_cov) <- colnames(group_1_cov) <- var_names
rownames(group_2_cov) <- colnames(group_2_cov) <- var_names
```

We can print the the means and covariances for both groups as follows:

```r
# Means for group 1.
print(group_1_means)
```

```
##   inf   sim   voc   com p_com p_arr b_des o_ass
## 10.09 12.07 10.25  9.96 10.90 11.24 10.30 10.44
```

```r
# Means for group 2.
print(group_2_means)
```

```
##   inf   sim   voc   com p_com p_arr b_des o_ass
##  10.1  10.3   9.8  10.1  10.1  10.1   9.9  10.2
```

```r
# Covariances for group 1.
print(group_1_cov)
```

```
##         inf    sim    voc   com p_com  p_arr  b_des o_ass
## inf   9.364  7.777  6.422 5.669 3.048  3.505  3.690 3.640
## sim   7.777 12.461  8.756 7.445 4.922  4.880  5.440 4.641
## voc   6.422  8.756 10.112 6.797 4.513  4.899  5.220 4.877
## com   5.669  7.445  6.797 8.123 4.116  5.178  3.151 3.568
## p_com 3.048  4.922  4.513 4.116 6.200  5.114  3.587 3.819
## p_arr 3.505  4.880  4.899 5.178 5.114 15.603  6.219 5.811
## b_des 3.690  5.440  5.220 3.151 3.587  6.219 11.223 6.501
## o_ass 3.640  4.641  4.877 3.568 3.819  5.811  6.501 9.797
```

```r
# Covariances for group 2.
print(group_2_cov)
```

```
##         inf   sim   voc   com  p_com  p_arr  b_des  o_ass
## inf   9.610 5.844 6.324 4.405  4.464  3.478  5.270  4.297
## sim   5.844 8.410 6.264 4.457  4.547  2.967  4.930  4.594
## voc   6.324 6.264 9.000 5.046  4.512  2.970  4.080  4.356
## com   4.405 4.457 5.046 8.410  3.712  2.871  3.254  3.158
## p_com 4.464 4.547 4.512 3.712 10.240  3.802  5.222  4.963
## p_arr 3.478 2.967 2.970 2.871  3.802 10.890  3.590  3.594
## b_des 5.270 4.930 4.080 3.254  5.222  3.590 11.560  6.620
## o_ass 4.297 4.594 4.356 3.158  4.963  3.594  6.620 10.890
```

Finally, as we can see in the documentation of `lavaan` for arguments `sample.cov`, `sample.mean`, and `sample.nobs`, for multiple group analysis we need to specify a list of mean vectors and covariance matrices:

> `sample.cov`: For a multiple group analysis, a list with a variance-covariance matrix for each group.
>
> `sample.mean`: For a multiple group analysis, a list with a mean vector for each group.
>
> `sample.nobs`: For a multiple group analysis, a list or a vector with the number of observations for each group.

We can create the required lists as follows:

```r
# Combine the mean vectors into a single list.
means <- list(
    group_1 = group_1_means,
    group_2 = group_2_means
)
```

```r
# Combine the covariance matrices into a single list.
covariances <- list(
    group_1 = group_1_cov,
    group_2 = group_2_cov
)

# Combine the sample sizes into a single list.
samples <- list(
    group_1 = 81,
    group_1 = 200
)
```

Specify which fit measures we are interested in:

```r
# Fit indices to print.
fit_indices <- c("chisq", "df", "pvalue", "cfi", "tli", "rmsea", "rmsea.pvalue", "srmr")
```

Part 2. Measurement invariance investigation.

a. Write down the model syntax for the model in *Figure 2*.

```r
# Model syntax.
model_ex_1 <- "
    # Measurement part.
    verbal =~ inf + sim + voc + com
    visual =~ p_com + p_arr + b_des + o_ass

    # Latent covariance.
    verbal ~~ visual
"
```

b. Manually fit the model in *Figure 2* for each group and allow the means of the observed variables to enter the model. Report and interpret the model fit.

To allow the observed means to enter the model, we need to set `meanstructure = TRUE`, in the `lavaan` function `sem` or `cfa`. First, we fit the model for the first group.

```r
# Fit the model for group 1.
model_ex_1_group_1_fit <- sem(
    model_ex_1,
    sample.cov = group_1_cov,
    sample.nobs = 81,
    sample.mean = group_1_means,
    meanstructure = TRUE
)

# Model summary.
summary(model_ex_1_group_1_fit, standardized = TRUE)
```

```
## lavaan 0.6-12 ended normally after 42 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        25
##
##   Number of observations                            81
##
## Model Test User Model:
##
##   Test statistic                                29.169
```

```
##   Degrees of freedom                                19
##   P-value (Chi-square)                           0.063
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal =~
##     inf               1.000                               2.347    0.772
##     sim               1.330    0.153    8.687    0.000    3.121    0.890
##     voc               1.189    0.138    8.613    0.000    2.791    0.883
##     com               1.015    0.125    8.129    0.000    2.382    0.841
##   visual =~
##     p_com             1.000                               1.788    0.723
##     p_arr             1.437    0.274    5.246    0.000    2.570    0.655
##     b_des             1.322    0.234    5.641    0.000    2.364    0.710
##     o_ass             1.285    0.220    5.830    0.000    2.297    0.738
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal ~~
##     visual            3.086    0.772    3.997    0.000    0.735    0.735
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf              10.090    0.338   29.861    0.000   10.090    3.318
##    .sim              12.070    0.390   30.965    0.000   12.070    3.441
##    .voc              10.250    0.351   29.191    0.000   10.250    3.243
##    .com               9.960    0.315   31.648    0.000    9.960    3.516
##    .p_com            10.900    0.275   39.643    0.000   10.900    4.405
##    .p_arr            11.240    0.436   25.769    0.000   11.240    2.863
##    .b_des            10.300    0.370   27.843    0.000   10.300    3.094
##    .o_ass            10.440    0.346   30.206    0.000   10.440    3.356
##     verbal            0.000                               0.000    0.000
##     visual            0.000                               0.000    0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf               3.742    0.673    5.564    0.000    3.742    0.405
##    .sim               2.564    0.605    4.237    0.000    2.564    0.208
##    .voc               2.200    0.502    4.379    0.000    2.200    0.220
##    .com               2.348    0.467    5.027    0.000    2.348    0.293
##    .p_com             2.926    0.592    4.945    0.000    2.926    0.478
##    .p_arr             8.806    1.631    5.398    0.000    8.806    0.571
##    .b_des             5.497    1.089    5.046    0.000    5.497    0.496
##    .o_ass             4.399    0.916    4.804    0.000    4.399    0.455
##     verbal            5.507    1.369    4.024    0.000    1.000    1.000
##     visual            3.198    0.924    3.462    0.001    1.000    1.000
```

```
# Fit measures.
fitMeasures(model_ex_1_group_1_fit, fit.measures = fit_indices)
```

```
##        chisq           df       pvalue          cfi          tli        rmsea rmsea.pvalue
##       29.169       19.000        0.063        0.971        0.958        0.081        0.184
##         srmr
##        0.047
```

Then, we fit the model for the second group.

```
# Fit the model for group 1.
model_ex_1_group_2_fit <- sem(
    model_ex_1,
    sample.cov = group_2_cov,
    sample.nobs = 200,
    sample.mean = group_2_means,
    meanstructure = TRUE
)

# Model summary.
summary(model_ex_1_group_2_fit, standardized = TRUE)
```

```
## lavaan 0.6-12 ended normally after 45 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        25
##
##   Number of observations                           200
##
## Model Test User Model:
##
##   Test statistic                                24.211
##   Degrees of freedom                               19
##   P-value (Chi-square)                          0.188
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal =~
##     inf               1.000                               2.440    0.789
##     sim               0.997    0.079   12.607    0.000    2.433    0.841
##     voc               1.045    0.082   12.778    0.000    2.550    0.852
##     com               0.768    0.083    9.301    0.000    1.874    0.648
##   visual =~
##     p_com             1.000                               2.182    0.684
##     p_arr             0.715    0.122    5.854    0.000    1.561    0.474
##     b_des             1.149    0.135    8.542    0.000    2.507    0.739
##     o_ass             1.100    0.130    8.464    0.000    2.401    0.730
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal ~~
##     visual            4.103    0.661    6.204    0.000    0.771    0.771
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf              10.100    0.219   46.192    0.000   10.100    3.266
##    .sim              10.300    0.205   50.355    0.000   10.300    3.561
##    .voc               9.800    0.212   46.314    0.000    9.800    3.275
##    .com              10.100    0.205   49.377    0.000   10.100    3.491
##    .p_com            10.100    0.226   44.748    0.000   10.100    3.164
##    .p_arr            10.100    0.233   43.392    0.000   10.100    3.068
##    .b_des             9.900    0.240   41.282    0.000    9.900    2.919
```

```
##    .o_ass            10.200    0.233   43.822    0.000   10.200    3.099
##     verbal            0.000                               0.000    0.000
##     visual            0.000                               0.000    0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf              3.609    0.455    7.928    0.000    3.609    0.377
##    .sim              2.450    0.354    6.925    0.000    2.450    0.293
##    .voc              2.453    0.370    6.635    0.000    2.453    0.274
##    .com              4.857    0.533    9.114    0.000    4.857    0.580
##    .p_com            5.426    0.675    8.034    0.000    5.426    0.533
##    .p_arr            8.398    0.897    9.364    0.000    8.398    0.775
##    .b_des            5.215    0.716    7.279    0.000    5.215    0.453
##    .o_ass            5.069    0.682    7.434    0.000    5.069    0.468
##     verbal           5.953    0.928    6.416    0.000    1.000    1.000
##     visual           4.763    0.952    5.006    0.000    1.000    1.000
```

```r
# Fit measures.
fitMeasures(model_ex_1_group_2_fit, fit.measures = fit_indices)
```

```
##        chisq           df       pvalue          cfi          tli        rmsea rmsea.pvalue
##       24.211       19.000        0.188        0.992        0.989        0.037        0.662
##         srmr
##        0.029
```

c. Investigate *configural* measurement invariance. Report and interpret the model fit.

We already investigated configural measurement invariance at point *(b)* by manually fitting the model in each group and checking the global fit and whether the loadings are significant for both groups. This time, instead of manually fitting the model to each group, we will pass the list of mean vectors, covariance matrices, and sample sizes to `lavaan`, which will automate the fitting for us.

```r
# Fit the model to both groups.
model_ex_1_configural_fit <- sem(
    model_ex_1,
    sample.cov = covariances,
    sample.nobs = samples,
    sample.mean = means
)

# Model summary.
summary(model_ex_1_configural_fit, standardized = TRUE)
```

```
## lavaan 0.6-12 ended normally after 93 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        50
##
##   Number of observations per group:
##     group_1                                         81
##     group_2                                        200
##
## Model Test User Model:
##
##   Test statistic                                53.380
##   Degrees of freedom                                38
##   P-value (Chi-square)                           0.050
##   Test statistic for each group:
##     group_1                                     29.169
##     group_2                                     24.211
```

```
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
##
## Group 1 [group_1]:
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal =~
##     inf               1.000                               2.347    0.772
##     sim               1.330    0.153    8.687    0.000    3.121    0.890
##     voc               1.189    0.138    8.613    0.000    2.791    0.883
##     com               1.015    0.125    8.129    0.000    2.382    0.841
##   visual =~
##     p_com             1.000                               1.788    0.723
##     p_arr             1.437    0.274    5.246    0.000    2.570    0.655
##     b_des             1.322    0.234    5.641    0.000    2.364    0.710
##     o_ass             1.285    0.220    5.830    0.000    2.297    0.738
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal ~~
##     visual            3.086    0.772    3.997    0.000    0.735    0.735
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf              10.090    0.338   29.861    0.000   10.090    3.318
##    .sim              12.070    0.390   30.965    0.000   12.070    3.441
##    .voc              10.250    0.351   29.191    0.000   10.250    3.243
##    .com               9.960    0.315   31.648    0.000    9.960    3.516
##    .p_com            10.900    0.275   39.643    0.000   10.900    4.405
##    .p_arr            11.240    0.436   25.769    0.000   11.240    2.863
##    .b_des            10.300    0.370   27.843    0.000   10.300    3.094
##    .o_ass            10.440    0.346   30.206    0.000   10.440    3.356
##     verbal            0.000                               0.000    0.000
##     visual            0.000                               0.000    0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf               3.742    0.673    5.564    0.000    3.742    0.405
##    .sim               2.564    0.605    4.237    0.000    2.564    0.208
##    .voc               2.200    0.502    4.379    0.000    2.200    0.220
##    .com               2.348    0.467    5.027    0.000    2.348    0.293
##    .p_com             2.926    0.592    4.945    0.000    2.926    0.478
##    .p_arr             8.806    1.631    5.398    0.000    8.806    0.571
##    .b_des             5.497    1.089    5.046    0.000    5.497    0.496
##    .o_ass             4.399    0.916    4.804    0.000    4.399    0.455
##     verbal            5.507    1.369    4.024    0.000    1.000    1.000
##     visual            3.198    0.924    3.462    0.001    1.000    1.000
##
##
## Group 2 [group_2]:
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal =~
##     inf               1.000                               2.440    0.789
```

```
##     sim               0.997    0.079   12.607    0.000    2.433    0.841
##     voc               1.045    0.082   12.778    0.000    2.550    0.852
##     com               0.768    0.083    9.301    0.000    1.874    0.648
##   visual =~
##     p_com             1.000                              2.182    0.684
##     p_arr             0.715    0.122    5.854    0.000    1.561    0.474
##     b_des             1.149    0.135    8.542    0.000    2.507    0.739
##     o_ass             1.100    0.130    8.464    0.000    2.401    0.730
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal ~~
##     visual           4.103    0.661    6.204    0.000    0.771    0.771
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf              10.100    0.219   46.192    0.000   10.100    3.266
##    .sim              10.300    0.205   50.355    0.000   10.300    3.561
##    .voc               9.800    0.212   46.314    0.000    9.800    3.275
##    .com              10.100    0.205   49.377    0.000   10.100    3.491
##    .p_com            10.100    0.226   44.748    0.000   10.100    3.164
##    .p_arr            10.100    0.233   43.392    0.000   10.100    3.068
##    .b_des             9.900    0.240   41.282    0.000    9.900    2.919
##    .o_ass            10.200    0.233   43.822    0.000   10.200    3.099
##     verbal            0.000                              0.000    0.000
##     visual            0.000                              0.000    0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf               3.609    0.455    7.928    0.000    3.609    0.377
##    .sim               2.450    0.354    6.925    0.000    2.450    0.293
##    .voc               2.453    0.370    6.635    0.000    2.453    0.274
##    .com               4.857    0.533    9.114    0.000    4.857    0.580
##    .p_com             5.426    0.675    8.034    0.000    5.426    0.533
##    .p_arr             8.398    0.897    9.364    0.000    8.398    0.775
##    .b_des             5.215    0.716    7.279    0.000    5.215    0.453
##    .o_ass             5.069    0.682    7.434    0.000    5.069    0.468
##     verbal            5.953    0.928    6.416    0.000    1.000    1.000
##     visual            4.763    0.952    5.006    0.000    1.000    1.000
```

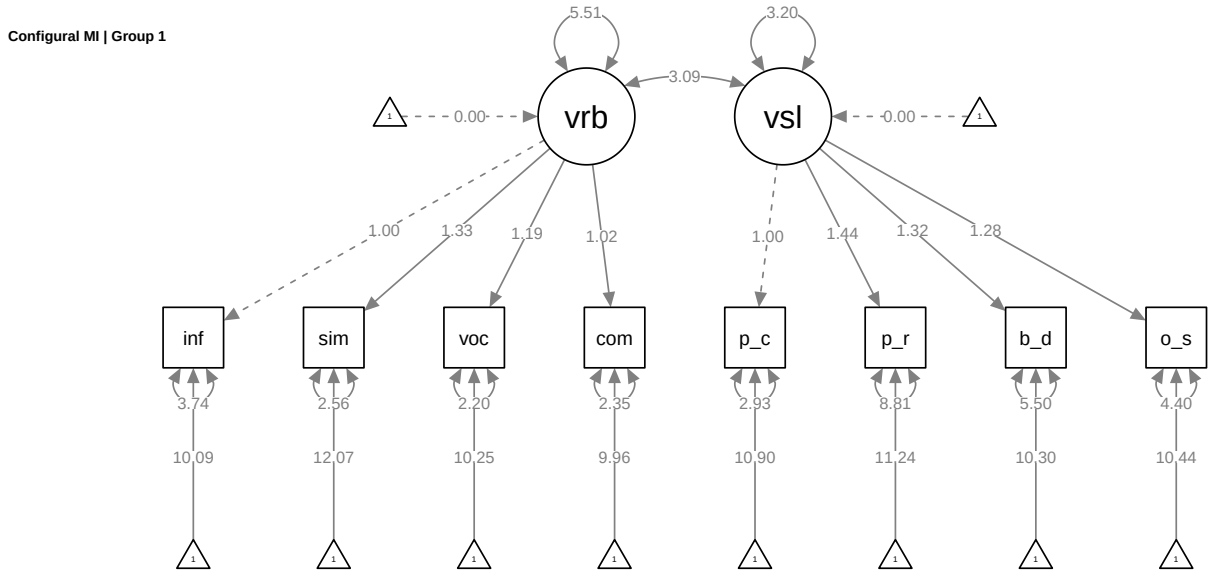
```
# Fit measures.
fitMeasures(model_ex_1_configural_fit, fit.measures = fit_indices)
```

```
##        chisq           df       pvalue          cfi          tli        rmsea rmsea.pvalue
##       53.380       38.000        0.050        0.985        0.978        0.054        0.402
##         srmr
##        0.034
```

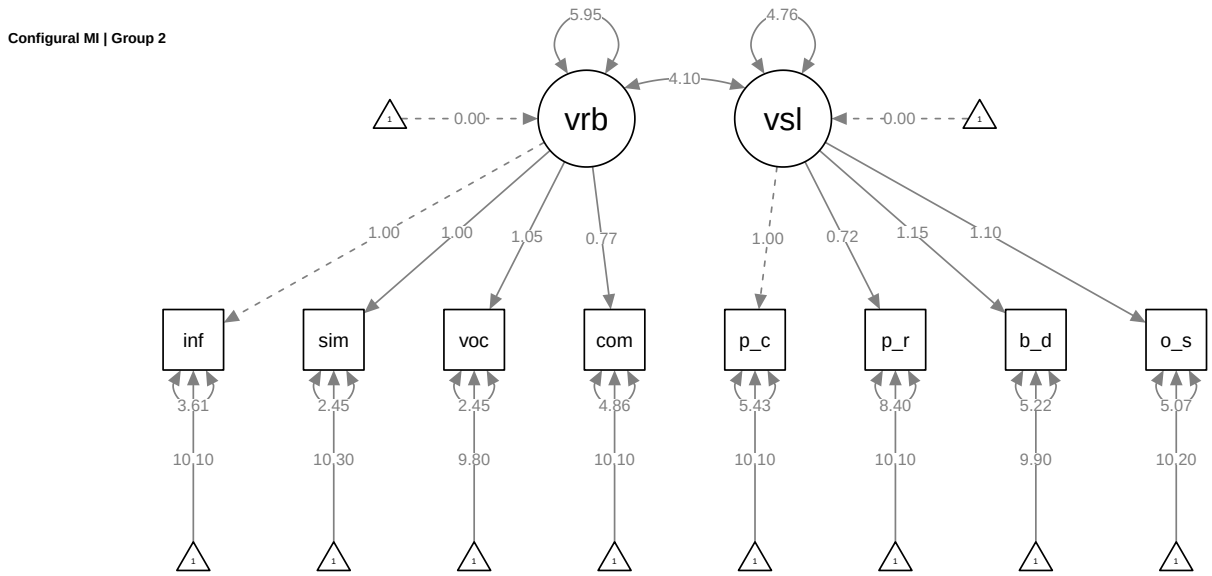
We see that the model has an adequate fit for each group. Before we continue, we can also plot the model for each group.

```
# Save the model plots for each groups as a list with two elements.
plots_configural <- semPaths(
    model_ex_1_configural_fit,
    what = "paths",
    whatLabels = "est",
    DoNotPlot = TRUE,
    ask = FALSE,
    title = FALSE
)
```

```
# Plot the model for the first group.
plot(plots_configural[[1]])
title("Configural MI | Group 1", adj = 0)
```




```
# Plot the model for the second group.
plot(plots_configural[[2]])
title("Configural MI | Group 2", adj = 0)
```



d. Investigate *weak* or *metric* measurement invariance. Report and interpret the model fit.

To investigate metric measurement invariance, we need to constrain the loadings to be equal in both groups. To achieve this, we can use the `group.equal` argument in `lavaan`.

```
# Fit the model.
model_ex_1_weak_fit <- sem(
    model_ex_1,
    sample.cov = covariances,
```

```
    sample.nobs = samples,
    sample.mean = means,
    group.equal = c("loadings")
)

# Model summary.
summary(model_ex_1_weak_fit, standardized = TRUE)
```

```
## lavaan 0.6-12 ended normally after 94 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        50
##   Number of equality constraints                     6
##
##   Number of observations per group:
##     group_1                                         81
##     group_2                                        200
##
## Model Test User Model:
##
##   Test statistic                                65.992
##   Degrees of freedom                                44
##   P-value (Chi-square)                           0.018
##   Test statistic for each group:
##     group_1                                     37.832
##     group_2                                     28.160
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
##
## Group 1 [group_1]:
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal =~
##     inf               1.000                               2.631    0.806
##     sim     (.p2.)    1.105    0.073   15.193    0.000    2.908    0.867
##     voc     (.p3.)    1.094    0.071   15.298    0.000    2.877    0.893
##     com     (.p4.)    0.864    0.068   12.720    0.000    2.273    0.825
##   visual =~
##     p_com             1.000                               1.958    0.758
##     p_arr   (.p6.)    0.888    0.118    7.554    0.000    1.739    0.480
##     b_des   (.p7.)    1.217    0.121   10.089    0.000    2.383    0.711
##     o_ass   (.p8.)    1.174    0.116   10.148    0.000    2.298    0.738
##
## Covariances:
```

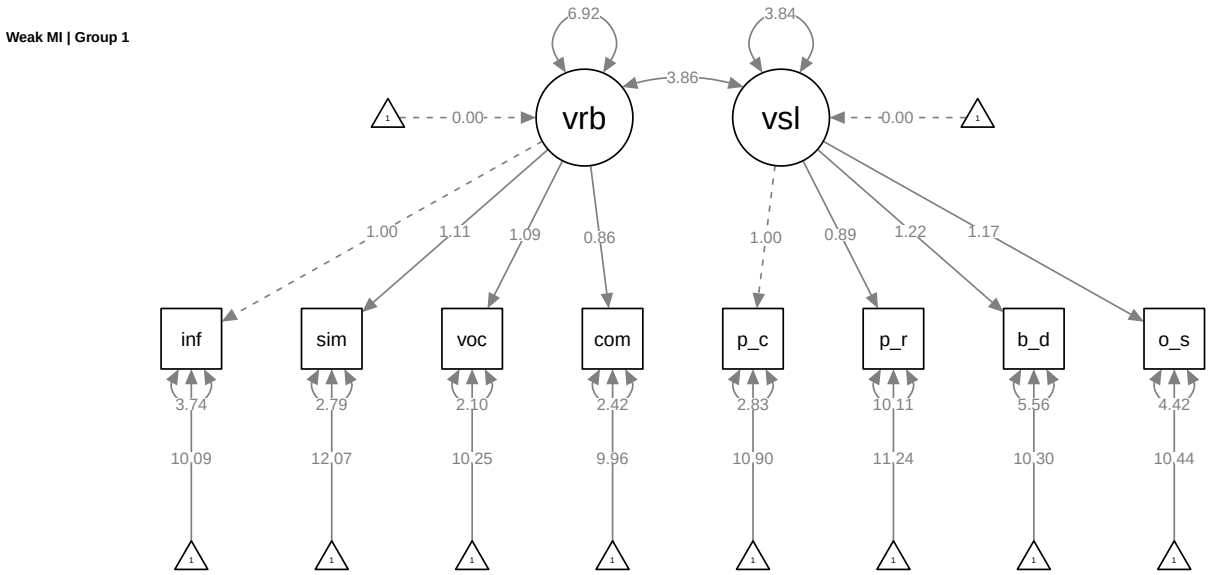
```
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal ~~
##     visual            3.865    0.845    4.571    0.000    0.750    0.750
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf              10.090    0.363   27.815    0.000   10.090    3.091
##    .sim              12.070    0.373   32.397    0.000   12.070    3.600
##    .voc              10.250    0.358   28.639    0.000   10.250    3.182
##    .com               9.960    0.306   32.548    0.000    9.960    3.616
##    .p_com            10.900    0.287   37.996    0.000   10.900    4.222
##    .p_arr            11.240    0.403   27.915    0.000   11.240    3.102
##    .b_des            10.300    0.373   27.646    0.000   10.300    3.072
##    .o_ass            10.440    0.346   30.170    0.000   10.440    3.352
##     verbal            0.000                              0.000    0.000
##     visual            0.000                              0.000    0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf               3.739    0.694    5.386    0.000    3.739    0.351
##    .sim               2.787    0.589    4.733    0.000    2.787    0.248
##    .voc               2.097    0.494    4.242    0.000    2.097    0.202
##    .com               2.419    0.464    5.217    0.000    2.419    0.319
##    .p_com             2.831    0.599    4.725    0.000    2.831    0.425
##    .p_arr            10.109    1.678    6.024    0.000   10.109    0.770
##    .b_des             5.564    1.079    5.158    0.000    5.564    0.495
##    .o_ass             4.417    0.895    4.936    0.000    4.417    0.455
##     verbal            6.920    1.362    5.081    0.000    1.000    1.000
##     visual            3.835    0.877    4.371    0.000    1.000    1.000
##
##
## Group 2 [group_2]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal =~
##     inf               1.000                              2.286    0.762
##     sim      (.p2.)   1.105    0.073   15.193    0.000    2.527    0.855
##     voc      (.p3.)   1.094    0.071   15.298    0.000    2.500    0.844
##     com      (.p4.)   0.864    0.068   12.720    0.000    1.975    0.669
##   visual =~
##     p_com             1.000                              2.047    0.655
##     p_arr    (.p6.)   0.888    0.118    7.554    0.000    1.817    0.534
##     b_des    (.p7.)   1.217    0.121   10.089    0.000    2.491    0.736
##     o_ass    (.p8.)   1.174    0.116   10.148    0.000    2.402    0.730
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal ~~
##     visual            3.607    0.558    6.461    0.000    0.771    0.771
##
```

```
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf              10.100    0.212   47.646    0.000   10.100    3.369
##    .sim              10.300    0.209   49.298    0.000   10.300    3.486
##    .voc               9.800    0.209   46.810    0.000    9.800    3.310
##    .com              10.100    0.209   48.367    0.000   10.100    3.420
##    .p_com            10.100    0.221   45.728    0.000   10.100    3.233
##    .p_arr            10.100    0.241   41.952    0.000   10.100    2.966
##    .b_des             9.900    0.239   41.391    0.000    9.900    2.927
##    .o_ass            10.200    0.233   43.843    0.000   10.200    3.100
##     verbal            0.000                             0.000    0.000
##     visual            0.000                             0.000    0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf               3.762    0.452    8.329    0.000    3.762    0.419
##    .sim               2.346    0.352    6.660    0.000    2.346    0.269
##    .voc               2.516    0.362    6.953    0.000    2.516    0.287
##    .com               4.821    0.532    9.056    0.000    4.821    0.553
##    .p_com             5.568    0.662    8.415    0.000    5.568    0.571
##    .p_arr             8.290    0.905    9.155    0.000    8.290    0.715
##    .b_des             5.238    0.705    7.432    0.000    5.238    0.458
##    .o_ass             5.055    0.671    7.535    0.000    5.055    0.467
##     verbal            5.225    0.770    6.783    0.000    1.000    1.000
##     visual            4.189    0.777    5.389    0.000    1.000    1.000
```

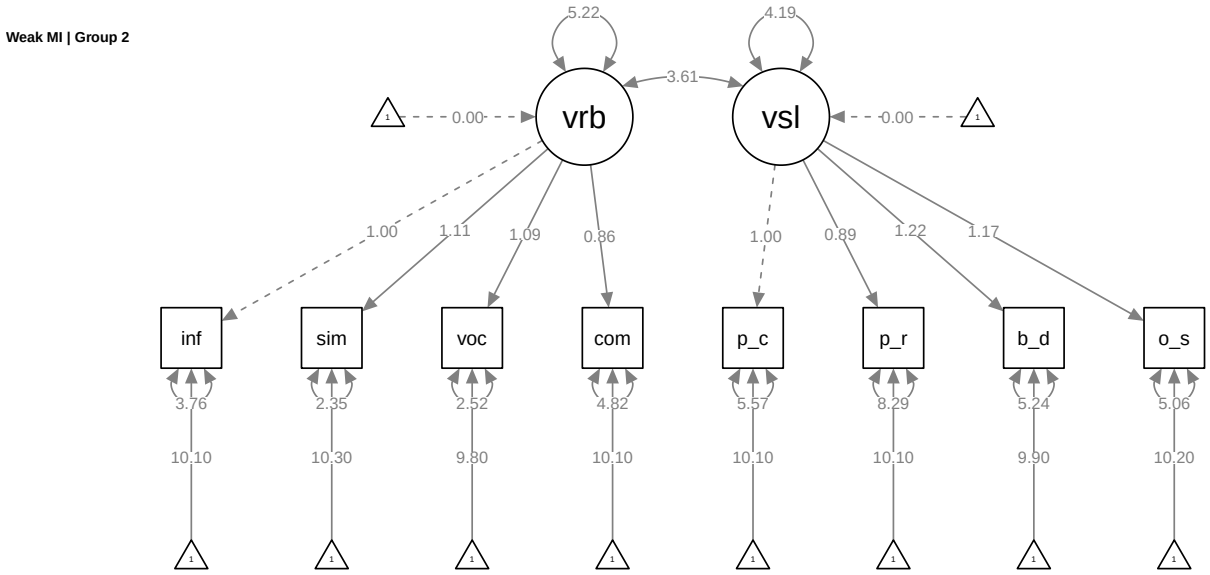
```r
# Fit measures.
fitMeasures(model_ex_1_weak_fit, fit.measures = fit_indices)
```

```
##        chisq           df       pvalue          cfi          tli        rmsea rmsea.pvalue
##       65.992       44.000        0.018        0.979        0.973        0.060        0.281
##         srmr
##        0.055
```

```r
# Save the model plots for each groups as a list with two elements.
plots_weak <- semPaths(
    model_ex_1_weak_fit,
    what = "paths",
    whatLabels = "est",
    DoNotPlot = TRUE,
    ask = FALSE,
    title = FALSE
)

# Plot the model for the first group.
plot(plots_weak[[1]])
title("Weak MI | Group 1", adj = 0)
```




```
# Plot the model for the second group.
plot(plots_weak[[2]])
title("Weak MI | Group 2", adj = 0)
```



e. Check if the the model fit at point *(d)* does not significantly worsen the fit compared to the model fit at point *(c)*.

Since these models are nested, can use the `anova()` function in `R` to perform a LRT.

```
# Perform LRT.
anova(model_ex_1_weak_fit, model_ex_1_configural_fit)
```

```
## Chi-Squared Difference Test
##
##                           Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## model_ex_1_configural_fit 38 10583 10765 53.380
## model_ex_1_weak_fit       44 10584 10744 65.992     12.613       6    0.04961 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Our findings show support for weak measurement invariance. We see a χ^2 difference value of 12.613 with a p-value $= 0.049$. As discussed during the lecture, the χ^2 difference test is often significant for the tiniest differences. A recommended practice is to also look at the decrease in *CFI* and increase in *RMSEA* when moving to a higher level of measurement invariance. For two groups, the decrease in *CFI* and increase in *RMSEA* should not exceed 0.01, which we see is the case.

f. Investigate *strong* or *scalar* measurement invariance. Report and interpret the model fit.

To investigate scalar measurement invariance, on top of constraining the loadings to be equal in both groups, we also need to constrain the intercepts. To achieve this, we can, again, use the `group.equal` argument in `lavaan`.

```
# Fit the model.
model_ex_1_strong_fit <- sem(
    model_ex_1,
    sample.cov = covariances,
    sample.nobs = samples,
    sample.mean = means,
    group.equal = c("loadings", "intercepts")
)

# Model summary.
summary(model_ex_1_strong_fit, standardized = TRUE)
```

```
## lavaan 0.6-12 ended normally after 105 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        52
##   Number of equality constraints                    14
##
##   Number of observations per group:
##     group_1                                         81
##     group_2                                        200
##
## Model Test User Model:
##
##   Test statistic                               109.088
##   Degrees of freedom                                50
##   P-value (Chi-square)                           0.000
##   Test statistic for each group:
##     group_1                                      70.405
##     group_2                                      38.684
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
##
```

```
## Group 1 [group_1]:
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal =~
##     inf               1.000                              2.585    0.791
##     sim     (.p2.)    1.118    0.077   14.588    0.000    2.891    0.818
##     voc     (.p3.)    1.110    0.073   15.103    0.000    2.869    0.900
##     com     (.p4.)    0.858    0.070   12.321    0.000    2.218    0.809
##   visual =~
##     p_com             1.000                              1.981    0.763
##     p_arr   (.p6.)    0.892    0.116    7.722    0.000    1.767    0.482
##     b_des   (.p7.)    1.188    0.117   10.188    0.000    2.353    0.703
##     o_ass   (.p8.)    1.140    0.112   10.222    0.000    2.258    0.726
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal ~~
##     visual            3.894    0.847    4.595    0.000    0.761    0.761
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf     (.20.)   10.477    0.321   32.647    0.000   10.477    3.205
##    .sim     (.21.)   11.115    0.351   31.644    0.000   11.115    3.144
##    .voc     (.22.)   10.338    0.341   30.287    0.000   10.338    3.242
##    .com     (.23.)   10.295    0.279   36.925    0.000   10.295    3.755
##    .p_com   (.24.)   10.749    0.265   40.565    0.000   10.749    4.139
##    .p_arr   (.25.)   10.736    0.283   37.941    0.000   10.736    2.928
##    .b_des   (.26.)   10.466    0.321   32.646    0.000   10.466    3.126
##    .o_ass   (.27.)   10.690    0.305   35.032    0.000   10.690    3.438
##     verbal            0.000                              0.000    0.000
##     visual            0.000                              0.000    0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf               4.007    0.743    5.391    0.000    4.007    0.375
##    .sim               4.146    0.800    5.184    0.000    4.146    0.332
##    .voc               1.936    0.501    3.861    0.000    1.936    0.190
##    .com               2.595    0.496    5.237    0.000    2.595    0.345
##    .p_com             2.823    0.605    4.667    0.000    2.823    0.418
##    .p_arr            10.327    1.716    6.017    0.000   10.327    0.768
##    .b_des             5.677    1.091    5.204    0.000    5.677    0.506
##    .o_ass             4.570    0.909    5.025    0.000    4.570    0.473
##     verbal            6.682    1.333    5.011    0.000    1.000    1.000
##     visual            3.923    0.893    4.391    0.000    1.000    1.000
##
##
## Group 2 [group_2]:
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
```

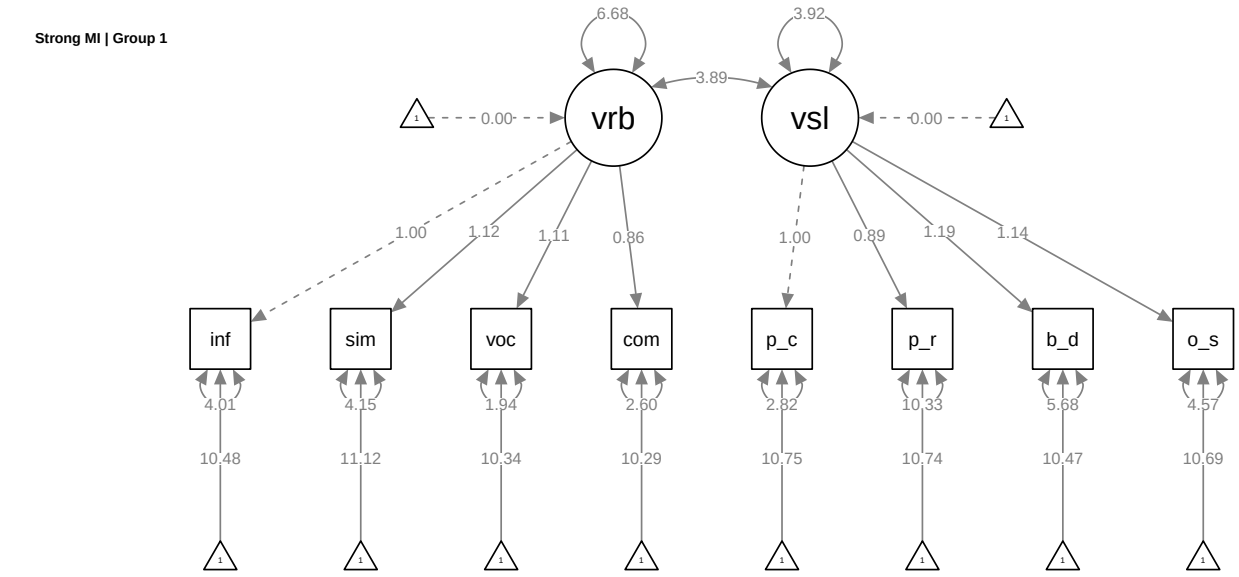
```
##   verbal =~
##     inf               1.000                           2.266    0.758
##     sim     (.p2.)    1.118    0.077   14.588    0.000    2.533    0.851
##     voc     (.p3.)    1.110    0.073   15.103    0.000    2.515    0.847
##     com     (.p4.)    0.858    0.070   12.321    0.000    1.944    0.660
##   visual =~
##     p_com             1.000                           2.079    0.662
##     p_arr   (.p6.)    0.892    0.116    7.722    0.000    1.854    0.541
##     b_des   (.p7.)    1.188    0.117   10.188    0.000    2.469    0.732
##     o_ass   (.p8.)    1.140    0.112   10.222    0.000    2.370    0.723
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal ~~
##     visual            3.639    0.562    6.478    0.000    0.773    0.773
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf     (.20.)   10.477    0.321   32.647    0.000   10.477    3.505
##    .sim     (.21.)   11.115    0.351   31.644    0.000   11.115    3.735
##    .voc     (.22.)   10.338    0.341   30.287    0.000   10.338    3.482
##    .com     (.23.)   10.295    0.279   36.925    0.000   10.295    3.495
##    .p_com   (.24.)   10.749    0.265   40.565    0.000   10.749    3.424
##    .p_arr   (.25.)   10.736    0.283   37.941    0.000   10.736    3.134
##    .b_des   (.26.)   10.466    0.321   32.646    0.000   10.466    3.101
##    .o_ass   (.27.)   10.690    0.305   35.032    0.000   10.690    3.261
##     verbal           -0.525    0.348   -1.510    0.131   -0.232   -0.232
##     visual           -0.529    0.301   -1.759    0.079   -0.255   -0.255
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf              3.799    0.455    8.352    0.000    3.799    0.425
##    .sim              2.440    0.364    6.704    0.000    2.440    0.275
##    .voc              2.489    0.364    6.836    0.000    2.489    0.282
##    .com              4.896    0.539    9.087    0.000    4.896    0.564
##    .p_com            5.534    0.663    8.346    0.000    5.534    0.562
##    .p_arr            8.301    0.911    9.117    0.000    8.301    0.707
##    .b_des            5.293    0.706    7.495    0.000    5.293    0.465
##    .o_ass            5.128    0.672    7.628    0.000    5.128    0.477
##     verbal           5.133    0.766    6.698    0.000    1.000    1.000
##     visual           4.321    0.791    5.459    0.000    1.000    1.000
```

```
# Fit measures.
fitMeasures(model_ex_1_strong_fit, fit.measures = fit_indices)
```

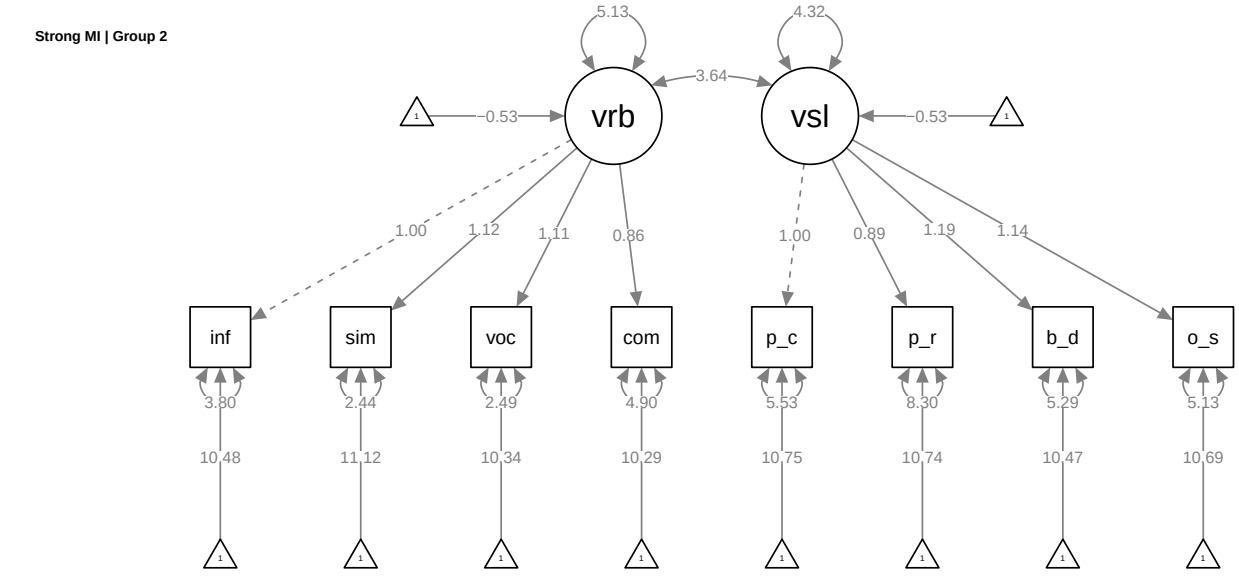
```
##        chisq           df       pvalue          cfi          tli        rmsea rmsea.pvalue
##      109.088       50.000        0.000        0.942        0.935        0.092        0.003
##         srmr
##        0.064
```

```
# Save the model plots for each groups as a list with two elements.
plots_strong <- semPaths(
    model_ex_1_strong_fit,
```

```
    what = "paths",
    whatLabels = "est",
    DoNotPlot = TRUE,
    ask = FALSE,
    title = FALSE
)

# Plot the model for the first group.
plot(plots_strong[[1]])
title("Strong MI | Group 1", adj = 0)
```




```
# Plot the model for the second group.
plot(plots_strong[[2]])
title("Strong MI | Group 2", adj = 0)
```



g. Check if the the model fit at point *(f)* does not significantly worsen the fit compared to the model fit at

point *(d)*.

Since these models are nested, can use the `anova()` function in `R` to perform a LRT.

```
# Perform LRT.
anova(model_ex_1_strong_fit, model_ex_1_weak_fit)
```

```
## Chi-Squared Difference Test
##
##                       Df   AIC   BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## model_ex_1_weak_fit   44 10584 10744  65.992
## model_ex_1_strong_fit 50 10615 10753 109.088     43.096       6  1.117e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Our findings do not show support for strong measurement invariance. We obtain a χ^2 difference value of 43.096 with a p-value < 0.001. Also, the rules of thumbs regarding the decrease in *CFI* and increase in *RMSEA* show that we do not have support for strong measurement invariance.

h. Based on the findings above, should you continue with investigating strict measurement invariance or not?

Perform again the LRT for all nested models investigated so far.

```
# LRT.
anova(model_ex_1_configural_fit, model_ex_1_weak_fit, model_ex_1_strong_fit)
```

```
## Chi-Squared Difference Test
##
##                           Df   AIC   BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## model_ex_1_configural_fit 38 10583 10765  53.380
## model_ex_1_weak_fit       44 10584 10744  65.992     12.613       6    0.04961 *
## model_ex_1_strong_fit     50 10615 10753 109.088     43.096       6  1.117e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We already see that we do not have support for strong measurement invariance. Hence, we should not continue with strict measurement invariance. However, we can identify which intercepts show misfit and allow those intercepts to be freely estimated in both groups. Doing so, we investigate partial strong measurement invariance.

i. Investigate which intercepts show misfit in the model fit at point *(f)*. Free the corresponding intercept to be freely estimated in both groups and compare the fit of this model with that fitted at point *(d)*.
   - Tip: check `lavaan` functions `parTable` and `lavTestScore`.

We check the misfit by looking at the discrepancy between the estimated intercepts by `lavaan` and the observed means for each group. Then, we can test if our fit would improve with those parameters being freely estimated.

To inspect the estimated intercepts, we use the `lavInspect` function in `lavaan`.

```
# Means discrepancies for group one.
lavInspect(model_ex_1_strong_fit, "mu")[[1]] - group_1_means
```

```
##    inf    sim    voc    com  p_com  p_arr  b_des  o_ass
##  0.387 -0.955  0.088  0.335 -0.151 -0.504  0.166  0.250
```

```
# Means discrepancies for group two.
lavInspect(model_ex_1_strong_fit, "mu")[[2]] - group_1_means
```

```
##    inf    sim    voc    com  p_com  p_arr  b_des  o_ass
## -0.139 -1.542 -0.496 -0.116 -0.680 -0.976 -0.463 -0.353
```

We can see that the misfit is most prominent for variable `sim`. We can now check the improvement in model fit if we freely estimate the intercepts for `sim` across groups (i.e., if we release them from the equality constraint). To do this, we use the `lavTestScore` function in `lavaan`. First, let us use the `parTable` function in `lavaan` to print the parameter table and learn the label `lavaan` automatically generated for the intercept of variable `sim`. Note that, behind the scenes, `lavaan` applies labels automatically to enforce the equality constraints requested via the argument `group.equal`.

```
# Show the parameter table.
parTable(model_ex_1_strong_fit)
```

```
##    id    lhs op    rhs user block group free ustart exo label plabel  start    est    se
## 1   1 verbal =~    inf    1     1     1    0      1   0         .p1.  1.000  1.000 0.000
## 2   2 verbal =~    sim    1     1     1    1     NA   0  .p2.   .p2.  1.340  1.118 0.077
## 3   3 verbal =~    voc    1     1     1    2     NA   0  .p3.   .p3.  1.148  1.110 0.073
## 4   4 verbal =~    com    1     1     1    3     NA   0  .p4.   .p4.  0.988  0.858 0.070
## 5   5 visual =~  p_com    1     1     1    0      1   0         .p5.  1.000  1.000 0.000
## 6   6 visual =~  p_arr    1     1     1    4     NA   0  .p6.   .p6.  1.592  0.892 0.116
## 7   7 visual =~  b_des    1     1     1    5     NA   0  .p7.   .p7.  1.432  1.188 0.117
## 8   8 visual =~  o_ass    1     1     1    6     NA   0  .p8.   .p8.  1.357  1.140 0.112
## 9   9 verbal ~~ visual    1     1     1    7     NA   0         .p9.  0.000  3.894 0.847
## 10 10    inf ~~    inf    0     1     1    8     NA   0        .p10.  4.624  4.007 0.743
## 11 11    sim ~~    sim    0     1     1    9     NA   0        .p11.  6.154  4.146 0.800
## 12 12    voc ~~    voc    0     1     1   10     NA   0        .p12.  4.994  1.936 0.501
## 13 13    com ~~    com    0     1     1   11     NA   0        .p13.  4.011  2.595 0.496
## 14 14  p_com ~~  p_com    0     1     1   12     NA   0        .p14.  3.062  2.823 0.605
## 15 15  p_arr ~~  p_arr    0     1     1   13     NA   0        .p15.  7.705 10.327 1.716
## 16 16  b_des ~~  b_des    0     1     1   14     NA   0        .p16.  5.542  5.677 1.091
## 17 17  o_ass ~~  o_ass    0     1     1   15     NA   0        .p17.  4.838  4.570 0.909
## 18 18 verbal ~~ verbal    0     1     1   16     NA   0        .p18.  0.050  6.682 1.333
## 19 19 visual ~~ visual    0     1     1   17     NA   0        .p19.  0.050  3.923 0.893
## 20 20    inf ~1           0     1     1   18     NA   0 .p20.  .p20. 10.090 10.477 0.321
## 21 21    sim ~1           0     1     1   19     NA   0 .p21.  .p21. 12.070 11.115 0.351
## 22 22    voc ~1           0     1     1   20     NA   0 .p22.  .p22. 10.250 10.338 0.341
## 23 23    com ~1           0     1     1   21     NA   0 .p23.  .p23.  9.960 10.295 0.279
## 24 24  p_com ~1           0     1     1   22     NA   0 .p24.  .p24. 10.900 10.749 0.265
## 25 25  p_arr ~1           0     1     1   23     NA   0 .p25.  .p25. 11.240 10.736 0.283
## 26 26  b_des ~1           0     1     1   24     NA   0 .p26.  .p26. 10.300 10.466 0.321
## 27 27  o_ass ~1           0     1     1   25     NA   0 .p27.  .p27. 10.440 10.690 0.305
## 28 28 verbal ~1           0     1     1    0      0   0        .p28.  0.000  0.000 0.000
## 29 29 visual ~1           0     1     1    0      0   0        .p29.  0.000  0.000 0.000
## 30 30 verbal =~    inf    1     2     2    0      1   0        .p30.  1.000  1.000 0.000
## 31 31 verbal =~    sim    1     2     2   26     NA   0  .p2.   .p31.  0.994  1.118 0.077
## 32 32 verbal =~    voc    1     2     2   27     NA   0  .p3.   .p32.  1.088  1.110 0.073
## 33 33 verbal =~    com    1     2     2   28     NA   0  .p4.   .p33.  0.783  0.858 0.070
## 34 34 visual =~  p_com    1     2     2    0      1   0        .p34.  1.000  1.000 0.000
## 35 35 visual =~  p_arr    1     2     2   29     NA   0  .p6.   .p35.  0.705  0.892 0.116
## 36 36 visual =~  b_des    1     2     2   30     NA   0  .p7.   .p36.  1.213  1.188 0.117
## 37 37 visual =~  o_ass    1     2     2   31     NA   0  .p8.   .p37.  1.172  1.140 0.112
## 38 38 verbal ~~ visual    1     2     2   32     NA   0        .p38.  0.000  3.639 0.562
## 39 39    inf ~~    inf    0     2     2   33     NA   0        .p39.  4.781  3.799 0.455
## 40 40    sim ~~    sim    0     2     2   34     NA   0        .p40.  4.184  2.440 0.364
## 41 41    voc ~~    voc    0     2     2   35     NA   0        .p41.  4.478  2.489 0.364
## 42 42    com ~~    com    0     2     2   36     NA   0        .p42.  4.184  4.896 0.539
```

```
## 43 43  p_com ~~  p_com    0     2     2   37     NA   0        .p43.  5.094  5.534 0.663
## 44 44  p_arr ~~  p_arr    0     2     2   38     NA   0        .p44.  5.418  8.301 0.911
## 45 45  b_des ~~  b_des    0     2     2   39     NA   0        .p45.  5.751  5.293 0.706
## 46 46  o_ass ~~  o_ass    0     2     2   40     NA   0        .p46.  5.418  5.128 0.672
## 47 47 verbal ~~ verbal    0     2     2   41     NA   0        .p47.  0.050  5.133 0.766
## 48 48 visual ~~ visual    0     2     2   42     NA   0        .p48.  0.050  4.321 0.791
## 49 49    inf ~1           0     2     2   43     NA   0 .p20.  .p49. 10.100 10.477 0.321
## 50 50    sim ~1           0     2     2   44     NA   0 .p21.  .p50. 10.300 11.115 0.351
## 51 51    voc ~1           0     2     2   45     NA   0 .p22.  .p51.  9.800 10.338 0.341
## 52 52    com ~1           0     2     2   46     NA   0 .p23.  .p52. 10.100 10.295 0.279
## 53 53  p_com ~1           0     2     2   47     NA   0 .p24.  .p53. 10.100 10.749 0.265
## 54 54  p_arr ~1           0     2     2   48     NA   0 .p25.  .p54. 10.100 10.736 0.283
## 55 55  b_des ~1           0     2     2   49     NA   0 .p26.  .p55.  9.900 10.466 0.321
## 56 56  o_ass ~1           0     2     2   50     NA   0 .p27.  .p56. 10.200 10.690 0.305
## 57 57 verbal ~1           0     2     2   51     NA   0        .p57.  0.000 -0.525 0.348
## 58 58 visual ~1           0     2     2   52     NA   0        .p58.  0.000 -0.529 0.301
## 59 59   .p2. ==  .p31.    2     0     0    0     NA   0               0.000  0.000 0.000
## 60 60   .p3. ==  .p32.    2     0     0    0     NA   0               0.000  0.000 0.000
## 61 61   .p4. ==  .p33.    2     0     0    0     NA   0               0.000  0.000 0.000
## 62 62   .p6. ==  .p35.    2     0     0    0     NA   0               0.000  0.000 0.000
## 63 63   .p7. ==  .p36.    2     0     0    0     NA   0               0.000  0.000 0.000
## 64 64   .p8. ==  .p37.    2     0     0    0     NA   0               0.000  0.000 0.000
## 65 65  .p20. ==  .p49.    2     0     0    0     NA   0               0.000  0.000 0.000
## 66 66  .p21. ==  .p50.    2     0     0    0     NA   0               0.000  0.000 0.000
## 67 67  .p22. ==  .p51.    2     0     0    0     NA   0               0.000  0.000 0.000
## 68 68  .p23. ==  .p52.    2     0     0    0     NA   0               0.000  0.000 0.000
## 69 69  .p24. ==  .p53.    2     0     0    0     NA   0               0.000  0.000 0.000
## 70 70  .p25. ==  .p54.    2     0     0    0     NA   0               0.000  0.000 0.000
## 71 71  .p26. ==  .p55.    2     0     0    0     NA   0               0.000  0.000 0.000
## 72 72  .p27. ==  .p56.    2     0     0    0     NA   0               0.000  0.000 0.000
```

We see that for the first group, `lavaan` attached the label `.p21.` for the parameter `sim ~ 1` (i.e., the intercept).

```
parTable(model_ex_1_strong_fit)[21, ]
```

```
##    id lhs op rhs user block group free ustart exo label plabel start    est    se
## 21 21 sim ~1        0     1     1   19     NA   0 .p21.  .p21. 12.07 11.115 0.351
```

In the second group, `lavaan` refers to the parameter `sim ~ 1` (i.e., the intercept) using the label `.p50.`.

```
parTable(model_ex_1_strong_fit)[50, ]
```

```
##    id lhs op rhs user block group free ustart exo label plabel start    est    se
## 50 50 sim ~1        0     2     2   44     NA   0 .p21.  .p50.  10.3 11.115 0.351
```

In order for `lavaan` to respect the equality constraints we requested, it set both of these labels to be equal, i.e., `.p21. == .p50.`. Therefore, when looking at the output of `lavTestScore` to understand which "release" results in the highest change in the χ^2 difference, we expect `.p21. == .p50.` to yield the highest change.

```
# Perform the score test for releasing constrained model parameters.
lavTestScore(model_ex_1_strong_fit)
```

```
## $test
##
## total score test:
##
##    test     X2 df p.value
## 1 score 50.655 14       0
##
## $uni
```

```
##
## univariate score tests:
##
##      lhs op   rhs     X2 df p.value
## 1   .p2. == .p31.  0.634  1   0.426
## 2   .p3. == .p32.  0.138  1   0.710
## 3   .p4. == .p33.  1.649  1   0.199
## 4   .p6. == .p35.  5.483  1   0.019
## 5   .p7. == .p36.  0.017  1   0.898
## 6   .p8. == .p37.  0.012  1   0.913
## 7  .p20. == .p49.  5.165  1   0.023
## 8  .p21. == .p50. 29.459  1   0.000
## 9  .p22. == .p51.  0.850  1   0.357
## 10 .p23. == .p52.  7.455  1   0.006
## 11 .p24. == .p53.  1.745  1   0.186
## 12 .p25. == .p54.  2.955  1   0.086
## 13 .p26. == .p55.  0.772  1   0.380
## 14 .p27. == .p56.  2.376  1   0.123
```

The overall `total score test` is a multivariate test indicating whether releasing (i.e., freeing) all constraints improves the fit over the base model. We reject the null hypothesis and proceed by looking at the univariate tests. Here, we indeed see that freeing the constraint `.p21. == .p50.` would result in the largest χ^2 difference (e.g., 29.459). We go ahead and refit the strong measurement invariance model with the `sim ~ 1` freely estimated in both groups. Then, we check again using the function `lavTestScore` whether additional constraints can be freed. To do so, we use the `group.partial` argument. In this case, we are are actually investigating is called *strong partial measurement invariance*.

```r
# Fit the model.
model_ex_1_strong_partial_fit <- sem(
    model_ex_1,
    sample.cov = covariances,
    sample.nobs = samples,
    sample.mean = means,
    group.equal = c("loadings", "intercepts"),
    group.partial = c("sim ~ 1")
)

# Fit measures.
fitMeasures(model_ex_1_strong_partial_fit, fit.measures = fit_indices)
```

```
##        chisq           df       pvalue          cfi          tli        rmsea rmsea.pvalue
##       75.924       49.000        0.008        0.974        0.970        0.063        0.218
##         srmr
##        0.058
```

```r
# Perform the score test.
lavTestScore(model_ex_1_strong_partial_fit)
```

```
## $test
##
## total score test:
##
##    test     X2 df p.value
## 1 score 22.189 13   0.053
##
## $uni
##
## univariate score tests:
##
```

```
##      lhs op   rhs    X2 df p.value
## 1   .p2. == .p31. 2.561  1   0.110
## 2   .p3. == .p32. 0.669  1   0.413
## 3   .p4. == .p33. 0.952  1   0.329
## 4   .p6. == .p35. 5.477  1   0.019
## 5   .p7. == .p36. 0.016  1   0.899
## 6   .p8. == .p37. 0.010  1   0.922
## 7  .p20. == .p49. 0.678  1   0.410
## 8  .p22. == .p51. 3.610  1   0.057
## 9  .p23. == .p52. 1.967  1   0.161
## 10 .p24. == .p53. 1.741  1   0.187
## 11 .p25. == .p54. 2.951  1   0.086
## 12 .p26. == .p55. 0.778  1   0.378
## 13 .p27. == .p56. 2.366  1   0.124
```

This time we see that the multivariate score test is not significant and looking at the univariate tests we cannot free any parameters. We perform again the *LRT* for the nested models investigated so far.

```
# LRT.
anova(model_ex_1_configural_fit, model_ex_1_weak_fit, model_ex_1_strong_partial_fit)
```

```
## Chi-Squared Difference Test
##
##                               Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## model_ex_1_configural_fit     38 10583 10765 53.380
## model_ex_1_weak_fit           44 10584 10744 65.992    12.6128       6    0.04961 *
## model_ex_1_strong_partial_fit 49 10584 10726 75.924     9.9317       5    0.07720 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Based on the χ^2 difference test above and the fit measures below we see some support for partial scalar measurement invariance.

```
# Combine all fit measures for all models into a data frame.
fit_measures_all_ex_1 <- rbind(
    configural = fitMeasures(model_ex_1_configural_fit, fit_indices),
    weak = fitMeasures(model_ex_1_weak_fit, fit_indices),
    strong = fitMeasures(model_ex_1_strong_fit, fit_indices),
    strong_partial = fitMeasures(model_ex_1_strong_partial_fit, fit_indices)
)

# Print the fit measures for all models with four decimals.
print(round(fit_measures_all_ex_1, 4))
```

```
##                   chisq df pvalue    cfi    tli  rmsea rmsea.pvalue   srmr
## configural      53.3796 38 0.0500 0.9850 0.9779 0.0537       0.4025 0.0344
## weak            65.9923 44 0.0175 0.9785 0.9727 0.0596       0.2805 0.0553
## strong         109.0882 50 0.0000 0.9423 0.9354 0.0917       0.0028 0.0638
## strong_partial  75.9240 49 0.0081 0.9737 0.9700 0.0625       0.2175 0.0579
```

j. What kind of comparisons can we make when we find support for strong (partial) measurement invariance?

When we find support for strong measurement invariance, we can compare the means of the latent variables across groups. For our example, we see that children in the second group (i.e., the norming group) perform worse than children in the first group (i.e., with manic symptoms) on both the verbal (i.e., -0.171) and visual (i.e., -0.53) components of intelligence. However, if we look at the p-values we see that neither of these differences are significant (e.g., $p = 0.633$ for verbal and $p = 0.078$ for visual).

Additional information on the p-values:

```
# Get the parameters.
par <- parameterTable(model_ex_1_strong_partial_fit)

# Filter the parameters for ease of read.
# Group ome should be reference, hence zero.
par[par$op == "~1" & par$group == 1 & (par$lhs == "verbal" | par$lhs == "visual"), ]
```

```
##    id    lhs op rhs user block group free ustart exo label plabel start est se
## 28 28 verbal ~1        0     1     1    0      0   0        .p28.     0   0  0
## 29 29 visual ~1        0     1     1    0      0   0        .p29.     0   0  0
```

```
# Group two should show be relative to group one.
par[par$op == "~1" & par$group == 2 & (par$lhs == "verbal" | par$lhs == "visual"), ]
```

```
##    id    lhs op rhs user block group free ustart exo label plabel start    est    se
## 57 57 verbal ~1        0     2     2   51     NA   0        .p57.     0 -0.171 0.358
## 58 58 visual ~1        0     2     2   52     NA   0        .p58.     0 -0.530 0.301
```

```
#    id    lhs op rhs user block group free ustart exo label plabel start    est    se
# 57 57 verbal ~1        0     2     2   51     NA   0        .p57.     0 -0.171 0.358
# 58 58 visual ~1        0     2     2   52     NA   0        .p58.     0 -0.530 0.301

# Or, we know that the intercepts are named `alpha` in SEM, hence we can look for
# that in the output of `inspect()`.
inspect(model_ex_1_strong_partial_fit, what = "est")
```

```
## $group_1
## $group_1$lambda
##       verbal visual
## inf    1.000  0.000
## sim    1.107  0.000
## voc    1.096  0.000
## com    0.862  0.000
## p_com  0.000  1.000
## p_arr  0.000  0.891
## b_des  0.000  1.188
## o_ass  0.000  1.139
##
## $group_1$theta
##       inf    sim    voc    com    p_com  p_arr  b_des  o_ass
## inf    3.750
## sim    0.000  2.768
## voc    0.000  0.000  2.160
## com    0.000  0.000  0.000  2.455
## p_com  0.000  0.000  0.000  0.000  2.814
## p_arr  0.000  0.000  0.000  0.000  0.000 10.333
## b_des  0.000  0.000  0.000  0.000  0.000  0.000  5.675
## o_ass  0.000  0.000  0.000  0.000  0.000  0.000  0.000  4.588
##
## $group_1$psi
##        verbal visual
## verbal 6.908
## visual 3.929  3.925
##
## $group_1$nu
##       intrcp
## inf   10.219
## sim   12.070
## voc   10.072
## com   10.120
## p_com 10.750
```

```
## p_arr 10.736
## b_des 10.466
## o_ass 10.690
##
## $group_1$alpha
##        intrcp
## verbal      0
## visual      0
##
##
## $group_2
## $group_2$lambda
##       verbal visual
## inf    1.000  0.000
## sim    1.107  0.000
## voc    1.096  0.000
## com    0.862  0.000
## p_com  0.000  1.000
## p_arr  0.000  0.891
## b_des  0.000  1.188
## o_ass  0.000  1.139
##
## $group_2$theta
##       inf   sim   voc   com   p_com p_arr b_des o_ass
## inf   3.768
## sim   0.000 2.343
## voc   0.000 0.000 2.528
## com   0.000 0.000 0.000 4.841
## p_com 0.000 0.000 0.000 0.000 5.533
## p_arr 0.000 0.000 0.000 0.000 0.000 8.302
## b_des 0.000 0.000 0.000 0.000 0.000 0.000 5.288
## o_ass 0.000 0.000 0.000 0.000 0.000 0.000 0.000 5.133
##
## $group_2$psi
##        verbal visual
## verbal 5.212
## visual 3.671  4.326
##
## $group_2$nu
##       intrcp
## inf   10.219
## sim   10.489
## voc   10.072
## com   10.120
## p_com 10.750
## p_arr 10.736
## b_des 10.466
## o_ass 10.690
##
## $group_2$alpha
##        intrcp
## verbal -0.171
## visual -0.530
```

```r
# $group_2$alpha
#        intrcp
# verbal -0.171
# visual -0.530

# Or, if you want the standard errors, replace `what = "est"` with `what = "se`.
inspect(model_ex_1_strong_partial_fit, what = "se")
```

```
## $group_1
## $group_1$lambda
##       verbal visual
## inf    0.000  0.000
## sim    0.073  0.000
## voc    0.072  0.000
## com    0.068  0.000
## p_com  0.000  0.000
## p_arr  0.000  0.115
## b_des  0.000  0.117
## o_ass  0.000  0.111
##
## $group_1$theta
##       inf   sim   voc   com   p_com p_arr b_des o_ass
## inf   0.697
## sim   0.000 0.589
## voc   0.000 0.000 0.505
## com   0.000 0.000 0.000 0.469
## p_com 0.000 0.000 0.000 0.000 0.604
## p_arr 0.000 0.000 0.000 0.000 0.000 1.717
## b_des 0.000 0.000 0.000 0.000 0.000 0.000 1.091
## o_ass 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.912
##
## $group_1$psi
##        verbal visual
## verbal 1.361
## visual 0.856  0.893
##
## $group_1$nu
##       intrcp
## inf    0.327
## sim    0.372
## voc    0.347
## com    0.284
## p_com  0.265
## p_arr  0.283
## b_des  0.321
## o_ass  0.305
##
## $group_1$alpha
##        intrcp
## verbal      0
## visual      0
##
##
## $group_2
## $group_2$lambda
##       verbal visual
## inf    0.000  0.000
## sim    0.073  0.000
## voc    0.072  0.000
## com    0.068  0.000
## p_com  0.000  0.000
## p_arr  0.000  0.115
## b_des  0.000  0.117
## o_ass  0.000  0.111
##
## $group_2$theta
##       inf   sim   voc   com   p_com p_arr b_des o_ass
## inf   0.452
## sim   0.000 0.352
```

```
## voc   0.000 0.000 0.363
## com   0.000 0.000 0.000 0.534
## p_com 0.000 0.000 0.000 0.000 0.663
## p_arr 0.000 0.000 0.000 0.000 0.000 0.910
## b_des 0.000 0.000 0.000 0.000 0.000 0.000 0.706
## o_ass 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.672
##
## $group_2$psi
##        verbal visual
## verbal 0.769
## visual 0.565  0.792
##
## $group_2$nu
##       intrcp
## inf    0.327
## sim    0.370
## voc    0.347
## com    0.284
## p_com  0.265
## p_arr  0.283
## b_des  0.321
## o_ass  0.305
##
## $group_2$alpha
##        intrcp
## verbal  0.358
## visual  0.301
```

```
# $group_2$alpha
#        intrcp
# verbal  0.358
# visual  0.301

# You can find the standard errors p-values in the summary output under the `Intercepts` section.
summary(model_ex_1_strong_partial_fit)
```

```
## lavaan 0.6-12 ended normally after 112 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        52
##   Number of equality constraints                    13
##
##   Number of observations per group:
##     group_1                                         81
##     group_2                                        200
##
## Model Test User Model:
##
##   Test statistic                                75.924
##   Degrees of freedom                                49
##   P-value (Chi-square)                           0.008
##   Test statistic for each group:
##     group_1                                     44.340
##     group_2                                     31.584
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
```

```
##
##
## Group 1 [group_1]:
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)
##   verbal =~
##     inf               1.000
##     sim     (.p2.)    1.107    0.073   15.181    0.000
##     voc     (.p3.)    1.096    0.072   15.253    0.000
##     com     (.p4.)    0.862    0.068   12.658    0.000
##   visual =~
##     p_com             1.000
##     p_arr   (.p6.)    0.891    0.115    7.722    0.000
##     b_des   (.p7.)    1.188    0.117   10.193    0.000
##     o_ass   (.p8.)    1.139    0.111   10.219    0.000
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)
##   verbal ~~
##     visual            3.929    0.856    4.592    0.000
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)
##    .inf     (.20.)   10.219    0.327   31.211    0.000
##    .sim              12.070    0.372   32.403    0.000
##    .voc     (.22.)   10.072    0.347   29.041    0.000
##    .com     (.23.)   10.120    0.284   35.622    0.000
##    .p_com   (.24.)   10.750    0.265   40.570    0.000
##    .p_arr   (.25.)   10.736    0.283   37.950    0.000
##    .b_des   (.26.)   10.466    0.321   32.647    0.000
##    .o_ass   (.27.)   10.690    0.305   35.049    0.000
##     verbal            0.000
##     visual            0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)
##    .inf               3.750    0.697    5.381    0.000
##    .sim               2.768    0.589    4.704    0.000
##    .voc               2.160    0.505    4.279    0.000
##    .com               2.455    0.469    5.231    0.000
##    .p_com             2.814    0.604    4.655    0.000
##    .p_arr            10.333    1.717    6.017    0.000
##    .b_des             5.675    1.091    5.200    0.000
##    .o_ass             4.588    0.912    5.029    0.000
##     verbal            6.908    1.361    5.076    0.000
##     visual            3.925    0.893    4.393    0.000
##
##
## Group 2 [group_2]:
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)
##   verbal =~
##     inf               1.000
##     sim     (.p2.)    1.107    0.073   15.181    0.000
##     voc     (.p3.)    1.096    0.072   15.253    0.000
##     com     (.p4.)    0.862    0.068   12.658    0.000
##   visual =~
##     p_com             1.000
##     p_arr   (.p6.)    0.891    0.115    7.722    0.000
```

```
##     b_des    (.p7.)    1.188    0.117   10.193    0.000
##     o_ass    (.p8.)    1.139    0.111   10.219    0.000
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   verbal ~~
##     visual            3.671    0.565    6.499    0.000
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)
##    .inf     (.20.)   10.219    0.327   31.211    0.000
##    .sim              10.489    0.370   28.342    0.000
##    .voc     (.22.)   10.072    0.347   29.041    0.000
##    .com     (.23.)   10.120    0.284   35.622    0.000
##    .p_com   (.24.)   10.750    0.265   40.570    0.000
##    .p_arr   (.25.)   10.736    0.283   37.950    0.000
##    .b_des   (.26.)   10.466    0.321   32.647    0.000
##    .o_ass   (.27.)   10.690    0.305   35.049    0.000
##     verbal           -0.171    0.358   -0.477    0.633
##     visual           -0.530    0.301   -1.761    0.078
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)
##    .inf              3.768    0.452    8.334    0.000
##    .sim              2.343    0.352    6.646    0.000
##    .voc              2.528    0.363    6.958    0.000
##    .com              4.841    0.534    9.065    0.000
##    .p_com            5.533    0.663    8.346    0.000
##    .p_arr            8.302    0.910    9.119    0.000
##    .b_des            5.288    0.706    7.494    0.000
##    .o_ass            5.133    0.672    7.637    0.000
##     verbal           5.212    0.769    6.776    0.000
##     visual           4.326    0.792    5.461    0.000
```

```
# Intercepts:
#                    Estimate  Std.Err  z-value  P(>|z|)
#       .                 .        .        .        .
#       .                 .        .        .        .
#       .                 .        .        .        .
#     verbal           -0.171    0.358   -0.477    0.633
#     visual           -0.530    0.301   -1.761    0.078

# Or, you can approximate the p-values yourself given that you have the standard errors.
# For `verbal` latent variable.
pnorm(-0.171 / 0.358) * 2
```

```
## [1] 0.6328968
```

```
# [1] 0.6328968

# For `visual` latent variable.
pnorm(-0.530 / 0.301) * 2
```

```
## [1] 0.07827271
```

```
# [1] 0.07827271
```

k. Investigate *strict* measurement invariance. Also, perform a *LRT* to compare the strict measurement invariance model to the strong partial measurement invariance model. Report and interpret model fit of the fitted model and the results of the *LRT*.

To investigate strict measurement invariance, on top of constraining the loadings and the intercepts to be

equal in both groups, we also need to constrain the residual covariances. To achieve this, we can, again, use the `group.equal` argument in `lavaan`.

```
# Fit the model.
model_ex_1_strict_fit <- sem(
    model_ex_1,
    sample.cov = covariances,
    sample.nobs = samples,
    sample.mean = means,
    group.equal = c("loadings", "intercepts", "residuals"),
    group.partial = c("sim ~ 1")
)

# Model summary.
summary(model_ex_1_strict_fit, standardized = TRUE)
```

```
## lavaan 0.6-12 ended normally after 79 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        52
##   Number of equality constraints                    21
##
##   Number of observations per group:
##     group_1                                         81
##     group_2                                        200
##
## Model Test User Model:
##
##   Test statistic                                94.881
##   Degrees of freedom                                57
##   P-value (Chi-square)                           0.001
##   Test statistic for each group:
##     group_1                                     59.688
##     group_2                                     35.193
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
##
## Group 1 [group_1]:
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal =~
##     inf               1.000                               2.627    0.805
##     sim     (.p2.)    1.109    0.073   15.242    0.000    2.912    0.880
##     voc     (.p3.)    1.096    0.072   15.245    0.000    2.878    0.880
##     com     (.p4.)    0.846    0.070   12.098    0.000    2.222    0.736
```

```
##   visual =~
##     p_com             1.000                               2.026    0.681
##     p_arr   (.p6.)    0.893    0.115    7.786    0.000    1.809    0.518
##     b_des   (.p7.)    1.179    0.115   10.244    0.000    2.389    0.720
##     o_ass   (.p8.)    1.128    0.110   10.218    0.000    2.284    0.718
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal ~~
##     visual            3.936    0.886    4.443    0.000    0.740    0.740
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf     (.20.)   10.229    0.328   31.139    0.000   10.229    3.136
##    .sim              12.070    0.368   32.834    0.000   12.070    3.648
##    .voc     (.22.)   10.074    0.349   28.828    0.000   10.074    3.081
##    .com     (.23.)   10.171    0.287   35.406    0.000   10.171    3.370
##    .p_com   (.24.)   10.700    0.281   38.130    0.000   10.700    3.598
##    .p_arr   (.25.)   10.759    0.286   37.633    0.000   10.759    3.083
##    .b_des   (.26.)   10.451    0.324   32.302    0.000   10.451    3.152
##    .o_ass   (.27.)   10.686    0.310   34.487    0.000   10.686    3.358
##     verbal            0.000                               0.000    0.000
##     visual            0.000                               0.000    0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf     (.10.)    3.737    0.382    9.778    0.000    3.737    0.351
##    .sim     (.11.)    2.465    0.312    7.906    0.000    2.465    0.225
##    .voc     (.12.)    2.411    0.305    7.912    0.000    2.411    0.225
##    .com     (.13.)    4.172    0.396   10.542    0.000    4.172    0.458
##    .p_com   (.14.)    4.744    0.502    9.442    0.000    4.744    0.536
##    .p_arr   (.15.)    8.908    0.821   10.848    0.000    8.908    0.731
##    .b_des   (.16.)    5.289    0.599    8.830    0.000    5.289    0.481
##    .o_ass   (.17.)    4.911    0.553    8.877    0.000    4.911    0.485
##     verbal            6.901    1.359    5.077    0.000    1.000    1.000
##     visual            4.103    0.981    4.184    0.000    1.000    1.000
##
##
## Group 2 [group_2]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal =~
##     inf               1.000                               2.292    0.764
##     sim      (.p2.)   1.109    0.073   15.242    0.000    2.540    0.851
##     voc      (.p3.)   1.096    0.072   15.245    0.000    2.511    0.850
##     com      (.p4.)   0.846    0.070   12.098    0.000    1.938    0.688
##   visual =~
##     p_com             1.000                               2.109    0.696
##     p_arr   (.p6.)    0.893    0.115    7.786    0.000    1.884    0.534
##     b_des   (.p7.)    1.179    0.115   10.244    0.000    2.487    0.734
```

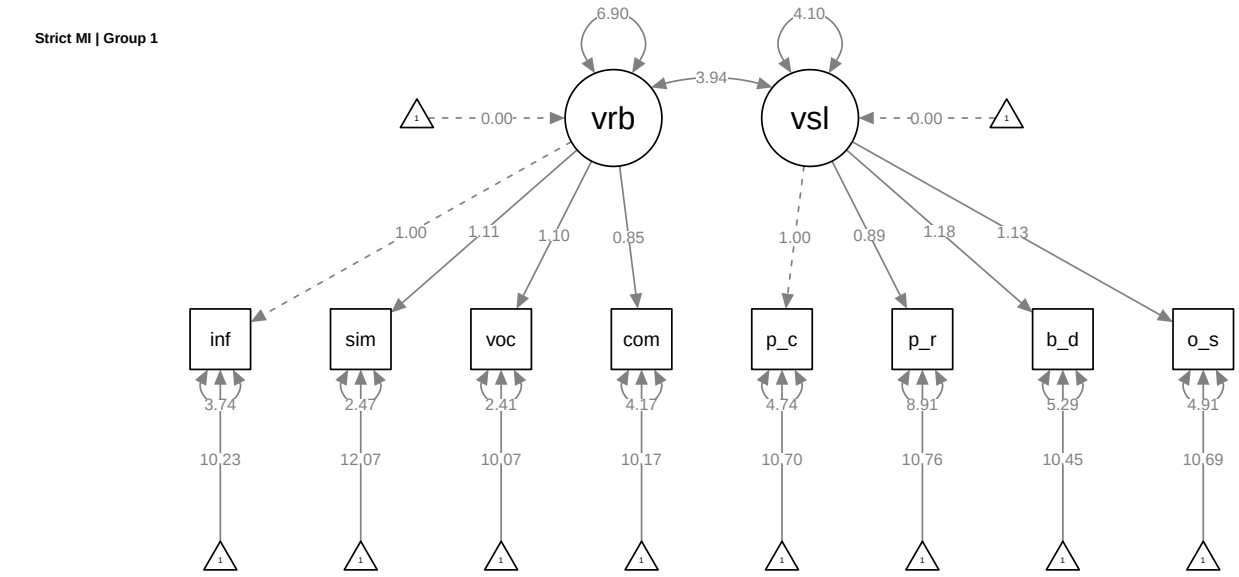
```
##     o_ass    (.p8.)    1.128    0.110   10.218    0.000    2.379    0.732
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal ~~
##     visual             3.712    0.567    6.546    0.000    0.768    0.768
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf     (.20.)   10.229    0.328   31.139    0.000   10.229    3.412
##    .sim              10.505    0.374   28.088    0.000   10.505    3.518
##    .voc     (.22.)   10.074    0.349   28.828    0.000   10.074    3.412
##    .com     (.23.)   10.171    0.287   35.406    0.000   10.171    3.612
##    .p_com   (.24.)   10.700    0.281   38.130    0.000   10.700    3.529
##    .p_arr   (.25.)   10.759    0.286   37.633    0.000   10.759    3.048
##    .b_des   (.26.)   10.451    0.324   32.302    0.000   10.451    3.085
##    .o_ass   (.27.)   10.686    0.310   34.487    0.000   10.686    3.287
##     verbal            -0.185    0.361   -0.513    0.608   -0.081   -0.081
##     visual            -0.520    0.308   -1.685    0.092   -0.246   -0.246
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf     (.10.)    3.737    0.382    9.778    0.000    3.737    0.416
##    .sim     (.11.)    2.465    0.312    7.906    0.000    2.465    0.276
##    .voc     (.12.)    2.411    0.305    7.912    0.000    2.411    0.277
##    .com     (.13.)    4.172    0.396   10.542    0.000    4.172    0.526
##    .p_com   (.14.)    4.744    0.502    9.442    0.000    4.744    0.516
##    .p_arr   (.15.)    8.908    0.821   10.848    0.000    8.908    0.715
##    .b_des   (.16.)    5.289    0.599    8.830    0.000    5.289    0.461
##    .o_ass   (.17.)    4.911    0.553    8.877    0.000    4.911    0.465
##     verbal            5.252    0.774    6.789    0.000    1.000    1.000
##     visual            4.449    0.794    5.606    0.000    1.000    1.000
```

```
# Fit measures.
fitMeasures(model_ex_1_strict_fit, fit.measures = fit_indices)
```

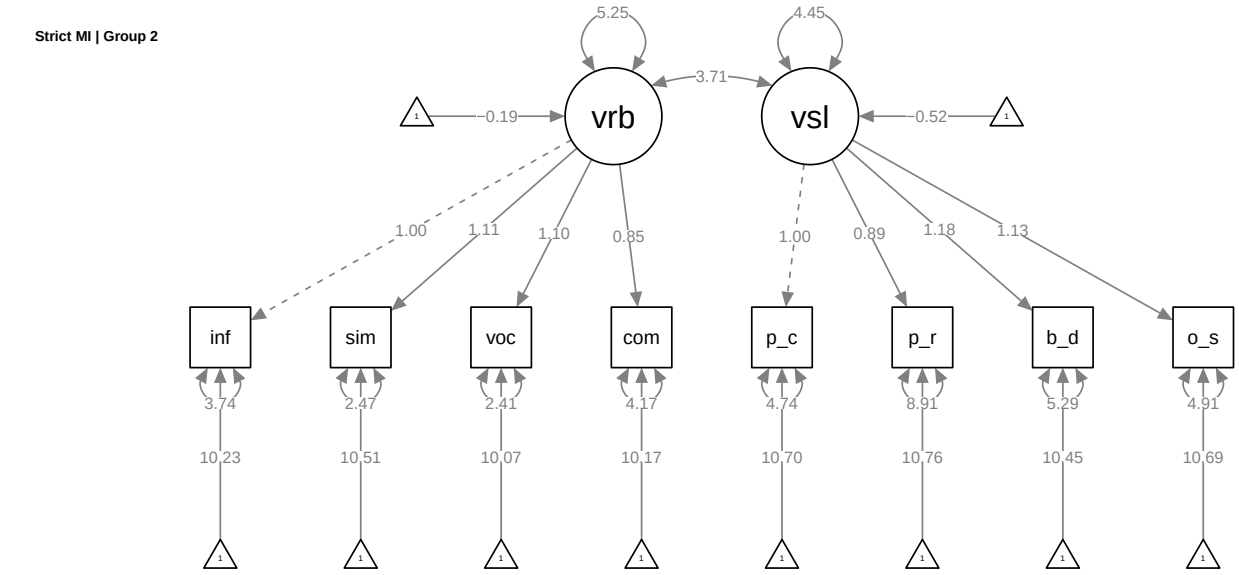
```
##        chisq           df       pvalue          cfi          tli        rmsea rmsea.pvalue
##       94.881       57.000        0.001        0.963        0.964        0.069        0.105
##         srmr
##        0.070
```

```
# Save the model plots for each groups as a list with two elements.
plots_strict <- semPaths(
    model_ex_1_strict_fit,
    what = "paths",
    whatLabels = "est",
    DoNotPlot = TRUE,
    ask = FALSE,
    title = FALSE
)

# Plot the model for the first group.
plot(plots_strict[[1]])
```

```
title("Strict MI | Group 1", adj = 0)
```




```
# Plot the model for the second group.
plot(plots_strict[[2]])
title("Strict MI | Group 2", adj = 0)
```



We can again combine the fit measures for the models investigated so far for convenience.

```
# Combine all fit measures.
fit_measures_all_ex_1 <- rbind(
    fit_measures_all_ex_1,
    strict = fitMeasures(model_ex_1_strict_fit, fit_indices)
)

# Print the fit measures for all models with four decimals.
print(round(fit_measures_all_ex_1, 4))
```

```
##                  chisq df pvalue    cfi    tli  rmsea rmsea.pvalue   srmr
## configural     53.3796 38 0.0500 0.9850 0.9779 0.0537       0.4025 0.0344
```

```
## weak             65.9923 44 0.0175 0.9785 0.9727 0.0596       0.2805 0.0553
## strong          109.0882 50 0.0000 0.9423 0.9354 0.0917       0.0028 0.0638
## strong_partial   75.9240 49 0.0081 0.9737 0.9700 0.0625       0.2175 0.0579
## strict           94.8814 57 0.0012 0.9630 0.9637 0.0688       0.1049 0.0702
```

We observe a $\chi^2 = 129.978$ with $DF = 57$ and a p-value $= 0.001$. The fit indices indicate some support for strict measurement invariance.

Finally, we perform a *LRT* between the strict model and the partial strong model.

```
# Perform LRT.
anova(
    model_ex_1_configural_fit,
    model_ex_1_weak_fit,
    model_ex_1_strong_partial_fit,
    model_ex_1_strict_fit
)
```

```
## Chi-Squared Difference Test
##
##                               Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## model_ex_1_configural_fit     38 10583 10765 53.380
## model_ex_1_weak_fit           44 10584 10744 65.992    12.6128       6    0.04961 *
## model_ex_1_strong_partial_fit 49 10584 10726 75.924     9.9317       5    0.07720 .
## model_ex_1_strict_fit         57 10587 10700 94.881    18.9574       8    0.01509 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Based on the χ^2 difference test and the fit measures above, we do not see support for strict measurement invariance. We could, of course, use the same strategy as above (i.e., using the function `lavTestScore`) to detect which parameters to free in order to increase the model fit, but this exercise is already getting too long...

l. What kinds of comparisons does strict measurement invariance allow you to make?

Strict measurement invariance is an important prerequisite if we want to make comparisons based on the raw scores of variables (e.g., sum scores of variables). The rationale behind this is that an observed score X consists of a true score T and and error score E (i.e., $X = T + E$; see this in the context of reliability). Therefore, with strict measurement invariance we insure that the error part (i.e., E) is the same across groups, allowing us to validly compare sum scores or other measures derived based the raw variable realizations.

Part 3. Generating some data.

Optional.

In *Part 2* of this exercise we used the mean and covariance vectors and matrices for multiple group *CFA*. In `lavaan`, we can also use the entire data for multiple group *CFA*, in which case we can specify the `group` argument to tell `lavaan` which variable to use to split our dataset. This implies that we have an additional variable in our data which indicates the group membership (e.g., manic vs. norming). In this part you will first learn how to generate a complete dataset using reported means and covariances, then you will see how the generated data can be used to test for configural measurement invariance. The reminder of the measurement invariance checks (i.e., weak, strong and strict measurement invariance) are left for you to do as an exercise.

Note that this part is optional and you can safely skip it. It is presented here as an example for those of you who are curious about generating data.

To generate our complete datasets, we will use the means and covariances presented in *Figure 1.* Since we already have the observed means and covariances, and we assume that our data comes from a multivariate normal distribution, we can "draw" a sample from this distribution using the observed means and covariances. Recall that, just like the normal distribution, the multivariate normal distribution depends on two parameters, the $\boldsymbol{\mu}$ vector of means and the $\boldsymbol{\Sigma}$ variance-covariance matrix:

$$p(x; \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{n/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left(-\frac{1}{2}(x-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(x-\boldsymbol{\mu})\right)$$

Conceptually, this means that we use our observed means and covariances as population parameters and we draw a sample from that population (e.g., imagine applying a questionnaire). If the sample we drew is sufficiently large, then the means and covariances we calculate on the generated data should be close to our observed means and covariances that we used as population parameters. Furthermore, since each group has its own values for the mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$, we consider that each group comes for its own population. Therefore, we draw two samples from the multivariate normal distribution, one per group.

That being said, we can go ahead and generate some data. First, let us check that we can indeed recover the means and covariances provided in *Figure 1* if our sample is sufficiently large. We do this as an example for the first group. As always, make sure to check the documentation for function `?rmvnorm` in the `R` package `mvtnorm`.

```
# Inspect the means for group one.
group_1_means
```

```
##   inf   sim   voc   com p_com p_arr b_des o_ass
## 10.09 12.07 10.25  9.96 10.90 11.24 10.30 10.44
```

```
# Inspect the covariances for group one.
group_1_cov
```

```
##         inf    sim    voc   com p_com  p_arr  b_des o_ass
## inf   9.364  7.777  6.422 5.669 3.048  3.505  3.690 3.640
## sim   7.777 12.461  8.756 7.445 4.922  4.880  5.440 4.641
## voc   6.422  8.756 10.112 6.797 4.513  4.899  5.220 4.877
## com   5.669  7.445  6.797 8.123 4.116  5.178  3.151 3.568
## p_com 3.048  4.922  4.513 4.116 6.200  5.114  3.587 3.819
## p_arr 3.505  4.880  4.899 5.178 5.114 15.603  6.219 5.811
## b_des 3.690  5.440  5.220 3.151 3.587  6.219 11.223 6.501
## o_ass 3.640  4.641  4.877 3.568 3.819  5.811  6.501 9.797
```

```
# Draw a sample of one million respondents for group one.
data_ex_1_group_1 <- rmvnorm(n = 1e6, mean = group_1_means, sigma = group_1_cov)

# Check that we indeed have one million respondents for eight variables.
dim(data_ex_1_group_1)
```

```
## [1] 1000000       8
```

```
# Compare the means of the variables in the generated data with the provided means.
round(group_1_means - colMeans(data_ex_1_group_1), 4)
```

```
##     inf     sim     voc     com   p_com   p_arr   b_des   o_ass
## -0.0062 -0.0054 -0.0052 -0.0029 -0.0056 -0.0035 -0.0068 -0.0055
```

```r
# Compare the covariances of the generated data with the provided covariance matrix.
round(group_1_cov - cov(data_ex_1_group_1), 3)
```

```
##          inf    sim    voc    com  p_com  p_arr  b_des  o_ass
## inf   -0.005 -0.005 -0.003 -0.008 -0.017 -0.029 -0.008 -0.020
## sim   -0.005 -0.010  0.000 -0.005 -0.008 -0.005  0.000 -0.015
## voc   -0.003  0.000 -0.001  0.001 -0.007 -0.001  0.003 -0.006
## com   -0.008 -0.005  0.001 -0.002 -0.010  0.000 -0.002 -0.010
## p_com -0.017 -0.008 -0.007 -0.010 -0.007 -0.003  0.002 -0.013
## p_arr -0.029 -0.005 -0.001  0.000 -0.003  0.017  0.008 -0.001
## b_des -0.008  0.000  0.003 -0.002  0.002  0.008  0.012  0.001
## o_ass -0.020 -0.015 -0.006 -0.010 -0.013 -0.001  0.001 -0.018
```

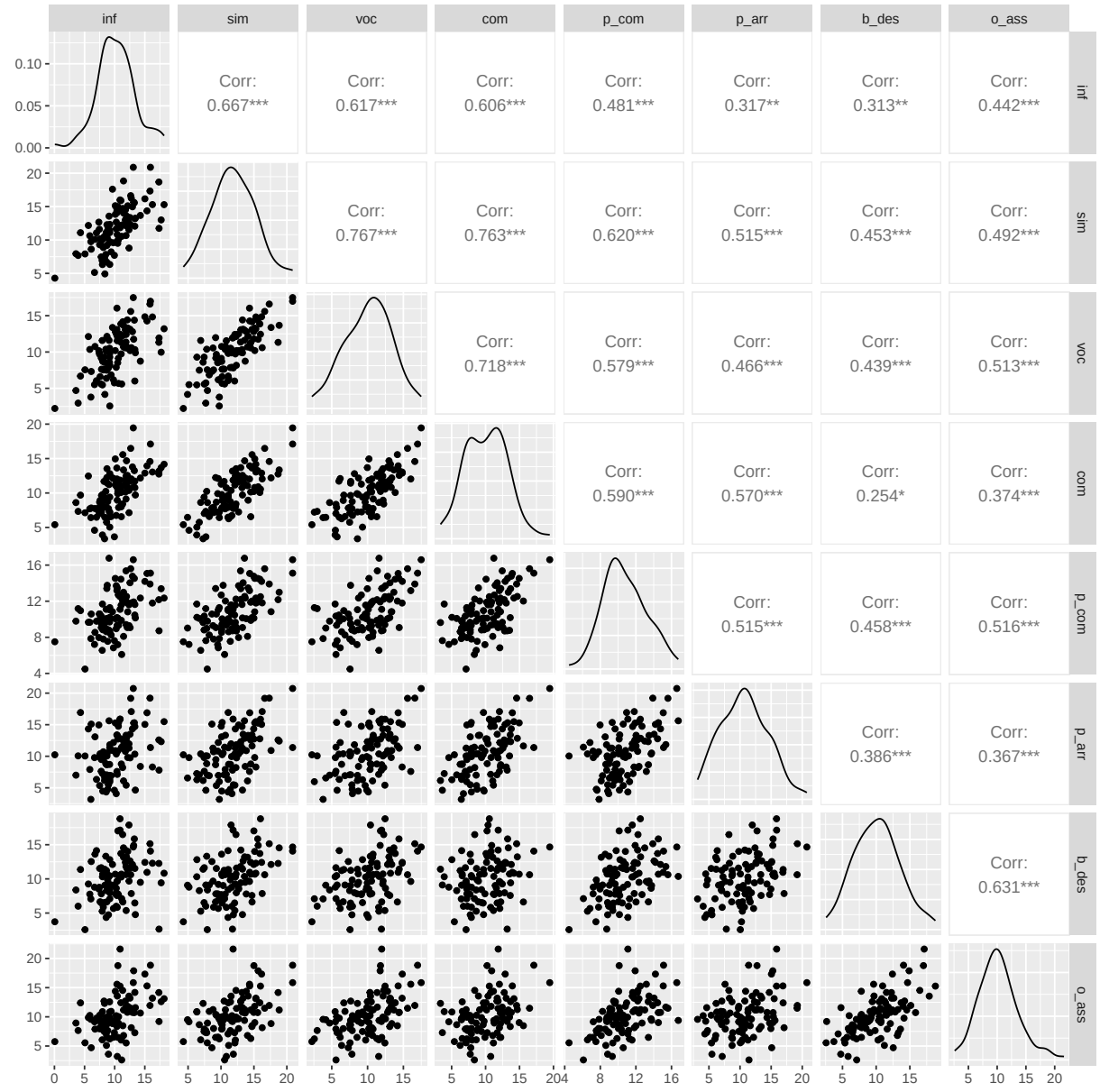
We can see that the differences are quite small. You can test that the more we increase the sample size (i.e., `n`), the smaller the differences become.

Now, let us go ahead and take more reasonably sized samples for both groups and combine them into a single data frame. We will take a sample of $N = 100$ for group one and $N = 300$ for group two. Since we are dealing with random number generation (e.g., see `?RNG`), we also set a "seed" so we can replicate the results.

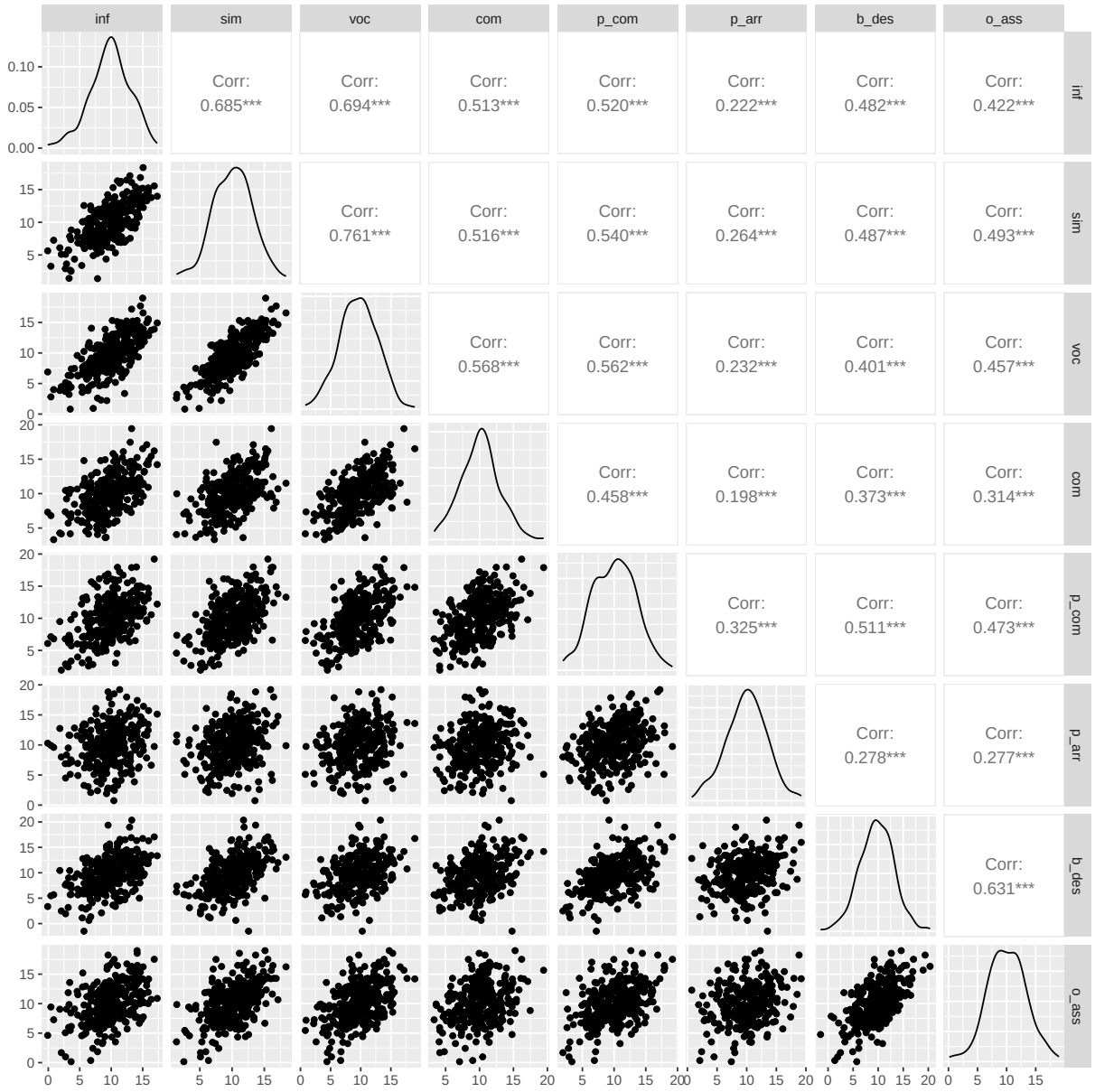
```r
# Set seed for the `RNG`.
set.seed(20031993)

# Data for group one with 100 cases.
data_ex_1_group_1 <- rmvnorm(n = 100, mean = group_1_means, sigma = group_1_cov)

# Data for group two with 300 cases.
data_ex_1_group_2 <- rmvnorm(n = 300, mean = group_2_means, sigma = group_2_cov)
```

Before combining the data, we may also want to visualize what we generated. We use the function `ggpairs` in package `GGally`. Note that the function expects the data to be provided as a data frame and not matrix object.

```r
# Generated data for group one.
ggpairs(as.data.frame(data_ex_1_group_1))
```




```
# Generated data for group two.
ggpairs(as.data.frame(data_ex_1_group_2))
```



We continue with combining the data for both groups into a single data frame.

```
# Combine the two datasets into a single dataset.
data_ex_1 <- data.frame(rbind(data_ex_1_group_1, data_ex_1_group_2))

# Create a variable that holds the group membership.
group <- as.factor(c(rep("manic", 100), rep("norming", 300)))

# Inspect the `group` variable.
str(group)
```

```
##  Factor w/ 2 levels "manic","norming": 1 1 1 1 1 1 1 1 1 1 ...
```

```
# Now we can add the `group` variable to the dataset.
data_ex_1 <- cbind(data_ex_1, group = group)

# Inspect the head of the data.
head(data_ex_1)
```

```
##         inf       sim       voc       com     p_com     p_arr     b_des     o_ass group
## 1 10.611565 10.037764  9.130394 11.283947 12.688104 14.753140  7.705091  8.936030 manic
## 2  3.934381  7.689681  2.964765  7.333193 11.178762 10.065857  6.005804  7.658062 manic
## 3  8.234517 10.474990  7.648771 11.680683  9.500099  7.070482 11.253965 12.700114 manic
## 4 17.341116 18.674477 11.327905 12.748632 12.170414 12.589906 12.265402 14.160554 manic
## 5 10.254932 11.662189  7.893027  6.740234  8.954935  4.073957  5.995267 10.199569 manic
## 6 11.477607 14.029962 10.700104 11.057523  8.102913  6.095811 13.159574  6.806254 manic
```

```
# Inspect the tail of the data.
tail(data_ex_1)
```

```
##           inf       sim       voc       com     p_com     p_arr     b_des     o_ass   group
## 395  9.645569 10.603463  8.045378  9.272881  9.107972 10.680583 10.918464  7.269432 norming
## 396  9.685306  9.747118  5.585436  8.525569  9.477196 17.859655 11.879686  9.528950 norming
## 397 15.868305 15.255730 15.334440 14.216598 13.083955 13.440027 12.467222 10.008781 norming
## 398  7.830651  5.914111  6.729447  4.816376  7.086313  2.400862  4.919694  1.856532 norming
## 399  5.884157  7.718305  3.908084  7.182868  5.671944 11.791069 10.473924  5.922491 norming
## 400 13.042982 13.321951  9.408113 14.672445 10.559942  8.451276  7.966637 12.787075 norming
```

With the data in hand, we can now perform the test for configural measurement invariance. This time we use the full data and make use of the `group` argument in `lavaan`. Note that the model syntax remains the same.

```
# Fit the model using the full data.
model_ex_1_configural_data_fit <- sem(model_ex_1, data_ex_1, group = "group")


# Model summary.
summary(model_ex_1_configural_data_fit, standardized = TRUE)
```

```
## lavaan 0.6-12 ended normally after 101 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        50
##
##   Number of observations per group:
##     manic                                          100
##     norming                                        300
##
## Model Test User Model:
##
##   Test statistic                                97.221
##   Degrees of freedom                                38
##   P-value (Chi-square)                           0.000
##   Test statistic for each group:
##     manic                                       43.234
##     norming                                     53.987
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
##
## Group 1 [manic]:
##
```

```
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal =~
##     inf               1.000                              2.280    0.725
##     sim               1.331    0.149    8.916    0.000    3.034    0.911
##     voc               1.222    0.146    8.386    0.000    2.786    0.853
##     com               1.125    0.137    8.207    0.000    2.564    0.835
##   visual =~
##     p_com             1.000                              1.905    0.786
##     p_arr             1.251    0.205    6.113    0.000    2.383    0.637
##     b_des             1.124    0.184    6.099    0.000    2.141    0.636
##     o_ass             1.247    0.187    6.682    0.000    2.376    0.694
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal ~~
##     visual            3.594    0.753    4.774    0.000    0.827    0.827
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf              10.240    0.314   32.574    0.000   10.240    3.257
##    .sim              11.875    0.333   35.643    0.000   11.875    3.564
##    .voc              10.082    0.327   30.852    0.000   10.082    3.085
##    .com              10.095    0.307   32.861    0.000   10.095    3.286
##    .p_com            10.773    0.242   44.457    0.000   10.773    4.446
##    .p_arr            10.677    0.374   28.562    0.000   10.677    2.856
##    .b_des            10.081    0.337   29.945    0.000   10.081    2.995
##    .o_ass            10.206    0.343   29.789    0.000   10.206    2.979
##     verbal            0.000                              0.000    0.000
##     visual            0.000                              0.000    0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf               4.683    0.731    6.408    0.000    4.683    0.474
##    .sim               1.895    0.462    4.101    0.000    1.895    0.171
##    .voc               2.915    0.538    5.414    0.000    2.915    0.273
##    .com               2.863    0.506    5.657    0.000    2.863    0.303
##    .p_com             2.243    0.455    4.925    0.000    2.243    0.382
##    .p_arr             8.297    1.341    6.186    0.000    8.297    0.594
##    .b_des             6.748    1.090    6.192    0.000    6.748    0.595
##    .o_ass             6.090    1.039    5.861    0.000    6.090    0.519
##     verbal            5.199    1.269    4.097    0.000    1.000    1.000
##     visual            3.629    0.834    4.351    0.000    1.000    1.000
##
##
## Group 2 [norming]:
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal =~
##     inf               1.000                              2.539    0.800
```

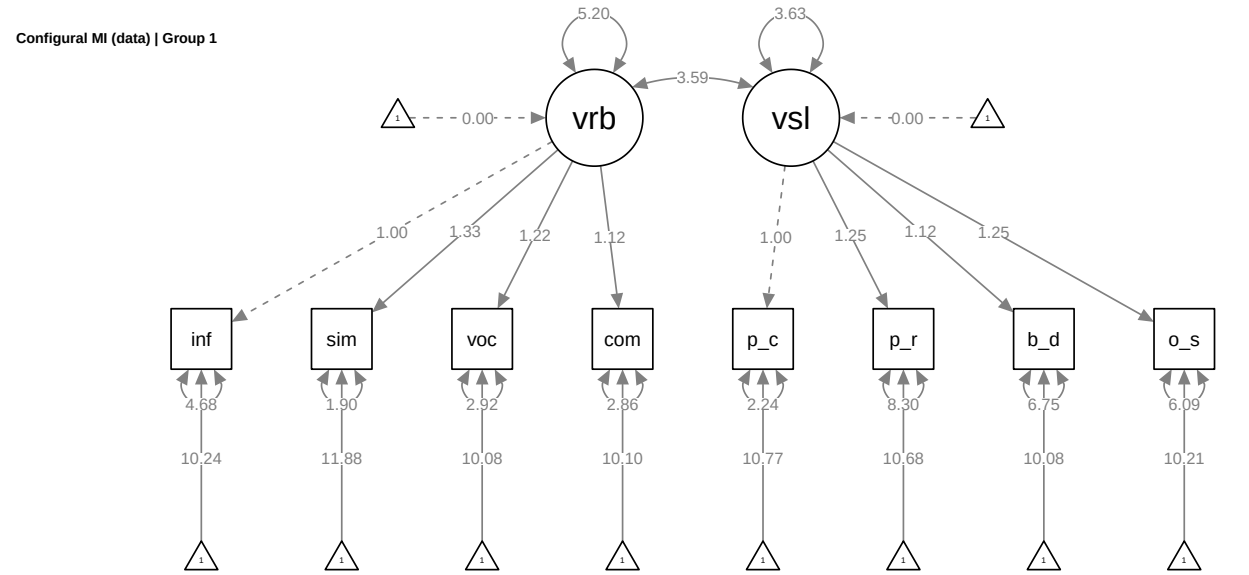
```
##     sim               1.011    0.061   16.611    0.000    2.568    0.865
##     voc               1.077    0.064   16.714    0.000    2.736    0.870
##     com               0.695    0.061   11.324    0.000    1.765    0.633
##   visual =~
##     p_com             1.000                               2.398    0.722
##     p_arr             0.549    0.090    6.106    0.000    1.316    0.390
##     b_des             1.070    0.095   11.292    0.000    2.565    0.753
##     o_ass             1.021    0.092   11.095    0.000    2.447    0.736
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   verbal ~~
##     visual           4.701    0.595    7.905    0.000    0.772    0.772
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf              9.810    0.183   53.545    0.000    9.810    3.091
##    .sim             10.106    0.171   58.946    0.000   10.106    3.403
##    .voc              9.589    0.182   52.797    0.000    9.589    3.048
##    .com             10.047    0.161   62.393    0.000   10.047    3.602
##    .p_com           10.053    0.192   52.411    0.000   10.053    3.026
##    .p_arr            9.985    0.195   51.269    0.000    9.985    2.960
##    .b_des            9.683    0.197   49.241    0.000    9.683    2.843
##    .o_ass           10.052    0.192   52.353    0.000   10.052    3.023
##     verbal           0.000                               0.000    0.000
##     visual           0.000                               0.000    0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .inf              3.621    0.368    9.828    0.000    3.621    0.360
##    .sim              2.222    0.273    8.130    0.000    2.222    0.252
##    .voc              2.409    0.303    7.946    0.000    2.409    0.243
##    .com              4.664    0.410   11.365    0.000    4.664    0.600
##    .p_com            5.288    0.561    9.426    0.000    5.288    0.479
##    .p_arr            9.647    0.818   11.796    0.000    9.647    0.848
##    .b_des            5.021    0.568    8.833    0.000    5.021    0.433
##    .o_ass            5.072    0.553    9.176    0.000    5.072    0.459
##     verbal           6.449    0.798    8.080    0.000    1.000    1.000
##     visual           5.750    0.868    6.621    0.000    1.000    1.000
```

```
# Fit measures.
fitMeasures(model_ex_1_configural_data_fit, fit.measures = fit_indices)
```

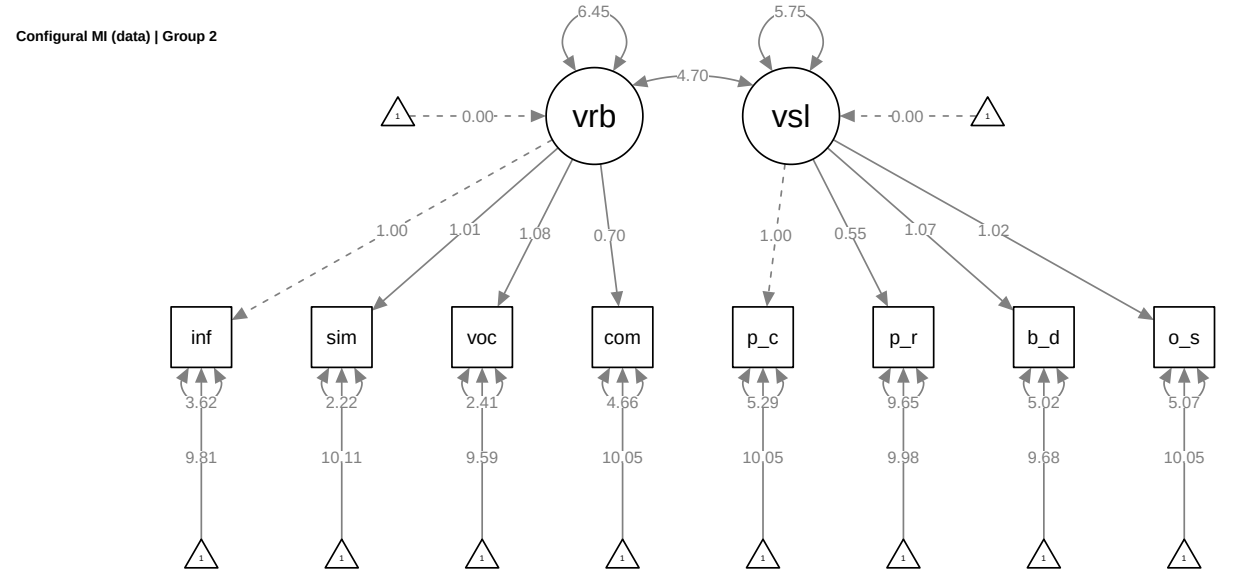
```
##        chisq           df       pvalue          cfi          tli        rmsea rmsea.pvalue
##       97.221       38.000        0.000        0.962        0.943        0.088        0.003
##         srmr
##        0.040
```

```
# Save the model plots for each groups as a list with two elements.
plots_configural_data <- semPaths(
    model_ex_1_configural_data_fit,
    what = "paths",
    whatLabels = "est",
```

```
    DoNotPlot = TRUE,
    ask = FALSE,
    title = FALSE
)

# Plot the model for the first group.
plot(plots_configural_data[[1]])
title("Configural MI (data) | Group 1", adj = 0)
```




```
# Plot the model for the second group.
plot(plots_configural_data[[2]])
title("Configural MI (data) | Group 2", adj = 0)
```



Exercise 2

Part 1. Investigating longitudinal measurement invariance.

In this part of *Exercise 2*, you are going to investigate measurement invariance using longitudinal data. This exercise differs a bit from the lecture, in the sense that we will not deploy measurement invariance tests to check whether we can validly compare models parameters across groups. *Instead, we test for measurement invariance to understand whether we can validly compare model parameters across time.* In other words, we are interested to understand whether our construct is measurement invariant from one measurement occasion to the other.

Since testing for measurement invariance for longitudinal data is slightly more involved than what we did during *Exercise 1*, I recommend you to take a look at *Chapter 2* from Newsom (2015) (i.e., attached on Canvas under the *References* heading for the current practical). For a quick overview of the parametrization required, you can take a look at *Figure 3.* The relationships depicted in *Figure 3* are also described below:

- **Weak measurement invariance**: when loadings are equal over time but intercepts, unique variances, latent means, and latent variances vary over time.
- **Strong measurement invariance**: when loadings and intercepts do not vary but unique variances, latent means, and latent variances vary over time.
- **Strict measurement invariance**: when loadings, intercepts, and measurement residuals are equal over time.
- **Structural invariance**: when factor means, factor variances, loadings, intercepts, and measurement residuals are equal over time.

Key points to keep in mind for this exercise:

- During *Exercise 1* we applied the constraints across groups (e.g., a loading in group one was constrained to be equal with the corresponding loading in group two).
- However, when we check for measurement invariance for longitudinal data, we think of the measurement occasions as our "groups". In this case, we
  1. fit the latent construct at both measurement occasions in the same model,
  2. and constrain the parameters to be equal across time points (e.g., a loading for the latent construct at time point one constrained to be equal to the corresponding loading for the latent construct at time point two).
- So, the same ideas that you discussed during the lecture apply, but instead of fitting one model per group, you now fit a single model (i.e., with a latent construct per time point), and you constrain the parameters to be equal across time points.

The data you will use consists of a set of three items (i.e., `w1vst1`, `w1vst2`, `w1vst3`, `w2vst1`, `w2vst2`, and `w2vst3`) measured at two time points (i.e., `w1` for wave one and `w2` for wave two), with a total sample size of $N = 574$. You can find the data in the folder for this practical on Canvas, and the baseline model that you will investigate is graphically represented in *Figure 4*.

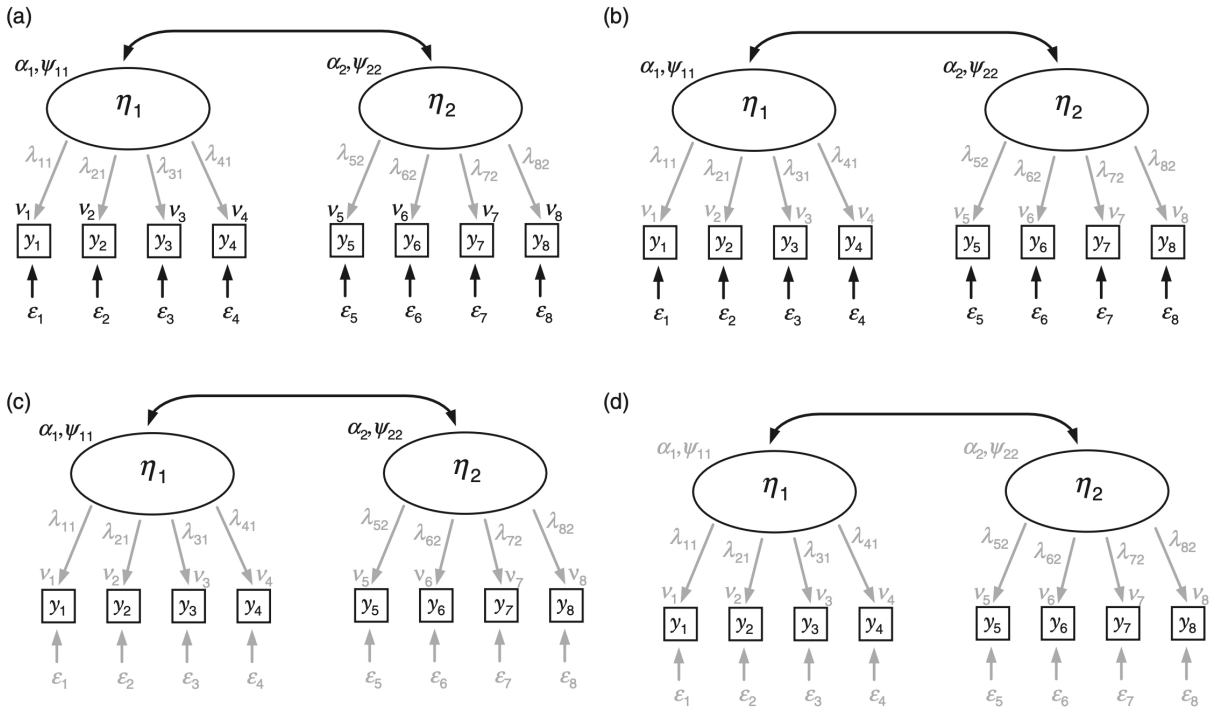


Figure 2.1 Graphic Depiction of Meredith's Factorial Invariance Definitions: (a) weak invariance; (b) strong invariance; (c) strict invariance; (d) structural invariance. Note: grayed lines and symbols represent parameters that are equal over time, where it is assumed that the equality is only between the longitudinal counterparts of each (e.g., $\lambda_{11}=\lambda_{52}$ or $\nu_1=\nu_5$).

Figure 3: Reproduction of *Figure 2.1* from Newsom (2015).

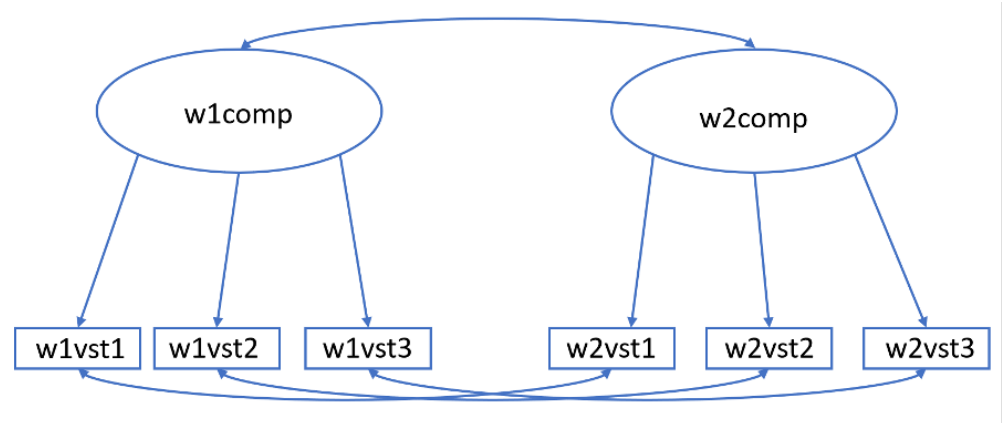


Figure 4: Model example for investigating longitudinal measurement invariance.

Start this exercise by loading the dataset `socex1.dat` in `R` and adding the following variable names to the data.

Set the working directory to the location where your data file has been downloaded and load the data.

```
# For example.
setwd("/Users/mihai/Downloads")

# Load data.
data_ex_2 <- read.csv("socex1.dat")

# Inspect the data.
View(data_ex_2)
```

Set the variable names.

```
# Variable names.
variable_ex_2_names <- c(
    "w1vst1", "w1vst2", "w1vst3", "w2vst1", "w2vst2",
    "w2vst3", "w3vst1", "w3vst2", "w3vst3", "w1unw1", "w1unw2", "w1unw3",
    "w2unw1", "w2unw2", "w2unw3", "w3unw1", "w3unw2", "w3unw3", "w1dboth",
    "w1dsad", "w1dblues", "w1ddep", "w2dboth", "w2dsad", "w2dblues", "w2ddep",
    "w3dboth", "w3dsad", "w3dblues", "w3ddep", "w1marr2", "w1happy", "w1enjoy",
    "w1satis", "w1joyful", "w1please", "w2happy", "w2enjoy", "w2satis", "w2joyful",
    "w2please", "w3happy", "w3enjoy", "w3satis", "w3joyful", "w3please", "w1lea",
    "w2lea", "w3lea"
)

# Set the names.
names(data_ex_2) <- variable_ex_2_names

# List variables.
str(data_ex_2)
```

```
## 'data.frame':    574 obs. of  49 variables:
##  $ w1vst1  : int  0 3 2 2 3 4 1 3 4 1 ...
##  $ w1vst2  : int  1 3 2 2 2 4 1 2 4 0 ...
##  $ w1vst3  : int  0 4 3 3 2 4 0 2 4 0 ...
##  $ w2vst1  : int  3 3 1 1 2 4 3 3 3 2 ...
##  $ w2vst2  : int  3 3 0 2 2 4 1 2 2 3 ...
##  $ w2vst3  : int  2 3 2 2 2 4 2 2 2 3 ...
##  $ w3vst1  : int  3 2 2 2 3 4 2 2 2 3 ...
##  $ w3vst2  : int  2 3 2 2 3 4 3 2 3 3 ...
##  $ w3vst3  : int  2 3 1 2 2 4 2 2 3 2 ...
##  $ w1unw1  : int  2 1 2 4 2 0 2 4 3 4 ...
##  $ w1unw2  : int  2 3 1 3 4 2 2 3 1 2 ...
##  $ w1unw3  : int  3 2 1 3 3 1 2 3 3 3 ...
##  $ w2unw1  : int  4 3 1 3 2 2 3 3 2 2 ...
##  $ w2unw2  : int  4 2 3 2 3 1 2 3 3 3 ...
##  $ w2unw3  : int  4 3 2 1 2 1 3 4 3 2 ...
##  $ w3unw1  : int  2 2 2 3 2 2 2 4 2 2 ...
##  $ w3unw2  : int  3 3 2 4 3 1 2 4 3 3 ...
##  $ w3unw3  : int  3 2 2 2 2 1 1 3 2 2 ...
```

```
##  $ w1dboth : int  0 0 0 2 1 0 0 0 0 0 ...
##  $ w1dsad  : int  0 1 0 0 2 0 0 2 0 0 ...
##  $ w1dblues: int  1 1 0 0 1 0 1 0 0 0 ...
##  $ w1ddep  : int  0 1 0 0 1 0 2 1 0 0 ...
##  $ w2dboth : int  0 0 1 1 0 0 0 2 0 0 ...
##  $ w2dsad  : int  1 1 1 0 1 0 0 0 1 1 ...
##  $ w2dblues: int  0 2 1 0 0 0 0 2 0 0 ...
##  $ w2ddep  : int  1 2 0 3 0 0 0 3 1 0 ...
##  $ w3dboth : int  0 1 0 1 0 1 0 2 0 1 ...
##  $ w3dsad  : int  0 0 0 1 0 0 0 2 0 1 ...
##  $ w3dblues: int  1 2 0 2 0 1 0 1 0 1 ...
##  $ w3ddep  : int  0 2 0 0 2 0 0 1 0 0 ...
##  $ w1marr2 : int  1 1 1 1 0 1 1 1 1 0 ...
##  $ w1happy : int  3 3 3 2 2 5 2 2 2 4 ...
##  $ w1enjoy : int  3 3 2 3 3 5 3 2 4 3 ...
##  $ w1satis : int  3 3 3 3 3 4 2 2 4 3 ...
##  $ w1joyful: int  3 3 2 3 2 4 3 2 3 3 ...
##  $ w1please: int  3 2 3 4 2 4 2 1 3 3 ...
##  $ w2happy : num  2.9 3.9 1.9 2.9 1.9 4.9 2.9 1.9 3.9 2.9 ...
##  $ w2enjoy : num  2.9 2.9 2.9 1.9 1.9 3.9 2.9 2.9 2.9 3.9 ...
##  $ w2satis : num  3.9 3.9 2.9 2.9 1.9 3.9 2.9 2.9 1.9 3.9 ...
##  $ w2joyful: num  2.9 2.9 2.9 3.9 2.9 3.9 2.9 1.9 3.9 2.9 ...
##  $ w2please: num  2.9 2.9 2.9 2.9 1.9 3.9 1.9 1.9 3.9 2.9 ...
##  $ w3happy : num  2.8 2.8 2.8 2.8 2.8 3.8 2.8 1.8 2.8 3.8 ...
##  $ w3enjoy : num  1.8 3.8 2.8 3.8 2.8 3.8 2.8 2.8 3.8 3.8 ...
##  $ w3satis : num  1.8 2.8 1.8 2.8 1.8 4.8 2.8 1.8 2.8 2.8 ...
##  $ w3joyful: num  2.8 2.8 1.8 2.8 2.8 4.8 1.8 1.8 2.8 2.8 ...
##  $ w3please: num  2.8 2.8 2.8 2.8 2.8 3.8 1.8 1.8 3.8 2.8 ...
##  $ w1lea   : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ w2lea   : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ w3lea   : int  0 0 0 0 1 0 0 1 0 0 ...
```

Specify which fit measures we are interested in:

```
# Fit indices to print.
fit_indices <- c("chisq", "df", "pvalue", "cfi", "tli", "rmsea", "rmsea.pvalue", "srmr")
```

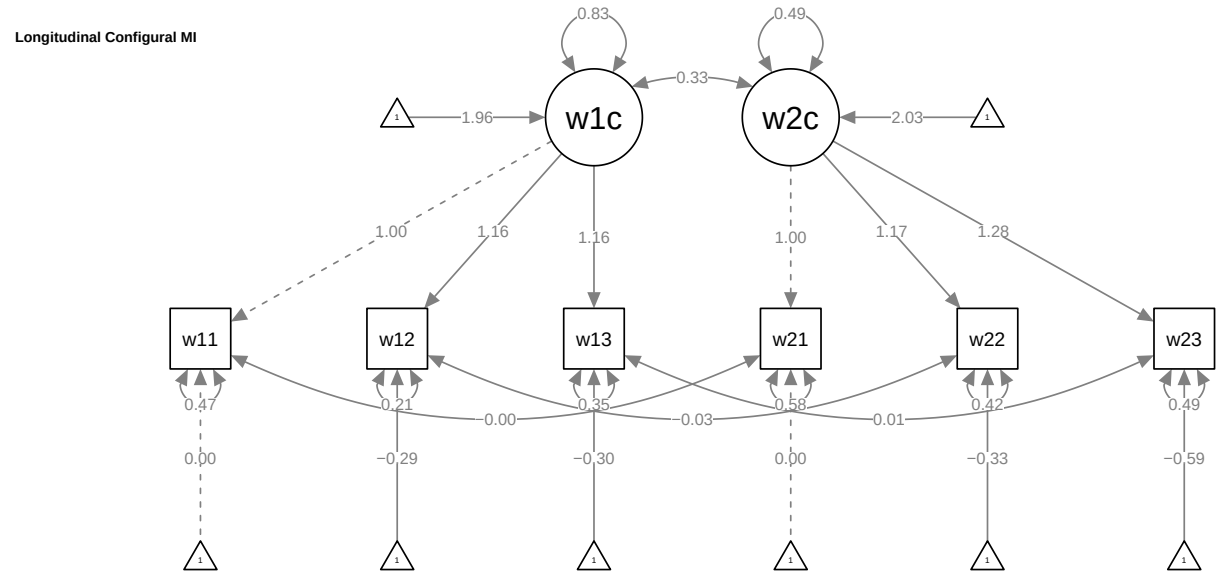
a. Investigate *configural* measurement invariance and also estimate the means of the latent variables. Report and interpret the model fit.

Contrary to *Exercise 1*, we now have to set the constraints manually. Few things to note:

- we use the default marker variable approach for setting the scales of latent variables `w1comp` and `w2comp`
- since we are going to estimate the means of the latent variables we must fix the first intercept at each time point to 0 for model identification

```
# Model syntax.
model_ex_2_configural <- "
    # Measurement part.
    w1comp =~ w1vst1 + w1vst2 + w1vst3
    w2comp =~ w2vst1 + w2vst2 + w2vst3

    # Covariance between latent variables.
```

```
    w2comp ~~ w1comp

    # Covariances between residuals across time.
    w1vst1 ~~ w2vst1
    w1vst2 ~~ w2vst2
    w1vst3 ~~ w2vst3

    # Fix first intercept at each time point to 0 for identification.
    w1vst1 ~ 0
    w2vst1 ~ 0

    # Freely estimate means of latent variables.
    w1comp ~ 1
    w2comp ~ 1
"

# Fit the model.
model_ex_2_configural_fit <- sem(model_ex_2_configural, data_ex_2)

# Visualize the model.
semPaths(model_ex_2_configural_fit, what = "paths", whatLabels = "est")

# Add a title to the plot.
title("Longitudinal Configural MI", adj = 0)
```




```
# Model summary.
summary(model_ex_2_configural_fit, standardized = TRUE)
```

```
## lavaan 0.6-12 ended normally after 47 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        22
##
```

```
##   Number of observations                           574
##
## Model Test User Model:
##
##   Test statistic                                 9.911
##   Degrees of freedom                                 5
##   P-value (Chi-square)                           0.078
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   w1comp =~
##     w1vst1            1.000                               0.913    0.800
##     w1vst2            1.159    0.048   24.360    0.000    1.059    0.917
##     w1vst3            1.156    0.049   23.659    0.000    1.056    0.874
##   w2comp =~
##     w2vst1            1.000                               0.698    0.674
##     w2vst2            1.173    0.081   14.419    0.000    0.819    0.784
##     w2vst3            1.284    0.088   14.556    0.000    0.897    0.788
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   w1comp ~~
##     w2comp            0.331    0.039    8.442    0.000    0.519    0.519
##  .w1vst1 ~~
##    .w2vst1           -0.001    0.026   -0.045    0.965   -0.001   -0.002
##  .w1vst2 ~~
##    .w2vst2           -0.030    0.021   -1.417    0.156   -0.030   -0.102
##  .w1vst3 ~~
##    .w2vst3            0.013    0.025    0.509    0.610    0.013    0.032
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .w1vst1            0.000                               0.000    0.000
##    .w2vst1            0.000                               0.000    0.000
##     w1comp            1.960    0.048   41.134    0.000    2.146    2.146
##     w2comp            2.031    0.043   47.003    0.000    2.909    2.909
##    .w1vst2           -0.293    0.101   -2.904    0.004   -0.293   -0.254
##    .w1vst3           -0.297    0.104   -2.851    0.004   -0.297   -0.246
##    .w2vst2           -0.333    0.172   -1.944    0.052   -0.333   -0.319
##    .w2vst3           -0.589    0.186   -3.165    0.002   -0.589   -0.518
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .w1vst1            0.469    0.034   13.798    0.000    0.469    0.360
##    .w1vst2            0.212    0.029    7.337    0.000    0.212    0.159
```

```
##    .w1vst3            0.346    0.033   10.511    0.000    0.346    0.237
##    .w2vst1            0.585    0.043   13.557    0.000    0.585    0.545
##    .w2vst2            0.419    0.042    9.946    0.000    0.419    0.385
##    .w2vst3            0.490    0.049    9.900    0.000    0.490    0.379
##     w1comp            0.834    0.074   11.206    0.000    1.000    1.000
##     w2comp            0.488    0.059    8.261    0.000    1.000    1.000
```

```r
# Fit measures.
fitMeasures(model_ex_2_configural_fit, fit.measures = fit_indices)
```

```
##        chisq           df       pvalue          cfi          tli        rmsea rmsea.pvalue
##        9.911        5.000        0.078        0.997        0.991        0.041        0.591
##         srmr
##        0.018
```

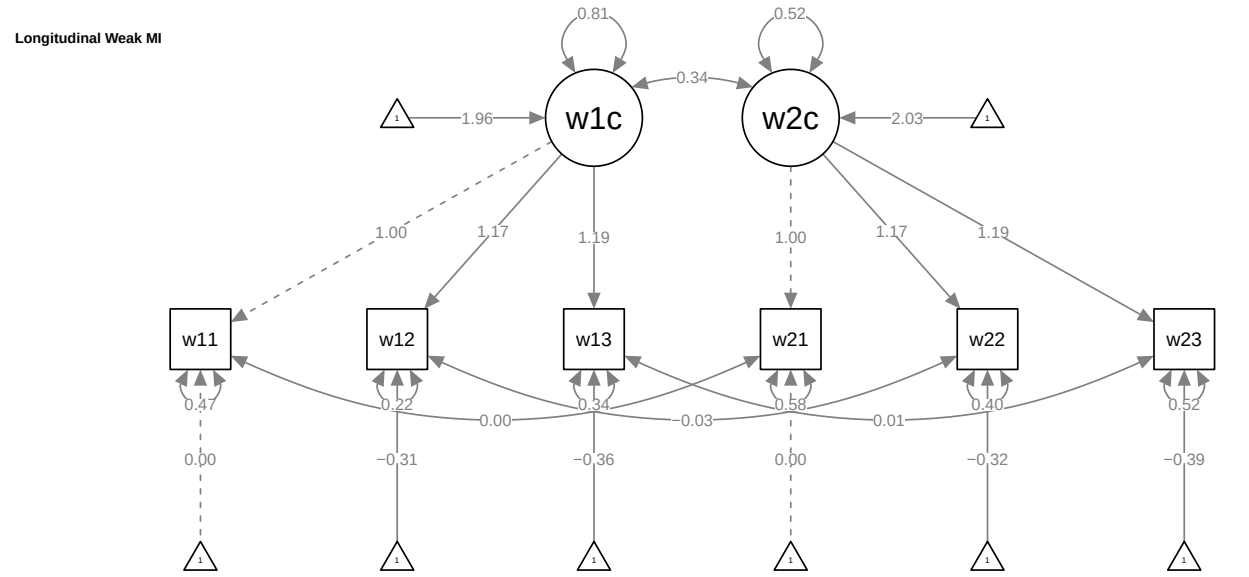
The configural measurement invariance model fits the data well and all loadings are significant. The other fit indices (e.g., *RMSEA* and *CFI*) also show indicate good fit. The estimated means of the latent variables are 1.960 for `w1comp` and 2.031 for `w2comp`. Note that we cannot yet compare these means.

b. Investigate *weak* or *metric* measurement invariance. Test if the additional constraints of the weak invariance model do not significantly worsen model fit in comparison to the configural model. Report and interpret the results.

For the metric measurement invariance, we need to constrain the loadings to be equal across time points. To do this, we can use labels. Note that the marker items do not need equality constraint, as those loadings are already set to 1 for identification purposes. Just as for point *(a)*, we leave the mean structure unconstrained.

```r
# Model syntax.
model_ex_2_weak <- "
    # Measurement part.
    w1comp =~ w1vst1 + a * w1vst2 + b * w1vst3
    w2comp =~ w2vst1 + a * w2vst2 + b * w2vst3

    # Covariance between latent variables.
    w2comp ~~ w1comp

    # Covariances between residuals across time.
    w1vst1 ~~ w2vst1
    w1vst2 ~~ w2vst2
    w1vst3 ~~ w2vst3

    # Fix first intercept at each time point to 0 for identification.
    w1vst1 ~ 0
    w2vst1 ~ 0

    # Freely estimate means of latent variables.
    w1comp ~ 1
    w2comp ~ 1
"

# Fit the model.
model_ex_2_weak_fit <- sem(model_ex_2_weak, data_ex_2)
```

```
# Visualize the model.
semPaths(model_ex_2_weak_fit, what = "paths", whatLabels = "est")

# Add a title to the plot.
title("Longitudinal Weak MI", adj = 0)
```




```
# Model summary.
summary(model_ex_2_weak_fit, standardized = TRUE)
```

```
## lavaan 0.6-12 ended normally after 38 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        22
##   Number of equality constraints                     2
##
##   Number of observations                           574
##
## Model Test User Model:
##
##   Test statistic                                12.077
##   Degrees of freedom                                 7
##   P-value (Chi-square)                           0.098
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   w1comp =~
##     w1vst1            1.000                               0.902    0.796
```

```
##     w1vst2     (a)    1.166    0.041   28.422    0.000    1.051    0.914
##     w1vst3     (b)    1.188    0.043   27.672    0.000    1.071    0.879
##   w2comp =~
##     w2vst1            1.000                               0.721    0.689
##     w2vst2     (a)    1.166    0.041   28.422    0.000    0.840    0.799
##     w2vst3     (b)    1.188    0.043   27.672    0.000    0.856    0.764
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   w1comp ~~
##     w2comp            0.335    0.039    8.613    0.000    0.516    0.516
##  .w1vst1 ~~
##    .w2vst1            0.001    0.026    0.025    0.980    0.001    0.001
##  .w1vst2 ~~
##    .w2vst2           -0.032    0.021   -1.513    0.130   -0.032   -0.110
##  .w1vst3 ~~
##    .w2vst3            0.014    0.025    0.533    0.594    0.014    0.032
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .w1vst1            0.000                               0.000    0.000
##    .w2vst1            0.000                               0.000    0.000
##     w1comp            1.960    0.047   41.425    0.000    2.173    2.173
##     w2comp            2.031    0.044   46.513    0.000    2.819    2.819
##    .w1vst2           -0.305    0.089   -3.424    0.001   -0.305   -0.266
##    .w1vst3           -0.359    0.094   -3.823    0.000   -0.359   -0.295
##    .w2vst2           -0.319    0.095   -3.363    0.001   -0.319   -0.303
##    .w2vst3           -0.393    0.100   -3.948    0.000   -0.393   -0.351
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .w1vst1            0.471    0.034   13.997    0.000    0.471    0.367
##    .w1vst2            0.218    0.028    7.775    0.000    0.218    0.165
##    .w1vst3            0.338    0.033   10.343    0.000    0.338    0.227
##    .w2vst1            0.576    0.042   13.777    0.000    0.576    0.526
##    .w2vst2            0.400    0.038   10.505    0.000    0.400    0.362
##    .w2vst3            0.522    0.044   11.869    0.000    0.522    0.416
##     w1comp            0.814    0.068   11.951    0.000    1.000    1.000
##     w2comp            0.519    0.046   11.219    0.000    1.000    1.000
```

```
# Fit measures.
fitMeasures(model_ex_2_weak_fit, fit.measures = fit_indices)
```

```
##        chisq           df       pvalue          cfi          tli        rmsea rmsea.pvalue
##       12.077        7.000        0.098        0.997        0.994        0.036        0.728
##         srmr
##        0.025
```

Perform a LRT between the week model and the configural model.

```
# Perform LRT.
anova(model_ex_2_weak_fit, model_ex_2_configural_fit)
```

```
## Chi-Squared Difference Test
##
##                             Df    AIC    BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## model_ex_2_configural_fit  5 8864.6 8960.3  9.9108
## model_ex_2_weak_fit        7 8862.7 8949.8 12.0767      2.166       2     0.3386
```

```r
# Combine the fit measures for the configural and weak model.
fit_measures_all_ex_2 <- rbind(
    configural = fitMeasures(model_ex_2_configural_fit, fit_indices),
    weak = fitMeasures(model_ex_2_weak_fit, fit_indices)
)

# Print fit measures with four decimals.
print(round(fit_measures_all_ex_2, 4))
```

```
##              chisq df pvalue    cfi    tli  rmsea rmsea.pvalue   srmr
## configural  9.9108  5 0.0778 0.9971 0.9914 0.0414       0.5909 0.0181
## weak       12.0767  7 0.0981 0.9970 0.9937 0.0355       0.7284 0.0251
```

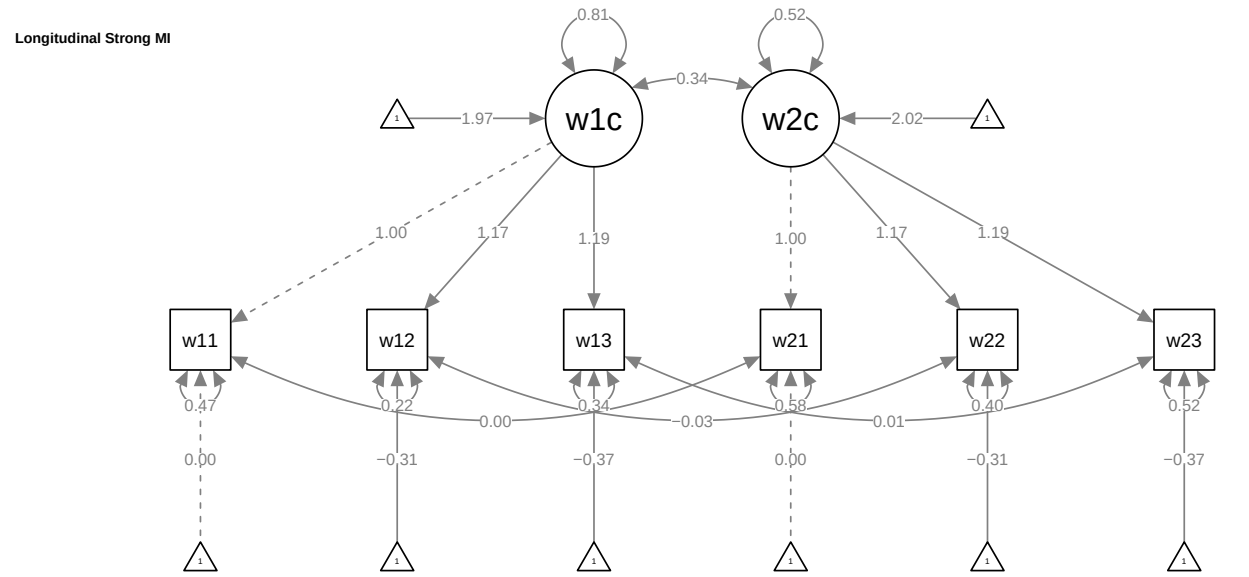
The results indicate good model fit with a $\chi^2 = 12.076$ with 7 degrees of freedom and a p-value $= 0.098$. The change in *CFI* between the weak model and the configural model is 0, and *RMSEA* has dropped from 0.0414 to .0355, in accordance with rules of thumb. The χ^2 difference test between the two models (i.e., weak and configural) is also non-significant with p-value $= 0.338$, indicating that the constrained model does not significantly worsen the fit. Based on these results, we can conclude that we have support for weak longitudinal measurement invariance and continue testing for the next level of measurement invariance.

c. Investigate *strong* or *scalar* measurement invariance, now with the necessary equality constraints on the intercepts. Test if the additional constraints of the strong invariance model do not significantly worsen model fit in comparison to the weak invariance model. Report and interpret the results.

For the scalar measurement invariance we need the constrain the intercepts to be equal across time. Since we are interested in the means of the latent variables, we must fix the intercept for the first indicator at each time point to 0 for identification purposes. Therefore, we can only constrain the remaining intercepts to be equal across time.

```r
# Model syntax.
model_ex_2_strong <- "
    # Measurement part.
    w1comp =~ w1vst1 + a * w1vst2 + b * w1vst3
    w2comp =~ w2vst1 + a * w2vst2 + b * w2vst3

    # Covariance between latent variables.
    w2comp ~~ w1comp

    # Covariances between residuals across time.
    w1vst1 ~~ w2vst1
    w1vst2 ~~ w2vst2
    w1vst3 ~~ w2vst3

    # Fix first intercept at each time point to 0 for identification.
    w1vst1 ~ 0
```

```
    w2vst1 ~ 0

    # Constrain the remaining intercepts to be equal across time.
    w1vst2 ~ c * 1
    w1vst3 ~ d * 1
    w2vst2 ~ c * 1
    w2vst3 ~ d * 1

    # Freely estimate means of latent variables.
    w1comp ~ 1
    w2comp ~ 1
"

# Fit the model.
model_ex_2_strong_fit <- sem(model_ex_2_strong, data_ex_2)

# Visualize the model.
semPaths(model_ex_2_strong_fit, what = "paths", whatLabels = "est")

# Add a title to the plot.
title("Longitudinal Strong MI", adj = 0)
```




```
# Model summary.
summary(model_ex_2_strong_fit, standardized = TRUE)
```

```
## lavaan 0.6-12 ended normally after 42 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        22
##   Number of equality constraints                     4
##
##   Number of observations                           574
##
```

```
## Model Test User Model:
##
##   Test statistic                                12.399
##   Degrees of freedom                                9
##   P-value (Chi-square)                          0.192
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   w1comp =~
##     w1vst1            1.000                               0.902    0.796
##     w1vst2     (a)    1.165    0.041   28.448    0.000    1.052    0.914
##     w1vst3     (b)    1.187    0.043   27.693    0.000    1.071    0.879
##   w2comp =~
##     w2vst1            1.000                               0.721    0.689
##     w2vst2     (a)    1.165    0.041   28.448    0.000    0.840    0.799
##     w2vst3     (b)    1.187    0.043   27.693    0.000    0.855    0.764
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   w1comp ~~
##     w2comp            0.336    0.039    8.615    0.000    0.516    0.516
##  .w1vst1 ~~
##    .w2vst1            0.001    0.026    0.023    0.982    0.001    0.001
##  .w1vst2 ~~
##    .w2vst2           -0.032    0.021   -1.513    0.130   -0.032   -0.110
##  .w1vst3 ~~
##    .w2vst3            0.014    0.025    0.531    0.595    0.014    0.032
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .w1vst1            0.000                               0.000    0.000
##    .w2vst1            0.000                               0.000    0.000
##    .w1vst2     (c)   -0.311    0.087   -3.600    0.000   -0.311   -0.271
##    .w1vst3     (d)   -0.372    0.091   -4.083    0.000   -0.372   -0.305
##    .w2vst2     (c)   -0.311    0.087   -3.600    0.000   -0.311   -0.296
##    .w2vst3     (d)   -0.372    0.091   -4.083    0.000   -0.372   -0.332
##     w1comp            1.967    0.044   44.577    0.000    2.180    2.180
##     w2comp            2.023    0.038   52.736    0.000    2.806    2.806
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .w1vst1            0.471    0.034   13.994    0.000    0.471    0.367
##    .w1vst2            0.218    0.028    7.770    0.000    0.218    0.165
##    .w1vst3            0.338    0.033   10.356    0.000    0.338    0.228
##    .w2vst1            0.576    0.042   13.775    0.000    0.576    0.526
```

```
##    .w2vst2           0.400    0.038   10.501    0.000    0.400    0.362
##    .w2vst3           0.523    0.044   11.878    0.000    0.523    0.417
##     w1comp           0.814    0.068   11.956    0.000    1.000    1.000
##     w2comp           0.520    0.046   11.222    0.000    1.000    1.000
```

```
# Fit measures.
fitMeasures(model_ex_2_strong_fit, fit.measures = fit_indices)
```

```
##        chisq           df       pvalue          cfi          tli        rmsea rmsea.pvalue
##       12.399        9.000        0.192        0.998        0.997        0.026        0.886
##         srmr
##        0.025
```

Perform a LRT between the strong model and the weak model.

```
# Perform LRT.
anova(model_ex_2_strong_fit, model_ex_2_weak_fit)
```

```
## Chi-Squared Difference Test
##
##                        Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## model_ex_2_weak_fit     7 8862.7 8949.8 12.077
## model_ex_2_strong_fit   9 8859.0 8937.4 12.399    0.32254       2     0.8511
```

```
# Combine the fit measures for the configural and weak model.
fit_measures_all_ex_2 <- rbind(
    fit_measures_all_ex_2,
    strong = fitMeasures(model_ex_2_strong_fit, fit_indices)
)

# Print fit measures with four decimals.
print(round(fit_measures_all_ex_2, 4))
```

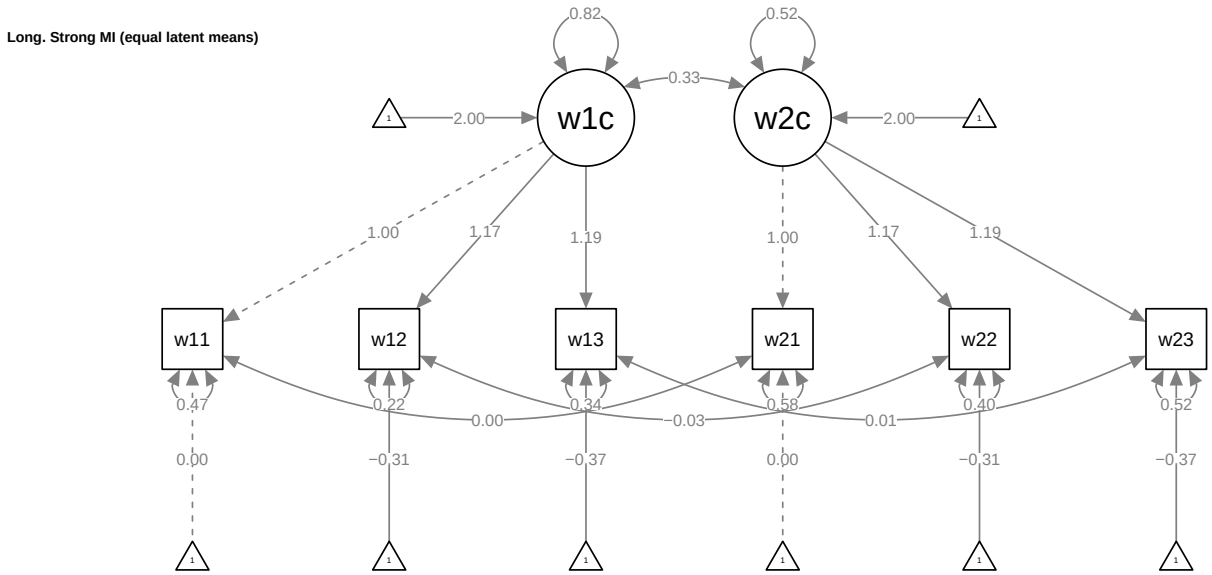
```
##              chisq df pvalue    cfi    tli  rmsea rmsea.pvalue   srmr
## configural  9.9108  5 0.0778 0.9971 0.9914 0.0414       0.5909 0.0181
## weak       12.0767  7 0.0981 0.9970 0.9937 0.0355       0.7284 0.0251
## strong     12.3993  9 0.1917 0.9980 0.9967 0.0257       0.8864 0.0253
```

The results indicate good model fit with a $\chi^2 = 12.399$ with 9 degrees of freedom and a p-value $= 0.192$. The change in *CFI* between the strong model and the weak model is 0.001, and *RMSEA* has dropped from 0.0355 to 0.0257, in accordance with rules of thumb. The χ^2 difference test between the two models (i.e., strong and weak) is also non-significant with p-value $= 0.851$, indicating that the constrained model does not significantly worsen the fit. Based on these results, we can conclude that we have support for strong longitudinal measurement invariance and continue testing for the next level of measurement invariance.

Since we now have support for strong measurement invariance, we can validly compare the means of the latent variables. We can, for example, test the hypothesis that the latent variable means are equal across time. To do this, we re-estimate the strong (i.e., scalar) measurement invariance model, with the additional constrain that the latent variable means should be equal over time.

```
# Model syntax.
model_ex_2_strong_equal_latent_means <- "
    # Measurement part.
```

```
    w1comp =~ w1vst1 + a * w1vst2 + b * w1vst3
    w2comp =~ w2vst1 + a * w2vst2 + b * w2vst3

    # Covariance between latent variables.
    w2comp ~~ w1comp

    # Covariances between residuals across time.
    w1vst1 ~~ w2vst1
    w1vst2 ~~ w2vst2
    w1vst3 ~~ w2vst3

    # Fix first intercept at each time point to 0 for identification.
    w1vst1 ~ 0
    w2vst1 ~ 0

    # Constrain the remaining intercepts to be equal across time.
    w1vst2 ~ c * 1
    w1vst3 ~ d * 1
    w2vst2 ~ c * 1
    w2vst3 ~ d * 1

    # Freely estimate means of latent variables.
    w1comp ~ h * 1
    w2comp ~ h * 1
"

# Fit the model.
model_ex_2_strong_equal_latent_means_fit <- sem(model_ex_2_strong_equal_latent_means, data_ex_2)

# Visualize the model.
semPaths(model_ex_2_strong_equal_latent_means_fit, what = "paths", whatLabels = "est")

# Add a title to the plot.
title("Long. Strong MI (equal latent means)", adj = 0)
```




```r
# Model summary.
summary(model_ex_2_strong_equal_latent_means_fit, standardized = TRUE)
```

```
## lavaan 0.6-12 ended normally after 42 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        22
##   Number of equality constraints                     5
##
##   Number of observations                           574
##
## Model Test User Model:
##
##   Test statistic                                14.439
##   Degrees of freedom                                10
##   P-value (Chi-square)                           0.154
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   w1comp =~
##     w1vst1           1.000                               0.903    0.796
##     w1vst2     (a)   1.165    0.041   28.446    0.000    1.052    0.914
##     w1vst3     (b)   1.187    0.043   27.693    0.000    1.072    0.879
##   w2comp =~
##     w2vst1           1.000                               0.721    0.689
##     w2vst2     (a)   1.165    0.041   28.446    0.000    0.840    0.799
##     w2vst3     (b)   1.187    0.043   27.693    0.000    0.856    0.764
```

```
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   w1comp ~~
##     w2comp            0.335    0.039    8.597    0.000    0.514    0.514
##  .w1vst1 ~~
##    .w2vst1            0.000    0.026    0.018    0.985    0.000    0.001
##  .w1vst2 ~~
##    .w2vst2           -0.033    0.021   -1.517    0.129   -0.033   -0.110
##  .w1vst3 ~~
##    .w2vst3            0.014    0.025    0.539    0.590    0.014    0.033
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .w1vst1            0.000                               0.000    0.000
##    .w2vst1            0.000                               0.000    0.000
##    .w1vst2     (c)   -0.313    0.087   -3.594    0.000   -0.313   -0.272
##    .w1vst3     (d)   -0.373    0.092   -4.077    0.000   -0.373   -0.306
##    .w2vst2     (c)   -0.313    0.087   -3.594    0.000   -0.313   -0.297
##    .w2vst3     (d)   -0.373    0.092   -4.077    0.000   -0.373   -0.333
##     w1comp     (h)    2.004    0.036   55.806    0.000    2.219    2.219
##     w2comp     (h)    2.004    0.036   55.806    0.000    2.779    2.779
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .w1vst1            0.471    0.034   13.995    0.000    0.471    0.366
##    .w1vst2            0.218    0.028    7.772    0.000    0.218    0.165
##    .w1vst3            0.338    0.033   10.345    0.000    0.338    0.227
##    .w2vst1            0.576    0.042   13.774    0.000    0.576    0.525
##    .w2vst2            0.400    0.038   10.502    0.000    0.400    0.362
##    .w2vst3            0.522    0.044   11.868    0.000    0.522    0.416
##     w1comp            0.815    0.068   11.956    0.000    1.000    1.000
##     w2comp            0.520    0.046   11.223    0.000    1.000    1.000
```

```r
# Fit measures.
fitMeasures(model_ex_2_strong_equal_latent_means_fit, fit.measures = fit_indices)
```

```
##        chisq           df       pvalue          cfi          tli        rmsea rmsea.pvalue
##       14.439       10.000        0.154        0.997        0.996        0.028        0.882
##         srmr
##        0.029
```

```r
# Combine the fit measures for the configural and weak model.
fit_measures_all_ex_2 <- rbind(
    fit_measures_all_ex_2,
    strong_equal_latent_means = fitMeasures(model_ex_2_strong_equal_latent_means_fit, fit_indices)
)

# Print fit measures with four decimals.
print(round(fit_measures_all_ex_2, 4))
```

```
##                          chisq df pvalue    cfi    tli  rmsea rmsea.pvalue   srmr
## configural              9.9108  5 0.0778 0.9971 0.9914 0.0414       0.5909 0.0181
```

```
## weak                        12.0767  7 0.0981 0.9970 0.9937 0.0355       0.7284 0.0251
## strong                      12.3993  9 0.1917 0.9980 0.9967 0.0257       0.8864 0.0253
## strong_equal_latent_means 14.4389 10 0.1539 0.9974 0.9961 0.0278       0.8816 0.0287
```

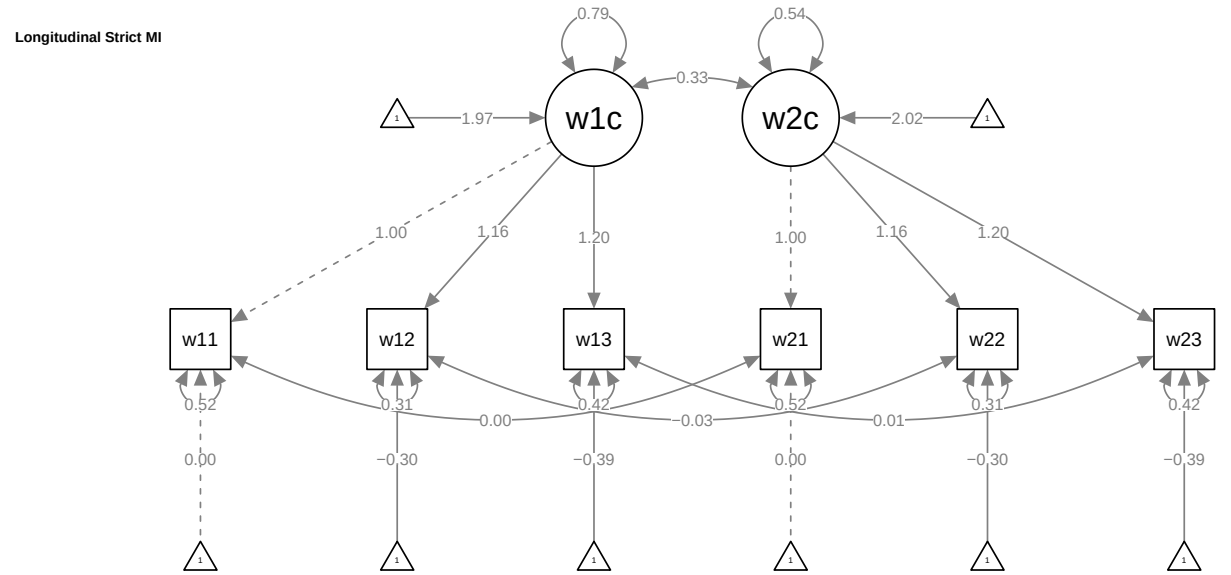
The estimate for the mean of the latent variable at both time points is 2.004. We see that our additional constraint on the latent means does not significantly worsen the model fit. Hence, we can say that we have support for the hypothesis of no change in the latent means across time.

d. Investigate *strict* measurement invariance. Test if the additional constraints of the strict invariance model do not significantly worsen model fit in comparison to the strong invariance model. Report and interpret the results.

For the strict level of measurement invariance, we must do everything we did for the strong model and, on top of that, also constrain the the measurement residuals to be equal across time.

```
# Model syntax.
model_ex_2_strict <- "
    # Measurement part.
    w1comp =~ w1vst1 + a * w1vst2 + b * w1vst3
    w2comp =~ w2vst1 + a * w2vst2 + b * w2vst3

    # Covariance between latent variables.
    w2comp ~~ w1comp

    # Covariances between residuals across time.
    w1vst1 ~~ w2vst1
    w1vst2 ~~ w2vst2
    w1vst3 ~~ w2vst3

    # Fix first intercept at each time point to 0 for identification.
    w1vst1 ~ 0
    w2vst1 ~ 0

    # Constrain the remaining intercepts to be equal across time.
    w1vst2 ~ c * 1
    w1vst3 ~ d * 1
    w2vst2 ~ c * 1
    w2vst3 ~ d * 1

    # Freely estimate means of latent variables.
    w1comp ~ 1
    w2comp ~ 1

    # Set measurement residuals to be equal across time.
    w1vst1 ~~ e * w1vst1
    w1vst2 ~~ f * w1vst2
    w1vst3 ~~ g * w1vst3
    w2vst1 ~~ e * w2vst1
    w2vst2 ~~ f * w2vst2
    w2vst3 ~~ g * w2vst3
"
```

```r
# Fit the model.
model_ex_2_strict_fit <- sem(model_ex_2_strict, data_ex_2)

# Visualize the model.
semPaths(model_ex_2_strict_fit, what = "paths", whatLabels = "est")

# Add a title to the plot.
title("Longitudinal Strict MI", adj = 0)
```




```r
# Model summary.
summary(model_ex_2_strict_fit, standardized = TRUE)
```

```
## lavaan 0.6-12 ended normally after 40 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        22
##   Number of equality constraints                     7
##
##   Number of observations                           574
##
## Model Test User Model:
##
##   Test statistic                                60.843
##   Degrees of freedom                                12
##   P-value (Chi-square)                           0.000
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
```

```
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   w1comp =~
##     w1vst1            1.000                              0.887    0.775
##     w1vst2     (a)    1.162    0.043   27.134    0.000    1.031    0.879
##     w1vst3     (b)    1.197    0.045   26.600    0.000    1.062    0.852
##   w2comp =~
##     w2vst1            1.000                              0.736    0.713
##     w2vst2     (a)    1.162    0.043   27.134    0.000    0.855    0.837
##     w2vst3     (b)    1.197    0.045   26.600    0.000    0.881    0.804
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   w1comp ~~
##     w2comp            0.334    0.039    8.552    0.000    0.512    0.512
##  .w1vst1 ~~
##    .w2vst1            0.001    0.026    0.029    0.977    0.001    0.001
##  .w1vst2 ~~
##    .w2vst2           -0.033    0.022   -1.488    0.137   -0.033   -0.105
##  .w1vst3 ~~
##    .w2vst3            0.014    0.026    0.537    0.591    0.014    0.033
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .w1vst1            0.000                              0.000    0.000
##    .w2vst1            0.000                              0.000    0.000
##    .w1vst2     (c)   -0.305    0.090   -3.375    0.001   -0.305   -0.260
##    .w1vst3     (d)   -0.395    0.095   -4.136    0.000   -0.395   -0.317
##    .w2vst2     (c)   -0.305    0.090   -3.375    0.001   -0.305   -0.298
##    .w2vst3     (d)   -0.395    0.095   -4.136    0.000   -0.395   -0.360
##     w1comp            1.968    0.044   44.875    0.000    2.218    2.218
##     w2comp            2.023    0.039   52.339    0.000    2.750    2.750
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .w1vst1     (e)    0.524    0.027   19.086    0.000    0.524    0.400
##    .w1vst2     (f)    0.314    0.025   12.311    0.000    0.314    0.228
##    .w1vst3     (g)    0.424    0.029   14.549    0.000    0.424    0.273
##    .w2vst1     (e)    0.524    0.027   19.086    0.000    0.524    0.492
##    .w2vst2     (f)    0.314    0.025   12.311    0.000    0.314    0.300
##    .w2vst3     (g)    0.424    0.029   14.549    0.000    0.424    0.354
##     w1comp            0.787    0.067   11.737    0.000    1.000    1.000
##     w2comp            0.542    0.048   11.367    0.000    1.000    1.000
```

```r
# Fit measures.
fitMeasures(model_ex_2_strict_fit, fit.measures = fit_indices)
```

```
##        chisq           df       pvalue          cfi          tli        rmsea rmsea.pvalue
##       60.843       12.000        0.000        0.972        0.965        0.084        0.003
##         srmr
##        0.034
```

Perform a LRT between the strict model and the strong model.

```r
# Perform LRT.
anova(model_ex_2_strict_fit, model_ex_2_strong_fit)
```

```
## Chi-Squared Difference Test
##
##                        Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## model_ex_2_strong_fit   9 8859.0 8937.4 12.399
## model_ex_2_strict_fit 12 8901.5 8966.8 60.843     48.444       3  1.713e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# Combine the fit measures for the configural and weak model.
fit_measures_all_ex_2 <- rbind(
    fit_measures_all_ex_2,
    strict = fitMeasures(model_ex_2_strict_fit, fit_indices)
)

# Print fit measures with four decimals.
print(round(fit_measures_all_ex_2, 4))
```

```
##                              chisq df pvalue    cfi    tli  rmsea rmsea.pvalue   srmr
## configural                  9.9108  5 0.0778 0.9971 0.9914 0.0414       0.5909 0.0181
## weak                       12.0767  7 0.0981 0.9970 0.9937 0.0355       0.7284 0.0251
## strong                     12.3993  9 0.1917 0.9980 0.9967 0.0257       0.8864 0.0253
## strong_equal_latent_means 14.4389 10 0.1539 0.9974 0.9961 0.0278       0.8816 0.0287
## strict                     60.8430 12 0.0000 0.9716 0.9645 0.0842       0.0035 0.0335
```

These results indicate that strict measurement invariance is not supported (i.e., the χ^2 is significant and the *CFI* and *RMSEA* worsen in comparison with the strong measurement invariance model).

e. Consider this statement: "The factor mean invariance test is really a test of the equality of the observed means for the marker variables at each time point." Is this statement true or false. Explain your answer.

Because factor means are a function of measurement intercepts, tests of factor means are not independent of intercept tests. With the marker variable approach (i.e., where we give the latent variable basically the scale of the marker variable), the mean of the factor is equal to the expected value of the indicator variable.

Part 2. A naïve approach?

We have seen that the `R` package `lavaan` has full support for multiple group analysis (i.e., as demonstrated in the `lavaan` tutorial and seen in *Exercise 1*). Consider how the data in the dataset `socex1.dat` are currently structured. Would it be possible to re-estimate the models investigated during *Exercise 2* with the multiple group analysis module? Justify your answer.

The data are in wide format. However, even if we transfer the data to long format and add a grouping variable to indicate the measurement occasions, we cannot use the automated procedure for multiple group analysis in `lavaan`. The reason for this is because we have longitudinal data and it makes sense to allow for the error terms to covary across time. Since the automated procedure in `lavaan` fits the model to each group separately (i.e., time point in our case), there is no way to estimate a covariance between the error terms of two separate models.

References

Beaujean, A. A. (2014). *Latent variable modeling using R: A step by step guide.* Routledge/Taylor & Francis Group.

Newsom, J. T. (2015). *Longitudinal structural equation modeling: A comprehensive introduction.* Routledge, Taylor and Francis Group.