Tutorial 8. Operatori liniari (în lucru)

-matricea asociată unui operator liniar $T : \mathbb{R}^n \to \mathbb{R}^n$ sau $T : \mathbb{C}^n \to \mathbb{C}^n$ într-o bază dată, comportarea la schimbări de baze;

-pentru orice $A \in \mathcal{M}_{n\times n}(\mathbb{C})$, există $Q \in \mathcal{M}_{n\times n}(\mathbb{C})$, astfel încât

$$A = Q^{-1}JQ, \tag{1}$$

unde J este forma canonică Jordan a matricei A.

$$e^{tA} = Q^{-1}e^{tJ}Q, \tag{2}$$

$$J = \operatorname{diag}\left(\tilde{J}_1, \tilde{J}_2, \dots, \tilde{J}_h\right), \ \tilde{J} = \lambda I + E \tag{3}$$

$$e^{tJ} = \operatorname{diag}\left(e^{t\tilde{J}_1}, e^{t\tilde{J}_2}, \dots, e^{t\tilde{J}_h}\right) \tag{4}$$

$$e^{t\tilde{J}} = e^{t\lambda I + tE} = e^{t\lambda I}e^{tE} = e^{t\lambda}Ie^{tE} = e^{\lambda t}e^{tE}. \tag{5}$$

$$e^{t\tilde{J}} = e^{\lambda t}\begin{pmatrix} 1 & \frac{t}{1!} & \frac{t^2}{2!} & \dots & \frac{t^{m_{pj}-1}}{(m_{pj}-1)!} \\ 0 & 1 & \frac{t}{1!} & \dots & \frac{t^{m_{pj}-2}}{(m_{pj}-2)!} \\ \vdots & & & & \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix} \tag{6}$$

Teorema 1 (structura matricei e^{tA})

$$\sum_{k=1}^{s} e^{\alpha_k t}\left(P_k(t)\cos\beta_k t + Q_k(t)\sin\beta_k t\right),$$