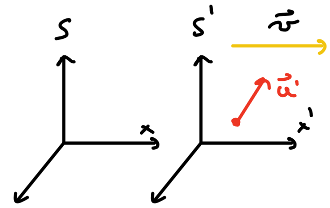
① Relativistična fizika

$\beta = \frac{v}{c}$ $\gamma = \frac{1}{\sqrt{1-\beta^2}}$



$u_x' = \frac{u_x - v}{1 - \frac{v u_x}{c^2}}$ $u_{y,z}' = \frac{u_y}{\gamma\left(1 - \frac{v u_x}{c^2}\right)}$

$a_x' = \frac{a_x}{\gamma^3 \left(1 - \frac{u_x v}{c^2}\right)^3}$

$a_{y,z}' = \frac{1}{\gamma^2 \left(1 - \frac{u_x v}{c^2}\right)^3} \left(\left(1 - \frac{u_x v}{c^2}\right) a_{y,z} + \frac{v}{c^2} u_{y,z} a_x \right)$

Lorentzove transformacije

$ct' = \gamma(ct - \beta x)$ $ct = \gamma(ct' + \beta x')$

$x' = \gamma(-\beta ct + x)$ $x = \gamma(+\beta ct' + x')$

$$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \underbrace{\begin{bmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{bmatrix}}_{L} \begin{bmatrix} ct \\ x \end{bmatrix}$$

$$\begin{bmatrix} ct \\ x \end{bmatrix} = \begin{bmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{bmatrix} \begin{bmatrix} ct' \\ x' \end{bmatrix}$$

$\tan\varphi' = \gamma \tan\varphi$

Dilatacija časa $t' = \gamma t$

Kontrakcija dolžine $L' = \frac{L}{\gamma}$

Skalarni produkt

$|\vec{r}|^2 = c^2t^2 - x^2 - y^2 - z^2 = r^\mu \cdot r_\mu$

Transformacije E in GK

$c\underline{p} = (E, p_x c, p_y c, p_z c)$

$c\underline{p}' = L\, c\underline{p}$

? Dolžino merimo pri istem času

$t_A' = t_B' = t_A = 0$

Relativna hitrost dveh sistemov (pazi predznake)

$\beta = \frac{\beta_1 + \beta_2}{1 + \beta_1\beta_2}$ $\rightarrow\rightarrow$ $-$; $\leftarrow\rightarrow$ $+$

Gibalna količina delca

$\vec{p} = \gamma_v m_0 \vec{v}$ (glede na mirovanje); m_0 — mirovna masa

Energija

$T = m_0 c^2 (\gamma - 1)$

$E_0 = m_0 c^2$

$E = E_0 + T = \gamma m_0 c^2$

$E^2 = m^2c^4 + \vec{p}^2 c^2$

$pc = \gamma\beta mc^2 = \beta E$

$pc = \sqrt{T(T + 2mc^2)}$

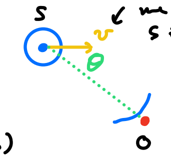
Invarianti

- dolžina $|\vec{r}|^2$
- GK $|c\underline{p}|^2$

Dopplerjev pojav

$\nu_0 = \nu_s \frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 \mp \frac{v}{c}\cos\theta}$

$\tan\theta' = \sin\theta / \gamma(\cos\theta \mp \beta)$



− približuje

+ oddaljuje

Gibanje v $\vec{E}$ in $\vec{B}$

$\mathcal{F} = \frac{dp^\mu}{d\tau} = e F^{\mu\nu} u_\nu$ $u^\mu = \frac{p^\mu}{m}$

τ — lastni čas

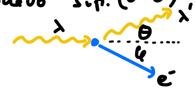
$$F^{\mu\nu} = \begin{bmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ \frac{E_x}{c} & 0 & -B_z & B_y \\ \frac{E_y}{c} & B_z & 0 & -B_x \\ \frac{E_z}{c} & -B_y & B_x & 0 \end{bmatrix}$$

$x^\mu = (ct, \vec{x})$ $x_\mu = (ct, -\vec{x})$

$\mathcal{F} = \left(\gamma \frac{\vec{F}\cdot\vec{u}}{c}, \gamma\vec{F}\right) = \gamma \frac{dp}{dt} = m \frac{du}{d\tau}$

$u = (c\gamma, \gamma\beta c) = \left(\frac{d(ct)}{d\tau}, \frac{dx}{d\tau}\right)$

Comptonovo sip. ($\gamma + e^-$) $\lambda' - \lambda = \frac{hc}{m_e c^2}(1 - \cos\theta)$

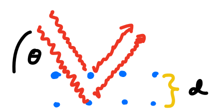


$\lambda_c = \frac{hc}{m_e c^2} \doteq 2\,\mathrm{pm}$

$hc = 1240\,\mathrm{eV\,nm}$ $\hbar c = 200\,\mathrm{eV\,nm}$

$m_e c^2 = 511\,\mathrm{keV}$

Braggovo sipanje $2d\sin\theta = n\lambda$



(konstruktivna interferenca)

② Kvantna fizika

$\hbar = \frac{h}{2\pi}$

de Broglijeva valovna dolžina

$\lambda = \frac{h}{p}$ $\nu = \frac{E}{h}$

Coulombski potencial

$V(r) = -\frac{ze^2}{4\pi\varepsilon_0 r}$

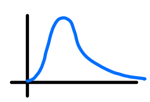
foton $E_f = h\nu = \frac{hc}{\lambda}$ $c = \nu\lambda$

$E_f = pc$

Bohrov radij $r_B = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} \doteq 52{,}9\,\mathrm{pm}$

Planckov zakon ?

$\frac{dj}{d\nu} = \frac{2\pi h}{c^2} \frac{\nu^3}{e^{h\nu/k_BT} - 1}$



$\frac{dj}{d\lambda} = -\frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_BT} - 1}$

Valovna funkcija $\Psi(x,t)$

$\rho(x,t) = |\Psi(x,t)|^2 = \Psi^*\Psi$

$\int_{-\infty}^{\infty} \Psi^*\Psi\, dx = 1$

Kroženje e^- v B

$pc = eBrc$

$k_B = 9 \cdot 10^{-5}\,\mathrm{eV/K}$

Razpad $N = N_0\, 2^{-\frac{t}{t_{1/2}}} = N_0 e^{-\frac{t}{\tau}}$

Valovni paket $\Psi(x,t) = a e^{-i(\omega t - kx)} \frac{\sin(\delta\omega t - \delta k x)}{\delta\omega t - \delta k x}$ (nenatančnost)

Gaussov val. paket $\psi(x) = \frac{\sqrt{\pi}}{2} \frac{1}{\sigma_x} A_0 \exp\left(-\frac{x^2}{4\sigma_x^2}\right) e^{ik_0 x}$ $\sigma_x = \frac{1}{2\sigma_k}$

$p = k\hbar$

$E = \omega\hbar$ $\Rightarrow$ $\omega = \frac{\hbar k^2}{2m}$

$k = \frac{2\pi}{\lambda}$

$\omega = 2\pi\nu$

$c = \omega / k$

Načelo nedoločenosti $\sigma_x \sigma_p \geq \frac{\hbar}{2}$ $\sigma_E \sigma_t \geq \frac{\hbar}{2}$

3 Kvantna mehanika

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

$$\hat{H} = \hat{T} + \hat{V} \qquad \hat{H}\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

$$\hat{T} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} \qquad \hat{p} = -i\hbar\frac{\partial}{\partial x} \qquad \hat{E} = i\hbar\frac{\partial}{\partial t}$$

Verjetnost zaznave $\int_{x_1}^{x_2} |\Psi|^2 dx$

Stacionarna stanja $\hat{H}\psi = E\psi$

$$\rightarrow \Psi_n = \psi_n e^{-iE_n t/\hbar}$$

$$|\Psi(x,t)|^2 = |\psi(x)|^2$$

$$\int_{-\infty}^{\infty} \psi_n^* \psi_m \, dx = \delta_{m,n} = \begin{cases} 0 \; ; & m \neq n \\ 1 \; ; & m = n \end{cases}$$

Korespondenčno načelo $(n \to \infty)$

preidemo v klasično limito

$$\Psi(x,t) = \sum_n c_n \psi_n(x) e^{-iE_n t/\hbar}$$

Razvoj po lastnih stanjih

$$c_n = \int_{-\infty}^{\infty} \psi_n^* \psi \, dx$$

$$\langle \hat{A} \rangle = \int_{-\infty}^{\infty} \Psi^* \hat{A} \Psi \, dx$$

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2 \quad \left(\begin{matrix} \text{V težiščnem} \\ \text{sistemu } \langle x \rangle = \langle p \rangle = 0 \end{matrix} \right)$$

$$\int \psi_\perp^* \psi \, dx = 0$$

$$\langle E \rangle = \sum_n |c_n|^2 E_n \qquad \langle p^2 \rangle = 2m\langle T \rangle = 2m\langle E \rangle$$

Neskončna pot. jama

$$V(x) = \begin{cases} 0 & \text{ознака} \\ \infty & \text{sicer} \end{cases}$$

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin k_n x \qquad k_n = \frac{n\pi}{a}$$

$$E_n = n^2 \frac{\hbar^2 \pi^2}{2ma^2} = n^2 E_1 \qquad n \geq 1$$

Končna potencialna jama

$$\psi(x) = \begin{cases} e^{kx} & x < 0 \\ e^{k(a-x)} & x > a \end{cases} \qquad k = \sqrt{2m(V-E)}/\hbar$$

Linearni harmonski oscilator

$$V(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega_0^2 x^2$$

$$\Psi_n(x,t) = \left(\frac{m\omega_0}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n(\xi) e^{-\xi^2/2} e^{-iE_n t/\hbar}$$

$\hookrightarrow$ Hermitovi polinomi

$$H_0(\xi) = 1$$
$$H_1(\xi) = 2\xi$$
$$H_2(\xi) = 4\xi^2 - 2$$
$$H_3(\xi) = 8\xi^3 - 12\xi$$

$$\xi = \sqrt{\frac{m\omega_0}{\hbar}}\, x \qquad \omega_0 = \sqrt{\frac{k}{m}}$$

$$E_n = \hbar\omega_0\left(n + \frac{1}{2}\right) \qquad n = 0, 1, \dots$$

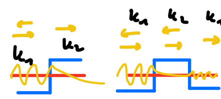
Potencialna plast

$$k_1 = \sqrt{2mE}/\hbar \qquad k_2 = \sqrt{2m(E-V)}/\hbar$$

Val Ae^{ikx} $\rightarrow$ $\qquad Ae^{-ikx}$ $\leftarrow$

$$T = \frac{1}{1 + \left(\frac{k_2^2 - k_1^2}{2k_1k_2}\right)^2 \sin^2 k_2 a}$$

Tuneljenje



$$k_2 = i\sqrt{2m(V-E)}/\hbar = i\varkappa$$

$$Ae^{-\varkappa x} \qquad Ae^{\varkappa x}$$

$$T = \frac{1}{1 + \left(\frac{k_1^2 + \varkappa^2}{2k_1\varkappa}\right)^2 \operatorname{sh}^2 \varkappa a}$$

Verjetnostni tok $\quad j = \frac{\hbar}{2mi}\left(\psi^* \frac{d\psi}{dx} - \frac{d\psi^*}{dx}\psi\right)$

$$T + R = 1$$

N. prehodil $\quad \psi_1 = \psi_2 \qquad \psi_1' = \psi_2'$

$$R = \frac{j_{odb}}{j_{vpad}} = \frac{\vec{B}|k_1|^2}{\vec{A}|k_1|^2} \qquad T = \frac{j_{prep}}{j_{vpad}} = \frac{\vec{C}|k_2|^2}{\vec{A}|k_1|^2}$$

Odvodi

$$x^{n\prime} = n x^{n-1}$$
$$e^{x\prime} = e^x$$
$$\ln x' = \frac{1}{x}$$
$$a^{x\prime} = a^x \ln a$$
$$\log_a x' = \frac{1}{x \ln a}$$
$$\tan x' = \frac{1}{\cos^2 x}$$
$$\cot x' = \frac{-1}{\sin^2 x}$$
$$\arcsin x' = \frac{1}{\sqrt{1-x^2}}$$
$$\arccos x' = \frac{-1}{\sqrt{1-x^2}}$$
$$\arctan x' = \frac{1}{1+x^2}$$
$$\operatorname{arccot} x' = \frac{-1}{1+x^2}$$

Integrali

$$\int x^n = \frac{x^{n+1}}{n+1}$$
$$\int e^x = e^x$$
$$\int a^x = \frac{a^x}{\ln a}$$
$$\int \frac{1}{x} = \ln|x|$$
$$\int \frac{1}{\cos^2 x} dx = \tan x$$
$$\int \frac{1}{\sin^2 x} dx = -\cot x$$
$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin\frac{x}{a}$$
$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\arctan\frac{x}{a}$$
$$\int \frac{1}{a^2 \mp x^2} dx = \frac{1}{2a}\ln\left|\frac{x \pm a}{x \mp a}\right|$$
$$\int \sqrt{a^2 \pm x^2}\, dx = \frac{x}{2}\sqrt{a^2 \pm x^2} \dots$$
$$\dots \pm \frac{a^2}{2}\ln\left|\sqrt{a^2 \pm x^2} + x\right|$$
$$\int_{-\infty}^{\infty} e^{-a(x+b)^2} dx = \sqrt{\frac{\pi}{a}}$$

Taylorjeve vrste

$$(1 \pm x)^m = 1 \pm mx + \frac{m(m-1)}{2!}x^2$$
$$(1 \pm x)^{-m} = 1 \mp mx + \frac{m(m+1)}{2!}x^2$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$
$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5$$
$$\operatorname{sh} x = x + \frac{x^3}{3!} + \frac{x^5}{5!}$$
$$\operatorname{ch} x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!}$$
$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$$
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3}$$
$$\arcsin x = x + \frac{x^3}{6} + \frac{3x^5}{40}$$
$$\arccos x = \frac{\pi}{2} - \arcsin x$$
$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5}$$
$$\operatorname{arccot} x = \frac{\pi}{2} - \arctan x$$
$$\tanh x = x - \frac{x^3}{3} + \frac{2}{15}x^5$$
$$\coth x = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45}$$

geometrijska $\quad S_n = a_0 \frac{1 - q^{n+1}}{1 - q}$

Hiperbolične funkcije

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\tanh x = \frac{\sinh x}{\cosh x}$$
$$\sinh' x = \cosh x$$
$$\cosh' x = \sinh x$$
$$\tanh' x = 1 - \tanh^2 x$$
$$\operatorname{sh} x = -i \sin ix$$
$$\operatorname{ch} x = i \cos ix$$
$$\operatorname{ch}^2 x - \operatorname{sh}^2 = 1$$
$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$
$$\cos x = \frac{e^{ix} + e^{-ix}}{2i}$$

Konstante

Bronštejn 846

Γ funkcija

$$\Gamma(s) = \int_0^\infty x^{s-1} e^{-x} dx$$
$$\Gamma(n) = (n-1)!$$
$$\Gamma(n+1) = n\Gamma(n)$$
$$\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$$
$$\Gamma(x+1) = \left(\frac{x}{e}\right)^x \sqrt{2\pi x}$$
$$\Gamma(s)\Gamma(1-s) = \frac{\pi}{\sin(\pi s)} = B(s, 1-s) \qquad 0 < s < 1$$

B funkcija

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$
$$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1} dx$$
$$B(a,b) = \int_0^\infty x^{a-1}/(1+x)^{a+b} dx$$
$$B(a,b) = 2\int_0^{\pi/2} \cos^{2a-1} x \sin^{2b-1} x \, dx$$

Linearni harmonski oscilator

$V(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega_0^2 x^2$

$$\Psi_n(x,t) = \left(\frac{m\omega_0}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n(\xi)\, e^{-\xi^2/2}\, e^{-iE_n t/\hbar}$$

H_n ↳ Hermitovi polinomi

$\xi = \sqrt{\frac{m\omega_0}{\hbar}}\,x \qquad \omega_0 = \sqrt{\frac{k}{m}}$

$E_n = \hbar\omega_0 (n + \frac{1}{2}) \qquad n = 0, 1, \ldots$

$H_0(\xi) = 1 \qquad H_2(\xi) = 4\xi^2 - 2$

$H_1(\xi) = 2\xi \qquad H_3(\xi) = 8\xi^3 - 12\xi$

$$H_n(x) = (-1)^n e^{x^2} \frac{d}{dx^n}\left(e^{-x^2}\right)$$

$$x H_n = \frac{1}{2}\left(H_{n+1} + 2n H_{n-1}\right)$$

H - atom

$$V(r) = -\frac{ze^2}{4\pi\varepsilon_0 r} = -\frac{\alpha\hbar c}{r} \qquad \alpha = \frac{1}{137} = \frac{e^2}{4\pi\varepsilon_0\hbar c}$$

↑ ali ↓

$$\psi(r, \varphi, \theta) = R_{nlm}(r)\, Y_{l,m}(\theta,\varphi)\, \chi_{m_s} \qquad n = 1, 2, \ldots$$

$0 \le l \le n-1$

$|m| \le l$

$$r_B = \frac{4\pi\varepsilon_0\hbar^2}{e^2 m_e} = 0{,}0529\,\mathrm{nm}$$

$$E_{nl} = -\frac{\alpha^2 m c^2}{2}\frac{z^2}{n^2} = \frac{E_0}{n^2} \qquad E_1 = -13{,}6\,\mathrm{eV}$$

za vodik

$l = 0$ S $\quad l = 1$ P $\quad l = 2$ D $\quad l = 3$ F

$dV = r^2 dr \sin\theta\, d\theta\, d\varphi$

$$\int \Psi^*_{n'l'm'} \Psi_{nlm}\, dV = \delta_{nn'}\,\delta_{l'l}\,\delta_{m'm}$$

Serijski spekter $\quad \frac{1}{\lambda} = R_y\left(\frac{1}{n^2} - \frac{1}{n'^2}\right)$

$R_y = 10{,}9 \cdot 10^6\,\mathrm{m}^{-1}$

Izbirno pravilo $\quad \Delta l = \pm 1 \qquad \Delta m = 0, \pm 1$

potenca r v $V(r)$ za LHO -1

Virialni teorem $\quad 2\langle T\rangle = n\langle V\rangle = -\frac{1}{2}\langle E\rangle$

Zeemanov pojav (H v zun. B)

① $B = 0$, samo LS sklopitev

② $\mu B \gg E_{ls}$, $E_{ls} \approx 0 \qquad \Delta E = -\langle\mu_z\rangle B = (m_l + 2m_s)\mu_B B$

③ $\mu B \overset{\approx}{\ll} E_{ls}$, $\langle\mu_z\rangle = -g\,\mu_B\, m_j \qquad \langle E_{mag}\rangle = -\langle\mu_z\rangle B$

Landéjevo giro. raz. $\quad g = \frac{3}{2} - \frac{l(l+1) - s(s+1)}{2j(j+1)}$

Dipolni prehodi in izbirna pravila

$$\frac{1}{\tau} = \frac{\omega_{12}^3 \left(\vec{p}_e^{(12)}\right)^2}{3\pi\varepsilon_0 c^3\hbar} = \frac{4}{3}\frac{\alpha}{\hbar}E_{12}^3\left(\frac{r_{12}}{\hbar c}\right)^2$$

$\Delta l = \pm 1 \qquad \Delta m_l = \pm 1, 0 \qquad \Delta m_s = 0$ Dovoljeno

$\Delta j = \pm 1, 0 \qquad \Delta m_j = \pm 1, 0$ Dovoljeno $\qquad j=0 \not\to j=0$

Za ∞ pot jamo Δn = liho

Za rotator $\Delta l = \pm 1 \qquad$ Za LHO $\Delta m = \pm 1$

Širina spektralne črte (ε) $\quad \frac{\hbar}{\tau}$

3D - ∞ pot jama

$$E_n = \frac{\hbar^2\pi^2}{2m}\left(\frac{n_x^2}{l_x^2} + \frac{n_y^2}{l_y^2} + \frac{n_z^2}{l_z^2}\right)$$

3D - HO

$$E_n = \hbar\omega_0\left(n_x + n_y + n_z + \frac{3}{2}\right)$$

Rotator

reducirana masa $\qquad \mu = \frac{mM}{m+M}$

sferični harmoniki

$$Y_{l,m}(\theta,\varphi) = (-1)^m\sqrt{\frac{(2l+1)}{4\pi}\frac{(l-m)!}{(l+m)!}}\,P_{lm}(\cos\theta)\,e^{im\varphi}$$

$m \in \mathbb{Z}$, $|m| \le l$ (z-komp VK), $l = 0, 1, 2, \ldots$ (KŠ tirne VK)

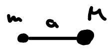
lastne vrednosti VK $\qquad$ ↑ če se ne vrti

$L^2 = \hbar l(l+1) \qquad L_z = m\hbar \qquad \hat{L}_z = -i\hbar\frac{d}{d\varphi}$

$P_{lm}(\cos\theta)$ Legendrovi polinomi

$l = 0$	$m = 0$	$Y_{00} = \frac{1}{\sqrt{4\pi}}$
$l = 1$	$m = 0$	$Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta$
	$m = \pm 1$	$Y_{1\pm1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{\pm i\varphi}$
$l = 2$	$m = 0$	$Y_{20} = \sqrt{\frac{5}{8\pi}}(3\cos^2\theta - 1)$
	$m = \pm 1$	$Y_{2\pm1} = \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{\pm i\varphi}$
	$m = \pm 2$	$Y_{2\pm2} = \sqrt{\frac{15}{32\pi}}\sin^2\theta\, e^{\pm 2i\varphi}$

$$E_{rot} = \frac{\hbar^2 l(l+1)}{2J} = \frac{\langle l^2\rangle}{2J} \qquad J = \mu a^2$$



Spin

$g_e = 1 \qquad \mu_B = 5{,}8 \cdot 10^{-5}\frac{\mathrm{eV}}{\mathrm{T}}$

$\hat{\vec{\mu}} = -\frac{1}{\hbar}g_e\mu_B\hat{L} \qquad \hat{\mu}_z = -\frac{1}{\hbar}g_e\mu_B\hat{L}_z$

$\langle\mu\rangle = g_e\sqrt{l(l+1)}\,\mu_B \qquad \langle\mu_z\rangle = g_e m_e \mu_B$

$E_{mag} = m_e g_e \mu_B B \qquad E_{nm_e} = -\frac{E_0}{n^2} + E_{mag}$ ni več degenerirano

$\langle\vec{S}^2\rangle = \hbar^2 s(s+1) \qquad \langle S_z\rangle = m_s\hbar \qquad s = \frac{1}{2} \qquad m_s = \pm\frac{1}{2}$

$\vec{\mu}_s = -\frac{1}{\hbar}g_s\mu_B\vec{S} \qquad g_s = 2$

$j = l \pm \frac{1}{2}$

Skupna VK $\quad \vec{j} = \vec{L} + \vec{S} \qquad |\vec{J}^2| = \hbar^2 j(j+1) \qquad J_z = \hbar m_j$

Spektroskopske oznake $\quad n\,^{2S+1}L_j \qquad L \in (S, P, D, F)$

Seštevanje VK

$\hat{J}^2 = \hat{L}^2 + 2\langle L, S\rangle + \hat{S}^2 \qquad j_{max} = l + s$

$\langle L, S\rangle = \frac{\hbar^2}{2}\left(j(j+1) - l(l+1) - s(s+1)\right) \qquad j_{min} = |l - s|$

$$E_{ls} = \frac{ze^2}{4\pi\varepsilon_0}\frac{\langle L,S\rangle}{m_e^2c^2}\left\langle\frac{1}{r^3}\right\rangle = \frac{\alpha\hbar c}{2m_e^2c^2}\left\langle\frac{1}{r^3}\right\rangle\langle l\cdot s\rangle$$

$$\langle E_{ls}\rangle = \frac{z^4\alpha^2}{n^3}E_0\,\frac{j(j+1) - l(l+1) - 5/4}{l(l+1)(2l+1)}$$

Hundova pravila za večelektronska stanja

① Najprej zapolnimo stanja z max S

② Najprej zapolnimo stanja z max L

LS razcep ③ Če elektroni zapolnijejo manj kot $\frac{1}{2}$ vseh stanj: zapolnimo min J; več: zapolnimo max J

$$x_{12} = \int \psi_1^*\, x\, \psi_2\, dx$$

$$\langle\hat{p}_{r_i}\rangle = e_0\int \psi^*\, r_i\, \psi\, dV$$

$$\int_0^{2\pi}\int_0^{\pi} Y^*_{l'm'}\, Y_{lm} \sin\theta\, d\theta\, d\varphi = \delta_{l'l}\,\delta_{m'm}$$

$$\int R_{n'l'}\, R_{nl}\, r^2 dr = \delta_{nn'}$$

$$\int Y^*_{l'm'}\, Y_{lm} = \delta_{l'l}\,\delta_{m'm} \qquad \text{ortonormiran set}$$

$$\langle L_x\rangle = \int Y^*_{l'm'}\, \hat{L}_x\, Y_{lm} = \frac{\hbar}{2}\sqrt{l(l+1) - m(m\pm1)}\;\delta_{ll'}\,\delta_{m'(m\pm1)}$$

$$\langle L_y\rangle = \int Y^*_{l'm'}\, \hat{L}_y\, Y_{lm} = \mp\frac{i\hbar}{2}\sqrt{l(l+1) - m(m\pm1)}\;\delta_{l'l}\,\delta_{m',(m\pm1)}$$

$$\langle L_z\rangle = \int Y^*_{l'm'}\, L_z\, Y_{lm} = \hbar m\, \delta_{l'l}\,\delta_{m'm}$$

$$\langle L^2\rangle = \int_\Omega Y^*_{l'm'}\, L^2\, Y_{lm} = \hbar^2 l(l+1)\,\delta_{l'l}\,\delta_{m'm}$$

$\hbar c = 200$ eV nm $\hbar = 6{,}58\cdot10^{-16}$ eVs

$hc = 1240$ eV nm $h = 4{,}14\cdot10^{-15}$ eVs

$m_e c^2 = 511$ keV $\mu_B = 5{,}78\cdot10^{-5}$ eV/T

$\alpha = \frac{1}{137}$

LHO $\quad \hat{x}\,\psi_n = \sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{n+1}\,\psi_{n+1} + \sqrt{n}\,\psi_{n-1}\right)$

V bazi $\quad n\, l\, s\, j\, m_j$

$$\langle l^2\rangle = \hbar^2 j(j+1)\,\delta_{j'j}\,\delta_{m_j'm_j}$$

$$\langle l_x\rangle = \frac{\hbar}{2}\sqrt{(j\mp m_j)(j\pm m_j+1)}\;\delta_{j'j}\,\delta_{m_j'm_j\pm1}$$

$$\langle l_y\rangle = \mp i\frac{\hbar}{2}\sqrt{(j\mp m_j)(j\pm m_j+1)}\;\delta_{j'j}\,\delta_{m_j'm_j\pm1}$$

$$\langle l_z\rangle = \hbar m_j\,\delta_{j'j}\,\delta_{m_j'm_j}$$

Neskončna pot. jama

$V(x) = \begin{cases} 0 & \text{očrca} \\ \infty & \text{sicer}\end{cases}$

$\psi_n(x) = \sqrt{\frac{2}{a}}\sin k_n x \qquad k_n = \frac{n\pi}{a}$

$E_n = n^2\frac{\hbar^2\pi^2}{2ma^2} = n^2 E_1 \qquad n\geq1$

Končna potencialna jama

$$\psi(x) = \begin{cases} e^{kx} & x<0 \\ e^{k(a-x)} & x>a\end{cases} \qquad k = \sqrt{2m(V-E)}/\hbar$$

Kvantna mehanika

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

$$\hat{H} = \hat{T} + \hat{V} \qquad \hat{H}\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

$$\hat{T} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} \qquad \hat{p} = -i\hbar\frac{\partial}{\partial x} \qquad \hat{E} = i\hbar\frac{\partial}{\partial t}$$

Verjetnost zaznave $\int_{x_1}^{x_2}|\Psi|^2 dx$

Stacionarna stanja $\hat{H}\psi = E\psi$

$$\rightarrow \Psi_n = \psi_n e^{-iE_n t/\hbar}$$

$$|\Psi(x,t)|^2 = |\psi(x)|^2$$

$$\int_{-\infty}^{\infty}\psi_n^*\,\psi_m\,dx = \delta_{m,n} = \begin{cases}0 ; & m\neq n\\ 1 ; & m=n\end{cases}$$

Korespondenčno načelo $(n\to\infty)$

preidemo v klasično limito

$$\Psi(x,t) = \sum_n c_n\,\psi_n(x)\,e^{-iE_nt/\hbar}$$

Razvoj po lastnih stanjih

$$c_n = \int_{-\infty}^{\infty}\psi_n^*\,\psi\,dx$$

$$\langle\hat{A}\rangle = \int_{-\infty}^{\infty}\Psi^*\hat{A}\,\Psi\,dx$$

$\sigma_x^2 = \langle x^2\rangle - \langle x\rangle^2 \quad$ (V težiščnem sistemu $\langle x\rangle = \langle p\rangle = 0$)

$$\int\psi_\perp^*\,\psi\,dx = 0$$

$$\langle E\rangle = \sum_n |c_n|^2 E_n \qquad \langle p^2\rangle = 2m\langle T\rangle = 2m\langle E\rangle$$

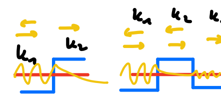
Potencialna plast

$k_1 = \sqrt{2mE}/\hbar \qquad k_2 = \sqrt{2m(E-V)}/\hbar$

Val $\quad Ae^{ikx} \rightarrow \qquad Ae^{-ikx}\leftarrow$

$$T = \frac{1}{1+\left(\frac{k_2^2-k_1^2}{2k_1k_2}\right)^2\sin^2 k_2 a}$$

Tuneliranje



$k_2 = i\sqrt{2m(V-E)}/\hbar = i\varkappa$

$A e^{-\varkappa x} \qquad Ae^{\varkappa x}$

$$T = \frac{1}{1+\left(\frac{k_1^2+\varkappa^2}{2k_1\varkappa}\right)^2\mathrm{sh}^2\varkappa a}$$

Verjetnostni tok $\quad j = \frac{\hbar}{2mi}\left(\psi^*\frac{d\psi}{dx} - \frac{d\psi^*}{dx}\psi\right)$

$T + R = 1 \qquad$ Na prehodih $\quad \psi_1 = \psi_2, \quad \psi_1' = \psi_2'$

$$R = \frac{j_{odb}}{j_{vpad}} = \frac{\overleftarrow{B}|k_1|^2}{\overrightarrow{A}|k_1|^2} \qquad T = \frac{j_{prep}}{j_{vpad}} = \frac{\overrightarrow{C}|k_2|^2}{\overrightarrow{A}|k_1|^2}$$

Γ funkcija

$\Gamma(s) = \int_0^\infty x^{s-1}e^{-x}dx$

$\Gamma(n) = (n-1)!$

$\Gamma(n+1) = n\Gamma(n)$

$\Gamma(\frac{1}{2}) = \sqrt{\pi}$

$\Gamma(x+1) = \left(\frac{x}{e}\right)^x\sqrt{2\pi x}$

$\Gamma(s)\Gamma(1-s) = \frac{\pi}{\sin(\pi s)} = B(s,1-s) \qquad 0<s<1$

B funkcija

$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$

$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}dx$

$B(a,b) = \int_0^\infty x^{a-1}/(1+x)^{a+b}\,dx$

$B(a,b) = 2\int_0^{\pi/2}\cos^{2a-1}x\,\sin^{2b-1}x\,dx$

Taylorjeve vrste

$(1\pm x)^m = 1\pm mx + \frac{m(m-1)}{2!}x^2$

$(1\pm x)^{-m} = 1\mp mx + \frac{m(m+1)}{2!}x^2$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$

$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5$

$\mathrm{sh}\,x = x + \frac{x^3}{3!} + \frac{x^5}{5!}$

$\mathrm{ch}\,x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!}$

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3}$

$\arcsin x = x + \frac{x^3}{6} + \frac{3x^5}{40}$

$\arccos x = \frac{\pi}{2} - \arcsin x$

$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5}$

$\mathrm{arccot}\,x = \frac{\pi}{2} - \arctan x$

$\tanh x = x - \frac{x^3}{3} + \frac{2}{15}x^5$

$\coth x = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45}$

geometrijska $\quad S_n = a_0\frac{1-q^{n+1}}{1-q}$

Hiperbolične funkcije

$\sinh x = \frac{e^x - e^{-x}}{2}$

$\cosh x = \frac{e^x + e^{-x}}{2}$

$\tanh x = \frac{\sinh x}{\cosh x}$

$\sinh' x = \cosh x$

$\cosh' x = \sinh x$

$\tanh' x = 1 - \tanh^2 x$

$\mathrm{sh}\,x = -i\sin ix$

$\mathrm{ch}\,x = \cos ix$

$\mathrm{ch}^2 x - \mathrm{sh}^2 x = 1$

$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$

$\cos x = \frac{e^{ix} + e^{-ix}}{2i}$

Konstante

Bronštejn 846