Structural Equation Modeling

P.04 - Estimation Methods in SEM

November 01, 2022

Lab Description

For this practical you will need the following packages:

- `lavaan`
- `semPlot`
- `psych`
- `ggplot2`

You can install and load these packages using the following code:

```r
# Install packages.
install.packages(c("lavaan", "semPlot", "psych", "ggplot2"))

# Load the packages.
library(lavaan)
library(semPlot)
library(psych)
library(ggplot2)
```

Exercise 1

Upon installing the `R` packages mentioned above perform the following:

    b. Import the dataset `ELEMM1.csv` that is available in the folder for this practical on Canvas.

Set the working directory to the location where your data file has been downloaded and load the data.

```r
# For example.
setwd("/Users/mihai/Downloads")

# Load data.
data_ex_1 <- read.csv("ELEMM1.csv")

# Inspect the data.
View(data_ex_1)
```

Quickly list the variables and their names.

```
# List the variables.
str(data_ex_1)
```

```
## 'data.frame':    372 obs. of  22 variables:
##  $ ITEM1 : int  4 2 6 7 6 2 6 4 6 4 ...
##  $ ITEM2 : int  4 2 6 7 6 2 6 6 5 6 ...
##  $ ITEM3 : int  5 1 7 7 6 2 6 3 5 2 ...
##  $ ITEM4 : int  4 7 7 7 6 7 7 7 7 4 ...
##  $ ITEM5 : int  4 2 3 1 4 6 2 2 4 2 ...
##  $ ITEM6 : int  2 1 5 1 2 2 7 3 3 2 ...
##  $ ITEM7 : int  7 7 6 7 6 7 7 7 7 6 ...
##  $ ITEM8 : int  2 2 6 7 2 1 7 3 4 1 ...
##  $ ITEM9 : int  7 7 5 7 7 7 4 7 6 7 ...
##  $ ITEM10: int  2 1 4 1 6 1 1 3 4 2 ...
##  $ ITEM11: int  2 1 5 1 6 1 1 3 4 2 ...
##  $ ITEM12: int  6 4 4 1 7 7 1 6 6 6 ...
##  $ ITEM13: int  3 2 4 1 4 3 6 3 3 3 ...
##  $ ITEM14: int  4 2 4 4 7 2 6 3 6 5 ...
##  $ ITEM15: int  1 1 2 1 5 1 1 3 4 1 ...
##  $ ITEM16: int  1 1 5 1 2 2 7 3 4 2 ...
##  $ ITEM17: int  7 7 5 7 7 7 6 7 6 6 ...
##  $ ITEM18: int  6 4 6 5 7 6 2 6 6 6 ...
##  $ ITEM19: int  6 6 6 7 7 5 4 6 7 6 ...
##  $ ITEM20: int  2 1 2 4 3 2 2 2 3 2 ...
##  $ ITEM21: int  6 6 4 7 5 6 6 6 6 2 ...
##  $ ITEM22: int  2 1 3 1 2 2 1 6 3 2 ...
```

c. Inspect the *skewness* and *kurtosis* of ITEM1 to ITEM22 using the psych package. Do you see indications of severe deviations from normality?

Check out the documentation for the package psych on how to compute descriptive measures for your variables by running ??psych. We can use the function psych::describe.

```
# Describe the data using `psych`.
describe(data_ex_1)
```

##		vars	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis	se
## ITEM1		1	372	4.37	1.66	4.0	4.36	2.97	1	7	6	-0.11	-1.17	0.09
## ITEM2		2	372	4.87	1.55	5.0	4.97	1.48	1	7	6	-0.50	-0.71	0.08
## ITEM3		3	372	3.53	1.73	3.0	3.49	1.48	1	7	6	0.32	-1.11	0.09
## ITEM4		4	372	6.30	1.00	7.0	6.50	0.00	2	7	5	-1.80	3.63	0.05
## ITEM5		5	372	2.20	1.49	2.0	1.92	1.48	1	7	6	1.32	0.91	0.08
## ITEM6		6	372	2.71	1.58	2.0	2.50	1.48	1	7	6	0.92	-0.01	0.08
## ITEM7		7	372	6.31	0.84	6.0	6.46	1.48	2	7	5	-1.64	3.77	0.04
## ITEM8		8	372	3.04	1.73	2.0	2.89	1.48	1	7	6	0.74	-0.61	0.09
## ITEM9		9	372	6.03	1.32	7.0	6.29	0.00	1	7	6	-1.54	1.84	0.07
## ITEM10		10	372	2.20	1.45	2.0	1.96	1.48	1	7	6	1.20	0.56	0.08
## ITEM11		11	372	2.24	1.53	2.0	1.97	1.48	1	7	6	1.27	0.80	0.08
## ITEM12		12	372	5.70	1.19	6.0	5.86	1.48	1	7	6	-1.31	1.84	0.06
## ITEM13		13	372	3.59	1.68	3.5	3.52	2.22	1	7	6	0.35	-0.79	0.09
## ITEM14		14	372	4.03	1.73	4.0	4.01	1.48	1	7	6	0.03	-0.94	0.09

```
## ITEM15   15 372 1.77 1.30    1.0    1.47 0.00   1   7    6   2.09      4.24 0.07
## ITEM16   16 372 2.47 1.44    2.0    2.28 1.48   1   7    6   0.97      0.16 0.07
## ITEM17   17 372 6.41 0.85    7.0    6.58 0.00   2   7    5  -1.97      5.06 0.04
## ITEM18   18 372 5.70 1.27    6.0    5.87 1.48   1   7    6  -1.23      1.34 0.07
## ITEM19   19 372 5.95 1.19    6.0    6.15 1.48   1   7    6  -1.48      2.21 0.06
## ITEM20   20 372 2.24 1.41    2.0    2.01 1.48   1   7    6   1.29      1.17 0.07
## ITEM21   21 372 5.85 1.27    6.0    6.06 1.48   2   7    5  -1.29      1.16 0.07
## ITEM22   22 372 2.58 1.58    2.0    2.35 1.48   1   7    6   1.06      0.18 0.08
```

There seems to be some indication of non-normality, but not too severe. This may warrant using a robust estimator.

d. Estimate the model in Figure 1 using the default Maximum Likelihood method.

```r
# Model syntax.
model_ex_1 <- "
    EMO =~ ITEM1 + ITEM2  + ITEM3  + ITEM6  + ITEM8  + ITEM13 + ITEM14 + ITEM16 + ITEM20
    DEP =~ ITEM5 + ITEM10 + ITEM11 + ITEM15 + ITEM22
    ACC =~ ITEM4 + ITEM7  + ITEM9  + ITEM12 + ITEM17 + ITEM18 + ITEM19 + ITEM21

    # Covariances between latent variables
    EMO ~~ DEP
    DEP ~~ ACC
    EMO ~~ ACC
"

# Estimate the model.
model_ex_1_fit_ml <- cfa(model_ex_1, data = data_ex_1, estimator = "ML")

# Visualize the model.
semPaths(model_ex_1_fit_ml, what = "paths", sizeMan = 3)
```
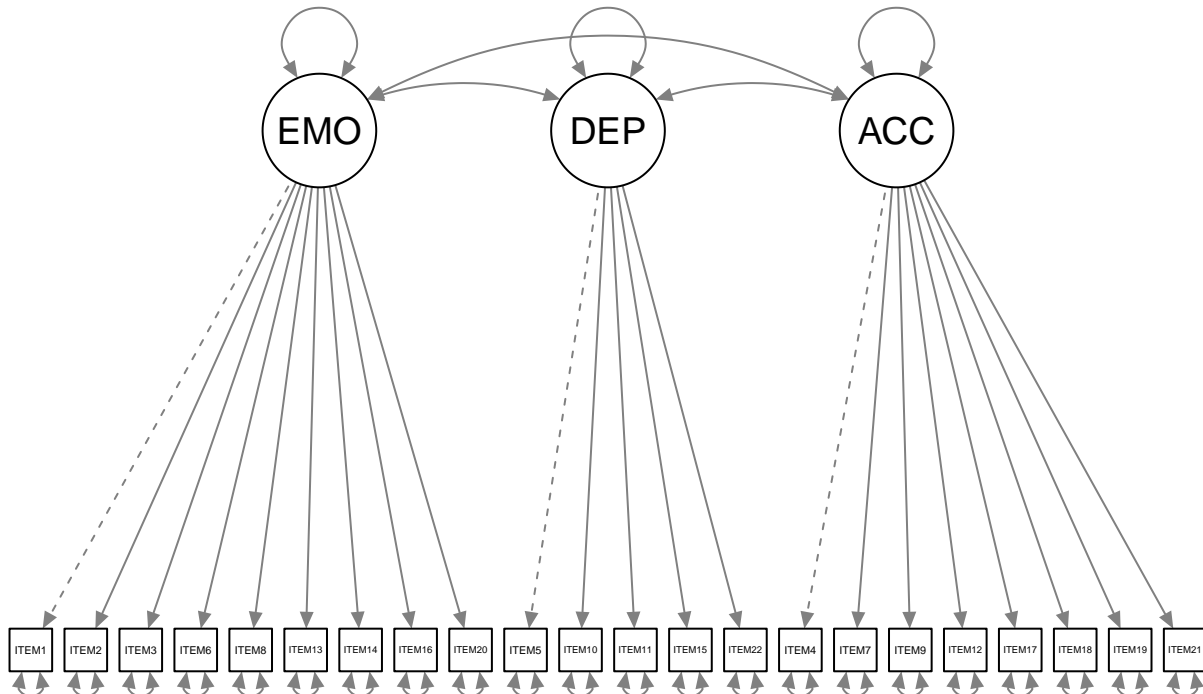
```
# Model summary for the `ML` estimator.
summary(model_ex_1_fit_ml)
```

```
## lavaan 0.6-12 ended normally after 46 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        47
##
##   Number of observations                           372
##
## Model Test User Model:
##
##   Test statistic                               695.719
##   Degrees of freedom                               206
##   P-value (Chi-square)                           0.000
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   EMO =~
##     ITEM1             1.000
##     ITEM2             0.887    0.061   14.621    0.000
##     ITEM3             1.021    0.068   15.085    0.000
##     ITEM6             0.764    0.064   12.013    0.000
##     ITEM8             1.143    0.066   17.299    0.000
##     ITEM13            1.017    0.065   15.544    0.000
##     ITEM14            0.848    0.069   12.251    0.000
##     ITEM16            0.715    0.058   12.410    0.000
##     ITEM20            0.753    0.056   13.410    0.000
##   DEP =~
##     ITEM5             1.000
##     ITEM10            1.142    0.127    8.986    0.000
##     ITEM11            1.353    0.142    9.511    0.000
##     ITEM15            0.905    0.109    8.318    0.000
##     ITEM22            0.768    0.121    6.361    0.000
##   ACC =~
##     ITEM4             1.000
##     ITEM7             0.970    0.150    6.482    0.000
##     ITEM9             1.780    0.254    7.007    0.000
##     ITEM12            1.499    0.221    6.769    0.000
##     ITEM17            1.348    0.181    7.463    0.000
##     ITEM18            1.918    0.262    7.329    0.000
##     ITEM19            1.716    0.238    7.205    0.000
##     ITEM21            1.356    0.218    6.219    0.000
##
```

```
## Covariances:
##                  Estimate  Std.Err  z-value  P(>|z|)
##   EMO ~~
##     DEP             0.701    0.099    7.061    0.000
##   DEP ~~
##     ACC            -0.172    0.035   -4.850    0.000
##   EMO ~~
##     ACC            -0.192    0.042   -4.537    0.000
##
## Variances:
##                  Estimate  Std.Err  z-value  P(>|z|)
##    .ITEM1           1.128    0.095   11.861    0.000
##    .ITEM2           1.105    0.090   12.214    0.000
##    .ITEM3           1.301    0.108   12.031    0.000
##    .ITEM6           1.553    0.121   12.888    0.000
##    .ITEM8           0.852    0.081   10.553    0.000
##    .ITEM13          1.142    0.097   11.821    0.000
##    .ITEM14          1.804    0.140   12.844    0.000
##    .ITEM16          1.235    0.096   12.812    0.000
##    .ITEM20          1.075    0.085   12.585    0.000
##    .ITEM5           1.503    0.125   12.026    0.000
##    .ITEM10          1.169    0.107   10.901    0.000
##    .ITEM11          1.044    0.112    9.330    0.000
##    .ITEM15          1.106    0.093   11.838    0.000
##    .ITEM22          2.076    0.160   12.958    0.000
##    .ITEM4           0.802    0.062   12.901    0.000
##    .ITEM7           0.523    0.042   12.572    0.000
##    .ITEM9           1.117    0.093   11.952    0.000
##    .ITEM12          0.987    0.080   12.287    0.000
##    .ITEM17          0.375    0.035   10.739    0.000
##    .ITEM18          0.909    0.081   11.224    0.000
##    .ITEM19          0.844    0.073   11.557    0.000
##    .ITEM21          1.245    0.098   12.764    0.000
##     EMO             1.625    0.190    8.551    0.000
##     DEP             0.705    0.132    5.321    0.000
##     ACC             0.193    0.048    4.047    0.000
```

e. Re-estimate the model, but now use the Satorra-Bentler estimator to estimate the *MFTS*. How does the scaling factors relate to the unscaled χ^2 value?

Now we are going to re-estimate the model using Maximum Likelihood with robust standard errors (SE) and a Satorra-Bentler scaled test statistic (i.e., χ^2). The estimator we are interested in is called MLM in lavaan.

```
# Re-estimate the model.
model_ex_1_fit_mlm <- cfa(model_ex_1, data = data_ex_1, estimator = "MLM")

# Model summary.
summary(model_ex_1_fit_mlm)
```

```
## lavaan 0.6-12 ended normally after 46 iterations
##
##   Estimator                                         ML
```

```
##    Optimization method                     NLMINB
##    Number of model parameters                   47
##
##    Number of observations                      372
##
## Model Test User Model:
##                                        Standard      Robust
##    Test Statistic                       695.719     567.753
##    Degrees of freedom                       206         206
##    P-value (Chi-square)                   0.000       0.000
##    Scaling correction factor                          1.225
##       Satorra-Bentler correction
##
## Parameter Estimates:
##
##    Standard errors                        Robust.sem
##    Information                              Expected
##    Information saturated (h1) model       Structured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)
##    EMO =~
##      ITEM1            1.000
##      ITEM2            0.887    0.040   22.391    0.000
##      ITEM3            1.021    0.053   19.310    0.000
##      ITEM6            0.764    0.070   10.974    0.000
##      ITEM8            1.143    0.059   19.366    0.000
##      ITEM13           1.017    0.062   16.340    0.000
##      ITEM14           0.848    0.058   14.584    0.000
##      ITEM16           0.715    0.066   10.826    0.000
##      ITEM20           0.753    0.061   12.303    0.000
##    DEP =~
##      ITEM5            1.000
##      ITEM10           1.142    0.152    7.509    0.000
##      ITEM11           1.353    0.162    8.368    0.000
##      ITEM15           0.905    0.123    7.366    0.000
##      ITEM22           0.768    0.122    6.284    0.000
##    ACC =~
##      ITEM4            1.000
##      ITEM7            0.970    0.128    7.563    0.000
##      ITEM9            1.780    0.322    5.529    0.000
##      ITEM12           1.499    0.241    6.232    0.000
##      ITEM17           1.348    0.200    6.757    0.000
##      ITEM18           1.918    0.298    6.435    0.000
##      ITEM19           1.716    0.287    5.978    0.000
##      ITEM21           1.356    0.227    5.984    0.000
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)
##    EMO ~~
##      DEP              0.701    0.106    6.608    0.000
```

```
##    DEP ~~
##       ACC                -0.172    0.036   -4.777    0.000
##    EMO ~~
##       ACC                -0.192    0.040   -4.796    0.000
##
## Variances:
##                        Estimate  Std.Err  z-value  P(>|z|)
##      .ITEM1               1.128    0.093   12.177    0.000
##      .ITEM2               1.105    0.088   12.506    0.000
##      .ITEM3               1.301    0.106   12.317    0.000
##      .ITEM6               1.553    0.134   11.550    0.000
##      .ITEM8               0.852    0.082   10.450    0.000
##      .ITEM13              1.142    0.124    9.173    0.000
##      .ITEM14              1.804    0.142   12.730    0.000
##      .ITEM16              1.235    0.110   11.278    0.000
##      .ITEM20              1.075    0.137    7.860    0.000
##      .ITEM5               1.503    0.179    8.381    0.000
##      .ITEM10              1.169    0.147    7.959    0.000
##      .ITEM11              1.044    0.141    7.398    0.000
##      .ITEM15              1.106    0.153    7.220    0.000
##      .ITEM22              2.076    0.184   11.266    0.000
##      .ITEM4               0.802    0.113    7.124    0.000
##      .ITEM7               0.523    0.075    7.010    0.000
##      .ITEM9               1.117    0.149    7.487    0.000
##      .ITEM12              0.987    0.126    7.852    0.000
##      .ITEM17              0.375    0.056    6.635    0.000
##      .ITEM18              0.909    0.143    6.376    0.000
##      .ITEM19              0.844    0.111    7.622    0.000
##      .ITEM21              1.245    0.133    9.338    0.000
##       EMO                 1.625    0.148   11.004    0.000
##       DEP                 0.705    0.158    4.452    0.000
##       ACC                 0.193    0.050    3.839    0.000
```

The Satorra-Bentler method adjusts the χ^2 downwards because due to non-normality it is otherwise overestimated. In other words, it takes into account kurtosis.

To obtain roughly the same $MFTS$ (i.e., χ^2) we observed when we used `estimator = "ML"` we need Satorra-Bentler MFTS $\times$ scaling correction factor

    f. Evaluate the fit of the model estimated in (e).

We observe a $\chi^2 = 567.753$ with $DF = 206$ and a p-value < 0.001. Hypothesis that model exactly reproduces data must be rejected.
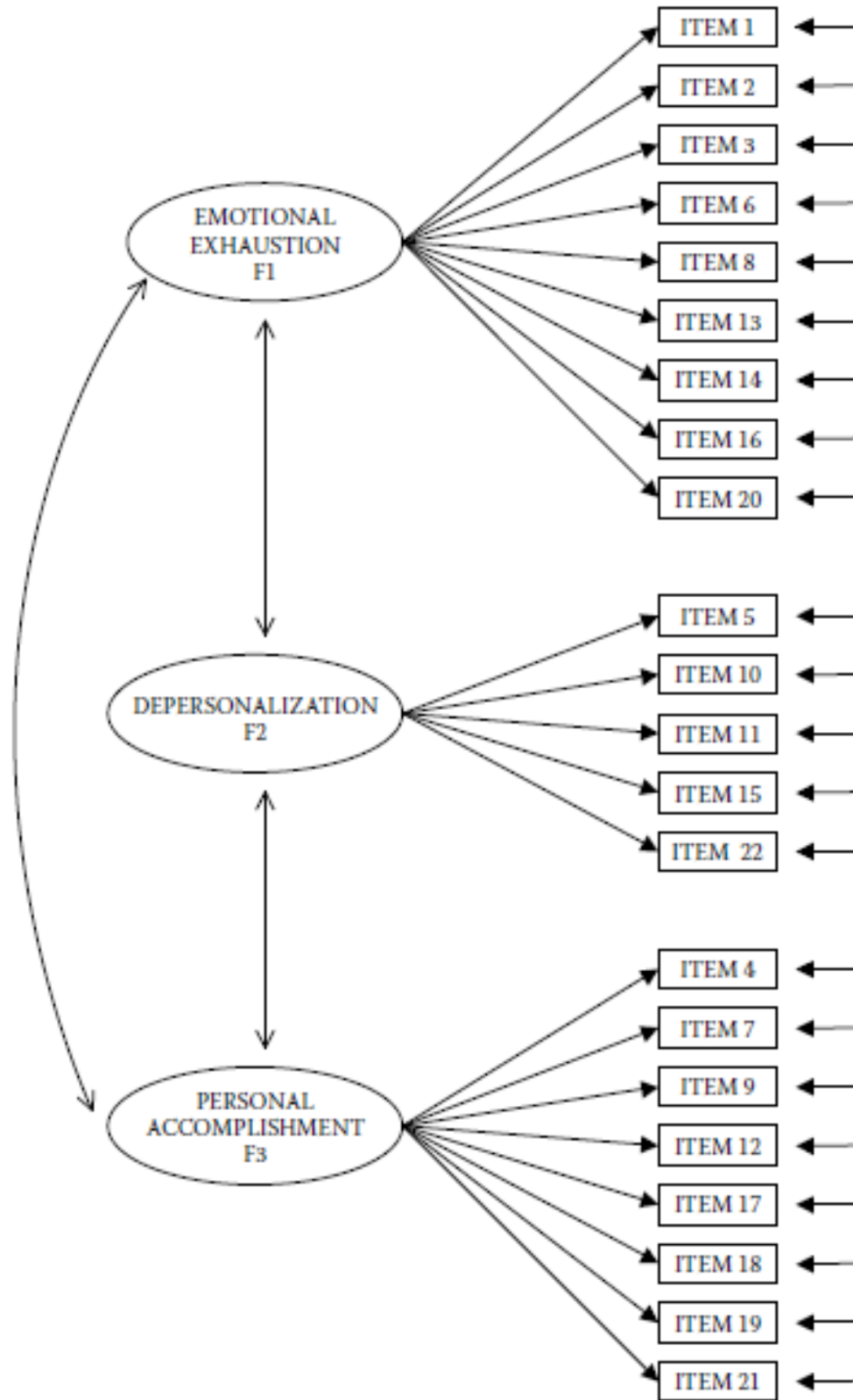
Figure 1: Hypothesized CFA model of factorial structure for the *Maslach Burnout Inventory* (MBI).

Exercise 2

a. Import the dataset `bdihk2c2.csv` that is available in the folder for this practical on Canvas.

Set the working directory to the location where your data file has been downloaded and load the data.

```
# For example.
setwd("/Users/mihai/Downloads")

# Load data.
data_ex_2 <- read.csv("bdihk2c2.csv")

# Inspect the data.
View(data_ex_2)
```

Quickly list the variables and their names.

```
# List the variables.
str(data_ex_2)
```

```
## 'data.frame':    486 obs. of  23 variables:
##  $ linkvar: int  102 107 139 165 166 171 179 180 189 190 ...
##  $ gender : int  2 2 2 2 2 2 2 2 2 2 ...
##  $ age    : int  14 14 14 14 14 14 14 14 14 14 ...
##  $ BDI2_1 : int  1 1 2 0 1 0 2 2 1 0 ...
##  $ BDI2_2 : int  0 0 0 0 0 0 2 1 0 0 ...
##  $ BDI2_3 : int  2 2 2 0 0 0 3 1 0 2 ...
##  $ BDI2_4 : int  0 0 0 0 0 0 1 1 0 0 ...
##  $ BDI2_5 : int  1 0 0 0 0 0 3 0 0 0 ...
##  $ BDI2_6 : int  0 0 1 0 0 0 2 0 1 0 ...
##  $ BDI2_7 : int  1 0 1 0 0 0 2 2 1 0 ...
##  $ BDI2_8 : int  3 0 1 0 0 0 1 0 1 0 ...
##  $ BDI2_9 : int  0 0 1 0 1 0 2 2 1 0 ...
##  $ BDI2_10: int  1 0 2 0 0 0 1 2 1 3 ...
##  $ BDI2_11: int  1 0 1 0 1 0 1 2 1 0 ...
##  $ BDI2_12: int  3 0 0 0 0 0 2 3 0 0 ...
##  $ BDI2_13: int  1 0 1 0 1 0 3 1 0 0 ...
##  $ BDI2_14: int  2 1 2 0 0 0 2 1 1 0 ...
##  $ BDI2_15: int  1 0 0 0 0 0 3 2 1 0 ...
##  $ BDI2_16: int  2 1 2 1 0 0 2 1 2 2 ...
##  $ BDI2_17: int  1 0 0 0 0 0 2 1 1 0 ...
##  $ BDI2_18: int  0 1 1 0 0 0 1 3 1 0 ...
##  $ BDI2_19: int  0 1 0 0 0 0 2 0 1 2 ...
##  $ BDI2_20: int  1 1 2 0 0 0 1 2 1 0 ...
```

b. Inspect the *skewness* and *kurtosis* of `BDI2_1` to `BDI2_20` using the `psych` package. Do you see indications of severe deviations from normality?

We can again use the `describe` function in the `psych` package.

```
# Describe the data using `psych`.
describe(data_ex_2)
```

```
##         vars   n    mean     sd median trimmed    mad min  max range  skew
## linkvar    1 486 1226.80 629.81 1189.5 1236.54 743.52 101 2275  2174 -0.05
```

```
## gender      2 486    1.51   0.50   2.0    1.52   0.00   1    2    1  -0.05
## age         3 486   15.74   1.18  15.0   15.67   1.48  14   18    4   0.42
## BDI2_1      4 486    0.76   0.87   0.0    0.67   0.00   0    3    3   0.69
## BDI2_2      5 486    0.49   0.70   0.0    0.37   0.00   0    3    3   1.43
## BDI2_3      6 486    0.83   0.88   1.0    0.76   1.48   0    3    3   0.51
## BDI2_4      7 486    0.49   0.70   0.0    0.37   0.00   0    3    3   1.46
## BDI2_5      8 486    0.48   0.76   0.0    0.32   0.00   0    3    3   1.69
## BDI2_6      9 486    0.62   0.96   0.0    0.41   0.00   0    3    3   1.44
## BDI2_7     10 486    0.54   0.81   0.0    0.37   0.00   0    3    3   1.47
## BDI2_8     11 486    0.50   0.77   0.0    0.34   0.00   0    3    3   1.63
## BDI2_9     12 486    0.26   0.54   0.0    0.14   0.00   0    3    3   2.17
## BDI2_10    13 486    0.45   0.88   0.0    0.23   0.00   0    3    3   1.92
## BDI2_11    14 486    0.90   0.82   1.0    0.82   1.48   0    3    3   0.63
## BDI2_12    15 486    0.53   0.72   0.0    0.40   0.00   0    3    3   1.26
## BDI2_13    16 486    0.59   0.72   0.0    0.47   0.00   0    3    3   1.12
## BDI2_14    17 486    0.52   0.77   0.0    0.38   0.00   0    3    3   1.29
## BDI2_15    18 486    0.77   0.80   1.0    0.68   1.48   0    3    3   0.79
## BDI2_16    19 486    0.99   0.77   1.0    0.96   1.48   0    3    3   0.34
## BDI2_17    20 486    0.64   0.75   0.0    0.53   0.00   0    3    3   0.88
## BDI2_18    21 486    0.62   0.82   0.0    0.47   0.00   0    3    3   1.27
## BDI2_19    22 486    0.93   0.83   1.0    0.86   1.48   0    3    3   0.56
## BDI2_20    23 486    0.93   0.71   1.0    0.89   0.00   0    3    3   0.47
##          kurtosis    se
## linkvar     -1.16 28.57
## gender      -2.00  0.02
## age         -0.79  0.05
## BDI2_1      -0.83  0.04
## BDI2_2       1.82  0.03
## BDI2_3      -1.10  0.04
## BDI2_4       2.07  0.03
## BDI2_5       2.48  0.03
## BDI2_6       0.88  0.04
## BDI2_7       1.39  0.04
## BDI2_8       2.26  0.03
## BDI2_9       4.47  0.02
## BDI2_10      2.50  0.04
## BDI2_11     -0.20  0.04
## BDI2_12      1.15  0.03
## BDI2_13      0.97  0.03
## BDI2_14      0.69  0.04
## BDI2_15     -0.04  0.04
## BDI2_16     -0.46  0.03
## BDI2_17     -0.12  0.03
## BDI2_18      0.97  0.04
## BDI2_19     -0.38  0.04
## BDI2_20      0.15  0.03
```

It looks like we see some signs of non-normality.

  c. Develop histograms (using the `ggplot2` package) for the variables `BDI2_1` and `BDI2_20`. What do you
     learn from the inspection of these histograms?

- *Tip: When working with `R` you will often encounter parts that you just don't know how to implement, so don't be ashamed to Google things (e.g., "how to create and histogram using `ggplot2` in `R`").*
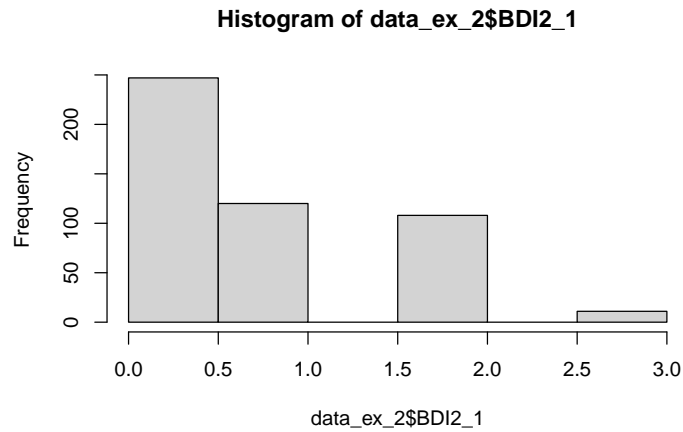
```
# Histogram for variable `BDI2_1`.
ggplot(data = data_ex_2) +
    geom_histogram(mapping = aes(BDI2_1), bins = 10) +
    theme_bw()
```
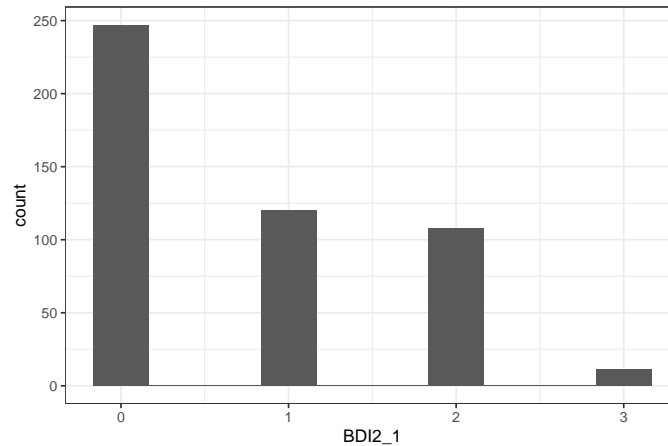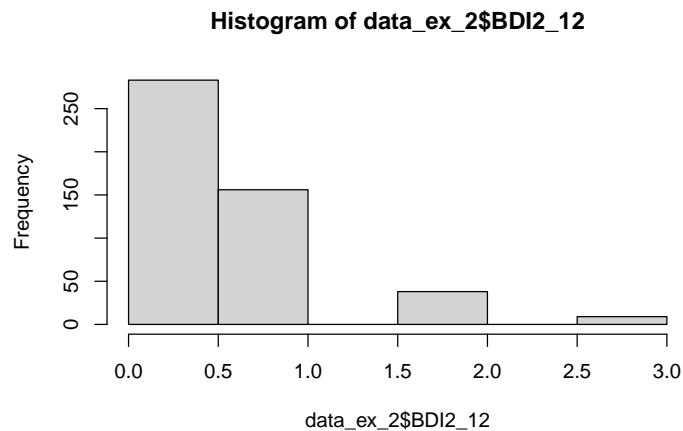


```
# Histogram for variable `BDI2_2`.
ggplot(data = data_ex_2) +
    geom_histogram(mapping = aes(BDI2_2), bins = 10) +
    theme_bw()
```
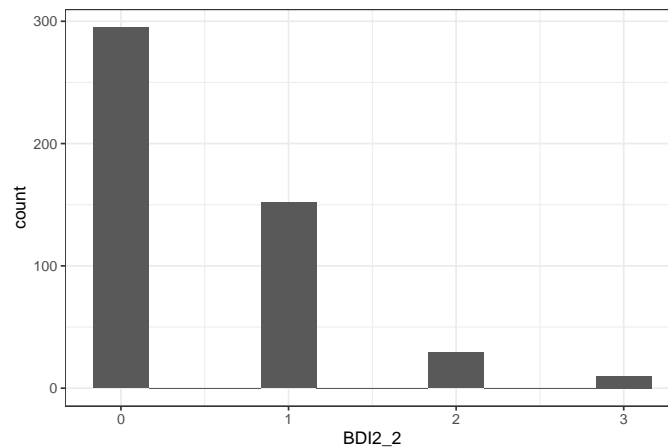


```
# Or you can also use the built-in function `hist` in `R` for this.
# Histogram for `BDI2_1` using `hist`
hist(data_ex_2$BDI2_1)
```

Histogram of data_ex_2$BDI2_1


```
# Histogram for `BDI2_2` using `hist`
hist(data_ex_2$BDI2_12)
```

Histogram of data_ex_2$BDI2_12

d. Estimate the model in Figure 2, but with the following additional constraints and model estimation specifications:

  1. Use `BDI2_3`, `BDI2_12`, and `BDI2_16` as marker variables.
  2. Constrain the variances of `F1`, `F2`, and `F3` to be equal.
  3. Fix the variance of `F4` to 1.
  4. Define the observed variables as ordered categorical variables.
  5. Use as estimator the *Mean and Variance Adjusted Weighted Least Squares* estimator (WLSMV).
  6. Evaluate the fit of this model.

Note: variables miss a `C` in the labeling, so `CBD` in picture is `BD` in the dataset.

First, we specify the model syntax.

```
# Model syntax.
model_ex_2 <- "
    # Measurement model.
    F1 =~ NA * BDI2_1 + BDI2_2 + 1 * BDI2_3 + BDI2_5 + BDI2_6 + BDI2_7 + BDI2_8 + BDI2_9 + BDI2_10 + BDI2_14
    F2 =~ NA * BDI2_4 + BDI2_11 + 1 * BDI2_12 + BDI2_13 + BDI2_17 + BDI2_19
    F3 =~ NA * BDI2_15 + 1 * BDI2_16 + BDI2_18 + BDI2_20


    # Structural model.
```

```
    F4 =~ NA * F1 + F2 + F3


    # Constrain the variance of the structural factor to 1.
    F4 ~~ 1 * F4


    # Add labels for the variances of the latent variables.
    F1 ~~ a1 * F1
    F2 ~~ a2 * F2
    F3 ~~ a3 * F3


    # Constrain the variances of the latent variables to be equal.
    a1 == a2
    a1 == a3
    a2 == a3
"
```

What other shorter syntax would allow us to constrain the variances of the latent variables?

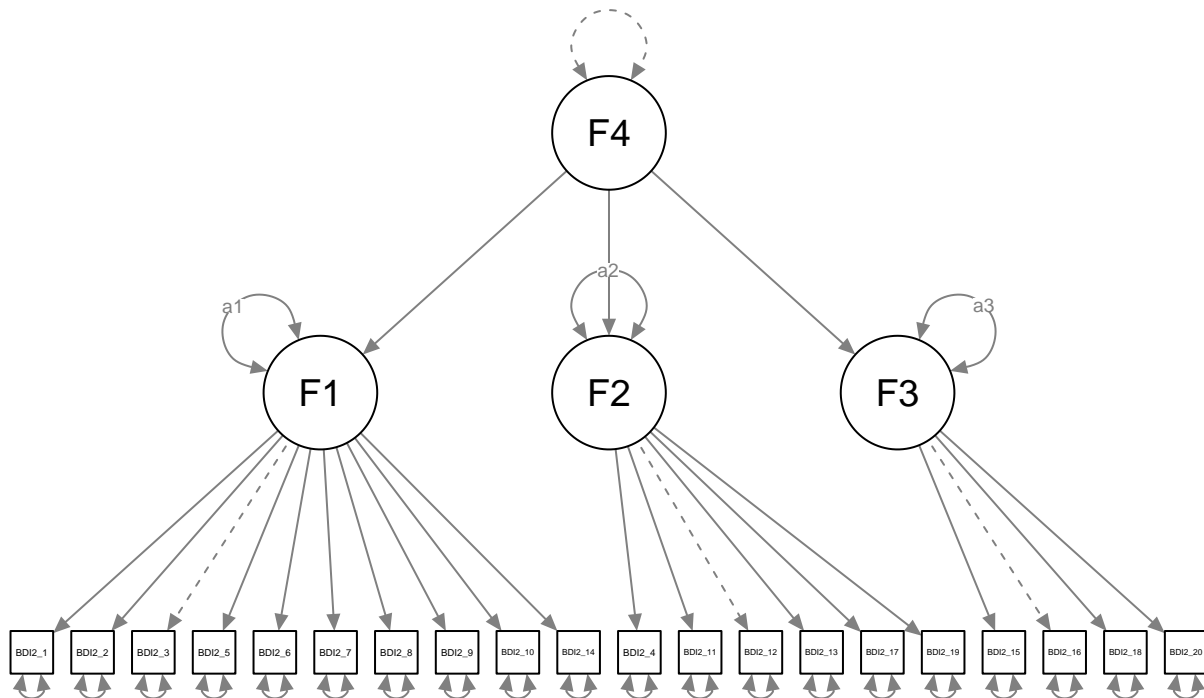Now we can estimate the model using the ML approach.

```
# Estimate the model using the `ML` approach.
model_ex_2_fit_ml <- cfa(model_ex_2, data = data_ex_2)

# Visualize the model.
semPaths(model_ex_2_fit_ml, what = "paths", sizeMan = 3)
```



```
# Model summary.
summary(model_ex_2_fit_ml)
```

```
## lavaan 0.6-12 ended normally after 45 iterations
##
##    Estimator                                         ML
```

```
##    Optimization method                        NLMINB
##    Number of model parameters                      43
##    Number of equality constraints                   3
##    Row rank of the constraints matrix               2
##
##    Number of observations                         486
##
## Model Test User Model:
##
##    Test statistic                             394.871
##    Degrees of freedom                             169
##    P-value (Chi-square)                         0.000
##
## Parameter Estimates:
##
##    Standard errors                           Standard
##    Information                               Expected
##    Information saturated (h1) model        Structured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)
##    F1 =~
##      BDI2_1           1.260    0.108   11.692    0.000
##      BDI2_2           1.026    0.087   11.793    0.000
##      BDI2_3           1.000
##      BDI2_5           0.880    0.088   10.006    0.000
##      BDI2_6           0.976    0.108    9.044    0.000
##      BDI2_7           1.231    0.102   12.094    0.000
##      BDI2_8           0.985    0.092   10.760    0.000
##      BDI2_9           0.600    0.062    9.643    0.000
##      BDI2_10          0.770    0.097    7.946    0.000
##      BDI2_14          1.193    0.098   12.226    0.000
##    F2 =~
##      BDI2_4           0.811    0.064   12.685    0.000
##      BDI2_11          1.080    0.075   14.387    0.000
##      BDI2_12          1.000
##      BDI2_13          0.943    0.066   14.334    0.000
##      BDI2_17          0.969    0.069   14.108    0.000
##      BDI2_19          0.949    0.076   12.450    0.000
##    F3 =~
##      BDI2_15          1.574    0.138   11.395    0.000
##      BDI2_16          1.000
##      BDI2_18          0.836    0.114    7.349    0.000
##      BDI2_20          1.307    0.118   11.088    0.000
##    F4 =~
##      F1               0.439    0.036   12.093    0.000
##      F2               0.493    0.030   16.256    0.000
##      F3               0.351    0.032   10.847    0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)
```

```
##    F4                1.000
##    .F1        (a1)   0.031    0.004    7.090    0.000
##    .F2        (a2)   0.031    0.004    7.090    0.000
##    .F3        (a3)   0.031    0.004    7.090    0.000
##    .BDI2_1           0.406    0.029   13.999    0.000
##    .BDI2_2           0.256    0.018   13.918    0.000
##    .BDI2_3           0.514    0.035   14.819    0.000
##    .BDI2_5           0.401    0.027   14.807    0.000
##    .BDI2_6           0.701    0.047   15.040    0.000
##    .BDI2_7           0.316    0.023   13.635    0.000
##    .BDI2_8           0.374    0.026   14.538    0.000
##    .BDI2_9           0.213    0.014   14.906    0.000
##    .BDI2_10          0.646    0.042   15.218    0.000
##    .BDI2_14          0.276    0.020   13.486    0.000
##    .BDI2_4           0.308    0.021   14.351    0.000
##    .BDI2_11          0.355    0.026   13.671    0.000
##    .BDI2_12          0.254    0.019   13.332    0.000
##    .BDI2_13          0.274    0.020   13.698    0.000
##    .BDI2_17          0.307    0.022   13.809    0.000
##    .BDI2_19          0.447    0.031   14.421    0.000
##    .BDI2_15          0.262    0.024   10.882    0.000
##    .BDI2_16          0.441    0.030   14.586    0.000
##    .BDI2_18          0.565    0.038   15.009    0.000
##    .BDI2_20          0.244    0.020   12.171    0.000
##
## Constraints:
##                                          |Slack|
##     a1 - (a2)                             0.000
##     a1 - (a3)                             0.000
##     a2 - (a3)                             0.000
```

Now we can specify which variables should be treated as ordered variables and use the `WLSMV` estimator. First, let's store the names of those variables in a vector for convenience.

```r
# Store the variable names.
ordinal_variables_ex_2 <- c(
    "BDI2_1", "BDI2_2", "BDI2_3", "BDI2_4",
    "BDI2_5", "BDI2_6", "BDI2_7", "BDI2_8",
    "BDI2_9", "BDI2_10", "BDI2_11", "BDI2_12",
    "BDI2_13", "BDI2_14", "BDI2_15", "BDI2_16",
    "BDI2_17", "BDI2_18", "BDI2_19", "BDI2_20"
)

# Print the variable names.
print(ordinal_variables_ex_2)
```

```
##  [1] "BDI2_1"  "BDI2_2"  "BDI2_3"  "BDI2_4"  "BDI2_5"  "BDI2_6"  "BDI2_7"
##  [8] "BDI2_8"  "BDI2_9"  "BDI2_10" "BDI2_11" "BDI2_12" "BDI2_13" "BDI2_14"
## [15] "BDI2_15" "BDI2_16" "BDI2_17" "BDI2_18" "BDI2_19" "BDI2_20"
```

Estimate the model using the `WLSMV` estimator and use the variables names stored in `ordinal_variables_ex_2` to indicate to `lavaan` which variables should be treated as ordinal.

```r
# Estimate the model using the `WLSMV` estimator.
model_ex_2_fit_wlsmv <- cfa(
    model_ex_2,
    data = data_ex_2,
    ordered = ordinal_variables_ex_2,
    estimator = "WLSMV"
)

# Model summary.
summary(model_ex_2_fit_wlsmv)
```

```
## lavaan 0.6-12 ended normally after 35 iterations
##
##   Estimator                                      DWLS
##   Optimization method                          NLMINB
##   Number of model parameters                       83
##   Number of equality constraints                    3
##   Row rank of the constraints matrix                2
##
##   Number of observations                          486
##
## Model Test User Model:
##                                       Standard      Robust
##   Test Statistic                       226.433     338.720
##   Degrees of freedom                       169         169
##   P-value (Chi-square)                   0.002       0.000
##   Scaling correction factor                          0.788
##   Shift parameter                                   51.254
##     simple second-order correction
##
## Parameter Estimates:
##
##   Standard errors                            Robust.sem
##   Information                                  Expected
##   Information saturated (h1) model         Unstructured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   F1 =~
##     BDI2_1            1.174    0.063   18.558    0.000
##     BDI2_2            1.179    0.064   18.371    0.000
##     BDI2_3            1.000
##     BDI2_5            0.976    0.070   13.866    0.000
##     BDI2_6            0.874    0.068   12.779    0.000
##     BDI2_7            1.234    0.068   18.080    0.000
##     BDI2_8            1.035    0.064   16.163    0.000
##     BDI2_9            0.959    0.075   12.723    0.000
##     BDI2_10           0.831    0.079   10.485    0.000
##     BDI2_14           1.233    0.068   18.040    0.000
##   F2 =~
##     BDI2_4            0.881    0.042   20.903    0.000
```

```
##      BDI2_11              0.936     0.040   23.162      0.000
##      BDI2_12              1.000
##      BDI2_13              0.928     0.042   21.899      0.000
##      BDI2_17              0.933     0.044   21.230      0.000
##      BDI2_19              0.805     0.044   18.133      0.000
##    F3 =~
##      BDI2_15              1.492     0.109   13.677      0.000
##      BDI2_16              1.000
##      BDI2_18              0.805     0.104    7.752      0.000
##      BDI2_20              1.400     0.110   12.776      0.000
##    F4 =~
##      F1                   0.606     0.033   18.464      0.000
##      F2                   0.770     0.027   28.794      0.000
##      F3                   0.503     0.040   12.657      0.000
##
## Intercepts:
##                       Estimate  Std.Err  z-value  P(>|z|)
##     .BDI2_1              0.000
##     .BDI2_2              0.000
##     .BDI2_3              0.000
##     .BDI2_5              0.000
##     .BDI2_6              0.000
##     .BDI2_7              0.000
##     .BDI2_8              0.000
##     .BDI2_9              0.000
##     .BDI2_10             0.000
##     .BDI2_14             0.000
##     .BDI2_4              0.000
##     .BDI2_11             0.000
##     .BDI2_12             0.000
##     .BDI2_13             0.000
##     .BDI2_17             0.000
##     .BDI2_19             0.000
##     .BDI2_15             0.000
##     .BDI2_16             0.000
##     .BDI2_18             0.000
##     .BDI2_20             0.000
##     .F1                  0.000
##     .F2                  0.000
##     .F3                  0.000
##      F4                  0.000
##
## Thresholds:
##                       Estimate  Std.Err  z-value  P(>|z|)
##     BDI2_1|t1            0.021     0.057    0.363    0.717
##     BDI2_1|t2            0.691     0.062   11.118    0.000
##     BDI2_1|t3            2.002     0.126   15.937    0.000
##     BDI2_2|t1            0.271     0.058    4.707    0.000
##     BDI2_2|t2            1.403     0.083   16.952    0.000
##     BDI2_2|t3            2.042     0.130   15.712    0.000
##     BDI2_3|t1           -0.088     0.057   -1.541    0.123
```

```
##      BDI2_3|t2       0.602    0.061     9.895    0.000
##      BDI2_3|t3       2.085    0.135    15.449    0.000
##      BDI2_5|t1       0.369    0.058     6.329    0.000
##      BDI2_5|t2       1.350    0.080    16.789    0.000
##      BDI2_5|t3       1.786    0.106    16.858    0.000
##      BDI2_6|t1       0.336    0.058     5.789    0.000
##      BDI2_6|t2       1.001    0.069    14.594    0.000
##      BDI2_6|t3       1.337    0.080    16.743    0.000
##      BDI2_7|t1       0.325    0.058     5.609    0.000
##      BDI2_7|t2       1.148    0.073    15.752    0.000
##      BDI2_7|t3       1.761    0.104    16.927    0.000
##      BDI2_8|t1       0.331    0.058     5.699    0.000
##      BDI2_8|t2       1.325    0.079    16.695    0.000
##      BDI2_8|t3       1.761    0.104    16.927    0.000
##      BDI2_9|t1       0.807    0.064    12.571    0.000
##      BDI2_9|t2       1.715    0.101    17.036    0.000
##      BDI2_9|t3       2.642    0.239    11.047    0.000
##     BDI2_10|t1       0.639    0.061    10.421    0.000
##     BDI2_10|t2       1.168    0.074    15.885    0.000
##     BDI2_10|t3       1.461    0.086    17.079    0.000
##     BDI2_14|t1       0.331    0.058     5.699    0.000
##     BDI2_14|t2       1.099    0.071    15.407    0.000
##     BDI2_14|t3       2.085    0.135    15.449    0.000
##      BDI2_4|t1       0.261    0.058     4.526    0.000
##      BDI2_4|t2       1.446    0.085    17.051    0.000
##      BDI2_4|t3       2.002    0.126    15.937    0.000
##     BDI2_11|t1      -0.380    0.058    -6.509    0.000
##     BDI2_11|t2       0.807    0.064    12.571    0.000
##     BDI2_11|t3       1.737    0.102    16.986    0.000
##     BDI2_12|t1       0.208    0.057     3.623    0.000
##     BDI2_12|t2       1.301    0.078    16.595    0.000
##     BDI2_12|t3       2.085    0.135    15.449    0.000
##     BDI2_13|t1       0.067    0.057     1.178    0.239
##     BDI2_13|t2       1.301    0.078    16.595    0.000
##     BDI2_13|t3       2.042    0.130    15.712    0.000
##     BDI2_17|t1       0.041    0.057     0.725    0.468
##     BDI2_17|t2       1.071    0.071    15.192    0.000
##     BDI2_17|t3       2.246    0.157    14.341    0.000
##     BDI2_19|t1      -0.397    0.059    -6.778    0.000
##     BDI2_19|t2       0.737    0.063    11.722    0.000
##     BDI2_19|t3       1.737    0.102    16.986    0.000
##     BDI2_15|t1      -0.176    0.057    -3.080    0.002
##     BDI2_15|t2       0.943    0.067    14.047    0.000
##     BDI2_15|t3       1.868    0.113    16.572    0.000
##     BDI2_16|t1      -0.596    0.061    -9.807    0.000
##     BDI2_16|t2       0.697    0.062    11.205    0.000
##     BDI2_16|t3       1.965    0.122    16.131    0.000
##     BDI2_18|t1       0.145    0.057     2.537    0.011
##     BDI2_18|t2       1.118    0.072    15.547    0.000
##     BDI2_18|t3       1.715    0.101    17.036    0.000
##     BDI2_20|t1      -0.620    0.061   -10.158    0.000
```

```
##      BDI2_20|t2        0.927    0.067   13.887     0.000
##      BDI2_20|t3        2.002    0.126   15.937     0.000
##
## Variances:
##                     Estimate  Std.Err  z-value  P(>|z|)
##      F4              1.000
##      .F1      (a1)   0.067    0.009    7.609     0.000
##      .F2      (a2)   0.067    0.009    7.609     0.000
##      .F3      (a3)   0.067    0.009    7.609     0.000
##      .BDI2_1          0.401
##      .BDI2_2          0.396
##      .BDI2_3          0.566
##      .BDI2_5          0.586
##      .BDI2_6          0.668
##      .BDI2_7          0.338
##      .BDI2_8          0.535
##      .BDI2_9          0.600
##      .BDI2_10         0.700
##      .BDI2_14         0.339
##      .BDI2_4          0.488
##      .BDI2_11         0.422
##      .BDI2_12         0.341
##      .BDI2_13         0.432
##      .BDI2_17         0.426
##      .BDI2_19         0.573
##      .BDI2_15         0.290
##      .BDI2_16         0.681
##      .BDI2_18         0.793
##      .BDI2_20         0.374
##
## Scales y*:
##                     Estimate  Std.Err  z-value  P(>|z|)
##      BDI2_1           1.000
##      BDI2_2           1.000
##      BDI2_3           1.000
##      BDI2_5           1.000
##      BDI2_6           1.000
##      BDI2_7           1.000
##      BDI2_8           1.000
##      BDI2_9           1.000
##      BDI2_10          1.000
##      BDI2_14          1.000
##      BDI2_4           1.000
##      BDI2_11          1.000
##      BDI2_12          1.000
##      BDI2_13          1.000
##      BDI2_17          1.000
##      BDI2_19          1.000
##      BDI2_15          1.000
##      BDI2_16          1.000
##      BDI2_18          1.000
```

```
##      BDI2_20             1.000
##
## Constraints:
##                                          |Slack|
##      a1 - (a2)                            0.000
##      a1 - (a3)                            0.000
##      a2 - (a3)                            0.000
```

The hypothesis that model exactly reproduces data must be rejected.

As you may have noticed, for some of the fit indices we also have a robust *version. Check out this question for more information: https://stats.stackexchange.com/q/241896/116619.*
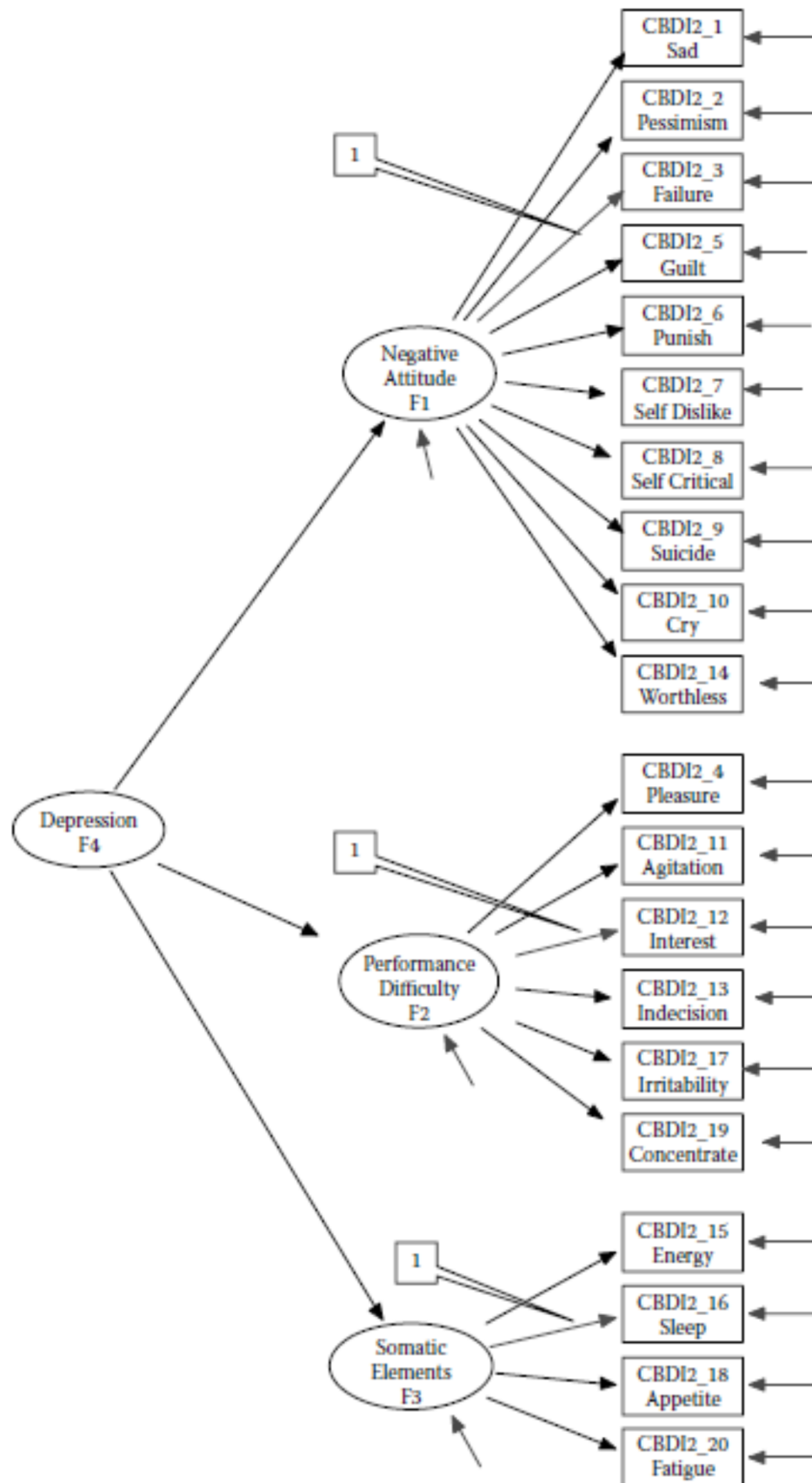
Figure 2: Hypothesized second-order model of factorial structure for the Chinese version of the *Beck Depression Inventory II.*