Cilindrični koord. sis.

$\nabla^2 u + \lambda u = 0$ $\quad \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}$

$\boxed{\nabla^2 u = 0 \Rightarrow \lambda = 0}$

nastavek: $u = R(r)\,\Phi(\varphi)\,Z(z)$

φ: $\frac{\Phi''}{\Phi} = -m^2$ $\quad m = 0 \ \{1, \varphi\}$ $\quad m \neq 0 \ \{\sin m\varphi, \cos m\varphi\}$

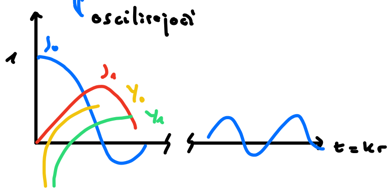
z: $\frac{Z''}{Z} = \pm\beta^2$ $\quad +\beta^2 \{\mathrm{sh}\,\beta z, \mathrm{ch}\,\beta z\}$ $\quad -\beta^2 \{\sin \beta z, \cos\beta z\}$ $\quad \beta = 0 \ \{1, z\}$

r: $r^2 R'' + rR' + [(\lambda \pm \beta^2) r^2 - m^2] R = 0$

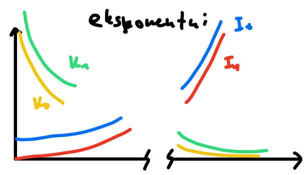
$\lambda \pm \beta^2 = 0$ $\quad m = 0 \ \{1, \ln r\}$ $\quad m \neq 0 \ \{r^m, r^{-m}\}$

$k^2 = \lambda \pm \beta^2 > 0$ $\quad \{J_m(kr), Y_m(kr)\}$ ← divergira v $r = 0$

oscilirajoči



$-k^2 = \lambda \pm \beta^2 < 0$ $\quad \{K_m(kr), I_m(kr)\}$



Splošno

$$u = \sum_{m=0}^{\infty} (A_m J_m(kr) + B_m Y_m(kr))(C_m \cos m\varphi + D_m \sin m\varphi)$$

Sferični koord. sis.

$\nabla^2 u + \lambda u = 0$ $\quad \nabla^2 u = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial u}{\partial r}\right) - \frac{L^2}{r^2} u$ $\quad u_{lm} = Y_l^m R_l$

$L^2 u = -\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial u}{\partial\theta}\right) - \frac{1}{\sin^2\theta}\frac{\partial^2 u}{\partial\varphi^2}$ $\quad L^2 Y_l^m = l(l+1) Y_l^m$

φ, θ: $Y_l^m = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}\, P_l^m(\cos\theta)\, e^{im\varphi}$

$\int_0^{2\pi} d\varphi \int_{-1}^{1} d\cos\theta \; Y_{l'}^{m'*} Y_l^m = \delta_{ll'}\delta_{mm'}$

r: $\lambda = 0$ $\quad \{r^l, r^{-(l+1)}\}$

$k^2 = \lambda > 0$ $\quad \{j_l(kr), y_l(kr)\} = \sqrt{\frac{\pi}{2kr}}\{J_{l+1/2}(kr), Y_{l+1/2}(kr)\}$

diversira $r \to 0$

$-k^2 = \lambda < 0$ $\quad \{i_l(kr), k_l(kr)\}$ $\quad$ divergira v $r \to \infty$; divergira v $r \to 0$

$$\int_{-1}^{1} P_n(\cos\theta) P_m(\cos\theta)\, d\cos\theta = \frac{2}{2n+1}\delta_{nm}$$

Enačbe

$\frac{\partial T}{\partial t} = D \nabla^2 T + \frac{q}{\rho c_p}$ $\quad D = \frac{\lambda}{\rho c_p}$

$P = \int_V q\, dV$

$u_{tt} = c^2 u_{xx}$
$\quad = c^2 \nabla^2 u$

$c^2 = \frac{\gamma}{\mu}$ opna $\quad$ na stiku $\frac{dF}{dl} = \gamma \frac{du}{dr}$ $\quad F = \gamma l \frac{du}{dr}$