Solucións á parte de cálculo do exame. Xan '16.

a)

$$\left|\frac{3x}{5}-1\right| \geq \frac{x}{5} ; \quad |3x-5| \geq x ; \quad \begin{cases} 3x-5 \geq x \text{ se } 3x-5 \geq 0 \\ \text{ou} \\ 3x-5 \leq -x \text{ se } 3x-5<0 \end{cases}$$

$$\begin{cases} x \geq \frac{5}{2} \text{ se } x \geq \frac{5}{3} \\ \text{ou} \\ x \leq \frac{5}{4} \text{ se } x < \frac{5}{3} \end{cases} \quad \begin{cases} x \in \left[\frac{5}{2},+\infty\right) \cap \left(\frac{5}{3},+\infty\right) \\ \text{unido a} \\ x \in \left(-\infty, \frac{5}{4}\right] \cap \left(-\infty, \frac{5}{3}\right) \end{cases} ; \quad x \in \left(-\infty, \frac{5}{4}\right] \cup \left[\frac{5}{2},+\infty\right)$$

b)

$$\cos(x-\pi) = \cos x \cos(-\pi) - \operatorname{sen}(x)\operatorname{sen}(-\pi) = -\cos x.$$

$$\operatorname{sen}\left(x+\frac{\pi}{2}\right) = \operatorname{sen} x \cos\frac{\pi}{2} + \cos x \operatorname{sen}\frac{\pi}{2} = \cos x.$$

Por tanto a fracción e -1.

c)

$$\lim_{x\to 0^+} \sqrt{\frac{1}{x}+2} - \sqrt{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}+2-\frac{1}{x}}{\sqrt{\frac{1}{x}+2}+\sqrt{\frac{1}{x}}} = 0$$

d) $y = \operatorname{arcsen}(x^2)$; $\operatorname{sen}(y) = x^2$

Derivando implícitamente, $\cos(y)\, y' = 2x$;

$$\sqrt{1-\operatorname{sen}^2(y)}\; y' = 2x ; \quad \sqrt{1-x^4}\; y' = 2x ; \quad y' = \frac{2x}{\sqrt{1-x^4}}$$

e) ningunha das anteriores; $\cos\left(\frac{\pi}{x(1-x)}\right)$ ten infinitas raíces por $\frac{\pi}{x(1-x)}$ toma tódolos valores en $(0,+\infty)$. Ademais, $\cos\left(\frac{\pi}{x(1-x)}\right)$ é función limitada.

f) $f(x) = \ln x;\quad f'(x) = \frac{1}{x};\quad f''(x) = -\frac{1}{x^2};\quad f'''(x) = +\frac{2}{x^3}$

$f(1) = 0\ ;\ f'(1) = 1\ ;\ f''(1) = -1\ ;\ f'''(x) = 2.$

$$T(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2}(x-1)^2 + \frac{f'''(1)}{6}(x-1)^3$$

$$= (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$$

Exercicio:

a) $x = 1,\ 2,\ 4.$

$f(x) = \log_2(1),\ \log_2(2),\ \log_2(4)$

Os nodos son $(1,0),\ (2,1),\ (4,2)$

O polinomio é $p(x) = 2\,l_2(x) + 4\,l_3(x) = 1\cdot\frac{(x-1)(x-4)}{(2-1)(2-4)} + 2\cdot\frac{(x-1)(x-2)}{(4-1)(4-2)}$

$$= -\frac{1}{2}(x^2-5x+4) + \frac{1}{3}(x^2-3x+2) = -\frac{x^2}{6} + \frac{3}{2}x - \frac{4}{3}$$

b) O erro máximo cometido será:

$$|f(x) - p(x)| = \left|\frac{f^{(3)}(c)}{6}(x-1)(x-2)(x-4)\right|$$

Como $\max_{c\in[1,4]} |f'''(c)| = \max_{c\in[1,4]} \left|\frac{2}{x^3}\frac{1}{\ln 2}\right| = \frac{2}{\ln 2}$ xa que $\frac{1}{x^3}$ é decrecente.

Por tanto,

$$|f(2{,}5) - p(2{,}5)| < \frac{\frac{2}{\ln 2}}{6}(2{,}5-1)(2{,}5-2)(4-2{,}5) < \frac{3/2}{3}\cdot\frac{3}{2}\,\frac{1}{2}\,\frac{3}{2} = \frac{9}{16}$$

Usando que $\frac{1}{\ln 2} < 3/2$.