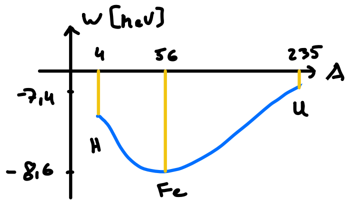
Lastnosti jeder

- masa
- porazdelitev gostote
- porazdelitev gostote naboja
- spin
- magnetni dipolni moment

Masa jedra

$$m(Z,A) = Z m_p + (A-Z) m_n + \frac{W}{c_0^2}$$ vezavna energija $W < 0$

Specifična vezavna energija: $w_0 = \frac{W}{A}$



$m_p = 0{,}9382 \frac{GeV}{c^2}$

$m_n = 0{,}9395 \frac{GeV}{c^2}$

Na nivoju GeV so $w \approx$ konst. → interakcija med nukleoni je kratkega dosega

Bethe-Weizsacker / kapljični model jedra

Predpostavke
- $w \simeq w_0$ konst $\left(\frac{W}{A}\right)$
- kapljice z radijem $r \propto A^{1/3}$
  površina $\propto A^{2/3}$
  rezultat je šibkejša vezava
- odboj protonov: $W \sim \frac{Z^2 e_0^2}{r}$
- št. n^0 in p^+ je podobno
  $Z \approx N$ ali $Z \approx (A-Z)$
- Z, N želimo, da so soda,
  zato, da je spin ↑↓

$$W(Z,A) = -w_0 A + w_1 A^{1/3} + w_2 \frac{Z^2}{A^{1/3}} + w_3 \frac{(2Z-A)^2}{A} + w_4 \frac{\delta(A,Z)}{A^{3/4}}$$

$$\delta(A,Z) = \begin{cases} -1 & \text{sodo – sodo} \\ 0 & \text{liho – sodo} \\ 1 & \text{liho – liho} \end{cases}$$

(št. protonov – št. nevtronov)

$w_0 = 15{,}6$ MeV

$w_1 = 17{,}2$ MeV

$w_2 = 0{,}7$ MeV

$w_3 = 23{,}2$ MeV

$w_4 = 12$ MeV

Od liho-liho obstajajo le $^{2}_{1}H$, $^{10}_{5}B$, $^{6}_{3}Li$, $^{14}_{7}N$

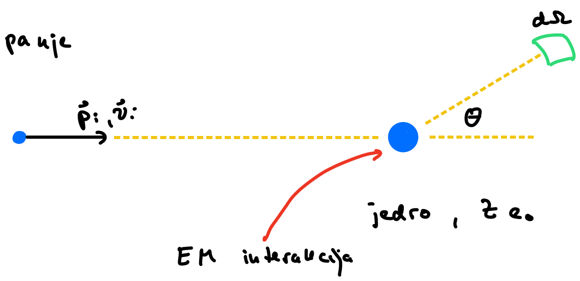
Magična jedra:

p: Z
n: $A-Z$ } 2, 8, 20, 28, 50

ta jedra so močneje vezana in odstopajo od semiempirične masne formule.

Porazdelitev naboja v jedru

Eksperiment: sipanje



$d\Omega = d\varphi \sin\theta \, d\theta$

simetrija po φ

$d\Omega = 2\pi \sin\theta \, d\theta$

Fermijevo zlato pravilo

matrični element $\langle \psi_f | \hat{H}' | \psi_i \rangle$

$\psi_i \to \psi_f$ velja $\quad W_{fi} = \frac{2\pi}{\hbar} |V_{fi}|^2 \rho_f(E_i)$

verjetnost za prehod na časovno enoto $[s^{-1}]$

gostota končnih stanj

Izpeljava $\quad \hat{H} = \hat{H}_0 + \hat{H}'$

motnja, interakcija

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle$$

$$|\psi(t)\rangle = \underbrace{e^{-\frac{i}{\hbar}\int_0^t \hat{H}\, dt}}_{U(t=0,\,t)} |\psi(t=0)\rangle$$

$$\downarrow$$

$$S(t=-\infty,\, t=\infty)$$

Perturbativno: $\quad H_0 |n\rangle = E_n |n\rangle$

Splošna rešitev $\quad |\psi(t)\rangle = \sum_n \underbrace{a_n(t)} e^{-i\frac{E_n}{\hbar}t} |n\rangle$

$P_n(t) = |a_n(t)|^2$

$$i\hbar \frac{\partial}{\partial t} \sum_n a_n(t)\, e^{-i\frac{E_n}{\hbar}t} |n\rangle = (H_0 + H') \sum_n a_n(t)\, e^{-i\frac{E_n}{\hbar}t} |n\rangle \qquad / \cdot \langle k | e^{i\frac{E_k}{\hbar}t}$$

$$i\hbar \frac{\partial a_k(t)}{\partial t} = \sum_n \underbrace{\langle k | H' | n \rangle}_{} a_n(t)\, e^{-i\frac{E_n - E_k}{\hbar}t}$$

Interakcija / prehod med stanji

$$a_k(t) = \frac{1}{i\hbar} \sum_n \int_0^t \langle k | H' | n \rangle a_n(t)\, e^{i\frac{E_k - E_n}{\hbar}t} \, dt$$

nov približek $\qquad$ star približek

Rešimo z iteracijo (Picardova iteracija)

Začetni približek: začetno stanje $n = i$, $\quad a_i(t=0) = 1$

$a_{n \neq i}(t=0) = 0$

$$a_n(t) = \frac{1}{i\hbar} \langle n | H' | i \rangle \left(e^{i\frac{E_n - E_i}{\hbar}t} - 1 \right) \frac{\hbar}{i(E_n - E_i)}$$

$$e^{ix} - 1 = 2i\, e^{i\frac{x}{2}} \left(\frac{e^{\frac{ix}{2}} - e^{-\frac{ix}{2}}}{2i} \right) = e^{i\frac{x}{2}}\, 2i \sin\left(\frac{x}{2}\right)$$

$$|a_n(t)|^2 = |\langle n | H' | i \rangle|^2 \, 4 \sin^2\left(\frac{E_n - E_i}{2\hbar} t \right) \frac{1}{(E_n - E_i)^2}$$

Končna stanja: – diskretna $\quad \sum_{n=f} |a_n(t)|^2 = P_f(t)$

– zvezna

$$P_f(t) = \int_{E_f} \underbrace{\rho(E_f)}_{\text{gostota končnih stanj}} dE_f \underbrace{|\langle f|H'|i\rangle|^2}_{V_{fi}} \frac{\sin^2\left(\frac{E_f-E_i}{2\hbar}t\right)}{(E_f-E_i)^2}\, 4$$

$$\int \underbrace{\frac{\sin^2 \alpha z}{z^2}}_{\sim \pi \alpha \delta(z)} dz = \pi \alpha$$

$z = E_f - E_i \qquad dz = dE_f \qquad \alpha = \frac{t}{2\hbar}$

$z \to 0$

$\downarrow$

$E_f = E_i$

elastično sipanje

$$P_f(t) = 4|V_{fi}|^2 \rho(E_f) \frac{\pi t}{2\hbar}$$

$$W_{fi} = \frac{dP}{dt} = \frac{2\pi}{\hbar}|V_{fi}|^2 \rho(E_f)$$

Gostota končnih stanj

$$dN_f = \frac{d^3r\, d^3p}{h^3} = V_n \frac{d^3p}{h^3}$$

nerelativistično

normalizacijski volumen

$$d^3p = p^2\, dp\, d\Omega$$

$$d\rho_f(E_f) = \frac{dN_f}{dE_f} = \frac{V_n}{h^3}\frac{p_f^2\, dp_f\, d\Omega}{dE_f} \, : \qquad E_f = \frac{p_f^2}{2m} \qquad dE_f = \frac{p_f\, dp_f}{m}$$

$$\frac{d\rho_f}{d\Omega} = \frac{V_n m}{h^3} p_f = \frac{V_n p_f m}{(2\pi\hbar)^3}$$

za elastično, nerelativistično sipanje velja

$p_f = p_i = m v_i \qquad E_f = E_i$

$$= V_n \frac{m^2 v_i}{(2\pi\hbar)^3}$$

Merljive količine pri sipanju

mera za verjetnost reakcije

$\frac{d\sigma}{d\Omega}$ – diferencialni sipalni presek po prostorskem kotu

$[b = 10^{-28} m^2 = 100\, fm^2]$

$$\frac{dN_f}{dt} = L\sigma$$

luminoznost (lastnost eksperimenta)

Integrirana luminoznost $\quad \int L\, dt \,, \quad L_{int}$

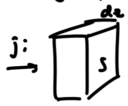
$$N_f = L_{int}\sigma$$

$$dN_f = N_i N_t \frac{\sigma}{S_0}$$

$$dN_f = N_i n_t \sigma\, dz$$

v času t

$$\xrightarrow{\frac{d}{dt}} \quad \frac{dN_f}{dt} = \underbrace{\frac{dN_i}{dt} n_t\, dz}_{L}\, \sigma$$



$$N_i = j_i S t$$

$$\frac{dN_f}{d\Omega} = N_i n_t \frac{d\sigma}{d\Omega} dz$$

$$W_{fi} = \frac{dN_f}{N_t\, t}$$

po definiciji

verjetnost za potek reakcije

$$\frac{d\sigma}{d\Omega} = \frac{dW_{fi}}{d\Omega}\frac{1}{j_i}$$

Matrični element

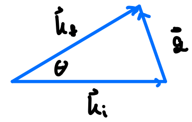
$$V_{fi} = \int_{V_p} \psi_f^*(\vec r)\, \overset{H'}{V(\vec r)}\, \psi_i(\vec r)\, d^3r$$

$$\psi_i = e^{i\vec k_i\cdot\vec r}\,\frac{1}{\sqrt{V_n}} \quad ; \quad \vec k_i = \frac{\vec p_i}{\hbar} \qquad \rho_i = |\psi_i|^2 = \frac{1}{V_n}$$

$$\psi_f = e^{i\vec k_f\cdot\vec r}\,\frac{1}{\sqrt{V_n}}$$

$$V_{fi} = \frac{1}{V_n}\int_{V_n} e^{-i\vec k_f\cdot\vec r}\, V(r)\, e^{i\vec k_i\cdot\vec r}\, d^3r$$

$$= \frac{1}{V_n}\int_{V_n} e^{i\vec q\cdot\vec r}\, V(\vec r)\, d^3r$$



$$|\vec k_i| = |\vec k_f|$$

$$V(\vec r) = \underset{\text{naboj izstrelka}}{e}\; \underset{\text{EM potencial tarče}}{U(\vec r)}$$

Greenova formula

$$\int_V u\nabla^2 v - v\nabla^2 u \; d^3r = \oint_{\partial V} u\nabla v - v\nabla u \; d\vec S$$

$$u = e^{i\vec q\cdot\vec r} \qquad v = e\,U(\vec r) \qquad \nabla^2 U = -\frac{\rho_e}{\varepsilon_0}$$

Tarča (jedro) $\int_V \rho_e \, dV = Z e_0$

daleč stran sta proti ničli

$$\frac{1}{V_n}\int_V (-q^2)\, e^{i\vec q\cdot\vec r}\, V \; d^3r - \frac{e}{V_n}\int e^{i\vec q\cdot\vec r}\,\underbrace{\nabla^2 U}_{-\rho_e/\varepsilon_0}\; d^3r = 0$$

$$V_{fi} = \frac{1}{V_n}\int e^{i\vec q\cdot\vec r}\, V(\vec r)\, d^3r = \frac{e}{\varepsilon_0 q^2}\,\frac{1}{V_n}\int_{V_n} e^{i\vec q\cdot\vec r}\,\rho(\vec r)\, d^3r$$

$$V_{fi} = \frac{e}{\varepsilon_0 q}\,\frac{1}{V_n}\underbrace{\int_{V_n} e^{i\vec q\cdot\vec r}\,\rho(\vec r)\, d^3r}_{F(\vec q)}$$

$F(\vec q)$ oblikovni faktor

$$\frac{d\sigma}{d\Omega} = \frac{dw_{fi}/d\Omega}{\rho_i\, v_i} \underset{V_n = \frac{1}{\rho_i}}{=} \frac{2\pi}{\hbar}|V_{fi}|^2\,\frac{d\rho_f}{d\Omega}\,\frac{V_n}{v_i} = \frac{2\pi}{\hbar}|V_{fi}|^2\, V_n\,\frac{m^2 v_f}{(2\pi\hbar)^3}\,\frac{V_n}{v_i} = \frac{2\pi}{\hbar}|V_{fi}|^2\, V_n^2\,\frac{m^2}{(2\pi\hbar)^3} =$$

$$\frac{d\sigma}{d\Omega} = \frac{2\pi}{\hbar}\left(\frac{m e}{\varepsilon_0 q^2}\right)^2 \frac{1}{(2\pi\hbar)^3}\,|F(\vec q)|^2$$

$$= \underbrace{\left(\frac{m e}{8\pi\varepsilon_0 p_i^2}\right)^2 \frac{1}{\sin^4\theta/2}}_{\text{Rutherford}}\; \underbrace{|F(\vec q)|^2}_{\text{približek}}$$

Jedrski razpadi

Razpad α

$$^A_Z X \longrightarrow {}^{A-4}_{Z-2} Y + \underbrace{{}^4_2 He}_{\alpha}$$

Kinematika: $T_\alpha + T_Y = \underbrace{(m_X - m_Y - m_\alpha)c^2}_{\text{semiemp. masna formula}}$

↳ $T_\alpha \geq 0$

⇒ Razpadi možni za približno $A > 150$

Eksperiment pokaže, da so α razpadi možni za $A > 200$ (svinec +)

Eksperimentalni fit na podatke

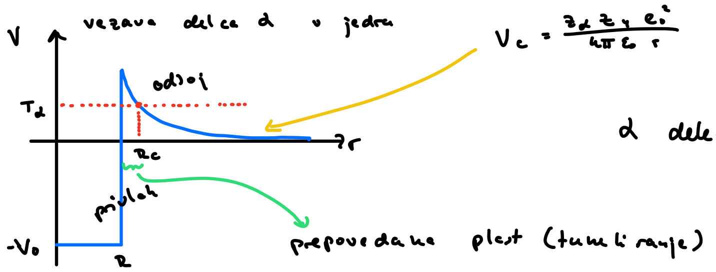
Geiger - Nuttallov zakon $\qquad t_{1/2} = \ln 2 \; \tau$

$$\log t_{1/2} = \frac{C_1}{\sqrt{T_\alpha}} + C_2 \qquad \tau \in [10^{-10}\, s, 10^{3}\, s]$$

↳ kinetična energija α delca

Model tuneliranja delca α

Red velikosti:
$R \sim 10$ fm
$T_\alpha \sim 5 - 10$ MeV



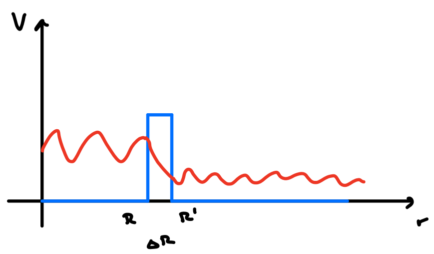
$$V_c = \frac{Z_\alpha Z_Y e_0^2}{4\pi\varepsilon_0 r}$$

~konst (sile kratkega dosega)

α delec je prost pri $r > R_c$

$$T_\alpha = \frac{e_0^2 Z_\alpha Z_Y}{4\pi\varepsilon_0 R_c}$$

Skica tuneliranja



Propustnost

$$P = \left(\frac{4kK}{k^2+K^2}\right)^2 e^{-2K\Delta R}$$

$$k^2 = \frac{2\mu}{\hbar} T_\alpha \qquad K^2 = \frac{2\mu}{\hbar^2}\left(V_c(R) - T_\alpha\right)$$

μ ... reducirana masa

$$\frac{1}{\mu} = \frac{1}{m_\alpha} + \frac{1}{m_Y}$$

Za majhen ΔR v R_i: (razrezano potencial)

$P_i \sim e^{-2K\Delta R}$

$$P_T = \prod P_i = \exp \sum -2K(R_i)\Delta R = e^{-2G}$$

Gamow faktor G

$$G = \int_R^{R_c} \kappa(r)\,dr = \int_R^{R_c} \sqrt{\frac{2\mu}{\hbar^2}\left(V_c(r) - T_\alpha\right)}\,dr \longrightarrow G \sim \frac{1}{\sqrt{T_\alpha}} \cdot Z$$

za $R \ll R_c$ (debele bariere)

Razpadi β

β^- : ${}^A_Z X \to {}^A_{Z+1} Y + e^- + \bar{\nu}_e$ oz. $n^0 \to p^+ e^- \bar{\nu}_e$

mogoč proces za prost n^0, $\tau = 890\,s$

β^+ : ${}^A_Z X \to {}^A_{Z-1} Y + e^+ + \nu_e$ oz. $p^+ \to n^0 e^+ \nu_e$

$\tau > 10^{36}$ let

Iz kinematike

β^- $\quad (m(Z,A) - m(Z+1,A) - m_e)\,c^2 \geq 0$

$$|W(Z+1,A)| - |W(Z,A)| \geq -0{,}78\,\mathrm{MeV}$$

β^+ $\quad |W(Z-1,A)| - |W(Z,A)| \geq 1{,}79\,\mathrm{MeV}$

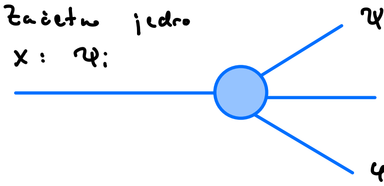
Verjetnost za razpad na časovno enoto

$$W_{fi} = \frac{1}{\tau} = \lambda = \frac{2\pi}{\hbar}|V_{fi}|^2 \rho_f(E_i)$$

Predpostavimo $\quad V_{fi} = G_F \int_V \psi_f^* \psi_e^* \psi_\nu^* \psi_i \, d^3r$

Začetno jedro

X : ψ_i



ψ_f : Y

ψ_e : e^-

ψ_ν : $\bar{\nu}_e$

splošen izraz

$V_{fi} = \langle \psi_f \psi_e \psi_\nu | H_{int} | \psi_i \rangle$

$H_{int} = G_F$

sklopitvena konstanta

točkovna interakcija

- $e, \nu \to \psi_e, \psi_\nu \to$ ravni valovi
  - ↳ nerelativistična K.M.
  - ↳ relativistična kinematika

$\hbar c = 197$ eVnm

$= 197$ MeV fm

$$\psi_e = \frac{1}{\sqrt{V_N}} e^{i\vec{k}_e\cdot\vec{r}} \qquad \psi_\nu = \frac{1}{\sqrt{V_N}} e^{i\vec{k}_\nu\cdot\vec{r}}$$

$$\psi_e^* \psi_\nu^* = \frac{1}{V_N} e^{-i\vec{k}\cdot\vec{r}} \qquad \vec{k} = \vec{k}_e + \vec{k}_\nu \qquad k_e = \frac{\sqrt{E_e^2 - m_e^2 c^4}}{\hbar c} = \frac{p}{\hbar}$$

$$V_{fi} = \frac{G_F}{V_N} \int_{V_N} \psi_f^* \psi_i \, e^{-i\vec{k}\cdot\vec{r}} \, d^3r$$

multipolni razvoj $\quad k \cdot r < 1$

$$= \frac{1\,\mathrm{MeV}}{197\,\mathrm{MeV\,fm}} \cdot 10\,\mathrm{fm} \approx 0{,}1$$

$$V_{fi} = \frac{G_F}{V_N} \int \psi_f^* \psi_i \left(1 - i\vec{k}\vec{r} + \frac{(i\vec{k}\vec{r})^2}{2!} - \dots\right) dV =$$

$$= \frac{G_F}{V_N}\left(\underbrace{\int \psi_f^* \psi_i \, dV}_{} - i \underbrace{\int \psi_f^* \psi_i \, \vec{k}\cdot\vec{r} \, dV}_{} + \dots \right)$$

$\neq 0$ dovoljeni razpad *

če prvi člen $= 0$ in tak $\neq 0$ je to enkrat prepovedan razpad *

Členi $\frac{(\vec{k}\cdot\vec{r})^\ell}{\ell!}$ ustrezajo izsevanju para $e\,\nu$ z vrtilno količino $L = \hbar\sqrt{\ell(\ell+1)}$ $\quad \ell = 0, 1, \dots$

* $\ell = 0 \quad Y \sim \frac{1}{\sqrt{4\pi}}$ izotropen

* $\ell \geq 1$

Te integrali so $\sim$ konst. (neodvisni od $E_e, E_\nu, \dots$)

Na primer dovoljen razpad ($\ell = 0$):

$$V_{fi} = \frac{G_F}{V_N} M_{fi} \qquad M_{fi} = \int \psi_f^* \psi_i \, dV$$

Splošno: fazni prostor N delcev v končnem stanju

$$\int \frac{d^3 p_1}{(2\pi\hbar)^3} \cdots \frac{d^3 p_N}{(2\pi\hbar)^3} \, \delta^{(3)}\Big(\vec{p} - \sum_{i=1}^{N} \vec{p}_i\Big) = \dots$$

začetna gib. kol.

omeji gibalno količino

$$= \delta^{(3)}\Big(\vec{p}_N - (\vec{p} - \sum^{N-1} \vec{p}_i)\Big)$$

$$\dots = \int \frac{d^3 p_1}{(2\pi\hbar)^3} \cdots \frac{d^3 p_{N-1}}{(2\pi\hbar)^3} \qquad \vec{p}_N = \vec{p} - \sum^{N-1} \vec{p}_i$$

Za naš primer $N = 3 \rightarrow \int \frac{d^3 p_e}{(2\pi\hbar)^3} \frac{d^3 p_\nu}{(2\pi\hbar)^3}$

Točno za ($\ell = 0$) dovoljeni razpad

$$dN_f = V_N^2 \frac{d^3 p_e \, d^3 p_\nu}{(2\pi\hbar)^6}$$

Predpostavimo izotropnost

$$dN_f = V_N^2 \frac{p_e^2 dp_e \, p_\nu^2 dp_\nu}{(2\pi\hbar)^6} (4\pi)^2$$

Dodamo še omejitev energije (povežemo p_e in p_ν)

$$E_i = m(Z,A)c^2 = E_f = m(Z-1,A)c^2 + \underbrace{E_e + E_\nu}_{E = \Delta m c^2}$$

$$dE_i = dE_f = dE$$

$$E_e^2 = p_e^2 c^2 + m_e^2 c^4 \qquad \Rightarrow \qquad p_\nu = \frac{E - E_e}{c} \qquad dp_\nu = \frac{dE}{c}$$

$$E_\nu = p_\nu c \quad (m_\nu \approx 0)$$

$$dN_f \sim p_e^2 dp_e (E - E_e)^2 dE$$

$$\hookrightarrow d\rho_f(E) = \frac{dN_f}{dE} \sim p_e^2 dp_e (E - E_e)^2 \Rightarrow \frac{d\rho_f}{dp_e} = p_e^2 (E - E_e)^2$$

merljivo $\rightarrow$ beta spekter

Točneje

$$\frac{d\rho_f}{dp_e} = \frac{16\pi^2 V_f^2}{(2\pi\hbar)^6 c^3} p_e^2 (E-E_e)^2$$

$$\frac{dW_{fi}}{dp_e} = \frac{d\lambda}{dp_e} = \frac{1}{2\pi^3}\frac{G_F^2 |M_{fi}|^2}{\hbar^7 c^4} p_e^2 (E-E_e)^2 \qquad \text{za } \ell = 0$$

brezdim. količine $\quad w = \frac{E_e}{m_e c^2} \quad p = \frac{p_e}{m_e c^2} \quad p = \sqrt{w^2-1} \quad w_0 = \frac{E}{m_e c^2}$

$$\Rightarrow \frac{d\lambda}{dp} = \frac{1}{2\pi^3} G_F^2 |M_{fi}|^2 \frac{m_e^5 c^4}{\hbar^7} p^2 (w_0 - w)^2$$

$$\Rightarrow \lambda \propto E_0^5$$

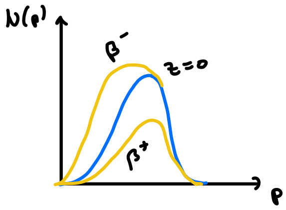
Primerjava s podatki

$$N(p) = \frac{d\lambda}{dp} \propto p^2 (W_0 - W)^2 \qquad \text{brezdim.}$$

Rišemo $\quad \left(\frac{N(p)}{p^2}\right)^{1/2} \propto W_0 - W \propto E_0 - E_e$

Dodamo še Fermijev faktor $F(Z,p)$ (coulomb correction)

↳ $G_F \to G_F\, F(Z,p)$



$$\lambda = \frac{1}{\tau} = \frac{1}{2\pi^3} G_{F_i} |M_{fi}|^2 \frac{m_e^5 c^4}{\hbar^7} f(Z,p)$$

$$f(Z,p) = \int_0^{p_0} p^2 F(Z,p) (w_0 - w)^2 \, dp$$

Rišemo $\quad \left(\frac{N(p)}{p^2 F(Z,p)}\right)^{1/2} \propto (W_0 - W)$

korekcija

Izbirna pravila za β razpade

- vrtilna kol. jeder se ohranja: $\vec{J}_i = \vec{J}_f + \vec{J}_{e\nu}$

sistem eν

$\vec{J}_{e\nu} = \vec{\ell} + \vec{s}$

kvantne št. $\quad s_e = \frac{1}{2} \quad s_\nu = \frac{1}{2}$

$S_{e\nu} = 0, 1$

- kategorije: $\quad S_{e\nu} = 0 \quad$ singletno stanje $\quad$ Fermijevi razpadi

$\quad S_{e\nu} = 1 \quad$ tripletno stanje $\quad$ Gamow-Teller razpadi

Fermi: $\quad J_{e\nu} = \ell$

GT: $\quad J_{e\nu} = \ell \pm 1, 0$

Parnost stanj se ohranja ? $\quad \hat{P}\psi_i(\vec{r}) = P_i \psi_i(\vec{r})$

$\hat{P}\psi_f(\vec{r}) = P_f \psi_f(\vec{r}) \qquad P_i = P_f \cdot \overset{(-1)^\ell}{\overset{\parallel}{P_{e\nu}}}$

Knjigovodstvo :

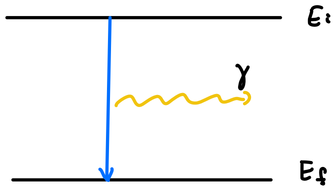
- $\Delta l = 0$ ni spremembe parnosti
  - $\Delta J = 0$ (F)
  - $\Delta J = 0, \pm 1$ (GT)

  } dovoljeni prehodi

- $\Delta l = 1$ sprememba parnosti
  - $\Delta J = 0, \pm 1$ (F)
  - $\Delta J = 0, \pm 1, \pm 2$ (GT)

  } prvi prepovedan prehod

Gama razpadi



$$w_e = \frac{1}{\tau} = \frac{\omega^3 |\langle f | \vec{p}_e | i \rangle|^2}{3\pi \varepsilon_0 c^3 \hbar} \qquad \omega = \frac{\Delta E}{\hbar} = \frac{E_i - E_f}{\hbar}$$

verjetnost za prehod / čas

$\vec{p}_e = e\vec{r}$

električno dipolno sevanje

Operator el. dip. $\vec{p}_e$

Magnetno dipolno sevanje

$$w_m = \frac{1}{\tau} = \frac{\mu_0 \omega^3 |\langle f | \vec{p}_m | i \rangle|^2}{3\pi c^3 \hbar}$$

$$\frac{w_m}{w_e} = \underbrace{\mu_0 \varepsilon_0}_{\frac{1}{c_0^2}} \frac{|\langle \vec{p}_m \rangle|^2}{|\langle \vec{p}_e \rangle|^2} \Rightarrow w_m \ll w_e$$

Ocena

$\vec{p}_e = e\vec{r}$: $\langle p_e \rangle \sim e R_j$

$p_m = \dots$: $\langle p_m \rangle \sim \mu_N$

nukleon $\nearrow$ $\mu_N = \frac{e_0 \hbar}{2 m_N}$

$$\frac{w_m}{w_e} \sim \frac{\mu_N^2}{e^2 R^2} = \frac{(\hbar c)^2}{(2 m_n c^2 R)^2} \sim 10^{-3} - 10^{-4}$$

Natančneje:

$$\langle f | \vec{p}_e | i \rangle = \int \psi_f^* \vec{p}_e \psi_i \, dV \qquad \vec{p}_e = e\vec{r}$$

$$\langle f | \vec{p}_m | i \rangle = \int \psi_f^* \vec{p}_m \psi_i \, dV \qquad \vec{p}_m = (g_l \vec{l} + g_s \vec{s}) \mu_N$$

$$g_l = \begin{cases} 1 & \text{proton} \\ 0 & \text{nevtron} \end{cases} \qquad g_s = \begin{cases} 5,6 & p^+ \\ -3,8 & n^0 \end{cases}$$

Tudi pri sevanju γ dobimo multipolne prehode

↪ Splošni izraz $\langle \vec{p} \rangle \to V_{fi} = \int \psi_f^* \left(\sum_l \hat{O}_l^{(m)} + \hat{O}_l^{(e)} \right) \psi_i \, dV$

↳ el. multipolni operator

l ... red multipola, rang tenzorja kot sferični ireducibilni tenzor

Sevanje γ - izpeljava

$$H = \frac{(\vec{p} - e\vec{A})^2}{2m} + eV \qquad \text{umeritev} \quad \nabla\vec{A} = 0$$

$$V = V_0(\vec{r}) + \varphi(\vec{r})$$

notranji stacionarni potencial — zunanji potencial (skupaj z $\vec{A}$)

$$H = \frac{\vec{p}^2 - 2e\vec{A}\vec{p} + e^2\vec{A}^2}{2m} + eV_0 + e\varphi$$

Pazi: $\vec{p}\cdot\vec{A} + \vec{A}\cdot\vec{p} = 2\vec{A}\cdot\vec{p} + \underbrace{[\vec{A},\vec{p}]}_{=0}$

$$\vec{p}\vec{A}\psi = -i\hbar\left(\underbrace{(\nabla\vec{A})}_{=0}\psi + \vec{A}\nabla\psi\right) = \vec{A}\vec{p}\psi$$

Uporabimo za opis interakcije z zun. EM valovanjem

Linearno polarizirano monokromatski ravni val

$$\varphi = 0$$

$$\vec{A} = A_0\vec{\varepsilon}\cos(\vec{k}\cdot\vec{r} - \omega t)$$

amplituda (A_0), $\vec{\varepsilon}$ polarizacija

A_0 majhen

$$\vec{E} = -\nabla\varphi - \frac{\partial\vec{A}}{\partial t} \qquad \vec{B} = \nabla\times\vec{A}$$

$$\vec{E} = -A_0\omega\vec{\varepsilon}\sin(\vec{k}\cdot\vec{r} - \omega t) \qquad \vec{B} = 2A_0\frac{\omega}{c}\underbrace{\left(\frac{\vec{k}}{k}\times\vec{\varepsilon}\right)}_{\vec{b}}\sin(\vec{k}\cdot\vec{r} - \omega t)$$

$$H = H_0 + H_{int} \qquad H_0 = \frac{p^2}{2m} + eV_0 \qquad H_{int} = -\underbrace{\frac{e\vec{A}\cdot\vec{p}}{m}}_{\sim A_0} + \underbrace{\cancel{\frac{e^2A^2}{2m}}}_{\sim A_0^2}$$

Zapišimo $\cos(\,)$ v $\mathbb{C}$ $\quad \cos z = \frac{e^{iz} + e^{-iz}}{2}$

$$H_{int} = -\frac{eA_0\,\vec{\varepsilon}\cdot\vec{p}}{2m}\left(\exp(i\vec{k}\vec{r} - i\omega t) + \exp(-i\vec{k}\cdot\vec{r} + i\omega t)\right)$$

$$= V e^{i\omega t} + V^{+} e^{-i\omega t} \qquad V^{+} = -\frac{eA_0\,\vec{\varepsilon}\cdot\vec{p}}{2m}e^{i\vec{k}\cdot\vec{r}}$$

Izpeljava zlatega pravila

$$i\hbar\, a_f(t) = \int_0^t \langle f|H_{int}|i\rangle e^{i\omega_{fi}t'}dt' \qquad \omega_{fi} = \frac{E_f - E_i}{\hbar}$$

$$a_f(t) = -\frac{i}{\hbar}\int_0^t \left(V_{fi}e^{i\omega t'} + V_{fi}^{+}e^{-i\omega t'}\right)e^{i\omega_{fi}t'}dt'$$

$$V_{fi} = \langle f|V|i\rangle$$
$$V_{fi}^{+} = \langle f|V^{+}|i\rangle = \langle i|V|f\rangle^*$$

$$\operatorname{sinc} z = \frac{\sin z}{z}$$

$$a_f(t) = -\frac{it}{\hbar}\Big(V_{fi}\exp\left(i\frac{(\omega+\omega_{fi})}{2}\right)\operatorname{sinc}\left(\frac{(\omega+\omega_{fi})t}{2}\right)$$
$$+ V_{fi}\exp\left(-i\frac{(\omega-\omega_{fi})}{2}\right)\operatorname{sinc}\left(\frac{(\omega-\omega_{fi})t}{2}\right)\Big)$$

Vrh pri $\omega + \omega_{fi} = 0$

$\omega = -\omega_{fi}$ $\omega = \omega_{fi}$

$\hbar\omega = E_i - E_f$ $E_f = E_i + \hbar\omega$

$E_i = E_f + \underbrace{\hbar\omega}$ absorpcija fotona

energija fotona

izsevanje foton

Zanima nas le sevanje.

$$|a_f|^2_{sevanje} = \frac{t^2}{\hbar^2}\frac{e^2|A_0|^2}{4m}\left|\langle f|\vec{\varepsilon}\cdot\vec{p}\, e^{-i\vec{k}\cdot\vec{r}}|i\rangle\right|^2 \operatorname{sinc}^2\left(\frac{(\omega+\omega_{fi})t}{2}\right)$$

$$P_{i\to f}(t) = \int |a_f|^2\, \rho(\omega)\, d\omega$$

Gostota valovanja $w_e = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\varepsilon_0\, \omega^2\, |A_0|^2$

$$P_{i\to f}(t) = \int_\omega \frac{t^2 e^2 \overbrace{w_e\, \rho(\omega)}^{\rho_e(\omega)}}{2\varepsilon_0 \hbar^2 m^2 \omega^2} |\langle f|\ldots|i\rangle|^2 \operatorname{sinc}^2(\ldots)\, d\omega$$

$$\rho_E(\omega) = \underbrace{\frac{\hbar\omega^3}{\pi^2c^3}}_{\text{gostota energije}} \frac{1}{\underbrace{e^{\hbar\omega/k_0T}-1}_{\text{zasedbeno število}}}$$ sevanje črnega telesa

$$\int \operatorname{sinc}^2 \alpha z\, dz = \int \frac{\pi}{\alpha}\delta(z)$$

Za naš primer $\frac{1}{\alpha} = \frac{2}{t}$ $z = \omega + \omega_{fi}$

vrh pri $\omega = -\omega_{fi} = \omega_\gamma$

$$\frac{1}{\tau} = \frac{P_{i\to f}}{t} = \frac{e^2\omega_\gamma}{\pi\varepsilon_0\hbar c^3 m^2}|\langle f|\underbrace{\vec{\varepsilon}\cdot\vec{p}\, e^{-i\vec{k}\cdot\vec{r}}}_{\text{multipolni razvoj}}|i\rangle|^2 \qquad |\vec{k}| = \frac{\omega_\gamma}{c}$$

$$e^{-i\vec{k}\cdot\vec{r}} = 1 - i\vec{k}\cdot\vec{r} + \frac{1}{2}(\vec{k}\cdot\vec{r})^2 + \ldots + \frac{(-i\vec{k}\cdot\vec{r})^n}{n!}$$

Vrtilna količina vsakega člena l = 1, 2, 3, ..., n+1

dipol, kvadrupol, oktopol

El $E1$ $E2$ $E3$

Električna dipolna aproksimacija

$$\langle f|\vec{\varepsilon}\cdot\vec{p}|i\rangle = \vec{\varepsilon}\cdot\langle f|\vec{p}|i\rangle =$$

$$[\vec{r}, H_0] = \frac{i\hbar\vec{p}}{m}$$

$$\langle f|\vec{r}H_0|i\rangle - \langle f|H_0\vec{r}|i\rangle = \langle f|\vec{r}|i\rangle(E_i - E_f)$$

$$\langle f|\vec{p}|i\rangle = -im\omega_\gamma\langle f|\vec{r}|i\rangle$$

Uporabimo še e iz polja izraza $\vec{p}_e = e\vec{r}$

$$\frac{1}{\tau_{dip}} = \frac{\omega_\gamma^3}{\pi \varepsilon_0 c^3 \hbar} \left| \vec{\varepsilon} \cdot \langle f | \vec{p}_e | i \rangle \right|^2$$

Povprečje po polarizaciji $\hat{\varepsilon}$

$\vec{k} = (0, 0, k)$

$\vec{p}_e = (p_e \sin\theta,\ 0,\ p_e \cos\theta)$

$\varepsilon = (\cos\varphi,\ \sin\varphi,\ 0)$

$\left| \hat{\varepsilon} \cdot \vec{p}_e \right|^2 = p_e^2 \sin^2\theta \cos^2\varphi$

Povprečje $\frac{1}{4\pi} \int d\Omega$

$\Rightarrow \frac{p_e^2}{3}$

$$\Rightarrow \quad \frac{1}{\tau_{dip}} = \frac{\omega^3}{3\pi \varepsilon_0 c^3 \hbar} \underbrace{\left| \langle f | \vec{p}_e | i \rangle \right|^2}_{p_e^2}$$

Magnetno dipolno sevanje

Naš račun ni upošteval $\vec{\mu} = \vec{p}_m$ sistema.

Spustili smo člen $\delta H = H_m = -\vec{p}_m \cdot \vec{B} = -\vec{\mu} \cdot \vec{B}$

$$\vec{B} = \nabla \times \vec{A} = \frac{2A_0 \omega}{c} \vec{b} \sin(\vec{k} \cdot \vec{r} - \omega t)$$

$$H_m = V_m e^{i\omega t} + V_m^\dagger e^{-i\omega t}$$

$$V_m = -\frac{i\omega A_0}{c} e^{-i\vec{k} \cdot \vec{r}}\, \vec{b} \cdot \vec{\mu} \qquad \text{razlika z } V \text{ je } \frac{1}{c}$$

$$\frac{1}{\tau_{dip}^m} = \frac{\omega_\gamma^3}{3\pi \varepsilon_0 c^3} \frac{1}{c^2} \left| \langle f | \vec{\mu} | i \rangle \right|^2$$

Izbirna pravila

$$\hat{p}_e \xrightarrow[\text{parnost}]{\hat{P}} -\hat{p}_e$$

$$\vec{\mu} \xrightarrow{\hat{P}} \hat{\mu}$$

$\langle f | \hat{p}_e | i \rangle$ zač. in končno stanje imata nasprotno parnost, da se $\langle\ \rangle$ ohrani pri parnosti.

E1 prehod: jedro spremeni parnost

$\langle f | \mu | i \rangle$ enaka parnost za mag. dip. prehod

M1 prehod: jedro ne spremeni parnosti

$E\ell$: parnost $(-1)^\ell$

$M\ell$: parnost $(-1)^{\ell+1}$

Jedro $|j - j'| \le \ell$

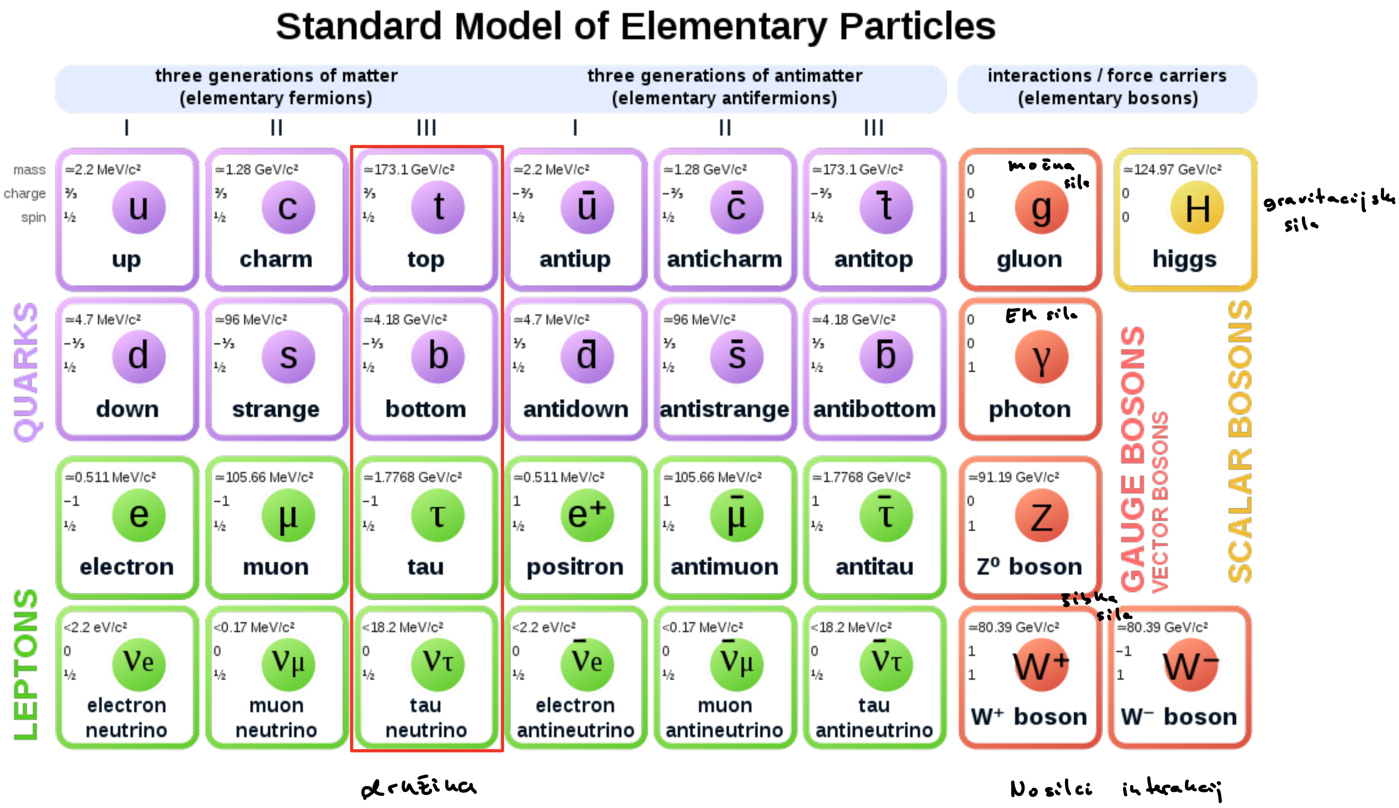
Prepovedan $j = j' = 0$ nemogoč

Hitrost razpada ($\frac{1}{\tau}$)

verjetnost razpada

	1	10^{-3}	10^{-6}	10^{-9}	
	E1	E2	E3	E4	
		M1	M2	M3	
Parnost	-	+	-	+	
J^P	1^-	2^+	3^-	4^+	za E
		1^+	2^-	3^+	za M

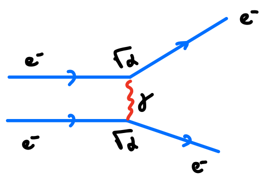
Osnovni delci



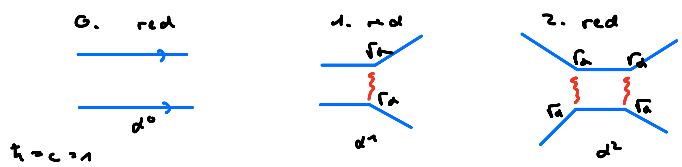
Nosilci interakcij
$S = 1$ (bozon)
razen Higgs $S=0$

Feynmanovi diagrami

EM sipanje



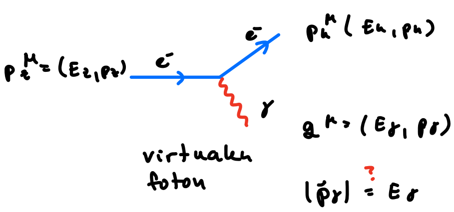
I. red - izmenjava fotona



$\hbar = c = 1$

$$|V_{fi}|^2 \sim \frac{e^2}{4\pi\varepsilon_0} = \alpha\hbar c \qquad \alpha = \alpha_{EM} = \frac{1}{137}$$

V vseh vertexih pripišemo $\sqrt{\alpha}$. Za matrični element vertexa zmnožimo



$$p_n^2 = p_n^\mu p_{n\mu} = E_n^2 - p_n^2 = m_e^2$$

$$p_n^2 = m_e^2$$

na masni lupini - realni delci

$$|\vec{p}_\gamma| \overset{?}{=} E_\gamma$$

$$q^2 \overset{?}{=} m_\gamma \sim 0 \quad \Rightarrow \quad q^2 = Q^2$$

Virtualni foton si izposodi maso iz vakuuma

$$\Delta E \, \tau \geq \hbar$$
$$\sqrt{|Q^2|}\, \tau \geq \hbar \qquad \tau \ldots \text{živ. čas virtualnega delca}$$

Valovna enačba za relativistične delce

Klasično
$$\hat{H} = \hat{E} = -i\hbar \frac{\partial}{\partial t}$$
$$\hat{H} = \hat{E} = \hat{T} + \hat{V} \qquad \hat{T} = \frac{p^2}{2m} = -\frac{\hbar^2}{2m}\nabla^2 \qquad \hat{p} = -i\hbar \nabla$$

Relativistično
$$E^2 = p^2 + m^2$$
$$\hat{E}^2 = \hat{p}^2 + \hat{m}^2$$
$$-\frac{\partial^2}{\partial t^2}\psi = -\nabla^2\psi + m^2\psi \qquad \text{Klein-Gordonova enačba}$$

za $m=0$ $\quad \left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\psi = 0 \quad$ valovna enačba ✓

EM valovanje, fotoni

v 3D $\nabla^2 = \Delta$ $\qquad$ v 4D $\frac{\partial^2}{\partial t^2} - \nabla^2 = \Box$

$$\partial_\mu = \left(\frac{\partial}{\partial t}, \nabla\right) \qquad \partial_\mu \partial^\mu = \frac{\partial^2}{\partial t^2} - \nabla^2$$

Poenostavljena Klein-Gordonova enačba

$$(\Box + m^2)\psi = 0$$

$\Rightarrow$ Diracova enačba, ki upošteva tudi spin.

Ker smo v relativnosti: $E^2 = m^2 + p^2 \Rightarrow E = \pm\sqrt{p^2+m^2}$

K-G enačba ima težave z negativnimi energijami

Kontinuitetna enačba $\quad \frac{\partial \rho}{\partial t} + \nabla \vec{j} = 0$

Schrödinger: verjetnostna gostota $\rho = |\psi|^2$

Iščemo $\frac{\partial \rho}{\partial t}$

$$i\frac{\partial \psi}{\partial t} = -\frac{1}{2m}\nabla^2\psi \qquad V=0 \qquad / \cdot -i\psi^*$$
$$\psi^*\frac{\partial \psi}{\partial t} = \frac{i}{2m}\psi^*\nabla^2\psi$$

c.c. $\quad \psi\frac{\partial \psi^*}{\partial t} = -\frac{i}{2m}\psi\nabla^2\psi^*$ $\quad +$

$$\psi^*\frac{\partial \psi}{\partial t} + \psi\frac{\partial \psi^*}{\partial t} = \frac{i}{2m}\left(\psi^*\nabla^2\psi - \psi\nabla^2\psi^*\right)$$

$$\frac{\partial}{\partial t}\psi\psi^* = \frac{\partial \rho}{\partial t} = -\frac{i}{2m}\nabla\left(\psi\nabla\psi^* - \psi^*\nabla\psi\right)$$

$$\Rightarrow \vec{j} = \frac{i}{2m}\left(\psi\nabla\psi^* - \psi^*\nabla\psi\right) \qquad \text{verjetnostni tok}$$

$$\psi \in \mathbb{R} \Leftrightarrow \vec{j} = 0 \qquad \text{vezana stanja}$$

Kontinuitetna enačba za K-G enačbo

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2} - m^2\right)\psi = 0 \qquad / -i\psi^*\cdot$$

$$-i\,\psi^*\nabla^2\psi + i\psi^*\frac{\partial^2\psi}{\partial t^2} + im^2\psi^*\psi = 0$$

c.c. $$i\,\psi\nabla^2\psi^* - i\psi\frac{\partial^2\psi^*}{\partial t^2} - im^2\psi\psi^* = 0$$

$$\Rightarrow \quad -i\left(\psi^*\nabla^2\psi - \psi\nabla^2\psi^*\right) = -i\left(\psi^*\frac{\partial^2\psi}{\partial t^2} - \psi\frac{\partial^2\psi^*}{\partial t^2}\right)$$

$$-i\nabla\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) = -i\frac{\partial}{\partial t}\left(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\right)$$

$$\nabla\vec{j} = -\frac{\partial\rho}{\partial t}$$

$$\vec{j} = -i\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) \qquad \rho = i\left(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\right)$$

Uporabimo ravni val $\psi = N e^{-i\omega t + i\vec{k}\cdot\vec{r}}$

$$= N e^{-\frac{i}{\hbar}(Et - \vec{p}\cdot\vec{r})} = N e^{-\frac{i}{\hbar}p^\mu x_\mu} \qquad p^\mu = \left(\frac{E}{c}, \vec{p}\right), \quad x^\mu = (ct, \vec{r})$$

$$= N e^{-ipx}$$

Reši K-G enačbo in dobimo $E^2 = p^2 + m^2$ (prost delec)

$$\psi^* = N^* e^{ipx}$$

$$\frac{\partial\psi}{\partial t} = -iE\psi \qquad \frac{\partial\psi^*}{\partial t} = iE\psi^*$$

$$\rho = i|N|^2(-iE)\cdot 2 = 2E|N|^2 \quad \Rightarrow \quad \frac{1}{2E}$$ velikost rel. faznega prostora

Če je $E<0$, $\rho<0$!! problem s K-G enačbo

Stacionarne rešitve K-G enačbe

- $m=0$, rotacijska simetrija (točkast izvor)

$\Rightarrow$ Sferične koordinate $\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial u}{\partial r} = \nabla^2 u = 0 \Rightarrow \psi(r) = u(r) = \frac{g}{r}$ potencial

Na primer foton $g = \frac{e^2}{4\pi\varepsilon_0} = \alpha$

$\Rightarrow$ Zveza med $m_\gamma = 0$ in dosegom interakcije $\sim \frac{1}{r}$ (dolg doseg)

- $m \neq 0$, rot. sim. $\Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial u}{\partial r} = m^2 u$; $u = u(r)$

Rešitev $u(r) = \frac{g}{r}e^{-r/R}$ $\qquad R = \frac{1}{m} = \left(\frac{\hbar}{mc_0}\right) \approx \lambda = \frac{h}{mc_0}$

(kratek doseg) $\qquad$ Comptonova val. dolžina

⇓

Sila med nukleoni

↓

Masivni nosilci interakcij

↓

Pioni $m_\pi \simeq 100$ MeV

Izmerili $m_\pi = 139$ MeV

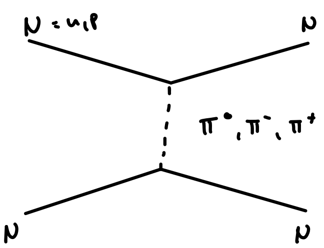
Tipičen doseg sile je R

naj bo to $R \sim 2$ fm

$\Rightarrow m = \frac{\hbar c}{R} = 100$ MeV

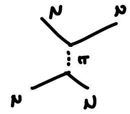
$\pi^+ = u\bar{d} \qquad \pi^- = d\bar{u}$

$\pi^0 = u\bar{u}$ ali $d\bar{d}$



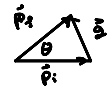
Ocena preseka za Yukawin potencial

$$u(r) = V_\pi(r) = \frac{g}{r} e^{-r/R}$$



$\Leftrightarrow \quad NN \xrightarrow{\pi} NN$

Recikliramo EM sipanje $\quad V_{fi} = \frac{1}{V_N} \int_{V_N} e^{i\vec{q}\cdot\vec{r}} V(\vec{r})\, d^3r \quad , \quad \vec{q} = \vec{p}_f - \vec{p}_i$



Za V_π

$$V_{fi} = \frac{1}{V_N} \int_{V_N} e^{i\vec{q}\vec{r}} V_\pi(r)\, d^3r =$$

$$= \frac{1}{V_N} \int_0^\infty \int_0^\pi e^{iqr\cos\theta} V_\pi(r) 2\pi r^2 dr \sin\theta d\theta$$

$$= \frac{2\pi g}{V_N} \int_0^\infty \int_0^\pi \frac{e^{-r/R}}{r} e^{iqr\cos\theta} r^2 dr \sin\theta d\theta$$

$$= -\frac{4\pi g}{V_N} \int_0^\infty r e^{-r/R} \frac{\sin qr}{qr} dr$$

$$= -\frac{4\pi g}{V_N q} \underbrace{\int_0^\infty e^{-r/R} \sin qr\, dr}_{\text{Laplaceova transformacija}}$$

$$= -\frac{4\pi g}{V_N} \frac{1}{q} \frac{q}{\frac{1}{R^2} + q^2}$$

$$V_{fi} \sim g \frac{1}{m^2 + q^2}$$

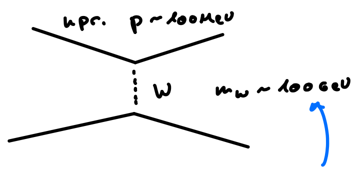
Presek $\quad \sigma(NN \xrightarrow{\pi} NN) \sim |V_{fi}|^2 \sim \frac{g^2}{(m^2+q^2)^2}$

Test: EM sipanje, $m=0$ $\qquad \sigma \sim \frac{(e^2)^2}{q^4}$ $\quad \checkmark$ Rutherfordovo sipanje

Limita $|q|^2 \ll |m|^2$ masivni interakcijski delec glede na kin. en. delcev v trku

$\Downarrow$

Šibka sila !



Virtualni delec,
energija si sposodimo

$$\sigma_W = \frac{g_W^2}{m_W^4} = G_F^2 \qquad g_W^2 \sim 0{,}1$$
$$m_W = 80{,}3 \text{ GeV}$$

$$= \frac{0{,}1}{(80{,}3)^4} \ll 1 \Rightarrow G_F^2$$

$\Downarrow$

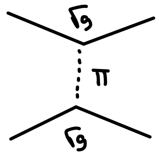
Zato je šibka sila

Ker smo reciklirali EM interakcijo: sipanje (odboj), elastično sipanje $E_i = E_f$

$$p_f^\mu = (E_f, \vec{p}_f)$$
$$p_i^\mu = (E_i, \vec{p}_i)$$
$$q^\mu = p_f^\mu - p_i^\mu = (0, \vec{q})$$
$$q^\mu q_\mu = Q^2 = -q^2 < 0 \; !$$

$$\Rightarrow V_{fi} \sim \frac{g}{Q^2 - m^2}$$

$\frac{1}{Q^2 - m^2}$ ... propagator v Feyn. diagramu in mat. el.



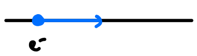
$\Rightarrow V_{fi} \sim (\sqrt{g})^2 \frac{1}{Q^2 - m^2}$

Nekaj več o sklopitvenih konstantah

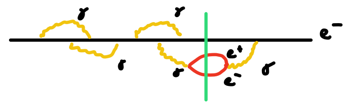
$\alpha \sim \frac{1}{100}$, $\alpha_w \sim \frac{1}{10}$, $\alpha_s \sim 1$ za nizke E

weak, strong

Klasična slika

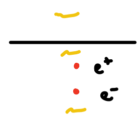


Rel. kvant. meh.



virtualni delci

os časa t

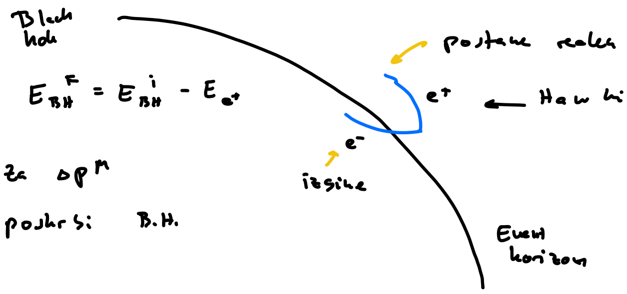


e^- skrit v oblaku γ in $e^- e^+$

Os časa t lahko izmerimo delce.

Energijo za obstoj virtualnih delcev dobimo iz meritve

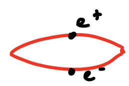
Hawking radiation



$E_{BH}^F = E_{BH}^i - E_{e^+}$

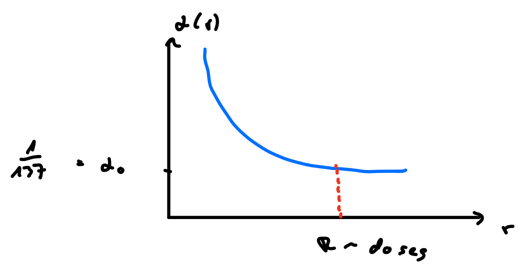
za Δp^μ poskrbi B.H.

Oblačenje v virtualne delce: polarizacija, senčenje vakuuma



virtualna

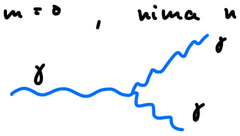
e^- •



- Bližje ko pridemo, večji bo viden (ne virtualni) naboj.
- Izstrelek večji p^μ (oz večji Q^2)
- Izmerimo odvisnost $\alpha(E)$

EM interakcija: nosilec foton, $m = 0$, nima naboja

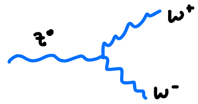
Niso možne reakcije:



Šibka sila: nosilec: šibki bozoni $W^+ W^- Z^0$, $m_W = 80.7\,GeV$ $m_Z = 91.1\,GeV$

nosijo "šibki naboj"

Možna reakcija



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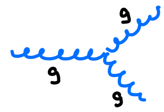
Močna sila: nosilci gluoni, $m_g = 0$, imajo "barvni naboj"



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Feynmanovi diagrami

Kontinuitetna (rel.) enačba $\partial^\mu = (\frac{\partial}{\partial t}, -\nabla)$ $\partial_\mu = (\frac{\partial}{\partial t}, \nabla)$

$$j^\mu = (\rho, \vec{j})$$

$$\partial_\mu j^\mu = \frac{\partial \rho}{\partial t} + \nabla \vec{j} = 0$$

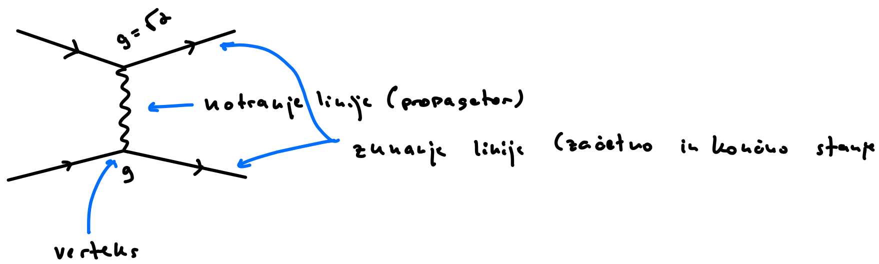
Za ravni val $\psi = N e^{ip\cdot x}$ $p\cdot x = p^\mu x_\mu = E\cdot t - \vec{p}\cdot\vec{r}$

$$j^\mu = 2|N|^2 p^\mu$$

$$(V_{fi}) = M_{fi} = M_1(\alpha^1) + M_2(\alpha^2) + M_3(\alpha^3) + \dots \qquad \alpha = g^2$$

Računaj v potenčno vrsto, želimo $\alpha < 1$

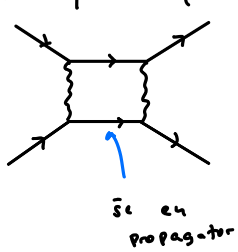
Feynman: diagrami in pravila prevedbe v M_i



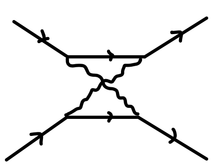
Zmnožimo vse vertekse in α, M_1

Teorija določa dovoljene vertekse (in pravila za njih), propagatorje in zun. linije

Naslednji red, α^2, M_2



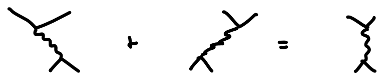
+



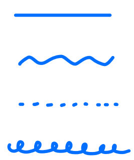
+ . . .

Splošno: - v vsakem verteksu se ohranja E in p

- vse zunanje linije predstavljajo realne delce (on shell) in proste rešitve sorazmerne z ravnim valom
- notranje linije (propagatorji) so virtualni delci
- Virtualni delci: sešteti vsi možni časovni poteki



Standardni model



spin $\frac{1}{2}$ fermioni (kvarki, e, ν)

spin 1 fotoni, včasih tudi drugi bozoni (Z^0, W^+, W^-, ...)

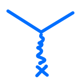
spin 1, 0 šibki bozoni, higgsov bozon

gluoni

Možni verteksi

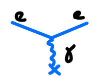
(MADGRAPH)

Interakcija s poljem / izvor



izvor polja

Npr. Lorentzova sila



Zunanji delci: usmerjene linije

Kaj z antidelci? Feynman - Stückelberg interpretacija

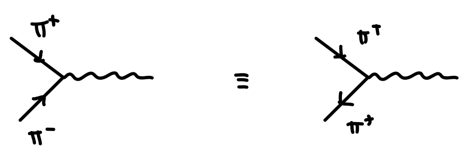
- zgled: π^+ (delec, $u\bar{d}$), π^- (antidelec, $\bar{u}d$)
- K. G. rešitev $\psi = N e^{-ip\cdot x}$ $p^\mu = (E = \sqrt{p^2+m^2}, \vec{p})$
- verjetnostni tok $j^\mu = 2|N|^2 p^\mu$

el. tok $j^\mu_e(\pi^+) = +e_0 j^\mu$

- el. tok za antidelec $j^\mu_e(\pi^-) = -e_0 j^\mu$

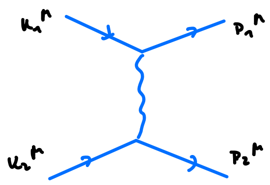
$$= e_0(-j^\mu)$$
$$= e_0\, 2|N|^2 (-p^\mu)$$

- Feynman: · sevanje (absorpcija) antidelca z g. k. p^μ je fizikalno ekvivalentna absorpciji (sevanju) delca z g. k. $(-p^\mu)$.
  - Pozitivne rešitve antidelca ($p^\mu > 0$), ki se giblje naprej v času so ekvivalentne negativnim $(-p^\mu)$ rešitvam delca, ki se giblje nazaj v času
  - Antidelci so delci, ki se gibljejo nazaj v času



Osnovne 2 → 2 topologije

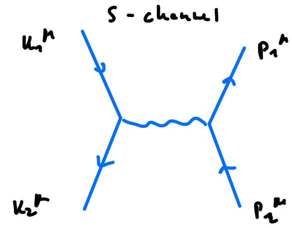
T- channel



Sipanje

$\underbrace{t = Q^2 = q^2}_{\text{ali}} = (p_1 - k_1)^2 = (p_2 - k_2)^2$

S - channel

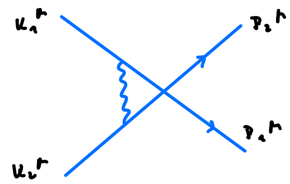


Anihilacija

$s = (k_1 + k_2)^2 = (p_1 + p_2)^2$

težiščna energija trkov

U-channel



Crossing

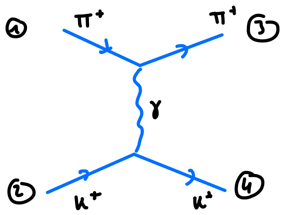
$u = (p_1 - k_2)^2 = (p_2 - k_1)^2$

t, s, u (Mandelstam variables)

$s + t + u = \sum_i m_i^2$

vsota vseh delcev v reakciji

Zgled: EM sipanje π^+ in K^+ (spin 0): KG enačba



$(\Box + m^2)\psi =$

$= \underbrace{-ie(\partial_\mu A^\mu + A^\mu \partial_\mu)\psi}_{-\hat V \psi}$ + višji redi

interakcija z EM poljem

(sevanje γ : $\hat p \to \hat p + e\hat A$
$\hat\nabla \to \hat\nabla + ie\hat A$)

$\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu$

$\Rightarrow \Box = \partial_\mu \partial^\mu \to D_\mu D^\mu$

$\to M_{fi} = \frac{1}{i}\int d^4x\, \psi_f^* \hat V \psi_i$

0-ta rešitev $(\Box + m^2)\psi = 0$

$\psi_1 = N_1 e^{-ik_1 x}$ enako za 2, 3, 4

1-red perturbacij $(\Box + m^2)\psi = -\hat V \psi$

$M_{fi} = \frac{1}{i}\int \psi_3^* \hat V \psi_1 \, d^4x$

$M_{fi} = e\int d^4x \left((-\partial_\mu \psi_3^*)\psi_1 + \psi_3^*(\partial_\mu \psi_1)\right) A^\mu$

$j_\mu(\pi^+) = ie\left(\psi_3^*(\partial_\mu \psi_1) - (\partial_\mu \psi_3^*)\psi_1\right)$

↳ prehodni tok med ψ_1 in ψ_3

$M_{fi} = -i\int j_\mu(\pi^+) A^\mu d^4x$

$j^\mu(K^+) = ie\left(\psi_4^*(\partial^\mu \psi_2) + (\partial^\mu \psi_4^*)\psi_2\right)$

Zveza z A^μ, uporabimo EMP $\Box A^\mu = j^\mu$

$\Rightarrow A^\mu = -\frac{1}{q^2} j^\mu(K^+)$

$$\Rightarrow M_{fi} = i \int j_\mu(\pi^+) \frac{1}{q^2} j^\mu(K^+) \, d^4x$$

$$M_{fi} = ie^2 N_1 N_2 N_3 N_4 \, (p_1+k_1)_\mu (p_2+k_2)^\mu \frac{1}{q^2} \underbrace{\int d^4x \, e^{i(p_1+p_2-k_1-k_2)}}_{(2\pi)^4 \delta^{(4)}(p_1+p_2-(k_1+k_2))}$$

ohranitev E in p

metrični tenzor

Propagator $\quad -\frac{i g^{\mu\nu}}{q^2}$

Vertekse $\quad -ie(p_1+k_1)_\mu$

$\quad -ie(p_2+k_2)_\mu$

Matrični element
dobimo iz zmnožka
propagator in verteksov

Sipalni presek

M_{fi} brez N_i

$$d\sigma = \frac{\overline{|T_{fi}|^2}}{4\left((k_1 k_2)^2 - m_1^2 m_2^2\right)\boxed{?}} \underbrace{(2\pi)^4 \delta^{(4)}\left(k_1 + k_2 - \sum_{i=1}^{N} p_i\right)}_{\text{ohranitev en. in gib. kol.}} \underbrace{\prod_{i=1}^{N} \frac{d^3 p_i}{(2E_i)} \frac{1}{(2\pi)^3}}_{dQ}$$

četverci

d Lips - Lorentz invariant phase space

$$\overline{|T_{fi}|^2} = \frac{1}{S} \sum_{\text{zač}} \sum_{\text{končni}} |T_{fi}|^2$$

začetni delci $\quad$ vse možne končne stanje

Tipično $\frac{1}{S}\sum = \frac{1}{(2S_1+1)(2S_2+1)} \sum_{S_1 S_2}$

Simetrije in ohranitveni zakoni

↳ količine, ki se s časom ohranjajo

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle$$

Formalna rešitev $\quad |\psi(t)\rangle = \underbrace{e^{-i\frac{\hat{H}t}{\hbar}}}_{U,\ \text{unitarna}} |\psi(0)\rangle \qquad U^+ U = I$

Opazljivka D z operatorjem $\hat{D}$ se s časom ohranja

$$\langle \psi(t) | \hat{D} | \psi(t) \rangle = \begin{cases} \langle \psi(0) | \hat{D} | \psi(0) \rangle = D_0 \\ \langle \psi(0) | U^+ \hat{D} U | \psi(0) \rangle \end{cases}$$

$$U^+ \hat{D} U = D_0$$
$$U U^+ \hat{D} U U^+ = U D_0 U^+$$
$$\hat{D} = U D_0 D^+ \quad \Big)\frac{d}{dt}$$

$$0 = \frac{d}{dt}\hat{D} = \frac{\partial U}{\partial t} D_0 U^+ + U D_0 \frac{\partial U^+}{\partial t} = \dots = -\frac{i\hat{H}}{\hbar} U D_0 U^+ + U D_0 \left(+\frac{i\hat{H}}{\hbar}\right) U^+$$

$$= -\frac{i}{\hbar}\left(\hat{H} U D_0 U^+ + U D_0 U^+ \hat{H}\right) = -\frac{i}{\hbar}[\hat{H}, \hat{D}]$$

Zgled: ohranitev $\vec{J}$, vzamemo $\vec{J}_z = \vec{J}_n$, velja $[\hat{H}, \hat{J}_z] = 0$ npr. H atom

→ Spomni se: $\hat{J}_z$ je generator rotacij: unitarni zapis

$$U(J_z, \varphi) = e^{-i\frac{J_z \varphi}{\hbar}} \quad \text{v } \mathbb{C}$$

$$\psi \to \psi' = U\psi$$

- Ohranjanje: diskretne količine (parnost, naboj, ...) in zvezne $(\vec{p}, \vec{J}, \dots)$

- Barionsko št.: Primer diskretne simetrije ("dosti" kvantno število sistema)

→ Barioni qqq $B=1$ $\hat{B}|\psi\rangle = B|\psi\rangle = 1|\psi\rangle$

↳ sim. operacija $U = e^{i\hat{B}\theta}$ → Poměšljivski kot

→ Antibarioni $\bar{q}\bar{q}\bar{q}$ $B=-1$ $\hat{B}|\psi\rangle = -|\psi\rangle$

→ Mezoni $q\bar{q}$ $B=0$

→ Damo na nivoju kvarkov: q $B=1/3$; $\bar{q}$ $B=-1/3$

Hadroni: Barioni (qqq, $B=1$) + Mezoni ($q\bar{q}$, $B=0$)

Leptoni ℓ: $B=0$

Ohranitev barionskega št.

$$pp \to pp\,p\bar{p} \;\checkmark$$

$$\begin{array}{cccc} pp & \to & p\bar{p} & \pi^+\pi^+ \;\times \\ 1\,1 & & 1\,-1 & 0\;\;0 \end{array}$$

- Ohranitev Leptonskega števila L
  - → Leptoni: $L=1$
  - → Antileptoni: $L=-1$
  - → ostali: $L=0$

Eksperimenti

- $$\begin{array}{lcccc} & p\,p & \to & e^+\,e^+ & \times \\ L: & 0\;0 & & -1\;-1 & \end{array}$$
- $$\begin{array}{lcccc} & e^+e^- & \to & \tau^+\tau^- & \checkmark \\ L: & -1\;+1 & & -1\;+1 & \end{array}$$
- $$\begin{array}{lcccc} & \nu_\mu\,n & \to & p\,\mu^- & \checkmark \\ L: & 1\;0 & & 0\;1 & \\ B: & 0\;1 & & 1\;0 & \end{array}$$
- $$\begin{array}{lcccc} & \nu_\mu\,n & \to & p\,e^- & \times \\ L: & 1\;0 & & 0\;1 & \\ B: & 0\;1 & & 1\;0 & \\ L_\mu: & 1\;0 & & 0\;0 & \\ L_e: & 0\;0 & & 0\;1 & \end{array}$$

→ Vpeljemo še leptonsko število po okusih L_μ, L_e, L_τ

Ohranitev $B-L$

→ razpad protona

$$\begin{array}{lcccc} & p & \to & e^+ & \pi^0 \\ B & 1 & & 0 & 0 \\ L & 0 & & -1 & 0 \\ B-L & 1 & = & 1 & \checkmark \end{array}$$

Intermezzo: Permutacijska simetrija funkcij stanja z več delci:

$N=2$, enodelčni stanji φ_a, φ_b

$$\psi_s(1,2) = \frac{1}{\sqrt{2}}\left(\varphi_a(1)\varphi_b(2) + \varphi_a(2)\varphi_b(1)\right)$$

Simetrija $\psi_s(1,2) = \psi_s(2,1)$ nedoločljivi delci npr. e^-e^-

$$\psi_A = \frac{1}{\sqrt{2}}\left(\varphi_a(1)\varphi_b(2) - \varphi_a(2)\varphi_b(1)\right)$$

Antisimetrija $\psi_A(1,2) = -\psi_A(2,1)$ fermioni $(s = \frac{1}{2}, \frac{3}{2}, \dots)$

paulijev prepoved

Velja za vsak sistem z N fermioni

Št možnih stanj = št. barionov: $N=3$ kvarki (u,d,s) : $3^3 = 27$ kombinacij

- Izospin I, I_3

Delec ψ, ki je v stanjih
$|I = 1/2, I_3 = 1/2 = \uparrow\rangle = |p\rangle$
$|I = 1/2, I_3 = -1/2 = \downarrow\rangle = |n\rangle$

Kot spinorji:
$\binom{1}{0} = |p\rangle$
$\binom{0}{1} = |n\rangle$

Dobro kvantno število, za jedrsko močno interakcijo

$$I_3 = \frac{1}{2}(Z - N) \quad \text{v enačbah jedra}$$

$$[\hat{H}, \hat{I}_3] = 0$$

Spinski formalizem

$I_3|p\rangle = \frac{1}{2}|p\rangle$ $\quad I_+|p\rangle = 0$ $\quad I_-|n\rangle = 0$

$I_3|n\rangle = -\frac{1}{2}|n\rangle$ $\quad I_+|n\rangle = |p\rangle$ $\quad I_-|p\rangle = |n\rangle$

Prevod v kvarkovsko sliko

$I_3|u\rangle = \frac{1}{2}|u\rangle$ $\quad u: I = \frac{1}{2}$ $\quad I_3 = \frac{1}{2}$ $\quad I_-|u\rangle = |d\rangle$

$I_3|d\rangle = -\frac{1}{2}|d\rangle$ $\quad d: I = \frac{1}{2}$ $\quad I_3 = -\frac{1}{2}$ $\quad I_+|d\rangle = |u\rangle$

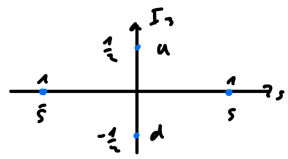
Sistematična konstrukcija barionskih stanj: okusov

$$\hat{I}_- = \sum_{i=1}^{3} I_{-i} \quad : \quad \hat{I}_- \overbrace{|uuu\rangle}^{\psi_{S1}} = \overbrace{|duu\rangle + |udu\rangle + |uud\rangle}^{\psi_{S3}}$$

$$\hat{I}_+ = \sum_{i=1}^{3} I_{+i} \quad : \quad \hat{I}_+ \underbrace{|ddd\rangle}_{\psi_{S2}} = \underbrace{|udd\rangle + |dud\rangle + |ddu\rangle}_{\psi_{S4}}$$

Dodajmo še kvark s

$$|s\rangle = |I = 0, I_3 = 0\rangle$$



$\Rightarrow$ Novo kvantno št: čudnost: $S = -1$ za $|s\rangle$
$S = 1$ za $|\bar{s}\rangle$

$$\pi^- p \to K^0 \Lambda^0$$

čudnost 0 0 1 -1

Čudnost se ohranja pri močni in EM interakciji, ne pa pri šibki

Dodamo še kvark s:

$$\psi_{S5} = \frac{1}{\sqrt{3}} \left(|suu\rangle + |usu\rangle + |uus\rangle \right)$$
$$\psi_{S6} = \frac{1}{\sqrt{3}} \left(|sdd\rangle + |dsd\rangle + |dds\rangle \right)$$
$$\psi_{S7} = \frac{1}{\sqrt{6}} \left(|dus\rangle + |dsu\rangle + |sdu\rangle + |uds\rangle + |sud\rangle + |usd\rangle \right)$$
$$\psi_{S8} = \frac{1}{\sqrt{3}} \left(|sus\rangle + |uss\rangle + |ssu\rangle \right)$$
$$\psi_{S9} = \frac{1}{\sqrt{3}} \left(|sds\rangle + |ssd\rangle + |dss\rangle \right)$$
$$\psi_{S10} = |sss\rangle$$

Dekuplet simetričnih stanj okusa

Ostane še 17 stanj

→ popolnoma antisim. stanje

$$\psi_{A1} = \frac{1}{\sqrt{6}} \left(-|dus\rangle - |dsu\rangle - |sdu\rangle + |uds\rangle + |sud\rangle + |usd\rangle \right)$$

Katerakoli dva dela zamenjamo dobimo −

$$\psi_A = \frac{1}{\sqrt{6}} \left(|\underset{u\,d\,s}{RGB}\rangle + \dots \right)$$ stanje barv kvarkov za vse barione

Mešana stanja: sim. na zamenjavo dveh & in antisim. na zamenjavo drugih dveh

ostali okusi

$$\psi_{MA1} = \frac{1}{\sqrt{2}} \left(|\underbrace{ud}_{A}u\rangle - |\underbrace{du}_{A}u\rangle \right)$$

(S, S)

$\vdots$

ψ_{MA8}

$$\psi_{MS1} = \frac{1}{\sqrt{6}} \left(|udu\rangle + |duu\rangle - 2|uud\rangle \right)$$

$\vdots$

ψ_{MS8}

Skupna funkcija: stanje barionov

Celotna funkcija mora biti antisim. (3 fermioni)

$$\psi_B \sim \underbrace{\psi(\vec{r})}_{\substack{\text{krajevni del}\\ \text{simetričen}}} \underbrace{\psi(\text{spin})\ \psi(\text{okus})}_{\text{simetrično}} \underbrace{\psi(\text{QCD})}_{\substack{\text{antisim.}\\ \text{barvni singlet}}}$$

Možnosti: ψ_{S1-S10} (okus) ψ_{S1-4} (spin) → spin $J = \frac{3}{2}$

Dekuplet barionov z $J = 3/2$

↳ zgodovina $\Delta^{++} = |uuu\rangle$ − simetričen

$J = 3/2$ izmerjen $|\uparrow\uparrow\uparrow\rangle$ − sim.

Mešane komb.

$$\psi_{M1} = \psi_{MS1}(\text{okus})\, \psi_{MS1}(\text{spin}) + \psi_{MA1}(\text{okus})\, \psi_{MA1}(\text{spin})$$

$\vdots$

ψ_{M8} ⇒ 8 barionov (oktet) s spinom $J = \frac{1}{2}$

Hiper naboj $\qquad$ Naboj

$$Y = B + \overset{\text{čudnost}}{S} \qquad Q = I_3 + \frac{B+S}{2}$$

Eksperimentni test:
magnetni momenti n in p

Račun iz modela, proton: $\psi_p = \psi_{MS1}(okus)\,\psi_{MS1}(spin) + \psi_{MA1}(okus)\,\psi_{MA1}(spin)$

spin $= \frac{1}{2}$

$$= \frac{1}{\sqrt{2}}\left(|udu\rangle - |duu\rangle\right)\left(|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle\right) + \frac{1}{\sqrt{6}}\left(|udu\rangle + |duu\rangle - |uud\rangle\right)\left(|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - |\downarrow\downarrow\uparrow\rangle\right)$$

$$= \frac{1}{\sqrt{18}}\left(2|u\uparrow u\uparrow d\downarrow\rangle - |u\uparrow u\downarrow d\uparrow\rangle + \dots\right)$$

Operator mag. momenta $\hat{\vec{\mu}}_i = g_s \frac{e_0 Q_i \hat{\vec{s}}_i}{2m_i}$

$g_s = 2$ za spin $\frac{1}{2}$
$\vec{s}_i$ - spin
m_i - masa kvarka
Q_i - delež osn. naboja

$$\hat{\mu}_z \psi_p = \mu_p \psi_p \qquad \mu_p = \frac{e_0}{2m_q}$$

$$\hat{\mu}_z \psi_n = \mu_n \psi_n \qquad \mu_n = -\frac{2}{3}\frac{e_0}{2m_q}$$

Merimo $\frac{\mu_n}{\mu_p} = -\frac{2}{3}$ (m_q in napake se pokrajšajo)

Eksperiment $\frac{\mu_n}{\mu_p} \doteq -0{,}685$

Mezoni

Stanja $q\bar{q}$ $\quad B=0$

Operator konjugacije naboja: $\hat{C}: |q\rangle \to |\bar{q}\rangle$, $\quad \hat{C}|q\rangle = \lambda|\bar{q}\rangle$

$\hat{C}^2|q\rangle = |\lambda|^2|q\rangle$

$|\lambda|^2 = 1 \qquad \overset{\text{izberemo}}{\lambda = \pm 1}$

Naj bo $\hat{C}|u\rangle = -|\bar{u}\rangle$

$\hat{C}|d\rangle = +|\bar{d}\rangle$

Kako je z izospinom

Delci $|u\rangle = |I=\tfrac{1}{2}, I_3 = \tfrac{1}{2}\rangle \qquad I_3|u\rangle = \tfrac{1}{2}|u\rangle$

$|d\rangle = |I=\tfrac{1}{2}, I_3 = -\tfrac{1}{2}\rangle$

Antidelci $I_3|\bar{u}\rangle = -\tfrac{1}{2}|\bar{u}\rangle$

$I_3|\bar{d}\rangle = \tfrac{1}{2}|\bar{d}\rangle$

$I_-|\bar{d}\rangle = -|\bar{u}\rangle$

$I_+|\bar{u}\rangle = -|\bar{d}\rangle$

Mezonski spekter:

Začnimo s π $(B=0,\ Y=B+S=S=0)$ spin $=0$

$$|\pi^+\rangle = |u\bar{d}\rangle = |I=1,\ I_3=1\rangle$$

dela

$$I_3 = \tfrac{1}{2} + \tfrac{1}{2} = 1$$

$$I = 1$$

$$I_-|u\bar{d}\rangle = \tfrac{1}{\sqrt{2}}\left(|d\bar{d}\rangle - |u\bar{u}\rangle\right) \qquad \pi^0 \quad |I=1,\ I_3=0\rangle$$

$$I_-|\pi^0\rangle = |d\bar{u}\rangle \qquad \pi^- \quad |I=1,\ I_3=-1\rangle$$

Imamo (u, d, s) = 9 kombinacij

$$|\pi^+\rangle \xrightarrow{d\to s} |u\bar{s}\rangle = |K^+\rangle \qquad \text{kaon}$$

$S=1$
$Y=1$
$I_3=1/2$

$$|\pi^+\rangle \xrightarrow{u\to s} |s\bar{d}\rangle = |\bar{K}^0\rangle$$

anti K^0

$S=-1$
$Y=-1$
$I_3=1/2$

$$|\pi^-\rangle \xrightarrow{d\to s} |s\bar{u}\rangle = |K^-\rangle \qquad I_3=-1/2 \qquad S=-1$$

$$|\pi^-\rangle \xrightarrow{u\to s} |d\bar{s}\rangle = |K^0\rangle \qquad I_3=-1/2 \qquad S=1$$

Manjkata še 2 stanji:

Popolnoma sim komb (singlet) v izospinu

$$\tfrac{1}{\sqrt{3}}\left(|d\bar{d}\rangle + |u\bar{u}\rangle + |s\bar{s}\rangle\right) = |\eta_0\rangle$$

Še ortogonalno stanje

$$\tfrac{1}{\sqrt{6}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle\right) = |\eta_8\rangle$$

V naravi so stanja mešanica obeh

$$|\eta\rangle = \sin\theta\,|\eta_0\rangle + \cos\theta\,|\eta_8\rangle$$

$$|\eta'\rangle = \cos\theta\,|\eta_0\rangle - \sin\theta\,|\eta_8\rangle$$

Spin 0: $\pi,\ K,\ \underbrace{\eta_0,\ \eta_8}_{\eta,\ \eta'}$

Spin 1: $\underbrace{\rho}_{\rho^+,\rho^-,\rho^0},\ K^*,\ \underbrace{\phi_0,\ \phi_8}_{\phi,\ \omega}$

$$|\phi\rangle = \sin\theta'\,|\phi_0\rangle + \cos\theta'\,|\phi_8\rangle \qquad |\omega\rangle = \cos\theta'\,|\phi_0\rangle - \sin\theta'\,|\phi_8\rangle$$

Dodamo težke kvarke :

→ kvark c (čar, charm, $C=1$) Mezoni: D ($D^+, D^-, D^{*+}, D^0, \bar{D}^0, \ldots$

$\bar{c}$ ($C=-1$)

b (beauty Beauty = 1) Mezoni: B ($B^+, B^-, B^0, B^{*+}, \ldots$

$\bar{b}$ (= -1)

		Čisto stanje
Y psi	Y/ψ	$\lvert c\bar{c}\rangle$
upsilon	Υ	$\lvert b\bar{b}\rangle$

Diracove enačbe

Diracove ideja: funkcija stanja ($\hat{H}$) je lin. v času ⟸ verj. gostota $\rho = |\psi|^2$
Želi si samo $p_0 = E > 0$ rešitve $\left(\frac{\partial \rho}{\partial t} \sim \psi^* \frac{\partial \psi}{\partial t} + \frac{\partial \psi^*}{\partial t}\psi\right)$
$(p_0, \vec{p}) = p^\mu$

Posledica za konstrukcijo: lin v $\frac{\partial}{\partial t}$ → lin v $\partial_\mu = \left(\frac{\partial}{\partial t}, \vec{\nabla}\right)$

$$\Rightarrow \quad * \quad i\frac{\partial \psi}{\partial t} = (\underbrace{-i\vec{\alpha}\cdot\vec{\nabla}}_{\vec{\alpha}\cdot\vec{p}} + \beta m)\psi = \hat{H}\psi \qquad \hat{H} = \vec{\alpha}\cdot\vec{p} + \beta m$$

Pogoj: ohranitev energije (K-G enačba, $(\Box + m^2)\psi = 0$)

Rešitev oblike $\psi = N\, w(p)\, e^{-ipx}$

Kvadriramo *

$$-\frac{\partial^2}{\partial t^2}\psi = (-i\vec{\alpha}\cdot\vec{\nabla} + \beta m)(-i\vec{\alpha}\cdot\vec{\nabla} + \beta m)\psi$$

$$= -\sum_{i=1}^{3} \alpha_i^2 \frac{\partial^2 \psi}{\partial x_i^2} - \sum_{i>j}(\alpha_i\alpha_j + \alpha_j\alpha_i)\frac{\partial^2 \psi}{\partial x_i \partial x_j} - im\sum_i(\alpha_i\beta + \beta\alpha_i)\frac{\partial \psi}{\partial x_i} + \beta^2 m^2 \psi$$

⇒ Pogoji za $\vec{\alpha}, \beta$. Primerjamo s K-G enačbo

- $\alpha_i\beta + \beta\alpha_i = \{\alpha_i, \beta\} = 0$
- $\{\alpha_i, \alpha_j\} = 0$
- $(\alpha_i)^2 = I \qquad \beta^2 = I$

Dim $\vec{\alpha}$ in β = 4

ψ oz w je 4-dim vektor $\psi = \begin{bmatrix}\psi_1\\ \vdots \\ \psi_4\end{bmatrix}$

Zahtevam ustreza $\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}$ α_i je 4×4 matrika
σ_i so Paulijeve matrike

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \qquad \beta = \begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix} = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}$$

Izbira ni enolična $\alpha_i' = U\alpha_i U^{-1}$ $\beta' = U\beta U^{-1}$ U-unitarna.
tudi rešitve

$$\psi = N\, w\, e^{-ipx}$$

bispinor $\begin{pmatrix}\varphi \\ \chi\end{pmatrix}$

$$\varphi = \begin{pmatrix}0\\1\end{pmatrix} \begin{pmatrix}1\\0\end{pmatrix}$$
$$\chi = \begin{pmatrix}0\\1\end{pmatrix} \begin{pmatrix}1\\0\end{pmatrix}$$

Zapis D. E. v kovariantni obliki

$$\gamma^\mu = \begin{cases} \gamma^0 = \beta \\ \gamma^i = \beta \alpha_i \end{cases} \qquad \gamma_0^2 = I \qquad (\gamma^i) = -I$$

Star zapis množimo z $\gamma^0 = \beta$

Diracova en. $$\boxed{(i\, \underbrace{\gamma^\mu \partial_\mu}_{\gamma^0 \frac{\partial}{\partial t} + \gamma^i \frac{\partial}{\partial x_i}} - m)\, \psi = 0}$$

naj bo $\gamma^\mu a = \not{a}$

$$\Rightarrow (i \not{\partial} - m)\psi = 0$$

$$\{\gamma^\mu, \gamma^\nu\} = 2 g^{\mu\nu} I \qquad g^{\mu\nu} = (1\ {-1}\ {-1}\ {-1})$$

Rešitev D. E.

def. $E = +\sqrt{p^2 + m^2}$ $\qquad p^0 = \pm E \qquad p^\mu = (p^0, \vec{p})$

Nastavek $\psi = N\, w(p)\, e^{-ipx}$

$$w = \begin{bmatrix}\varphi \\ \chi\end{bmatrix}$$

Vstavimo v D.E.

$$(i\gamma^\mu \partial_\mu - m)\, w(p)\, e^{-ipx} = 0$$

$$(\gamma^\mu p_\mu - m)\, w(p) = 0$$

Množimo z $\gamma^0 = \beta$

$$(\vec{\alpha}\cdot\vec{p} + \beta m) w = p_0 w$$

$$\begin{bmatrix} m I_2 & \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -m I_2 \end{bmatrix} \begin{bmatrix}\varphi \\ \chi\end{bmatrix} = p_0 \begin{bmatrix}\varphi \\ \chi\end{bmatrix}$$

2 enačbi za 2 spinorja

$$(\vec{\sigma}\cdot\vec{p})\chi = (p_0 - m)\varphi$$
$$(\vec{\sigma}\cdot\vec{p})\varphi = (p_0 + m)\chi \qquad \Rightarrow \chi = \frac{\vec{\sigma}\cdot\vec{p}}{p_0 + m}\varphi$$

Vstavimo v 1. en

$$\underbrace{(\vec{\sigma}\cdot\vec{p})^2}_{\vec{p}^2} \varphi = (p_0 + m)(p_0 - m)\varphi$$

$$p^2 \varphi = (p_0^2 - m^2)\varphi$$

$$\Rightarrow p_0 = \pm\sqrt{\vec{p}^2 + m^2} = \pm E$$

Še vedno možna stanja z $\pm E$.

Za $p_0 > +E$

$$\omega^{(1,2)} = N \begin{pmatrix} \varphi^{(1,2)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\varphi^{(1,2)} \end{pmatrix} \qquad \varphi^{(1,2)} \text{ - spinor} \qquad \varphi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \varphi_\uparrow \quad \varphi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \varphi_\downarrow$$

Normalizacija

$$\omega^\dagger\omega = |N|^2 \left(1 + \frac{(\vec{\sigma}\cdot\vec{p})^2}{(E+m)^2}\right) = |N|^2 \left(\frac{E^2+2mE+m^2+p^2}{(E+m)^2}\right) = |N|^2 \frac{2E^2+2mE}{(E+m)^2}$$

$$\varphi^\dagger\varphi = 1 \qquad\qquad = |N|^2 \frac{2E}{E+m}$$

Tipično $N = \sqrt{E+m}$ $\quad \omega^\dagger\omega = 2E$

$$\Updownarrow$$

$\int \rho \, dV = 2E$ kot ravni val

Za $p_0 = E > 0$

$$\omega \to u(p,s) = (E+m)^{1/2} \begin{pmatrix} \varphi^{(s)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\varphi^{(s)} \end{pmatrix} \qquad s = 1,2$$

$$\underset{\text{Delec}}{\psi(e^-)} = \frac{1}{\sqrt{V}} u(p,s) \, e^{-ipx}$$

Negativna energija $p_0 = -E < 0$

$$\underset{\text{anti-delec}}{\omega(p_0 = -E, -\vec{p}, s)} = v(p,s) = (E+m)^{1/2} \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(s)} \\ \chi^{(s)} \end{pmatrix}$$

$\chi^{(1)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \chi^{(2)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

ravno obratno kot pri φ.

$$\psi(e^+) = \frac{1}{\sqrt{V}} v(p,s) \, e^{ipx}$$

$$p = (\underset{\overset{\vee}{0}}{E}, \vec{p})$$

Izpeljava:

$$(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) = (\vec{a}\cdot\vec{b}) I_2 + i\vec{\sigma}\cdot(\vec{a}\times\vec{b})$$

$$[\sigma_i, \sigma_j] = 2i\varepsilon_{ijk}\sigma_k$$

$$\{\sigma_i, \sigma_j\} = 2\delta_{ij} I_2$$

$$(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) = \sigma_j a_j \sigma_k b_k = a_j b_k \sigma_j\sigma_k = \dots$$

$$\sigma_j\sigma_k = \frac{1}{2}\{\sigma_j, \sigma_k\} + \frac{1}{2}[\sigma_j, \sigma_k]$$

$$\dots = a_j b_k \left(\frac{1}{2} 2\delta_{jk} I_2 + \frac{1}{2} 2i\varepsilon_{jki}\sigma_i\right)$$

$$= a_j b_k \delta_{jk} I_2 + i\varepsilon_{jki}\sigma_i a_j b_k$$

$$= (\vec{a}\cdot\vec{b}) I_2 + i\vec{\sigma}\cdot(\vec{a}\times\vec{b}) \qquad \Rightarrow \quad (\vec{\sigma}\cdot\vec{p})^2 = p^2 + 0$$

Analiza rešitev D.E.

- $\vec{p}=0$ (pri miru) $\quad p_0 = \pm m = \pm E$

$p_0 = +m > 0$

$$u^{(1)} = u(p=(m,0), s=1) = m\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \qquad \psi \sim \begin{pmatrix}1\\0\\0\\0\end{pmatrix} e^{-imt}$$

$$u^{(2)} = m\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$p_0 = -m < 0$

$$v^{(1)} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix} m \qquad v^{(2)} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} m \qquad \psi \sim v\, e^{imt}$$

Posplošen operator spina

$$\vec{S} = \frac{1}{2}\vec{\sigma} \Rightarrow \frac{1}{2}\vec{\Sigma} = \frac{1}{2}\begin{pmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{pmatrix}$$

spin: $\quad \left[\frac{1}{2}\Sigma_i, \frac{1}{2}\Sigma_j\right] = i\,\frac{1}{2}\Sigma_k$

$$\left(\frac{1}{2}\Sigma_i\right)^2 = \frac{3}{4} I$$

V mirovnem sistemu

$$\frac{1}{2}\Sigma_z u^{(1,2)} = \pm\frac{1}{2} u^{(1,2)} \qquad \text{spin } \uparrow\downarrow$$

$$\frac{1}{2}\Sigma_z v^{(1,2)} = \mp\frac{1}{2} v^{(1,2)}$$

V splošnem $\vec{p} \neq 0$

$$\hat{H} = \vec{\alpha}\cdot\vec{p} + \beta\cdot m \Rightarrow \left[\hat{H}, \frac{1}{2}\vec{\Sigma}\right] = i(\vec{\alpha}\times\vec{p}) \neq 0$$

u. i. Σijk di pj

spin ni dobro kvantno število, ni lastna baza

Prost delec v $\vec{B}=0$: relevantna smer $\vec{p}$

$\rightarrow (\vec{\sigma}\cdot\vec{p})$ - projekcija spina na $\vec{p}$

helicity - sučnost, vijačnost $\qquad S_z \rightarrow \lambda$

$\lambda = +\frac{1}{2}$ $\xrightarrow{\quad}$ $\vec{p}$ $\quad = +1$

$\lambda = -\frac{1}{2}$ $\xrightarrow{\quad}$ $\vec{p}$ $\quad = -1$

Operator sučnosti

$$h(\vec{p}) = \begin{pmatrix}\frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|} & 0\\ 0 & \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}\end{pmatrix}$$

$$[\hat{H}, h] = 0 \quad ✓$$

Sedaj so tudi druge lastne funkcije

$p_0 = E > 0$

$$h(\vec p)\begin{pmatrix}\varphi\\ \frac{\vec\sigma\cdot\vec p}{E+m}\varphi\end{pmatrix} = (\pm1)\begin{pmatrix}\varphi\\ \frac{\vec\sigma\cdot\vec p}{E+m}\varphi\end{pmatrix} \to \varphi_+ , \varphi_- : \quad \frac{\vec\sigma\cdot\vec p}{|\vec p|}\varphi_{+,-} = \pm1\,\varphi_{+,-}$$

$$\psi_+ = (E+m)^{1/2}\begin{pmatrix}1\\0\\ \frac{|\vec p|}{E+m}\\0\end{pmatrix} = u_R = u(p, \lambda = \overset{+\frac{1}{2}}{+1})$$

desno sučni

$$\psi_- = (E+m)^{1/2}\begin{pmatrix}0\\1\\0\\ -\frac{|\vec p|}{E+m}\end{pmatrix} = u_L = u(p, \lambda = -1)$$

Podobno za

$$v_L = v(p, \lambda = -1) = (E+m)^{1/2}\begin{pmatrix}\frac{|\vec p|}{E+m}\\0\\1\\0\end{pmatrix} \qquad v_R = v(p, \lambda = +1) = (E+m)^{1/2}\begin{pmatrix}0\\ -\frac{|\vec p|}{E+m}\\0\\1\end{pmatrix}$$

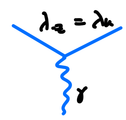
Posebej: $m \sim 0$ oz ultrarelativistična limita $E \gg m$

$$u_R = \sqrt{E}\begin{pmatrix}1\\0\\1\\0\end{pmatrix} \qquad u_L = \sqrt{E}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix} \qquad v_L = \sqrt{E}\begin{pmatrix}1\\0\\1\\0\end{pmatrix} \qquad v_R = \sqrt{E}\begin{pmatrix}0\\-1\\0\\1\end{pmatrix}$$

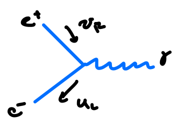
Razumski pristop: Helicity formalizem

V verteksih se vedno ohranja heliciteta

Sipanje



Anihilacija



Polna vrtilna količina

- Velja že $[\hat H, \frac{1}{2}\vec\Sigma] = i(\vec\alpha \times \vec p) \neq 0$
- Angl: orbitalna vrt. količina $\vec L = \vec r \times \vec p$ $\qquad [\hat H, \vec L] = -i(\vec\alpha \times \vec p) \neq 0$
- Definiramo: polna vrtilna količina $\vec J = \vec L + \frac{1}{2}\vec\Sigma$

$[\hat H, \vec J] = 0$ ✓

E. M. Interakcija - nerel. limita (Schröd. en. v EM polju)

EM $\quad \partial_\mu \to D_\mu = \partial_\mu + ieA_\mu \overset{e^-}{=} \partial_\mu - ie_0A_\mu$

D.E. $\quad (i\gamma^\mu D_\mu - m)\psi = 0$

$(i\gamma^\mu\partial_\mu + e_0\gamma^\mu A_\mu - m)\psi = 0$ $\quad$ pomnožimo z γ^0 in primerjamo, Schröd. en.

$$i\frac{\partial\psi}{\partial t} = [\vec\alpha\cdot\vec p + m]\psi + V_0\psi$$

$$V_0 = -e_0\gamma^0\gamma^\mu A_\mu = e\begin{pmatrix}-A_0 I_2 & \vec\sigma\cdot\vec A\\ \vec\sigma\cdot\vec A & -A_0 I_2\end{pmatrix} \qquad A_0 = U$$

Vaje z bispinorji

$$\begin{bmatrix}(m - e_0A_0)I_2 & \vec\sigma(\vec p + e_0\vec A)\\ \vec\sigma(\vec p + e_0\vec A) & (-m - e_0A_0)I_2\end{bmatrix}\begin{bmatrix}\varphi\\ \chi\end{bmatrix} = p_0\begin{bmatrix}\varphi\\ \chi\end{bmatrix}$$

Recimo $p_0 = E > 0$

$$(p_0 - m + e_0 A_0)\varphi = \vec{\sigma}\cdot(\vec{p} + e_0\vec{A})\chi$$
$$(p_0 + m + e_0 A_0)\chi = \vec{\sigma}\cdot(\vec{p} + e_0\vec{A})\varphi$$

$$\Rightarrow (p_0 - m + e_0 A_0)(p_0 + m + e_0 A_0)\varphi = [\vec{\sigma}\cdot(\vec{p} + e_0\vec{A})]^2\varphi$$

Low-energy $\overset{NR}{(\text{nerel. limit})}$ limit: $m \gg |p|$, $m \gg e_0 A_0$ $\quad$ $p_0 \approx m$, $p_0 = (p_0 = E, \vec{p})$

$$E_{NR} = p_0 - m$$

Aprox: $p_0 + m + e_0 A_0 \approx 2m$ $\quad$ 0-ti red

$p_0 - m + e_0 A_0 \approx E_{NR} + e_0 A_0$ $\quad$ 1-vi red

$$\Rightarrow 2m(E_{NR} + e_0 A_0) = [\vec{\sigma}\cdot\underbrace{(\vec{p} + e_0\vec{A})}_{\vec{p}\,'}]^2\varphi$$

$$(\vec{\sigma}\cdot\vec{p}\,')^2\varphi = p'^2\varphi + i\vec{\sigma}\cdot\underbrace{(\vec{p}\,'\times\vec{p}\,')\varphi} = \dots$$

$$\underbrace{(\vec{p}\times\vec{p})}_{=0}\varphi + [e_0(\vec{p}\times\vec{A}) + e_0(\vec{A}\times\vec{p})]\varphi + e_0^2\underbrace{\vec{A}\times\vec{A}}_{=0}\varphi =$$

$$\underset{\vec{p} = -i\nabla}{=} -ie_0\nabla\times\vec{A}\varphi - ie_0\vec{A}\times\nabla\varphi =$$

$$= -ie_0\left((\nabla\times\vec{A})\varphi - \vec{A}\times\nabla\varphi + \vec{A}\times\nabla\varphi\right)$$

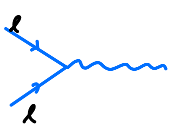
$$= -ie_0\vec{B}\varphi$$

$$\dots = p'^2\varphi + e_0\vec{\sigma}\cdot\vec{B}\varphi$$

$$\Rightarrow E_{NR}\varphi = \left(\frac{(\vec{p} + e_0\vec{A})^2}{2m} + \underbrace{\frac{e_0}{m}\frac{\vec{\sigma}}{2}\cdot\vec{B}}_{\vec{\mu}\cdot\vec{B} = g_s\,\mu_B\,\vec{s}\cdot\vec{B}} - e_0 A_0\right)\varphi$$

$$g_s = 2, \quad \mu_B = \frac{e_0}{2m}$$

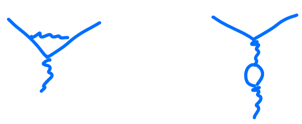
Popravki višjih redov



0. red

lepton

Višji redi
višji popravki



...

$\Rightarrow$ $a = \frac{g-2}{2}$ anomalni moment

$a = 11{,}65\ldots 10^{-4}$

Še nekaj formalizma

- kontinuitetna enačba $\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0 \Leftrightarrow \partial_\mu j^\mu = 0$

DE. $\rho = |\psi|^2 = \psi^\dagger \psi = \sum_{i=1}^{4} |\psi_i|^2 \geq 0$ ✓

$$\vec{j} = \psi^\dagger \vec{\alpha} \psi = \underbrace{\psi^\dagger \gamma^0}_{\bar{\psi}} \underbrace{\gamma^0 \vec{\alpha}}_{\vec{\gamma}} \psi = \bar{\psi}\, \vec{\gamma}\, \psi$$

ψ bar

Tudi $\rho = \psi^\dagger \psi = \psi^\dagger \gamma^0 \gamma^0 \psi = \bar{\psi} \gamma^0 \psi$

$\Rightarrow j^\mu = \bar{\psi} \gamma^\mu \psi$

za zraven $\gamma^{0\dagger} = \gamma^0 \quad (\gamma^i)^\dagger = -\gamma^i$

$\bar{\gamma}_\nu = \gamma_0 \gamma_\nu^\dagger \gamma_0 = \gamma_\nu$

Velja tudi

$$u^\dagger u = 2E \qquad \bar{u} = u^\dagger \gamma^0 \qquad \bar{u} u = 2m$$
$$v^\dagger v = 2E \qquad \bar{v} = v^\dagger \gamma^0 \qquad \bar{v} v = -2m$$

$(\not{p} - m)\, u = 0 \rightarrow (\ )^\dagger, \gamma^0 \Rightarrow \bar{u}(\not{p} - m) = 0$

$\bar{v}(\not{p} + m) = 0$

Velja tudi $\sum_{s=1,2} u^{(s)} \bar{u}^{(s)} = \not{p} + m \qquad \sum_{s=1,2} v^{(s)} \bar{v}^{(s)} = \not{p} - m$

$= \gamma^\mu p_\mu + m I_4$

EM interakcija dalje

$j^\mu_{EM}(e^-) = -e_0 \bar{\psi} \gamma^\mu \psi \qquad \psi \sim u\, e^{-ip\cdot x}$

$j^\mu_{EM}(e^+) = e_0 \bar{\psi} \gamma^\mu \psi \qquad \psi \sim v\, e^{ip\cdot x}$

$\Rightarrow$ Feyn. diagrami: $M_{fi} = -i \int \psi_f^\dagger (p_1, \underbrace{s_f, x)\, \hat{V}_D\, \psi_i (k_1, s_i}_{\text{v resnici } \lambda_f,\ \lambda_i}, x)\, d^4x = -i \int \psi_f^\dagger \hat{V}_D \psi_i\, d^4x$

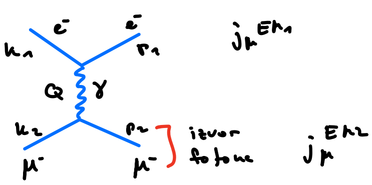
$$M_{fi} = -i \int \psi_f^\dagger (-e_0) \gamma^0 \gamma^\mu A_\mu \psi_i\, d^4x$$

$$= -i \int (-e_0)\, \bar{\psi}_f\, \gamma^\mu\, \psi_i\, A_\mu\, d^4x$$

Prehodni (EM) tok: $j^\mu_{EM} = -e_0\, \bar{\psi}_f \gamma^\mu \psi_i$

$$M_{fi} = -i \int j^\mu_{EM} A_\mu\, d^4x$$

Izvor fotona (A^μ)



$j_\mu^{EM2} = -e_0\, \bar{u}(p_2)\, \gamma_\mu\, u(k_2)\, e^{iqx}$

$q = p_2 - k_2$ izsev

$q = k_1 - k_2$ absorpcija

Reši

$$\Box A_\mu = j_\mu^{EM2} \qquad \text{izvor fotona}$$

$$\hookrightarrow c\, e^{iQx}$$

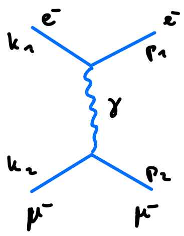
$$\Box e^{iQx} = -Q^2 e^{iQx}$$

$$\Rightarrow A_\mu = -\frac{1}{Q^2} j_\mu^{EM2}$$

$$M_{fi} = -i \int j^\mu_{EM} \left(-\frac{1}{Q^2}\right) j_\mu^{EM2}\, d^4x$$

current - current interaction

Sipanje $e^-\mu^- \to e^-\mu^-$



$$M_{fi} = -\int j^{(e)\nu} \left(-\frac{1}{Q^2}\right) j_\nu^{(\mu)} d^4x$$

$$j^{(e)\nu} = -e_0\, \bar{u}(p_1)\, \gamma^\nu\, u(k_1)\, e^{-i(k_1-p_1)\cdot x}$$

$$j_\nu^{(\mu)} = -e_0\, \bar{u}(p_2)\, \gamma_\nu\, u(k_2)\, e^{-i(k_2-p_2)\cdot x}$$

$$M_{fi} = \underbrace{-i\left(-e_0 \bar{u}(p_1)\gamma^\nu u(k_1)\right)\left(-\frac{1}{Q^2}\right)\left(-e_0\bar{u}(p_2)\gamma_\nu u(k_2)\right)}_{-i T_{fi}} \underbrace{\int e^{i(p_1+p_2-k_1-k_2)x} d^4x}_{(2\pi)^4 \delta(p_1+p_2-(k_1+k_2))}$$

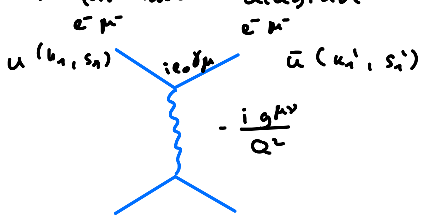
globalno ohranitev gib. kol.

$$\hookrightarrow \sigma \propto \overline{\sum |T_{fi}|^2}$$

$$\overline{\sum |T|^2} = \frac{1}{(2s_a+1)(2s_b+1)} \sum_{s_a s_b} \sum_{\substack{s_1\\ s_2}} |T_{s_a s_b s_1 s_2}|^2$$

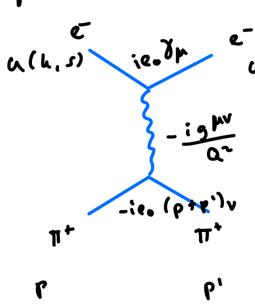
povprečimo po začetnem stanju (a,b) $\leftarrow$ spin ali heliciteta

Feynmanov diagram



Zgledi za izračun $\overline{\sum |T|^2}$, σ

ⓐ Sipanje $e^-\pi^+$



$$-iT_{ss'} = -e_0\, \overset{\bar{u}'}{\bar{u}(k',s')}\, \gamma_\mu\, \overset{u}{u(k,s)} \left(-\frac{g^{\mu\nu}}{Q^2}\right) e_0 (p+p')_\nu$$

$$\bar{u}' = u'^+\gamma^0$$

$$|T_{ss'}|^2 = T_{ss'} T_{ss'}^* = \left(\frac{e_0^2}{Q^2}\right)^2 \left(\bar{u}\gamma_\mu u (p+p')^\mu\right)\underbrace{\left(\bar{u}'\gamma_\nu u (p+p')^\nu\right)^+}_{*}$$

$$* = \left(u^+ \gamma_\nu^+ \gamma_0^+ u'\right)(p+p')^\nu =$$

$u^+\gamma_0 = \bar{u}$ $\qquad \gamma_0\gamma_0 = I \qquad \gamma_0^+ = \gamma_0$

$$[ABC]^+ = C^+ B^+ A^+$$

$$= \left(\bar{u} \left(\gamma_0 \gamma_\nu^+ \gamma_0 \right) u' \right) (p+p')^\nu$$

$$= (\bar{u} \; \gamma_\nu \, u') \, (p+p')^\nu$$

$$\overline{\sum |T_{ss'}|^2} = \left(\frac{e_0^2}{Q^2}\right)^2 \underbrace{\frac{1}{2} \sum_{ss'} (\bar{u}' \gamma_\mu u)(\bar{u} \gamma^\nu u')}_{L_{\mu\nu}} \underbrace{(p+p')^\mu (p+p')^\nu}_{T^{\mu\nu}}$$

Leptonski tenzor — Hadronski tenzor

Že vemo

$$\sum_{s=1,2} u(k,s) \, \bar{u}(k,s) = \not{k} + m = k^\mu \gamma_\mu + m I_4$$

vnanji produkt $\sum u_b \bar{u}_c = (\not{k} + m)_{bc}$

$$L_{\mu\nu} = \frac{1}{2} \sum_s \sum_{s'} \bar{u}'_a (\gamma_\mu)_{ab} u_b \, \bar{u}_c (\gamma_\nu)_{cd} u'_d =$$

$$= \frac{1}{2} (\not{k}' + m)_{da} (\gamma_\mu)_{ab} (\not{k} + m)_{bc} (\gamma_\nu)_{cd} =$$

$$= \frac{1}{2} \mathrm{Tr} \left((\not{k}' + m) \, \gamma_\mu \, (\not{k} + m) \, \gamma_\nu \right) = \ldots$$

Trace teorem:
$$\{ \gamma^\mu, \gamma^\nu \} = 2 g^{\mu\nu} I_4$$

$$\mathrm{Tr} \, I = 4$$
$$\mathrm{Tr} \, (\text{liho } \gamma) = 0$$
$$\mathrm{Tr} \, (\not{a} \, \not{b}) = 4 \, (a \cdot b)$$
$$\mathrm{Tr} \, (\not{a} \, \not{b} \, \not{c} \, \not{d}) = 4 \left((a \cdot b)(c \cdot d) + (a \cdot d)(b \cdot c) - (a \cdot c)(b \cdot d) \right)$$

$$\ldots = \frac{1}{2} \mathrm{Tr} \, (\not{k}' \gamma_\mu \not{k} \gamma_\nu) + \frac{1}{2} \mathrm{Tr} \, (m^2 \gamma_\mu \gamma_\nu) + 0 = \ldots$$

mešani členi, 3 γ

Pomožni postopek

$$\gamma_\mu a^\mu = \not{a} \qquad \gamma_\mu b^\mu = \not{b}$$
$$\mathrm{Tr} \, (\not{a} \, \not{b}) = 4 \, a \cdot b$$
$$a^\mu \mathrm{Tr} \, (\gamma_\mu \gamma_\nu) \, b^\nu = 4 \, a^\mu g_{\mu\nu} b^\nu$$

$$\mathrm{Tr} \, (\gamma_\mu \gamma_\nu) = 4 \, g_{\mu\nu}$$

$$\ldots \; L_{\mu\nu} = 2 \left(k'_\mu k_\nu + k'_\nu k_\mu + \frac{Q^2}{2} g_{\mu\nu} \right)$$

$$Q^2 = (k - k')^2 = 2 m_e^2 - 2 k k' = t$$

$$\overline{\sum |T|^2} = \Big(\underbrace{\frac{4\pi \alpha}{Q^2}}_{e_0^2 = 4\pi\alpha} \Big)^2 L_{\mu\nu} T^{\mu\nu} = \left(\frac{4\pi\alpha}{Q^2} \right)^2 8 \left(2 (p \cdot k)(p \cdot k') + \frac{Q^2}{2} M^2 \right)$$

masa piona

$m_e \neq 0$

V težiščnem sistemu piona $(M_\pi \gg m_e)$

$$\frac{d\sigma}{d\Omega} = \frac{\overline{\sum |T|^2} \, p_f}{64 \pi^2 p_i W^2}$$

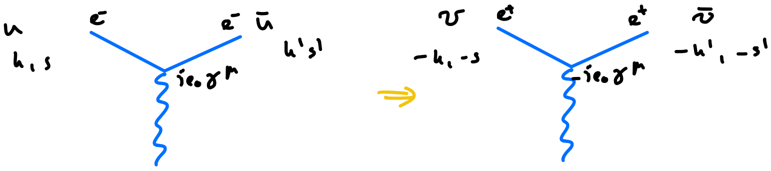
p_f — končna g.k. izstrelka (e^-)
p_i — začetna

$W^2 = E_{cm}^2 = s$
$p_\pi^\mu = (M, \vec{0})$

$$Q^2 = -4kk'\sin^2\frac{\theta}{2} \qquad k = |\vec{k}| = |\vec{p}_i|, \quad k' = |\vec{k}'| = |\vec{p}_f|$$

$$\Rightarrow \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4k^2\sin^4\frac{\theta}{2}}\cos^2\left(\frac{\theta}{2}\right)\frac{k'}{k} = \left(\frac{d\sigma}{d\Omega}\right)_{MOTT} \qquad \frac{k}{k'} = 1 + \frac{2k}{M}\sin^2\frac{\theta}{2}$$

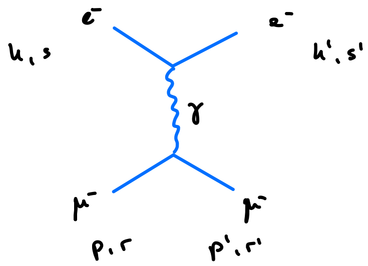
Spremembe v računu za $e^+\pi^+ \to e^+\pi^+$



$$T_{fi} \sim \bar{v}(k', s')(-ie_0\gamma_\mu)\, v(k, s)$$

$$\hookrightarrow \sum_{s=1,2} v\bar{v} = \not{k} - m$$

(b) sipanje $e^-\mu^- \to e^-\mu^-$



$$L^{(e)}_{\mu\nu} = 2\left(k'_\mu k_\nu + k'_\nu k_\mu + \frac{Q^2}{2} g_{\mu\nu}\right)$$

lažja notacija

$$L^{(\mu)\mu\nu} = M^{\mu\nu} = 2\left(p'^\mu p^\nu + p'^\nu p^\mu + \frac{Q^2}{2} g^{\mu\nu}\right)$$

$$Q^2 = (k - k')^2 = (p - p')^2$$

$$Q^2 = 2m_e^2 - 2kk'$$

$$Q^2 = 2m_\mu^2 - 2pp'$$

$$\overline{\sum |T|^2} = \left(\frac{e_0^2}{Q^2}\right)^2 L_{\mu\nu} M^{\mu\nu}$$

$$= 8\frac{e_0^4}{Q^4}\left((k'\cdot p')(k\cdot p) + (k'\cdot p)(k\cdot p') - m_e^2(p\cdot p') - m_\mu^2(k\cdot k') + 2m_e^2 m_\mu^2\right)$$

V težiščnem sistemu μ (μ je tarča) $\qquad p^\mu = (m_\mu = M, 0)$

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{MOTT}\left(1 - \frac{Q^2\tan^2\theta}{2M^2}\right)$$

V skupnem težiščnem sistemu $\qquad k = \left(\frac{W}{2}, \vec{p}_i\right) \quad p = \left(\frac{W}{2}, -\vec{p}_i\right)$

$\qquad k' = \left(\frac{W}{2}, \vec{p}_f\right) \quad p' = \left(\frac{W}{2}, -\vec{p}_f\right)$

Ultra-rel. limita $m_e = m_\mu = 0$

$$t = Q^2 = (k' - k)^2 = (0, \vec{p}_f - \vec{p}_i)^2 = -\frac{W^2}{2}(1 - \cos\theta)$$

$$\vec{p}_i \cdot \vec{p}_f = \frac{W^2}{4}\cos\theta$$

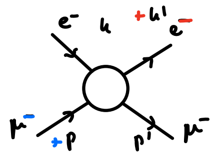
$$k\cdot p = \frac{W^2}{4} + p_i^2 = \frac{W^2}{2} = k'\cdot p'$$

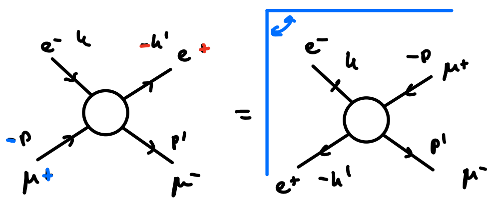
$$k'\cdot p = \frac{W^2}{4} - \vec{p}_f\cdot(-\vec{p}_i) = \frac{W^2}{4}(1 + \cos\theta) = k\cdot p'$$

$$\Rightarrow \overline{\sum |T|^2} = \frac{2e_0^4\,(4 + (1 + \cos\theta)^2)}{(1 - \cos\theta)^2}$$

$$\frac{d\sigma}{d\Omega} = \alpha^2 \frac{1}{2} \frac{1}{\omega^2} \frac{(4+(1+\cos\theta)^2)}{(1-\cos\theta)^2} \sim \frac{1}{s}$$

Proces $e^+e^- \to \mu^+\mu^-$ "crossing / križanje"





		Vhodni		Izhodni	
$e^-\mu^- \to e^-\mu^-$		p_a	p_b	p_1	p_2
		k	p	k'	p'
$e^-e^+ \to \mu^-\mu^+$		k	$-k'$	$-p$	p'

Samo zamenjamo g.k.

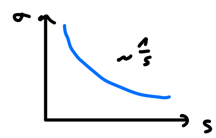
$$Q^2 = (p+k)^2 = \omega^2 = s$$

$$\overline{\sum |T|^2} = \frac{8e_0^4}{Q^4}\left((-p\cdot p')(-k\cdot k') + (-p\cdot(-k'))(k\cdot p') + \underset{\text{---}}{\overset{\text{zanemarimo mase}}{}} \right)$$

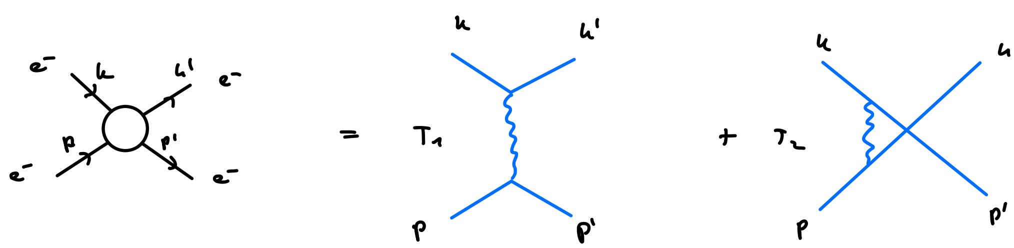
$$p\cdot p' = \frac{\omega^2}{4}(1-\cos\theta) = k\cdot k'$$

$$\frac{d\sigma}{d\Omega} = \underbrace{\frac{e_0^4}{64\pi^2}}_{\sim\alpha^2} \underbrace{\frac{1}{\omega^2}}_{\sim\frac{1}{s}} (1+\cos^2\theta)$$

$$\sigma = \alpha^2 \frac{4\pi}{3}\frac{1}{s}$$

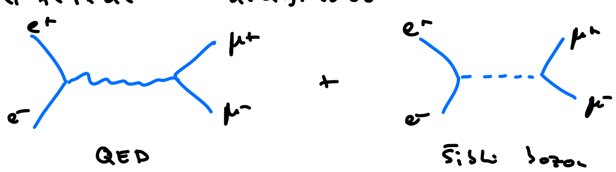


Zgled $e^-e^- \to e^-e^-$

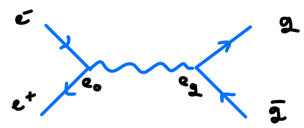


$$T = T_1 + T_2$$
$$\downarrow$$
$$\overline{\sum |T|^2}$$

Interferenca diagramov



Sipanje $e^-e^+ \to q\bar{q}$



lahki kvarki $m_q \approx 0$

$$\frac{\sigma(e^-e^+ \to q\bar{q})}{\sigma(e^-e^+ \to \mu^+\mu^-)} = \frac{e_0^2\, e_q^2}{e_0^4} = \frac{e_q^2}{e_0^2} \; (\cdot 3)$$

za enake kvarke

zaradi barv

Kateri kvarki lahko nastajajo?

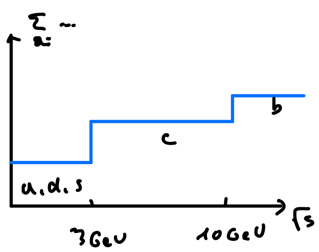
- kinematično (al.) : ustvarjeno kvarke q_i , $\sqrt{s} \geq 2 m_{q_i}$

npr. $\sqrt{s} < 3\,\mathrm{GeV}$ $\quad q_i = u, d, s$

$$\Rightarrow \sum_{q_i} 3 \left(\frac{e_{q_i}}{e_0}\right)^2 = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) = 2$$

$3\,\mathrm{GeV} < \sqrt{s} < 10\,\mathrm{GeV}$ $\quad q_i = u, d, s, c$ $\quad \Rightarrow \quad 3{,}3$

Meritev



Šibka interakcija

- Razpad π : $\tau(\pi^-) \doteq 2{,}6 \cdot 10^{-8}\,\mathrm{s}$

$\tau(\pi^0) \doteq 8{,}4 \cdot 10^{-17}\,\mathrm{s}$ $\quad$ hitrejši razpad $\Leftrightarrow$ močnejša interakcija

lepton μ, e

$\pi^- \to l\, \bar{\nu}_e$ $\quad$ šibka interakcija

$\pi^0 \to \gamma\gamma$ $\quad$ EM interakcija

- Mejniki: Fermi opiše razpad β : $^{10}C \to {}^{10}Be\; e^+ \nu_e$

$p \to n\, e^+ \nu_e$

- EM, močna int. ohranjata parnost, šibka pa ne

$^{60}_{27}Co \to {}^{60}_{28}Ni \quad e^- \quad \bar{\nu}_e \quad 2\gamma$

v močnem $\vec{B}$ $\quad \uparrow \vec{B} \quad \underset{J_z=5}{\overset{^{60}Co}{\uparrow}} \;\to\; \underset{J_z=4}{\overset{^{60}Ni}{\uparrow}} \quad \underset{J_z=\frac{1}{2}}{\overset{e^-}{\uparrow}} \quad \underset{J_z=\frac{1}{2}}{\overset{\bar{\nu}_e}{\uparrow}}$

Smer letenja $\quad \overset{e^-}{\vec{s}\uparrow\uparrow} \quad \overset{\bar{\nu}_e}{\vec{s}\uparrow\downarrow} \qquad \overset{e^-}{\vec{s}\uparrow\downarrow} \quad \overset{\bar{\nu}_e}{\uparrow\uparrow\vec{s}} \qquad \uparrow \vec{B}$

Sučnost / helicity λ $\quad \underbrace{1 \quad -1}_{\text{tega niso našli}} \qquad -1 \quad 1$

Helicity (sučnost) fermionov : $h = \vec{\Sigma} \cdot \hat{p} = \begin{bmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{bmatrix} \frac{\vec{p}}{p_0}$

$\downarrow$ operator parnosti

$h \to -h$

Če interakcija ohranja parnost, potem morajo vsa stanja slučnosti nastopati enakovredno (simetrično). Je kršena za šibke sile, parnost maksimalno kršena

! V šibkih interakcijah se $\hat{P}$ (parnost) ne ohranja, $\hat{C}\hat{P}$ (charge parity) pa se.

[tudi $\hat{C}\hat{P}$ kršena (zelo malo), v razpadih K^0]

→ Rešitev / razlaga : nova količina : ročnost
Zakaj obstajeta le levo-sučni neutrino in desno-sučni antineutrino ?

V Diracovi enačbi, γ^μ : $\hat{P} = \gamma_0$ operator parnosti na rešitvi D.E.

$$\gamma_0 \, u((p_0,\vec{p}),s) = u((p_0,-\vec{p}),s)$$

Vpeljemo operator ročnosti $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

$$\gamma_5 = \begin{pmatrix} 0 & I_{2\times2} \\ I_{2\times2} & 0 \end{pmatrix} \quad (\gamma_5)^2 = I \quad (\gamma_5)^+ = \gamma_5 \quad \{\gamma_5, \gamma_\mu\} = 0 \quad \gamma_0 \vec{\gamma} \gamma_0 = -\vec{\gamma}$$

V D.E. el. tok res vektorski tok: $\vec{j}_e \xrightarrow{\hat{P}} -\vec{j}_e$

$$\vec{j}_e \sim \bar{u}(\vec{k}',s')\,\vec{\gamma}\,u(\vec{k},s)$$
$$\Downarrow \hat{P}$$
$$-\bar{u}(-\vec{k}',s')\,\vec{\gamma}\,u(-\vec{k},s) = -\bar{u}(\vec{k}',s')\,\gamma_0\,\vec{\gamma}\,\gamma_0\,u(k,s) = \bar{u}(\vec{k}',s)\,\vec{\gamma}\,u(k,s)$$

Aksialni tok

$$A^\mu \sim \bar{u}\gamma^\mu\gamma_5 u$$
$$\Downarrow \hat{P}$$
$$\bar{u}\,\gamma_0\,\gamma^\mu\,\gamma_5\,\gamma_0\,u$$

$$A'^\mu = (-A_0, \vec{A})$$

Pseudo-skalar $\bar{u}\gamma_5 u$, skalar $\bar{u} I u$

- Z rešitvijo D.E. $u(k,s)$ lahko skonstruiramo

$$u = \underbrace{\left(\frac{1+\gamma_5}{2}\right)}_{P_R} u + \underbrace{\left(\frac{1-\gamma_5}{2}\right)}_{P_L} u$$

desno in levo ročni projektor : chirality projector

$P_R u = u_R$ desno ročna komponenta $\quad P_L u = u_L$ levo ročna komponenta

$\gamma_5 u_{L,R} = ?$

$$\gamma_5 u_R = \gamma_5\left(\frac{I+\gamma_5}{2}\right)u = \frac{1}{2}(\gamma_5 + 1)u = u_R \checkmark$$

lastna stanja ročnosti

$$\gamma_5 u_L = -u_L \checkmark$$

Velja tudi: $P_L^2 = P_L$, $P_R^2 = P_R$ ker so projektorji

$$P_L P_R = P_R P_L = 0$$

- Zveza med ročnostjo in sučnostjo
  $m \approx 0$ ultra-rel. lim.

$$(\not p - m)u = 0 \Rightarrow \not p u = 0 \Rightarrow \gamma^\mu p_\mu u = 0 \Rightarrow (\gamma^0 p_0 - \vec\gamma\cdot\vec p)u = 0$$

$\gamma^5\gamma^0 \cdot /$ $\qquad (\gamma^5 p_0 - \vec\Sigma\cdot\vec p)\,u = 0 \qquad \vec\Sigma = \gamma^5\gamma^0\vec\gamma = \begin{pmatrix}\vec\sigma & 0\\ 0 & \vec\sigma\end{pmatrix}$

$p_0 : /$ $\qquad \gamma^5 u = \vec\Sigma\,\hat p\, u$

ročnost $\qquad$ sučnost

- Brezmasni nevtrino: $u_L = \overbrace{\frac12(1-\gamma_5)}^{P_L \text{ za delce}}u$ levo-ročni nevtrino $\Rightarrow$ levo-sučni $(\lambda = -1)$

  u_R ne obstaja

  $v_R = \underbrace{\frac12(1-\gamma_5)}_{P_R \text{ za antidelce}}v$ desno-ročni antinevtrino $\rightarrow$ desno-sučni $(\lambda = 1)$

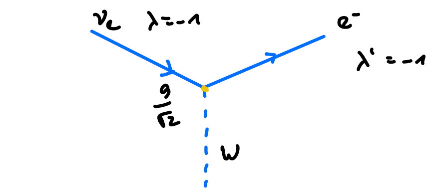
- Transition current: EM: $j_\mu^{EM} = \bar u\gamma_\mu u$

$$= (u_L^\dagger + u_R^\dagger)\gamma_0\gamma_\mu(u_L + u_R)$$

$$= \bar u_R\gamma_\mu u_R + \bar u_L\gamma_\mu u_L$$

chirality helicity conserved

- Kaj pa za šibko int.



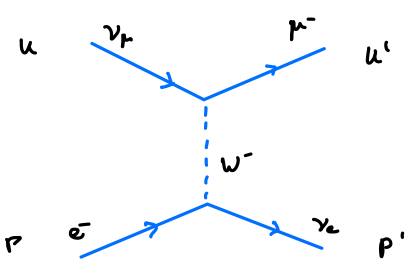
levo sučni

ν_{eL} , e_L samo

šibka int. $\frac{g}{\sqrt2}\bar u_L\gamma^\mu u_L = \frac{g}{\sqrt2}\bar u(e^-)\gamma^\mu\frac12(1-\gamma_5)u(\nu_e) \sim j_W^\mu$

šibki prehodni tok

- Primer: $e^-\nu_\mu \to \mu^-\nu_e$



Propagator za $W^\pm$: $\dfrac{-g^{\mu\nu} + \frac{Q^\mu Q^\nu}{M_W^2}}{Q^2 - M_W^2}$

$M_W \doteq 80$ GeV

Če $|Q^2| \ll M_W^2$ $\Rightarrow$ Fermijev model: $\frac{G_F}{\sqrt2} = \frac{g^2}{8M_W}$

mat. element $\quad T = \frac{G_F}{\sqrt2}\,\bar u(\mu^-)\gamma_\mu(1-\gamma_5)u(\nu_\mu)\,g^{\mu\nu}\,\bar u(\nu_e)\gamma_\nu(1-\gamma_5)u(e^-)$

$$\overline{\sum |T|^2} = \frac{1}{2} \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} |T|^2$$

lahko seštevamo po vseh sučnostih, ker napačne, dajo projekcijo 0.

$$= \frac{G_F^2}{2} \underbrace{\mathrm{Tr}\left(\not k' \gamma_\mu (1-\gamma_5) \not k \gamma_\nu (1-\gamma_5)\right)}_{N_{\mu\nu} \quad \text{mion } \nu_\mu \to \mu^-} \cdot \underbrace{\frac{1}{2}\mathrm{Tr}\left(\not p' \gamma^\mu (1-\gamma_5) \not p \gamma^\nu (1-\gamma_5)\right)}_{E^{\mu\nu} \quad \text{elektron } e^- \to \nu_e}$$

$$= \frac{G_F^2}{2} N_{\mu\nu} E^{\mu\nu}$$

Trik: $(1-\gamma_5)^2 = 2(1-\gamma_5)$

$P^2 = P \qquad P = \frac{1-\gamma_5}{2}$

$$N_{\mu\nu} = 2\mathrm{Tr}\left(\not k' \gamma_\mu (1-\gamma_5) \not k \gamma_\nu \right) = 2\mathrm{Tr}\left(\not k' \gamma_\mu \not k \gamma_\nu \right) - 2\mathrm{Tr}\left(\gamma_5 \not k \gamma_\nu \not k' \gamma_\mu \right)$$

$$= 8\left(k'_\mu k_\nu + k'_\nu k_\mu + \frac{Q^2}{2} g_{\mu\nu} \right) - 8i\, \varepsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta$$

$$E^{\mu\nu} = 4\left(p'^\mu p^\nu + p'^\nu p^\mu + \frac{Q^2}{2} g^{\mu\nu} \right) - 4i\, \varepsilon^{\mu\nu\gamma\delta} p_\gamma p'_\delta$$

Še en trik $\quad p' = p + Q \quad \Rightarrow \quad Q^\mu N_{\mu\nu} = Q^\nu N_{\mu\nu} = 0$

$\rightarrow$ se znebimo p' v $E^{\mu\nu}$

$\vdots$

$$\overline{\sum |T|^2} = \frac{G_F^2}{2} \cdot 32 s^2$$

$s = (k+p)^2 = (k'+p')^2$ kvadrat težiščne energije

brezmasno

$s = k^2 + p^2 + 2k\cdot p = 2k\cdot p$

analogno

$t = Q^2 = (k-k')^2 = -2k\cdot k'$

V težiščnem sistemu

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \overline{\sum |T|^2}$$

$t = -2p^2(1-\cos\theta) \quad ; \quad p = |\vec p|$

$s = 4p^2$

$\frac{d\sigma}{dt} = ? \qquad dt = \frac{1}{2\pi} 2p^2 \, d\Omega \qquad \frac{d}{dt} = \frac{\pi}{p^2}\frac{d}{d\Omega} = \frac{4\pi}{s}\frac{d}{d\Omega}$

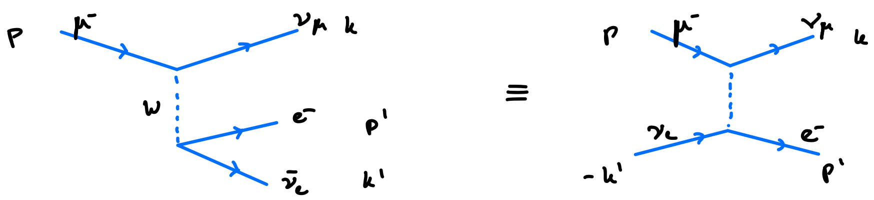
$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} \overline{\sum |T|^2}$$

Za $\nu_\mu e^- \to \mu^- \nu_e$: $\frac{d\sigma}{dt} = \frac{G_F^2}{\pi} \quad \Rightarrow \quad \sigma = \frac{4 G_F^2}{\pi} p^2$ (Lab) : $\sigma \sim s$

$p^2 = \frac{1}{2} m_e E_\nu$

$\Rightarrow \sigma \sim E_\nu$

Razpad miona



$v(k') = u(-k')$

$$T = \frac{G_F}{\sqrt{2}}\left(\bar u(k)\gamma^\mu(1-\gamma_5)u(p)\right)\left(\bar u(p')\gamma_\mu(1-\gamma_5)v(k')\right)$$

če $m_\nu = 0$ $\quad \overline{\sum |T|^2}$ odpadejo ostale mase $(m_\nu^2 m_e^2 \to 0)$

$$\overline{\sum |T|^2} = 64\, G_F^2\, (k\cdot p')(p\cdot k')$$

↳ $d\Gamma = \frac{\overline{\sum |T|^2}}{2M}\, dLIPS$ fazni prostor končnega stanja

masa delca, ki razpade

Konkreten račun: $M = m_\mu$: T.S. μ^- ; $m_e = 0$

μ^-: $p = (m_\mu, 0)$

ν: $k' = (\omega', \vec k')$ $\qquad (\vec k')^2 = 0$

$k = (\omega, \vec k)$ $\qquad (\vec k)^2 = 0$

e^-: $p' = (E', \vec p')$ $\qquad (\vec p')^2 = 0$

brezmasni

$(k-p')^2 = -2k\cdot p' \Rightarrow k\cdot p' = -\frac{1}{2}(k-p')^2$

$p\cdot k' = m_\mu \omega'$

$p'\cdot k = \frac{1}{2} m_\mu - m_\mu\omega'$

$$\overline{\sum |T|^2} = G_F^2\, 32\, (m_\mu^2 - 2m_\mu\omega')(m_\mu\omega')$$

$$= G_F^2\, 32\, m_\mu^2 (m_\mu - 2\omega')\omega'$$

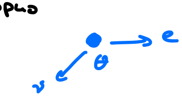
$$dLIPS = \frac{d^3p'}{2E'}\frac{d^3k}{2\omega}\frac{d^3k'}{2\omega'}\frac{1}{(2\pi)^5}\delta^{(4)}(p-p'-k-k')$$

Nadomestimo nazaj $\quad \int \frac{d^3k}{2\omega} = \int \theta(\omega)\,\delta(k^2)\, \underset{d\omega d^3k}{d^4k}$

Integriramo po d^4k : $\frac{d^3p'}{2E'}\frac{d^3k'}{2\omega'}\frac{1}{(2\pi)^5}\int d^4k\, \theta(\omega)\delta(k^2)\delta^4(p-p'-k-k') \overset{\text{zaradi } \delta}{=}$

$$= \frac{d^3p'}{2E'}\frac{d^3k'}{2\omega'}\frac{1}{(2\pi)^5}\,\theta(E-E'-\omega')\,\delta\left((p-p'-k')^2\right)$$

$d^3p' = 4\pi E'^2\, dE'$ $\quad (m_e = 0)$ $\quad e^-$ leti izotropno



$d^3k' = 2\pi\omega'\, d\omega'\, d\cos\theta$

gleda na prvi delec

$\delta(ax) = \frac{1}{|a|}\delta(x) \Rightarrow \delta\left((p-p'-k')^2\right) = \delta\left(m_\mu^2 - 2m_\mu E' - 2m_\mu\omega' + 2E'\omega'(1-\cos\theta)\right)$

$$= \frac{1}{2E'\omega'}\delta(\cos - A) \quad ; \quad A = \frac{m_\mu^2 - 2m_\mu E' + 2m_\mu\omega'}{2E'\omega'} + 1$$

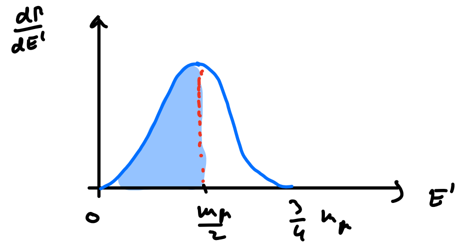
$$d\Gamma = \frac{G_F^2}{2\pi^3} m_\mu \omega' (m_\mu - 2\omega') \, dE' \, d\omega' \, \delta(\cos\theta - 1) \, d(\cos\theta)$$

Meje: $E' \in [0, \frac{m_\mu}{2}]$ $\quad \omega' \in [\frac{m_\mu}{2} - E', \frac{m_\mu}{2}]$

$$d\Gamma = \frac{G_F^2}{2\pi^3} m_\mu \, dE' \int_{\frac{m_\mu}{2} - E'}^{\frac{m_\mu}{2}} d\omega' (m_\mu - 2\omega') \quad = \quad \frac{d\Gamma}{dE'} = \frac{G_F^2}{12\pi^3} m_\mu^2 E'^2 \left(3 - \frac{4E'}{m_\mu}\right)$$

$$\Gamma = \frac{G_F^2}{192\pi^3} m_\mu^5$$

$$\tau = \frac{(\hbar)}{\Gamma} = 2{,}1\,\mu s$$

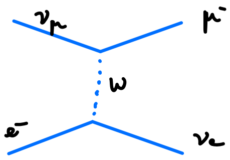


Analogno

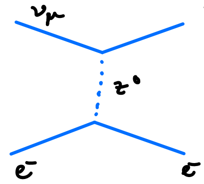
$$\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = \frac{G_F^2}{192\pi^3} m_\tau^5 = \Gamma_\tau \quad \Rightarrow \quad \frac{\Gamma_\mu}{\Gamma_\tau} = \frac{m_\mu^5}{m_\tau^5} \left(\frac{G_F^2}{G_F^2}\right)$$

Standardni model pravi
$G_F(\tau) = G_F(\mu)$
univerzalnost leptonske okuse

Charged weak current: CC



Neutral weak current: NC



$$j_\mu^{NC} = g_N \bar{u}_f \gamma_\mu \left(c_L \frac{(1-\gamma_5)}{2} + c_R \frac{(1+\gamma_5)}{2}\right) u_i$$

Odvisno od okusa fermionov $\quad c_L^\nu = \frac{1}{2}$

$c_R^\nu = 0$

$l = e^-, \mu^-, \tau^-$

$c_L^l = -\frac{1}{2} + a$

$c_R^l = a$

$W(CC)$: $\quad g = \frac{e_0}{\sin\theta_W}$

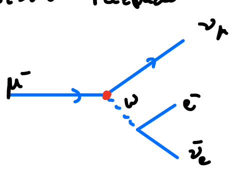
$g_N = \frac{g}{\cos\theta_W}$

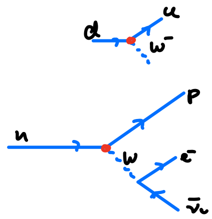
$a = \sin^2\theta_W = 0{,}23$

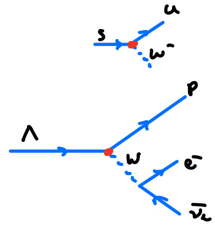
Weinbergov kot

Šibki procesi s kvarki

Šibki razpadi







Sklopitve so različne

$\Delta S = 0$ (čudnost) $\qquad \Delta S = 1$ čudnost se spremeni

$e \to \nu_e : g$ $\qquad d \to u: \; g \cos\theta_C$ $\qquad s \to u: \; g \sin\theta_C$

$\mu \to \nu_\mu : g$

θ_C ... Cabibbov kot $\quad \sin\theta_C \doteq 0{,}23 \qquad \theta_C = 13{,}7°$

Matrični zapis Cabibbovega modela

$$j^{W^+} = \frac{g}{\sqrt2} \overbrace{\left(\underset{\text{kvark}}{\bar u} \,,\, \underset{\bar u(d)}{\bar d} \cos\theta_c + \underset{\bar u(s)}{\bar s} \sin\theta_c \right)}^{\bar{\mathfrak q}} \overset{\frac{1-\gamma_5}{2}}{P_L} \underbrace{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}_{\sigma_+} \overbrace{\begin{pmatrix} u \\ \underset{u(d)}{d}\cos\theta_c + \underset{u(s)}{s}\sin\theta_c \end{pmatrix}}^{\mathfrak q}$$

$$j^{W^+} = \underbrace{\frac{g}{\sqrt2}\, \bar u P_L d \cos\theta_c}_{\pi^- \to e^- \bar\nu_e} + \underbrace{\frac{g}{\sqrt2}\, \bar u P_L s \sin\theta_c}_{K^- \to e^- \bar\nu_e} \qquad \sigma_\pm = \frac{\sigma_1 \pm i\sigma_2}{2}$$

$$j^{W^-} = \frac{g}{\sqrt2}\, \bar{\mathfrak q}\, P_L \underbrace{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}}_{\sigma_-} \mathfrak q$$

$$j^{W^+} = \bar{\mathfrak q} P_L \sigma^+ \mathfrak q = \bar{\mathfrak q}_L \sigma^+ \mathfrak q_L \qquad j^{W^-} = \bar{\mathfrak q} P_L \sigma^- \mathfrak q = \bar{\mathfrak q}_L \sigma^- \mathfrak q_L$$

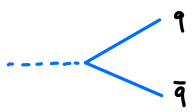
$$\mathfrak q = \begin{pmatrix} u \\ d\cos\theta + s\sin\theta \end{pmatrix} = \begin{pmatrix} u \\ d_c \end{pmatrix}$$

lastno stanje prostega hamiltonjana / kin. energije

Cabibbo rotated state

Eksperiment $\quad \dfrac{\Gamma(K^+ \to \mu^+\nu_\mu)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} = \tan^2\theta_c$

Nevtralni tokovi kvarkov (NC)



q: $u, d, s, \dots, e^-, \mu^-, \nu_e$

$\bar q$: $\bar u, \bar d, \bar s, \dots, e^+, \mu^+, \bar\nu_e$

$$j^{NC} = g_N\, \bar u(q) \underbrace{\gamma_\mu \left(c_L^q \frac{1-\gamma_5}{2} + c_R^q \frac{1+\gamma_5}{2} \right)}_{P^{NC}} u(q)$$

$$g_N = \frac{g}{\cos\theta_W} \qquad c_L^u = \frac12 - \frac23 a \,,\quad c_L^d = -\frac12 + \frac13 a \qquad a = \sin^2\theta_W = 0{,}23$$

up-like: u, c, t $\qquad$ down-like: d, c, s

$$c_R^u = -\frac23 a \qquad c_R^d = \frac13 a$$

NC + Cabibbo = Glashow-Iliopoulos-Maiani (GIM) mehanizem

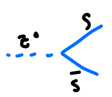
$$j^{W^+} \sim \bar{\mathfrak q} \frac{\sigma^+}{2} \mathfrak q \qquad j^{W^-} \sim \bar{\mathfrak q} \frac{\sigma^-}{2} \mathfrak q \qquad \text{CC + Cabibbo}$$

$$\hookrightarrow \begin{pmatrix} u \\ d_c \end{pmatrix}$$

$$\Rightarrow \text{NC} \qquad \frac{\sigma^+}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \frac{\sigma^-}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad \Rightarrow \quad \left[\frac{\sigma^+}{2}, \frac{\sigma^-}{2} \right] = \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \quad j^{NC} \sim \bar{\mathfrak q}\, \sigma_3\, \mathfrak q = (\bar u, \bar d_c) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} u \\ d_c \end{pmatrix} = \bar u \overset{P^{NC}}{u} - \bar d_c \overset{P^{NC}}{d_c} =$$

$$= \bar u u - \left(\bar d d \cos^2\theta_c + \bar s s \sin^2\theta_c + \bar s d \sin\theta_c \cos\theta_c + s \bar d \sin\theta_c \cos\theta_c \right)$$



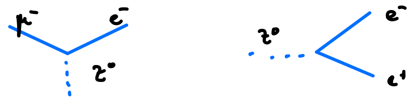
✓ enak okus

Kaj pa

različen okus

Flavour changing neutral currents (FCNC) $\quad$ // ne obstaja.

Analogno za leptone



lepton flavour violation (LFV) // ne obstaja

GIM mehanizem: obstaja še en prispevek, ki * (FCNC) odšteje

$$j^{NC} \sim \bar{q}\,\sigma_3\,q + \bar{q}'\,\sigma_3\,q' \qquad q' = \begin{pmatrix} c \\ s_c \end{pmatrix}$$

odkritje pozneje

$$s_c = -d\sin\theta_c + s\cos\theta_c$$

$$= \bar{u}u + \bar{c}c + \bar{d}d + \bar{s}s \quad ✓$$

Sedaj imamo dva dubleta $\begin{pmatrix} u \\ d_c \end{pmatrix} \begin{pmatrix} c \\ s_c \end{pmatrix}$ $\overset{\text{kompaktno}}{\Rightarrow}$ $\begin{pmatrix} d_c \\ s_c \end{pmatrix} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$ $\quad c = \cos\theta_c$, $s = \sin\theta_c$

$$j^{W^+} \sim \frac{g}{\sqrt{2}} \underbrace{(\bar{u}, \bar{c})}_{\bar{u}} \gamma^\mu \frac{1-\gamma_5}{2} \underbrace{\begin{pmatrix} d_c \\ s_c \end{pmatrix}}_{D}$$

$\overset{\text{danes še}}{\Rightarrow}$ $\begin{pmatrix} t \\ b \end{pmatrix} \Rightarrow \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underline{V} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$ $\qquad \underline{V} = V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$

Po SM je V_{CKM} unitarna $(V^\dagger V = V^{-1} V = I)$ $\Rightarrow$ ima 4 parametre

$$j^{W+} = \frac{g}{\sqrt{2}}\, \bar{u}\, \gamma^\mu \frac{1-\gamma_5}{2} V_{CKM}\, D \qquad u = \begin{pmatrix} u \\ c \\ t \end{pmatrix} \quad D = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\Gamma \sim |V_{CKM}|^2$$

Wolfensteinova parametrizacija CKM

$$\begin{bmatrix} 1-\lambda^2 & \lambda & A\lambda^3(\rho + i\eta) \\ -\lambda & 1-\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & A\lambda^2 & 1 \end{bmatrix}$$

$A = 0{,}876$

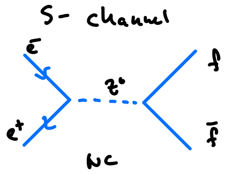
$\lambda = \sin\theta_c = 0{,}23$

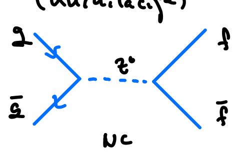
$\rho = 0{,}121$

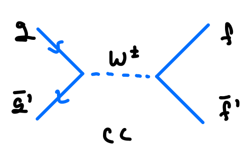
$\eta = 0{,}355$

Resonančna produkcija šibkih bozonov

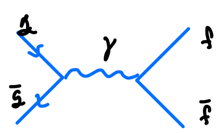
S- channel procesi: (anihilacija)







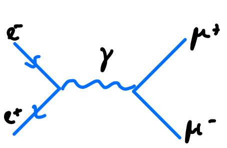
Zraven še EM (QED)

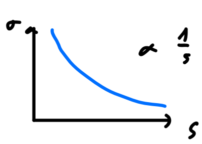


interferira z NC

$$iT = i\underset{Z^0}{T_{NC}} + i\underset{\gamma}{T_{EM}}$$

EM





Masivna vmesna stanja

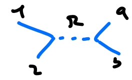
$M^2_{virt} = s$ masa virtualnega delca (nestabilno stanje)

Propagator oblike $\sim \frac{1}{s-M^2+iM\Gamma}$ M ... masa delca

Γ ... razpadna širina $(\sim \frac{1}{\tau})$ $\Gamma = \sum_{\text{razpadi}} \Gamma_i$

Števec odvisen od vrste delca (spin, ...)
Šibki bozoni s spinom s=1 $\sim \frac{-(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{M^2})}{Q^2-M^2+iM\Gamma}$ $Q^2=s>0$, $Q^\mu = p_1^\mu+p_2^\mu$

Relativistična formula za nastanek in razpad resonance R

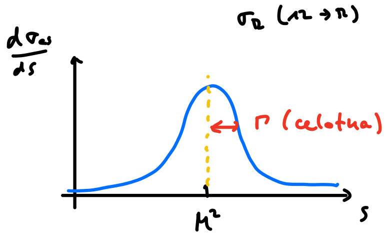


$$\frac{d\sigma_{ab}}{ds} = \underbrace{\frac{d\sigma_R}{ds}}_{\substack{\text{presek za}\\ \text{nastanek}\\ \text{delca /}\\ \text{resonance}\\ \sigma_R\,(12\to R)}} \frac{\pi^{-1}\sqrt{s}\;\Gamma_{R\to ab}(\sqrt{s})}{(s-M^2)^2+M^2\Gamma^2(\sqrt{s})}$$

Γ(√s) ... celotna razpadna širina

R → ab
cd
ef
⋮

Relativistična oblika
(BW) Breit-Wignerjeve funkcije



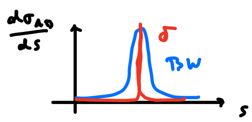
Nerelativistična $\frac{\pi^{-1} M}{(s-M^2)^2+M^2\Gamma^2(M)}$

Interferenca: $Z^0+\gamma$

BW naloži na $1/s$

Narrow width approximation

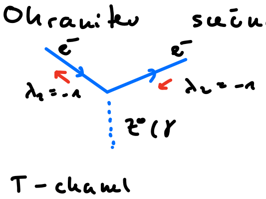
→ širina resonance je zanemarljiva $\frac{\Gamma}{M} \ll 1$

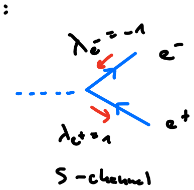


Kotne porazdelitve pri resonančni produkciji: (EM + Z^0 + ...)

Helicity formalism: pri velikih E oz. $m\to 0$: sučnost ⇔ ročnost

Ohranitev sučnosti:

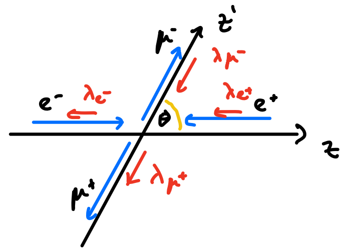




Lab: težiščni sistem

$e^+e^- \to \mu^+\mu^-$

$S_{e^-}^z = -\frac{1}{2}$



$S_{e^+}^z = -\frac{1}{2}$

$\longrightarrow$ $J^z = -1$

dobro kvantno število pred reakcijo

$J_z' = -1$ dobro kvantno število po reakciji

rot okoli osi y

Iščem $\underset{J_z \to J_z'}{iT(z \to z')}$ = rotacijska matrika $\sim e^{-iJ_y\theta}$

$J = 1$ vmesno stanje s spin 1 (γ, Z^0)

$$iT = \underbrace{\langle J J_z' |\, e^{-i\theta \hat{J}_y} \,| J J_z \rangle}_{d^J_{J_z J_z'}(\theta)}$$

Wignerjeva funkcija

$$d^1_{11}(\theta) = d^1_{-1-1}(\theta) = \frac{1}{2}(1+\cos\theta)$$
$$d^1_{-11}(\theta) = d^1_{1-1}(\theta) = \frac{1}{2}(1-\cos\theta)$$

$iT_{\lambda_1\lambda_2\lambda_3\lambda_4} \sim d^1_{J_zJ_z'}$

$J_z = \frac{1}{2}(\lambda_1 - \lambda_2)$
$J_{z'} = \frac{1}{2}(\lambda_3 - \lambda_4)$

$$\overline{\sum_\lambda |T|^2} \sim \sum_{\text{dovoljene možnosti}} (d)^2$$

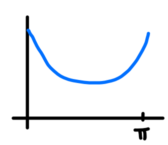
V tež. sistemu $\frac{d\sigma}{d\Omega} = \frac{1}{2\pi}\frac{d\sigma}{d\cos\theta} = \frac{1}{64\pi^2 s}\overline{\sum|T|^2}$

kotna odvisnost

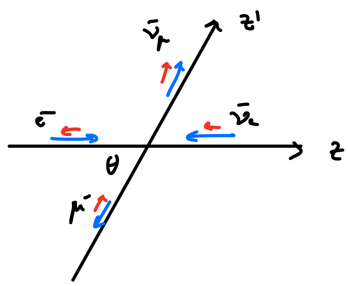
$$\overline{\sum|T|^2} = f(\sqrt{s}) \sum d^2$$

Zgled: EM, vse $\lambda_i = \pm 1$ dovoljene, dokler se sučnost ohranja $(J=1)$, vse možnosti so enakovredne

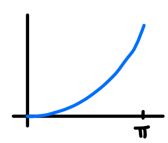
$\sum_{J_zJ_z'} |d^1_{J_zJ_z'}|^2 \sim (1+\cos^2\theta)$ $\Rightarrow$ kotna odvisnost $\theta \in [0,\pi]$



Zgled: šibka sila $^{(CC)}$, dovoljena le ena konfiguracija, le levoročni



$\frac{d\sigma}{d\cos\theta} \sim (1-\cos\theta)^2$



Za NC Z^0/γ $\frac{d\sigma}{d\cos\theta} \sim \underbrace{(1+\cos\theta)^2}_{EM} + A_{FB}\cos\theta$ $\qquad A_{FB} \sim \frac{c_L^2 - c_R^2}{c_L^2 + c_R^2}$

Forward-backward asymmetry