

Exame FuMa. xau-15. Parte de Cálculo.

a) Recta entre $(\frac{1}{3}, 0)$, $(3, 1)$: $\frac{x - 1/3}{3 - 1/3} = \frac{y - 0}{1 - 0}$; $y = \frac{3}{8}(x - \frac{1}{3})$

Recta por $(\frac{1}{3}, 0)$ con pendiente $-\frac{3}{8}$: $y = -\frac{3}{8}(x - \frac{1}{3})$

Corte co eixo OY: $y = -\frac{3}{8}(0 - \frac{1}{3})$; $y = \frac{1}{8}$

b) $\sin(x + \frac{5\pi}{6}) = \sin x \cdot \cos \frac{5\pi}{6} + \cos x \cdot \sin \frac{5\pi}{6} = \sin x \cdot (-\cos \frac{\pi}{6}) + \cos x \cdot \sin \frac{\pi}{6} =$
 $= -\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x$

c) $\lim_{x \rightarrow 0^-} \frac{1}{3 + 2^{1/x}} = \frac{1}{3}$ pues $\lim_{x \rightarrow 0^-} 2^{1/x} = 2^{-\infty} = 0$.

$\lim_{x \rightarrow 0^+} \frac{1}{3 + 2^{1/x}} = 0$ pues $\lim_{x \rightarrow 0^+} 2^{1/x} = 2^{\infty} = \infty$.

O limite non existe.

d) $x^3y + xy^3 = 2$; para $x=1$, $y^3 + y - 2 = 0$;

Por Ruffini, $\begin{array}{r|rrrr} 1 & 1 & 0 & 1 & -2 \\ & & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & 0 \end{array}$

O polinomio $y^2 + y + 2$ non ten raíces reais.

Derivando implícitamente:

$3x^2y + x^3y' + y^3 + 3xy^2y' = 0$; $y' = -\frac{3x^2y + y^3}{x^3 + 3xy^2}$

$y'(1,1) = -\frac{3+1}{1+3} = -1$.

e) $y' = \frac{1}{a} \frac{1}{1 + \frac{1}{a^2} \tan^2 x} \frac{1}{a} (1 + \tan^2 x) = \frac{1 + \tan^2 x}{a^2 + \tan^2 x} = \frac{1 + \frac{\sin^2 x}{\cos^2 x}}{a^2 + \frac{\sin^2 x}{\cos^2 x}} =$

$= \frac{\cos^2 x + \sin^2 x}{a^2 \cos^2 x + \sin^2 x} = \frac{1}{a^2 \cos^2 x + \sin^2 x}$

$$f) y = \cos x$$

$$y' = -\sin x$$

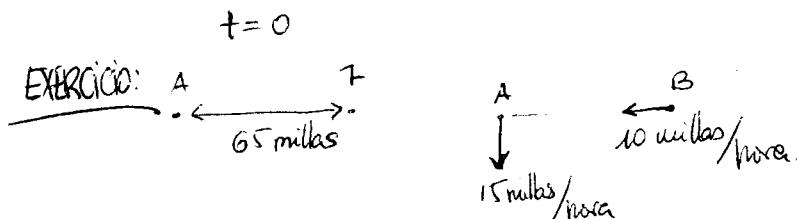
$$y'' = -\cos x$$

$$y''' = \sin x$$

$$y^{(4)} = \cos x$$

$$R_4(x) = \frac{\cos x(0)}{4!} (x-0)^4.$$

$$\left| R_4\left(\frac{1}{2}\right) \right| \leq \frac{1}{4!} \left(\frac{1}{2}\right)^4 = \frac{1}{3 \cdot 2^7}$$



$$D(t) = d(t)^2 = (65 - 10t)^2 + (15t)^2$$

$$\frac{dD}{dt} = 2(65 - 10t)(-10) + 2 \cdot 15^2 t = 0; -1300 + 200t + 450t; t = \frac{1300}{650} = 2$$

Como B para na vertical de A, a distância mede indefinidamente, logo podemos buscar o mínimo em $[0, 6.5]$, dando como resultado que se dá em $t=2$.