

Mandelstamove sprem.

$$p, k \rightarrow p', k'$$

$$s = (p+k)^2 = (p'+k')^2$$

$$t = (p-p')^2$$

$$u = (p-k')^2 = (p'-k)^2$$

Spin I, I₃

$$I_i = \frac{S_i}{2} \quad [I_i, I_j] = i\epsilon_{ijk} I_k$$

$$|p\rangle = |1/2, 1/2\rangle_I = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |u\rangle = |1/2, -1/2\rangle_I = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$I_{\pm} = I_{\pm} \pm i I_{\mp}$$

$$I^2 |I, I_3\rangle = I(I+1) |I, I_3\rangle$$

$$I_3 |I, I_3\rangle = I_3 |I, I_3\rangle$$

$$I_{\pm} |I, I_3\rangle = \sqrt{I(I \pm 1) - I_3(I_3 \pm 1)} |I, I_3 \pm 1\rangle$$

$$I_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad I_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Triplet $|1, 1\rangle = |p\rangle |p\rangle$
 $I=1 \quad |1, 0\rangle = \frac{1}{\sqrt{2}} (|p\rangle |u\rangle + |u\rangle |p\rangle)$
 $|1, -1\rangle = |u\rangle |u\rangle$

Singlet $|0, 0\rangle = \frac{1}{\sqrt{2}} (|p\rangle |u\rangle - |u\rangle |p\rangle)$
 $I=0$

Za vse fermione je $|\psi\rangle$ antisim.

Zamenjava delcev $1 \leftrightarrow 2$

$$|\psi\rangle_S \rightarrow (-1)^{I+1} |\psi\rangle_S$$

$$|\psi\rangle_S \rightarrow (-1)^{I+1} |\psi\rangle_S$$

$$|\psi\rangle_S \rightarrow (-1)^I |\psi\rangle_S$$

$$|\psi\rangle \rightarrow (-1)^{I+S+L} |\psi\rangle$$

$$J = S + L$$

Magnetni moment

$$\hat{\mu}_z = \frac{e\hbar}{2m} g \hat{S}_z \quad \hat{S}_z |S, S_z\rangle = S_z |S, S_z\rangle$$

$$\hat{\mu}_z^2 = \frac{e^2 \hbar^2}{4m^2} \hat{S}_z^2$$

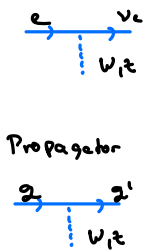
$$\mu_z^u = \frac{e\hbar}{2m_u} \quad \mu_z^d = \frac{-e\hbar}{2m_d}$$

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Šibka interakcija



$$\text{Propagator} = \frac{-i(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_W^2})}{q^2 - m_W^2}$$

$$P_L = \frac{1-\gamma^5}{2}$$

$$- \frac{i g}{\sqrt{2}} \gamma^\mu P_L \quad - \frac{i g}{\sqrt{2}} \gamma^\mu P_L V_{\text{CKM}}$$

Diracova enačba

$$(i \gamma^\mu \partial_\mu - m) \psi = 0$$

$$\gamma^\mu \partial_\mu = \vec{\gamma} \cdot \nabla + \gamma^0 \frac{\partial}{\partial t}$$

$$(\vec{\alpha} \cdot \vec{p} + \beta m) \psi = i \frac{\partial \psi}{\partial t}$$

$$\vec{\gamma} = \gamma^0 \vec{\alpha} \quad \gamma^0 = \beta$$

$$H = \gamma^0 (\vec{\gamma} \cdot \vec{p} + m)$$

$$\bar{\psi} = \psi^\dagger \gamma^0$$

$$j^\mu = e \bar{\psi} \gamma^\mu \psi \quad \text{gostota el. toka}$$

$$\partial_\mu j^\mu = 0$$

$$\not{p} = \gamma_\mu p^\mu = E \gamma_0 - \vec{\gamma} \cdot \vec{p}$$

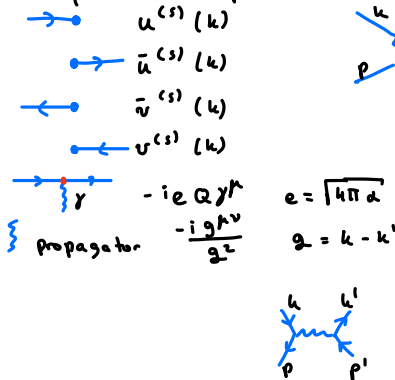
$$\not{p} \not{p} = p \cdot p = p^2$$

Sučunost / helicity

$$h = \frac{\vec{p}}{|\vec{p}|} \cdot \vec{s} \quad \vec{s} = \frac{1}{2} \begin{pmatrix} \sigma_x & \sigma_y & \sigma_z \end{pmatrix}$$

last. vada. $\pm \frac{1}{2}$

Feynmanova pravila



$$u^{(s)}(k) \quad \bar{u}^{(s)}(k) \quad \bar{v}^{(s)}(k) \quad v^{(s)}(k)$$

$$-ieQ\gamma^\mu \quad -\frac{ig^{\mu\nu}}{q^2}$$

$$e = \sqrt{4\pi\alpha} \quad q = k - k'$$

$$-iM = \bar{u}(k') [-ieQ\gamma^\mu] u(k) \left\{ \frac{-ig^{\mu\nu}}{q^2} \right\} \bar{u}(p') [-ieQ\gamma^\nu] u(p)$$

$$q^2 = 2m_e^2 - 2k \cdot k' = -8m_e T_e \sin^2 \frac{\theta}{2}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{1}{|M|^2} P^{\pm}/p_z$$

$$-iM = \bar{u}(p') [\dots] u(k) \{ \dots \} \bar{u}(k') [\dots] v(p')$$

Lastnosti γ matrik

$$(\gamma^0)^2 = 1 \quad (\gamma^i)^2 = -1 \quad i=1,2,3$$

$$(\gamma^0)^\dagger = \gamma^0 \quad (\gamma^i)^\dagger = -\gamma^i$$

$$(\gamma^\mu)^\dagger = \gamma^0 \gamma^\mu \gamma^0 \quad \gamma_\mu = g^{\mu\nu} \gamma_\nu$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

$$\gamma^\mu \gamma^\nu = -\gamma^\nu \gamma^\mu \quad \text{če } \mu \neq \nu$$

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_{4 \times 4} \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

$$\text{Tr } \gamma^\mu = 0$$

$$\text{Tr } \gamma^5 = 0$$

$$\text{Tr } \gamma^\mu \gamma^\nu \dots \gamma^\lambda = 0 \quad \text{št. } \gamma \text{ je liho}$$

$$\text{Tr } \gamma^5 \gamma^\mu \dots \gamma^\nu = 0 \quad \text{št. } \gamma \text{ je sodo}$$

$$\text{Tr } \gamma^\mu \gamma^\nu = 4g^{\mu\nu}$$

$$\text{Tr } \gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta = 4(g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha})$$

$$\text{Tr}((\not{p} - m) \gamma^\mu (\not{k} + m) \gamma^\nu) = 4(-m^2 g^{\mu\nu} + p^\mu k^\nu - (p \cdot k) g^{\mu\nu} + p^\nu k^\mu)$$

Rešitve Diracove enačbe

$$\psi_p^s = N e^{-i p \cdot x} u^s(\vec{p})$$

Delcev $u^s(\vec{p}) = N \begin{pmatrix} \chi^s \\ \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \chi^s \end{pmatrix} \quad \chi^{\uparrow} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \uparrow$
 $\chi^{\downarrow} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \downarrow$

Antidelcev $v^s(\vec{p}) = N \begin{pmatrix} \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \chi^{-s} \\ \chi^{-s} \end{pmatrix} \quad \chi^{\uparrow} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \uparrow$
 $\chi^{\downarrow} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \downarrow$

$$N = \sqrt{E+m}$$

$$\int \mathcal{L} dV = \int \bar{\psi} \psi dV = 2E$$

Lastnosti rešitev

$$u^{(r)}(\vec{p})^\dagger u^{(s)}(\vec{p}) = \delta^{rs} 2E$$

$$v^{(r)}(\vec{p})^\dagger v^{(s)}(\vec{p}) = \delta^{rs} 2E$$

$$\bar{u}^{(r)}(\vec{p})^\dagger u^{(s)}(\vec{p}) = \delta^{rs} 2m$$

$$\bar{v}^{(r)}(\vec{p})^\dagger v^{(s)}(\vec{p}) = \delta^{rs} 2m$$

$$\sum_s u^{(s)}(\vec{p}) \bar{u}^{(s)}(\vec{p}) = \not{p} + m$$

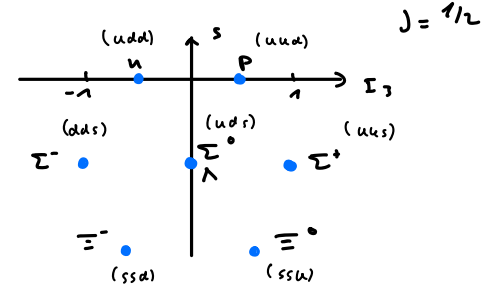
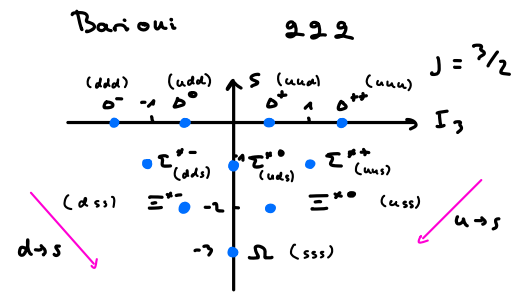
$$\sum_s v^{(s)}(\vec{p}) \bar{v}^{(s)}(\vec{p}) = \not{p} - m$$

mass → $\approx 2.3 \text{ MeV}/c^2$ charge → $2/3$ spin → $1/2$	mass → $\approx 1.275 \text{ GeV}/c^2$ charge → $2/3$ spin → $1/2$	mass → $\approx 173.07 \text{ GeV}/c^2$ charge → $2/3$ spin → $1/2$	0 0 1	mass → $\approx 126 \text{ GeV}/c^2$ 0 0 0
u up	c charm	t top	g gluon	H Higgs boson
mass → $\approx 4.8 \text{ MeV}/c^2$ charge → $-1/3$ spin → $1/2$	mass → $\approx 95 \text{ MeV}/c^2$ charge → $-1/3$ spin → $1/2$	mass → $\approx 4.18 \text{ GeV}/c^2$ charge → $-1/3$ spin → $1/2$	0 0 1	
d down	s strange	b bottom	γ photon	
mass → $0.511 \text{ MeV}/c^2$ charge → -1 spin → $1/2$	mass → $105.7 \text{ MeV}/c^2$ charge → -1 spin → $1/2$	mass → $1.777 \text{ GeV}/c^2$ charge → -1 spin → $1/2$	91.2 GeV/c ² 0 1	
e electron	μ muon	τ tau	Z Z boson	
mass → $< 2.2 \text{ eV}/c^2$ charge → 0 spin → $1/2$	mass → $< 0.17 \text{ MeV}/c^2$ charge → 0 spin → $1/2$	mass → $< 15.5 \text{ MeV}/c^2$ charge → 0 spin → $1/2$	80.4 GeV/c ² ± 1 1	
ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	

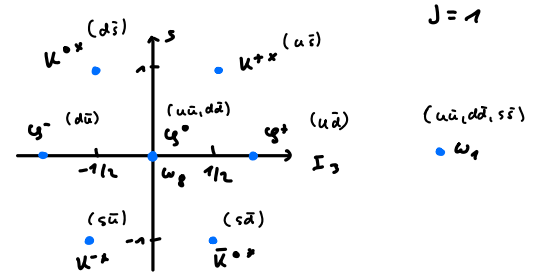
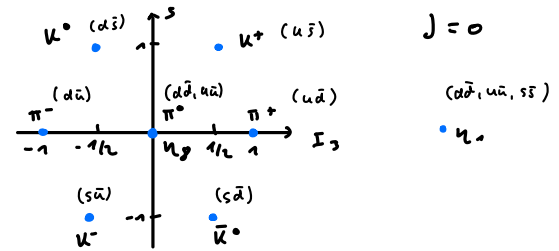
QUARKS

LEPTONS

GAUGE BOSONS



Mezoni $2 \bar{2}$



$J = 3/2$ antisimetrična

$$|\chi\rangle_0 = \frac{1}{\sqrt{6}} (RGB - GRB + \dots) = \frac{1}{\sqrt{6}} \epsilon_{ijk} |ijk\rangle$$

$$|\chi\rangle_3 = |\frac{3}{2} \frac{3}{2}\rangle = |\uparrow\uparrow\uparrow\rangle$$

$$|\chi\rangle_{-\frac{3}{2}} = |\downarrow\downarrow\downarrow\rangle$$

$uuu \quad |\Delta^{++}\rangle = |uuu\rangle$
 $uud \quad |\Delta^+\rangle = \frac{1}{\sqrt{3}} (uud + udu + duu)$
 $udd \quad |\Delta^0\rangle = \frac{1}{\sqrt{3}} (udd + ddu + duu)$
 $ddd \quad |\Delta^-\rangle = |ddd\rangle$
 $uus \quad |\Sigma^{*+}\rangle = \frac{1}{\sqrt{3}} (uus + usu + suu)$
 $uds \quad |\Sigma^{*0}\rangle = \frac{1}{\sqrt{6}} (uds + dus + dsu + usd + sud + sdu)$

$J = 1$

$u\bar{d} \quad |\pi^+\rangle = u\bar{d}$
 $u\bar{u}, d\bar{d} \quad |\pi^0\rangle = \frac{1}{\sqrt{2}} (u\bar{u} - d\bar{d})$
 $d\bar{u} \quad |\pi^-\rangle = d\bar{u}$
 $u\bar{s} \quad |K^+\rangle = u\bar{s}$
 $u\bar{u}, d\bar{d}, s\bar{s} \quad |\eta_1\rangle = \frac{1}{\sqrt{3}} (u\bar{u} + d\bar{d} + s\bar{s})$
 $u\bar{u}, d\bar{d}, s\bar{s} \quad |\eta_8\rangle = \frac{1}{\sqrt{6}} (u\bar{u} + d\bar{d} - 2s\bar{s})$

$|\chi\rangle_3 = |1, 1\rangle = \uparrow\uparrow$
 $|\chi\rangle_0 = \frac{1}{\sqrt{2}} (\uparrow\downarrow + \downarrow\uparrow)$
 $|\chi\rangle_{-1} = \downarrow\downarrow$
 $|\chi\rangle_0 = \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)$

Relativnost

$J = 1/2$

$$|\chi\rangle_3 = 2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow = |\frac{1}{2} \frac{1}{2}\rangle$$

$uud \quad |p\rangle = \frac{1}{\sqrt{6}} (uud + udu + duu)$
 $udd \quad |n\rangle = \frac{1}{\sqrt{6}} (udd + ddu + duu)$
 $uus \quad |\Sigma^+\rangle = \frac{1}{\sqrt{3}} (uus + usu + suu)$
 $uds \quad |\Sigma^0\rangle = \frac{1}{\sqrt{6}} (dus + uds + dsu + usd + sud + sdu)$
 $uds \quad |\Lambda^0\rangle = \frac{1}{\sqrt{12}} (s(u\bar{d} - d\bar{u}) + \uparrow(\uparrow\downarrow - \downarrow\uparrow) + (us\bar{d} - ds\bar{u})(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow) + (ud - du)s(\uparrow\downarrow - \downarrow\uparrow)\uparrow)$

$J_3 = 1/2$

$$\beta = \frac{v}{c} \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$p^\mu = (E, \vec{p})$$

$$E = m + T = \gamma m$$

$$E^2 = m^2 + p^2$$

$$p = \gamma \beta m = \beta E$$

$$T = (\gamma - 1)m$$