$H\Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$ $\qquad H = \frac{p^2}{2m} + V(x) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$ $\qquad p = -i\hbar\frac{\partial}{\partial x}$

$H\psi_n(x) = E_n \psi_n(x)$

Energijski nivo ni degeneriran če za en en. nivo obstaja le ena ψ.
En. nivo je n-krat deg. če je n lin. neodv. last. funk. za isto En.

zvezni del spektra

⋮

E_2 1. vzbujeno stanje

E_1 osnovno stanje

diskretni del spektra

Delec v konst. potencialu

za $V(x) = V_0 =$ konst.

$E > V_0$

$$\psi(x) = A e^{ikx} + B e^{-ikx} = C \sin kx + D \cos kx = F\cos(k(x-x_0))$$

$$k = \sqrt{\frac{2m(E-V_0)}{\hbar^2}} \qquad E = V_0 + \frac{\hbar^2 k^2}{2m}$$

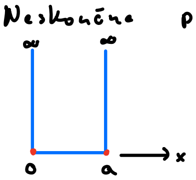
zvezen spekter

$E < V_0$

$$\psi(x) = A e^{-\kappa x} + B e^{\kappa x} = C \operatorname{ch} \kappa x + D \operatorname{sh} \kappa x$$

$$\kappa = \sqrt{\frac{2m(V_0-E)}{\hbar^2}} \qquad E = V_0 - \frac{\hbar^2 \kappa^2}{2m}$$

Neskončna potencialna jama



$\psi(0) = \psi(a) = 0$ robni pogoj

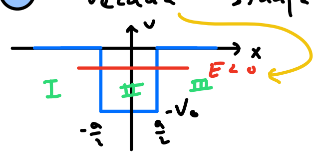
Diskretni spekter

$$\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a} \qquad n = 1, 2, \dots$$

normalizacija $\qquad \frac{n\pi}{a} = k \qquad \int_{-\infty}^{\infty} |\psi|^2 dx = 1$

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2}$$

1 Vezana stanja končne potencialne jame



I $\quad \psi_1 = A e^{\kappa x} + B e^{-\kappa x}$

II $\quad \psi_2 = C e^{ikx} + D e^{-ikx}$

III $\quad \psi_3 = F e^{\kappa x} + G e^{-\kappa x}$

$$\kappa = \sqrt{\frac{-2mE}{\hbar^2}} \qquad k = \sqrt{\frac{2m(V_0+E)}{\hbar^2}}$$

$\psi(\pm\infty) = 0 \Rightarrow G = 0 \quad B = 0$

$\psi_1(-\frac{a}{2}) = \psi_2(-\frac{a}{2})$

$\psi_2(\frac{a}{2}) = \psi_3(\frac{a}{2})$

$\psi_1'(-\frac{a}{2}) = \psi_2'(-\frac{a}{2})$

$\psi_2'(\frac{a}{2}) = \psi_3'(\frac{a}{2})$

$\Rightarrow$ Homogen sistem $\Rightarrow \det = 0 \Rightarrow E \Rightarrow A, C, D, F$

...

Izrek: če je $V(x)$ soda funkcija lahko najdemo take last funk. ki so ali soda ali liha.

Dokaz $\quad H\psi(x) = E\psi(x) \qquad H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$

$x \to -x$

$H\psi(-x) = E\psi(-x) \quad \Rightarrow \quad \psi(-x)$ tudi last funk.

→ En. nivo ni degeneriran $\Rightarrow \quad \psi(-x) = e^{i\alpha}\psi(x) \qquad |e^{i\alpha}| = 1$

$x \to -x$

$\psi(x) = e^{i\alpha}\psi(-x)$

$\psi(-x) = e^{2i\alpha}\psi(-x) \qquad \Rightarrow \quad e^{2i\alpha} = 1$

$\Rightarrow \quad e^{i\alpha} = \pm 1$

$\Rightarrow \quad \psi(-x) = \pm\psi(x)$

→ En. nivo je degeneriran : $\psi(x)$ in $\psi(-x)$ sta lahko lin. neodv.

$\psi_+ \propto \psi(x) + \psi(-x)$ $\quad$ soda

$\psi_- \propto \psi(x) - \psi(-x)$ $\quad$ liha

Našli smo bazo v kateri sta lastni funkciji sodi in lihi $\quad \square$

...

Soda

$A = F \qquad \psi_1 = Ae^{\kappa x} \qquad \psi_3 = Ae^{-\kappa x}$

$\psi_2 = B\cos(kx)$

$Ae^{-\kappa\frac{a}{2}} = B\cos\left(\frac{ka}{2}\right) \qquad x = -\frac{a}{2}$

$-A\kappa e^{-\kappa\frac{a}{2}} = -Bk\sin\left(k\frac{a}{2}\right)$

$:/ \qquad \kappa = k\tan\left(\frac{ka}{2}\right)$

Liha

$\psi_1 = Ae^{\kappa x} \qquad \psi_3 = -Ae^{-\kappa x}$

$\psi_2 = D\sin(kx)$

$-Ae^{-\kappa\frac{a}{2}} = D\sin\left(k\frac{a}{2}\right)$

$A\kappa e^{-\kappa\frac{a}{2}} = Dk\cos\left(k\frac{a}{2}\right)$

$-\kappa = k\cot\left(\frac{ka}{2}\right)$

$u = ka$

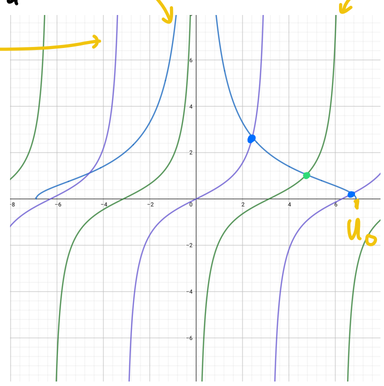
$\tan\left(\frac{u}{2}\right) = \frac{\kappa a}{u} \qquad\qquad -\cot\left(\frac{u}{2}\right) = \frac{\kappa a}{u}$

$\kappa^2 + k^2 = \frac{2m}{\hbar^2}(-E + V_0 + E) = \underbrace{\frac{2mV_0}{\hbar^2}}_{u_0^2} \qquad /\cdot a^2$

$\kappa^2a^2 + k^2a^2 = u_0^2$

$\tan\left(\frac{u}{2}\right) = \sqrt{\frac{u_0^2}{u^2} - 1} \qquad\qquad -\cot\left(\frac{u}{2}\right) = \sqrt{\frac{u_0^2}{u^2} - 1}$

Rešimo grafično



približno

$u \doteq n\pi$

$E = -V_0 + \frac{\hbar^2k^2}{2m} = -V_0 + \frac{\hbar^2u^2}{2ma^2}$

$= -V_0 + \frac{\hbar^2n^2\pi^2}{2ma^2}$

② Poglejmo si limitni primer naloge od zadnjič, ko $a \to 0$, $V_0 \to \infty$, $aV_0 = \lambda = \text{konst.}$
Takšnemu potencialu pravimo delta funkcija: $V(x) = -\lambda\,\delta(x)$.

$$\Rightarrow \quad u_0 = \sqrt{\frac{2mV_0a^2}{\hbar^2}} \xrightarrow[a\to 0]{V_0\to\infty} 0, \qquad u_0 \ll 1$$

Vidimo, da bo vezano stanje največ eno (in sicer s sodo simetrijo).

$$\tan\frac{u}{2} = \sqrt{\left(\frac{u_0}{u}\right)^2 - 1}$$

$$\Rightarrow \quad u = u_0 - \varepsilon \qquad \varepsilon \ll u_0$$

$$\frac{u}{2} = \frac{u_0-\varepsilon}{2} = \sqrt{\left(\frac{u_0}{u_0-\varepsilon}\right)^2 - 1} = \sqrt{\left(\frac{1}{1-\varepsilon/u_0}\right)^2 - 1} \overset{\text{Taylor}}{=} \sqrt{1+2\,\varepsilon/u_0 - 1} = \sqrt{\frac{2\varepsilon}{u_0}}$$

Želimo izračunati ε

$$\frac{u_0^2}{4} = \frac{2}{u_0}\varepsilon \Rightarrow \varepsilon = \frac{u_0^3}{8}$$ Upravičeno smo sklepali, da je $\varepsilon \ll u_0$

Sledi $$u = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}\,a$$

$$\left(\frac{u}{a}\right)^2\hbar^2 = 2m(E+V_0)$$

temu členu ne moremo zanemarit.

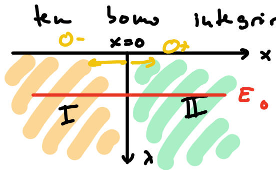
$$E = -V_0 + \frac{\hbar^2}{2m}\left(\frac{u}{a}\right)^2 = -V_0 + \frac{\hbar^2}{2m}\left(\frac{u_0 - u_0^3/8}{a}\right)^2 = -V_0 + \frac{\hbar^2}{2m}\frac{u_0^2}{a^2}\left(1 - \frac{u_0^3}{4} + \frac{u_0^4}{64}\right)$$

Vstavimo V_0

$$E = -V_0 + V_0\left(1 - \frac{u_0^2}{4}\right)$$

$$E = -\frac{m\lambda^2}{2\hbar^2}$$

Po tem bomo integrirali:



Čeprav imamo neskončen potencial, smo dobili končno energijo vezanega stanja.

Želimo najti še valovno funkcijo tega vezanega stanja

Od zadnjič $$\psi(|x|>a/2) = A e^{-\kappa|x|}$$
$$\psi(|x|<a/2) = B\cos(kx)$$

Naš (limitni) primer:

$$\psi_0(x) = A e^{-\kappa_0|x|}, \quad \kappa_0 = \sqrt{\frac{-2mE_0}{\hbar^2}} = \sqrt{\frac{2m^2\lambda^2}{2\hbar^2\cdot\hbar^2}} = \frac{m\lambda}{\hbar^2}$$

Normalizacija VF: $\int_{-\infty}^{\infty}\psi\cdot\psi^*\,dx = 1$. S pomočjo tega določimo A:

$$\int_{-\infty}^{\infty}\psi\cdot\psi^*\,dx = 2\cdot\int_{0}^{\infty}\psi\cdot\psi^*\,dx = 2\cdot\int_{0}^{\infty}Ae^{-\kappa_0 x}\cdot A^*\cdot e^{-\kappa_0 x}\,dx = 1$$

$$AA^*\int_{0}^{\infty}e^{-2\kappa_0 x}\,dx = \frac{1}{2}$$

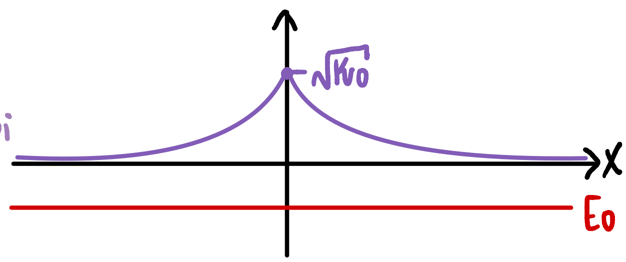
$$AA^*\cdot\frac{1}{(-2\kappa_0)}e^{-2\kappa_0 x}\Big|_{0}^{\infty} = AA^*\cdot\frac{1}{(-2\kappa_0)}\cdot\left[\frac{1}{e^{\infty}}-\frac{1}{e^{0}}\right] = \frac{AA^*}{(+\cancel{2}\kappa_0)}\cdot(+1) = \frac{1}{\cancel{2}}$$

$\Rightarrow |A|^2 = \kappa_0$ oz. $|A| = \sqrt{\kappa_0}$

$\Rightarrow A = \sqrt{\kappa_0}\cdot e^{i\alpha}$; $\alpha\in\mathbb{R}$, toda lahko vzamemo kar $A=\sqrt{\kappa_0}$

Naša rešitev: $\boxed{\psi_0(x) = \sqrt{\kappa_0}\cdot e^{-\kappa_0|x|}}$

Pove, kako hitro padajo exp. repi



Po našem izračunu v $x=0$ naša funkcija ni več zvezno odvedljiva, v naravi pa je.

To poskusimo izračunati še po drugi poti:

$$-\frac{\hbar^2}{2m}\cdot\frac{\partial^2\psi}{\partial x^2} - \lambda\cdot\delta(x)\psi = E\cdot\psi \qquad \text{SSE}$$

$$\psi_1(x) = Ae^{\kappa x} + \cancel{Be^{-\kappa x}}$$

Da bo ψ v $\pm\infty$ konvergirala

$$\psi_2(x) = \cancel{Ce^{\kappa x}} + De^{-\kappa x}$$

Robni pogoji: (*)

$$-\int_{0-}^{0+}\frac{\hbar^2}{2m}\cdot\frac{\partial^2\psi}{\partial x^2}\,dx - \lambda\cdot\int_{0-}^{0+}\delta(x)\psi\,dx = E\cdot\int_{0-}^{0+}\psi\,dx$$

Predpostavimo, da je končen ko jo bomo integrirali po infinitezimalno majhnem intervalu, bomo dobili 0.

$$\boxed{-\frac{\hbar^2}{2m}\cdot\frac{\partial\psi}{\partial x}\Big|_{0-}^{0+} - \lambda\cdot\psi(0) = 0}$$

Izpeljali smo robni pogoj namesto zveznosti odvoda in sicer razliko med levim in desnim odvodom.

Drug robni pogoj je še vedno $\boxed{\psi(0-) = \psi(0+)}$ → skoka v VF ne sme biti!

Vstavimo robna pogoja:

→ $\psi_1(0) = \psi_2(0) \Rightarrow \underline{A = D}$

→ $-\frac{\hbar^2}{2m}(-\kappa\cdot D - \kappa A) - \lambda\psi = 0$

$$\frac{\hbar^2}{2m}\kappa\cdot 2A = \lambda A$$

$$\lambda = \kappa\cdot\frac{\hbar^2}{m} \Rightarrow \underline{\kappa = \frac{m\lambda}{\hbar^2} = \kappa_0}$$

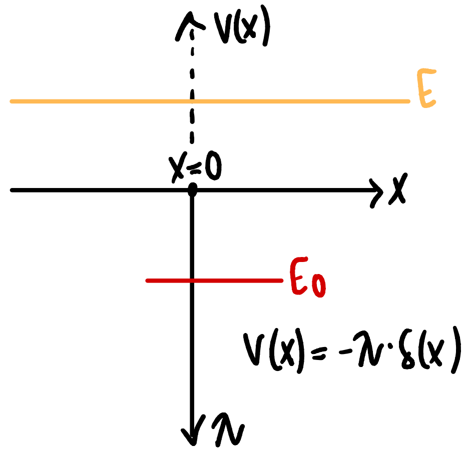
, po drugi strani pa iz nastavka vemo, da je $\kappa = \sqrt{-2mE/\hbar^2}$

$$\Rightarrow -\frac{2\cancel{m}E}{\hbar^2} = \frac{m^{\cancel{2}}\lambda^2}{\hbar^{\cancel{4}2}} \Rightarrow \underline{E = -\frac{m\lambda^2}{2\hbar^2} = E_0}$$

$$\psi(x) = A\cdot e^{-\kappa_0|x|}$$

✓ Dobimo isto kot prej

Zdaj se bomo ukvarjali še z nevezanimi oz. sipalnimi stanji.



Zanimale nas bodo verjetnosti za odboj in prepustnost.

$$\psi(x) = Ae^{ikx} + Be^{-ikx} \quad ; \quad k = \sqrt{\frac{2mE}{\hbar^2}} \quad (*)$$

verjetnostni tok delcev v tej VF:

$$j(x) = \frac{\hbar}{2mi}\left(\psi^*(x)\cdot\psi'(x) - \underbrace{\psi(x)\cdot\psi'^*(x)}_{\text{konjugiran 1. člen}}\right)$$

$$\Updownarrow$$

$$j(x) = \frac{\hbar}{m}\cdot\mathrm{Im}(\psi^*(x)\cdot\psi'(x)) =$$

$$= \frac{\hbar}{m}\cdot\mathrm{Im}\left(\left[A^*e^{-ikx} + B^*e^{ikx}\right]\left[Aike^{ikx} - Bike^{-ikx}\right]\right) =$$

$$= \frac{\hbar}{m}\cdot\mathrm{Im}\left[AA^*ik + B^*Aike^{2ikx} - A^*Bike^{-2ikx} - B^*Bik\right] =$$

$$= \frac{\hbar}{m}\cdot k\cdot\left(|A|^2 - |B|^2 + \cancelto{0}{\mathrm{Im}(-\|-)}\right) =$$

$$= \underbrace{\frac{\hbar k}{m}}_{v}\left(|A|^2 - |B|^2\right)$$

$$z - z^* = 2i\cdot\mathrm{Im}\,z$$
$$z + z^* = 2\cdot\mathrm{Re}\,z$$

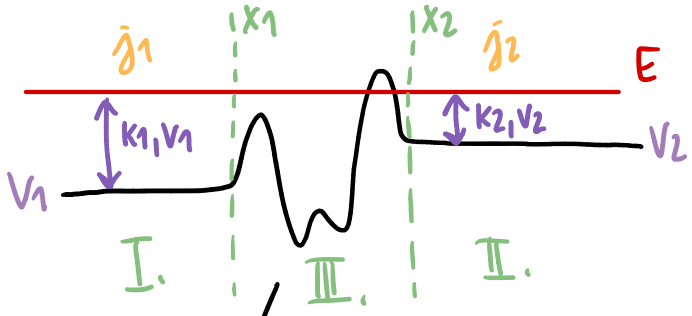
Od zdaj naprej VF ne bomo več pisali v obliki (*), ampak kot:

$$\psi(x) = A\frac{e^{ikx}}{\sqrt{v}} + B\frac{e^{-ikx}}{\sqrt{v}} \quad !$$

$$j(x) = |A|^2 - |B|^2$$

Ravne valove smo normirali na enoto verjetnostnega toka!

Obravnavamo zdaj nek splošni potencial, na katerem se delec lahko sipje.



Lepše nariši to sliko!

$$v_1 = \frac{\hbar k_1}{m}, \quad v_2 = \frac{\hbar k_2}{m}$$

$$\Psi_1 = A \cdot \frac{e^{ik_1x}}{\sqrt{v_1}} + B\frac{e^{-ik_1x}}{\sqrt{v_1}}$$

$$\Psi_2 = C \cdot \frac{e^{ik_2x}}{\sqrt{v_2}} + D\frac{e^{-ik_2x}}{\sqrt{v_2}}$$

V vmesnem območju velja DE: $-\frac{\hbar^2}{2m} \cdot \Psi''(x) + V_3(x) \cdot \Psi(x) = E \cdot \Psi(x)$

$\Downarrow$

Sledi: $\Psi_3 = \alpha \cdot \varphi(x) + \beta \cdot \phi(x)$

Linearna navadna dif. en.
$\Rightarrow$ Rešitev je linearna kom. lin. neodvisnih rešitev te enačbe ($\exists 2$ rešitvi)

Dogovorimo se še nekaj: $A, B \to A_1, B_1$; $C, D \to A_2, B_2$ } A-ja vpadata proti potencialu, B-ja pa stran od njega

Robni pogoji za tak problem:

$$\Psi_1(x_1) = \Psi_3(x_1) \qquad \Psi_2(x_2) = \Psi_3(x_2)$$
$$\Psi_1'(x_1) = \Psi_3(x_1) \qquad \Psi_2'(x_2) = \Psi_3'(x_2)$$

→ A_1 in A_2 sta v resnici vhodna parametra, ki ju pri eksperimentu poznamo in torej želimo izračunati le B_1 in B_2. Ko rešimo cel sistem enačb, bomo lahko dobili B_1, B_2, α, β.

Vpeljimo $\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$ in $\begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$. Ugotovili smo, da obstaja linearna preslikava iz $A \to B$. Sledi:

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = S \cdot \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}, \quad S \in Mat(2 \times 2, \mathbb{C})$$ sipalna matrika

$$B = SA$$

$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \xrightarrow{\text{hermitska}} [B_1^*, B_2^*] = B^\dagger = B^h$

hermitska ~~konjugacija?~~ ~~an~~jungacija? ADJUNGAC.

Procesov, kjer delci nastajajo ali izginjajo, še ne znamo opisati (2. stopnja → kvantna teorija polja), zato bomo mi predpostavili, da se število delcev ohranja. Verjetnostni tok ravnih valov, ki se širi proti sipalcu, mora biti enak verjetnostnemu toku, ki se širi stran od sipalca:

$$|A_1|^2 + |A_2|^2 = |B_1|^2 + |B_2|^2$$
$$\Updownarrow$$
$$|\vec{A}|^2 = |\vec{B}|^2$$

→ Toliko delcev, kot jih pošljemo noter, jih pride tudi ven

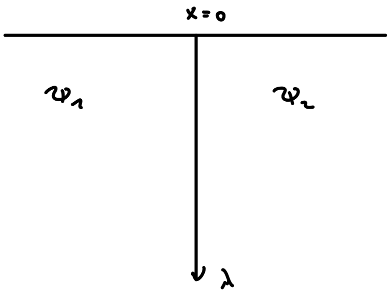
$$\underbrace{A^\dagger A} = B^\dagger B = \underbrace{A^\dagger S^\dagger S A}$$

$^*(SA)^\dagger = A^\dagger S^\dagger$

to mora veljati za vsak A! $\Rightarrow S^\dagger S = Id \to$ Unitarna!!!!

Če sipalna matrika ne bi bila unitarna, se število delcev ne bi ohranjalo.

③ Sipanje na delta funkciji



$V = -\lambda\,\delta(x)$

$v = \frac{\hbar k}{m}$

$A_1 \rightarrow$ $\quad$ $A_2 \leftarrow$

$\leftarrow B_1$ $\quad$ $\rightarrow B_2$

• $\psi_1 = \frac{e^{ikx}}{\sqrt{v}} + r\frac{e^{-ikx}}{\sqrt{v}}$

odbojnost

enote verjetnostnega toka

$\psi_2 = t\frac{e^{ikx}}{\sqrt{v}}$

prepustnost

$A_1 = 1 \qquad A_2 = 0$

val prihaja iz leve

posiljamo eno enoto sipalnega toka iz leve

$r, t = ?$

$\psi_1(0) = \psi_2(0)$

$\frac{1}{\sqrt{v}} + \frac{r}{\sqrt{v}} = \frac{t}{\sqrt{v}}$

$1 + r = t$

$K_0 = \frac{m\lambda}{\hbar^2}$

$-\frac{\hbar^2}{2m}\left(\psi_2'(0) - \psi_1'(0)\right) - \lambda\psi_1(0) = 0$

$-\frac{\hbar^2}{2m}\left(t\frac{ik}{\sqrt{v}} - \frac{ik}{\sqrt{v}} + r\frac{ik}{\sqrt{v}}\right) - \lambda\frac{t}{\sqrt{v}} = 0$

$-\frac{\hbar^2 ik}{2m}(t - 1 + r) - \lambda t = 0$

$-\frac{ik}{2}(t - 1 + r) - K_0 t = 0$

$\frac{ik}{K_0}(t-1) + t = 0$

$t = \frac{ik}{ik + K_0} \qquad r = \frac{K_0}{K_0 + ik}$

• $A_1 = 0 \quad A_2 = 0$ izkaže se za ta primer da $r' = r \quad t' = t$

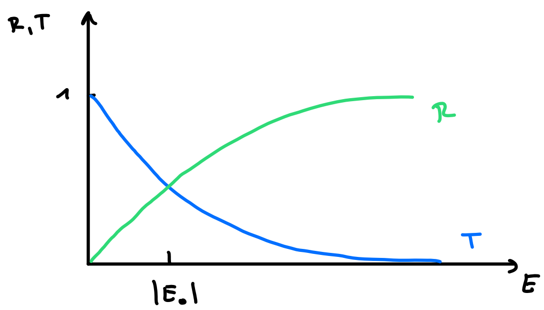
$S = \frac{1}{K_0 + ik}\begin{pmatrix} K_0 & ik \\ ik & K_0 \end{pmatrix}$

R ... odbojnost, $\quad R = |r|^2$

T ... prepustnost, $\quad T = |t|^2 \qquad R + T = 1$

$R = \frac{K_0^2}{(ik + K_0)(-ik + K_0)} = \frac{K_0^2}{K_0^2 + k^2} \qquad T = \frac{k^2}{K_0^2 + k^2} \Rightarrow R + T = 1$

$S^+ S = \begin{pmatrix} r^* & t^* \\ t'^* & r'^* \end{pmatrix}\begin{pmatrix} r & t' \\ t & r' \end{pmatrix} = \begin{pmatrix} rr^* + tt^* & r^*t' + t^*r' \\ rt'^* + tr'^* & t't'^* + r'r'^* \end{pmatrix} = I$



$k = \sqrt{\frac{2mE}{\hbar^2}} \qquad R = \frac{\frac{\hbar^2}{2m}K_0^2}{E + \frac{\hbar^2}{2m}K_0^2} \qquad T = \frac{E}{E + \frac{\hbar^2}{2m}K_0^2}$

$E_0 = -\frac{\hbar^2 K_0^2}{2m}$

$R = \frac{|E_0|}{E + |E_0|} \qquad T = \frac{E}{E + |E_0|}$

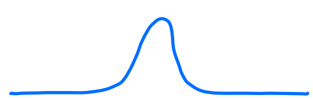
Sodost potenciala in vpliv na sipalno matriko S

$V(-x) = V(x)$

$H\psi(x) = E\psi(x)$

$x \rightarrow -x$ na H zamenjava ne vpliva

$H\psi(-x) = E\psi(-x)$



$\frac{A_1 e^{ikx}}{\sqrt{v}} + \frac{B_1 e^{-ikx}}{\sqrt{v}} \qquad \frac{A_2 e^{-ikx}}{\sqrt{v}} + \frac{B_2 e^{ikx}}{\sqrt{v}}$

$x \rightarrow -x \qquad \frac{A_2 e^{ikx}}{\sqrt{v}} + \frac{B_2 e^{-ikx}}{\sqrt{v}} \qquad \frac{A_1 e^{-ikx}}{\sqrt{v}} + \frac{B_1 e^{ikx}}{\sqrt{v}}$

$$\begin{pmatrix} A_2 \\ A_1 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{\sigma_x} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$

$$\sigma_x \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = S\, \sigma_x \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$

$$\sigma_x B = S \sigma_x A \quad / \sigma_x \qquad \sigma_x^2 = I$$

$$B = \sigma_x S \sigma_x A \qquad B = SA$$

$$\Rightarrow \quad S = \sigma_x S \sigma_x$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} r & t' \\ t & r' \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} t' & r \\ r' & t \end{pmatrix} = \begin{pmatrix} r' & t \\ t' & r \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \Rightarrow \begin{matrix} r = r' \\ t = t' \end{matrix}$$

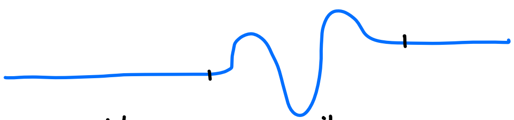
za $V(x) = -\lambda \delta(x) \longrightarrow S = \frac{1}{\kappa_0 + ik}\begin{pmatrix} -\kappa_0 & ik \\ ik & -\kappa_0 \end{pmatrix}$; $\kappa_0 = \frac{m\lambda}{\hbar^2}$

4

$$H\psi(x) = E\psi(x) \quad /^* \qquad H = \frac{p^2}{2m} + V(x) = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) = H^*$$

$$H\psi^*(x) = E\psi^*(x)$$

Če je ψ lastna funk. je tudi ψ^* last. fun.



$$A_2^* \frac{e^{ik_2 x}}{\sqrt{v_2}} + B_2^* \frac{e^{-ik_2 x}}{\sqrt{v_2}}$$

$$A_1^* \frac{e^{-ik_1 x}}{\sqrt{v_1}} + B_1^* \frac{e^{ik_1 x}}{\sqrt{v_1}}$$

$$\begin{pmatrix} A_1^* \\ A_2^* \end{pmatrix} = S\begin{pmatrix} B_1^* \\ B_2^* \end{pmatrix}$$

$$S^+ / \quad A^* = S B^*$$

$$S^+ A^* = S^+ S B^*$$

$$S^+ A^* = B^* \quad /^*$$

$$B = (S^+ A^*)^*$$

$$B = S^T A \qquad \text{in od prej } B = SA$$

$$\Rightarrow \quad S^T = S \quad \Rightarrow t = t'$$

Primer ko $H \neq H^*$, delec v magnetnem polju

$$H = \frac{(\vec{p} - e\vec{A})^2}{2m} + V(\vec{r}) = \frac{(-i\hbar\nabla - e\vec{A})^2}{2m} + V(x) \neq H^*$$

5 $\quad V(x) = -\lambda\delta(x) \qquad S = \frac{1}{\kappa_0 + ik}\begin{pmatrix} -\kappa_0 & ik \\ ik & -\kappa_0 \end{pmatrix}$

S ima pol pri $\kappa_0 + ik = 0$ oz. $k = i\kappa_0$

$\Rightarrow E = \frac{\hbar^2 k^2}{2m} = -\frac{\hbar^2 \kappa_0^2}{2m} = E_0$ energija vez. stanja je pri polu.

pri $k = i\kappa_0$: $S = \frac{1}{\underbrace{\kappa_0 + ik}_{\to\, \infty}}\begin{pmatrix} -\kappa_0 & -\kappa_0 \\ -\kappa_0 & -\kappa_0 \end{pmatrix}$

$k \to i\kappa_0$

$e^{ikx} + re^{-ikx} \qquad te^{ikx}$

$\Rightarrow e^{-\kappa_0 x} + \infty(-\kappa_0)e^{\kappa_0 x} \qquad \infty(-\kappa_0)e^{-\kappa_0 x}$

lahko zanemarimo

normiramo : $Ae^{-\kappa_0|x|} \longrightarrow \sqrt{\kappa_0}\, e^{-\kappa_0|x|}$

valovna funkcija vezanega stanja

⑥ Heisenbergovo načelo nedoločnosti

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$

Poišči vse valovne funkcije $\psi(x)$ za katere velja $\sigma_x \sigma_p = \frac{\hbar}{2}$

① Dokaz načela nedoločnosti

$\sigma A \sigma B$ A in B sta opazljivki ; $A^\dagger = A$, $B^\dagger = B$

$\sigma A = \sqrt{\langle (A - \langle A \rangle)^2 \rangle}$

$$\sigma A^2 \sigma B^2 = \langle (A - \langle A\rangle)^2\rangle \langle (B - \langle B\rangle)^2\rangle$$
$$= \langle \tilde{A}^2 \rangle \langle \tilde{B}^2 \rangle$$
$$= \langle \psi | \tilde{A}^2 \psi\rangle \langle \psi | \tilde{B}^2 \psi \rangle$$
$$= \langle \tilde{A}\psi | \tilde{A}\psi\rangle \langle \tilde{B}\psi | \tilde{B}\psi\rangle \geq \dots$$

vpeljemo $\tilde{A} \equiv A - \langle A \rangle$ $\tilde{B} = B - \langle B \rangle$

$\tilde{A}^\dagger = (A - \langle A\rangle)^\dagger = A^\dagger + (-\langle A \rangle)^* = A - \langle A \rangle = \tilde{A}$

za hermitske op. $\langle A \rangle = \langle A \rangle^*$

v splošnem $\langle \psi | c \psi \rangle = \langle c^\dagger \psi | \psi \rangle$

za herm. to $\langle \psi | c\psi \rangle = \langle c\psi | \psi\rangle$

Cauchy-Schwartzova neenakost

$$\dots \geq |\langle \tilde{A}\psi | \tilde{B}\psi\rangle|^2 =$$
$$= |\langle \psi | \tilde{A}\tilde{B}\psi\rangle|^2 =$$
$$= \left|\left\langle \psi \left| \frac{\tilde{A}\tilde{B} - \tilde{B}\tilde{A} + \tilde{A}\tilde{B} + \tilde{B}\tilde{A}}{2} \psi\right.\right\rangle\right|^2 =$$
$$= \left|\frac{1}{2}\langle \psi | ([\tilde{A},\tilde{B}] + \{\tilde{A},\tilde{B}\})\psi\rangle\right|^2 =$$
$$= \left|\frac{1}{2}(\underbrace{\langle \psi | [\tilde{A},\tilde{B}]\psi\rangle}_{\in i\mathbb{R}} + \underbrace{\langle \psi | \{\tilde{A},\tilde{B}\}\psi\rangle}_{\in \mathbb{R}})\right|^2 =$$

$|x+iy|^2 = x^2 + y^2$

$$= \frac{1}{4}|\langle \psi | [\tilde{A},\tilde{B}]\psi\rangle|^2 + \frac{1}{4}\langle \psi | \{\tilde{A},\tilde{B}\}\psi\rangle^2 \geq$$
$$\geq \left|\frac{1}{2}\langle \psi | [\tilde{A},\tilde{B}]\psi\rangle\right|^2 =$$
$$= \left|\frac{1}{2}\langle \psi | [A,B]\psi\rangle\right|^2$$

- $[\tilde{A},\tilde{B}] = \tilde{A}\tilde{B} - \tilde{B}\tilde{A}$ komutator
- $\{\tilde{A},\tilde{B}\} = \tilde{A}\tilde{B} + \tilde{B}\tilde{A}$ antikomutator
- $[A,B]^\dagger = (AB - BA)^\dagger = B^\dagger A^\dagger - A^\dagger B^\dagger = BA - AB = -[A,B]$
- $\{A,B\}^\dagger = B^\dagger A^\dagger + A^\dagger B^\dagger = BA + AB = \{A,B\}$

  Antikomutator dveh hermitskih operatorjev je hermitski operator. Komutator pa je antihermitski.
- Naj bo $C^\dagger = -C$ (antihermitski)

  $\langle C \rangle = \langle \psi | C\psi\rangle = \langle C^\dagger \psi, \psi\rangle = -\langle C\psi, \psi\rangle =$
  $= -\langle \psi | C\psi\rangle^* = -\langle C\rangle^* \Rightarrow \langle C \rangle \in i\mathbb{R}$
- $[\tilde{A},\tilde{B}] = [A - \langle A\rangle, B - \langle B \rangle] =$
  $= (A - \langle A\rangle)(B - \langle B\rangle) - (B - \langle B\rangle)(A - \langle A\rangle)$
  $= AB - A\cancel{\langle B\rangle} - \cancel{\langle A\rangle}B + \cancel{\langle A\rangle\langle B\rangle}$
  $- BA + B\cancel{\langle A\rangle} + \cancel{\langle B\rangle}A - \cancel{\langle B\rangle\langle A\rangle} = [A,B]$

$A = x$, $B = p$, $[x,p] = i\hbar$

$$\sigma A \sigma B \geq \frac{1}{2}|\langle \psi | [A,B]\psi\rangle| =$$
$$= \frac{1}{2}|\langle \psi | i\hbar \psi\rangle| =$$
$$= \frac{1}{2}\hbar \langle \psi | \psi \rangle = \frac{\hbar}{2}$$

Kdaj velja $\sigma A \sigma B = \frac{1}{2}|\langle \psi | [A,B]\psi\rangle|$?

① $\langle A\psi | A\psi\rangle\langle B\psi | B\psi\rangle = |\langle A\psi | B\psi\rangle|^2 \Rightarrow |B\psi\rangle = \lambda |A\psi\rangle$ $\lambda \in \mathbb{C}$ sta vzporedna

② $\langle \psi | \{A,B\}\psi\rangle = 0$

$\hookrightarrow \langle \psi | \{\tilde{A},\tilde{B}\}\psi\rangle = \langle \psi | (\tilde{A}\tilde{B} + \tilde{B}\tilde{A})\psi\rangle =$

$= \langle \psi | \tilde{A}\tilde{B}\psi\rangle + \langle \psi | \tilde{B}\tilde{A}\psi\rangle = \langle \tilde{A}\psi | \tilde{B}\psi\rangle + \langle \tilde{B}\psi | \tilde{A}\psi\rangle =$

$= \langle \tilde{A}\psi | \lambda\tilde{A}\psi\rangle + \langle \lambda\tilde{A}\psi | \tilde{A}\psi\rangle = \lambda\langle \tilde{A}\psi, \tilde{A}\psi\rangle + \lambda^*\langle \tilde{A}\psi | \tilde{A}\psi\rangle =$

$= (\lambda + \lambda^*)\langle \tilde{A}\psi | \tilde{A}\psi\rangle = 0$

$\hookrightarrow \lambda + \lambda^* = 0 \rightarrow \lambda \in i\mathbb{R}$

ali: $\langle \tilde{A}\psi | \tilde{A}\psi\rangle = 0 = \sigma^2 A$

$\tilde{B}|\psi\rangle = \lambda \tilde{A}|\psi\rangle$

$\tilde{B}|\psi\rangle = i\mu \tilde{A}|\psi\rangle$ $\mu \in \mathbb{R}$

$A = x$, $B = p$

$(p - \langle p \rangle)|\psi\rangle = i\mu(x - \langle x\rangle)|\psi\rangle$

$(-i\hbar\frac{\partial}{\partial x} - \langle p \rangle)\psi(x) = i\mu(x - \langle x\rangle)\psi(x)$

$$(-\langle p\rangle - i\mu x + i\mu\langle x\rangle)\psi(x) = i\hbar\frac{\partial}{\partial x}\psi(x)$$

$$\int(-\langle p\rangle - i\mu x + i\mu\langle x\rangle)\,dx = i\hbar\int\frac{1}{\psi}\,d\psi$$

$$-\langle p\rangle x - i\mu\frac{x^2}{2} + i\mu\langle x\rangle x + C = i\hbar\ln\psi$$

$$-\frac{\langle p\rangle}{i\hbar}x - \frac{\mu}{\hbar 2}x^2 + \frac{\mu}{\hbar}\langle x\rangle x + C = \ln\psi$$

$$-\frac{\mu}{2\hbar}(x-\langle x\rangle)^2 + \frac{\mu}{2\hbar}\langle x\rangle^2 + \frac{i\langle p\rangle}{\hbar}x + C = \ln\psi$$

$$\psi = C\exp\left(-\frac{\mu}{2\hbar}(x-\langle x\rangle)^2\right)\exp\left(\frac{i\langle p\rangle}{\hbar}x\right)$$

Normalizacija $1 = \int_{-\infty}^{\infty} C^2 e^{\frac{i\langle p\rangle}{\hbar}x + \left(-\frac{\mu}{2\hbar}(x-\langle x\rangle)^2\right)^2 + -\frac{i\langle p\rangle}{\hbar}x}\,dx$

$$1 = \int_{-\infty}^{\infty} C^2 e^{-\frac{\mu}{\hbar}(x-\langle x\rangle)^2}\,dx$$

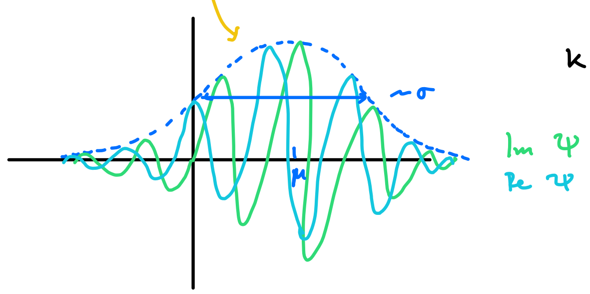
$$\Downarrow$$

$$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\langle x\rangle)^2}{2\sigma^2}}$$

$$C = (2\pi\sigma^2)^{-\frac{1}{4}} \qquad 2\sigma^2 = \frac{\hbar}{\mu} \qquad \sigma = \frac{\hbar}{2\mu}$$

$$C = \sqrt[4]{\frac{\mu}{\pi\hbar}}$$

$\Rightarrow$ $\boxed{\psi(x) = \underbrace{\sqrt[4]{\frac{1}{2\pi\sigma^2}}}\, e^{\frac{i\langle p\rangle x}{\hbar}}\, \underbrace{e^{-\frac{(x-\langle x\rangle)^2}{4\sigma^2}}}}$ Gaussovi valovni paketi



$$k = \frac{\langle p\rangle}{\hbar}$$

Časovni razvoj

$$H = \frac{p^2}{2m}$$

$$H\psi = E\psi \qquad \psi_k = \frac{1}{\sqrt{2\pi}}e^{ikx} \qquad E_k = \frac{\hbar^2k^2}{2m}$$

Razvijemo ψ po last. funk. H in dodamo časovne faktorje

$$\psi(x) = \int dk\, c(k)\, e^{ikx}\frac{1}{\sqrt{2\pi}}$$

$$c(k) = \int dx\, \psi(x, t=0)\, e^{-ikx}\frac{1}{\sqrt{2\pi}}$$

$$\Rightarrow \quad \psi(x,t) = \int dk\, c(k)\, e^{-i\frac{E_k t}{\hbar}}\, e^{ikx}\frac{1}{\sqrt{2\pi}}$$

$\langle x, t=0\rangle = \langle x\rangle$

$\langle p, t=0\rangle = \langle p\rangle$

$\sigma_x(t=0) = \sigma \qquad \sigma_x\sigma_p = \frac{\hbar}{2}$

$\sigma_p(t=0) = \frac{\hbar}{2\sigma}$

...

$\langle A, t\rangle = ? = \langle \psi, t | A | \psi, t\rangle$

↳ Računanje v Schrödingerjevi sliki

$$|\psi, t\rangle = e^{-\frac{iHt}{\hbar}} |\psi, 0\rangle$$

$$\langle A, t\rangle = \langle \psi, t | A \, e^{-\frac{iHt}{\hbar}} |\psi, 0\rangle$$

$$\langle \psi, t| = (|\psi, t\rangle)^{+} = \langle \psi, 0| e^{\frac{iHt}{\hbar}}$$

$$\langle A, t\rangle = \langle \psi, 0| \underbrace{e^{\frac{iHt}{\hbar}} A \, e^{-\frac{iHt}{\hbar}}}_{A(t)} |\psi, 0\rangle$$

↳ Rešujemo v Heisenbergovi sliki

$$(AB)(t) = e^{\frac{iHt}{\hbar}} A \underset{I}{\cdot} B \, e^{-\frac{iHt}{\hbar}} = e^{\frac{iHt}{\hbar}} A \, e^{-\frac{iHt}{\hbar}} e^{\frac{iHt}{\hbar}} B \, e^{-\frac{iHt}{\hbar}} = A(t) B(t)$$

$$(\alpha A + \beta B)(t) = \alpha A(t) + \beta B(t)$$

$$\frac{d}{dt} A(t) = e^{\frac{iHt}{\hbar}} \left(\frac{iH}{\hbar} A - A \frac{iH}{\hbar} \right) e^{-\frac{iHt}{\hbar}} = \frac{i}{\hbar} e^{\frac{iHt}{\hbar}} [H, A] \, e^{-\frac{iHt}{\hbar}} = \frac{i}{\hbar} [H, A](t)$$

...

$$\frac{d}{dt} x(t) = \frac{i}{\hbar} [H, x](t) = \frac{i}{\hbar} \left[\frac{p^2}{2m}, x \right](t) = \frac{p(t)}{m}$$

$$\frac{d}{dt} p(t) = \frac{i}{\hbar} [H, p](t) = \frac{i}{\hbar} [p^2/2m, p](t) = 0$$

$$\left[\frac{p^2}{2m}, x \right] = \frac{1}{2m} p [p, x] + \frac{1}{2m} [p, x] p \qquad [x, p] = i\hbar$$

$$= \frac{1}{2m} p (-i\hbar) + \frac{1}{2m} (-i\hbar) p$$

$$= -\frac{i\hbar}{m} p$$

$$\frac{d}{dt} x(t) = \frac{1}{m} p(t) \Rightarrow x(t) = \frac{p}{m} t + C = \frac{p}{m} t + x \qquad x(0) = x \Rightarrow C = x$$

$$\frac{d}{dt} p(t) = 0 \Rightarrow p(t) = C = p(0) = p$$

$$\langle p, t\rangle = \langle \psi, 0 | p(t) | \psi, 0\rangle = \langle \psi, 0 | p | \psi, 0\rangle = \langle p, 0\rangle$$

$$\langle x, t\rangle = \langle \psi, 0 | x(t) | \psi, 0\rangle = \langle \psi, 0 | \tfrac{p}{m} t + x | \psi, 0\rangle = \frac{\langle p, 0\rangle t}{m} + \langle x, 0\rangle = \frac{\langle p\rangle}{m} t + \langle x\rangle$$

Teorija / ponovitev

$$\psi(x, t=0) = \frac{1}{\sqrt[4]{2\pi\sigma^2}} e^{i\frac{\langle p\rangle x}{\hbar}} e^{-\frac{(x - \langle x\rangle)^2}{4\sigma^2}}$$

$$\langle A, t\rangle = \langle \psi, t | A | \psi, t\rangle = \langle \psi, 0 | A(t) | \psi, 0\rangle \qquad A(t) = e^{i\frac{H}{\hbar}t} A \, e^{-i\frac{H}{\hbar}t}$$

$$\frac{d}{dt} A(t) = \hbar [H, A](t) \quad \text{in zač. pog.} \quad A(0) = A$$

$$(AB)(t) = A(t) B(t)$$

$$\delta x^2(t) = \langle x^2, t\rangle - \langle x, t\rangle^2$$

$$\langle x^2, t\rangle = \langle \psi, 0 | x^2(t) | \psi, 0\rangle = \langle \psi, 0 | (x(t))^2 | \psi, 0\rangle = \langle \psi, 0 | (\tfrac{pt}{m} + x)^2 | \psi, 0\rangle$$

$$= \langle \psi, 0 | \frac{p^2 t^2}{m^2} + \frac{t}{m}(px + xp) + x^2 | \psi, 0\rangle =$$

$$= \frac{t^2}{m^2} \langle p^2 | 0\rangle + \langle x^2, 0\rangle + \frac{t}{m} \langle px + xp | 0\rangle$$

$$\delta_p(t)^2 = \langle p^2, t\rangle - \langle p, t\rangle^2$$

$$\langle p^2, t\rangle = \langle \psi, 0 | p^2(t) | \psi, 0\rangle = \langle \psi, 0 | (p(t))^2 | \psi, 0\rangle = \langle p^2, 0\rangle$$

Se ne spreminja s časom

$$\delta p(t)^2 = \langle p^2, 0\rangle - \langle p, 0\rangle^2 = \delta p(0)^2$$

$$\delta x(t)^2 = \frac{t^2}{m^2}\langle p^2, 0\rangle + \langle x^2, 0\rangle + \frac{t}{m}\langle px + xp, 0\rangle - \frac{t^2}{m^2}\langle p, 0\rangle^2 - \langle x, 0\rangle^2 - 2\frac{t}{m}\langle p, 0\rangle\langle x, 0\rangle$$

$$= \sigma_x(0)^2 + \frac{t^2}{m^2}\sigma_p(0)^2 + \ldots$$

$$= \sigma^2 + \frac{t^2}{m^2}\frac{\hbar^2}{4\sigma^2} + \ldots$$

$$\langle xp + px, 0\rangle \underset{x^\dagger = x \quad p^\dagger = p}{=} \langle xp + (xp)^\dagger, 0\rangle = \langle xp, 0\rangle + \langle (xp)^\dagger, 0\rangle = \langle xp, 0\rangle + \langle xp, 0\rangle^* = 2\,\mathrm{Re}\langle xp, 0\rangle$$

$$(px)^\dagger = x^\dagger p^\dagger = xp \qquad \langle A^\dagger\rangle = \langle \psi | A^\dagger \psi\rangle = \langle A\psi | \psi\rangle = \langle \psi | A\psi\rangle^* = \langle A\rangle^*$$

$$\langle xp, 0\rangle = \langle \psi | xp | \psi\rangle = \quad \ldots$$

$$p\psi = -i\hbar\frac{\partial}{\partial x}\psi = -i\hbar\left(i\frac{\langle p\rangle}{\hbar} - \frac{(x - \langle x\rangle)}{4\sigma^2}2\right)\psi = \left(\langle p\rangle + \frac{i\hbar}{2\sigma^2}(x - \langle x\rangle)\right)\psi$$

$$\ldots = \mathrm{Re}\left(\int_{-\infty}^{\infty}\psi^*\left(x\langle p\rangle + \underbrace{\frac{i\hbar}{2\sigma^2}(x - \langle x\rangle)x}_{\mathrm{Im}}\right)\psi\,dx\right) = \langle p\rangle\int\psi^* x\psi\,dx = \langle p\rangle\langle x\rangle$$

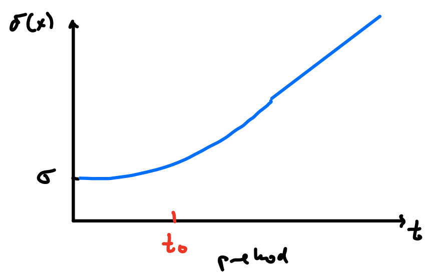
$$\Rightarrow \langle xp + px, 0\rangle = 2\langle p\rangle\langle x\rangle$$

$$\Rightarrow \sigma_x(t)^2 = \sigma^2 + \frac{t^2}{m^2}\frac{\hbar^2}{4\sigma^2} + \frac{t}{m}\cdot 2\langle x\rangle\langle p\rangle - \frac{t}{m}2\langle x\rangle\langle p\rangle$$

$$\delta x(t) = \sigma\sqrt{1 + \frac{t^2\hbar^2}{4m^2\sigma^4}}$$

$$\delta x\,\delta p = \sigma\sqrt{1 + \frac{t^2\hbar^2}{4m^2\sigma^4}}\;\frac{\hbar}{2\sigma} = \frac{\hbar}{2}\sqrt{1 + \frac{t^2\hbar^2}{4m^2\sigma^4}} \geq \frac{\hbar}{2}$$

le ob $t=0$



prehod med režimoma

$$\frac{t_0^2\hbar^2}{4m^2\sigma^4} = 1$$

$$t_0 = \frac{2m\sigma^2}{\hbar}$$

Ⓣ Harmonski oscilator

$$H = \frac{p^2}{2m} + \frac{1}{2}kx^2 = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) \quad ; \quad \omega = \sqrt{\frac{k}{m}}$$

$$a = \frac{1}{\sqrt{2}}\left(\frac{x}{x_0} + i\frac{p}{p_0}\right) \qquad x_0 = \sqrt{\frac{\hbar}{m\omega}} \qquad p_0 = \frac{\hbar}{x_0}$$

anihilacijski

$$a^\dagger = \frac{1}{\sqrt{2}}\left(\frac{x}{x_0} - i\frac{p}{p_0}\right) \qquad \Rightarrow \quad x = \frac{x_0}{\sqrt{2}}(a + a^\dagger)$$

kreacijski

$$p = \frac{p_0}{\sqrt{2}i}(a - a^\dagger)$$

Lastna stanja

$$H|n\rangle = \overbrace{\hbar\omega\left(n + \frac{1}{2}\right)}^{E_n}|n\rangle \qquad n = 0, 1, \ldots \qquad \langle n | m\rangle = \delta_{nm}$$

$$a|n\rangle = \sqrt{n}\,|n-1\rangle \qquad a|0\rangle = 0$$

$$a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$a^\dagger a|n\rangle = n|n\rangle$$ operator števila $\qquad [a, a^\dagger] = 1$

(7) $|\psi,0\rangle = \frac{1}{\sqrt2}|0\rangle + \frac{i}{\sqrt2}|1\rangle$

$|\psi,t\rangle = ?$
$\langle x,t\rangle = ?$
$\langle p,t\rangle = ?$
$\sigma_x(t) = ?$

$$|\psi,t\rangle = \frac{1}{\sqrt2}e^{-i\frac{E_0}{\hbar}t}|0\rangle + \frac{i}{\sqrt2}e^{-i\frac{E_1}{\hbar}t}|1\rangle = \frac{1}{\sqrt2}e^{-i\frac{\omega}{2}t}|0\rangle + \frac{i}{\sqrt2}e^{-i\frac{3}{2}\omega t}|1\rangle$$

$$\langle x,t\rangle = \langle \frac{x_0}{\sqrt2}(a+a^\dagger),t\rangle = \frac{x_0}{\sqrt2}(\langle a,t\rangle + \langle a^\dagger,t\rangle) = \frac{x_0}{\sqrt2}(\langle a,t\rangle + \langle a,t\rangle^*) =$$

$$\boxed{\langle x,t\rangle = x_0\sqrt2\,\mathrm{Re}\langle a,t\rangle}$$ Splošno za harm. osc.

$$\langle p,t\rangle = \frac{p_0}{\sqrt2 i}(\langle a,t\rangle - \langle a^\dagger,t\rangle) = \sqrt2 p_0\,\mathrm{Im}\langle a,t\rangle$$

$$\boxed{\langle p,t\rangle = \sqrt2 p_0\,\mathrm{Im}\langle a,t\rangle}$$

$\langle\psi| = (|\psi\rangle)^*$

Schrödingerjeva slika

$$\langle a,t\rangle = \langle\psi,t|a|\psi,t\rangle =$$

$$= \left(\frac{1}{\sqrt2}e^{i\frac{\omega}{2}t}\langle 0| - \frac{i}{\sqrt2}e^{i\frac{3}{2}\omega t}\langle 1|\right)\underbrace{a\left(\frac{1}{\sqrt2}e^{-i\frac{\omega}{2}t}|0\rangle\right.}_{0} + \underbrace{\left.\frac{i}{\sqrt2}e^{-i\frac{3}{2}\omega t}|1\rangle\right)}_{\frac{i}{\sqrt2}e^{-i\frac{3}{2}\omega t}1|0\rangle}$$

$$= \frac{1}{\sqrt2}e^{i\frac{\omega}{2}t}\frac{i}{\sqrt2}e^{-i\frac{3}{2}\omega t}\overset{1}{\overset{\shortparallel}{\langle 0|0\rangle}} - \frac{i}{\sqrt2}e^{i\frac{3}{2}\omega t}\frac{i}{\sqrt2}e^{-i\frac{3}{2}\omega t}\underset{0}{\underset{\shortparallel}{\langle 1|0\rangle}}$$

$$= \frac{i}{2}e^{-i\omega t} = \frac{1}{2}(i\cos(\omega t) + \sin(\omega t))$$

$$\langle x,t\rangle = x_0\frac{\sqrt2}{2}\sin\omega t \qquad \langle p,t\rangle = p_0\frac{\sqrt2}{2}\cos\omega t$$

Ehrenfestov teorem

$$\boxed{\begin{array}{l}\frac{d}{dt}\langle x,t\rangle = \frac{\langle p,t\rangle}{m}\\[2ex] \frac{d}{dt}\langle p,t\rangle = \langle -\frac{d}{dx}V(x)\rangle\end{array}}$$

$$\longrightarrow \frac{d}{dt}\sin\omega t\,\frac{\sqrt2}{2}x_0 = \frac{1}{m}\frac{\sqrt2}{2}p_0\cos\omega t$$

$$\cos\omega t\,\frac{\sqrt2}{2}\omega x_0 = \frac{p_0}{m}\frac{\sqrt2}{2}\cos\omega t \quad ✓$$

$$= \langle -\tfrac{1}{2}k\,2x\rangle = -k\langle x\rangle$$

$$\omega x_0 = \frac{p_0}{m}$$

$$\sqrt{\frac{\hbar}{m\omega}} = \frac{\hbar}{\sqrt{\frac{\hbar}{m\omega}}\,m\omega} \quad ✓$$

Heisenbergova slika

$$\langle x,t\rangle = \sqrt2 x_0\,\mathrm{Re}\langle a(t),0\rangle \qquad \langle p,t\rangle = \sqrt2 p_0\,\mathrm{Im}\langle a(t),0\rangle$$

$$\boxed{a(t) = e^{i\frac{H}{\hbar}t}\,a\,e^{-i\frac{H}{\hbar}t}}$$

$$\boxed{\frac{d}{dt}a(t) = \frac{i}{\hbar}[H,a](t) \qquad \text{z. p.} \quad a(0) = a}$$

$$\boxed{[AB,C] = A[B,C] + [A,C]B}$$

$$\boxed{(AB)(t) = A(t)B(t) \qquad (\alpha A + \beta B)(t) = \alpha A(t) + \beta B(t)}$$

$$[H,a] = \hbar\omega[a^\dagger a, a] + \hbar\omega\underbrace{[\tfrac{1}{2},a]}_{0} = \hbar\omega\left(a^\dagger\underbrace{[a,a]}_{0} + \underbrace{[a^\dagger,a]}_{-1}a\right) = -\hbar\omega\, a(t)$$

$$\frac{d}{dt}a = -i\omega a$$

$$\frac{da}{a} = -i\omega\, dt \qquad \ln a = -i\omega t \qquad a(t) = Ce^{-i\omega t} \qquad C = a(0)$$

$$\langle a(t)\rangle = \langle\psi|a(t)|\psi\rangle = \left(\frac{1}{\sqrt2}\langle 0| - \frac{i}{\sqrt2}\langle 1|\right)\underbrace{\underbrace{a e^{-i\omega t}\left(\frac{1}{\sqrt2}|0\rangle\right.}_{=0} + \left.\frac{i}{\sqrt2}|1\rangle\right)}_{=|0\rangle} =$$

$$= e^{-i\omega t}\left(\frac{1}{\sqrt2}\langle 0| - \frac{i}{\sqrt2}\langle 1|\right) \frac{i}{\sqrt2}|0\rangle = \frac{i}{2} e^{-i\omega t}$$ ✓

$$\langle x^2, t\rangle = \langle \frac{x_0^2}{2}(a+a^\dagger)^2, t\rangle = \frac{x_0^2}{2}\langle a^2 + aa^\dagger + a^\dagger a + a^\dagger a^\dagger, t\rangle = \dots$$

Normalni vrstni red $(a^\dagger)^n a^m$ $\qquad aa^\dagger = 1 + a^\dagger a$

$$\dots = \frac{x_0^2}{2}\langle a^2 + 1 + 2a^\dagger a + a^{\dagger 2}, t\rangle = \frac{x_0^2}{2}\left(2\,\mathrm{Re}\langle a^2,t\rangle + 1 + 2\langle a^\dagger a, t\rangle\right)$$

V Heisenbergovi sliki

$a^2(t) = a(t)^2 \qquad a^\dagger a(t) = a^\dagger(t)\, a(t)$

$$\boxed{\begin{aligned} a(t) &= a e^{-i\omega t} \\ a^\dagger(t) &= a^\dagger e^{i\omega t}\end{aligned}}$$

$$A^\dagger(t) = e^{i\frac{H}{\hbar}t} A^\dagger e^{-i\frac{H}{\hbar}t} = \left(e^{i\frac{H}{\hbar}t} A e^{-i\frac{H}{\hbar}t}\right)^\dagger = (A(t))^\dagger$$

zamenjamo vrstni red in konjugiramo

$a^2(t) = a^2 e^{-2i\omega t} \qquad a^\dagger a(t) = a^\dagger a$

$$\langle x^2,t\rangle = \frac{x_0^2}{2}\left(2\,\mathrm{Re}\langle a^2 e^{-2i\omega t}\rangle + 1 + 2\langle a^\dagger a\rangle\right) = x_0^2\left(\langle a^2\rangle \cos(2\omega t) + \frac12 + \langle a^\dagger a\rangle\right)$$

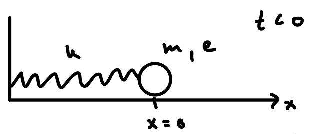
Za naš primer

$$\langle a^2\rangle = \langle\psi|a^2|\psi\rangle = \left(\frac{1}{\sqrt2}\langle0| - \frac{i}{\sqrt2}\langle1|\right) a^2 \left(\underbrace{\frac{1}{\sqrt2}|0\rangle}_{0} + \frac{i}{\sqrt2}|1\rangle\right) = 0$$

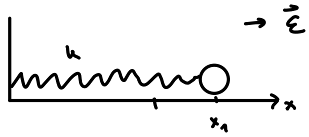
$$\langle a^\dagger a\rangle = \left(\frac{1}{\sqrt2}\langle0| - \frac{i}{\sqrt2}\langle1|\right)a^\dagger a\left(\frac{1}{\sqrt2}|0\rangle + \frac{i}{\sqrt2}|1\rangle\right) = \left(\frac{1}{\sqrt2}\langle0| - \frac{i}{\sqrt2}\langle1|\right)\frac{i}{\sqrt2}|1\rangle = \frac12$$

$$\delta x(t)^2 = \langle x^2,t\rangle - \langle x,t\rangle^2 = x_0^2\left(0 + \frac12 + \frac12\right) - \left(x_0 \frac{\sqrt2}{2}\sin\omega t\right)^2 = \frac{x_0^2}{2}(2 - \sin^2\omega t)$$

8 Vzmetno nihalo



$\rightarrow \vec{\mathcal{E}}$ električno polje



Kroglica bo nihala okoli x_1 z amplitudo $x_1 - 0 = x_1$

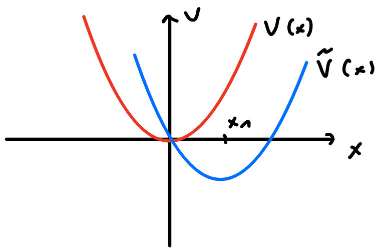
$$x(t) = x_1(1-\cos\omega t)$$

$$x_1 = \frac{e\mathcal{E}}{k}$$

$$H = \frac{p^2}{2m} + \frac12 kx^2 \qquad t<0$$
$$\tilde H = \frac{p^2}{2m} + \frac12 kx^2 - e\mathcal{E}x \qquad t>0$$

$$\tilde V = \frac12 k\left(\left(x - \frac{e\mathcal E}{k}\right)^2 - \left(\frac{e\mathcal E}{k}\right)^2\right) \qquad x_1 = \frac{e\mathcal E}{k}$$
$$= \frac12 k\left((x-x_1)^2 - x_1^2\right)$$

Sistem smo mirovati pri $t=0$ zato je v osnovnem stanju $|\psi,0\rangle = |0\rangle$, ker je nedoločenost $\hat x$ in $\hat p$ najmanjša v osnovnem stanju



Naj bo $\tilde x = x - x_1$, $\quad \tilde p = -i\hbar\frac{d}{d\tilde x} = -i\hbar\frac{d}{dx} = p$

$$\tilde H = \frac{\tilde p^2}{2m} + \frac12 k\tilde x^2 - \frac12 kx_1^2$$
$$= \hbar\omega\left(\tilde a^\dagger\tilde a + \frac12\right) - \frac12 kx_1^2 \qquad \tilde a = \frac{1}{\sqrt2}\left(\frac{\tilde x}{x_0} + i\frac{\tilde p}{p_0}\right)$$

$$a|0\rangle = 0$$
$$a|\psi,0\rangle = 0$$

$$\tilde{a} = \frac{1}{\sqrt{2}}\left(\frac{\hat{x}}{x_0} + i\frac{\hat{p}}{p_0}\right) = \frac{1}{\sqrt{2}}\left(\frac{x}{x_0} + \frac{p}{p_0}\right) - \frac{1}{\sqrt{2}}\frac{x_1}{x_0} = a - \frac{x_1}{\sqrt{2}x_0}$$

$$\left(\tilde{a} + \frac{x_1}{\sqrt{2}x_0}\right)|\psi,0\rangle = 0$$

Koherentno stanje $\tilde{a}|\psi,0\rangle = -\frac{x_1}{\sqrt{2}x_0}|\psi,0\rangle$ $\Rightarrow$ Osnovno stanje začetnega a je $\hat{a}$ tudi lastno stanje.

Koherentno stanje je lastno stanje a.
$$\hat{a}|z\rangle = z|z\rangle \qquad z \in \mathbb{C}$$
$$\hat{a} \neq \hat{a}^{\dagger} \quad \text{ni hermitski} \rightarrow z \in \mathbb{C}$$

Ⓣ Lastnosti koherentnih stanj

$$a|z\rangle = z|z\rangle$$

$$|z,t\rangle = ?$$

$$|z\rangle = \sum_{n=0}^{\infty} c_n |n\rangle \quad \Big\} \text{ v enačbo}$$

$$\sum c_n a|n\rangle = \sum_n z c_n |n\rangle$$
$n-1=m$
$$\sum_{n=1}^{\infty} c_n\sqrt{n}|n-1\rangle = \sum_{n=0}^{\infty} z c_n |n\rangle$$
$$\sum_{m=0} c_{m+1}\sqrt{m+1}|m\rangle = \sum_{n=0}^{\infty} z c_n |n\rangle \qquad \langle l|$$
$$c_{m+1}\sqrt{m+1}\,\delta_{ml} = z c_n \delta_{nl}$$
$$c_{l+1}\sqrt{l+1} = z c_l$$

$$c_{l+1} = \frac{z}{\sqrt{l+1}} c_l$$

Členi $c_0 \quad c_1 = \frac{z}{\sqrt{1}}c_0 \quad c_2 = \frac{z^2}{\sqrt{2\cdot 1}}c_0 \quad \dots \quad c_n = \frac{z^n}{\sqrt{n!}}c_0$

$$|z\rangle = c_0 \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!}}|n\rangle$$

$\hookrightarrow$ c_0 iz normalizacije $1 = \langle z|z\rangle = c_0^* c_0 \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{z^{*n}}{\sqrt{n!}}\frac{z^m}{\sqrt{m!}}\langle n,m\rangle$

$$1 = c_0^* c_0 \sum_{n=0}^{\infty}\frac{z^{*n} z^n}{n!}$$

$$1 = |c_0|^2\sum_{n=0}^{\infty}\frac{|z|^{2n}}{n!} = |c_0|^2 e^{|z|^2} \Rightarrow c_0 = e^{-\frac{|z|^2}{2}}$$

$$\Rightarrow |z\rangle = e^{-\frac{|z|^2}{2}}\sum_{n=0}^{\infty}\frac{z^n}{\sqrt{n!}}|n\rangle$$

$$|z,t\rangle = e^{-\frac{|z|^2}{2}}\sum_{n=0}^{\infty}\frac{z^n}{\sqrt{n!}}e^{-i\omega(n+\frac{1}{2})t}|n\rangle$$
$$= e^{-\frac{|z|^2}{2}-i\omega t/2}\sum_{n=0}^{\infty}\frac{1}{\sqrt{n!}}\left(z e^{-i\omega t}\right)^n|n\rangle$$
$$= \underbrace{e^{-\frac{|ze^{i\omega t}|^2}{2}}}\, e^{-i\omega t/2}\underbrace{\sum_{n=0}^{\infty}\frac{1}{\sqrt{n!}}\left(z e^{-i\omega t}\right)^n|n\rangle} = \text{koherentno stanje}$$
$$= \underline{\underline{e^{-i\omega t/2}\,|z e^{-i\omega t}\rangle}}$$

Lastnosti

$$\langle a \rangle = \langle z | \underbrace{a | z}_{z|z\rangle} \rangle = z \langle z | z \rangle = z$$

$$\langle x \rangle = \sqrt{2}\, x_0 \operatorname{Re} z$$

$$\langle p \rangle = \sqrt{2}\, p_0 \operatorname{Im} z$$

pazi vrstni red

$$\langle z | (a^+)^n a^m | z \rangle = z^m \langle z | (a^+)^n | z \rangle = z^m \langle a^n z | z \rangle = z^{*n} z^m$$

$$[a, a^+] = 1$$

$$\langle x^2 \rangle = \frac{x_0^2}{2} \langle aa + aa^+ + a^+a + a^+a^+ \rangle \overset{\text{v normalni vrstni red } a, a^+}{=} \frac{x_0^2}{2} \langle a^2 + a^+a + 1 + a^+a + (a^+)^2 \rangle$$

$$= \frac{x_0^2}{2} \left(z^2 + 2z^*z + 1 + z^{*2} \right) = \frac{x_0^2}{2} \left(1 + (z + z^*)^2\right) = \frac{x_0^2}{2}\left(1 + 4(\operatorname{Re} z)^2\right)$$

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2 = \frac{x_0^2}{2}\left(1 + 4(\operatorname{Re} z)^2\right) - (\sqrt{2}\, x_0 \operatorname{Re} z) = \frac{x_0^2}{2}$$

$$\sigma_p^2 = \overset{\text{DN}}{\ldots} = \frac{p_0^2}{2}$$

$$\sigma_x \sigma_p = \frac{1}{2} x_0 p_0 = \frac{1}{2} x_0 \frac{\hbar}{x_0} = \frac{\hbar}{2}$$

Koherentno stanje je Gaussov valovni paket ker je $\sigma_x \sigma_p$ minimalna

Zapišemo v koordinatnem zapisu

$$\sigma_x = \sigma \qquad \left(\text{Gaussov val. paket} \quad \psi(x) = \frac{1}{\sqrt[4]{2\pi\sigma^2}} e^{-(x - \langle x \rangle)^2/4\sigma^2} e^{i \langle p \rangle / \hbar}\, x \right.$$

$$\psi_z(x) = \frac{1}{\sqrt[4]{\pi x_0^2}} e^{-(x - \sqrt{2} x_0 \operatorname{Re} z)^2 / 2x_0^2} e^{i \frac{\sqrt{2} p_0}{\hbar} \operatorname{Im} z\, x}$$

Gaussov val. paket se s časom ne spreminja v harmonskem potencialu.

$$\langle H \rangle = \langle z | H | z \rangle = \langle z | \hbar\omega (a^+a + \tfrac{1}{2}) | z \rangle = \hbar\omega z^* z + \hbar\omega \tfrac{1}{2} = \hbar\omega (|z|^2 + \tfrac{1}{2})$$

$$\langle H^2 \rangle = \hbar^2\omega^2 \langle z | (a^+a)^2 + a^+a + \tfrac{1}{4} | z \rangle = \hbar^2\omega^2 (z^{*2} z^2 + 2z^*z + \tfrac{1}{4}) = \hbar^2\omega^2 (|z|^4 + 2|z|^2 + \tfrac{1}{4})$$

$$\hookrightarrow a^+aa^+a = a^+(a^+a + 1)a = a^{+2}a^2 + a^+a$$

$$\sigma^2_H = \langle H^2 \rangle - \langle H \rangle^2 = \hbar^2\omega^2 \left(|z|^4 + 2|z|^2 + \tfrac{1}{4} - |z|^4 - |z|^2 - \tfrac{1}{4} \right) = \hbar^2\omega^2 |z|^2$$

$$\sigma_H = \hbar\omega |z|$$

$$\frac{\sigma_H}{\langle H \rangle} = \frac{|z|}{|z|^2 + \frac{1}{2}} \qquad \frac{\sigma_x}{\langle x \rangle} = \frac{x_0/\sqrt{2}}{\sqrt{2}\, x_0 \operatorname{Re}(z)} = \frac{1}{2\operatorname{Re}(z)}$$

$$z(0) = -\frac{x_1}{\sqrt{2} x_0} \qquad \tilde{a} |\psi, 0\rangle = -\frac{x_1}{\sqrt{2} x_0} |\psi, 0\rangle$$

$$z(t) = -\frac{x_1}{\sqrt{2} x_0} e^{-i\omega t}$$

$$\langle x, t \rangle = ?$$

$$\langle \tilde{x}, t \rangle = \sqrt{2}\, x_0 \operatorname{Re} z = -\operatorname{Re} x_1 e^{-i\omega t} = -x_1 \cos \omega t \qquad \tilde{x} = x - x_1$$

$$\langle x, t \rangle = x_1 - x_1 \cos \omega t$$

Do tu za test

9 2D harmonski oscilator

$$H = \frac{p^2}{2m} + \frac{1}{2}k_x x^2 + \frac{1}{2}k_y y^2 \qquad p^2 = (\vec{p})^2 \qquad \vec{p} = (p_x, p_y) = \left(-i\hbar\frac{\partial}{\partial x}, -i\hbar\frac{\partial}{\partial y}\right) = -i\hbar\nabla$$

$$= H_x + H_y \qquad H_x = \frac{p_x^2}{2m} + \frac{1}{2}k_x x^2$$

↑ kadar lahko ločimo tako, uporabimo separacijo spremenljivk

$$H\,\Psi(x,y) = E\,\Psi(x,y) \qquad \text{Nastavek } \Psi(x,y) = \varphi(x)\,\chi(y) \qquad \text{kjer } H_x\varphi_n(x) = E_n\varphi_n(x)$$

$$\text{tedaj } H\varphi_n\chi_m = (E_n + E_m)\,\varphi_n(x)\,\chi_m(x) \qquad H_y\chi_m(y) = E_m\chi_m(y)$$

$$H|n\rangle_x|m\rangle_y = (E_n + E_m)|n\rangle_x|m\rangle_y$$

$$= \quad H|n\,m\rangle = (E_n + E_m)|n\,m\rangle$$

$$H_x|n\rangle = \hbar\omega_x\left(n + \tfrac{1}{2}\right)|n\rangle \qquad \omega_x = \sqrt{\frac{k_x}{m}}$$

$$H_y|m\rangle = \hbar\omega_y\left(m + \tfrac{1}{2}\right)|m\rangle$$

$$H|n\,m\rangle = \hbar\left(\omega_x\left(n+\tfrac{1}{2}\right) + \omega_y\left(m+\tfrac{1}{2}\right)\right)|n\,m\rangle$$

$k_x > 0,\ k_y > 0$

Kaj pa če $k_x > 0$, $k_y = 0$

H_x je enak kot prej

$H_y = \frac{p_y^2}{2m} + 0$ delec v konst potencialu, rešitve ravni valovi

$$p_y\,e^{iqy} = \hbar q\,e^{iqy}$$

$$H_y\,e^{iqy} = \frac{\hbar^2 q^2}{2m}\,e^{iqy}$$

$$H_y|q\rangle = \frac{\hbar^2 q^2}{2m}|q\rangle$$

$$H|n, q\rangle = \left(\frac{\hbar^2 q^2}{2m} + \hbar\omega_x\left(n+\tfrac{1}{2}\right)\right)|n, q\rangle$$

Nadaljujmo z $k_x > 0$, $k_y > 0$ in $k_x = k_y = k$

$$H = \frac{p^2}{2m} + \frac{1}{2}k r^2 = \frac{p^2}{2m} + V(r)$$

centralni potencial

$$H|n\,m\rangle = \hbar\omega\,(n + m + 1)\,|n\,m\rangle$$

E[ħω]
- 4: $|30\rangle\ |21\rangle\ |12\rangle\ |03\rangle$
- 3: $|20\rangle\ |02\rangle\ |11\rangle$
- 2: $|10\rangle, |01\rangle \longrightarrow \alpha|10\rangle + \beta|01\rangle$
- 1: $|00\rangle$
- 0

Kolikšna je energija superpozicije lastnih stanj

$\alpha|10\rangle + \beta|01\rangle$ kjer $|\alpha|^2 + |\beta|^2 = 1$

$$H(\alpha|10\rangle + \beta|01\rangle) = \alpha H|10\rangle + \beta H|01\rangle =$$

$$= \alpha\hbar\omega(1+0+1)|10\rangle + \beta\hbar\omega(0+1+1)|01\rangle$$

$$= 2\hbar\omega\,(\alpha|10\rangle + \beta|01\rangle)$$

↑ tudi to je lastno stanje z isto energijo

Pojavi se degeneracija, ki je ni bilo v 1D.

Za centralni potencial velja:

$$[H, L_z] = 0 \qquad L_z = x p_y - y p_x = -i\hbar\frac{\partial}{\partial\varphi}$$

če operatorja komutirata lahko najdemo skupne lastne funkcije obeh operatorjev

$$L_z|\Psi\rangle = \lambda|\Psi\rangle$$

$$-i\hbar\frac{\partial}{\partial\varphi}\Psi(\varphi) = \lambda\,\Psi(\varphi)$$

$$\int\frac{d\Psi}{\Psi} = \int -\frac{\lambda}{i\hbar}\,d\varphi$$

$$\Psi = C\,e^{i\frac{\lambda}{\hbar}\varphi}$$

Ψ je periodična ⇒ R.P. $\Psi(\varphi) = \Psi(2\pi n + \varphi) \Rightarrow \frac{\lambda}{\hbar} = m \Rightarrow \lambda = \hbar m \quad m \in \mathbb{Z}$

Normalizacija $\int_0^{2\pi} |\psi|^2 d\varphi = 1 \Rightarrow C = \frac{1}{\sqrt{2\pi}}$

Lastne funkcije L_z $\psi(\varphi) = \frac{1}{\sqrt{2\pi}} e^{im\varphi}$ z lastnimi vrednostmi $\lambda = m\hbar$

$$\psi(x,y) = \alpha \psi_{10}(x,y) + \beta \psi_{01}(x,y)$$
$$\alpha\, \psi_1(x)\psi_0(y) + \beta\, \psi_0(x)\psi_1(y)$$

$$\psi_0(x) = \frac{1}{\sqrt[4]{\pi x_0^2}} e^{-x^2/2x_0^2}$$

$a|0\rangle = 0|0\rangle$ koherentno stanje $\Rightarrow$ gausov val. paket

$a^+|0\rangle = |1\rangle$ $a^+ = \frac{1}{\sqrt{2}}\left(\frac{x}{x_0} - i\frac{p}{p_0}\right)\psi_0(x) = \psi_1(x)$

$$\psi_1(x) = \frac{1}{\sqrt{2}}\left(\frac{x}{x_0} - i\frac{p}{p_0}\right)\frac{1}{\sqrt[4]{\pi x_0^2}} e^{-x^2/2x_0^2} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt[4]{\pi x_0^2}} e^{-x^2/2x_0^2}\left(\frac{x}{x_0} - i\frac{(-i\hbar)}{p_0}\frac{(-2x)}{2x_0^2}\right)$$

$$= \frac{1}{\sqrt{2}\sqrt[4]{\pi x_0^2}} e^{-x^2/2x_0^2}\left(\frac{x}{x_0} + \underbrace{\frac{\hbar x}{x_0^2 p_0}}_{\frac{x}{x_0}}\right) \qquad p_0 = \frac{\hbar}{x_0}$$

$$= \frac{\sqrt{2}}{\sqrt[4]{\pi x_0^2}} e^{-x^2/2x_0^2}\frac{x}{x_0} = \sqrt{2}\frac{x}{x_0}\psi_0(x)$$

$$\psi_{10}(x,y) = \sqrt{2}\frac{x}{x_0}\psi_0(x)\psi_0(y) = \sqrt{\frac{2}{\pi}}\frac{x}{x_0^2} e^{-(x^2+y^2)/2x_0^2} \qquad x_0 = \sqrt{\frac{\hbar}{m\omega}}$$
$$\psi_{01}(x,y) = \sqrt{2}\frac{y}{y_0}\psi_0(x)\psi_0(y) = \sqrt{\frac{2}{\pi}}\frac{y}{x_0^2} e^{-(x^2+y^2)/2x_0^2}$$

$$\psi_{10}(r,\varphi) = \sqrt{\frac{2}{\pi}}\frac{r\cos\varphi}{x_0^2} e^{-r^2/2x_0^2} = \cos\varphi\, f(r)$$
$$\psi_{01}(r,\varphi) = \sqrt{\frac{2}{\pi}}\frac{r\sin\varphi}{x_0^2} e^{-r^2/2x_0^2} = \sin\varphi\, f(r)$$

$f(r)(\cos\varphi + i\sin\varphi) = e^{i\varphi} f(r)$ $(m=1)$

$f(r)(\cos\varphi - i\sin\varphi) = e^{-i\varphi} f(r)$ $(m=-1)$

$= \alpha|10\rangle + \beta|01\rangle$ Samo te dve sta lastni stanji obeh operatorjev H in L_z.

$$|m=1\rangle = \frac{1}{\sqrt{2}}\left(1|10\rangle + i|01\rangle\right)$$
$$|m=-1\rangle = \frac{1}{\sqrt{2}}\left(1|10\rangle - i|01\rangle\right)$$

normalizacija

Alternativni postopek

$|\psi\rangle = \alpha|10\rangle + \beta|01\rangle$

$L_z|\psi\rangle = \lambda|\psi\rangle$

$L_z(\alpha|10\rangle + \beta|01\rangle) = \lambda(\alpha|10\rangle + \beta|01\rangle)$

$\alpha L_z|10\rangle + \beta L_z|01\rangle = \alpha\lambda|10\rangle + \lambda\beta|01\rangle$ $\langle nm|n'm'\rangle = \delta_{nn'}\delta_{mm'}$

$\alpha\langle 10|L_z|10\rangle + \beta\langle 10|L_z|01\rangle = \alpha\lambda$ $/\ \langle 10|\cdot$

$\alpha\langle 01|L_z|10\rangle + \beta\langle 01|L_z|01\rangle = \beta\lambda$ $/\ \langle 01|\cdot$

$$\begin{bmatrix}\langle 10|L_z|10\rangle & \langle 10|L_z|01\rangle \\ \langle 01|L_z|10\rangle & \langle 01|L_z|01\rangle\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \lambda\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$$

matrični elementi

$L_z = xp_y - yp_x = -i\hbar\frac{\partial}{\partial\varphi}$ $x = \frac{x_0}{\sqrt{2}}(a + a^+)$ $p = \frac{p_0}{\sqrt{2}i}(a - a^+)$

$x_0 = \sqrt{\frac{\hbar}{m\omega}}$ $p_0 = \frac{\hbar}{x_0}$

$$H = H_x + H_y = \hbar\omega(a_x a_x^+ + \tfrac{1}{2}) + \hbar\omega(a_y a_y^+ + \tfrac{1}{2})$$

$\Rightarrow$ $y = \frac{x_0}{\sqrt{2}}(a_y + a_y^+)$ $p_y = \frac{p_0}{\sqrt{2}}(a_y - a_y^+)$

$$L_z = \frac{x_0}{\sqrt{2}}\frac{p_0}{\sqrt{2}\,i}\Big((a_x + a_x^\dagger)(a_y - a_y^\dagger) - (a_y + a_y^\dagger)(a_x - a_x^\dagger)\Big)$$

$$= \frac{\hbar}{2i}\Big(a_x a_y - a_x a_y^\dagger + a_x^\dagger a_y - a_x^\dagger a_y^\dagger - a_y a_x + a_y a_x^\dagger - a_y^\dagger a_x + a_y^\dagger a_x^\dagger \Big)$$

$$= \frac{\hbar}{2i}\Big(\underbrace{[a_x, a_y]}_{=0} + 2\, a_y a_x^\dagger - 2\, a_x a_y^\dagger + \underbrace{[a_y^\dagger, a_x^\dagger]}_{=0} \Big) = \frac{\hbar}{i}\left(a_y a_x^\dagger - a_x a_y^\dagger \right)$$

$$[a_x\ a_y] = [a_x\ a_y^\dagger] = [a_x^\dagger\ a_y] = \dots = 0$$

$$\boxed{L_z = \frac{\hbar}{i}\left(a_x^\dagger a_y - a_x a_y^\dagger\right)}\quad \text{2D LHO}$$

$$\langle \overset{x}{1}\overset{y}{0} |\, L_z \,| 10\rangle = \frac{\hbar}{i}\langle 10 |\, a_x^\dagger a_y - a_x a_y^\dagger \,| 10\rangle = \frac{\hbar}{i}\Big(\langle 10 | a_x^\dagger a_y | 10\rangle - \langle 10 | a_x a_y^\dagger | 10\rangle\Big) = \dots$$

$$a|n\rangle = \sqrt{n}\,|n-1\rangle$$
$$a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$a_x^\dagger a_y |10\rangle = a_x^\dagger a_y |1\rangle_x |0\rangle_y = a_x^\dagger |1\rangle_x\, a_y |0\rangle_y = 0$$

$$= \frac{\hbar}{i}\left(0 - \langle 10 | 01\rangle\right) = 0$$

$$\langle 10 |\, L_z \,| 01\rangle = \frac{\hbar}{i}\langle 10 |\, a_x^\dagger a_y - a_x a_y^\dagger \,| 01\rangle = \frac{\hbar}{i}\langle 10 | \left(|10\rangle - 0\right) = \frac{\hbar}{i}$$

$$\langle 01 |\, L_z \,| 10\rangle = \frac{\hbar}{i}\langle 01 |\, a_x^\dagger a_y - a_x a_y^\dagger \,| 10\rangle = \frac{\hbar}{i}\langle 01 | \left(0 - |01\rangle\right) = -\frac{\hbar}{i}$$

$$\langle 01 |\, L_z \,| 01\rangle = \frac{\hbar}{i}\langle 01 |\, a_x^\dagger a_y - a_x a_y^\dagger \,| 01\rangle = \frac{\hbar}{i}\langle 01 | \left(|10\rangle - 0\right) = 0$$

$$i\hbar \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \lambda \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \Rightarrow \begin{array}{l} -\alpha = \frac{i\lambda}{\hbar}\beta \\ \beta = \frac{i\lambda}{\hbar}\alpha \end{array} \quad -\alpha = -\frac{\lambda^2}{\hbar^2}\alpha \Rightarrow \lambda^2 = \hbar^2, \quad \lambda = \pm\hbar$$

$$\Downarrow$$

$$\det A - \lambda I = 0 \Rightarrow \lambda = \pm\hbar$$

$$\Downarrow \ \lambda = \hbar \qquad\qquad \lambda = -\hbar$$
$$\alpha = -i\beta \qquad\qquad \alpha = i\beta$$

Normalizacija $|\alpha|^2 + |\beta|^2 = 1$

$$|\beta|^2 + |\beta|^2 = 1 \qquad\qquad |\beta|^2 + |\beta|^2 = 1$$
$$|\beta|^2 = \frac{1}{2} \qquad\qquad \Rightarrow \beta = \frac{1}{\sqrt{2}}$$
$$\beta = \frac{1}{\sqrt{2}} \qquad\qquad \alpha = \frac{i}{\sqrt{2}}$$
$$\alpha = -\frac{i}{\sqrt{2}}$$

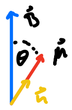
$$\Rightarrow |\psi_1\rangle = -\frac{i}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|01\rangle \qquad |\psi_2\rangle = \frac{i}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|01\rangle$$
$$m = 1 \qquad\qquad m = -1$$

$$L_z|m\rangle = m\hbar\,|m\rangle$$

Rezultat od prej je le zamaknjen za fazo $(-i)$

$$|m=1\rangle = \frac{1}{\sqrt{2}}\left(1|10\rangle + i|01\rangle\right)$$
$$|m=-1\rangle = \frac{1}{\sqrt{2}}\left(1|10\rangle - i|01\rangle\right)$$

10



magnetni moment $\qquad H = -\vec\mu\cdot\vec B \qquad \vec\mu = -\mu_0 \frac{\vec L}{\hbar}$

$$\Downarrow$$

$H = \lambda \vec L\cdot\vec B \qquad$ za $l = 1$ $\;\to\;$ baza $|11\rangle \quad |10\rangle \quad |1\,{-1}\rangle$

$|l\,m\rangle$

$L^2|l\,m\rangle = \hbar^2 l(l+1)|l\,m\rangle$

$L_z|l\,m\rangle = \hbar m|l\,m\rangle \qquad l \to m = -l,\dots,l$

Začetni pogoj

$\vec L\cdot\vec n\,|\psi,0\rangle = \hbar|\psi,0\rangle \qquad \vec n = (\sin\theta,\ 0,\ \cos\theta)$

$|\psi,t\rangle = ?$

$|11\rangle \ : \ L_z|11\rangle = \hbar|11\rangle$ kaže v smer z.

$\vec L\,\hat e_z|11\rangle = \hbar|11\rangle$

$$|\psi,0\rangle = a|11\rangle + b|10\rangle + c|1\,{-1}\rangle$$

$$\vec L\cdot\vec n = L_x\sin\theta + L_z\cos\theta$$

$$\left(L_x\sin\theta + L_z\cos\theta\right)\left(a|11\rangle + b|10\rangle + c|1\,{-1}\rangle\right) = \hbar\left(a|11\rangle + b|10\rangle + c|1\,{-1}\rangle\right)$$

$$\boxed{\begin{array}{l} L_\pm = L_x \pm iL_y \qquad L_x = \frac12(L_+ + L_-) \qquad L_y = \frac{1}{2i}(L_+ - L_-) \\ L_\pm|l\,m\rangle = \hbar\sqrt{l(l+1) - m(m\pm1)}\ |l\ m\pm1\rangle \end{array}}$$

$$L_x|11\rangle = \frac12(L_+ + L_-)|11\rangle = \frac12\hbar\Big(\underbrace{\sqrt{2-2}}_{0}\,\underbrace{|12\rangle}_{0} + \sqrt{2-0}\,|10\rangle\Big) = \frac{\sqrt2}{2}\hbar|10\rangle$$

$$L_x|10\rangle = \frac12(L_+ + L_-)|10\rangle = \frac12\hbar\Big(\sqrt{2-0}\,|11\rangle + \sqrt2\,|1\,{-1}\rangle\Big) = \frac{\sqrt2}{2}\hbar\left(|11\rangle + |1\,{-1}\rangle\right)$$

$$L_x|1\,{-1}\rangle = \frac{\sqrt2}{2}\hbar|10\rangle$$

$$\sin\theta\Big(a\frac{\sqrt2}{2}\hbar|10\rangle + b\frac{\sqrt2}{2}\hbar\left(|11\rangle + |1\,{-1}\rangle\right) + c\frac{\sqrt2}{2}\hbar|10\rangle\Big) + \cos\theta\Big(a\hbar\,1|11\rangle + b\hbar\,0|10\rangle + c\hbar(-1)|1\,{-1}\rangle\Big)$$

$$= \sin\theta\frac{\sqrt2}{2}\hbar\Big(a|10\rangle + b|11\rangle + b|1\,{-1}\rangle + c|10\rangle\Big) + \cos\theta\,\hbar\Big(a|11\rangle - c|1\,{-1}\rangle\Big)$$

$$\equiv \hbar\left(a|11\rangle + b|10\rangle + c|1\,{-1}\rangle\right)$$

$$\Rightarrow \quad \begin{aligned} \cancel{\hbar}a &= \sin\theta\frac{\sqrt2}{2}\cancel{\hbar}b + \cos\theta\cancel{\hbar}a & \Rightarrow\quad a &= \frac{\sin\theta\frac{\sqrt2}{2}}{1-\cos\theta}b \\ \cancel{\hbar}b &= \sin\theta\frac{\sqrt2}{2}\cancel{\hbar}(a+c) & & \\ \cancel{\hbar}c &= \sin\theta\frac{\sqrt2}{2}\cancel{\hbar}b - c\cos\theta\cancel{\hbar} & \Rightarrow\quad c &= \frac{\sin\theta\frac{\sqrt2}{2}}{1+\cos\theta}b \end{aligned}$$

$$\sin\theta = 2\sin\frac\theta2\cos\frac\theta2 \qquad 1+\cos\theta = 2\cos^2\frac\theta2$$
$$1-\cos\theta = 2\sin^2\frac\theta2$$

$$\Rightarrow \quad a = \frac{2\sin\frac\theta2\cos\frac\theta2}{2\sin^2\frac\theta2}\frac{\sqrt2}{2}b = \frac{\sqrt2}{2}b\cot\frac\theta2 \qquad c = \frac{\sqrt2}{2}b\tan\frac\theta2$$

Normalizacija $\quad 1 = a^2 + b^2 + c^2 = b^2 + \frac12 b^2\cot^2\frac\theta2 + \frac12 b^2\tan^2\frac\theta2 =$

$$= b^2\frac{1}{2\sin^2\frac\theta2\cos^2\frac\theta2}\left(2\sin^2\frac\theta2\cos^2\frac\theta2 + \cos^4\frac\theta2 + \sin^4\frac\theta2\right) =$$

$$= \frac{b^2\left(\sin^2\frac\theta2 + \cos^2\frac\theta2\right)^2}{2\sin^2\frac\theta2\cos^2\frac\theta2} = \frac{b^2}{2\sin^2\frac\theta2\cos^2\frac\theta2}$$

$$b = \sqrt2\sin\frac\theta2\cos\frac\theta2$$

$$a = \cos^2\frac\theta2 \qquad c = \sin^2\frac\theta2$$

$$\Rightarrow \quad |\psi,0\rangle = \cos^2\frac\theta2|11\rangle + \sqrt2\sin\frac\theta2\cos\frac\theta2|10\rangle + \sin^2\frac\theta2|1\,{-1}\rangle$$

Časovni razvoj

- $|l\,m\rangle$ razvijemo po lastnih stanjih hamiltoniana

$$H = \lambda \vec{L}\cdot\vec{B} = \lambda B L_z \qquad \vec{B} = (0,0,B)$$

$|l\,m\rangle$ je tudi baza H.

$$H|11\rangle = \lambda B\hbar\,|11\rangle \qquad H|l\,m\rangle = \lambda B\hbar m\,|l\,m\rangle$$
$$H|10\rangle = 0\,|10\rangle$$
$$H|1\,{-1}\rangle = -\lambda B\hbar\,|1\,{-1}\rangle$$

$$|\psi,t\rangle = \cos^2\frac{\theta}{2}\, e^{-i\frac{\lambda B\hbar}{\hbar}t}|11\rangle + \sqrt{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\, e^{-i\frac{0}{\hbar}t}|10\rangle + \sin^2\frac{\theta}{2}\, e^{-i\frac{(-\lambda B\hbar)}{\hbar}t}|1\,{-1}\rangle$$

$$|\psi,t\rangle = \cos^2\frac{\theta}{2}\, e^{-i\lambda Bt}|11\rangle + \sqrt{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}|10\rangle + \sin^2\frac{\theta}{2}\, e^{i\lambda Bt}|1\,{-1}\rangle$$

$$\langle \vec{L},t\rangle = ?$$

$L_z|l\,m\rangle = \hbar m|l\,m\rangle$
$L_x = \frac{1}{2}(L_+ + L_-)$
$L_y = \frac{1}{2i}(L_+ - L_-)$
$L_+ = L_x + iL_y$
$L_- = L_x - iL_y$
$L_- = L_+^\dagger$
$L_\pm|l\,m\rangle = \hbar\sqrt{l(l+1) - m(m\pm1)}\,|l\,m\rangle$

$$\langle L_z,t\rangle = \langle\psi,t|L_z|\psi,t\rangle = \langle\psi,t|\Big(\cos^2\frac{\theta}{2}e^{-i\lambda Bt}\hbar|11\rangle + 0 + \sin^2\frac{\theta}{2}e^{i\lambda Bt}(-\hbar)|1\,{-1}\rangle\Big)$$
$$= \cos^4\frac{\theta}{2}\hbar - \sin^4\frac{\theta}{2}\hbar = \hbar\Big(\cos^4\frac{\theta}{2} - \sin^4\frac{\theta}{2}\Big) = \hbar\Big(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\Big)\cdot 1 = \hbar\cos\theta$$

$$\langle L_x,t\rangle = \Big\langle\frac{L_+ + L_-}{2},t\Big\rangle = \frac{1}{2}\big(\langle L_+,t\rangle + \langle L_-,t\rangle\big) = \frac{1}{2}\big(\langle L_+,t\rangle + \langle L_+,t\rangle^*\big) = \mathrm{Re}\langle L_+,t\rangle$$

$$\langle L_y,t\rangle = \Big\langle\frac{L_+ - L_-}{2i},t\Big\rangle = \frac{1}{2i}\big(\langle L_+,t\rangle - \langle L_-,t\rangle\big) = \mathrm{Im}\langle L_+,t\rangle$$

$$L_+|1\,{-1}\rangle = \hbar\sqrt{2}|10\rangle \qquad L_+|1\,0\rangle = \sqrt{2}\hbar|11\rangle \qquad L_+|11\rangle = 0$$

$$\langle L_+,t\rangle = \langle\psi,t|L_+|\psi,t\rangle = \langle\psi,t|\Big(0 + \sqrt{2}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\hbar\sqrt{2}|11\rangle + \sin^2\frac{\theta}{2}e^{i\lambda Bt}\hbar\sqrt{2}|10\rangle\Big)$$
$$= 2\sin\frac{\theta}{2}\cos^3\frac{\theta}{2}\hbar\, e^{i\lambda Bt} + 2\hbar\sin^3\frac{\theta}{2}\cos\frac{\theta}{2}e^{i\lambda Bt} =$$
$$= 2\hbar\sin\frac{\theta}{2}\cos\frac{\theta}{2}e^{i\lambda Bt} = \hbar\sin\theta\, e^{i\lambda Bt}$$

$$\langle L_x,t\rangle = \hbar\sin\theta\cos\lambda Bt$$
$$\langle L_y,t\rangle = \hbar\sin\theta\sin\lambda Bt$$

$$\langle\vec{L},t\rangle = \hbar\begin{bmatrix}\sin\theta\cos\lambda Bt\\ \sin\theta\sin\lambda Bt\\ \cos\theta\end{bmatrix}$$

$\vec{L}$ kroži v xy ravnini, dobimo klasično precesijo.
Frekvenca kroženja
$$\omega_L = \lambda B \quad \text{Larmor}$$

Kaj se zgodi, če izmerimo L_z ob času t?
Razvijemo ψ po lastnih stanjih operatorja (npr. L_z) ✓

lastna vrednost stanja

Rezultat meritve	Verjetnost $\lvert c_n\rvert^2$	ψ ob t^+ (stanje po meritvi)
$\hbar$	$\cos^4\frac{\theta}{2}$	$\lvert 11\rangle$
0	$2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$	$\lvert 10\rangle$
$-\hbar$	$\sin^4\frac{\theta}{2}$	$\lvert 1\,{-1}\rangle$

Povprečje meritev $\quad \overline{L_z} = \hbar\cos^4\frac{\theta}{2} + 0\cdot 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \hbar\sin^4\frac{\theta}{2} = \hbar\cos\theta$

(11) Spin 2D

$$H = \frac{p^2}{2m} + \underbrace{\lambda(p_x S_y - p_y S_x)}_{\text{Rashbova sklopitev}}$$

$\vec{p} = (p_x, p_y)$

$H|\psi\rangle = E|\psi\rangle$?

$[H, \vec{p}] = ?$

$$[H, p_x] = \underbrace{\left[\frac{p_x^2}{2m}, p_x\right]}_{=0} + \underbrace{\left[\frac{p_y^2}{2m}, p_x\right]}_{=0} + \lambda \underbrace{[p_x S_y, p_x]}_{=0} - \lambda \underbrace{[p_y S_x, p_x]}_{=0} = 0$$

ker p in S delujeta v različnih Hilbertovih prostorih, komutirata

$[H, p_y] = 0$ $\quad [H, \vec{p}] = 0$ $\quad$ Ker komutirata so lastna stanja $\vec{p}$ enaka H.

$\vec{p}|\vec{k}\rangle = \hbar\vec{k}|\vec{k}\rangle$ $\quad$ last. stanja $\vec{p}$ so ravni valovi

$\psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}$

dvodimenzionalen Hilbertov prostor

Naj bo $|\psi\rangle = |\vec{k}\rangle|\chi\rangle$ in naj bo spin $s = 1/2$

neskončno-dim. Hil. prostor

$$H|\vec{k}\rangle|\chi\rangle = E|\vec{k}\rangle|\chi\rangle$$

$$\left(\frac{p^2}{2m} + \lambda(p_x S_y - p_y S_x)\right)|\vec{k}\rangle|\chi\rangle = \left(\frac{p^2}{2m}|\vec{k}\rangle\right)|\chi\rangle + \lambda\left(p_x|\vec{k}\rangle S_y|\chi\rangle - p_y|\vec{k}\rangle S_x|\chi\rangle\right) = E|\vec{k}\rangle|\chi\rangle$$

$$\frac{\hbar^2}{2m}k^2|\vec{k}\rangle|\chi\rangle + \lambda\left(\hbar k_x|\vec{k}\rangle S_y|\chi\rangle - \hbar k_y|\vec{k}\rangle S_x|\chi\rangle\right) = E|\vec{k}\rangle|\chi\rangle$$

$$\frac{\hbar^2 k^2}{2m}|\chi\rangle + \lambda\hbar\left(k_x S_y|\chi\rangle - k_y S_x|\chi\rangle\right) = E|\chi\rangle$$

$s = \frac{1}{2}$ $\quad |\tfrac{1}{2}\,\tfrac{1}{2}\rangle = |\uparrow\rangle$

$|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle = |\downarrow\rangle$

$S_x = \frac{S_+ + S_-}{2}$

$S_y = \frac{S_+ - S_-}{2i}$

$|\chi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$

$S_+|\uparrow\rangle = 0$

$S_+|\downarrow\rangle = \hbar\sqrt{\tfrac{1}{2}\tfrac{3}{2} - (-\tfrac{1}{2})(\tfrac{1}{2})}\,|\uparrow\rangle = \hbar|\uparrow\rangle$

$S_-|\uparrow\rangle = \hbar|\downarrow\rangle$

$S_-|\downarrow\rangle = 0$

$S_x|\uparrow\rangle = \frac{\hbar}{2}|\downarrow\rangle$

$S_x|\downarrow\rangle = \frac{\hbar}{2}|\uparrow\rangle$

$S_y|\uparrow\rangle = -\frac{\hbar}{2i}|\downarrow\rangle$

$S_y|\downarrow\rangle = \frac{\hbar}{2i}|\uparrow\rangle$

$$\frac{\hbar^2 k^2}{2m}(\alpha|\uparrow\rangle + \beta|\downarrow\rangle) + \lambda\hbar\left(k_x\left[-\alpha\frac{\hbar}{2i}|\downarrow\rangle + \beta\frac{\hbar}{2i}|\uparrow\rangle\right] - k_y\left[\frac{\hbar}{2}\alpha|\downarrow\rangle + \frac{\hbar}{2}\beta|\uparrow\rangle\right]\right) = E(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)$$

$$\frac{\hbar^2 k^2}{2m}\alpha + \lambda\hbar\left(k_x\beta\frac{\hbar}{2i} - k_y\frac{\hbar}{2}\beta\right) = E\alpha$$

$$\frac{\hbar^2 k^2}{2m}\beta + \lambda\hbar\left(-k_x\alpha\frac{\hbar}{2i} - k_y\frac{\hbar}{2}\alpha\right) = E\beta$$

$$\alpha\frac{\hbar^2 k^2}{2m} - \beta\frac{\hbar^2}{2}\lambda(ik_x + k_y) = E\alpha$$

$$\beta\frac{\hbar^2 k^2}{2m} - \alpha\frac{\hbar^2}{2}\lambda(-ik_x + k_y) = E\beta$$

$$\frac{\hbar^2}{2}\underbrace{\begin{bmatrix} \frac{k^2}{m} & -\lambda(ik_x + k_y) \\ -\lambda(-ik_x + k_y) & \frac{k^2}{m} \end{bmatrix}}_{\text{je hermitska } \checkmark}\begin{bmatrix}\alpha \\ \beta\end{bmatrix} = E\begin{bmatrix}\alpha \\ \beta\end{bmatrix}$$

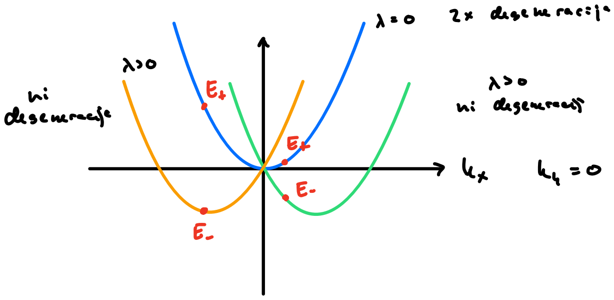
$$\det\begin{vmatrix} \frac{k^2}{m} - E\frac{2}{\hbar^2} & -\lambda(ik_x + k_y) \\ -\lambda(-ik_x + k_y) & \frac{k^2}{m} - E\frac{2}{\hbar^2} \end{vmatrix} = 0$$

$$\left(\frac{k^2}{m} - \frac{2}{\hbar^2}E\right)^2 - \lambda^2(ik_x + k_y)(-ik_x + k_y) = 0$$

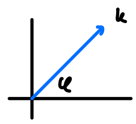
$$\left(\frac{k^2}{m} - \frac{2}{\hbar^2}E\right)^2 = \lambda^2\underbrace{(k_x^2 + k_y^2)}_{=k^2}$$

$$\frac{k^2}{m} - \frac{2}{\hbar^2}E = \pm\lambda k \qquad k = |\vec{k}|$$

$E_- = \frac{\hbar^2}{2}\left(\frac{k^2}{m} - \lambda k\right)$ $\quad E_+ = \frac{\hbar^2}{2}\left(\frac{k^2}{m} + \lambda k\right)$ Dobimo dve lastni energiji pri določenem k.



Iščemo lastne vektorje



$$k_x + i k_y = X = k e^{i\varphi}$$

$$\frac{\hbar^2}{2}\begin{bmatrix} \frac{k^2}{m} & -\lambda(i k_x + k_y) \\ -\lambda(-i k_x + k_y) & \frac{k^2}{m} \end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = E\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$$

E_1:

$$\frac{\hbar^2}{2}\begin{bmatrix} \frac{k^2}{2m} & -\lambda i k e^{-i\varphi} \\ \lambda i k e^{i\varphi} & \frac{k^2}{2m} \end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = E_1\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$$

Poglejmo k prvo enačbo

$$\frac{\hbar^2}{2}\frac{k^2}{2m}\alpha - \frac{\hbar^2}{2}k\lambda i e^{-i\varphi}\beta = \frac{\hbar^2}{2}\left(\frac{k^2}{m} - \lambda k\right)\alpha$$

$$\frac{\hbar^2}{2}\lambda k \alpha - \frac{\hbar^2}{2}k\lambda i e^{-i\varphi}\beta = 0$$

$$\beta = \frac{\lambda k\, \hbar^2/2}{\hbar^2/2\, \lambda i e^{-i\varphi}}\alpha = -i e^{i\varphi}\alpha$$

$\Rightarrow$ E_- $$|\psi\rangle_{E_-} = \frac{|\uparrow\rangle - i e^{i\varphi}|\downarrow\rangle}{\sqrt{2}}$$

E_+ $$|\psi\rangle_{E_+} = \frac{|\uparrow\rangle + i e^{i\varphi}|\downarrow\rangle}{\sqrt{2}}$$

$s = 1/2$

$\vec{S}\cdot\vec{n}\,|\psi\rangle = \frac{\hbar}{2}|\psi\rangle$ če to velja, spin kaže v smeri $\vec{n}$

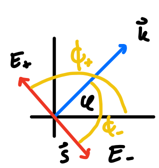
$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle = \cos\frac{\theta}{2}|\uparrow\rangle + \sin\frac{\theta}{2}e^{i\phi}|\downarrow\rangle$$ Alternativni zapis

$|\alpha|^2 + |\beta|^2 = 1$

$$|\psi\rangle_{E_2} = \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{i}{\sqrt{2}}e^{i\varphi}|\downarrow\rangle$$ $|\psi\rangle_{E_1}$

$\cos\frac{\theta}{2} = \frac{\sqrt{2}}{2} \Rightarrow \theta = \frac{\pi}{2}$ $\quad\quad$ $\theta = \frac{\pi}{2}$

$e^{i\phi} = i e^{i\varphi} \Rightarrow \phi = \varphi + \frac{\pi}{2}$ $\quad\quad$ $\phi = \varphi - \frac{\pi}{2}$



(12) Ista naloga kot prej, narejena s Paulijevimi matrikami

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle \longrightarrow \begin{pmatrix}\alpha\\\beta\end{pmatrix}$$

Dirac — Spinor

$$\vec{S} = (S_x, S_y, S_z) = \frac{\hbar}{2}\vec{\sigma} = \frac{\hbar}{2}(\sigma_x, \sigma_y, \sigma_z)$$

$$\sigma_x = \begin{bmatrix}0&1\\1&0\end{bmatrix} \quad \sigma_y = \begin{bmatrix}0&-i\\i&0\end{bmatrix} \quad \sigma_z = \begin{bmatrix}1&0\\0&-1\end{bmatrix}$$

$$H = \frac{p^2}{2m} + \lambda(p_x S_y - p_y S_x) = \frac{p^2}{2m} I + \lambda\left(p_x \frac{\hbar}{2}\sigma_y - p_y \frac{\hbar}{2}\sigma_x\right)$$

deluje na spine

$$|\psi\rangle = |\vec{k}\rangle|\chi\rangle$$

spinsko stanje

apliciramo $|k\rangle$

$$H = \frac{\hbar^2 k^2}{2m} + \frac{\lambda\hbar^2}{2}(k_x\sigma_y - k_y\sigma_x)$$

$$H = \frac{\hbar^2}{2}\begin{bmatrix}\frac{k^2}{m} & \lambda(-ik_x - k_y)\\ \lambda(ik_x - k_y) & \frac{k^2}{m}\end{bmatrix}$$

Invariantnost na obrat časa

Operator časa $\quad T = i\sigma_y K$

Operator kompleksne konjugacije

$$K e^{i\vec{k}\cdot\vec{r}} = e^{-i\vec{k}\cdot\vec{r}}$$

$$K\begin{pmatrix}\alpha\\\beta\end{pmatrix} = \begin{pmatrix}\alpha^*\\\beta^*\end{pmatrix}$$

T je antiunitaren

$$\langle T\phi|T\psi\rangle = \langle\phi|\psi\rangle^*$$

$$[H, T] \overset{?}{=} 0 \qquad H \text{ je invarianten na obrat časa}$$

$$-\frac{\hbar^2}{2m}\nabla^2 \quad -i\hbar\frac{\partial}{\partial x} \quad -i\hbar\frac{\partial}{\partial y}$$

$$TH = (i\sigma_y K)\left(\frac{p^2}{2m} + \lambda\frac{\hbar}{2}(p_x\sigma_y - p_y\sigma_x)\right) = i\sigma_y\left(\frac{p^2}{2m} + \frac{\lambda\hbar}{2}(-p_x(-\sigma_y) - (-p_x)\sigma_x)\right)K$$

$$\begin{pmatrix}0&-i\\i&0\end{pmatrix} \qquad \begin{pmatrix}0&1\\1&0\end{pmatrix}$$

$$K\sigma_y = \sigma_y^* K = -\sigma_y K$$

$$i\sigma_y\sigma_y = \sigma_y i\sigma_y$$

$$i\sigma_y\sigma_x = i(-\sigma_x\sigma_y) = -\sigma_x i\sigma_y$$

$$\{\sigma_\alpha, \sigma_\beta\} = 2\delta_{\alpha\beta}$$

$$= \left(\frac{p^2}{2m} + \frac{\lambda\hbar}{2}(p_x\sigma_y - p_y\sigma_x)\right)i\sigma_y K = HT \qquad \Rightarrow \text{komutirata}$$

Naj bo $\quad H|\psi\rangle = E|\psi\rangle$

$$HT|\psi\rangle = TH|\psi\rangle = ET|\psi\rangle \qquad T|\psi\rangle \text{ tudi lastno stanje}$$

Kako se razlikujeta $|\psi\rangle$ in $T|\psi\rangle$

$$T^2 = (i\sigma_y K)(i\sigma_y K) = -\sigma_y(-1)K(\sigma_y K) = +\sigma_y(-\sigma_y)KK = -\sigma_y^2 K^2 = -\sigma_y^2 = -I$$

$$\langle\psi|T\psi\rangle = \langle T\psi|T^2\psi\rangle^* = \langle T\psi|-\psi\rangle^* = -\langle\psi, T\psi\rangle \quad \Rightarrow \langle\psi, T\psi\rangle = 0$$

Kramersova degeneracija (Kramersov dublet)

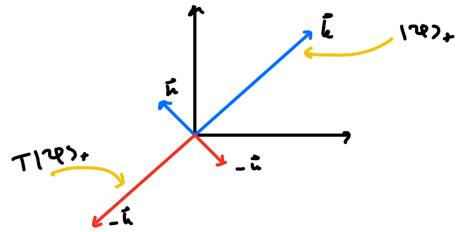
Če je H invarianten na čas imamo 2x degenerirana stanja $|\psi\rangle$ in $T|\psi\rangle$

ψ in $T\psi$ sta različni, ortogonalni

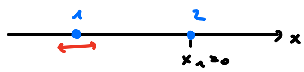
$$T e^{i\vec{k}\cdot\vec{r}} \frac{\binom{1}{0} + i e^{i\varphi}\binom{0}{1}}{\sqrt{2}} = T e^{i\vec{k}\cdot\vec{r}} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ i e^{i\varphi}/\sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{2}} e^{-i\vec{k}\cdot\vec{r}} \underbrace{i\sigma_y}_{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}} K \begin{pmatrix} 1 \\ i e^{i\varphi} \end{pmatrix} = \frac{1}{\sqrt{2}} e^{-i\vec{k}\cdot\vec{r}} \begin{pmatrix} -i e^{-i\varphi} \\ -1 \end{pmatrix} =$$

$$= \frac{1}{\sqrt{2}} e^{-i\vec{k}\cdot\vec{r}} \begin{pmatrix} 1 \\ -i e^{-i\varphi} \end{pmatrix}$$

$|\tilde{\psi}\rangle = e^{i\gamma}|\psi\rangle$



13 1D, dva delca, $s_1 = \frac{1}{2}$, $s_2 = 1$



$$H = \frac{p_1^2}{2m} - \frac{\lambda}{\hbar^2}\delta(x_1)\,\vec{S}_1\cdot\vec{S}_2 \qquad \lambda > 0$$

Vezana stanja? Sipalna stanja?

Skupna vrtilna količina / spin:

$$\vec{S} = \vec{S}_1 + \vec{S}_2$$ enako velja tudi za $\vec{L}$

$$S^2 = S_1^2 + S_2^2 + 2\,\vec{S}_1\cdot\vec{S}_2$$

$$\vec{S}_1\cdot\vec{S}_2 = \frac{1}{2}(S^2 - S_1^2 - S_2^2)$$

$$H = \frac{p_1^2}{2m} - \frac{\lambda}{2\hbar^2}\delta(x_1)(S^2 - S_1^2 - S_2^2)$$

$$\Rightarrow [H, S^2] = [H, S_1^2] = [H, S_2^2] = [H, S_z] = 0$$

$$[S^2, S_1^2] = [S^2, S_2^2] = [S^2, S_z] = 0$$

$$[S_1^2, S_2^2] = [S_1^2, S_z] = 0$$

$$[S_2^2, S_z] = 0$$

$H, S^2, S_1^2, S_2^2, S_z$ → vsi ti komutirajo

1. delec $s_1 = 1/2$

baza $|\uparrow\rangle = |1/2\ 1/2\rangle$

$|\downarrow\rangle = |1/2\ -1/2\rangle$

S_1 S_{1z}

2. delec $s_2 = 1$

baza $|1 1\rangle$

$|1 0\rangle$

$|1 -1\rangle$

S_2 S_{2z}

1. in 2. delec — naj bo

$|\uparrow\rangle|1 1\rangle = |\uparrow\rangle|1\rangle$

$|\uparrow\rangle|1 0\rangle = |\uparrow\rangle|0\rangle$

$|\uparrow\rangle|1 -1\rangle = |\uparrow\rangle|-1\rangle$

$|\downarrow\rangle|1 1\rangle = |\downarrow\rangle|1\rangle$

$|\downarrow\rangle|1 0\rangle = |\downarrow\rangle|0\rangle$

$|\downarrow\rangle|1 -1\rangle = |\downarrow\rangle|-1\rangle$

Clebsch - Gordonovi koeficienti nam pretvorijo eno bazo v drugo

produktna baza

lastna stanja $S_1^2, S_{1z}, S_2^2, S_{2z}$

baza z dobrim skupnim spinom

S^2, S_z, S_1^2, S_2^2 ← želimo last. stanja teh operatorjev

vsaka baza je lastna funkcija teh operatorjev

$S_1, S_2 \Rightarrow S = |S_1 - S_2|, \ldots, S_1 + S_2$

$1/2\ ,\ 3/2$ za naš primer

$|S_1\ S_2\ S\ S_z\rangle$ naj bo

$|1/2\ 1\ 1/2\ -1/2\rangle = |\frac{1}{2}\ -\frac{1}{2}\rangle$

$|1/2\ 1\ 1/2\ 1/2\rangle = |1/2\ 1/2\rangle$

$|1/2\ 1\ 3/2\ -3/2\rangle = |3/2\ -3/2\rangle$

$|1/2\ 1\ 3/2\ -1/2\rangle = |3/2\ -1/2\rangle$

$|1/2\ 1\ 3/2\ 1/2\rangle = |3/2\ 1/2\rangle$

$|1/2\ 1\ 3/2\ 3/2\rangle = |3/2\ 3/2\rangle$ — $|s\ s_z\rangle$

pokrova 6 stanj

$S_z = -S, \ldots, S$

$S_z = -3/2, -1/2, 1/2, 3/2$ za $S = 3/2$

$H|\psi\rangle = E|\psi\rangle$

$|\psi\rangle = |\phi\rangle |s\, s_z\rangle$

krajevni del

$$H|\phi\rangle|s s_z\rangle = \left(\frac{p_1^2}{2m} - \frac{\lambda}{2\hbar^2}\delta(x_1)\left(S^2 - S_1^2 - S_2^2\right)\right)|\phi\rangle|s s_z\rangle =$$

$$= \left(\frac{p_1^2}{2m}|\phi\rangle\right)|s s_z\rangle - \frac{\lambda}{2\hbar^2}\left(\delta(x_1)|\phi\rangle\right)\underbrace{\left(S^2 - S_1^2 - S_2^2\right)|s s_z\rangle}$$

$S^2|s s_z\rangle = \hbar^2 s(s+1)|s s_z\rangle$

$S_1^2|s s_z\rangle = \hbar^2 \frac{1}{2}\frac{3}{2}|s s_z\rangle = \frac{3}{4}\hbar^2|s s_z\rangle$

$S_2^2|s s_z\rangle = \hbar^2\; 1\cdot 2\,|s s_z\rangle = 2\hbar^2|s s_z\rangle$

$$\frac{p_1^2}{2m}|\phi\rangle|s s_z\rangle - \frac{\lambda}{2\hbar^2}\delta(x_1)|\phi\rangle\left(\hbar^2 s(s+1) - \frac{11}{4}\hbar^2\right)|s s_z\rangle = E|\phi\rangle|s s_z\rangle$$

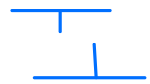
$$\frac{p_1^2}{2m}|\phi\rangle - \frac{\lambda}{2}\left(s(s+1) - \frac{11}{4}\right)\delta(x_1)|\phi\rangle = E|\phi\rangle$$

skupen spin

Imamo dva H, za $s = 1/2$ in $s = 3/2$

$$H_{3/2} = \frac{p_1^2}{2m} - \frac{\lambda}{2}\left(\frac{3}{2}\frac{5}{2} - \frac{11}{4}\right)\delta(x_1) = \frac{p_1^2}{2m} - \frac{\lambda}{2}\delta(x_1)$$

$$H_{1/2} = \frac{p_1^2}{2m} - \frac{\lambda}{2}\left(\frac{1}{2}\frac{3}{2} - \frac{11}{4}\right)\delta(x_1) = \frac{p_1^2}{2m} + \lambda\,\delta(x_1)$$



ima vezano stanje

ima sipalno stanje

ta proslem že poznamo

Za $H_{3/2}$: $E_0 = -\frac{m\lambda^2}{8\hbar^2}$ $\quad \kappa = \frac{m\lambda}{2\hbar}$

$\psi_0 = \sqrt{\kappa}\, e^{-\kappa|x|} \leftarrow |\phi\rangle$

stanje je 4x degenerirano, 4 vezana stanja

$|\phi\rangle|\frac{3}{2}\ \frac{3}{2}\rangle$

$|\frac{3}{2}\ \frac{1}{2}\rangle$

$|\frac{3}{2}\ -\frac{1}{2}\rangle$

$|\frac{3}{2}\ -\frac{3}{2}\rangle$

Sipalno stanje:

za $E>0$ $\quad e^{ikx} + r e^{-ikx} \downarrow \quad t e^{ikx}$ $\qquad k = \sqrt{\frac{2mE}{\hbar^2}}$ $\qquad S = \frac{1}{\kappa + ik}\begin{bmatrix} -\kappa & ik \\ ik & -\kappa \end{bmatrix}$

$S = \begin{bmatrix} r & t' \\ t & r' \end{bmatrix}$

Za $H_{1/2}$: $S_{1/2} = \dots$ $\quad$ uporabimo tvorno formulo

$H_{3/2}$: $S_{3/2} = \dots$

Poglejmo k sipalnim stanjem za vpadni val iz leve pri $E>0$:

$s = 3/2$ $\quad \left(e^{ikx_1} + r_{3/2}e^{-ikx_1}\right)|\frac{3}{2}\, s_z\rangle \downarrow \quad t_{3/2}\, e^{ikx_1}|\frac{3}{2}\, s_z\rangle$ $\qquad$ 4 stanja

$s = 1/2$ $\quad \left(e^{ikx_1} + r_{1/2}e^{-ikx_1}\right)|\frac{1}{2}\, s_z\rangle \downarrow \quad t_{1/2}\, e^{ikx_1}|\frac{1}{2}\, s_z\rangle$ $\qquad$ 2 stanji

valovi, ki prihajajo iz desne.

$2x \Rightarrow 12$ stanj

Kaj se zgodi, ko se prvi delec sipje na drugem

$$e^{ikx_1}\underset{\text{1. delec}}{|\uparrow\rangle}\underset{\text{2. delec}}{|0\rangle} \qquad E>0 \qquad k=\sqrt{\frac{2mE}{\hbar^2}}$$

produktno bazo želimo pretvoriti v bazo z dobrim skupnim spinom

Uporabimo Klebsch-Gordanove koef.

S_2 S_1

$1\times 1/2$	3/2, +3/2	3/2, +1/2	1/2, +1/2	3/2, −1/2	1/2, −1/2	3/2, −3/2
+1 +1/2	1					
+1 −1/2		1/3	2/3			
0 +1/2		2/3	−1/3			
0 −1/2				2/3	1/3	
−1 +1/2				1/3	−2/3	
−1 −1/2						1

stanje v produktni bazi:

$|0\rangle|\uparrow\rangle$

baza z dobrim skupnim spinom

jih moramo še $\sqrt{\ }$

$$e^{ikx_1}|0\rangle|\uparrow\rangle = \left(\sqrt{\tfrac{2}{3}}\,|\tfrac32\ \tfrac12\rangle - \sqrt{\tfrac13}\,|\tfrac12\ \tfrac12\rangle\right)e^{ikx_1}$$

$$\underbrace{\sqrt{\tfrac23}}_{}\left(e^{ikx_1}+r_{3/2}e^{-ikx_1}\right)|\tfrac32\ \tfrac12\rangle - \sqrt{\tfrac13}\left(e^{ikx_1}+r_{1/2}e^{-ikx_1}\right)|\tfrac12\ \tfrac12\rangle$$

Sipalno stanje

$= e^{ikx_1}|\uparrow\rangle|0\rangle$

$x_1=0$ $\qquad x_1$

$$\sqrt{\tfrac23}\,t_{3/2}\,e^{ikx_1}|\tfrac32\ \tfrac12\rangle - \sqrt{\tfrac13}\,t_{1/2}\,e^{ikx_1}|\tfrac12\ \tfrac12\rangle$$

Želimo priti nazaj v produktno bazo

$$|\tfrac32\ \tfrac12\rangle = \sqrt{\tfrac13}\,|1\rangle|-\tfrac12\rangle + \sqrt{\tfrac23}\,|0\rangle|\tfrac12\rangle = \sqrt{\tfrac13}\,|1\rangle|\downarrow\rangle + \sqrt{\tfrac23}\,|0\rangle|\uparrow\rangle$$

$$|\tfrac12\ \tfrac12\rangle = \sqrt{\tfrac23}\,|\downarrow\rangle|1\rangle - \sqrt{\tfrac13}\,|\uparrow\rangle|0\rangle$$

Leva stran

$$\sqrt{\tfrac23}\left(e^{ikx_1}+r_{3/2}e^{-ikx_1}\right)\left(\sqrt{\tfrac13}\,|\downarrow\rangle|1\rangle + \sqrt{\tfrac23}\,|\uparrow\rangle|0\rangle\right)$$

$$-\sqrt{\tfrac13}\left(e^{ikx_1}+r_{1/2}e^{-ikx_1}\right)\left(\sqrt{\tfrac23}\,|\downarrow\rangle|1\rangle - \sqrt{\tfrac13}\,|\uparrow\rangle|0\rangle\right) =$$

$$=\left(\sqrt{\tfrac13}\sqrt{\tfrac23}\left(e^{ikx_1}+r_{3/2}e^{-ikx_1}\right) - \sqrt{\tfrac13}\sqrt{\tfrac23}\left(e^{ikx_1}+r_{1/2}e^{-ikx_1}\right)\right)|\downarrow\rangle|1\rangle +$$

$$\left(\sqrt{\tfrac23}^{\,2}\left(e^{ikx_1}+r_{3/2}e^{-ikx_1}\right) + \sqrt{\tfrac13}^{\,2}\left(e^{ikx_1}+r_{1/2}e^{-ikx_1}\right)\right)|\uparrow\rangle|0\rangle =$$

$$= \underbrace{e^{ikx_1}|\uparrow\rangle|0\rangle}_{\text{vpadni val, kot v navodilih}} + \underbrace{e^{-ikx_1}\underbrace{\left(\tfrac{\sqrt2}{3}r_{3/2} - \tfrac{\sqrt2}{3}r_{1/2}\right)}_{r_{\downarrow\uparrow}}|\downarrow\rangle|1\rangle + e^{-ikx_1}\underbrace{\left(\tfrac23 r_{3/2} + \tfrac13 r_{1/2}\right)}_{r_{\uparrow\uparrow}}|\uparrow\rangle|0\rangle}_{\text{odbiti val}}$$

odbije v $\downarrow$ output

začne v $\uparrow$ input

velja le za 1. delec

Desna stran

$$e^{ikx_1}\underbrace{\left(\tfrac{\sqrt2}{3}t_{3/2} - \tfrac{\sqrt2}{3}t_{1/2}\right)}_{t_{\downarrow\uparrow}}|\downarrow\rangle|1\rangle + e^{ikx_1}\underbrace{\left(\tfrac23 t_{3/2} + \tfrac13 t_{1/2}\right)}_{t_{\uparrow\uparrow}}|\uparrow\rangle|0\rangle$$

Merljive količine

$$R_{\uparrow\uparrow} = |r_{\uparrow\uparrow}|^2 = \left|\tfrac23 r_{3/2} + \tfrac13 r_{1/2}\right|^2 = \tfrac49|r_{3/2}|^2 + \tfrac19|r_{1/2}|^2 + \tfrac49\,\mathrm{Re}\left(r_{3/2}^*\,r_{1/2}\right)$$

$$R_{\downarrow\uparrow} = |r_{\downarrow\uparrow}|^2 = \left|\tfrac{\sqrt2}{3} r_{3/2} - \tfrac{\sqrt2}{3} r_{1/2}\right|^2 = \tfrac29|r_{3/2}|^2 + \tfrac29|r_{1/2}|^2 - \tfrac49\,\mathrm{Re}\left(r_{3/2}^*\,r_{1/2}\right)$$

$$R = R_{\uparrow\uparrow} + R_{\downarrow\uparrow} = \tfrac23|r_{3/2}|^2 + \tfrac13|r_{1/2}|^2$$

$T_{\uparrow\uparrow} = |t_{\uparrow\uparrow}|^2 = \dots$

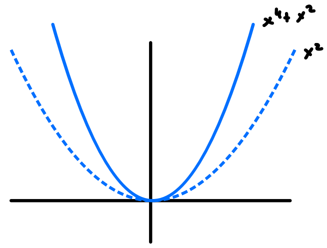
$T_{\downarrow\uparrow} = |t_{\downarrow\uparrow}|^2 = \dots$

$T = T_{\uparrow\uparrow} + T_{\downarrow\uparrow} = \frac{2}{3}|t_{3/2}|^2 + \frac{1}{3}|t_{1/2}|^2$

$$T + R = \frac{2}{3}|r_{3/2}|^2 + \frac{1}{3}|r_{1/2}|^2 + \frac{2}{3}|t_{3/2}|^2 + \frac{1}{3}|t_{1/2}|^2 = \frac{2}{3}\big(\underbrace{|r_{3/2}|^2 + |t_{3/2}|^2}_{1}\big) + \frac{1}{3}\big(\underbrace{|r_{1/2}|^2 + |t_{1/2}|^2}_{1}\big) = 1$$

$\Rightarrow$ ohranja se število delcev

(14) Teorija motenj



$$H = \underbrace{\frac{p^2}{2m} + \frac{kx^2}{2}}_{H_0} + \underbrace{\lambda x^4}_{H'} \qquad \lambda > 0$$

Nemoten H_0: $\quad H_0|n\rangle^0 = E_n^0 |n\rangle^0$

$E_n^0 = \hbar\omega(n + \frac{1}{2}) \qquad n = 0, 1, 2, \dots$

$$E_n = E_n^0 + {}^0\langle n|H'|n\rangle^0 + \dots$$

1. red teorije motenj $\leftarrow$ če je $=0$ gremo na naslednji člen

za nedegenerirana stanja

$|n\rangle^0 \Rightarrow |n\rangle$

$$\langle n|H'|n\rangle = \langle n|\lambda x^4|n\rangle = \lambda\langle x^2 n|x^2 n\rangle \qquad x = \frac{x_0}{\sqrt{2}}(a^\dagger + a)$$

$$x^2 = \frac{x_0^2}{2}(a^\dagger a^\dagger + a^\dagger a + a a^\dagger + a a) = \frac{x_0^2}{2}(a^\dagger a^\dagger + aa + 2a^\dagger a + 1)$$

$$|x^2 n\rangle = \frac{x_0^2}{2}(a^\dagger a^\dagger + aa + 2a^\dagger a + 1)|n\rangle =$$

$a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$

$a|n\rangle = \sqrt{n}\,|n-1\rangle$

$$= \frac{x_0^2}{2}\left(\sqrt{n+1}\sqrt{n+2}\,|n+2\rangle + \sqrt{n(n-1)}\,|n-2\rangle + 2n|n\rangle + |n\rangle\right)$$

$$\lambda\langle x^2 n|x^2 n\rangle = \frac{\lambda x_0^4}{4}\left(\sqrt{(n+1)(n+2)}\,\langle n+2| + \sqrt{n(n-1)}\,\langle n-2| + (2n+1)\langle n|\right)\cdot$$
$$\cdot\left(\sqrt{n+1}\sqrt{n+2}\,|n+2\rangle + \sqrt{n(n-1)}\,|n-2\rangle + 2n|n\rangle + |n\rangle\right)$$

$$= \frac{\lambda x_0^4}{4}\left((n+1)(n+2) + n(n-1) + (2n+1)^2\right)$$

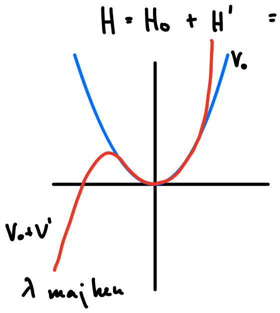
$$= \frac{\lambda x_0^4}{4}\left(n^2 + 3n + 2 + n^2 - n + 4n^2 + 4n + 1\right)$$

$$= \frac{\lambda x_0^4}{4}(6n^2 + 6n + 3) = \frac{3\lambda x_0^4}{2}(n^2 + n + \tfrac{1}{2})$$

$$E_n = \hbar\omega(n + \tfrac{1}{2}) + \underbrace{\tfrac{3}{2}\lambda x_0^4(n^2 + n + \tfrac{1}{2})}_{\text{energija se dvigne}}$$

(15)

$$H = H_0 + H' = \frac{p^2}{2m} + \frac{1}{2}kx^2 + \lambda x^3 \qquad 0 < \lambda \ll 1$$



(T) Nedegenerirana stanja

$$E_n = E_n^0 + {}^0\langle n|H'|n\rangle^0 + \sum_{m\neq n}\frac{|{}^0\langle m|H'|n\rangle^0|^2}{E_n^0 - E_m^0} + \dots$$

$$|n\rangle = |n\rangle^0 + \sum_{m\neq n}\frac{\langle m|H'|n\rangle}{E_n^0 - E_m^0}|m\rangle^0$$

$$\langle n|H'|n\rangle = \int_{-\infty}^{\infty}\underbrace{|\psi_n^0(x)|^2}_{\text{vedno soda}}\lambda x^3\,dx = 0$$

$$\langle m|H'|n\rangle = \lambda\langle m|x^3|n\rangle = \lambda\langle xm|x^2 n\rangle = \dots \qquad x = \frac{x_0}{\sqrt{2}}(a^\dagger + a)$$

$$x^2|n\rangle \overset{\text{prejšnja naloga}}{=} \dots = \frac{x_0^2}{2}\left(\sqrt{n+1}\sqrt{n+2}\,|n+2\rangle + \sqrt{n(n-1)}\,|n-2\rangle + (2n+1)|n\rangle\right)$$

$$x|m\rangle = \frac{x_0}{\sqrt{2}}(a^\dagger + a)|m\rangle = \frac{x_0}{\sqrt{2}}\left(\sqrt{m+1}\,|m+1\rangle + \sqrt{m}\,|m-1\rangle\right)$$

$$\ldots = \lambda \frac{x_0^3}{2\sqrt{2}} \left(\sqrt{m+1} \langle m+1| + \sqrt{m} \langle m-1| \right) \left(\sqrt{n+1}\sqrt{n+2}\, |n+2\rangle + \sqrt{n(n-1)}\, |n-2\rangle + (2n+1)|n\rangle \right)$$

$$= \frac{\lambda x_0^3}{2\sqrt{2}} \Big[\sqrt{(n+2)(n+1)(n+2)}\, \delta_{m,n+1} + \sqrt{(n+1)(n+2)(n+3)}\, \delta_{m,n+3}$$

$$+ \sqrt{n(n-1)(n-2)}\, \delta_{m,n-3} + \sqrt{n(n-1)(n-1)}\, \delta_{m,n-1}$$

$$+ (2n+1)\sqrt{n}\, \delta_{m,n-1} + (2n+1)\sqrt{n+1}\, \delta_{m,n+1} \Big]$$

$$= \frac{\lambda x_0^3}{2\sqrt{2}} \Big[\sqrt{n(n-1)(n-2)}\, \delta_{m,n-3} + 3n\sqrt{n}\, \delta_{m,n-1} + \sqrt{n+1}\,(3n+3)\, \delta_{m,n+1} + \sqrt{(n+1)(n+2)(n+3)}\, \delta_{m,n+3}$$

$$E_n = E_n^0 + \frac{|\langle n-3|H'|n\rangle|^2}{E_n - E_{n-3}} + \frac{|\langle n-1|H'|n\rangle|^2}{E_n - E_{n-1}} + \frac{|\langle n+1|H'|n\rangle|^2}{E_n - E_{n+1}} + \frac{|\langle n+3|H'|n\rangle|^2}{E_n - E_{n+3}} =$$

$$E_n = \hbar\omega\left(n + \tfrac{1}{2}\right)$$

$$= \hbar\omega\left(n+\tfrac{1}{2}\right) + \left(\frac{\lambda x_0^3}{2\sqrt{2}}\right)^2 \frac{1}{\hbar\omega} \left[\frac{n(n-1)(n-2)}{n-n+3} + \frac{9n^3}{1} + \frac{9(n+1)^3}{-1} + \frac{(n+1)(n+2)(n+3)}{-3} \right)$$

$$= \hbar\omega\left(n+\tfrac{1}{2}\right) + \left(\frac{\lambda x_0^3}{2\sqrt{2}}\right)^2 \frac{1}{3\hbar\omega} \left(n(n-1)(n-2) + 27n^3 - 27(n+1)^3 - (n+1)(n+2)(n+3) \right)$$

$$= \hbar\omega\left(n+\tfrac{1}{2}\right) - \left(\frac{\lambda x_0^3}{2\sqrt{2}}\right)^2 \frac{1}{\hbar\omega} \left(30n^2 + 30n + 11\right)$$

$$|0\rangle = |0\rangle^0 + \frac{\lambda x_0^3}{2\sqrt{2}} \Big[\sum^{n=0} \frac{\sqrt{n+1}\,(3n+3)\,\delta_{m,n+1}}{-\hbar\omega} |n\rangle + \frac{\sqrt{(n+1)(n+2)(n+3)}\,\delta_{m,n+3}}{-3\hbar\omega} |3\rangle \Big] =$$

$$= |0\rangle^0 - \frac{\lambda x_0^3}{2\sqrt{2}\hbar\omega} \left(3|1\rangle^0 + \frac{\sqrt{6}}{3}|3\rangle^0 \right)$$

$$a|n\rangle = \sqrt{n}\,|n-1\rangle$$

$$\langle x \rangle = \sqrt{2}\, x_0 \,\mathrm{Re}\langle a \rangle$$

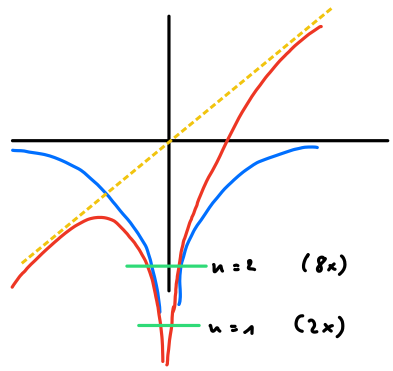
$$a|0\rangle = 0 - \frac{\lambda x_0^3}{2\sqrt{2}\hbar\omega} \left(3|0\rangle + \sqrt{2}|2\rangle \right)$$

$$\langle 0|a|0\rangle = \left(\langle 0| - \frac{\lambda x_0^3}{2\sqrt{2}\hbar\omega} \left(3\langle 1| + \frac{\sqrt{6}}{3}\langle 3| \right) \right) \left(- \frac{\lambda x_0^3}{2\sqrt{2}\hbar\omega} \right) \left(3|0\rangle + \sqrt{2}|2\rangle \right) =$$

$$= - \frac{\lambda x_0^3}{2\sqrt{2}\hbar\omega}\, 3$$

$$\langle x \rangle = -\frac{3}{2} \frac{x_0^4}{\hbar\omega}$$

16 Vodikov atom v električnem polju



$$H = H_0 + H'$$

$$= \underbrace{\frac{p^2}{2m} - \frac{e^2}{4\pi\varepsilon_0 r}}_{[H_0, L^2] = 0} \underbrace{- e\mathcal{E} z}_{[H', L_z] = 0}$$

os na simetrija zato komutira z L_z

Kako se razcepi prvo vzbujeno stanje ($n = 2$)?

$$E_n^0 = -\frac{R_y}{n^2}$$ Rydbergova konst. 13,6 eV

Kvantna števila

$n = 2$

$\ell = \{0, 1\}$ $\quad \ell < n$

$m = 0$ $\quad m = \{-1, 0, 1\}$ $\quad$ 4 stanja

$$\underbrace{m_s = \pm \tfrac{1}{2} \quad \text{spin}}_{|\ell\, m\, m_s\rangle} \quad \Rightarrow \quad 8 \text{ stanj, } 8\times \text{ degenerirano}$$

n je konstanten

Baza lastnih stanj

$\lvert 0\ 0\ \tfrac{1}{2}\rangle$	$\lvert 1\ 0\ \tfrac{1}{2}\rangle$	$\lvert 1\ 1\ \tfrac{1}{2}\rangle$	$\lvert 1\ {-1}\ \tfrac{1}{2}\rangle$
$\lvert 0\ 0\ {-\tfrac{1}{2}}\rangle$	$\lvert 1\ 0\ {-\tfrac{1}{2}}\rangle$	$\lvert 1\ 1\ {-\tfrac{1}{2}}\rangle$	$\lvert 1\ {-1}\ {-\tfrac{1}{2}}\rangle$

Valovne funk. H atoma $\quad \psi_{nlm}(\vec r) = R_{nl}(r)\underbrace{Y_{lm}(\theta,\varphi)}_{|lm\rangle}$

$$L^2|lm\rangle = \hbar^2 l(l+1)|lm\rangle$$
$$L_z|lm\rangle = \hbar m|lm\rangle$$

matrika 8×8

$\langle l m m_s|H'|l' m' m_s'\rangle = \qquad |lmm_s\rangle = |lm\rangle|m_s\rangle$

$= \langle lm|H'|l'm'\rangle\langle m_s|m_s'\rangle$ ker H' ne vpliva na spin

$= \langle lm|H'|l'm'\rangle\,\delta_{m_s m_s'}$

$$\begin{array}{c|cc} & |\uparrow\rangle & |\downarrow\rangle \\ \hline \langle\uparrow| & & 0 \\ \langle\downarrow| & 0 & \end{array}$$

enaka bloka

$$\Rightarrow \quad \begin{array}{c|cccc} & |00\rangle & |11\rangle & |10\rangle & |1-1\rangle \\ \hline \langle 00| & & & & \\ \langle 11| & & & & \\ \langle 10| & & & & \\ \langle 1-1| & & & & \end{array}$$

ker $[H',L_z]=0$

$\langle lm|[H',L_z]|l'm'\rangle = 0$

$= \langle lm|H'L_z|l'm'\rangle - \langle lm|L_zH'|l'm'\rangle =$

$= m'\hbar\langle lm|H'|l'm'\rangle - \langle L_z lm|H'|l'm'\rangle =$

$= m'\hbar\langle lm|H'|l'm'\rangle - m\hbar\langle lm|H'|l'm'\rangle$

$= \langle lm|H'|l'm'\rangle\,\hbar(m'-m) = 0$

Če $m \neq m' \Rightarrow \langle lm|H'|l'm'\rangle = 0 \Rightarrow \begin{bmatrix} \square & 0 & \square & 0 \\ 0 & \square & 0 & 0 \\ \square & 0 & \square & 0 \\ 0 & 0 & 0 & \square \end{bmatrix}$

Operator parnosti $P: \vec r \to -\vec r$

$PH_0 = H_0P \Rightarrow [P,H_0]=0$

$PH' = -H'P \Rightarrow \{P,H'\}=0$

$P\psi(\vec r) = \lambda\psi(\vec r) \qquad \lambda = \pm 1$ soda parnost / liha parnost

Velja $PY_{lm}(\theta,\varphi) = (-1)^l Y_{lm}(\theta,\varphi) \qquad \psi_{nlm} = R_{nl}Y_{lm}$

soda parnost

ker je hermitski

$\langle lm|\{H',P\}|l'm'\rangle = \langle lm|H'P+PH'|l'm'\rangle = \langle lm|H'P|l'm'\rangle + \langle lm|PH'|l'm'\rangle =$

$= (-1)^{l'}\langle lm|H'|l'm'\rangle + (-1)^l\langle lm|H'|l'm'\rangle =$

$= \left((-1)^{l'}+(-1)^l\right)\langle lm|H'|l'm'\rangle = 0$

Če $(-1)^l+(-1)^{l'} \neq 0 \Rightarrow \langle lm|H'|l'm'\rangle = 0$

$\Rightarrow l = l'$

$$\begin{bmatrix} 0 & 0 & M & 0 \\ 0 & 0 & 0 & 0 \\ M^* & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Ker je matrika hermitska rabimo izračunati le en člen.

$M = \langle 00|H'|10\rangle = \int_V R_{20}^* Y_{00}^* R_{21} Y_{10}(-e\varepsilon)z\,dx\,dy\,dz =$

$= \int_0^\infty r^2dr\int_0^\pi \sin\theta\,d\theta\int_0^{2\pi}d\varphi\ \frac{2}{(2r_B)^{3/2}}\left(1-\frac{r}{2r_B}\right)e^{-\frac{r}{2r_B}}\ \frac{1}{\sqrt{4\pi}}\ \frac{1}{\sqrt3}\frac{1}{(2r_B)^{3/2}}\frac{r}{r_B}e^{-\frac{r}{2r_B}}\ \sqrt{\frac{3}{4\pi}}\cos\theta\ (-e\varepsilon)\,r\cos\theta$

$$= -\frac{2}{(2r_s)^{3/2}} \frac{1}{\sqrt{4\pi}} \frac{1}{\sqrt{3}} \frac{1}{(2r_s)^{3/2}} \sqrt{\frac{3}{4\pi}}\, e\mathcal{E} \int_0^\infty r^3 \left(1-\frac{r}{2r_s}\right) e^{-\frac{r}{r_s}} \frac{r}{r_s}\, dr \int_0^\pi \sin\theta\cos^2\theta\, d\theta \int_0^{2\pi} d\varphi$$

$$= -\frac{e\mathcal{E}}{16\pi}\, 2\pi \int_0^\infty \left(\frac{r}{r_s}\right)^4 \left(1-\frac{r}{2r_s}\right) e^{-\frac{r}{r_s}}\, dr \underbrace{\int_{-1}^{1} \cos^2\theta\, d\cos\theta}_{2/3} =$$

$u = \frac{r}{r_s}$ $\quad r_s\, du = dr$

$$\int_0^\infty u^t e^{-u}\, du = \Gamma(t-1) = t!$$

$$= -\frac{e\mathcal{E} r_s}{12} \int_0^\infty u^4 \left(1-\frac{u}{2}\right) e^{-u}\, du =$$

$$= -\frac{e\mathcal{E} r_s}{12} \int_0^\infty u^4 e^{-u} - \frac{1}{2} u^5 e^{-u}\, du =$$

$$= -\frac{e\mathcal{E} r_s}{12} \left(4! - \frac{1}{2} 5!\right) = 3e\mathcal{E} r_s = -3|e|\mathcal{E} r_s$$

Diagonalizacija matrike

zamenjamo stolpce

$$\begin{array}{c|cccc} & 00 & 10 & 11 & 1{-}1 \\ \hline 00 & 0 & M & & \\ 10 & M & 0 & & \\ 11 & & & 0 & \\ 1{-}1 & & & & 0 \end{array}$$

$\Rightarrow$ dve lastni vrednosti sta 0

energija se ne spremeni

kako se spremeni spekter od nemotenega stanja

lastna vektorja

$|11\rangle$, $|1-1\rangle$

Iščemo lastne vrednosti $\begin{bmatrix} 0 & M \\ M & 0 \end{bmatrix}$

$$\det \begin{pmatrix} -\lambda & M \\ M & -\lambda \end{pmatrix} = 0$$

$$\lambda^2 - M^2 = 0 \qquad \lambda = \pm M =$$

① $\lambda = -M = 3|e|\mathcal{E} r_s$

$$\begin{bmatrix} M & M \\ M & M \end{bmatrix} \sim \begin{bmatrix} M & M \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = 0$$

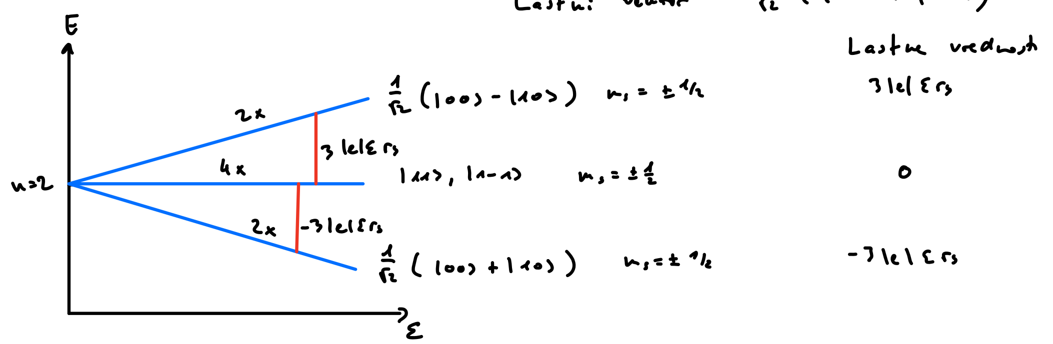
$\alpha + \beta = 0 \qquad \beta = -\alpha \qquad \Rightarrow \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Lastni vektor $\frac{1}{\sqrt{2}}(|00\rangle - |10\rangle)$

② $\lambda = M = -3|e|\mathcal{E} r_s \quad \begin{bmatrix} -M & M \\ M & -M \end{bmatrix} \sim \begin{bmatrix} -M & M \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = 0$

$\alpha - \beta = 0 \qquad \alpha = \beta \qquad \Rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Lastni vektor $\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$



$$\langle z \rangle = \frac{1}{2}(\langle 00| \mp \langle 10|)\, z\, (|00\rangle \mp |10\rangle) \qquad \overset{0}{\scriptstyle\parallel}\ \text{vemo od prej} \qquad \ell = \ell' \Rightarrow 0$$

$$= \frac{1}{2}\left(\mp \langle 00|z|10\rangle \mp \langle 10|z|00\rangle + \ldots\right)$$

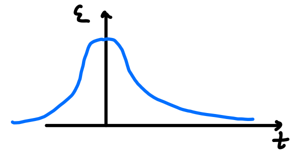
$$= \mp\frac{1}{2}\left(\langle 00|z|10\rangle + \langle 00|z|10\rangle^*\right) = \mp \mathrm{Re}\, \langle 00|z|10\rangle = -3r_s$$

Premakne se bolj v desno

$$\langle 00|H'|10\rangle = 3e\mathcal{E} r_s \Rightarrow \langle 00|z|10\rangle = \frac{1}{e\mathcal{E}} \langle 00|H'|10\rangle = -3r_s$$

Stanje z višjo energijo se premakne bolj desno

17 Časovno odvisna motnja



$$H = H_0 + H'(t) = \frac{p^2}{2m} - \frac{e^2}{4\pi\varepsilon_0 r} - e\,\mathcal{E}(t)\,z \qquad \mathcal{E}(t) = \mathcal{E}_0 \frac{1}{1+\left(\frac{t}{\tau}\right)^2}$$

$t = -\infty \qquad |\psi, -\infty\rangle = |1\,0\,0\rangle$ ($n\ l\ m$) osnovno stanje H atoma

$t = \infty \qquad P_{n=2} = ?$ verjetnost da je v prvem vzbujenem stanju

n	l	m
2	1	1
2	1	0
2	1	-1
2	0	0

spina ne upoštevamo

(T) $H_0|n\rangle = E_n|n\rangle$ lastna stanja H_0

$$|\psi, t\rangle = \sum_n c_n(t)\,|n, t\rangle \qquad |n,t\rangle = |n\rangle e^{-i\frac{E_n}{\hbar}t}$$

$$c_n(t) = c_n(t_0) - \frac{i}{\hbar}\int_{t_0}^{t} dt' \sum_m \langle n\,t'|H'(t)|m, t'\rangle\, c_m(t')$$

po vseh stanjih

ob nekem času poznamo koeficient

1. red teorije motnje $\rightarrow c_m(t_0)$

$c_{2lm}(-\infty)$, $c_{100}(-\infty) = 1$, ostali so 0

$$c_{2lm} = 0 - \frac{i}{\hbar}\int_{-\infty}^{\infty} dt'\ 1\ \langle 2lm, t'|\ -e\mathcal{E}(t)z\ |100, t'\rangle$$

$$= -\frac{i}{\hbar}\int_{-\infty}^{\infty}\langle 2lm|\,e^{i\frac{E_2}{\hbar}t}\left(-e\frac{\mathcal{E}_0 z}{1+\left(\frac{t}{\tau}\right)^2}\right)e^{-i\frac{E_1}{\hbar}t}\,|100\rangle\,dt$$

Za H atom $E_n = -\frac{R_y}{n^2}$

$$= \frac{ie\mathcal{E}_0}{\hbar}\int_{-\infty}^{\infty} e^{-i\frac{E_1-E_2}{\hbar}t}\frac{1}{1+\left(\frac{t}{\tau}\right)^2}\,dt\ \langle 2lm|z|100\rangle$$

$\langle 2lm|z|100\rangle = ?$ Od prej vemo, da so matrični elementi enaki nič če:

$$m \neq m'$$

$$m=0 \quad l=1 \quad \Leftarrow \quad (-1)^l + (-1)^{l'} \neq 0$$

$$\langle 210|z|100\rangle = \int \psi_{210}^* \, z\, \psi_{100}\, dV$$

$$= \int_0^\infty r^2 dr \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\varphi\ R_{21}(r)\,Y_{10}(\theta,\varphi)\ r\cos\theta\ R_{10}(r)\,Y_{00}(\theta,\varphi) =$$

$$= \int_0^\infty r^2 dr \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\varphi\ \frac{r}{r_b}\frac{1}{\sqrt{3}\,(2r_b)^{3/2}}e^{-\frac{r}{2r_b}}\sqrt{\frac{3}{4\pi}}\cos\theta\ \frac{2}{r_b^{3/2}}e^{-\frac{r}{r_b}}\ \frac{1}{\sqrt{4\pi}} =$$

$$= \cancel{2\pi}\int_{-1}^{1}\cos^2\theta\, d\cos\theta \int_0^\infty r^4 e^{-\frac{3}{2}\frac{r}{r_b}}\,dr\ \frac{1}{\cancel{\sqrt{3}}}\frac{1}{(2r_b)^{3/2}}\frac{1}{r_b}\sqrt{\frac{\cancel{3}}{\cancel{4\pi}}}\frac{\cancel{2}}{r_b^{3/2}}\frac{1}{\sqrt{4\pi}}$$

$$= \frac{1}{2^{3/2} r_b^4}\ \frac{2}{3}\left(\frac{2}{3}r_b\right)^5\int_0^\infty u^4 e^{-u}\,du = \frac{2^6}{3^6}\frac{r_b}{2^{3/2}}\ 4! = \frac{2^{9-3/2}}{3^5}r_b = \frac{2^7\sqrt{2}}{3^5}r_b$$

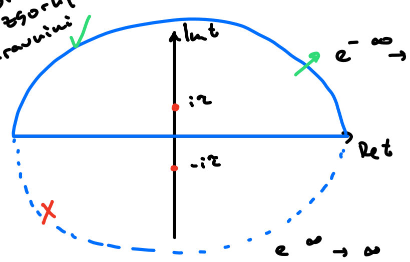
integriramo po zgornji polravnini

$$\int_{-\infty}^{\infty}\frac{e^{-i\frac{E_1-E_2}{\hbar}t}}{\tau^2+t^2}\tau^2\,dt = \int_{-\infty}^{\infty}\frac{\boxed{\tau^2 e^{-i\frac{E_1-E_2}{\hbar}t}}}{\boxed{(t+i\tau)}(t-i\tau)}\,dt = 2\pi i\ \mathrm{Res}(t = i\tau) = 2\pi i\left.\frac{\tau^2 e^{-i\frac{E_1-E_2}{\hbar}t}}{t+i\tau}\right|_{t=i\tau} =$$

Residuum

izven ostalih residuih

$$= \pi\tau\, e^{\frac{E_1-E_2}{\hbar}\tau}$$



$$c_{2lm}(t) = \frac{ie\mathcal{E}_0}{\hbar}\ \pi\tau\, e^{\frac{E_1-E_2}{\hbar}\tau}\ \frac{2^7\sqrt{2}}{3^5}r_b\ \delta_{l,1}\delta_{m,0}$$

$$P_{n=2}(t=\infty) = \sum_{lm}|c_{2lm}(\infty)|^2 = |c_{210}(\infty)|^2$$

$$= \pi^2\frac{2^{15}}{3^{10}}\left(\frac{e\mathcal{E}_0 r_b \tau}{\hbar}\right)^2 e^{\frac{E_1-E_2}{\hbar}2\tau}$$

Kakšen je optimalen τ, da bo $P_{n=2}$ največja?

$$\frac{\partial P}{\partial \tau} = 0 = \pi^2 \frac{2^{15}}{3^{10}} \left(\frac{e r_0 r_1}{\hbar}\right) \frac{\partial}{\partial \tau} \tau^2 e^{\frac{E_1 - E_2}{\hbar} 2\tau}$$

$$= 2\tau e^{\frac{E_1 - E_2}{\hbar} 2\tau} + \tau^2 \frac{E_1 - E_2}{\hbar} 2 e^{\frac{E_1 - E_2}{\hbar} 2\tau} = 0$$

$$\Rightarrow \quad 2\tau = -2\tau^2 \frac{E_1 - E_2}{\hbar}$$

$$\tau = \frac{\hbar}{E_2 - E_1}$$