Valovna enačba

zunanja sila

$$u_{tt} = c^2 u_{xx} + f(x,t)$$

Masa (m) na struni   R.P. $u_1(0,t) = u_2(0,t)$, $m u_{tt} = F\left(\frac{\partial u}{\partial x}\Big|_{0^+} - \frac{\partial u}{\partial x}\Big|_{0^-}\right)$ $\frac{F_y}{F} = \frac{\partial u}{\partial x}$

Val. en. za opno

$$z_{tt} - c^2 \nabla^2 z = 0 \qquad c^2 = \frac{\gamma^2}{\rho} \qquad z_{tt} = \frac{\overset{\frac{dP}{dS}}{P}}{\rho h} + \underbrace{\frac{\gamma}{\rho h}}_{c^2} \nabla^2 z$$

Veza $F_y = \gamma^2 \frac{dz}{dx}$ $F_s = -F_0 \frac{\partial u}{\partial x}$ R.P. $u_{1x} = u_{2x}$ $u_1 = u_2 \big|_{x=0}$

$$\boxed{u(x,t) = \sum_n T_n X_n = \sum (A_n \cos \omega_n t + B_n \sin \omega_n t)(C_n \cos k_n x + D_n \sin k_n x)}$$

Besselova DE

$$z^2 u_{zz} + z u_z + (z^2 - n^2) u = 0$$

$$u(z) = A J_n(z) + B Y_n(z)$$

Legendrova DE

$$(1 - x^2) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + \ell(\ell+1) y = 0$$

$$y(x) = P_n(x)$$

d'Alembertova rešitev (na neskončni struni) $u(x,0)$, $u'(x,0)$

Rešitev $u(x,t) = \frac{1}{2}\left(u(x-ct,0) + u(x+ct,0)\right) + \frac{1}{2c}\int_{x-ct}^{x+ct} u_t(x,0)\, dx$

Difuzijska enačba

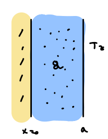
izvori toplote (volumska gostota)

$$\frac{\partial T}{\partial t} = D \nabla^2 T + \frac{q}{\rho c_p} \qquad D = \frac{\lambda}{\rho c_p}$$

izolacija

R.P. ① $\frac{\partial T}{\partial x}\Big|_{x=0} = 0$ ② $j = \sigma T^4 \big|_{x=a} - \sigma T_e^4 = -\lambda \frac{\partial T}{\partial x}\Big|_{x=a}$

okolica peče



Greenova funkcija

nehomogenost

$$u(\vec r, t) = \int_{-\infty}^{t} d\tau \int_D d^3 \vec r_0\, G(\vec r, \vec r_0, t-\tau)\, f(\vec r_0, \tau)$$

začetni pogoj

$$+ \int d^3 \vec r_0\, u_0(\vec r_0)\, G(\vec r, \vec r_0, t)$$

$$- \int_{-\infty}^{t} d\tau \int_{\partial D} g(\vec r_0, \tau) \frac{\partial G}{\partial n}(\vec r, \vec r_0, t-\tau)\, dS$$

robni pogoj

Greenova funkcija za difuzijo v 1D prostoru

$$G(x, x_0, t) = \frac{1}{\sqrt{4\pi D t}}\, e^{-(x-x_0)^2/4Dt}$$

Laplaceova enačba

$$\nabla^2 \varphi = 0$$

Splošna rešitev $\nabla^2 \varphi = 0$ v cilindričnih koordinatah

$$\phi(r, \varphi) = (A_0 + B_0 \ln r)(a_0 + b_0 \varphi) + \sum_{n=1}^{\infty} \left(A_n r^n + B_n \frac{1}{r^n}\right)(C_n \sin n\varphi + D_n \cos n\varphi)$$

Hidrodinamika - Navier-Stokesova enačba

$$\rho \left(\frac{D\vec{v}}{Dt}\right) = \underbrace{\rho \vec{f}}_{\text{gostota sil}} - \underbrace{\nabla p}_{\text{tlak}} + \underbrace{\eta}_{\text{viskoznost}} \nabla^2 \vec{v} + (\underbrace{\zeta}_{\text{volumska viskoznost}} + \eta/3) \nabla(\nabla \vec{v})$$ Ohranitev gibalne količine

$$\frac{D\rho}{Dt} = -\rho \nabla \vec{v}$$ Ohranitev mase

$$\frac{D\gamma}{Dt} = \frac{\partial \gamma}{\partial t} + (\vec{v} \nabla)\gamma$$

Nestisljiva tekočina $\nabla \vec{v} = 0$

Opna

$$\nabla^2 u + \frac{f}{\gamma} = \underbrace{\frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}}_{=0 \text{ stacionarno stanje}}$$

$$f = \frac{dF_g}{dS} = -\underbrace{\frac{dm}{dS}}_{\mu} g = -\mu g$$

μ površinska gostota