# Benchmark Optimization with Genetic Algorithms

Mihai-Cristian Farcaș

May 2025

## 1 Introduction

This project explores the application of Genetic Algorithms (GAs) for finding the minima of benchmark optimization functions. It implements two multimodal benchmark functions in Python, optimizes them using different GA configurations, and conducts a statistical analysis of the performance results.

## 2 Function Selection

Two multimodal benchmark functions were selected for this study:

### 2.1 Rastrigin Function

**Domain**: $[-5.12, 5.12] \times [-5.12, 5.12]$

**Formula**:

$$f(x, y) = 20 + x^2 + y^2 - 10(\cos(2\pi x) + \cos(2\pi y))$$

The Rastrigin function is highly multimodal, with many local minima arranged in a regular grid-like manner. It has a global minimum of 0 at $(0, 0)$.

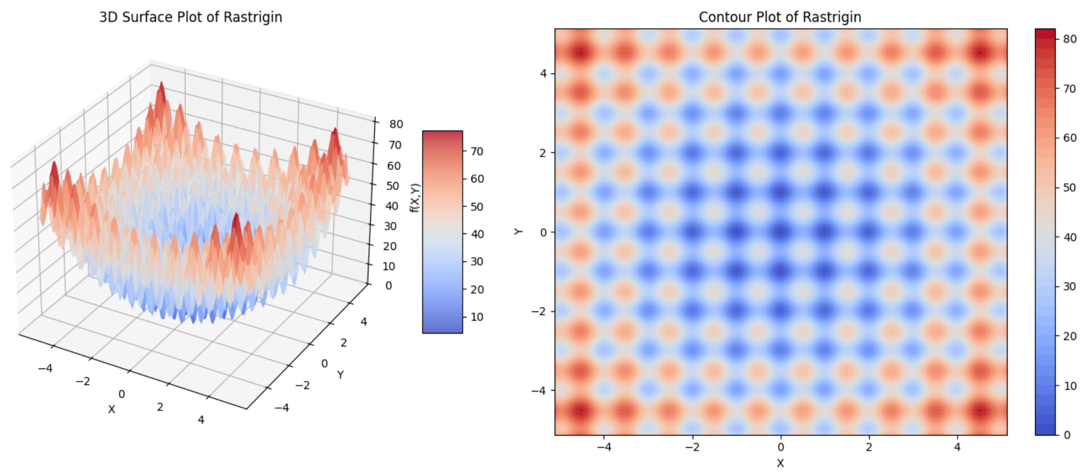


Figure 1: Surface plot of the Rastrigin function

## 2.2 Ackley Function

**Domain**: $[-5, 5] \times [-5, 5]$
**Formula**:

$$f(x, y) = -20 \cdot \exp\left(-0.2 \cdot \sqrt{0.5(x^2 + y^2)}\right) - \exp\left(0.5(\cos(2\pi x) + \cos(2\pi y))\right) + e + 20$$

The Ackley function combines exponential terms with cosine modulation, creating a surface with many local minima and a narrow global minimum at $(0, 0)$.

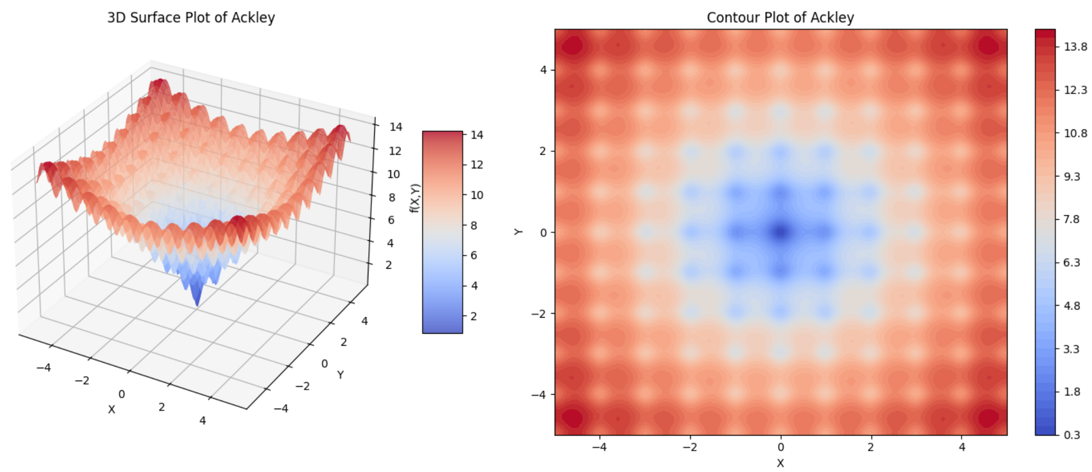


Figure 2: Surface plot of the Ackley function

# 3 Genetic Algorithm Implementation

## 3.1 Overview

The GA implementation is modular and configurable:

- **Encoding**: Binary and real-valued
- **Selection**: Tournament
- **Crossover**: Several types depending on encoding
- **Mutation**: Bit-flipping or Gaussian perturbation
- **Elitism**: Preserving top individuals

## 3.2 Encoding Options

### 3.2.1 Binary Encoding

Each variable is encoded with 16 bits and decoded to a real value in the function domain.

### 3.2.2 Real-Valued Encoding

Individuals are represented directly by $[x, y]$ vectors, enabling more precise solutions.

### 3.3 Crossover Types

#### 3.3.1 Binary Encoding Crossovers

- **One-Point Crossover**
- **Two-Point Crossover**

#### 3.3.2 Real-Valued Encoding Crossovers

- **Arithmetic Crossover**
- **BLX-$\alpha$ Crossover**

### 3.4 Mutation Operators

- **Binary**: Bit flip with a given probability
- **Real-valued**: Add Gaussian noise scaled to domain

### 3.5 Selection and Elitism

Tournament selection is used, and the best individuals are preserved using elitism.

## 4 Experimental Setup

### 4.1 Parameters

- Population size: 100
- Crossover rate: 0.8
- Mutation rate: 0.1
- Elite size: 2
- Runs: 30 per configuration
- Evaluations: 5000 per run

### 4.2 Experimental Configurations

Combinations tested:

- Functions: Rastrigin, Ackley
- Encodings: Binary, Real-valued
- Crossovers: One-point, Two-point (Binary); Arithmetic, BLX-$\alpha$ (Real)

# 5 Results and Analysis

## 5.1 Performance Metrics

- Best Fitness
- Convergence Generation
- Best Solution Coordinates

## 5.2 Results Visualization

**Comparison of Encoding Types:**

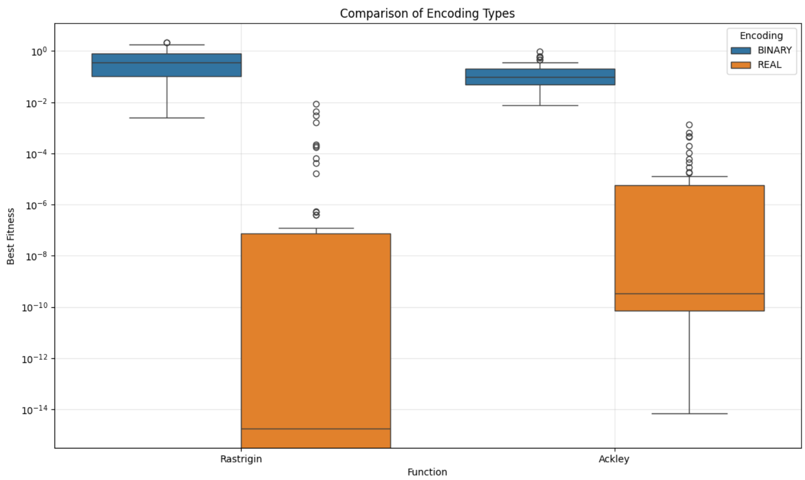


Figure 3: Comparison of binary vs real-valued encoding performance

**Comparison of Crossover Methods:**

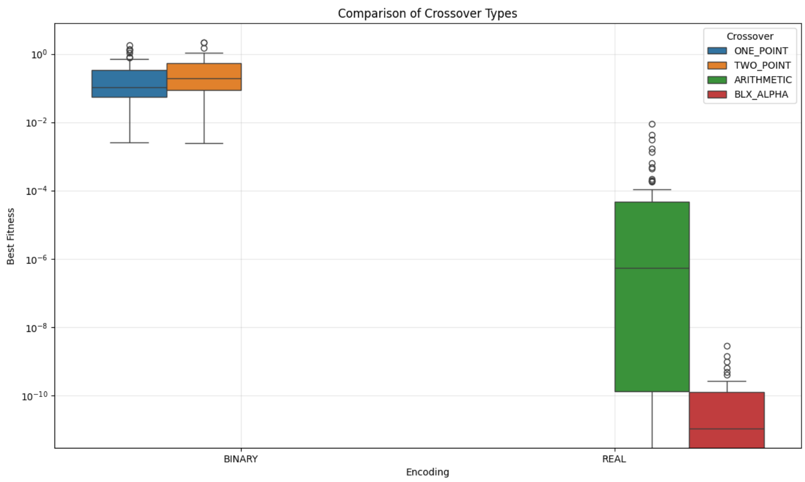


Figure 4: Comparison of different crossover methods

**Average Best Fitness by Configuration:**

| Function | Encoding | Crossover | Avg. Best Fitness | Std. Dev |
|---|---|---|---|---|
| Ackley | Real | BLX-$\alpha$ | 3.096758e-10 | 5.604689e-10 |
| Ackley | Real | Arithmetic | 1.145163e-04 | 2.841697e-04 |
| Ackley | Binary | One-Point | 1.318021e-01 | 1.173744e-01 |
| Ackley | Binary | Two-Point | 1.875472e-01 | 2.231714e-01 |
| Rastrigin | Real | BLX-$\alpha$ | 1.030879e-12 | 5.631603e-12 |
| Rastrigin | Real | Arithmetic | 6.230941e-04 | 1.847987e-03 |
| Rastrigin | Binary | One-Point | 4.328597e-01 | 4.974397e-01 |
| Rastrigin | Binary | Two-Point | 6.107105e-01 | 5.746422e-01 |

Table 1: Average best fitness results per configuration

## 5.3 Statistical Comparison

We used independent sample t-tests with 95% confidence intervals to compare:

**Rastrigin Function Analysis:**

- **Encoding Comparison**: Real encoding significantly outperformed binary ($t = 7.4754$, $p = 0.0000$)
- **Binary Crossovers**: No significant difference between one-point and two-point ($p = 0.2052$)
- **Real Crossovers**: No significant difference between arithmetic and BLX-$\alpha$ ($p = 0.0750$)

**Ackley Function Analysis:**

- **Encoding Comparison**: Real encoding significantly outperformed binary ($t = 6.9071$, $p = 0.0000$)
- **Binary Crossovers**: No significant difference between one-point and two-point ($p = 0.2324$)
- **Real Crossovers**: BLX-$\alpha$ significantly outperformed arithmetic crossover ($t = 2.2072$, $p = 0.0354$)

**Key Observations:**

- Real-valued encoding achieved extremely low fitness values (close to 0) for both functions
- BLX-$\alpha$ crossover outperformed arithmetic crossover on the Ackley function (statistically significant)
- The Rastrigin function was more challenging for binary encoding, with higher fitness values and standard deviations
- Real-valued encoding showed consistent superiority with extremely small standard deviations

# 6 Conclusion

This study illustrates the effectiveness of GAs on multimodal benchmark functions. Through comparative analysis, we assess how encoding and crossover strategies affect performance.

**Key Insights:**

- Real-valued encoding is significantly more effective than binary encoding for continuous optimization problems
- BLX-$\alpha$ crossover shows a significant advantage over arithmetic crossover on the Ackley function
- The choice of crossover operator has more impact on the Ackley function than on the Rastrigin function
- The Rastrigin function presents a more challenging optimization landscape than Ackley, especially for binary encoding

# 7 References


1. Handbook of Test Problems in Local and Global Optimization
2. Holland, J. H. (1975). *Adaptation in natural and artificial systems*
3. Goldberg, D. E. (1989). *Genetic algorithms in search, optimization, and machine learning*
4. Mitchell, M. (1998). *An introduction to genetic algorithms*