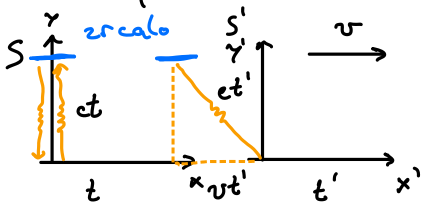
Posebna teorija relativnosti

- linearna transformacija, ki poveže $t \leftrightarrow (x, y, z)$
- hitrost svetlobe je konst. za vse inerc. sisteme $c = 3 \cdot 10^8 \frac{m}{s}$
- $\beta = \frac{v}{c}$ $\begin{cases} \sim 0 & \text{nerelativistična fizika} \\ \sim 1 & \text{PTR} \end{cases}$
- podaljšanje časa



$$(ct')^2 = (vt')^2 + (ct)^2$$

$$t'^2 (c^2 - v^2) = c^2 t^2 \quad /: c^2$$

$$t' = \frac{t}{\sqrt{1-\beta^2}} = \gamma t$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \begin{cases} 1 - \frac{1}{2}\beta^2 & \beta \sim 0 \\ \frac{1}{\sqrt{2\varepsilon}} & \beta = 1 - \varepsilon \quad \varepsilon = \text{majhen} \end{cases}$$

- Lorentzova transformacija

$$x^\mu = (ct, x, y, z) \; ; \; \mu = 0, 1, 2, 3$$

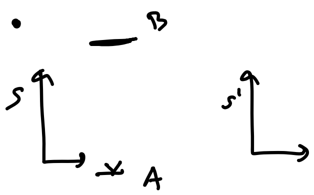
$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

$$ct' = \gamma(ct - \beta x)$$
$$x' = \gamma(x - \beta ct)$$
$$y' = y$$
$$z' = z$$

Obratna L.T $\beta \to -\beta$

$$ct = \gamma(ct' + \beta x')$$
$$x = \gamma(x' + \beta ct')$$

-



$A = (0, 0)$ $\quad A' = (0, 0)$

$B = (ct, 0)$ $\quad B' = (\gamma ct, -\gamma\beta c^2 t)$

$$ct' = \Delta ct_{A'B'} = \gamma ct$$

$$t' = \gamma t$$

1 Letenje mionov

a) $v_\mu = 0{,}994c$
$\tau_\mu = 2{,}2\,\mu s$

$\tau_\mu' = ?$
$l_\mu' = ?$

$$\tau_\mu' = \gamma\, \tau_\mu \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} = 9{,}14$$

$$\tau_\mu' = 20{,}11\,\mu s$$

$$l_\mu' = v\,\tau_\mu' = 6\,km$$

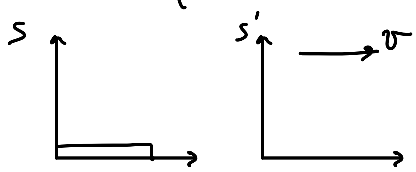
b) $N(t=0) = 568\, s^{-1} = N_0$
$h = 2100\,m$
$N_{merilo} = 412\, s^{-1}$

$$N(t') = N_0\, e^{-\frac{vt'}{v\tau'}} \qquad vt' = h, \quad v\tau' = 6\,km$$ S podaljšanje časa

$$N(t') = 400\, s^{-1}$$

$$N(t) = N_0\, e^{-\frac{vt}{v\tau}} = 25\, s^{-1}$$ Brez podaljšanja časa

T Skrčenje dolžin



$A(0,0)$ $\quad A'(0,0)$
$B($ $\quad B'(0, L')$

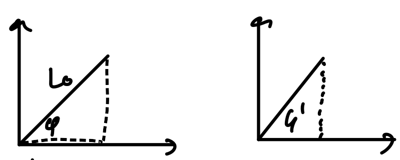
Ker se v S ne premika jo lahko izmerimo ob različnih časih

Obratna L.T.

$$ct = \gamma\,(ct' + \beta x')$$
$$x = \gamma\,(x' + \beta ct')$$
$$y = y'$$
$$z = z'$$

$$\Rightarrow B(\gamma\beta L', \gamma L')$$
$$\Downarrow$$
$$L = \gamma L'$$

Poševni palica



$L_x = L\cos\varphi$
$L_y = L\sin\varphi$

$$\boxed{\tan\varphi' = \gamma\tan\varphi}$$

$$\tan\varphi = \frac{L_y}{L_x} \qquad \tan\varphi' = \frac{L_y'}{L_x'} = \frac{\gamma L_y}{L_x} = \gamma\tan\varphi$$

2

$v = 0{,}8c$
$\alpha' = 30°$
$\alpha = ?$
$L = 1m$
$L' = ?$

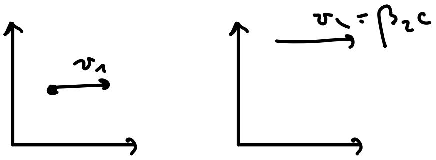
$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{0{,}6}$$

$$\tan\alpha = \frac{\tan\alpha'}{\gamma} = 0{,}6\cdot\tan 30° \Rightarrow \alpha = 19°$$

$$L'^2 = L_x'^2 + L_y'^2 \Rightarrow L' =$$

$$L_x' = \frac{L_x}{\gamma} \qquad L_y' = L_y$$
$$L_x = L\cos\varphi \qquad L_y = L\sin\varphi$$

① Relativistično seštevanje hitrosti



Klasično $\quad v_1' = v_1 - v_2$

$$\frac{v_1'}{c} = \frac{dx'}{c\,dt'} = \frac{\gamma(dx - \beta_2 c\,dt)}{c\,\gamma(c\,dt - \beta_2 dx)} \;\begin{matrix}/:dt\\=\\/:dt\end{matrix}\; \frac{v_1 - \beta_2 c}{c\left(1 - \beta_2 \frac{v_1}{c}\right)}$$

Relativistično

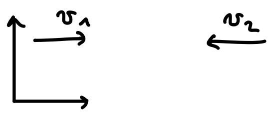
$$\boxed{\begin{aligned} v_1' &= \frac{v_1 - v_2}{1 - \frac{v_1 v_2}{c^2}} \\ \beta_1' &= \frac{\beta_1 - \beta_2}{1 - \beta_1\beta_2} \quad \text{Oddaljevanje} \\ \beta_1' &= \frac{\beta_1 + \beta_2}{1 + \beta_1\beta_2} \quad \text{Približevanje} \end{aligned}}$$

$\xrightarrow{v_1} \quad \xleftarrow{v_2} \qquad \beta_1' = \dfrac{\beta_1 + \beta_2}{1 + \beta_1\beta_2}$

$\xrightarrow{v_1} \quad \xrightarrow{v_2} \qquad \beta_1' = \dfrac{\beta_1 - \beta_2}{1 - \beta_1\beta_2}$

$v_2 \to -v_2$

③ $v = 0{,}99c \qquad \beta = 0{,}99$



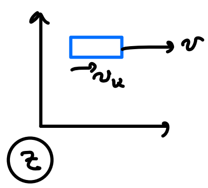
Klasično $\quad v_1' = v_1 - v_2$

Relativistično $\quad \beta_1' = \dfrac{\beta_1 + \beta_2}{1 + \beta_1\beta_2} < 1$

$$\beta_1' = \frac{2(1 - 0{,}01)}{1 + (1 - 0{,}02 + 10^{-4})} \doteq \frac{1}{1 + \frac{1}{2}10^{-4}} = 0{,}99995$$

④ $L = 100\,m$
$v = 0{,}5\,c$
$v_{kz} = 0{,}9\,c$

$t_{kl} = ?$



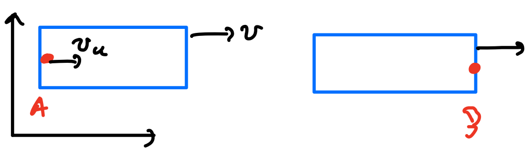
① Transformacija hitrosti

$$\beta_{kl} = \frac{\beta_{kz} - \beta}{1 + \beta_k\beta} = \frac{0{,}9 - 0{,}5}{1 - 0{,}9 \cdot 0{,}5} = \frac{0{,}4}{0{,}55} =$$

Vse količine so v sistemu ladje

$$L = \beta_{kl}\, c\, t_{kl} \rightarrow t_{kl} = \frac{L}{\beta_{kl}\, c} = \frac{L}{c}\,\frac{1 - \beta\beta_{kz}}{\beta_{kz} - \beta} = 0{,}46\,\mu s$$

② Z dogodki in L.T.



$$ct' = \gamma(ct - \beta x)$$
$$x' = \gamma(x - \beta ct)$$

če gremo v smeri x osi drugače
$\beta \to -\beta$

A $(0,0)$
B $(ct_z, v_z t_z)$

V sistemu Zemlje

$\xRightarrow{L.T.}$

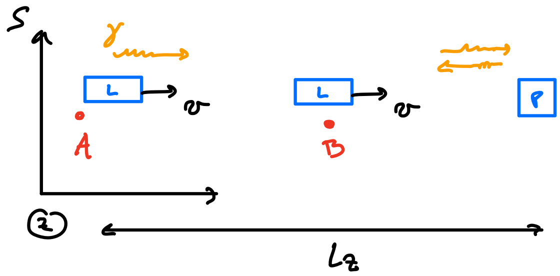
A' $(0,0)$
B' $(ct_{kl}, L) =$
$= \left.\begin{matrix}(\gamma(ct_z - \beta v_z t_z), \\ \gamma(v_z t_z - \beta ct_z))\end{matrix}\right\}$

$$ct_{kl} = ct_z\,\gamma(1 - \beta\beta_k)$$
$$L = ct_z\,\gamma(\beta_k - \beta)$$

$$t_{kl}\frac{c}{L} = \frac{1 - \beta\beta_k}{\beta_k - \beta}$$

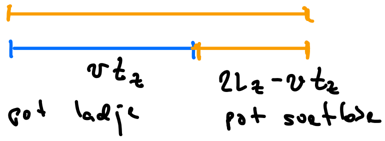
9

$L_z = 4\, c\, td$
$t_L = 6\, td$
$v = ?$
$t_{LP} = ?$



1

a) Postavimo se v sistem Zemlje



$$2L_z - v t_z = c t_z$$
$$2L_z = \gamma t_L (c+v)$$

Dilatacija časa
$t_z = \gamma t_L$

$$\frac{2L_z}{c t_L} = \sqrt{\frac{1+\beta}{1-\beta}}$$

$$(1-\beta)\left(\frac{2L_z}{c t_L}\right)^2 = 1+\beta$$

$$\beta = \frac{\left(\frac{2L_z}{ct_L}\right)^2 - 1}{\left(\frac{2L_z}{ct_L}\right)^2 + 1} = \frac{\frac{16}{9} - 1}{\frac{16}{9}+1} = \frac{7}{25} = 0{,}28$$

b) $t_{LP} = ?$

Z: $t_z = L_z / \beta c \qquad t_z = \gamma t_L$

$$t_{LP} = \frac{L_z}{\gamma \beta c} = \underbrace{\frac{4\, c\, td}{0{,}28\, c}}_{100\ \text{dni}} \underbrace{\sqrt{1-\beta^2}}_{1 - \frac{1}{2} 0{,}28^2 \simeq 0{,}96} = 96\ \text{dni}$$

2 z dogodki

S: $A(0,0)$
$B(ct_z,\ vt_z)$
$vt_z = 2L_z - ct_z$

S': $A'(0,0)$
$B'(\gamma(ct_z - \beta v t_z),\ \gamma(v t_z - \beta c t_z))$
$B'(c t_L,\ v_L t_L)$

$$t_L = \gamma t_z (1-\beta^2) = t_z \sqrt{1-\beta^2} = \frac{t_z}{\gamma}$$

T Gibalna količina in kinetična energija

Klasično: $\vec{p} = m\vec{v} \qquad W_k = \frac{mv^2}{2}$

Relativistično: $(ct, x) = x^\mu$

$$\boxed{\begin{array}{l} cp^\mu = (E,\ c\vec{p}) \\ E = \gamma m c^2 \qquad cp = \gamma \beta m c^2 \end{array}}$$

V sistemu S

$$\boxed{cp' = L\, cp}$$

V sistemu S' $\quad cp'^\mu = (E',\ cp')$

$$= (\gamma(E - \beta cp),\ \gamma(cp - \beta E))$$

Poglejmo limite $\gamma = \frac{1}{\sqrt{1-\beta^2}} \simeq 1 + \frac{1}{2}\beta^2$

(a) Nerelativistična limita

$$E = \underbrace{mc^2}_{} + \underbrace{\frac{1}{2}mv^2 + \dots}_{T}$$

polna energija

relativistična kinetična energija

mirovna energija -- v klasični je to le neka konstanta, ki jo lahko postavimo na 0

$$cp = \gamma\beta mc^2 \rightarrow cmv$$

$$E = mc^2 + T = \gamma mc^2$$

$$\boxed{T = (\gamma - 1)mc^2}$$

Invarianta cp^μ

$$\boxed{(cp^\mu)(cp_\mu) = (cp^0)^2 - (c\vec{p})^2}$$

$$= E^2 - (c\vec{p})^2 =$$

$$= (\gamma mc^2)^2 - (\gamma\beta mc^2)^2 = (mc^2)^2(\cancel{\gamma^2 - \gamma^2\beta^2})^{1} = m^2c^4$$

(b) Relativistična limita

$\beta \rightarrow 1 \qquad \gamma \gg 1$

$$E \approx cp \quad \Rightarrow \quad E^2 - (cp)^2 = m^2c^4$$

Brezmasni delci $m = 0$ (fotoni)

$$E = cp$$

17

$$\frac{T}{W_k} = R = (1.01,\ 1.1,\ 5)$$

nerelativistično, relativistično

$\beta = ?$

$$\frac{(\gamma - 1)mc^2}{\frac{1}{2}mc^2\beta^2} = R \qquad \frac{1}{\sqrt{1-\beta^2}} = \frac{R\beta^2}{2} + 1$$

$$\frac{1}{1-\beta^2} = \left(\frac{R\beta^2}{2} + 1\right)^2$$

Rešimo kvadratno enačbo

$$\beta = \sqrt{\frac{R - 4 + \sqrt{R(R+8)}}{2R}}$$

$R \sim 1 \rightarrow \beta \sim 0$

$R \gg 1 \rightarrow \beta \sim 1$

18

$m_p c^2 = 940\ \mathrm{MeV}$

$p c = 900\ \mathrm{MeV}$

$$E = \underbrace{T}_{\text{kin. en.}} + mc^2$$

$$E^2 = (pc)^2 + (mc^2)^2$$

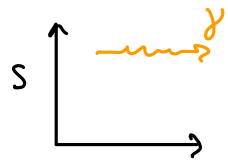
$$(T + mc^2)^2 = (pc)^2 + m^2c^4$$

$$T = \sqrt{(pc)^2 + m^2c^4} - mc^2$$

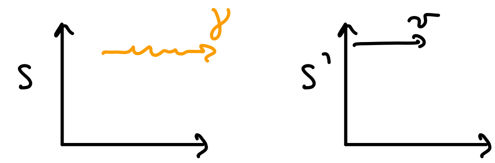
$$\boxed{T = mc^2\left(\sqrt{1 + \frac{(pc)^2}{(mc^2)^2}} - 1\right)}$$

$$T \doteq 0{,}3\ \mathrm{Ge}$$

T Dopplerjev pojav



$$p^\mu = (E, cp) = (h\nu, h\nu) = h\nu(1,1)$$



$m_\gamma = 0$

$E^2 = (pc)^2 - 0$

$E = pc = h\nu$

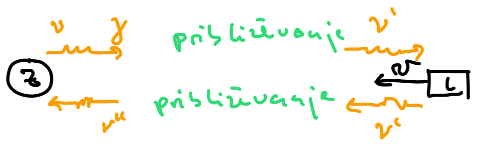
$$S':\ cp'^\mu = (\gamma(E - \beta(cp)), \gamma(cp - \beta E))$$
$$= (\gamma(h\nu - \beta h\nu), \gamma(h\nu - \beta h\nu))$$
$$= \gamma h\nu(1-\beta, 1-\beta)$$
$$= (E', cp')$$

$$\nu' = \gamma\nu(1-\beta)$$

Opazovalec se oddaljuje $\quad \nu' = \nu\sqrt{\frac{1-\beta}{1+\beta}}$

približuje $\quad \nu' = \nu\sqrt{\frac{1+\beta}{1-\beta}}$

61



$\nu = 10\ \mathrm{GHz}$

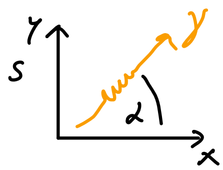
$\nu'' = 15\ \mathrm{GHz}$

$v = ?$

$$\nu' = \nu\sqrt{\frac{1+\beta}{1-\beta}} \qquad \nu'' = \nu'\sqrt{\frac{1+\beta}{1-\beta}}$$

$$\nu'' = \nu\,\frac{1+\beta}{1-\beta} \qquad \beta = \frac{\nu'' - \nu}{\nu'' + \nu} = 0{,}2$$

T Dopplerjev pojav pod kotom

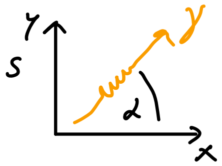


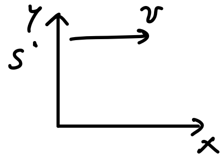
$$cp^\mu = (E, cp_x, cp_y, 0) = h\nu(1, \cos\alpha, \sin\alpha, 0)$$

$$E^2 - (cp_x)^2 - (cp_y)^2 = 0 \quad (m = 0)$$

$E = h\nu \qquad |cp|^2 = E^2$

$cp_x = h\nu\cos\alpha \qquad cp_y = h\nu\sin\alpha$





S': $cp'^\mu = h\nu'(1, \cos\alpha', \sin\alpha')$

$= h\nu\big(\gamma(1-\beta\cos\alpha),$
$\gamma(\cos\alpha-\beta), \sin\alpha, 0\big)$

0: $\nu' = \gamma\nu(1-\beta\cos\alpha)$

1: $\nu'\cos\alpha' = \gamma\nu(\cos\alpha-\beta)$ Opazovalec se oddaljuje

2: $\nu'\sin\alpha' = \nu\sin\alpha$

62 $\beta = 0{,}6$ $\nu = 100\ \text{MHz}$

$\alpha = 30°$

$\nu' = ?$

$\alpha' = ?$

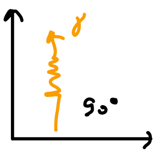
$$\tan\alpha' = \frac{\sin\alpha}{\gamma(\cos\alpha+\beta)} = 0{,}27 \Rightarrow \alpha' = 15°$$

Opazovalec se približuje

$$\nu' = \gamma\nu(1+\beta\cos\alpha) = 190\ \text{MHz}$$

T Transverzalni Dopplerjev pojav

1



$\alpha = 90° \Rightarrow \nu' = \gamma\nu$ $\cos\alpha' = -\frac{\gamma\beta\nu}{\nu'} = -\beta$

$h\nu'(1, \cos\alpha', \sin\alpha', 0) = h\nu(\gamma, -\beta\gamma, 1, 0)$

2 $\alpha' = 90°$ Obratna L.T.

$h\nu(1, \cos\alpha, \sin\alpha, 0)$

$= h\nu'\big(\gamma(1+\beta\cos\alpha'), \gamma(\cos\alpha'+\beta), \sin\alpha', 0\big)$

$\nu = \gamma\nu'$

22 Svetlobna raketa

Ohranitev GK

$$\sum_z p^\mu = \sum_k p^\mu$$

$$p^\mu = p'^\mu_{\text{ladje}} + p^\mu_\gamma$$

Na začetku ladja miruje

$cp^\mu = (mc^2, 0, 0, 0)$

Na koncu je masa ladje $m/2$, β

$p'^\mu_{\text{lad}} = (\gamma \frac{m}{2}c^2, \gamma\beta\frac{m}{2}c^2, 0, 0)$

$cp_\gamma^\mu = E_\gamma(1, -1)$

↑ ker sveti γ v $\leftarrow$

0: $mc^2 = \gamma\frac{m}{2}c^2 + E_\gamma$

1: $0 = \gamma\beta\frac{m}{2}c^2 - E_\gamma$

$mc^2 = \gamma\frac{m}{2}c^2(1+\beta)$

$1 = \frac{\gamma}{2}(1+\beta)$

$2 = \sqrt{\frac{1+\beta}{1-\beta}}$

$4 - 4\beta = 1+\beta$

$\beta = \frac{3}{5} = 0{,}6$

1 Gibanje nabitega delca v elektromagnetnem polju $\vec{E}$ ali $\vec{B}$

Klasično $\vec{F} = m\vec{a} = \frac{\partial \vec{G}}{dt}$

$\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$

Invarianta je lastni čas

$x^\mu = (ct, \vec{x})$

$dx^a = (cdt, d\vec{x})$

$dx^a dx_a = (cdt)^2 - dx^2$

$= (cd\tau)^2$

Če se ne premikamo $d\vec{x} = 0$

$\Rightarrow dt = d\tau$

$$\boxed{\frac{dp^\mu}{d\tau} = e F^{\mu\nu} u_\nu \qquad u^\mu = \frac{p^\mu}{m}}$$

tenzor elektromagnetnega polja

$$u^\mu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} u_\nu$$

$$\begin{pmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} u^0 \\ -u^1 \\ -u^2 \\ -u^3 \end{pmatrix}$$

!

$$F^{\mu\nu} = \begin{bmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ \frac{E_x}{c} & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{bmatrix}$$

Antisimetrična

$x^\mu = (ct, \vec{x})$ kontravariantni vektor

$x_\mu = (ct, -\vec{x})$ kovariantni vektor

23

e^- $\bullet\longrightarrow$ $\vec{E}$

$E = 1{,}76 \ \frac{kV}{m}$ $\quad \vec{E} = (E, 0, 0)$

$\beta = ?$

$t = 1\mu s$ laboratorijski čas

$v_0 = 0$

$l = ?$ pot

$T = ?$ kin. en.

$$m \frac{du^\mu}{d\tau} = e F^{\mu\nu} u_\nu$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -\frac{E}{c} & 0 & 0 \\ \frac{E}{c} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$m \begin{pmatrix} \frac{du^0}{d\tau} \\ \frac{du^1}{d\tau} \end{pmatrix} = e \begin{pmatrix} 0 & -\frac{E}{c} \\ \frac{E}{c} & 0 \end{pmatrix} \begin{pmatrix} u_0 = u^0 \\ u_1 = -u^1 \end{pmatrix}$$

$m\dot{u}^0 = \frac{e(-E)}{c}(-u^1)$

$m\dot{u}^1 = \frac{eE}{c} u^0$

$\alpha = \frac{eE}{mc}$

$\dot{u}^0 = \alpha u^1$

$\dot{u}^1 = \alpha u^0$ odvajamo

$\ddot{u}^0 = \alpha \dot{u}^1$

$\ddot{u}^0 = \alpha^2 u^0$

$u^0 = A \operatorname{ch} \alpha\tau + B \operatorname{sh} \alpha\tau$

$\dot{u}^0 = A\alpha \operatorname{sh} \alpha\tau + B\alpha \operatorname{ch} \alpha\tau$

$\dot{u}^0 = \alpha u^1$

Robni pogoji (A, B)

$\tau = 0 \Rightarrow t = 0 \quad v = 0$

$$u^\mu = \frac{dx^\mu}{d\tau} = \gamma \frac{dx^\mu}{dt} = \gamma \left(c\frac{dt}{dt}, \frac{dx}{dt} \right) = \gamma (c, \beta c)$$

$dt = \gamma \, d\tau$

(a) $\tau = 0 \quad v = 0 \quad \beta = 0 \quad \gamma = 1$

$u^\mu = (c, 0)$

vstavim v rešitve enačbe

$u^0(\tau = 0) = A = c$

$u^1(\tau = 0) = B = 0$

$u^0(\tau) = c \operatorname{ch} \alpha\tau$

$u^1(\tau) = c \operatorname{sh} \alpha\tau$

$u^0 = \frac{dx}{d\tau} = c \operatorname{ch} \alpha\tau$

$ct = \int_0^\tau c \operatorname{ch} \alpha\tau' \, d\tau'$

$= c \left. \frac{\operatorname{sh} \alpha\tau'}{\alpha} \right|_0^\tau$

$= \frac{c}{\alpha} \operatorname{sh} \alpha\tau$

$\alpha t = \operatorname{sh} \alpha\tau \underset{\text{pri } \alpha\tau \ll 1}{\approx} \alpha\tau$

$t = \tau$

opomba

$u^1 = \frac{dx}{d\tau} = c \operatorname{sh} \alpha\tau$

$\int_0^L dx = L = \left. \frac{c}{\alpha} \operatorname{ch} \alpha\tau' \right|_0^\tau$

$L = \frac{c}{\alpha} (\operatorname{ch} \alpha\tau - 1)$

$\operatorname{ch}^2 \alpha\tau = 1 + \underbrace{\operatorname{sh}^2 \alpha\tau}_{\alpha t}$

$L(t) = \frac{c}{\alpha} \left(\sqrt{1 + (\alpha t)^2} - 1 \right)$

$\alpha = \frac{eE}{m_e c^2} \quad t_c = \frac{1{,}35 \text{ keV/m}}{511 \text{ keV}} \quad \mu s \cdot 8 \cdot 10^8 \frac{m}{s}$

$\alpha t = 1{,}07$

$L(t) = 126 \text{ m}$

Drug način

z uporabo las. časa

$\mu = 1 \qquad m \frac{du^1}{d\tau} = e \frac{E}{c} u^0 = e \frac{E}{c} \frac{dct}{d\tau}$

$u^1 = \frac{eE}{mc} c t$

$p_1 = m\gamma\beta c$

$u^1 = \frac{p_1}{m} = \gamma v \qquad u^1 = \gamma v$

$\Rightarrow \gamma\beta = \alpha t \quad |^2$

$\frac{\beta^2}{1-\beta^2} = (\alpha t)^2$

$\beta^2 (1 + (\alpha t)^2) = (\alpha t)^2$

$\beta = \frac{\alpha t}{\sqrt{1 + (\alpha t)^2}} = \frac{dx}{dt} \frac{1}{c}$

$\int_0^L dx = L = c\alpha \int_0^t \frac{t \, dt}{\sqrt{1 + (\alpha t)^2}}$

$2u \, du = \alpha^2 2t \, dt$

$L = \frac{c}{\alpha} \int_1^{\sqrt{1+(\alpha t)^2}} \frac{\alpha t' \, \alpha dt'}{\sqrt{u^2}}$

$L = \frac{c}{\alpha} \int_1^{\sqrt{1+(\alpha t)^2}} \frac{u \, du}{u}$

$L = \frac{c}{\alpha} \left(\sqrt{1 + (\alpha t)^2} - 1 \right)$

?

O

Kakšna je kinetična energija

$$T = eU = eE \cdot L$$
$$T = mc^2(\gamma - 1) = mc^2\left(\sqrt{1+(\alpha t)^2} - 1\right)$$
$$\Rightarrow L = \frac{mc^2}{eE}\left(\sqrt{1+(\alpha t)^2} - 1\right)$$

$$\gamma^2 \beta^2 = (\alpha t)^2$$
$$(\gamma^2 - 1) = (\alpha t)^2$$
$$\gamma = \sqrt{1+(\alpha t)^2}$$

① z uporedo τ

$\int u^0(\tau)$, $\int u^1(\tau)$

$\Downarrow$ $\tau \Leftrightarrow t$, $\Downarrow$ $L \Leftrightarrow \tau$

$\Downarrow$

$L \Leftrightarrow t$

② z uporedo t

$\int \beta(t) \Rightarrow L$

③ z uporedo ohranitev energije

$T = eU = eEL$

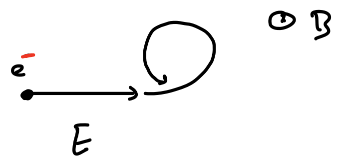
27 Kroženje v mag. polju

$t=0$, $\tau = 0$, $v=0$, $\gamma = 1$, $\beta = 0$

ga pospeši napetost $U = 1\,\mathrm{MV} \Rightarrow T = eU = 1\,\mathrm{MeV}$

in nato prileti v prečno polje $B = 0{,}048\,\mathrm{T}$

$\tau_r = ?$



Enačbe gibanja

$$m\frac{du^\mu}{d\tau} = e\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -B & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} u_\nu$$

$$m\frac{du_0}{d\tau} = 0 \Rightarrow u_0 = \text{konst.}$$
$$\gamma c \Rightarrow \gamma = \text{konst.} \quad \beta = \text{konst.}$$

$$m\dot{u}_1 = e(-B)(-u_2)$$
$$m\dot{u}_2 = eB(-u_1)$$
$$\frac{eB}{m} = \omega$$

$$\dot{u}_1 = \omega u_2$$
$$\dot{u}_2 = -\omega u_1$$

$$\frac{d^2u^1}{d\tau^2} = \ddot{u}_1 = \omega \dot{u}_2 = -\omega^2 u_1$$
$$\ddot{u}_2 = -\omega^2 u_2$$

Nastavek

$$u_1 = A\sin\omega\tau + D\cos\omega\tau$$

$$u_1 = \frac{dx}{d\tau} \qquad u_2 = \frac{dy}{d\tau}$$

Robni pogoji

- $t = \tau = 0 \qquad v_y = 0$

$$u_2(\tau=0) = 0$$

- $u_1(\tau=0) = \frac{cp}{mc} = p/m$

$$\boxed{cp = \sqrt{T(T+2mc^2)}}$$

pri $\tau = 0$

$$u_1 = B = \frac{p}{m}$$

$$\dot{u}_1 = \underbrace{A\cos\omega t\, \omega}_{=\omega u_2} - \frac{p}{m}\omega \sin\omega t$$

$$u_2 = A\cos\omega t - \frac{p}{m}\sin\omega t$$

- robni pogoj 2

$$u_2(\tau=0) = A = 0$$

$$\frac{dx}{d\tau} = u_1 = \frac{p}{m}\cos\omega\tau \quad \Rightarrow \quad x(\tau) = \frac{p}{m}\frac{\sin\omega\tau}{\omega}$$

$$\frac{dy}{d\tau} = u_2 = -\frac{p}{m}\sin\omega\tau \qquad y(\tau) = \frac{p}{m\omega}\cos\omega\tau$$

radij krožnice, ki je tudi enak amplitudi

$$r = \frac{p}{m\omega} = \frac{pm}{meB} \Rightarrow$$

$$\boxed{p = erB}$$

$$2r = \frac{2\sqrt{T(T+2mc^2)}}{eBc}$$

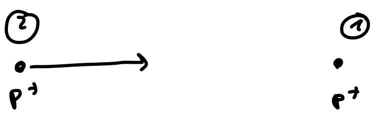
$$2r = \frac{2\sqrt{1\,\mathrm{keV}\,(1\,\mathrm{keV} + 2\,\mathrm{keV})}}{0{,}05\,\frac{\mathrm{eVs}}{\mathrm{m}^2}\cdot 3\cdot 10^8\,\frac{\mathrm{m}}{\mathrm{s}}}$$

$$2r = 0{,}2\,\mathrm{m}$$

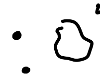
34

$T = 6\,\mathrm{GeV}$

$m_p c^2 \doteq \mathrm{GeV}$



2 p^+ 1 p^+



novi delci

Kolikšen del en. je na voljo za nove delce

max en. bo ko bodo vsi na miru

$$cp_1^\mu = (m_p c^2, 0, 0, 0)$$

$$cp_2^\mu = (\gamma m_p c^2, \gamma\beta m_p c^2, 0, 0)$$

$$cp_{1+2}^\mu = m_p c^2(\gamma+1, \gamma\beta)$$

V končnem sis. bomo imeli največ en. za formacijo delcev

$$c\sum p_k^\mu = (2m_p c^2 + E_{delci}, 0)$$

$$(p_1+p_2)^2 = (p_1+p_2)^\mu (p_1+p_2)_\mu = p_k^\mu p_{k\mu}$$

$$m_p^2 c^4\left((\gamma+1)^2 - \gamma^2\beta^2\right) = (2m_p c^2 + E_{delci})^2 - 0^2$$

$$m_p^2 c^4\left((\gamma+1)^2 - (\gamma^2-1)\right) =$$

$$m^2 c^4(\gamma^2 + 2\gamma + 1 - \gamma^2 + 1) =$$

$$2m_p^2 c^4(\gamma+1) =$$

$$\sqrt{2m^2c^4(\gamma+1)} = 2mc^2 + E_x$$

$$\sqrt{2m^2c^4\left(\frac{T}{mc^2}+2\right)} - 2mc^2 = E_x$$

$$2mc^2\left(\sqrt{\frac{T}{2mc^2}+1} - 1\right) = E_x$$

$$E_x = 2\ \mathrm{GeV}\left(\sqrt{\frac{6\ \mathrm{GeV}}{2\ \mathrm{GeV}}+1} - 1\right) = 2\ \mathrm{GeV}$$

$$\frac{E_x}{T} = \frac{2\ \mathrm{GeV}}{6\ \mathrm{GeV}} = \frac{1}{3}$$

$\frac{1}{3}T$ je na raspolaso za nova dela

$$T = m_p c^2(\gamma - 1)$$

$$\gamma = \frac{T}{mc^2} + 1$$

55

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$

$T_\pi = 100\ \mathrm{MeV}$

$T_{\mu^+} = 80\ \mathrm{MeV}$

$\alpha_\mu = ?$

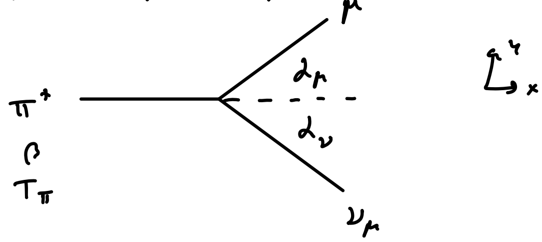
$\alpha_\nu = ?$

$E_\nu = ?$

$m_\pi c^2 = 140\ \mathrm{MeV}$

$m_\mu c^2 = 100\ \mathrm{MeV}$

$m_\nu c^2 = 0$



$$p_\pi^\mu = p_\mu^\mu + p_\nu^\mu$$

$$cp_\pi^\mu = (E_\pi = m_\pi c^2 + T_\pi,\ cp_\pi,\ 0,\ 0)$$

$$E_\pi = 240\ \mathrm{MeV}$$

$$cp_\mu^\mu + cp_\nu^\mu = (E_\mu + E_\nu,\ cp_\mu \cos\alpha_\mu + cp_\nu \cos\alpha_\nu,\ cp_\mu \sin\alpha_\mu - cp_\nu \sin\alpha_\nu,\ 0)$$

$\mu = 0$

$$E_\pi = E_\mu + E_\nu$$

$$T_\pi + m_\pi c^2 - T_\mu - m_\mu c^2 = E_\nu$$

$$240\ \mathrm{MeV} - 180\ \mathrm{MeV} = E_\nu$$

$$E_\nu = 60\ \mathrm{MeV}$$

x: $cp_\pi = cp_\mu \cos\alpha_\mu + E_\nu \cos\alpha_\nu$

y: $cp_\mu \sin\alpha_\mu = E_\nu \sin\alpha_\nu \quad /^2$

$$cp_\pi - cp_\mu \cos\alpha_\mu = E_\nu \cos\alpha_\nu \quad)^2$$

$$(cp_\pi)^2 - 2cp_\pi cp_\mu \cos\alpha_\mu + (cp_\mu)^2 = E_\nu^2$$

$$cp_\pi = \sqrt{T_\pi(T_\pi + 2m_\pi c^2)}$$

$$= 196\ \mathrm{MeV}$$

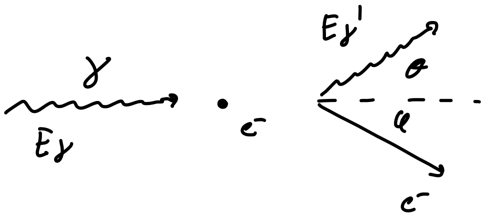
$$cp_\mu = 162\ \mathrm{MeV}$$

$$\cos\alpha_\mu = \frac{-E_\nu^2 + (cp_\mu)^2 + (cp_\pi)^2}{2cp_\pi\, cp_\mu}$$

$$\alpha_\mu = 14^\circ$$

$$\alpha_\nu = 37.3^\circ$$

35



$$0: \quad E_\gamma + m_e c^2 = E_\gamma' + E_e$$

$$x: \quad E_\gamma + 0 = E_\gamma' \cos\theta + cp\cos\varphi \quad /^2$$

$$y: \quad E_\gamma' \sin\theta = cp \sin\varphi \quad /^2 \quad \Big] +$$

foton $m=0$

$E_\gamma = cp_\gamma$

$$(E_\gamma - E_\gamma' \cos\theta)^2 + E_\gamma'^2 \sin^2\theta = (cp)^2 = E_e^2 - m_e^2 c^4$$

$$E_\gamma^2 - 2E_\gamma E_\gamma' \cos\theta + E_\gamma'^2 = E_e^2 - m_e^2 c^4$$

$$0: \quad E_e^2 = (E_\gamma - E_\gamma' + m_e c^2)^2 =$$

$$= E_\gamma^2 - E_\gamma E_\gamma' + E_\gamma m_e c^2 - E_\gamma' E_\gamma + E_\gamma'^2 - E_\gamma' m_e c^2 + m_e c^2 E_\gamma - m_e c^2 E_\gamma' + m_e^2 c^4$$

$$-2E_\gamma E_\gamma' \cos\theta = -2E_\gamma E_\gamma' + 2m_e c^2 (E_\gamma - E_\gamma')$$

$$E_\gamma E_\gamma' (1 - \cos\theta) = m_e c^2 (E_\gamma - E_\gamma')$$

$$E_\gamma = h\nu = \frac{hc}{\lambda}$$

$$\frac{(hc)^2}{\lambda\lambda'}(1 - \cos\theta) = \left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right) mc^2$$

$$\boxed{\lambda' - \lambda = \underbrace{\frac{hc}{mc^2}}_{\lambda_c}(1 - \cos\theta)} \quad \text{Comptonovo sipanje}$$

$$\lambda_c = \frac{hc}{m_e c^2} \approx 2\,\mu m \qquad hc = 1240 \text{ eVnm}$$

$$E_\gamma \gg mc^2, \quad T_e^{max}$$

$$E_\gamma + mc^2 = E_\gamma' + mc^2 + T_e \quad : \quad T_e^{max} \Rightarrow E_\gamma'^{min} \qquad \frac{hc}{E_\gamma'} = \frac{hc}{E_\gamma} + \frac{hc}{mc^2}(1 - \cos\theta)$$

Najveći prenos energije na elektron $\theta = 180°$

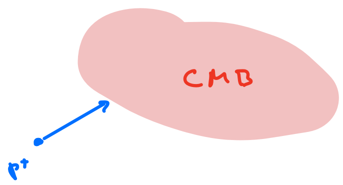
$\cos\theta = -1$

$$T_e = E_\gamma - E_\gamma' = E_\gamma - \frac{1}{\frac{1}{E_\gamma} + \frac{2}{mc^2}} = E_\gamma - \frac{E_\gamma}{1 + \frac{2E_\gamma}{mc^2}}$$

$$= E_\gamma \frac{1}{1 + \frac{mc^2}{2E_\gamma}} \approx E_\gamma\left(1 - \frac{mc^2}{2E_\gamma}\right) \approx E_\gamma$$

Kvantna fizika

Sipanje p^+ na kozmičnem ozadju CMB



$T_p = 2{,}7\,K$ $\qquad E_p = k_B T_p = 10^{-4}\,\frac{eV}{K}\cdot 2{,}7\,K = 2{,}7\cdot 10^{-4}\,eV$

ⓐ $p^+ + \gamma \rightarrow e^+ e^- p^+$

ⓑ $p^+ + \gamma \rightarrow p^+ + \pi^0$ $\qquad c^2 m_\pi = 140\,MeV$

ⓐ

$p^+ \longrightarrow \quad \leftsquigarrow \gamma$ } na začetku

$\Downarrow$

$p^+ \;\cdot\; \cdot e^-$ } težišče sistem

$\cdot e^+$

$$\left(\sum p_z\right)^2 = \left(\sum p_k\right)^2$$

$$\left((E, pc) + (E_\gamma, -E_\gamma)\right)^2 = \left((m_p + 2m_e)c^2, 0\right)^2$$

$$(E + E_\gamma)^2 - (pc - E_\gamma)^2 = (m_p + 2m_e)^2 c^4$$

$$2(E + pc)E_\gamma = 4m_e(m_p + m_e)c^4$$

$$(E + pc)E_\gamma = 2m_e m_p c^4 \quad ; \quad E_\gamma = 10^{-4}\,eV \qquad m_e c^2 = 511\,keV$$

$$E \gg m_p c^2 \quad , \quad E \approx pc$$

$$E_\gamma 2E = 2m_e m_p c^4$$

$$E = \frac{m_e c^2\, m_p c^2}{E_\gamma} \doteq \frac{5}{2}\cdot 10^{19}\,eV \doteq 10^{10}\,GeV \qquad E_\gamma = \frac{3}{2}k_B T$$

namesto $e^+ e^- p$ imamo $\pi^0 p^+$

$$2EE_\gamma = 2m_\pi m_p c^4$$

$$E = \frac{m_\pi c^2\, m_p c^2}{2E_\gamma} \sim 2{,}5\cdot 10^{20}\,eV$$

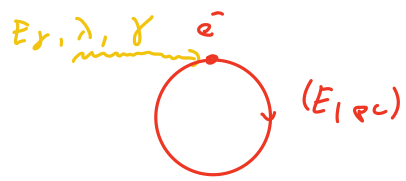
11/6 Comptonov pojav v magnetnem polju

$B = 0{,}002\,T$

$\theta = 90°$

$r = 2\,cm$

$\lambda = ?$



$\boxed{\text{Kroženje } e^- \text{ v } \vec{B} \quad pc = eBrc}$

$$pc = 2\cdot 10^{-3}\,\frac{eVs}{m^2}\; 2\cdot 10^{-2}\,m \cdot 3\cdot 10^{8}\,\frac{m}{s}$$

$$pc = 12\cdot 10^{3}\,eV \ll m_e c^2 \quad \text{ni relativistično}$$

$$T_e \sim \frac{p^2}{2m} = \frac{(pc)^2}{2mc^2} = 0{,}14\,keV$$

$$E_\gamma + m_e c^2 = E_\gamma' + m_e c^2 + T_e$$

$$\lambda' - \lambda = \lambda_c(1 - \cos\theta) = \lambda_c$$

$$\frac{hc}{\lambda} = \frac{hc}{\lambda'} + T_e \qquad \lambda = \lambda' - \lambda_c$$

$$hc\lambda' - hc\lambda = T_e \lambda\lambda'$$
$$hc\lambda_c = T_e \lambda\lambda'$$
$$hc\lambda_c = T_e \lambda(\lambda + \lambda_c)$$
$$\lambda^2 + \lambda_c\lambda - \frac{hc\lambda_c}{T_e} = 0$$

$$\lambda = \frac{1}{2}\left(-\lambda_c + \sqrt{\lambda_c^2 + \frac{4hc\lambda_c}{T_e}}\right)$$

$$\lambda = \frac{\lambda_c}{2}\left(-\cancel{1} + \sqrt{\cancel{1} + \frac{4hc}{T_e\lambda_c}}\right) \qquad \frac{4hc}{T_e\lambda_c} = \frac{4\,mc^2\,hc}{T_e\,hc} \gg 1$$

$$\lambda = \frac{hc}{2mc^2}\, 2\sqrt{\frac{mc^2}{T_e}} = 0{,}1\ \text{nm}$$

11/7

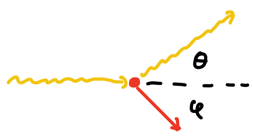
$E_\gamma = 1\ \text{MeV}$ povzroča Comptonov pojav v vodi

$\theta = ?$

Čerenkov

$n = 4/3$

$\varphi = ?$



$$\lambda' - \lambda = \lambda_c(1 - \cos\theta)$$

1 Sevanje Čerenkova

$$v \geq c_v = \frac{c}{n} = \frac{3}{4}c$$
$$\Downarrow$$
$$E_e = ?$$

$$\frac{v}{c} = \beta = \frac{3}{4} \Rightarrow \gamma = \frac{1}{\sqrt{1 - \frac{3^2}{4^2}}} = \frac{4}{\sqrt{7}}$$

$$T_e = mc^2(\gamma - 1)$$
$$= \left(\frac{4}{\sqrt{7}} - 1\right) m_e c^2 = \left(\frac{4}{\sqrt{7}} - 1\right) 511\ \text{keV} \doteq \frac{1}{4}\ \text{MeV}$$

2 Ohranitev energije

$$E_\gamma + mc^2 = E_\gamma' + mc^2 + T_e$$
$$E_\gamma' = E_\gamma - T_e = \frac{3}{4}\ \text{MeV}$$

3

$$\lambda' - \lambda = \lambda_c(1 - \cos\theta)$$
$$\frac{hc}{E_\gamma'} - \frac{hc}{E_\gamma} = \frac{hc}{mc^2}(1 - \cos\theta)$$

$$1 - \cos\theta = \frac{mc^2(E_\gamma - E_\gamma')}{E_\gamma E_\gamma'}$$

$$1 - \cos\theta = \frac{0{,}5\ \text{MeV} \cdot \frac{1}{4}\ \text{MeV}}{\frac{3}{4}\ \text{MeV} \cdot 1\ \text{MeV}} = \frac{1}{6}$$

$$\cos\theta = \frac{5}{6} \qquad \theta \doteq 35^\circ$$

4 Ohranitev GK v y smeri

$$E_\gamma' \sin\theta = \sqrt{T_e(T_e + 2mc^2)}\,\sin\varphi$$

$$\sin\varphi = \frac{E_\gamma' \sin\theta}{\sqrt{T_e(T_e + 2mc^2)}}$$

$$\varphi = 47^\circ$$

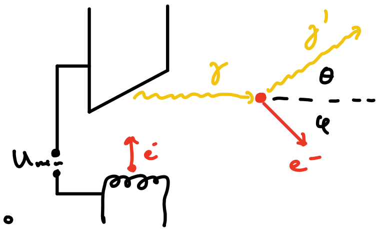
11/8 Zavorno sevanje in Comptonov pojav

$\lambda = 0{,}071\,nm$

$\theta = 45°$

$(pc)_e = ?$

$U_{min} = ?$ na rentgenski cevi, da dobimo to svetlobo



ⓑ $eU_{min} = E_\gamma = \frac{hc}{\lambda}$

$$U_{min} = \frac{hc}{\lambda e} = \frac{1240\,eV\,nm}{0{,}071\,nm\;e} = 17{,}4\,kV$$

ⓐ $\lambda' - \lambda = \lambda_c (1-\cos\theta) \quad /:hc$

$$\frac{1}{E_\gamma'} - \frac{1}{E_\gamma} = \frac{1}{mc^2}(1-\cos\theta)$$

$E_\gamma' = \dots$

$E_\gamma' \sim E_\gamma$

$(pc)_e$

$x:\; E_\gamma + 0 = (cp)\cos\varphi + E_\gamma'\cos\theta \quad /^2$

$+$

$y:\; E_\gamma'\sin\theta = cp\sin\varphi \quad /^2$

$$(E_\gamma - E_\gamma'\cos\theta)^2 + (E_\gamma'\sin\theta)^2 = (cp)^2$$

$$E_\gamma^2 - 2E_\gamma E_\gamma'\cos\theta + E_\gamma'^2 = (cp)^2$$

$E_\gamma' \sim E_\gamma$

$$2E_\gamma^2(1-\cos\theta) = (cp)^2$$

$$cp = E_\gamma\sqrt{2(1-\cos\theta)} = 17{,}4\,keV\sqrt{2(1-\tfrac{\sqrt{2}}{2})} = 13{,}3\,keV$$

11/8 $v = ?$ de Broglie λ_{DB}

Compton λ_c

$e^-:\quad \lambda_c = \frac{hc}{m_ec^2} \qquad \lambda_{DB} = \frac{hc}{pc}$

$\lambda_c = \lambda_{DB}$

$mc^2 = pc$

$pc = \gamma\beta mc^2 = mc^2$

$\gamma\beta = 1 \quad /^2$

$$\frac{\beta^2}{1-\beta^2} = 1$$

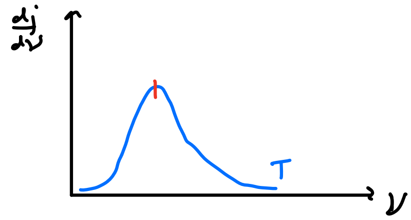
$2\beta^2 = 1$

$\beta = \frac{1}{\sqrt{2}}$

10/14 $T = 1000\,K$ $\quad \lambda = 668\,nm$ $\quad \lambda' = 670\,nm$ $\quad j' = 2j$ $\quad \Delta T = ?$

$$\boxed{\frac{dj}{d\nu} = \frac{2\pi h}{c^2}\frac{\nu^3}{e^{h\nu/kT}-1}}$$

Svetloba prihaja v paketih / kvantih $\quad E_\gamma = h\nu$



$$\frac{dj}{d\lambda} = \frac{dj}{d\nu}\frac{d\nu}{d\lambda}$$

$c = \nu\lambda$
$\nu = \frac{c}{\lambda}$

$$= \frac{dj}{d\nu}\left(-\frac{c}{\lambda^2}\right)$$

$$\frac{dj}{d\lambda} = -\frac{2\pi h}{c^2}\frac{c^3\lambda^{-3}\lambda^{-2}c}{e^{hc/\lambda kT}-1}$$

$\Delta\lambda = 2\,nm$ $\quad \lambda_0 = 669\,nm$

$\hookrightarrow \quad \Delta j = \left.\frac{dj}{d\lambda}\right|_T \Delta\lambda$

λ je enaka v obeh primerih

$T' = T + \Delta T$

$$\boxed{\frac{dj}{d\lambda} = \frac{-2\pi h c^2 \lambda^{-5}}{e^{hc/\lambda kT}-1}}$$

$$\frac{\Delta j(T')}{\Delta j(T)} = \frac{e^{hc/\lambda_0 kT}-1}{e^{hc/\lambda_0 kT'}-1}$$

$hc = 1240\,eV\,nm$
$k = 9\cdot 10^{-5}\,eV/K$

$$\frac{hc}{\lambda kT} \doteq 21{,}5 \gg 1$$ lahko zanemarimo -1

$$\frac{\Delta j(T')}{\Delta j(T)} = e^{\frac{hc}{\lambda_0 kT}\left(1-\frac{T}{T'}\right)} = 2$$

$$\frac{hc}{\lambda_0 kT}\left(1-\frac{T}{T+\Delta T}\right) = \ln 2$$

$$1-\left(1-\frac{\Delta T}{T}\right)$$

$$\frac{hc}{\lambda_0 kT}\frac{\Delta T}{T} = \ln 2$$

$$\Delta T = T\frac{\ln 2}{20} = 37\,K$$

11/11 $\lambda = 435{,}8\,nm$

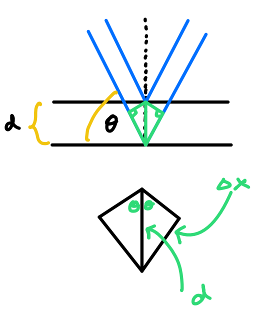
e^- Cs

$E_i = 1{,}9\,eV$

$$E^e_{max} = \frac{hc}{\lambda} - E_i$$

$$E^e_{max} = 0{,}94\,eV$$

11/13 Braggovo sipanje z delci



Ojačitev

$\Delta l = 2\Delta x$
$= 2d\sin\theta$
$= n\lambda$

$$\boxed{n\lambda = 2d\sin\theta}$$

$d = 0{,}21\,nm$
$T_e^{min} = ?$ $\quad m_e c^2 = 511\,keV$
$T_n^{min} = ?$ $\quad m_n c^2 = 1\,GeV$

$$2d\sin\theta = n\lambda = \frac{nh}{p}$$

$$cp^{min} = \frac{nhc}{2d\sin\theta} \overset{min}{=} \frac{hc}{2d} = \frac{1240\,eV\,nm}{0{,}4\,nm} = 3\,keV$$

$\ll m c^2$
ni relativistično

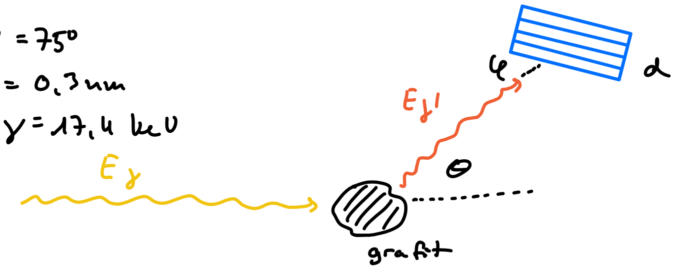
$$T_e = \frac{p^2}{2m} = \frac{p^2c^2}{2mc^2} = \frac{9\,keV^2}{2\cdot 0{,}5\,MeV} = 9\,eV$$

Svetlobno valovanje $\quad E_\gamma = \frac{hc}{\lambda}$

Delčno valovanje (De Broglijevo) $\quad \lambda_{DB} = \frac{h}{p}$

$$T_n = 4{,}5\cdot 10^{-7}\,eV$$

(14) $\theta = 75°$
$d = 0{,}3\,\mathrm{nm}$
$E_\gamma = 17{,}4\,\mathrm{keV}$



Compton

$$\lambda' - \lambda = \lambda_c (1 - \cos\theta)$$

(a) Nastavimo Braggov vrh na λ

$$2d \sin\alpha = n\lambda = nhc/E_\gamma$$

$$\sin\alpha = \frac{hc}{2dE_\gamma} = \frac{1240\,\mathrm{eV\,nm}}{0{,}6\,\mathrm{nm} \cdot 17{,}4\,\mathrm{keV}} \doteq 0{,}12° \qquad \alpha = 6{,}9°$$

(b) Nastavim Braggov vrh na λ'

$$\lambda' = \lambda + \lambda_c (1 - \cos\theta) \qquad \theta = 75°$$

$$\lambda = \frac{hc}{E_\gamma} = 0{,}07\,\mathrm{nm} \qquad \lambda_c = \frac{hc}{m_e c^2} = 2{,}4 \cdot 10^{-3}\,\mathrm{nm}$$

$$\lambda' = \qquad \rightarrow \quad \alpha'$$

(T) Princip nedoločenosti

$$\boxed{\delta x\, \delta p \geq \frac{\hbar}{2} \qquad \delta t\, \delta E \geq \frac{\hbar}{2}}$$

$$(\delta\sigma)^2 = \langle \sigma^2 \rangle - \langle \sigma \rangle^2$$

(1) e^- pospešim z $U = 0{,}1\,\mathrm{MV}$, $\delta U = 50\,\mathrm{V}$, $\delta x = ?$
Relativistično $T_e = eU = 0.1\,\mathrm{MeV} \approx m_e c^2 = 0{,}5\,\mathrm{MeV}$
$\delta T_e = 50\,\mathrm{eV}$

$$pc = \sqrt{T(T + 2mc^2)}$$

$$\delta pc = \frac{dp}{dT}\,\delta T = \frac{1}{2}\,\frac{2T + 2mc^2}{\sqrt{T(T+2mc^2)}}\,\delta T_e = \frac{0{,}1\,\mathrm{MeV} + 0{,}5\,\mathrm{MeV}}{\sqrt{0{,}1(0{,}1+1)\,\mathrm{MeV}^2}}\,50\,\mathrm{eV} = 90\,\mathrm{eV}$$

$$\delta x > \frac{\hbar}{2}\,\frac{1}{\delta p} = \frac{\hbar c}{2\,\delta pc} \doteq 1{,}1\,\mathrm{nm} \qquad \hbar c = \frac{hc}{2\pi} \doteq 197\,\mathrm{eV\,nm} \approx 200\,\mathrm{eV\,nm}$$

(2) Razpad η mezon

$$\tau_\eta = 7 \cdot 10^{-19}\,\mathrm{s} \qquad m_\eta = 550\,\mathrm{MeV}/c^2$$

$$\frac{\delta m_\eta}{m_\eta} = ? \qquad \delta E\, \delta t \geq \frac{\hbar}{2}, \quad \delta t \;\hat{=}\; \tau \qquad \delta E \geq \frac{\hbar c}{2 \tau_\eta c} = \frac{200\,\mathrm{eV\,nm}}{2 \cdot 7 \cdot 10^{-19}\,\mathrm{s} \cdot 3 \cdot 10^{8}\,\frac{\mathrm{m}}{\mathrm{s}}} \doteq 470\,\mathrm{eV}$$

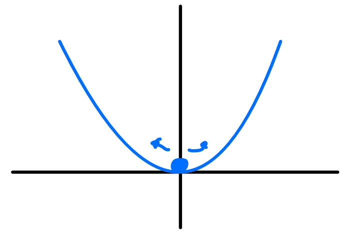
$$\frac{\delta m_\eta c^2}{m_\eta c^2} = \frac{470\,\mathrm{eV}}{550 \cdot 10^{6}\,\mathrm{eV}} \doteq 10^{-6}$$

(T) $\Psi(\vec{x},t)$

Schröd. en. $\quad i\hbar \frac{d\Psi}{dt} = \hat{H}\Psi$

Operator: $\hat{O}$; $\hat{x}$, $\hat{p}$, $\hat{H}$

(P1) Harmonski oscilator, ocena s principom nedol.



$$\hat{H} = \hat{T} + \hat{V} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2$$

$$\sigma_p \sigma_x \geq \frac{\hbar}{2}$$

$\sigma_p^2 = \langle p^2\rangle - \langle p\rangle^2$, težiščni sistem $\langle p\rangle = 0$

in izhodišče si lahko izberemo $\langle x\rangle = 0$

$$E_n = \langle \hat{H}\rangle = \frac{\langle p^2\rangle}{2m} + \frac{1}{2} m\omega^2 \langle x^2\rangle \qquad \sigma_p = \frac{\hbar}{2\sigma_x} \text{ ocena}$$

$$E_0 = \frac{\sigma_p^2}{2m} + \frac{1}{2} m\omega^2 \sigma_x^2 = \frac{\hbar^2}{8m\sigma_x^2} + \frac{1}{2} m\omega^2 \sigma_x^2$$

Iščemo minimum E_0

$$\frac{dE_0}{dx^2} = -\frac{\hbar^2}{8m\sigma_x^4} + \frac{1}{2} m\omega^2 = 0$$

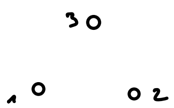
$$\frac{\hbar^2}{4m\sigma_x^4} = m\omega^2 \Rightarrow \sigma_x = \sqrt[4]{\frac{\hbar^2}{4m^2\omega^2}} = \sqrt{\frac{\hbar}{2m\omega}}$$

$$E_0 = \frac{\hbar^{\cancel{2}}}{8\cancel{m}} \frac{2\cancel{m}\omega}{\cancel{\hbar}} + \frac{1}{2}\cancel{m}\omega^{\cancel{2}} \frac{\hbar}{2\cancel{m}\cancel{\omega}} = \frac{\hbar\omega}{2}$$

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) \qquad n = 0, 1, 2, 3 \ldots$$

(P2) Ocena mase protona kot vezanega stanja treh kvarkov $p = (u\ u\ d)$

$V \propto k \circ \hat{r}$, $k = 0{,}09 \frac{GeV^2}{\hbar c}$ iz meritve



$$\hat{H} = \sum_{i=1,2,3} \frac{\hat{p}_i^2}{2m} + k\left((\hat{r}_3 - \hat{r}_1) + (\hat{r}_2 - \hat{r}_1) + (\hat{r}_3 - \hat{r}_2)\right)$$

Izberemo $\langle r_3\rangle > \langle r_2\rangle > \langle r_1\rangle$ v tež. sistemu $\langle p\rangle = 0$ $\quad \sigma_{p_i}^2 = \langle p_i^2\rangle$

$m_u \doteq m_d = 340 \frac{MeV}{c^2}$ $\quad$ Predpostavka $\sigma_{r_i} \sim \langle r_i\rangle$

$$\langle \hat{H}\rangle = \sum_{i=1}^{3} \frac{\langle p_i^2\rangle}{2m} + k\left(\langle r_3\rangle - \langle r_1\rangle + \langle r_2\rangle - \langle r_1\rangle + \langle r_3\rangle - \langle r_2\rangle\right) =$$

$$= \sum_{i=1}^{3} \frac{\langle p_i^2\rangle}{2m} + 2k\left(\langle r_3\rangle - \langle r_1\rangle\right)$$

Ocena $\quad \sigma_r = \frac{\hbar}{2\sigma_p}$

$$E = \sum_i \frac{\sigma_{p_i}^2}{2m} + 2k\frac{\hbar}{2}\left(\frac{1}{\sigma_{p_3}} - \frac{1}{\sigma_{p_1}}\right)$$

$$\frac{dE}{d\sigma_{p_1}} = \frac{\sigma_{p_1}}{m} + k\hbar\left(+\frac{1}{\sigma_{p_1}^2}\right) = 0 \qquad \sigma_{p_1}^3 = -\hbar k m$$

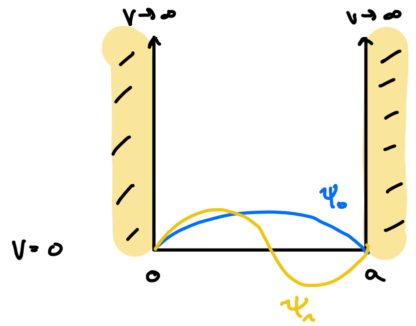
$$\frac{dE}{d\sigma_{p_2}} = \frac{\sigma_{p_2}}{m} = 0 \qquad \sigma_{p_2} = 0$$

$$\frac{dE}{d\sigma_{p_3}} = \frac{\sigma_{p_3}}{m} - k\hbar\frac{1}{\sigma_{p_3}^2} = 0 \qquad \sigma_{p_3}^3 = \hbar k m$$

Vemo $m_p \sim 938 \frac{MeV}{c^2}$

$$E_{min} = \frac{1}{2m}\left(2\sqrt[3]{\hbar k m}^{\,2}\right) + 2k\hbar\frac{1}{\sqrt[3]{\hbar k m}} = \sqrt[3]{\frac{\hbar^2 k^2}{m}} + 2\sqrt[3]{\frac{k^2\hbar^2}{m}} = 3\sqrt[3]{\frac{k^2\hbar^2}{m}} = \ldots = 0{,}9\ GeV$$

(P3) Neskončna potencialna jama



$$\hat{H} = \frac{\hat{p}^2}{2m} + \hat{V}, \quad \hat{V} = 0$$

$$\psi(x) \;; \quad \hat{p} = -i\hbar \frac{d}{dx} \qquad \hat{p}^2 = -\hbar^2 \frac{d^2}{dx^2}$$

$$\hat{H}\psi_n = E_n \psi_n$$

$$-\hbar^2 \psi''_n = E_n 2m \psi_n$$

$$\psi''_n = -\underbrace{\frac{2mE_n}{\hbar^2}}_{\omega^2} \psi_n$$

$$\psi_n = A \sin\sqrt{\frac{2mE_n}{\hbar^2}}\, x + B \cos\sqrt{\frac{2mE_n}{\hbar^2}}\, x$$

$$\psi_n(x=0) = 0 \Rightarrow B_n = 0$$

$$\psi_n(x=a) = 0 \Rightarrow \sqrt{\frac{2mE_n}{\hbar^2}}\, a = n\pi$$

$$E_n = \left(\frac{n\pi}{a}\right)^2 \frac{\hbar^2}{2m}$$

$$P_v = \int_0^a |\psi_n|^2 dx = 1$$

$$= A_n^2 \int_0^a \sin^2 \frac{n\pi}{a} x \, dx = \dots = A_n^2 \frac{a}{2}$$

$$A_n = \sqrt{\frac{2}{a}}$$

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$$

(34)

$m = 1\,\mu g$

$a = 1\,cm$

$E_{0,1} = \dots$

$E_n = 10^{-7}\,J \qquad n = ?$

$\Delta E_n = ?$

$$E_n = \left(\frac{n\pi\hbar}{a}\right)^2 \frac{1}{2m} = \left(\frac{nh}{2a}\right)^2 \frac{1}{2m} \qquad h = 6{,}6 \cdot 10^{-34}\,Js$$

$$E_n = n^2 E_1 \qquad E_1 = 6 \cdot 10^{-53}\,J$$

$$n = \sqrt{\frac{E_n}{E_1}} = \sqrt{\frac{10^{-7}}{6 \cdot 10^{-53}}} \doteq 10^{23}$$

$$\Delta E_n = E_{n+1} - E_n = (n+1)^2 E_1 - n^2 E_1 = E_1(2n+1) \overset{n \gg 1}{=} 2nE_1 \doteq 10^{-32}\,J$$

(37)

$\psi(x)$, $U(x)$ poznamo

$$\boxed{\langle \hat{O} \rangle = \int_V \psi^*(x)\, \hat{O}\, \psi(x)\, dx}$$

$2x_0 = a = 1\,nm$

$m_e c^2 = 0{,}5\,MeV$

$\psi(x) = A(x_0^2 - x^2)$

$\langle E \rangle = ?$

V pot. jami:

$$\boxed{\hat{H} = \frac{\hat{p}^2}{2m} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}}$$

$$\langle E \rangle = \langle \hat{H} \rangle = -\frac{\hbar^2}{2m} \int_{-x_0}^{x_0} \psi \cdot \psi''\, dx =$$

$$= +\frac{\hbar^2}{2m} A^2 \int_{-x_0}^{x_0} (x_0^2 - x^2)\, 2\, dx = \frac{\hbar^2}{m} A^2\, 2\left(x_0^3 - \frac{x_0^3}{3}\right)$$

$$= \frac{\hbar^2 A^2}{m} \frac{4}{3} x_0^3 = \frac{\hbar^2\, 4\, x_0^3\, 15}{m\, 3\, 16\, x_0^5} = \frac{5\, \hbar^2}{4\, m\, x_0^2}$$

$$\langle E \rangle = \dots = 0{,}4\,eV$$

Normalizacija

$$P_v = 1 = \int_{-x_0}^{x_0} |\psi|^2 dx = A^2 \int_{-x_0}^{x_0} (x_0^2 - x^2)^2 dx$$

$$= 2A^2 \int_0^{x_0} x_0^4 - 2x^2 x_0^2 + x^4\, dx$$

$$= 2A^2 \left(x_0^5 - \frac{2}{3}x_0^5 + \frac{x_0^5}{5}\right) =$$

$$= \frac{16}{15} A^2 x_0^5 \Rightarrow A = \sqrt{\frac{15}{16 x_0^5}}$$

$$\boxed{\langle x \rangle = \int_{-x_0}^{x_0} \psi \, \hat{x} \, \psi \, dx} = \int_{-x_0}^{x_0} A^2 (x_0^2 - x^2)^2 \, x \, dx = 0$$

$$\langle p \rangle = \int_{-x_0}^{x} A^2 (x_0^2 - x^2)^2 (-2x) \, dx = 0$$

$$\langle x^2 \rangle = \ldots = \frac{x_0^2}{7}$$

$$\langle p^2 \rangle = 2m \langle E \rangle = \frac{2m \hbar^2 5}{4m x_0^2} = \frac{5\hbar^2}{2x_0^2}$$

$$\sigma_x = \frac{x_0}{\sqrt{7}} \qquad \sigma_p = \sqrt{\frac{5}{2}} \frac{\hbar}{x_0} \qquad \sigma_x \sigma_p = \sqrt{\frac{5}{14}} \hbar = \frac{\hbar}{2} \underbrace{\sqrt{\frac{10}{7}}}_{1{,}2} > \frac{\hbar}{2}$$

42 Nev. pot. jama in e^-

$\psi = A(\psi_1 + 3\psi_3)$

$\psi_\perp = ?$

$\langle E \rangle_\psi = ?$

$\langle E \rangle_{\psi_\perp} = ?$

Normiraj

ker imamo ortonormiran sistem

$$1 = \int \psi^2 dx = A^2 \int \psi_1^2 + 9\psi_3^2 + \emptyset \, dx$$

$$1 = A^2 (1 + 9)$$

$$A = 1/\sqrt{10}$$

Razvoj po lastnih stanjih

$$\psi = \sum_n c_n \psi_n$$

$$\hat{H} \psi_n = E_n \psi_n \qquad E_n = n^2 E_1$$

$$\langle E \rangle = \int \psi^* \hat{H} \psi \, dx = \int A(\psi_1 + 3\psi_3) A (E_1 \psi_1 + 3E_3 \psi_3) \, dx$$

$$= A^2 (E_1 + 9E_3) = \frac{82}{10} E_1 = \frac{41}{5} E_1$$

Ker so lastna stanja med seboj ortogonalna so mešani členi 0.

Časovni razvoj valovne funkcije

$$\psi = \sum_n c_n \psi_n \qquad E_n \ldots \text{dobimo ob meritvi} \qquad \int \psi_m^* \psi_n = \delta_{mn}$$

$$\langle E \rangle = \langle \hat{H} \rangle = \int \psi^* \hat{H} \psi \, dx = \sum_n |c_n|^2 E_n$$

- $\int_{\partial V} \psi_\perp^* \psi \, dx = 0 \qquad \psi_\perp = A_\perp (\psi_1 + b\,\psi_3)$

$$= A_\perp \int (\psi_1 + b\,\psi_3) \frac{1}{\sqrt{10}} (\psi_1 + 3\psi_3) dx$$

$$= \frac{A_\perp}{\sqrt{10}} (1 + 3b) = 0 \quad \Rightarrow \quad b = -\frac{1}{3}$$

- $\int \psi_\perp^* \psi_\perp \, dx = 1$

$$= \int A_\perp^2 (\psi_1 - \tfrac{1}{3}\psi_3)(\psi_1 - \tfrac{1}{3}\psi_3) \, dx = A_\perp^2 (1 + \tfrac{1}{9}) = \tfrac{10}{9} A_\perp^2 = 1 \quad \Rightarrow \quad A_\perp = \frac{3}{\sqrt{10}}$$

$$\Rightarrow \quad \psi_\perp = \frac{3}{\sqrt{10}} (\psi_1 - \tfrac{1}{3}\psi_3)$$

$$\psi = \frac{1}{\sqrt{10}} (\psi_1 + 3\psi_3)$$

$$\langle E_\perp \rangle = \frac{9}{10} E_1 + \frac{1}{10} \underbrace{E_3}_{9E_1} = \frac{9}{5} E_1$$

Časovni razvoj

$\Psi(x, t=0)$

$\rightarrow$ Schrödinger: $i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi$ $\quad \psi = \sum_n c_n \psi_n, \quad \hat{H}\psi = E_n \psi$

$\hookrightarrow$ za $\psi_n = i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi_n = E_n \psi_n$

$$\frac{\partial \psi_n}{\psi_n} = \frac{iE_n}{\hbar} dt$$

$$\psi_n = e^{-\frac{iE_n t}{\hbar}} \psi_n$$

$\rightarrow$ $t=0$ $\quad \psi(x, t=0) = \sum_n c_n \psi_n(x, 0)$

$\rightarrow$ $t>0$ $\quad \psi(x,t) = \sum_n c_n e^{-iE_n t/\hbar} \psi_n(x, 0)$

Naš primer

$$\psi = \frac{1}{\sqrt{10}} (\psi_1 + 3\psi_3) = \psi(x, t=0)$$

$$\psi(x,t) = \frac{1}{\sqrt{10}} \left(e^{-i\frac{E_1 t}{\hbar}} \psi_1 + 3e^{-i\frac{E_3 t}{\hbar}} \psi_3 \right)$$

- $\langle E \rangle = \int \psi^* H \psi \, dx \qquad \psi = \sum_n c_n \psi_n$

$$= \int \sum_{n,m} \left(c_m^* \psi_m^* e^{+\frac{iE_m t}{\hbar}} \cdot E_n \cdot c_n \psi_n e^{-iE_n t/\hbar} \right)$$

$$= \sum_n |c_n|^2 E_n e^{i(E_m - E_n)t/\hbar}$$

- $\langle p \rangle = \int \psi^* \hat{p} \psi \, dx = 0 \qquad$ ker $\frac{\partial}{\partial t} \psi = 0$

$\langle p^2 \rangle = 2m \langle E \rangle$

- $\langle x \rangle = \int \psi^* x \psi \, dx$

Naš primer

$$\langle x \rangle = \frac{1}{10} \int \left(\psi_1 e^{iE_1 t/\hbar} + 3\psi_3 e^{iE_3 t/\hbar} \right) x \left(\psi_1 e^{-iE_1 t/\hbar} + 3\psi_3 e^{-iE_3 t/\hbar} \right) = \frac{a}{2}$$

$$\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

$$\langle x^2 \rangle = \frac{1}{10} \int_{-\infty}^{\infty} \left(\psi_1 e^{iE_1 t/\hbar} + 3\psi_3 e^{iE_3 t/\hbar} \right) x^2 \left(\psi_1 e^{-iE_1 t/\hbar} + 3\psi_3 e^{-iE_3 t/\hbar} \right) dx$$

$$= \frac{1}{10} \int_{-\infty}^{\infty} x^2 \left(\psi_1^2 + \psi_3^2 + 3\psi_1\psi_3 \left(e^{i(E_1 - E_3)t/\hbar} + e^{-i(E_1 - E_3)t/\hbar} \right) dx \right.$$

$$= \frac{1}{10} \int_{-\infty}^{\infty} x^2 \left(\psi_1^2 + \psi_3^2 + 6\psi_1\psi_3 \cos\left((E_1 - E_3)\frac{t}{\hbar}\right) \right) dx$$

$$= \frac{a^2}{10\pi^2} \left(10\pi^2 - \pi + 27\cos\left(\frac{E_3 - E_1}{\hbar} t\right) \right)$$

46

$$\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$$

$$\sigma_x \sigma_p = \dots (n) \leftarrow \langle x \rangle, \langle x^2 \rangle, \langle p \rangle, \langle p^2 \rangle$$

$$\langle x \rangle_n = \int_0^a \psi_n^* \, x \, \psi_n \, dx = \frac{2}{a} \int_0^a \sin^2\left(\frac{n\pi x}{a}\right) x \, dx$$

$$x = \frac{at}{n\pi}$$

$$dx = \frac{a}{n\pi} dt$$

$$= \frac{2}{a} \int_0^{n\pi} \sin^2 t \quad t \left(\frac{a}{n\pi}\right)^2 dt = \dots = \frac{a}{2}$$

$$\langle x^2 \rangle = \frac{a^2}{(n\hbar)^3} \int_0^{n\pi} (1 - \cos 2t) \, t^2 \, dt = \frac{a^2}{(n\hbar)^3} \left(\left(\frac{t^3}{3}\right) \Big|_0^{n\pi} - \right.$$

$$\langle p \rangle = 0 \qquad \alpha \int_0^{n\pi} \sin t \cos t \, dt = 0$$

$$\langle p^2 \rangle = \frac{2m}{\hbar^2} \langle E \rangle = \left(\frac{\hbar n \pi}{a}\right)^2$$

$$\sigma_x \sigma_p = \sqrt{\frac{a^2}{3}\left(1 - \frac{3}{2(n\pi)^2} - \frac{a^2}{4}\right)} \quad \frac{\hbar n \pi}{a} = \frac{\hbar}{2}\sqrt{\frac{(n\hbar)^2}{3} - \frac{1}{2}} \qquad \text{zu} \quad n \gg 1 \qquad \alpha\, n$$

$$n = 1 \Rightarrow 1{,}13$$

$$n = 2 \Rightarrow 3{,}74$$

① Harmonski oscilator

$$\hat{H} = \underbrace{\frac{\hat{p}^2}{2m}}_{\hat{T}} + \underbrace{\frac{1}{2} m\omega^2 \hat{x}^2}_{\hat{V}} \qquad \text{Za harmonski so} \qquad E_n = \hbar\omega\left(n+\tfrac{1}{2}\right)$$

Lastne funkcije $\quad \psi_n = \dfrac{1}{\sqrt{2^n n! \sqrt{\pi}\, a}} H_n(y)\, e^{-y^2/2} \qquad y = \dfrac{x}{a} \qquad a = \sqrt{\dfrac{\hbar}{m\omega}}$

$$\int_{-\infty}^{\infty} \psi_m^* \psi_n \, dx = \delta_{mn}$$

$$H_n(y) = (-1)^n e^{y^2} \frac{\partial^n}{\partial y^n} e^{-y^2}$$

Sodi → sodi polinomi:

$H_0 = 1$

$H_2 = 4y^2 - 2$

$H_4 = 16y^4 - 48y^2 + 12$

Lihi → lihi polinomi

$H_1 = 2y$

$H_3 = 8y^3 - 12y$

$H_5 = 32y^5 - 160y^3 - 1120y$

(50) ob $t=0 \quad \psi(x,0) = A(2y^2 + iy)\, e^{-y^2/2} \qquad y = \dfrac{x}{a} \qquad a = \sqrt{\dfrac{\hbar}{m\omega}}$

Harmonski oscilator

(a) $\langle E \rangle = ?$

(b) $\psi(x,t) = ?$

(a) Zanimajo nas koef. v razvoju

Razvoj bo le do C_2

$$\psi = \sum_{n=0}^{2} c_n \psi_n \qquad \psi_0 = \frac{1}{\sqrt{\sqrt{\pi}\, a}} e^{-y^2/2}$$

$$\psi_1 = \sqrt{2}\, y\, \psi_0$$
$$\psi_2 = \frac{1}{\sqrt{2}} (2y^2 - 1)\, \psi_0$$

$$\sqrt{2}\,\psi_2 + \psi_0 = 2y^2 \psi_0$$
$$\frac{i}{\sqrt{2}} \psi_1 = iy\, \psi_0$$

$$\psi = B\left(\sqrt{2}\,\psi_2 + \psi_0 + \frac{i}{\sqrt{2}}\psi_1\right)$$

$$\int_{-\infty}^{\infty} \psi^* \psi \, dx = B^2\Big(\overset{\psi_0^2}{1} + \overset{\psi_1^2}{\tfrac{1}{2}\cdot 1} + \overset{\psi_2^2}{2\cdot 1}\Big) = 1 \qquad B = \sqrt{\frac{2}{7}}$$

$$\psi = \sqrt{\frac{2}{7}}\left(\psi_0 + \frac{i}{\sqrt{2}}\psi_1 + \sqrt{2}\,\psi_2\right)$$

$$c_0 = \sqrt{\frac{2}{7}} \qquad c_1 = \frac{i}{\sqrt{7}} \qquad c_2 = \frac{2}{\sqrt{7}}$$

$$\langle E \rangle = \sum_n |c_n|^2 E_n \qquad E_n = \hbar\omega\left(n+\tfrac{1}{2}\right)$$

$$= \frac{2}{7}E_0 + \frac{1}{7}E_1 + \frac{4}{7}E_2 = \frac{\hbar\omega}{7}\left(\quad \dots \quad \right) = \frac{25\hbar\omega}{14}$$

b) Po Schröd.
$$\psi_n(x,t) = e^{-i\frac{E_n t}{\hbar}}\psi_n(x,0)$$
$$= e^{-i\omega(n+\frac{1}{2})t}\psi_n(x,0)$$

$$\psi = \sum_n c_n \psi_n$$
$$\psi = \sqrt{\frac{2}{7}}\, e^{-i\frac{\omega t}{2}}\psi_0 + \frac{i}{\sqrt{7}} e^{-i\frac{3}{2}\omega t}\psi_1 + \frac{2}{\sqrt{7}} e^{-i\frac{5}{2}\omega t}\psi_2$$

T) Splošna formula za razvoj po last. stanjih $\quad c_n = ?$

$$\psi(x) = \sum_n c\psi_n \quad \Big/ \cdot \int_{-\infty}^{\infty}\psi_m^* \, dx$$
$$\int_{-\infty}^{\infty}\psi_m^*\psi\, dx = \sum_n c_n \int_{-\infty}^{\infty}\psi_m^*\psi_n\, dx = \sum_n c_n \delta_{mn} = c_m$$

$$\boxed{c_n = \int_{-\infty}^{\infty}\psi_n^*\psi\, dx}$$

61) $m = 10^{-30}\,kg$

$V = \frac{1}{2}kx^2 \qquad k = 50\,\frac{eV}{nm^2}$

$$\psi(x) = \sqrt{\frac{\alpha}{\sqrt{\pi}}}\, e^{-\frac{\alpha^2(x-a)^2}{2}} \qquad a = 0{,}3\,nm \qquad \alpha = \sqrt[4]{\frac{mk}{\hbar^2}}$$
$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}kx^2 \qquad y = \alpha x$$
$$\psi_n = \sqrt{\frac{\alpha^2}{2^n n!\sqrt{\pi}}}\, H_n(y)\, e^{-y^2/2}$$
$$\psi = \sqrt{\frac{\alpha}{\sqrt{\pi}}}\, e^{-\frac{(y-\alpha a)^2}{2}}$$

Iz navodil $\quad \int_{-\infty}^{\infty} H_n(y)\, e^{-y^2+yy_0-y_0^2/2}\, dy = \sqrt{\pi}\, y_0^n\, e^{-\frac{y_0^2}{4}}$

$y = \alpha x$

$dy = \alpha dx$

$$c_n = \int_{-\infty}^{\infty}\psi_n\psi\, dx = \sqrt{\frac{\alpha^2}{2^n n!\sqrt{\pi}^2}}\int_{-\infty}^{\infty} H_n(y)\, e^{-y^2/2}\, e^{-\frac{y^2}{2}-y\alpha a-\frac{\alpha a^2}{2}}\, \underbrace{dx}_{\frac{1}{\alpha}dy}$$
$$= \sqrt{\frac{1}{2^n n!\,\pi}}\int_{-\infty}^{\infty} H_n(y)\, e^{-y^2+yy_0-y_0^2/2}\, dy$$
$$= \sqrt{\frac{1}{2^n n!\,\pi}}\,\sqrt{\pi}\, y_0^n\, e^{-y_0^2/4}$$
$$= \frac{1}{\sqrt{2^n n!}}\, y_0^n\, e^{-y_0^2/4} \qquad \text{za } \forall n \text{ je } c_n \neq 0$$

$$\boxed{\sum_{n=0}^{\infty}\frac{x^n}{n!} = e^x}$$

$$\int_{-\infty}^{\infty}|\psi|^2 dx = 1 \Rightarrow \sum_{n=0}^{\infty}|c_n|^2 = 1$$
$$\sum_{n=0}^{\infty}\frac{1}{2^n n!}\, y_0^{2n}\, e^{-y_0^2/2} = e^{-y_0^2/2}\sum\left(\frac{y_0^2}{2}\right)^n\frac{1}{n!} = e^{-y_0^2/2}\cdot e^{y_0^2/2} = 1$$

$$\langle E \rangle = \sum_n c_n^2 E_n = \sum_{n=0}^{\infty} \frac{1}{2^n n!} y_0^{2n} e^{-y_0^2/2} \hbar\omega \left(n+\tfrac{1}{2}\right) \qquad E_n = \hbar\omega\left(n+\tfrac{1}{2}\right), \quad \omega = \sqrt{\tfrac{k}{m}}$$

$$= \hbar\omega\, e^{-y_0^2/2} \left(\frac{1}{2} e^{y_0^2/2} + \frac{y_0^2}{2} \sum_{n=0}^{\infty} \frac{y_0^{2n-2}}{2^{n-1}(n-1)!} \right)$$

$$= \hbar\omega\, e^{-y_0^2/2} \left(\frac{1}{2} e^{y_0^2/2} + \frac{y_0^2}{2} e^{y_0^2/2} \right)$$

$$= \frac{\hbar\omega}{2} \left(1 + y_0^2\right)$$

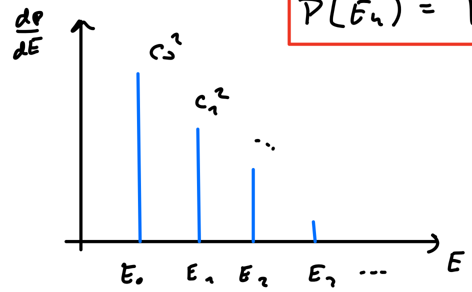
$$y_0 = a\alpha = a \sqrt[4]{\frac{mc^2 k}{(\hbar c)^2}} = \dots = 1{,}55$$

$$\langle E \rangle = \frac{\hbar\omega}{2}\left(1 + 1{,}55^2\right) = 7{,}2\,\text{eV}$$

$$\hat{H} \xrightarrow{\text{meritev}} \psi \xrightarrow{\text{kolaps}} \psi_n$$

verjetnost

Izmerimo E_n, $P_n = |c_n|^2$



$$\boxed{P(E_n) = |c_n|^2}$$

$$P\left(E_1 = \hbar \sqrt{\tfrac{k}{m}}\, \tfrac{3}{2}\right) = |c_1|^2 \qquad E_n = \hbar\omega \underbrace{\left(n + \tfrac{1}{2}\right)}_{\frac{3}{2}} \qquad n = 1$$

$$c_n = \frac{1}{\sqrt{2^n n!}} y_0^n e^{-y_0^2/4}$$

$$|c_1|^2 = \left(\frac{1}{\sqrt{2}} y_0 e^{-y_0^2/4} \right)^2 = \frac{y_0^2}{2} e^{-y_0^2/2}$$

$$|c_1|^2 = 0{,}36$$

- Skica direktnega izračuna

$$\langle E \rangle = \int_{-\infty}^{\infty} \psi^* \hat{H} \psi \, dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{2m}(-\hbar^2) \frac{\partial^2 \psi}{\partial x^2} \psi + \frac{1}{2} k x^2 \psi^2 \, dx$$

$$\alpha(x-a) = t$$

$$= \frac{1}{\sqrt{\pi}} \frac{\hbar\alpha^2}{2m} \underbrace{\int_{-\infty}^{\infty} e^{-t^2} dt}_{\sqrt{\pi}} \left(1 + (\alpha a)^2\right)$$

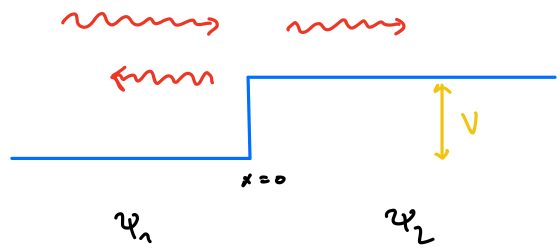
$$= \frac{(\hbar\alpha)^2}{2m} \left(1 + y_0^2\right)$$

23 Curek e^- na potencialno plast

$E = 10\,eV$

$V = 7\,eV$

$R = ?$



$$\hat{H} = \frac{\hat{p}^2}{2m} + \hat{V}$$

$$E_k = \frac{(\hbar k)^2}{2m}$$

$$\psi_1 = e^{\pm ikx} \qquad k = \frac{\sqrt{2mE}}{\hbar}$$

$$\psi_2 = e^{ik'x} \quad , \quad k' = \frac{\sqrt{2m(E-V)}}{\hbar}$$

vpadni val   odbit val

$$\psi_1 = Ae^{ikx} + Be^{-ikx}$$
$$\psi_2 = C\,e^{ik'x}$$

prepuščen val

$$\boxed{j = \frac{\hbar}{2mi}\left(\psi^* \frac{d\psi}{dx} - \frac{\partial \psi^*}{\partial x}\psi\right)}$$ verjetnostni tok

$$j_{vp} = \frac{\hbar}{2mi}\left(e^{-ikx} A^* A\, ik\, e^{ikx} - (-ik)|A|^2\right) = \frac{\hbar k}{m}|A|^2$$

$$j_{odb} = \frac{\hbar k}{m}|B|^2$$

$$j_{prep} = \frac{\hbar k'}{m}|C|^2$$

$$R = \frac{j_{odb}}{j_{vpad}} = \frac{|B|^2}{|A|^2} = \left|\frac{B}{A}\right|^2$$

Pri skoku $x=0$ upoštevamo robni pogoj

$$\psi_1(x=0) = \psi_2(x=0) \qquad \psi_1 = Ae^{ikx} + Be^{-ikx} \qquad k = \frac{\sqrt{2mE}}{\hbar} \quad k' = \frac{\sqrt{2m(E-V)}}{\hbar}$$
$$\psi_1'(x=0) = \psi_2'(x=0) \qquad \psi_2 = Ce^{ik'x}$$

$$A + B = C \qquad :A$$
$$ikA - ikB = ik'C \qquad :A$$

$$\left.\begin{aligned} 1 + \frac{B}{A} &= \frac{C}{A} \\ \frac{k}{k'}\left(1 - \frac{B}{A}\right) &= \frac{C}{A} \end{aligned}\right] =$$

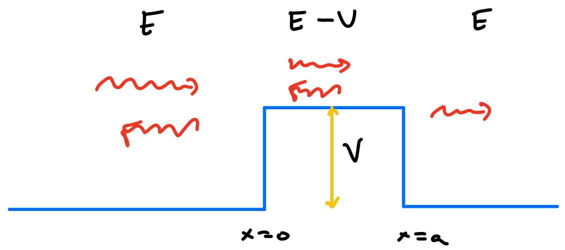
$$\left(1 + \frac{B}{A}\right) = \frac{k}{k'}\left(1 - \frac{B}{A}\right)$$

$$\frac{B}{A} = \frac{\frac{k}{k'} - 1}{\frac{k}{k'} + 1} = \frac{k - k'}{k + k'}$$

$$R = \left(\frac{\sqrt{E} - \sqrt{E-V}}{\sqrt{E} + \sqrt{E-V}}\right)^2 = 8 \cdot 10^{-3}$$ verjetnost da se val odbije

$$T = 1 - R$$

T



$$k = \frac{\sqrt{2mE}}{\hbar}$$

$$k' = \frac{\sqrt{2m(E-V)}}{\hbar}$$

ψ_1 ψ_2 ψ_3

$$\psi_1 = A e^{ikx} + B e^{-ikx}$$
$$\psi_2 = C e^{ik'x} + D e^{-ik'x}$$
$$\psi_3 = F e^{ikx}$$

$$T = \frac{j_{prep}}{j_{vpad}} = \left|\frac{F}{A}\right|^2$$

Robni pogoji

$x=0$: $A + B = C + D$
$ik(A-B) = ik'(C-D)$] $\pm$

$x=a$: $C e^{ik'a} + D e^{-ik'a} = F e^{ika}$
$ik'(C e^{ik'a} - D e^{ik'a}) = ikF e^{ika}$] $\pm$

$$2Ce^{ik'a} = F\left(1+\frac{k}{k'}\right)e^{ika} \Rightarrow C = \frac{F}{2}\left(1+\frac{k}{k'}\right)e^{i(k-k')a}$$
$$De^{ik'a} = F\left(1-\frac{k}{k'}\right)e^{ika} \Rightarrow D = \frac{F}{2}\left(1-\frac{k}{k'}\right)e^{i(k+k')a}$$
$$2A = C\left(1+\frac{k'}{k}\right) + D\left(1-\frac{k'}{k}\right)$$

$$\frac{F}{A} = \frac{2F}{C\left(1+\frac{k'}{k}\right) + D\left(1-\frac{k'}{k}\right)}$$

$$\frac{F}{A} = \frac{4}{\left(1+\frac{k'}{k}\right)\left(1+\frac{k}{k'}\right)e^{i(k-k')a} + \left(1-\frac{k'}{k}\right)\left(1-\frac{k}{k'}\right)e^{i(k+k')a}}$$

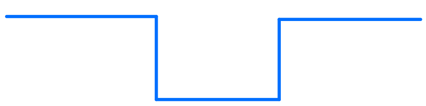
$$\frac{F}{A} = \frac{4kk'e^{i(k-k')a}}{(k+k')^2 - (k-k')^2 e^{2ik'a}}$$

$$\left|\frac{F}{A}\right|^2 = \frac{16k^2k'^2}{(k+k')^4 + (k'-k)^4 - (k^2-k'^2) - 2\cos 2k'a}$$

$$\boxed{T = \left|\frac{F}{A}\right|^2 = \frac{1}{1 + \left(\frac{k'^2-k^2}{2kk'}\right)^2 \sin^2 k'a}}$$

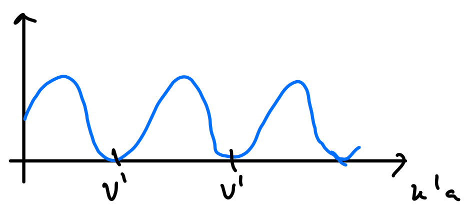
27

$a = 0{,}41\,nm$
$E = 0{,}7\,eV$
$V = ?$



$V \to -V$
$$k' = \frac{\sqrt{2m(E+V)}}{\hbar}$$

$$T = \frac{1}{1 + \left(\frac{k'^2-k^2}{2kk'}\right)^2 \sin^2 k'a}$$

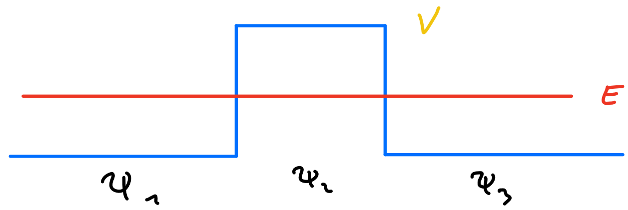


Ko je $\sin^2 k'a = 0 \Rightarrow T = 1$

$\Rightarrow$ $k'a = \pi n$
$$\frac{\sqrt{2m(E+V)}}{\hbar}a = n\pi$$

Resonančni pogoj pri $n=1$
$$V = \left(\frac{n\pi\hbar}{a}\right)^2\frac{1}{2m} - E = \frac{\pi^2(\hbar c)^2}{2a^2\, m_e c^2} - E = \frac{\pi^2 (200\,eVnm)^2}{2\cdot(0{,}4nm)^2\; 511\,keV} - 0{,}7\,eV = 1{,}53\,eV$$

(T) Tuneliranje $E < V$



$$k' = \frac{\sqrt{2m(E-V)}}{\hbar} =$$

$$= i\varkappa \qquad \varkappa = \frac{\sqrt{2m(V-E)}}{\hbar}$$

$$\psi_n = C e^{-\varkappa x} + D e^{\varkappa x}$$

V enačbi z. T

$$\sin ix = \frac{e^{-x} - e^{x}}{2} = i \operatorname{sh} x$$

$$T = \frac{1}{1 + \left(\frac{k'^2 - k^2}{2kk'}\right)^2 \sin^2 k'a} = \frac{1}{1 + \left(\frac{-\varkappa^2 - k^2}{2i\varkappa k}\right)^2 \sin^2 i\varkappa a}$$

$$T = \frac{1}{1 + \left(\frac{k^2 + \varkappa^2}{2k\varkappa}\right)^2 \frac{i^2}{i^2} \operatorname{sh}^2 \varkappa a}$$

$$\boxed{T = \frac{1}{1 + \left(\frac{k^2 + \varkappa^2}{2k\varkappa}\right)^2 \operatorname{sh}^2 \varkappa a}}$$

$$\operatorname{sh} x \xrightarrow{x \gg 1} \frac{e^x}{2}$$

$$\varkappa = \frac{\sqrt{2m(V-E)}}{\hbar}$$

$$T \propto e^{-\varkappa a}$$

Verjetnost za tuneliranje exp. pada

(28)

$E = 10\,eV$
$V = 12\,eV$
$a = 0{,}1\,nm$
$b = 1\,nm$
$c = 0{,}1\,\mu m$

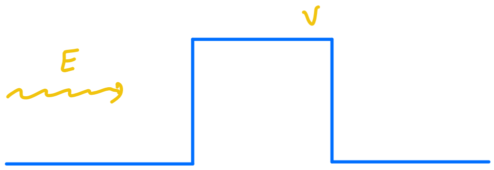
$$k = \frac{\sqrt{2mE}}{\hbar} = \frac{\sqrt{2mc^2 E}}{\hbar c} = \frac{\sqrt{2 \cdot 500\,keV \cdot 10\,eV}}{200\,eV\,nm} = \frac{1}{2} \cdot 10^2 \frac{1}{nm}$$

$$\varkappa = \frac{\sqrt{2m(V-E)}}{\hbar} \Rightarrow \varkappa a = \quad 0{,}7 \qquad 7{,}1 \qquad 710$$

$$T = \quad 10{,}4 \qquad 8{,}7 \cdot 10^{-4} \qquad 4 \cdot 10^{-311}$$

$$40\% \qquad 1\% \qquad \sim 0\%$$

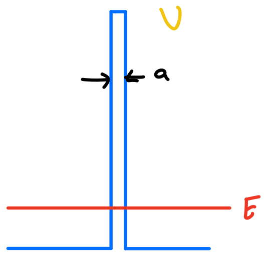
Tuneliranje, pobeg, rotator (vrtilna količina)



$$k = \sqrt{2mE}/\hbar$$
$$\varkappa = \sqrt{2m(V-E)}/\hbar$$

$$T = \frac{1}{1 + \left(\frac{k^2+\varkappa^2}{2k\varkappa}\right)^2 \mathrm{sh}^2 \varkappa a}$$ transmisivnost

32 e^-, $v = 2000\,\frac{m}{s}$, $Va = 10^{-3}\,eVnm$, $T = ?$



$V \gg E$

$$T = \frac{1}{1 + \left(\frac{k^2+\varkappa^2}{2k\varkappa}\right)^2 \mathrm{sh}^2 \varkappa a} = \frac{1}{1 + \left(\frac{\cancel{E} + V - \cancel{E}}{2\sqrt{E(V-\cancel{E})}}\right)^2 \mathrm{sh}^2 \varkappa a} =$$

$$\doteq \frac{1}{1 + \frac{V^2}{4EV}\left(\sqrt{2mVa}/\hbar\,\sqrt{a}\right)^2} =$$

$$\varkappa a = \sqrt{2m(V-E)}/\hbar\, a \to \sqrt{2mVa}/\hbar\sqrt{a} \ll 1$$

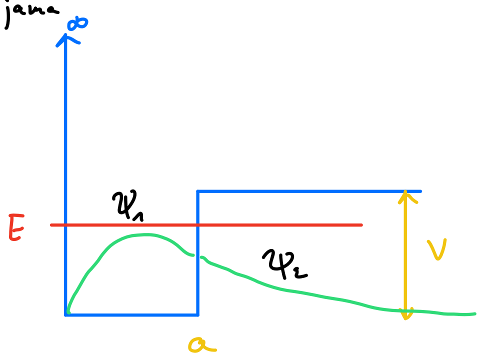
$$\beta = \frac{2\cdot 10^3\,\frac{m}{s}}{3\cdot 10^8\,\frac{m}{s}} = \frac{2}{3}\cdot 10^{-5}\,\frac{m}{s} \ll 1$$

$$E = \frac{1}{2}mv^2 = \frac{1}{2}mc^2\beta^2$$

$$\frac{2E}{mc^2} = \beta^2$$

$$= \frac{1}{1 + \frac{V}{4E}\frac{2mVa^2}{\hbar^2}} = \frac{1}{1 + \frac{mc^2(Va)^2}{2E\hbar^2c^2}} = \frac{1}{1 + \frac{\beta^2(Va)^2}{\hbar^2c^2}} = \frac{1}{1+\left(\frac{3}{4}\right)^2} = \frac{16}{25} = 64\%$$

39 1D pot. jama



$$\psi_1 = A \sin kx \qquad ka = \frac{2\pi}{3}$$
$$E = \frac{3}{4}V$$

Verjetnost, da je delec zunaj jame?

$$P(V) = \int_V |\psi|^2\, dV$$

$$P(\text{zunaj jame}) = \int_a^\infty |\psi_2|^2\, dx$$

$$\psi_1(a) = \psi_2(a)$$
$$\psi_1'(a) = \psi_2'(a)$$

Nastavek $\psi_2 = Be^{-\varkappa x}$

$$k = \sqrt{2mE}/\hbar$$
$$\varkappa = \sqrt{2m(V-E)}/\hbar$$

$$A\sin(ka) = Be^{-\varkappa a} \qquad B = Ae^{\varkappa a}\sin ka$$
$$kA\cos(ka) = -B\varkappa e^{-\varkappa a} = -\varkappa A e^{\varkappa a}e^{-\varkappa a}\sin ka$$

$$\tan \varkappa a = -\frac{k}{\varkappa}$$

$$\tan\frac{2\pi}{3} = -\sqrt{3} \Rightarrow k = \sqrt{3}\varkappa \qquad \varkappa a = \frac{ka}{\sqrt{3}} = \frac{2\pi}{3\sqrt{3}}$$

Določimo A

$$P = \underbrace{\int_0^a |\psi_1|^2 dx}_{I_1} + \underbrace{\int_a^\infty |\psi_2|^2 dx}_{I_2} = 1$$

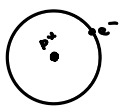
$$I_1 = A^2 \int_0^a \sin^2 kx \, dx = A^2 \int_0^a \frac{1-\cos 2kx}{2} dx = \frac{A^2}{2}\left(a - \frac{\sin 2ka}{k}\right)$$

$$I_2 = A^2 e^{2\varkappa a} \sin^2 ka \int_a^\infty e^{-2\varkappa x} dx = A^2 e^{2\varkappa a} \sin^2 ka \, \frac{e^{-2\varkappa a}}{2\varkappa} = \frac{A^2}{2\varkappa} \sin^2 ka$$

$$P(\text{zunaj}) = \frac{I_2}{I_1 + I_2} = \frac{1}{1 + \frac{I_1}{I_2}}$$

$$\frac{I_1}{I_2} = \frac{a - \frac{\sin 2ka}{k}}{\frac{\sin^2 ka}{\varkappa}} = \frac{\varkappa a - a\varkappa \frac{\sin 2ka}{ka}}{\sin^2 ka} = \ldots = 0{,}34$$

Atom vodika



$$\hat{H} = \hat{T} + \hat{V} = \frac{\hat{p}^2}{2m} - \frac{\alpha \hbar c}{r} \qquad \alpha = \frac{1}{137} = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$$

$$r^2 = x^2 + y^2 + z^2$$

Vedno ko imamo sferično simetrijo, zato lahko ločimo na radialni in orbitalni del.

$$\psi = R_{nl}(r)\, Y_{lm}(\theta, \varphi)$$

sferični harmoniki

$n = 1, 2, 3, \ldots$ Vseh stanj z istim l je $2l+1$

$0 \le l \le n-1$

$|m| \le l$

$$E_n = -\frac{\alpha^2 mc^2}{2} \frac{1}{n^2} \qquad \hat{H}\psi_{nlm} = E_n \psi_{nlm}$$

$$E_1 = -13{,}6\,\text{eV}$$

$$\int \psi^*_{n'l'm'} \psi_{nlm} \underbrace{dV}_{r^2 dr \sin\theta d\theta d\varphi} = \delta_{nn'} \delta_{ll'} \delta_{mm'}$$

$$\int_0^{2\pi}\int_0^{\pi} Y^*_{l'm'} Y_{lm} \sin\theta \, d\theta \, d\varphi = \delta_{ll'} \delta_{mm'}$$

$$\int_{-\infty}^{\infty} R^*_{n'l'}(r) R_{nl}(r)\, r^2 dr = \delta_{nn'}$$

Sferični harmoniki

Legendrovi polinomi

$$Y_{l,m}(\theta, \varphi) = (-1)^m \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}}\, P_{lm}(\cos\theta)\, e^{im\varphi}$$

Normalizacija

$l=0 \qquad m=0 \qquad Y_{00} = \frac{1}{\sqrt{4\pi}}$

$l=1 \qquad m=1$

$m=0 \qquad Y_{10} = \sqrt{\frac{3}{4\pi}} \cos\theta$

$m=-1 \qquad Y_{1\pm1} = \mp \sqrt{\frac{3}{8\pi}} \sin\theta \, e^{\pm i\varphi}$

$l=2 \qquad m=0 \qquad Y_{20} = \sqrt{\frac{5}{8\pi}} (3\cos^2\theta - 1)$

$\pm 1 \qquad Y_{2\pm1} = \mp \sqrt{\frac{15}{8\pi}} (\sin\theta\cos\theta) e^{\pm i\varphi}$

$\pm 2 \qquad Y_{2\pm2} = \sqrt{\frac{15}{32\pi}} \sin^2\theta \, e^{\pm 2i\varphi}$

$\vec{L} = \vec{r} \times \vec{p} \longrightarrow \hat{L}^2$

$\hat{L}_x, \hat{L}_y, \hat{L}_z$

$\hat{L}^2 Y_{lm} = \hbar^2 l(l+1) Y_{lm}$ $\qquad L^2 = \hbar\, l(l+1)$ last. vrednost vrt. kol.

$\hat{L}_z Y_{lm} = \hbar m Y_{lm}$ $\qquad L_z = \hbar m$ lastna vrednost projekcije vrt. kol. na z os.

$\vec{L} = \vec{r} \times (-i\hbar \vec{\nabla})$

$\downarrow$ iz kartezičnih v sferične

$\hat{L}_z = -i\hbar \frac{d}{d\varphi}$

54 $\psi = \frac{1}{2\sqrt{\pi}} f(r,\theta) (\cos\varphi + i\sqrt{3}\sin\varphi)$

$L_z = -i\hbar \frac{d}{d\varphi} \rightarrow \langle L_z \rangle = ?$

$\int |f|^2 r^2 dr \sin\theta d\theta = 1$

$$\langle L_z \rangle = \int_V \psi^* \hat{L}_z \psi \, dV = \frac{1}{4\pi} \int f^2 r^2 dr \sin\theta d\theta \int_0^{2\pi} (\cos\varphi - i\sqrt{3}\sin\varphi)(-i\hbar)(-\sin\varphi + i\sqrt{3}\cos\varphi)$$

$$= \frac{-i\hbar}{4\pi} \int_0^{2\pi} i\sqrt{3}\cos^2\varphi + i\sqrt{3}\sin^2\varphi - \sin\varphi\cos\varphi + 3\sin\varphi\cos\varphi \, d\varphi$$

$$= \frac{-i\hbar}{4\pi} \int_0^{2\pi} i\sqrt{3} + \sin 2\varphi \, d\varphi = -\frac{i\hbar}{4\pi} i\sqrt{3}\, 2\pi = \frac{\hbar\sqrt{3}}{2} = \frac{\hbar}{2}\sqrt{3}$$

Rotator / kotna odvisnost

$$\psi = \frac{1}{\sqrt{2}} \left(Y_{10} + Y_{11} \right) = \sqrt{\frac{3}{8\pi}} \left(\cos\theta - \frac{1}{\sqrt{2}} \sin\theta \, e^{i\varphi} \right)$$

$$\langle L_x \rangle, \quad \langle L_y \rangle, \quad \langle L_z \rangle \quad = ?$$

$$\hat{L}_x = i\hbar \left(s_\varphi \frac{d}{d\theta} + \frac{c_\varphi}{t_\theta} \frac{d}{d\varphi} \right)$$

$$\langle \hat{L}_x \rangle = \frac{3i\hbar}{8\pi} \int_{-1}^{1} \int_0^{2\pi} \left(c_\theta - \frac{1}{\sqrt{2}} s_\theta e^{-i\varphi} \right) \left(\frac{e^{i\varphi} - e^{-i\varphi}}{2i} \left(-\sin\theta - \frac{1}{\sqrt{2}} \cos\theta \, e^{i\varphi} \right) + \frac{c_\varphi}{t_\theta} \left(-\frac{i}{\sqrt{2}} \sin\theta \, e^{i\varphi} \right) \right)$$

$$\int_0^{2\pi} e^{\pm i\varphi} \, d\varphi = 0 \qquad \int_0^{2\pi} d\varphi = 2\pi$$

$$= \frac{3i\hbar}{8\pi} \int_{-1} d\cos\theta \left(\frac{-i \, c_\theta^2}{2\sqrt{2}} + \frac{s_\theta^2}{2\sqrt{2}\, i} + \frac{c_\theta^2}{2\sqrt{2}\, i} \right) 2\pi$$

$$= \frac{3\hbar}{8\sqrt{2}} \int_{-1}^{1} \left(1 + \cos^2\theta \right) d\cos\theta = \frac{3\hbar}{8\sqrt{2}} \left(2 + \frac{c_\theta^3}{3} \right) \Big|_{-1}^{1} = \frac{\hbar}{\sqrt{2}}$$

$$\langle L_y, \quad L_z \rangle$$

$$\hat{L}_y = i\hbar \left(-c_\varphi \frac{d}{d\theta} + s_\varphi \frac{c_\theta}{s_\theta} \frac{d}{d\varphi} \right)$$

$$\hat{L}_z = -i\hbar \frac{d}{d\varphi}$$

$$\langle L_y \rangle = \frac{3i\hbar}{8\pi} \int_0^{2\pi} d\varphi \int_{-1} dc_\theta \left(c_\theta - \frac{1}{\sqrt{2}} s_\theta e^{-i\varphi} \right) \left(- \frac{e^{i\varphi} + e^{-i\varphi}}{2} \cdot \left(-s_\theta - \frac{1}{\sqrt{2}} c_\theta e^{i\varphi} \right) + \frac{e^{i\varphi} - \cancel{e^{-i\varphi}}}{2} \cdot \frac{c_\theta}{\cancel{s_\theta}} \frac{-i}{\sqrt{2}} \cancel{s_\theta} e^{i\varphi} \right)$$

$$= \frac{3i\hbar}{8\pi} 2\pi \int_{-1}^{1} dc_\theta \, \frac{1}{2\sqrt{2}} 2 c_\theta^2 - \frac{1}{2\sqrt{2}} s_\theta^2 = \frac{3i\hbar}{8\sqrt{2}} \int_{-1} 2 c_\theta^2 - 1 + c_\theta^2 \, dc_\theta$$

$$= \frac{3i\hbar}{8\sqrt{2}} \int_{-1}^{1} 3 c_\theta^2 - 1 \, dc_\theta = \frac{3i\hbar}{8\sqrt{2}} \left(3 \frac{c_\theta^3}{3} \Big|_{-1}^{1} - c_\theta \Big|_{-1}^{1} \right) = 0$$

$$\langle L_z \rangle = - \frac{3\cancel{i}\hbar}{8\pi} \int_0^{2\pi} d\varphi \int_{-1}^{1} dc_\theta \left(c_\theta - \frac{1}{\sqrt{2}} s_\theta e^{-i\varphi} \right) \frac{\cancel{-i}}{\sqrt{2}} s_\theta e^{i\varphi}$$

$$= -\frac{3\hbar}{8\pi} \int_0^{2\pi} \int_{-1}^{1} \frac{1}{\sqrt{2}} s_\theta s_\theta \underbrace{e^{i\varphi}}_{= 0} - \frac{1}{2} s_\theta^2 =$$

$$= + \frac{3\hbar}{8\pi} 2\pi \frac{1}{2} \int_{-1}^{1} 1 - c_\theta^2 \, dc_\theta = \frac{3\hbar}{8} \left(2 - \frac{2}{3} \right) = \frac{\hbar}{2}$$

Z uporabo pravil za lastna stanja

$$\int_\Omega Y^*_{l'm'} Y_{lm} = \delta_{l'l}\,\delta_{m'm} \qquad \text{ortonormiran set}$$

$$\int_\Omega Y^*_{l'm'} \hat{L}_x Y_{lm} = \frac{\hbar}{2}\sqrt{l(l+1)-m(m\pm1)}\;\delta_{ll'}\,\delta_{m'(m\pm1)}$$

$$\int_\Omega Y^*_{l'm'} \hat{L}_y Y_{lm} = \mp\frac{i\hbar}{2}\sqrt{l(l+1)-m(m\pm1)}\;\delta_{l'l}\,\delta_{m',(m\pm1)}$$

$$\int_\Omega Y^*_{l'm'} \underbrace{\hat{L}_z Y_{lm}}_{\hbar m Y_{lm}} = \hbar m\,\delta_{l'l}\,\delta_{m'm}$$

$$\int_\Omega Y^*_{l'm'} \hat{L}^2 Y_{lm} = \hbar^2 l(l+1)\,\delta_{l'l}\,\delta_{m'm}$$

Naš primer $\quad \psi = \frac{1}{\sqrt{2}}(Y_{10}+Y_{11})$

$$\langle L_x\rangle = \frac{1}{2}\int (Y^*_{10}+Y^*_{11})\hat{L}_x(Y_{10}+Y_{11})$$

členi z istim m so 0.

$$= \frac{1}{2}\int \underset{m'=0}{Y^*_{10}}\hat{L}_x \underset{m=1}{Y_{11}} + \underset{m'=1}{Y^*_{11}}\hat{L}_x \underset{m=0}{Y_{10}} =$$

$$= \frac{\hbar}{4}\left(\sqrt{1(1+1)-1\cdot(1-1)} + \sqrt{1(1+1)-0\cdot(0+1)}\right) = \frac{\hbar\sqrt{2}}{2} = \frac{\hbar}{\sqrt{2}}$$

$$\langle L_y\rangle = \frac{1}{2}\int Y^*_{10}\hat{L}_y Y_{11} + Y^*_{11}\hat{L}_y Y_{10}$$

$$= \frac{i\hbar}{4}\left(+\sqrt{2}-\sqrt{2}\right) = 0$$

$$\langle L_z\rangle = \frac{1}{2}\int Y^*_{10}\hat{L}_z Y_{10} + Y^*_{11}\hat{L}_z Y_{11} =$$

$$= \frac{1}{2}(\hbar\cdot 0+\hbar\cdot 1) = \frac{\hbar}{2}$$

$$\langle L^2\rangle = \frac{1}{2}\int_\Omega Y^*_{10} L^2 Y_{10} + Y^*_{11} L^2 Y_{11}$$

$$= \frac{1}{2}\hbar^2\left(1\cdot(1+1)+1(1+1)\right) = 2\hbar^2$$

55 $\quad P(r>\langle r\rangle)$ elektron na radiju, ki so večji od povprečnega. e^- je v os. stanju n=1 l,m=0

$$\psi_{nlm} = R_{nl}(r)\,Y_{lm}(\theta,\varphi) \Rightarrow \psi_{100} = \underbrace{\frac{1}{\sqrt{4\pi}}}_{Y_{00}}\frac{2}{r_B^{3/2}}e^{-r/r_B} \qquad r_B = \frac{\hbar c}{\alpha m_e c^2}$$

$$\bullet\ \langle r\rangle = \int_V \psi^* r\,\psi\,dV = \overset{r^2dr\ \underbrace{\sin\theta\, d\theta\, d\varphi}_{4\pi}}{=}$$

$$= \int_0^\infty \frac{4}{r_B^3}e^{-\frac{2r}{r_B}}\,r\,r^2\,dr = \qquad t=\frac{2r}{r_B} \qquad \frac{r_B}{2}dt = dr$$

$$= \int_0^\infty \frac{4}{r_B^3}e^{-t}\left(\frac{r_B}{2}\right)^3 t^3\,\frac{r_B}{2}dt = \frac{r_B}{4}\Gamma(4) = \frac{3}{2}r_B$$

• $P(r > \langle r\rangle) = \int_V |\psi|^2 dV = \int_{\langle r\rangle}^{\infty} |\psi|^2 dV = \int_{\langle r\rangle}^{\infty} \frac{4}{r_B^3} e^{-\frac{2r}{r_B}} r^2 dr =$

$= \frac{4}{r_B^3} \int_3^{\infty} e^{-t} \left(\frac{r_B}{2}\right)^3 t^2 dt = \frac{1}{2} \int_3^{\infty} e^{-t} t^2 dt =$

$x = t-3 \qquad t = x+3$

$= \frac{1}{2} \int_0^{\infty} e^{-3-x} (x+3)^2 dx = \frac{e^{-3}}{2} \int_0^{\infty} e^{-x} (x^2 + 6x + 9) dx =$

$= \frac{e^{-3}}{2} \left(\Gamma(3) + 6\,\Gamma(2) + 9\,\Gamma(1) \right) = e^{-3} \frac{17}{2} \simeq 0{,}42$

57 $n=2 \quad l=1 \quad m=0$

$$\psi_{210} = \frac{1}{\sqrt{32\pi r_B^3}} \frac{r}{r_B} e^{-\frac{r}{2r_B}} \cos\theta$$

(a) $\langle V \rangle = ?$

$$\hat{V} = -\alpha\hbar c \,\hat{\frac{1}{r}}$$

$$\langle V \rangle = -\alpha\hbar c \int_V \frac{1}{32\pi r_B^3} \left(\frac{r}{r_B}\right)^2 e^{-\frac{r}{r_B}} \cos^2\theta \, \frac{1}{r} \, 2\pi \, d\cos\theta \, r^2 dr =$$

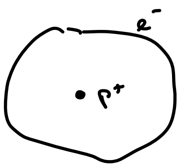
$t = \frac{r}{r_B}$

$$= -\frac{\alpha\hbar c}{16 r_B^2} \int_{-1}^{1} c_\theta^2 \, dc_\theta \int_0^{\infty} \frac{r^3}{r_B^3} e^{-\frac{r}{r_B}} dr =$$

$r_B = \frac{\hbar c}{mc^2 \alpha}$

$$= -\frac{\alpha\hbar c}{16 r_B} \frac{2}{3} 3! = -\frac{\alpha\hbar c}{4 r_B} = -\frac{mc^2\alpha^2}{4}$$

(T) Bohrov model atoma H



$\lambda_{DB} = \frac{h}{p}$

$2\pi r = n\lambda = \frac{nh}{p}$

$p = \frac{nh}{2\pi r} = \frac{n\hbar}{r}$

$F_c = F_e$

$\frac{mv^2}{r} = \frac{p^2}{mr} = \frac{\alpha\hbar c}{r^2}$

$\frac{n^2\hbar^2}{mr^3} = \frac{\alpha\hbar c}{r^2}$

$r = \frac{n^2\hbar c}{\alpha m c^2} = n^2 r_B$

(b) $\langle V \rangle_{Bohr} = ?$

$$\langle V \rangle_B = -\frac{\alpha\hbar c}{r} = -\frac{\alpha\hbar c}{n^2 r_B} = -\frac{\alpha^2 mc^2}{n^2} \qquad n=2 \qquad \langle V \rangle_{Bohr} = -\frac{\alpha^2 mc^2}{4}$$

(T) $\hat{H} = \hat{T} + \hat{V} = \frac{\hat{p}^2}{2m} - \alpha\hbar c \,\hat{\frac{1}{r}}$

$$\hat{p}^2 = -\hbar^2 \left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} \right) + \frac{\hat{L}^2}{2mr}$$

v lastnem stanju

Virialni teorem: $2\langle T \rangle_{\psi_{nlm}} = a \langle U \rangle_{\psi_{nlm}} \qquad V \propto r^a$

H-atom: $a = -1 \qquad 2\langle T \rangle = -\langle V \rangle$

Osnovno stanje $\quad \psi_{100} = \frac{1}{\sqrt{4\pi}} \frac{2}{r_B^{3/2}} e^{-\frac{r}{r_B}}$

$$\langle T \rangle = -\frac{\hbar^2}{2m} \int \psi_{100} \left(\psi_{100}'' + \frac{2}{r} \psi_{100}' + \overset{L^2=0}{0} \right)$$

$$= -\frac{\hbar^2}{4\pi \cdot 2m} \int \frac{4}{r_B^3} e^{-\frac{r}{r_B}} \left(\left(-\frac{1}{r_B}\right)^2 e^{-\frac{r}{r_B}} + \frac{2}{r}\left(-\frac{1}{r_B}\right) e^{-\frac{r}{r_B}} \right)$$

$$= -\frac{2\hbar^2}{m r_B^3} \int_0^\infty e^{-2r/r_B} \frac{1}{r_B^2} \left(1 - \frac{2 r_B}{r}\right) r^2 dr = \qquad t = \frac{2r}{r_B} \qquad dr = \frac{r_B}{2} dt$$

$$= -\frac{2\hbar^2}{m r_B^5} \int_0^\infty e^{-t} \left(1 - \frac{4}{t}\right) \left(\frac{r_B}{2}\right)^3 t^2 dt =$$

$$= -\frac{\hbar^2}{4m r_B^2} \int_0^\infty e^{-t} t^2 - e^{-t} \cdot 4t \, dt =$$

$$= -\frac{\hbar^2}{4m r_B^2} (2! - 4\,1!) = \frac{\hbar^2}{2m r_B^2} = \frac{(\hbar c)^2 (mc^2)^2 \alpha^2}{2mc^2 (\hbar c)^2} = \frac{\alpha^2 m c^2}{2} \qquad \alpha = \frac{1}{137}$$

$$\langle r^a \rangle = \frac{1}{4\pi} \left(\frac{2}{r_B^{3/2}}\right)^2 4\pi \int_0^\infty e^{-2r/r_B} r^a r^2 dr =$$

$$= \frac{4}{r_B^3} \int_0^\infty e^{-t} \left(\frac{r_B}{2}\right)^{a+3} t^{a+2} dt =$$

$$= \frac{r_B^a}{2^{a+1}} \int_0^\infty e^{-t} t^{a+2} dt = \frac{r_B^a}{2^{a+1}} (a+2)!$$

$$\langle V \rangle = -\alpha \hbar c \langle r^{-1} \rangle = -\alpha \hbar c \frac{r_B^{-1}}{2^0} 1! = -\frac{\alpha \hbar c}{r_B} = -\alpha^2 m c^2$$

Virialni teorem $\quad 2\langle T \rangle = -\langle V \rangle$

$$2 \frac{\alpha^2 m c^2}{2} = -(-\alpha^2 m c^2) \checkmark$$

Nedoločenost $\quad \sigma_r \sigma_p$

$$\langle p \rangle = 0 \qquad \langle p^2 \rangle = 2m\langle T \rangle = m^2 c^2 \alpha^2 \qquad \sigma_p = \alpha m c$$

$$\langle r \rangle = \frac{r_B}{2^2} 3! = \frac{3}{2} r_B \qquad \langle r^2 \rangle = \frac{r_B^2}{2^3} 4! = 3 r_B^2$$

$$\sigma_r = r_B \sqrt{3 - \frac{9}{4}} = r_B \frac{\sqrt{3}}{2}$$

$$\sigma_r \sigma_p = \frac{\hbar c}{\alpha m c^2} \frac{\sqrt{3}}{2} \alpha m c = \sqrt{3} \frac{\hbar}{2}$$

59 $\quad \psi = A(R_{10} + 2R_{20}) \Rightarrow \langle E \rangle_\psi \quad \langle r \rangle$

$\psi_\perp \Rightarrow \langle E \rangle_{\psi_\perp} \quad \langle r \rangle$

$$\int_0^\infty r^2 R_{n'l'}^* R_{nl} dr = \delta_{n'n}$$

$$\int_0^\infty \psi^* \psi r^2 dr = 1$$

$$\int_0^\infty A^2 (R_{10}^2 + 4R_{20}^2) r^2 dr = 1$$

$$A^2 (1 + 4) = 1 \qquad A = \frac{1}{\sqrt{5}}$$

$$\int \psi_\perp^* \psi r^2 dr = 0 \qquad \psi_\perp = B(R_{10} + cR_{20})$$

$$\psi_\perp = \frac{1}{\sqrt{5}} \left(\overset{+ \leftrightarrow -}{2R_{10} - R_{20}} \right)$$

zamenjamo

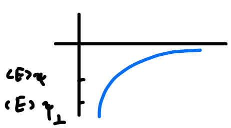
Uistavs / recept za $\psi_\perp$

$$E_n = -\frac{E_{Ry}}{n^2} \qquad E_{Ry} = 13,6 \text{ eV}$$

$$\langle E \rangle_\psi = \sum_n |c_n|^2 E_n = -\frac{1}{5} \frac{E_{Ry}}{1} - \frac{4}{5} \frac{E_{Ry}}{2^2} = -\frac{2}{5} E_{Ry}$$

$$\langle E \rangle_{\psi_\perp} = \frac{1}{5} 4E_1 + \frac{1}{5 \cdot 4} E_1 = -\frac{17}{20} E_{Ry}$$

$$\frac{\langle E \rangle_\psi}{\langle E \rangle_{\psi_\perp}} = \frac{8}{17}$$



$$\langle r\rangle = \int_0^\infty \psi^* r\, \psi\, r^2\, dr = \int_0^\infty \frac{1}{5}(R_{10}+2R_{20})\, r\, (R_{10}+2R_{20})\, r^2 dr$$

$$= \frac{1}{2}\big(\langle 1|r|1\rangle + 4\langle 2|r|2\rangle + 4\langle 1|r|2\rangle\big) =$$

$$\langle 1|r|1\rangle = \int R_{10}\, r\, R_{10}\, r^2\, dr$$

prehodni matrični element

$$\langle r\rangle_1 = \frac{1}{5}\int_0^\infty (2R_{10}-R_{20})\, r\, (2R_{10}-R_{20})\, r^2 dr$$

$$= \frac{1}{5}\big(4\langle 1|r|1\rangle - 4\langle 1|r|2\rangle + 2\langle 2|r|2\rangle\big) =$$

$$R_{10} = \frac{2}{r_B^{3/2}}\, e^{-r/r_B} \qquad R_{20} = \frac{2}{(2r_B)^{3/2}}\left(1-\frac{r}{2r_B}\right)e^{-\frac{r}{2r_B}}$$

$$\langle 1|r|1\rangle = \frac{4}{r_B^3}\int_0^\infty e^{-2r/r_B}\, r^3\, dr = \dots = \frac{3}{2} r_B$$

$$\langle 1|r|2\rangle = \frac{4}{2\sqrt{2}\, r_B^3}\int_0^\infty \left(1-\frac{r}{2r_B}\right)e^{-\frac{3}{2}\frac{r}{r_B}}\, r^3\, dr \qquad t = \frac{3}{2}\frac{r}{r_B} \qquad dr = \frac{2}{3} r_B\, dt$$

$$= \frac{\sqrt{2}}{r_B^3}\int_0^\infty \left(1-\frac{t}{3}\right)e^{-t}\left(\frac{2}{3}r_B\right)^3 t^3 \left(\frac{2}{3}r_B\right) dt =$$

$$\overset{\Gamma}{=} \frac{2^4\sqrt{2}}{3^4}\, r_B \left(3! - \frac{1}{3}\,4!\right) = -\frac{2^5\sqrt{2}\, r_B}{3^4} \doteq -0{,}56\, r_B$$

$$\langle 2|r|2\rangle = \frac{4}{(2r_B)^3}\int_0^\infty \left(1-\frac{r}{2r_B}\right)^2 e^{-\frac{r}{r_B}}\, r^3 dr \qquad \frac{r}{r_B} = t$$

$$= \frac{1\, r_B^4}{2 r_B^3}\int_0^\infty \left(1-\frac{1}{2}t\right)^2 e^{-t}\, t^3\, dt =$$

$$= \frac{r_B}{2}\int_0^\infty \left(t^3 - t^4 + \frac{1}{4}t^5\right)e^{-t}\, dt =$$

$$= \frac{r_B}{2}\left(3! - 4! + \frac{1}{4}\,5!\right) = 6\, r_B$$

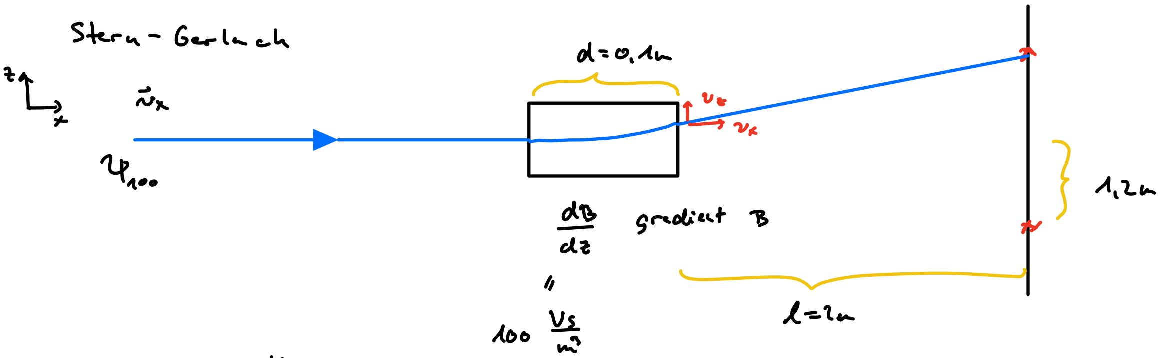
$$\langle r\rangle_\psi = \frac{1}{5}\,\frac{3}{2}\, r_B + \frac{4}{5}\, 6\, r_B - \frac{4}{5}\cdot 0{,}56\, r_B \doteq 4{,}7\, r_B$$

✓ pričakovan saj $E_{\psi_1} < E_\psi$

$$\langle r\rangle_{\psi_1} = \frac{r_B}{5}\left(4\cdot\frac{3}{2} - 4(-0{,}56) + 6\right) \doteq 2{,}85\, r_B$$

(T) Spin

Stern - Gerlach



$$F_z = p_{mz}\frac{dB}{dz}$$

dipolni moment $\quad \vec p_m = \frac{e}{2}\,\vec r\times\vec v = \frac{e}{2}\,\frac{\vec r\times\vec p}{m} = \frac{e}{2m}\vec L = \frac{e}{2m}\vec L_z \overset{QM}{\Longrightarrow} \hat\mu_z = \frac{e}{2m}\hat L_z$

$F_z = \langle \hat{\mu}_z \rangle \frac{dB}{dz}$ $\langle L_z \rangle_m = \hbar m$

$= \underbrace{\frac{e\hbar}{2m}}_{\mu_B} \frac{\langle \hat{L}_z \rangle}{\hbar} \frac{dB}{dz} g_e$

bohrov magneton

$F_z = \mu_B m_\ell g_\ell \frac{dB}{dz}$

Os. stanje: $\ell = 0 \to m = 0 \to F_z = 0$ ker pa se deli v vseh primerih mora obstajati neka druga vrtilna količina (spin)

Intrinzična VK – Spin

$\hat{L}$ $\hat{S}$

$\langle \hat{L}^2 \rangle = \hbar^2 \ell(\ell+1)$ $\langle \hat{S}^2 \rangle = \hbar s(s+1)$

$\langle \hat{L}_z \rangle = \hbar m_\ell$ $\langle \hat{S}_z \rangle = \hbar m_s$ Št. stanj (degeneracija) $2s+1$

$m_s = -s, \dots, s$

$s = \frac{1}{2}$

$F_z = -\frac{\mu_B}{\hbar} \left(g_\ell \langle L_z \rangle + g_s \langle S_z \rangle \right) \frac{dB}{dz}$

$\overset{OS}{=} -\frac{\mu_B}{\hbar} g_s \hbar m_s \frac{dB}{dz} = \pm \mu_B g_s \frac{1}{2} \frac{dB}{dz}$

$\int F_z \, dt = (\Delta p)_z = m_p v_z$ $\frac{v_z}{v_x} = \frac{z}{\ell}$

$\pm \mu_B \frac{1}{2} g_s \frac{dB}{dz} \left(\frac{d}{v_x} \right) = m_p v_x \frac{z}{\ell}$

$g_s = \frac{2 m_p v_x^2 z}{\mu_B \frac{dB}{dz} d \ell} \doteq 2$

v okviru napake

$s = \frac{1}{2}$ za H-atom $m_s = \pm \frac{1}{2}$

Zahtevek $\psi_{n\ell m_\ell} \to \psi_{n\ell m_\ell s m_s}$

T L-S sklopitev

$$\langle \Delta E_{LS} \rangle = \frac{\alpha \hbar c}{2(mc)^2} \left\langle \frac{1}{r^3} \right\rangle \langle LS \rangle$$

$$\hat{j} = \hat{L} + \hat{S}$$

$$\hat{j}^2 = \hat{L}^2 + 2\hat{L \cdot S} + \hat{S}^2$$

$$\hat{L \cdot S} = \frac{1}{2}(\hat{j}^2 - \hat{L}^2 - \hat{S}^2)$$

$$\psi_{n l m_l s m_s} \rightarrow \psi_{n l s j m_j} = \tilde{\psi}$$

$$\hat{j}^2 \tilde{\psi} = \hbar^2 j(j+1) \tilde{\psi} \qquad \hat{L}^2 \tilde{\psi} = \hbar^2 l(l+1) \tilde{\psi}$$

$$\hat{j}_z \tilde{\psi} = \hbar m_j \tilde{\psi} \qquad \hat{S}^2 \tilde{\psi} = \hbar^2 s(s+1) \tilde{\psi}$$

$$\langle LS \rangle = \frac{\hbar^2}{2}\left(j(j+1) - l(l+1) - s(s+1) \right)$$

Kako seštejemo dva vrt. količini

$$j_{max} = l + s$$
$$j_{min} = |l - s|$$

vmes so vsi j ki se razlikujejo za 1 enoto

Za vsak j $\exists\, m_j = -j, \dots, j$

Osnovno stanje $n=1$, $l=0$, $m=0$ $s = 1/2$ $m_s = \pm 1/2$

$$j_{max} = 0 + \frac{1}{2} = \frac{1}{2} \qquad j = \frac{1}{2} \qquad m_j = \pm\frac{1}{2}$$
$$j_{min} = \left|0 - \frac{1}{2}\right| = \frac{1}{2}$$

$$\psi_{100\frac{1}{2}\pm\frac{1}{2}} \rightarrow \psi_{n l s j m_j} = \psi_{10\frac{1}{2}\frac{1}{2}\pm\frac{1}{2}}$$

$$\langle L \cdot S \rangle = \frac{\hbar^2}{2}\left(\frac{1}{2}\left(\frac{1}{2}+1\right) - 0(0+1) - \frac{1}{2}\left(\frac{1}{2}+1\right)\right) = 0$$

ni razcepa v osnovnem stanju

$n=2$ $l=1$ $m_l = \pm 1, 0$ $s = \frac{1}{2}$ $m_s = \pm 1/2$

št. stanj
$(2l+1)(2s+1) = 6$

$$j_{max} = 1 + \frac{1}{2} = \frac{3}{2}$$

$$j_{min} = \left|1 - \frac{1}{2}\right| = \frac{1}{2}$$

$j = \left(\frac{3}{2}, \frac{1}{2}\right)$

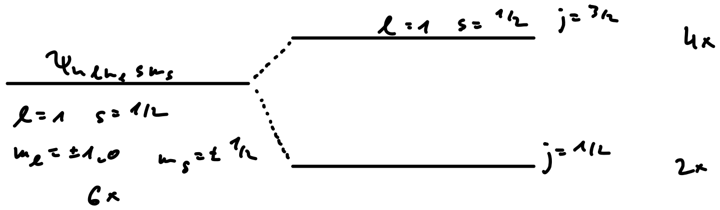
$m_j = \pm\frac{3}{2}, \pm\frac{1}{2}$ $\qquad$ $m_j = \pm\frac{1}{2}$

$$\langle L\cdot S\rangle = \frac{\hbar^2}{2}\left(j(j+1) - l(l+1) - s(s+1)\right)$$

$j=\frac{3}{2}$ $\quad l=1 \quad s=\frac{1}{2}$ $\qquad \langle L\cdot S\rangle = \frac{\hbar^2}{2}\left(\frac{3}{2}\frac{5}{2} - 1\cdot 2 - \frac{1}{2}\frac{3}{2}\right) = \frac{\hbar^2}{2}$

$j=\frac{1}{2}$ $\qquad \langle L\cdot S\rangle = \frac{\hbar^2}{2}\left(\frac{1}{2}\frac{3}{2} - 1\cdot 2 - \frac{1}{2}\frac{3}{2}\right) = -\hbar^2$

$$\langle L\cdot S\rangle = \hbar^2 \begin{cases} \frac{1}{2} & j=\frac{3}{2} \\ -1 & j=\frac{1}{2}\end{cases}$$



(60) $n=2 \quad l=1 \quad m_l=1$

$$\psi_{211} = \frac{1}{8\sqrt{\pi r_B^3}}\,\frac{r}{r_B}\,\sin\theta\, e^{-\frac{r}{2r_B}}\, e^{i\varphi}$$

$$\langle E_{ls}\rangle = \frac{\alpha\hbar c}{2(mc)^2}\left\langle\frac{1}{r^3}\right\rangle\langle LS\rangle \qquad \langle LS\rangle = \hbar^2\begin{cases}\frac{1}{2} & j=3/2\\ -1 & j=1/2\end{cases}$$ od prej

$$\left\langle\frac{1}{r^3}\right\rangle = \frac{1}{64\pi r_B^3}\int\left(\frac{r}{r_B}\right)^2 \sin^2\theta\, e^{-\frac{r}{r_B}}\, e^{i\varphi - i\varphi}\,\frac{1}{r^3}\, r^2\, dr\, d\Omega =$$

$$= \frac{1}{64\pi r_B^5}\int 2\pi\, r\, \sin^2\theta\, e^{-\frac{r}{r_B}}\, d\cos\theta\, dr =$$

$$= \frac{2\pi}{64\pi r_B^5}\int_{-1}^{1}(1-\cos^2\theta)\, d\cos\theta \int_0^\infty \frac{r}{r_B}\, e^{-\frac{r}{r_B}}\,\frac{dr}{r_B}\, r_B^2 =$$

$$= \frac{1}{32 r_B^3}\left(2-\frac{2}{3}\right) 1! = \frac{1}{24 r_B^3} \qquad\qquad r_B = \frac{\hbar c}{\alpha m c^2}$$

$$\langle E_{LS}\rangle = \frac{\alpha\cancel{\hbar c}}{2m^2\cancel{c^4}}\;\frac{1}{24}\frac{\alpha^3(m_e c^2)^{\cancel{3}}}{\cancel{(\hbar c)^3}}\;\frac{\cancel{\hbar^2}}{\cancel{c^2}}\begin{cases}1/2\\ -1\end{cases}$$

$$= \frac{\alpha^4}{96}\, m_e c^2 \begin{cases}1\\ -2\end{cases} \qquad\qquad \alpha = \frac{1}{137} \qquad mc^2 = 511\,\mathrm{keV}$$

$$= \begin{cases}1{,}5\cdot 10^{-5}\,\mathrm{eV}\\ -3\cdot 10^{-5}\,\mathrm{eV}\end{cases}$$

$$E_2 = -\frac{13{,}6\,\mathrm{eV}}{4} + \langle E_{LS}\rangle = -\frac{13{,}6\,\mathrm{eV}}{4} + \begin{cases}1{,}5\cdot 10^{-5}\,\mathrm{eV}\\ -3\cdot 10^{-5}\,\mathrm{eV}\end{cases}$$

T Zeemanov pojav

$$\hat{H} = \hat{H}_0 + \underbrace{\hat{H}_{ls}}_{\text{LS sklopitev}} - \underbrace{\hat{\mu}_z B}_{\text{Zeemanov pojav}}$$

a) $B \neq 0$ samo ls
b) $\mu B \gg \Delta E_{ls}$ močno B
c) $\mu B \ll \Delta E_{ls}$ šibko B

a) $B \neq 0$ $\quad \Delta E_{ls} \propto \langle ls \rangle \propto \frac{\hbar^2}{2}\left(j(j+1) - l(l+1) - s(s+1)\right)$

$n=1$ $l=0$ $m_l=0$ ... $\Delta E_{ls} = 0$ en. nivoji se ne razcepijo
$s=\frac{1}{2}$ $m_s = 1/2$
$\psi_{nlm_lsm_s} \longrightarrow \psi_{nlsjm_j}$

$n=2$ $l=0,1$ $\quad l=0$ isto kot zgoraj

$l=1$ $\quad j^{max} = l+s = \frac{3}{2}$
$j^{min} = |l-s| = \frac{1}{2}$

$$\langle l s \rangle = \frac{\hbar^2}{2}\left(\frac{3}{2}\left(\frac{5}{2}\right) - 2 - 3/4\right) = \frac{\hbar^2}{2} \quad \text{za } j = \frac{3}{2}$$

$$\langle l s \rangle = \frac{\hbar^2}{2}\left(\frac{3}{4} - 2 - \frac{3}{4}\right) = -\hbar \quad \text{za } j = \frac{1}{2}$$

za $l=1$ $\quad m_l = \pm 1, 0$
$s = 1/2$ $\quad m_s = \pm 1/2$

št. stanj ne razcepljenih $(2l+1)(2s+1) = 3 \cdot 2 = 6$

$l=1$ $j=\frac{3}{2}$ $m_j = \pm\frac{3}{2}, \pm\frac{1}{2}$ $\quad 4\times$
$l=0$ $m_j = \pm 1$ $\quad 2\times$
$l=1$ $j=1/2$ $m_j = \pm 1/2$ $\quad 2\times$

8 degen. stanj

b) Močno B

$$\Delta E \sim -\langle \mu_z \rangle B + \cancel{\Delta E_{ls}}^{\sim 0}$$

$$= \left(\langle l_z \rangle + 2\langle s_z \rangle\right) \frac{\mu_B}{\hbar} B$$

Dobra KŠ $\quad \psi_{nlm_lsm_s}$

$$\Delta E_B = (m_l + 2m_s)\,\mu_B B$$

$n=1$ $l=0$ $m_l=0$ $m_s = \pm 1/2$

$m_s = 1/2$ $\quad \mu_B B$
$m_s = -1/2$ $\quad -\mu_B B$

$$\Delta E_B = \pm \frac{1}{2}(-2)\,\mu_B B = \pm \mu_B B$$

Popoln razcep stanj

$n=2$ $l=0,1$ za $l=0$ ostane enako

za $l=1$ $m_l = \pm 1, 0$

$$\Delta E_B = (m_l + 2m_s)\,\mu_B B = \left((0, \pm 1) \pm 2\tfrac{1}{2} \right)\mu_B B = (\pm 1, 2, 0, -2, 0)\,\mu_B B$$

$l=1$ $s=1/2$
6x degenerirano

m_l	m_s	
1	$1/2$	1x
0	$1/2$	1x
$(1, -1/2)$	$(-1, +1/2)$	2x
0	$-1/2$	1x
-1	$-1/2$	1x

ⓒ Šibko B

$$\Delta E \propto \langle H_{ls} \rangle - \langle \hat{\mu}_z \rangle B$$

$$\hat{\mu}_z = \frac{\mu_B}{\hbar}(\hat{l}_z + 2\hat{s}_z)B = \frac{\mu_B}{\hbar}\, g_{lsj}\, \hbar m_j B = \underline{\mu_B\, g_{lsj}\, m_j B}$$

$$\psi_{n l m_l s m_s} \to \psi_{n l s j m_j}$$

$$\langle l_z + 2 s_z \rangle = \frac{\langle \hat{j}^2 \rangle + \langle \hat{s}\hat{j} \rangle}{\langle \hat{j}^2 \rangle} \cdot \langle j_z \rangle$$

Landéjev giromagnetni faktor g_{lsj}

$$\langle j^2 \rangle = \hbar^2 j(j+1)$$

$$\hat{j} = \hat{l} + \hat{s} \qquad \hat{j} - \hat{s} = \hat{l} \quad /^2$$

$$\hat{j}^2 - 2\hat{j}\hat{s} + \hat{s}^2 = \hat{l}^2$$

$$\langle \hat{j}\hat{s} \rangle = \frac{1}{2}\left(\langle j^2 \rangle - \langle l^2 \rangle + \langle s^2 \rangle \right)$$

$$= \frac{\hbar^2}{2}\left(j(j+1) - l(l+1) + s(s+1) \right)$$

$$g_{lsj} = \frac{\hbar^2 j(j+1) + \frac{\hbar^2}{2}\left(j(j+1) - l(l+1) + s(s+1) \right)}{\hbar^2 j(j+1)}$$

$$= \underline{\frac{3}{2} - \frac{l(l+1) - s(s+1)}{2j(j+1)}}$$

$n=1$ $l=0$ $m_l=0$ $s=1/2$ $m_s=\pm 1/2$ Os. st.

$j^{max} = \frac{1}{2}$ $j_{min} = \frac{1}{2}$ $m_j = \pm 1/2$

$$\Delta E = \Big(\underbrace{\frac{3}{2} - \frac{0 - 3/4}{2\frac{1}{2}\frac{3}{2}}}_{g_{0\frac{1}{2}\frac{1}{2}}} \Big)(\pm 1/2)\,\mu_B B = \pm \mu_B B$$

$n=2$ $l=0,1$ $s=1/2$ 1. vz. st.

za $l=0$ enako kot prej

$l=1$ $\quad j^{max} = l+s = 1+1/2 = 3/2$

$\quad j^{min} = |l-s| = 1/2$

$$g_{l,j} = g_{1,1/2,3/2} = \frac{3}{2} - \frac{2-3/4}{2\cdot 3/2\cdot 5/2} = \frac{4}{3}$$

$$g_{1,1/2,1/2} = \frac{3}{2} - \frac{2-3/4}{2\cdot 1/2\cdot 3/2} = \frac{2}{3}$$

$$\Delta E_{šibko}^{j=1/2} = \frac{2}{3}\left(\pm\frac{1}{2}\right)\mu_B B = \pm\frac{1}{3}\mu_B B$$

$n=2$ $l=1$ $m_l \pm 1, 0$ $s=1/2$

$j=1/2$ $m_j = +\frac{1}{2}$ $\quad +\frac{1}{3}$

$m_j = -\frac{1}{2}$ $\quad -\frac{1}{3}$

$2p_{1/2}$

Spektroskopske oznake

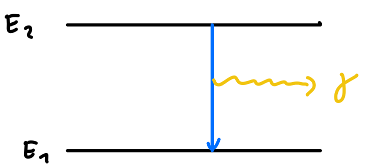
$n\ l$

	s	p	d	f
$l=$	0	1	2	3

$j=\frac{3}{2}$ $\quad g_{1,1/2,3/2} = \frac{4}{3}$, $\quad m_j = \pm\frac{3}{2}, \pm\frac{1}{2}$

$$\Delta E_B^{j=3/2} = \frac{4}{3}\left(\pm\frac{3}{2}, \pm\frac{1}{2}\right)\mu_B B = \left(\pm 2, \pm\frac{2}{3}\right)\mu_B B$$

(T) Dipolni sevalni prehodi



$$E_\gamma = E_2 - E_1 = E_{12} \qquad \frac{1}{\tau} = \frac{2}{3}\alpha E_{12}^3 \left(\frac{x_{12}}{\hbar c}\right)^2 \qquad \alpha = \frac{1}{137}$$

$$x_{12} = \langle 1 | x | 2 \rangle = \int \psi_1^* x \psi_2 \, dV$$

(64) $n=2 \longrightarrow n=1$, $a = 0{,}3$ nm $\quad \infty$ pot. jama

$$\Delta E \tau = \frac{\hbar}{2} \Rightarrow \Delta E = \frac{\hbar}{2}\frac{1}{\tau}$$

Pri računanju prehodov je treba izračunati E prehoda in $\langle x \rangle$

$$E_n = \frac{p^2}{2m} = \frac{1}{2m}\left(\frac{n\pi\hbar}{a}\right)^2$$

$$E_{12} = E_2 - E_1 = \frac{\pi^2\hbar^2}{2ma^2}(4-1) = \frac{3\pi^2\hbar^2}{2ma^2}$$

$$x_{12} = \int_0^a \psi_1^* x \psi_2 \, dx \qquad \psi_n = \sqrt{\frac{2}{a}} \sin\frac{n\pi x}{a}$$

$$= \frac{2}{a}\int_0^a \sin\frac{\pi x}{a}\, x\, \sin\frac{2\pi x}{a}\, dx \qquad \frac{\pi x}{a} = t \qquad dx = \frac{a}{\pi} dt$$

$$= \frac{2a}{\pi^2}\int_0^\pi \sin t \;\; t \;\; \sin 2t \;\; dt$$

$$= \frac{4a}{\pi^2}\int_0^\pi t \sin^2 t \cos t \; dt$$

$$= \frac{4a}{\pi^2}\int_0^\pi (t\cos t - t\cos^3 t)\, dt$$

$$\int_0^\pi t\cos t\, dt = \sin t \; t \Big|_0^\pi - \int_0^\pi \sin t\, dt = 0 + \cos\Big|_0^\pi = -2$$

$du = dt \qquad v = \sin t$

$$\int_0^\pi t\cos^3 t\, dt = t\sin t\left(1 - \frac{1}{3}\sin^2 t\right)\Big|_0^\pi - \int_0^\pi \sin t - \frac{1}{3}\sin^3 t\, dt =$$

$du = dt \qquad \cos^3 t\, dt = dv$

$\sin t - \frac{1}{3}\sin^3 t = v$

$$= \cos t\Big|_0^\pi - \frac{1}{3}\int (1-\cos^2 t)\, d\cos t$$

$$= -2 - \frac{1}{3}\int 1 - c^2\, dc$$

$$= -2 - \frac{2}{3} + \frac{1}{3}\frac{2}{3} = -\frac{14}{9}$$

$$x_{12} = \frac{4a}{\pi^2}\left(-2 + \frac{14}{9}\right) = -\frac{16a}{9\pi^2} \qquad \delta E = \frac{\hbar}{2}\frac{1}{\tau} = \dots = \frac{\hbar\alpha\, 2^5\pi^2\,\hbar^{10}}{3^2\, a^4 m^3 c^2} = \frac{2^6\pi^2}{3^2}\alpha\left(\frac{\hbar c}{amc^2}\right)^4 mc^2$$

$$= \dots = 10^{-6}\, eV$$

(65) $\tau_H = ?$ $\quad n=2,\ l=1,\ m_l = 0 \longrightarrow n=1,\ l=0,\ m_l=0$

Izbirna pravila:

Dipolni sevalni prehod nosi:

$$\boxed{\begin{matrix} \Delta l = \pm 1 & \Delta m_l = \pm 1, 0 & \\ \Delta m_s = 0 & & \text{To ne gre} \\ \Delta j = \pm 1, 0 & \Delta m_j = \pm 1, 0 & j=0 \not\to j=0 \end{matrix}}$$

$$\boxed{\frac{1}{\tau} = \frac{4}{3}\frac{\alpha}{\hbar} E_{12}^3 \left(\frac{r_{12}}{\hbar c}\right)^2}$$

$$E_n = -\frac{1}{2}\alpha^2 mc^2 \frac{1}{n^2} = -E_{Ry}\frac{1}{n^2} \qquad \text{za H-atom}$$

$$E_{12} = E_2 - E_1 = E_{Ry}\left(-\frac{1}{4} - (-1)\right) = \frac{3}{4}E_{Ry}$$

$$E_{12}^3 = \frac{3^3}{2^6}\frac{1}{2^3}\alpha^6 (mc^2)^3$$

$$\langle \psi_{100} | \vec{r} | \psi_{210} \rangle = \int \psi_{100}^* \vec{r}\, \psi_{210}\, r^2 dr\, d\cos\theta\, d\phi$$

$$\psi_{100} = \frac{2}{\sqrt{4\pi r_B^3}} e^{-r/r_B}$$

$$\psi_{210} = \frac{1}{(4\pi(2r_B)^3)^{1/2}} \frac{r}{r_B} e^{-r/2r_B} \cos\theta$$

$$\vec r = (x, y, z) = (r\sin\theta \underbrace{\cos\varphi}_{0}, r\sin\theta \underbrace{\sin\varphi}_{0}, r\cos\theta)$$

integral gre v nič ostane samo z-smer

$$r_{12} = \frac{2}{24\pi\sqrt{2}\, r_B^3} \int_0^\infty \frac{r}{r_B} e^{-\frac{3}{2}\frac{r}{r_B}} r\, r^2 dr \int_{-1}^{1} \cos^2\theta \, d\cos\theta \int_0^{2\pi} d\varphi =$$

$$= \frac{1}{2\, 2\sqrt{2}\pi} \int_0^\infty \left(\frac{r}{r_B}\right)^4 e^{-\frac{3}{2}\frac{r}{r_B}} dr \quad \frac{3}{2}\cdot 2\pi =$$

$$= \frac{2}{2\, 3\sqrt{2}\, r_B^4} \int_0^\infty e^{-t} t^4 \, dt \left(\frac{2r_B}{3}\right)^5 = \frac{2^6 r_B}{2\, 3^6 \sqrt{2}} 4! = \frac{2^8 r_B}{\sqrt{2}\, 3^5}$$

$$r_{12} = \frac{\sqrt{2}\, 2^7}{3^5} \frac{\hbar c}{\alpha m c^2}$$

$$\frac{1}{\tau} = \frac{4}{3} \frac{\alpha}{\hbar} \frac{3^7}{2^9} (\alpha^2 m c^2)^3 \left(\frac{\sqrt{2}\, 2^7}{3^5} \frac{1}{\alpha m c^2} \right)^2$$

$$\frac{1}{\tau} = \frac{3^2}{2^7} \frac{\alpha}{\hbar} \alpha^4 m c^2 \frac{2 \cdot 2^{14}}{3^{10}} = \frac{\alpha^5 2^8}{3^8 \hbar} m c^2$$

$$\tau = 1{,}6 \text{ ns}$$

$$\delta E = \frac{\hbar}{\tau} = \left(\frac{2}{3}\right)^8 \alpha^5 m c^2 = 0{,}4 \,\mu\text{eV}$$

62 Harmonski oscilator

$\langle m | x | n \rangle = ?$, $\langle n | x^2 | n \rangle = ?$

$$\hat H = \frac{\hat p^2}{2m} + \frac{1}{2} m\omega^2 \hat x^2$$

$$E_n = \hbar\omega \left(n + \frac{1}{2}\right)$$

$$\psi_n = \frac{1}{\sqrt{2^n n! \sqrt{\pi} a}} H_n(y) e^{-y^2/2} \qquad y = \frac{x}{a} \qquad a = \sqrt{\frac{\hbar}{m\omega}}$$

$$\int \psi_m^* \psi_n \, dx = \delta_{mn}$$

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}\left(e^{-x^2}\right)$$

$$x H_n = \frac{1}{2}\left(H_{n+1} + 2n H_{n-1}\right)$$

$$dx = a\, dy$$

$$\langle m | x | n \rangle = \frac{1}{\sqrt{2^{n+m} n! m! \pi a^2}} \int H_m(y) \overset{ay}{\overset{\shortparallel}{x}} H_n(y) e^{-y^2} dx$$

$$= \frac{a^2}{a\sqrt{2^{n+m} n! m! \pi}} \int_{-\infty}^{\infty} H_m(y) \frac{1}{2} \left(H_{n+1}(y) + 2n H_{n-1}(y) \right) e^{-y^2} dy = \ldots$$

$$\delta_{mn} = \langle m | n \rangle = \frac{1}{\sqrt{2^{n+m} n! m! \pi}} \int_{-\infty}^{\infty} H_n(y) H_m(y) e^{-y^2} dy$$

$$\ldots \frac{a}{2} \left(\frac{\sqrt{2(n+1)}}{\sqrt{2^{n+m+1} n! m! \pi (\ )}} \int H_m H_{n+1} e^{-y^2} dy + \frac{2n}{\sqrt{2n}} \frac{1}{\sqrt{2^{n+m-1}(n-1)! m! \pi}} \int H_m H_{n-1} e^{-y^2} dy \right)$$

$$= \frac{a}{2}\left(\sqrt{2(n+1)}\,\delta_{m\,n+1} + \sqrt{2n}\,\delta_{m\,n-1}\right)$$

$$\langle m|x|n\rangle = \frac{a}{\sqrt{2}}\left(\sqrt{n+1}\,\delta_{m\,n+1} + \sqrt{n}\,\delta_{m\,n-1}\right)$$

Npr. $\langle 1|x|0\rangle = \frac{a}{\sqrt{2}}$ Izbirno pravilo za HO $m-n=\pm 1$ $\Delta n = \pm 1$

$\langle n|x^2|n\rangle = ?$

$$x H_n = \frac{1}{2}\left(H_{n+1} + 2n H_{n-1}\right) \quad /\cdot x$$

$$x^2 H_n = \frac{1}{4}\left(H_{n+2} + 2(n+1) H_n + 2n\left(H_n + 2(n-1) H_{n-2}\right)\right)$$

mešani členi izginejo

$$\langle n|x^2|n\rangle = \frac{1\cdot a^3}{2^n n!\sqrt{\pi}\,a}\int H_n(y)\, y^2\, H_n(y)\, e^{-y^2}dy =$$

$$= \frac{a^2}{2^n n!\sqrt{\pi}}\int H_n(y)\,\frac{1}{2}\left(2(n+1)H_n + 2n H_n\right)e^{-y^2}dy$$

$$= \frac{a^2}{2^n n!\sqrt{\pi}}\int_{-\infty}^{\infty} H_n\left(n+\frac{1}{2}\right)H_n\, e^{-y^2}dy =$$

$$= a^2\left(n+\frac{1}{2}\right)\underbrace{\frac{1}{2^n n!\sqrt{\pi}}\int_{-\infty}^{\infty} H_n H_n\, e^{-y^2}dy}_{\delta_{nn}=1} = a^2\left(n+\frac{1}{2}\right)$$

Virialni teorem

$$2\langle T\rangle = p\langle V\rangle \qquad V \propto x^p$$

HO $p=2$

$$\langle T\rangle = \langle V\rangle$$

$$\rightarrow \langle E\rangle = \langle V\rangle + \langle T\rangle = 2\langle V\rangle$$

$$\langle E_n\rangle = E_n = \hbar\omega\left(n+\frac{1}{2}\right)$$

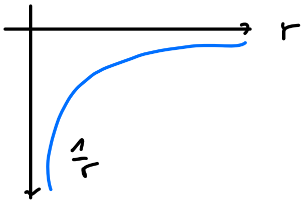
$$\langle V\rangle = \frac{1}{2}m\omega^2\langle x^2\rangle = \frac{1}{2}m\omega^2 a^2\left(n+\frac{1}{2}\right)$$

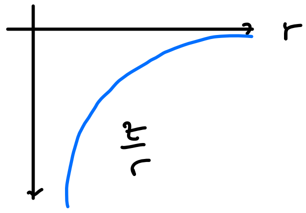
$$a = \sqrt{\frac{\hbar}{m\omega}}$$

$$\langle V\rangle = \frac{1}{2}\hbar\omega\left(n+\frac{1}{2}\right) = \frac{1}{2}\langle E\rangle$$

T Večelektronska stanja

$$\hat{H} = \sum_i \frac{p_i^2}{2m} - \underbrace{Z\frac{\alpha\hbar c}{r_i}}_{\text{privlak med elektroni}} + \underbrace{\sum_{j<i} \frac{\alpha\hbar c}{|r_j - r_i|}}_{\text{odboj med } e^-}$$





stran od jedra ~ H atom

manjši l : orbitala bližje : bolj vezana

večji l : orbitala dlje : manj vezana

Hundova pravila

$$\hat{S} = \sum_i \hat{s}_i \quad , \quad \hat{L} = \sum_i \hat{l}_i \quad , \quad \hat{J} = \sum_i \hat{j}_i$$

1. Stanje z max S najprej zapolnimo (ima najnižjo energijo)
2. Stanje z max L najprej zapolnimo (ima najnižjo energijo)
3. e^- zapolnjeni: nad $\frac{1}{2}$ stanj : max J
   pod $\frac{1}{2}$ stanj : min J

1

$$^{2S+1}L_J$$

L → cel. orbitalna

J → cel. skupna VK

$2S+1$ → v.š. cel. spina

1. elektron n_1 $l_1 = 1$ $s_1 = \frac{1}{2}$

2. e^- n_2 $l_2 = 2$, $s_2 = \frac{1}{2}$

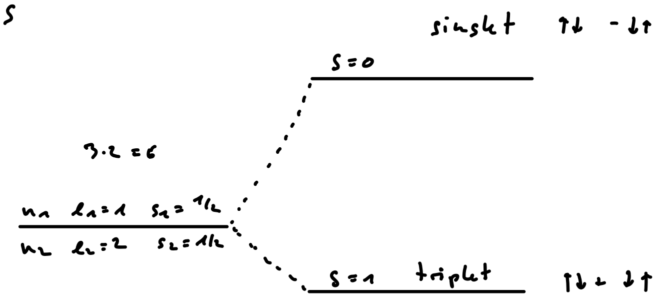
Velja LS sklopitev

$|L S\rangle$, najprej $l_1 + l_2 = L$
$s_1 + s_2 = S$

1. max S seštejemo $s_1 + s_2$

$$\hat{S} = \hat{s}_1 + \hat{s}_2$$
$$S_{max} = \tfrac{1}{2} + \tfrac{1}{2} = 1$$
$$S_{min} = 0$$

$3 \cdot 2 = 6$



$(2l+1)(2s+1) = 10$

$6 \cdot 10 = 60$ stanj

$$H_S \propto \mu \langle s_1 \cdot s_2 \rangle = \frac{\mu}{2}\left(\hat{S}^2 - \hat{s}_1^2 - \hat{s}_2^2\right)$$

$\Rightarrow \quad s_1 + s_2 = 1 \qquad \langle E \rangle = \frac{\mu}{4}$

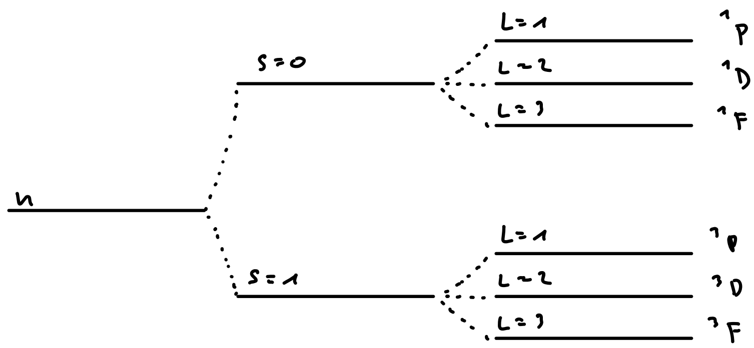
$\langle E \rangle_{S=1} = -\frac{\mu}{4}$

② max L ⇒ najnižji

$$\left.\begin{aligned} &l_1 = 1 \qquad l_2 = 2 \\ &L_{max} = 1 + 2 = 3 \\ &L_{min} = |1-2| = 1 \end{aligned}\right\} L = 3,\ 2,\ 1$$

$L=0$	S
1	P
2	D
3	F

$^{2S+1}L_J$



③ LS razcep

$$\Delta E \propto \langle L S \rangle \propto \frac{\hbar^2}{2}\left(J(J+1) - L(L+1) - S(S+1)\right)$$

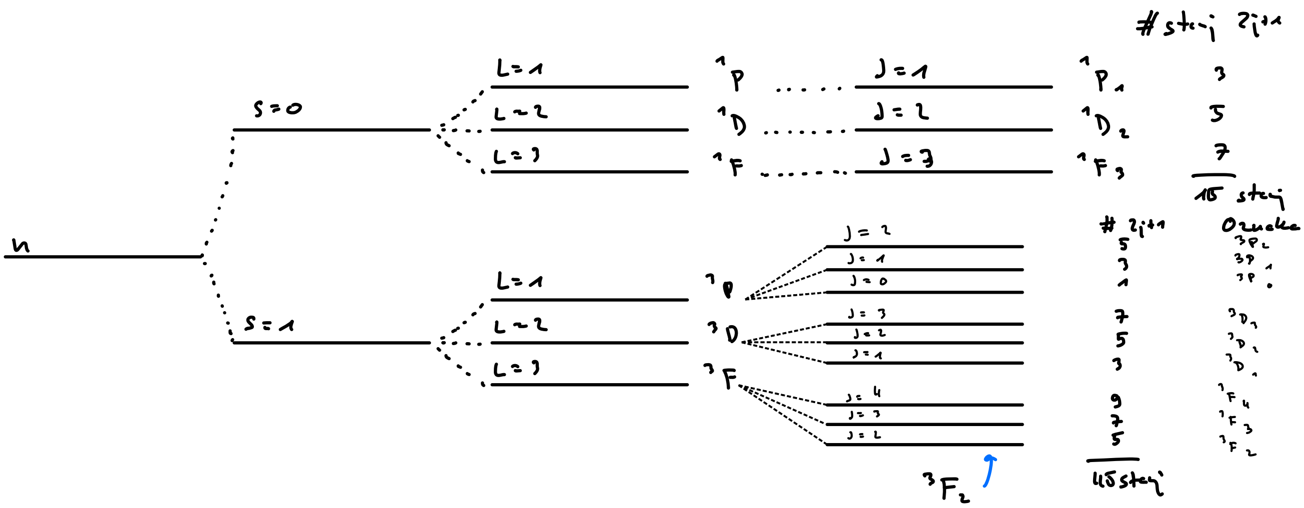
$S=0: \quad J = L$

⇓

ni LS razcepa

$S=1$

	max	min
$L=1$	$J \in (2, 1, 0)$	
$L=2$	$J \in (3, 2, 1)$	
$L=3$	$J \in (4, 3, 2)$	



$L = l_1 + l_2$ $\qquad S = s_1 + s_2$ $\qquad L + S = J$

J - J sklop

$j_1 = l_1 + s_1$ $\qquad j_2 = l_2 + s_2$

$\Downarrow \qquad \Downarrow$

$\Delta E = \Delta E_{j_1} + \Delta E_{j_2}$

$\propto \frac{\hbar^2}{2}\left(j(j+1) - l(l+1) - s(s+1)\right)$

$j = l + s$

$j(j+1) = l(l+1) + s(s+1) + 2\langle l \cdot s\rangle$

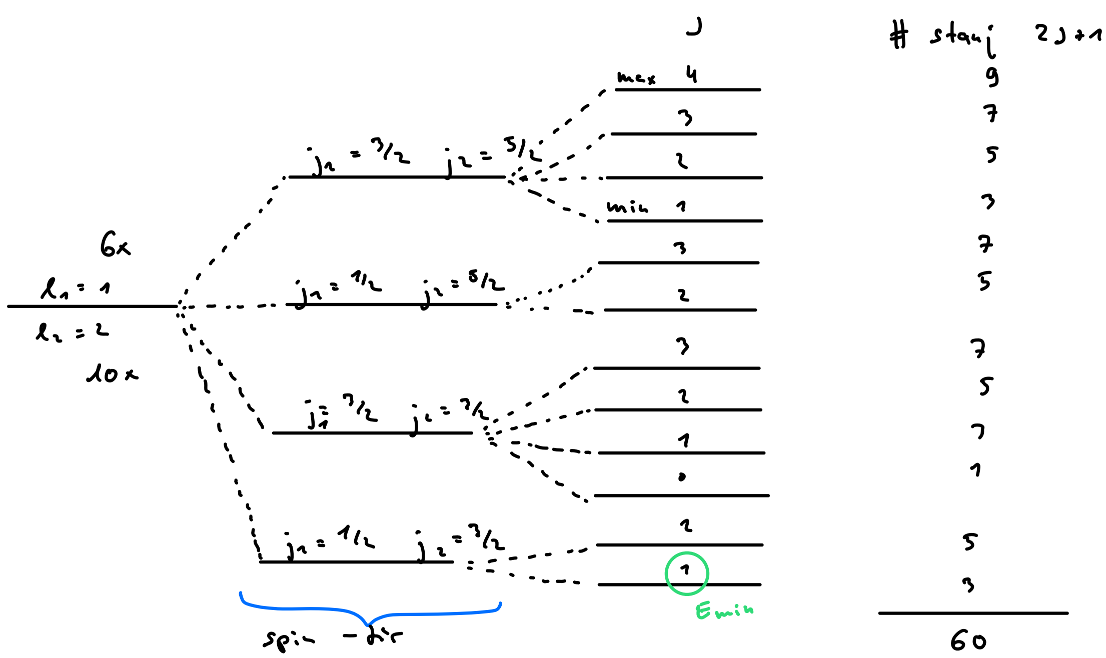
3

$l_1 = 1$ $\qquad j_1^{max} = \frac{3}{2}$ $\qquad \Delta E_{j=\frac{3}{2}} \propto \frac{3}{2}\,\frac{5}{2} - 2 - \frac{3}{4} = 1$

$s_1 = \frac{1}{2}$ $\qquad j_1^{min} = \frac{1}{2}$ $\qquad \Delta E_{j=\frac{1}{2}} \propto -2$

$l_2 = 2$ $\qquad j_2^{max} = \frac{5}{2}$ $\qquad \Delta E_{j=\frac{5}{2}} \propto \frac{35}{4} - \frac{24}{4} - \frac{3}{4} = 2$

$s_2 = \frac{1}{2}$ $\qquad j_2^{min} = \frac{3}{2}$ $\qquad \Delta E_{j=\frac{3}{2}} \propto -3$



• $j_1 + j_2 = J \xRightarrow[\text{pravilo}]{\text{Hundovo}} \min J$

(14) $\underbrace{{}^3P_1}_{S=1,\ L=1,\ J=1} \to \underbrace{{}^3D_2}_{S=1,\ L=2,\ J=2}$ ${}^{2S+1}L_J$ $B = 0{,}05\ T$ = šibko polje

(a) Razcep v šibkem polju ⇒ Landé faktor

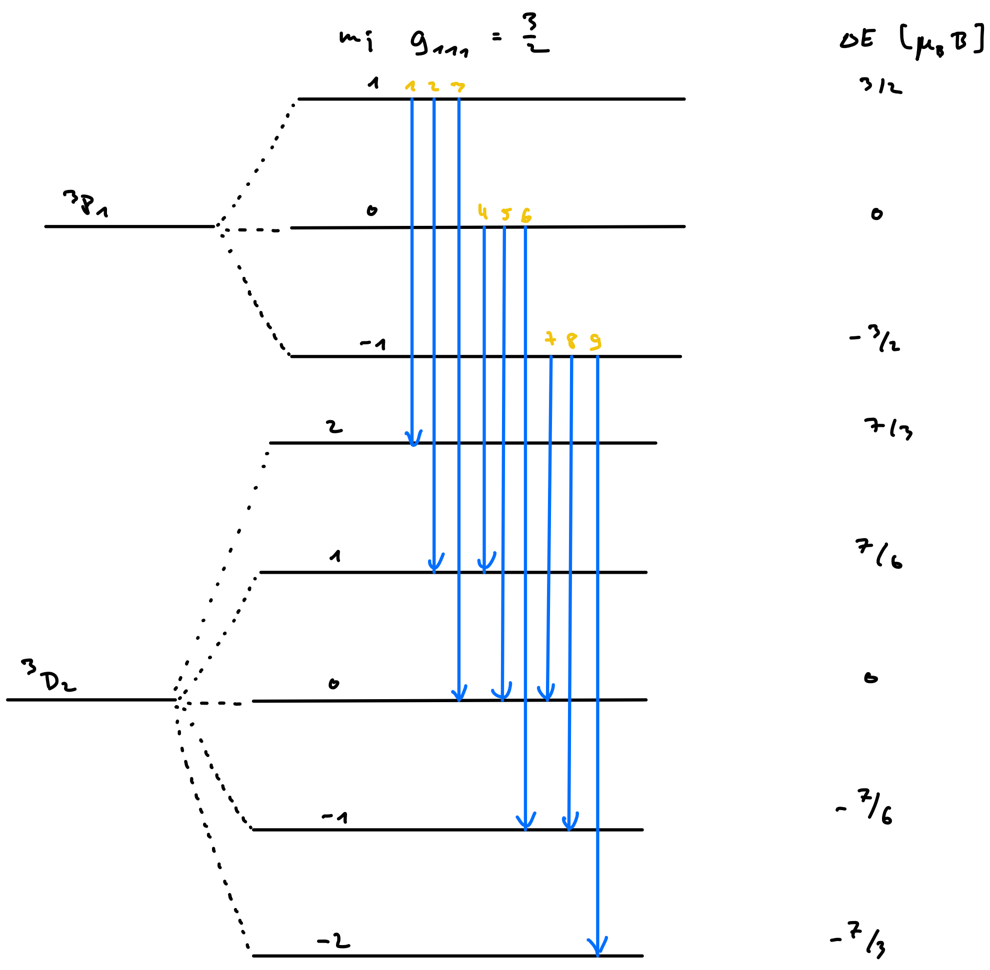
$$\Delta E_B = g_{LSJ}\, m_j\, \mu_B B$$

• $J=1,\ L=1,\ S=1$ $m_j \in [-1, 0, 1]$

$$g_{LSJ} = \frac{3}{2} - \frac{L(L+1) - S(S+1)}{2J(J+1)} = \frac{3}{2}$$

• $J=2,\ L=2,\ S=1$ $m_j \in [-2, -1, 0, 1, 2]$

$$g_{LSJ} = \frac{3}{2} - \frac{2\cdot 3 - 1\cdot 2}{2\cdot 2\cdot 3} = \frac{3}{2} - \frac{1}{3} = \frac{7}{6}$$



Dipolni sevalni prehodi in izbirna pravila

$\Delta L = \pm 1$ ✓ $P \to D$

$\Delta S = 0$ ✓

$\Delta J = \pm 1, 0$

$\Delta m_j = \pm 1, 0$

Spektar fotonov

$$E_f = \Delta E \; [\overset{E_0}{\overset{\shortparallel}{\mu_B B}}] \qquad E_0 = \frac{e\hbar}{2m_e} B$$

$$E_1 = \frac{3}{2} - \frac{7}{3} = -\frac{5}{6} E_0$$

$E_2 = \ldots$

$E_3 = \ldots$

$\vdots \quad \ldots$

$E_9 \quad \ldots$

$$\Delta E_f = (\pm 9, \pm 7, \pm 5, \pm 3, 0) \frac{E_0}{6}$$

Molekule

39

$$V_{odbojni} = \frac{c}{r^{35}} \qquad V_{privlačni} = -\frac{\alpha \hbar c}{r_0}$$

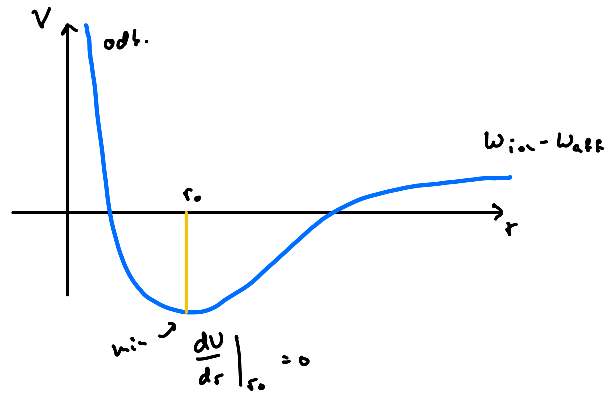
$W_{ion.}^{Na} = 5{,}1 \, eV \qquad r^{mol} = (1 - 0{,}11)\, r^{krist.}$

$W_{aff}^{Cl} = 3{,}8 \, eV \qquad M_{Na} = 23 \, kg \qquad M_{Cl} = 35 \, kg$

$\rho_{NaCl} = 2160 \, \frac{kg}{m^3}$

Obravnavaj kot harmonski oscilator

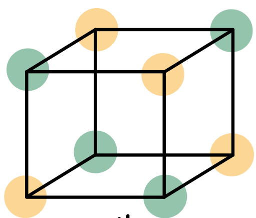
$$W_{vez} = W_{ion} - W_{aff} - \frac{\alpha \hbar c}{r_0} + \frac{c}{r^{35}} + W_{nih}^{HO}$$



- $\left.\frac{dU}{dr}\right|_{r_0} = \frac{\alpha \hbar c}{r_0^2} - \frac{35 c}{r_0^{36}} = 0$

$$\frac{\alpha \hbar c}{35 \, r_0^2} \, r_0^{36} = c$$

- $r^{kristal}$



$r^{kristal}$

$V = r_k^3$

$$m = \frac{1}{8}(4 m_{Na} + 4 m_{Cl}) = \frac{m_{Na} + m_{Cl}}{2} =$$

$$= \frac{1}{2}(35 + 23) m_p = 29 \, m_p$$

$$\rho = \frac{m}{V} = \frac{29 \, m_p}{r_k^3} \Rightarrow r_k = \sqrt[3]{\frac{29 \, m_p}{\rho}}$$

$$r_k = 0{,}28 \, nm$$

$$\Rightarrow r_0 = 0{,}89 \, r_k$$

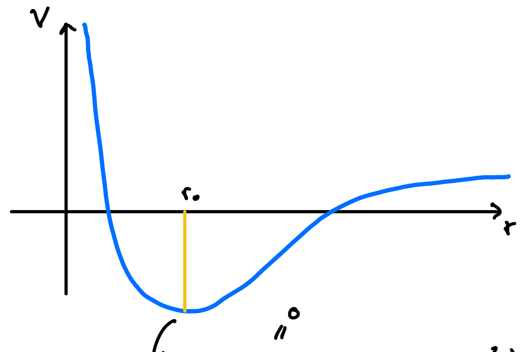
$$W_{vez}^0 = W_{ion} - W_{aff} - \frac{\alpha \hbar c}{r_0} + \frac{1}{r^{35}} \frac{\alpha \hbar c}{35 \, r_0^2} r_0^{36}$$

$$= W_{ion} - W_{aff} - \frac{\alpha \hbar c}{r_0}\left(1 - \frac{1}{35}\right)$$

$$= W_{ion} - W_{aff} - 1{,}11 \frac{\alpha \hbar c}{r_k} \frac{34}{35}$$

$$= \ldots = -4{,}16 \, eV$$

Obravnavamo mol. kot HO – Prispevek nihajne energije



$$V = -V_0 + \overset{=0}{\frac{dU}{dr}\Big|_{r_0}} + \frac{1}{2}\frac{d^2U}{dr^2}\Big|_{r_0}(r-r_0)^2$$

$$V^{HO} = \frac{1}{2} m\omega^2 (x-x_0)^2$$

$$m\omega^2 = \frac{d^2U}{dr^2}\Big|_{r_0}$$

$$m\omega^2 = -\frac{2d\hbar c}{r_0^3} + \frac{35\cdot 36}{r_0^{37}}\,\frac{d\hbar c}{35}\,r_0^{34}$$

$$\frac{1}{m} = \frac{1}{m_{Na}} + \frac{1}{m_{Cl}} = \left(\frac{1}{23}+\frac{1}{35}\right)\frac{1}{m_p}$$

$$m = \frac{23\cdot 35}{58}\, m_p$$

$$\longrightarrow m\omega^2 = 34\,\frac{d\hbar c}{r_0^3}$$

$$E_{min} = \hbar\omega\left(n+\frac{1}{2}\right) \quad \text{osn. st. } n=0$$

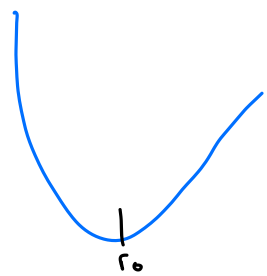
$$E^0_{min}: \frac{\hbar}{2}\omega = \frac{\hbar}{2}\sqrt{\frac{1}{m}\frac{d^2U}{dr^2}\Big|_{r_0}}$$

$$= \frac{\hbar}{2}\sqrt{\frac{34\,d\hbar c}{r_0^3\, m}} \approx 0{,}05\,eV$$

$$W_{vez} = -4{,}16\,eV + 0{,}05\,eV = -4{,}11\,eV$$

38 $V(r) = V_0\left(e^{-2(r-r_0)/a} - 2e^{-(r-r_0)/a}\right)$ $\quad V_0 = 3\,eV \quad a = 0{,}12\,nm \quad r_0 \dots$ ravnovesna razdalja med jedroma

W_{min} za $n=0$, $n=1$, $M=16$
Obravnavamo kot HO



$$e^{-x} = 1 - x + \frac{x^2}{2} + O(x^3)$$

$$\frac{r-r_0}{a} = x$$

$$V = V_0\left(1 - 2x + \frac{4x^2}{2} - 2\left(1 - x + \frac{x^2}{2}\right)\right)$$

$$V \doteq -V_0 + \overset{0}{\frac{dU}{dr}\Big|_{r_0}} + \overset{0}{\frac{d^2V}{dr^2}\Big|_{r_0}}$$

$$= V_0\left(-1 + \overset{\checkmark\ \frac{dU}{dr}=0}{0\cdot x} + x^2\right) = V_0(x^2-1) = -V_0 + \underbrace{V_0\frac{(r-r_0)^2}{a^2}}_{\frac{1}{2}m\omega^2(r-r_0)^2}$$

$$\frac{1}{2}m\omega^2 = \frac{V_0}{a^2}$$

$$\omega = \frac{1}{a}\sqrt{\frac{2V_0}{m}}$$

$16 m_p = m_0$

$$\frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2} = \frac{2}{m_0}$$

$$m = 8 m_p$$

$$W_{nih} = \hbar\omega \left(n + \frac{1}{2}\right) = \frac{\hbar}{a}\sqrt{\frac{2V_0}{m}}\left(n + \frac{1}{2}\right) = \frac{\hbar c}{a}\sqrt{\frac{2V_0}{8 m_p c^2}}\left(n + \frac{1}{2}\right)$$

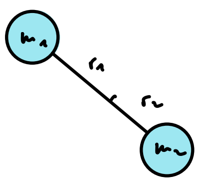
$$E_0 = \frac{1}{2}\hbar\omega = 27\ \mathrm{meV}$$
$$E_1 = \frac{3}{2}\hbar\omega = 69\ \mathrm{meV}$$

30 H_2 , HD , D_2 D... devterij $r_0 = 0{,}074$ nm   (Rotacijski spektri)

$$W_{rot}^{klas} = \frac{\Gamma^2}{2J}$$
J... vztr. mom.
Γ... vrtilna količ.

$$\Downarrow \mathrm{QM}$$

$$\langle E_{rot} \rangle = \frac{\langle L^2 \rangle}{2J_0} = \frac{\hbar^2 l(l+1)}{2J_0} \quad ; \quad l = 0, 1, \ldots$$



$$r_0 = r_1 + r_2$$
$$J_0 = J_1 + J_2$$
$$J = m r_0^2 \qquad \frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2}$$

$$E_r^{H_2} = \frac{\hbar^2}{2J} = \frac{\hbar^2}{2 m r_0^2} \overset{m = \frac{m_p}{2}}{=} \frac{\hbar^2 c^2}{m_p c^2 r_0^2} = 7{,}6\ \mathrm{meV}$$

$$E_r^{HD} = \frac{\hbar^2}{2 m r_0^2} = \left| \frac{1}{m_{HD}} = \frac{1}{m_p} + \frac{1}{2m_p} \Rightarrow m_{HD} = \frac{2}{3} m_p \right| = \frac{3\hbar^2 c^2}{4 m_p c^2 r_0^2} = \frac{3}{4} E_r^{H_2}$$

$$E_r^{D_2} = \frac{1}{2} E_r^{H_2}$$

deg	l	$E_r^{H_2} l(l+1)$	$E_r^{HD} l(l+1)$	$E_r^{D_2} l(l+1)$
1	0	0	0	0
3	1	15 eV	11 eV	7,5 eV
5	2	45 eV	33 eV	22,5 eV
7	3	90 eV	66 eV	45 eV

degeneracija $2l+1$