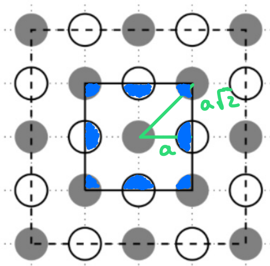
4.1 Madelungova konstanta

4.1 Spodnja skica prikazuje mrežo dvodimenzionalnega ionskega kristala iz N pozitivnih in N negativnih ionov z nabojem $\pm e_0$ na kvadratni mreži. Izračunaj prva dva člena vrste za izračun Madelungove konstante, pri čemer za prvi člen upoštevaj le del naboja znotraj manjšega kvadrata, za drugi člen pa del naboja, ki se nahaja zunaj manjšega in znotraj večjega kvadrata. Določi ravnovesno razdaljo med najbližjimi sosedi, če veš, da je vezavna energija kristala $2N \cdot 5$ eV, ionizacijska energija 1,5 eV ter elektronska afiniteta 1,0 eV. Odbojni del potenciala med najbližjimi sosedi ima obliko $1/r^{12}$.

$W_i = 1{,}5\,eV$

$W_a = 1\,eV$

Vezavna energija $E = ?$



$$W = -\frac{e_0^2}{4\pi\varepsilon_0}\,\frac{1}{2}\underbrace{\sum_{ij}\frac{1}{|\vec r_i-\vec r_j|}}_{\text{označimo } \alpha}$$

$$\alpha_1 = \left(4\,\frac{1}{2}\,\frac{1}{a} - 4\,\frac{1}{4}\,\frac{1}{a\sqrt{2}}\right) = \frac{1}{a}\cdot 1{,}29 \qquad \text{Prvi kvadrat}$$

$$\alpha_2 = \left(4\,\frac{1}{2}\,\frac{1}{a} - 4\,\frac{3}{4}\,\frac{1}{a\sqrt{2}} - 4\,\frac{1}{2}\,\frac{1}{2a} + 8\,\frac{1}{2}\,\frac{1}{a\sqrt{5}} - 4\,\frac{1}{4}\,\frac{1}{2a\sqrt{2}}\right) = \frac{1}{a}\cdot 0{,}314$$

$$\alpha_1+\alpha_2 = 1{,}6\cdot\frac{1}{a}$$

$$W = -\frac{e_0^2}{4\pi\varepsilon_0}\,\frac{1}{2}\,\frac{1}{a}\,1{,}6$$

$$-2N\,W_0 = N W_i - N W_a - \frac{e^2\tilde\alpha}{4\pi\varepsilon_0 r}\,2N\,\frac{1}{2} + \frac{A}{r^{12}} \qquad \tilde\alpha = \alpha\cdot a$$

Ravnovesni pogoj

$$\left.\frac{\partial W(r)}{\partial r}\right|_{r=a} = 0$$

$$-2N\,\frac{\partial W_0}{\partial r} = -\frac{e_0^2 N\tilde\alpha}{4\pi\varepsilon_0}\left(-\frac{1}{r^2}\right) + \left.\frac{(-12)A}{r^{13}}\right|_{r=a} = 0$$

$$\frac{N\tilde\alpha e^2}{4\pi\varepsilon_0}\,\frac{1}{a^2} = \frac{12A}{a^{13}}$$

$$A = \frac{N\tilde\alpha e^2}{4\pi\varepsilon_0}\,\frac{a^{11}}{12}$$

$$-2N\,W_0 = N W_i - N W_a - \frac{N e^2\tilde\alpha}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{a^{11}}{12a^{12}}\right)$$

$$\Rightarrow \quad a = \frac{11}{12}\,\frac{e^2}{4\pi\varepsilon_0}\,\frac{\tilde\alpha}{W_i - W_a + 2W_0} = \frac{11}{12}\,\underbrace{\frac{e^2}{4\pi\varepsilon_0\hbar c}}_{\alpha=\frac{1}{137}}\,\frac{\tilde\alpha\hbar c}{W_i - W_a + 2W_0} = 0{,}2\,nm$$

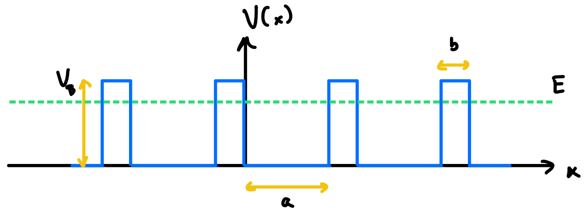
4.3 Kronig - Penney model

4.3 Elektroni se nahajajo v enodimenzionalnem kristalu, katerega periodični potencial opišemo z vsoto delta funkcij

$$V(x) = Ub[\cdots + \delta(x+a) + \delta(x) + \delta(x-a) + \cdots],$$

med katerimi je razdalja $a = 0{,}3$ nm. Kolikšna je energijska vrzel med najnižjima energijskima pasovoma, če je produkt višine U in širine b ($b \ll a$) posamezne potencialne plasti $Ub = 25$ meV nm?

☞ Upoštevaj, da je energijska vrzel majhna v primerjavi s širino posameznega pasu!



$$V(x) = V_B \sum_{l=-\infty}^{\infty} \delta(x - l(a+b)) \qquad \lim_{b\to 0} V_B b = \text{konst.} = 25 \text{ meVnm}$$

Energijska vrzel med pasoma?

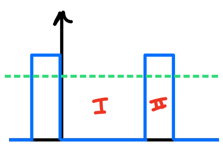
Blochov izrek

Če imamo periodičen potencial, potem lahko ψ zapišemo kot en val krat u, kjer je u periodičen.

$$\psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\vec{r}} u_{\vec{k}}(\vec{r}) \qquad u_{\vec{k}}(\vec{r}+\vec{R}) = u_{\vec{k}}(\vec{r})$$

oz. $$\psi_{\vec{k}}(\vec{r}+\vec{R}) = e^{i\vec{k}\vec{R}} \psi_{\vec{k}}(\vec{r})$$

Obravnavajmo le en odsek periodičnega potenciala



I $\quad \psi_1 = Ae^{ikx} + Be^{-ikx} \qquad k = \frac{\sqrt{2mE}}{\hbar} \qquad E = \frac{\hbar^2 k^2}{2m}$

II $\quad \psi_2 = Ce^{\varkappa x} + De^{-\varkappa x} \qquad \varkappa = \frac{\sqrt{2m(V-E)}}{\hbar}$

Robni pogoji

Blochov izrek

$$\Rightarrow \psi_2(x+a+b) = \psi_2(x) e^{ik_L(a+b)}$$

$$k_L = \frac{2\pi}{L} l \qquad l = -\frac{N}{2}, \dots, \frac{N}{2}$$

$$\begin{aligned}
\psi_1(0) &= \psi_2(0) & A + B &= C + D \\
\psi_1'(0) &= \psi_2'(0) & Aik - Bik &= C\varkappa - D\varkappa \\
\psi_1(a) &= \psi_2(a) & Ae^{ika} + Be^{-ika} &= \psi_2(-b) e^{ik_L(a+b)} = (Ce^{-\varkappa b} + De^{\varkappa b}) e^{ik_L(a+b)} \\
\psi_1'(a) &= \psi_2'(a) & ik(Ae^{ika} - Be^{-ika}) &= e^{ik_L(a+b)} \varkappa (Ce^{-\varkappa b} - De^{\varkappa b})
\end{aligned}$$

Z računalnikom rešimo sistem enačb

Pogoj

$$\frac{\varkappa^2 - k^2}{2\varkappa k} \sinh(\varkappa b) \sin(ka) + \cosh(\varkappa b)\cos(ka) = \cos(k_L(a+b))$$

Opombe:

ⓐ $\psi_{nk_L}(\vec r) = e^{i\vec k_L \vec r}\, u_{nk_L}(\vec r)$ in $u_{nk_L}(\vec r + \vec a) = u_{nk_L}(\vec r)$

velikost osnovne celice

ali

$\psi_{k_L}(\vec r + \vec a) = e^{i\vec k_L \vec a}\, \psi(\vec r)$ $\qquad k_L = \frac{2\pi n}{L}$ $\qquad n = -\frac{N}{2}, \dots, \frac{N}{2}$ ← število celic

L ← velikost celotnega kristala

ⓑ $A\vec x = 0 \Rightarrow \det A = 0$ za $\vec x \neq 0$

Pogoji $\quad V \to \infty \quad b \to 0 \quad Vb \to$ konst

$$\varkappa = \frac{\sqrt{2m(V-E)}}{\hbar} \qquad k = \frac{\sqrt{2mE}}{\hbar} \qquad \varkappa \gg k$$

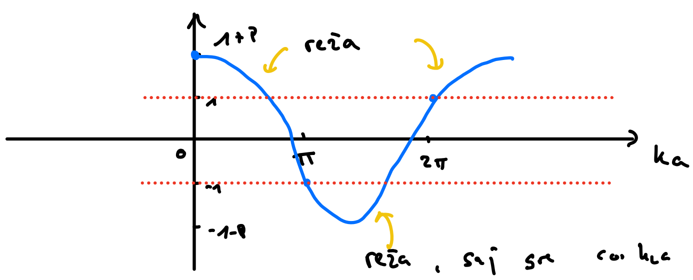
$$\sinh(\varkappa b) = \sinh\left(\sqrt{\tfrac{2m}{\hbar}}\,\underset{\text{konst}}{\sqrt{Vb}}\,\underset{\to 0}{\sqrt{b}}\right) \overset{\text{Taylor}}{=} \sqrt{\tfrac{2m}{\hbar}}\sqrt{Vb}\sqrt{b} = \varkappa b$$

$$\cosh(\varkappa b) \doteq 1$$

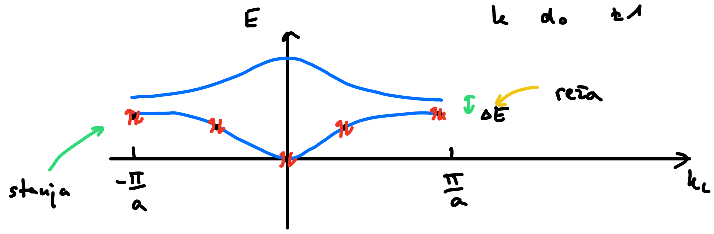
$$\Rightarrow \underbrace{\frac{\varkappa a}{2}\varkappa b}_{P} \frac{\sin(ka)}{ka} + \cos(ka) = \cos(k_L a) \qquad k = \frac{\sqrt{2mE}}{\hbar}$$

$$P\frac{\sin ka}{ka} + \cos ka = \cos k_L a$$

Numerično rešimo, da dobimo $E(k_L)$



reža, saj se $\cos k_L a$ le do ± 1



$k_L = \frac{2\pi}{L} n$

sodo atomov

število valenčnih elektronov na osnovno celico → $N = 2p$ → izolator

$N = 2p+1$ → prevodnik

Reža $\Delta E = ?$

$$ka = \pi + \tilde k a \qquad \sin(x+\pi) = -x + \dots$$

$$k_L a = \pi \qquad \cos(x+\pi) = -1 + \frac{x^2}{2} - \dots$$

$$P\frac{\sin ka}{ka} + \cos ka = \cos k_L a$$

$$\frac{P}{\pi + \tilde k a}(-\tilde k a) + \left(-1 + \frac{(ka)^2}{2}\right) = -1$$

$$P\frac{\tilde k a}{\pi + \tilde k a} - \frac{(\tilde k a)^2}{2} = 0$$

$$\tilde k a\left(\frac{P}{\pi + \tilde k a} - \frac{\tilde k a}{2}\right) = 0$$

① $\tilde{k}a = 0 \rightarrow k_1 a = \pi$

② $\tilde{k}a = \dfrac{2P}{\pi + \tilde{k}a} \overset{\tilde{k}a \ll 1}{\approx} \dfrac{2P}{\pi}$ $\qquad k_2 a = \pi + \dfrac{2P}{\pi}$

$$\Delta E = \frac{\hbar^2}{2m}\left(k_2^2 - k_1^2\right) = \frac{\hbar^2}{2m}\left(\left(\frac{\pi}{a} + \frac{2P}{\pi a}\right)^2 - \frac{\pi^2}{a^2}\right) = \frac{\hbar^2}{2m}\left(\frac{4P}{a^2} + \frac{4P^2}{\pi^2 a^2}\right)$$

$$\approx \frac{\hbar^2 4P}{2ma^2} \approx 0{,}17\,\mathrm{eV}$$

4.24 Kolikšna je $\langle v \rangle$ potovanja e^- v bakru v zun. el. polju $E = 1\,\frac{V}{m}$?
$\frac{\langle v \rangle}{v_F} = ?$ Gibljivost e^- je $\beta_e = 0{,}0032\ \mathrm{m^2/Vs}$ $\quad W_F = 7{,}0\,\mathrm{eV}$ Fermijeva energija.

Elektrone obravnavamo klasično.

prispevek sipanj e^- na nepravilnostih

$$\text{2.N.Z.}\qquad \frac{d\langle p \rangle}{dt} = eE - \frac{\langle p \rangle}{\tau}$$

- zunanje el. polje pospešuje
- napake, nečistoče (kvaliteta kristala)
- termično nihanje atomov v kristalni strukturi (temperatura)
- sipanje med e^-

τ ... povprečen čas med trki e^-

Stacionarno stanje

$$eE = \frac{\langle p \rangle}{\tau} \qquad \langle p \rangle = m\langle v \rangle = eE\tau$$

def. gibljivosti

ⓐ $\langle v \rangle = \boxed{\dfrac{e\tau}{m}}\, E = \beta_e E = 3{,}2\ \frac{mm}{s}$

ⓑ $\dfrac{\langle v \rangle}{v_F} = ?$ $\qquad W = \dfrac{mv^2}{2} = \dfrac{\hbar^2 k^2}{2m} \Rightarrow W_F = \dfrac{m v_F^2}{2}$ $\qquad v_F = \sqrt{\dfrac{2W_F}{m_e}}$

$$= \frac{3{,}2\ \frac{mm}{s}}{1{,}6\cdot 10^6\ \frac{m}{s}} = 2{,}0 \cdot 10^{-9} \qquad\qquad = 1{,}6\cdot 10^6\ \frac{m}{s}$$

4.23 Izračunaj povprečno prosto pot prevodniških e^- v bakru in natriju.
Prevodnost bakra $\sigma_{Cu} = 5{,}9 \cdot 10^5\ 1/\Omega m$, natrija pa $\sigma_{Na} = 2{,}2\cdot 10^5\ 1/\Omega m$
$M_{Cu} = 63$, $M_{Na} = 23$, $\rho_{Cu} = 8{,}9\ \mathrm{g/cm^3}$, $\rho_{Na} = 0{,}97\ \mathrm{g/cm^3}$
Cu in Na imata 1 valenčni e^-.

$L_e = ?$ $\qquad \boxed{L_e = \tau v_F}$

$$\langle v \rangle = \frac{e\tau}{m} E = \beta_e E$$

$e n_e$ ... gostota naboja valenčnih e^-

n_e ... številska gostota valenčnih e^- $\qquad n_e = n_a \cdot z$

gostota el. toka $\quad j_e = e n_e \langle v \rangle = \dfrac{e^2 n_e \tau}{m} E = \sigma E$

specifična el. prevodnost $\quad \sigma = e\beta_e n_e = \dfrac{e^2 n_e \tau}{m}$

$$\frac{m}{M} = \frac{N}{N_A} \qquad \rho = \frac{m}{V} = \frac{NM}{VN_A} = n_a \frac{M}{N_A} \Rightarrow n_e = \frac{\rho N_A}{M} z$$

$N_A = 6{,}02\cdot 10^{26}\ \frac{1}{kmol}$

$n_{e,Cu} = 8,5\cdot10^{28}\,\frac{1}{m^3}$ $\quad n_{e,Na} = 2,54\cdot10^{28}\,\frac{1}{m^3}$

$$\tau_{Cu} = \frac{m_e\sigma}{e_0^2\, n_e} = \frac{9,1\cdot10^{-31}\cdot 5,9\cdot10^{5}}{(1,6\cdot10^{-19})^2\; 8,5\cdot10^{28}} = 2,5\cdot10^{-16}\,s$$

$\tau_{Na} = \dots = 3,1\cdot10^{-16}\,s$ $\qquad$ 2 spina $\qquad$ τ tipično $\sim 0,1$ ps

$v_F =$ $\qquad N = 2\,\frac{V_F}{V_1}$

$V_F = \frac{4\pi k_F^3}{3}$ volumen Fermijeve krogle

$k_x = \frac{2\pi n}{L}$ $\quad \Delta k = \frac{2\pi}{L}$ $\quad (\Delta k)^3 = V_1 = \left(\frac{2\pi}{L}\right)^3$ volumen enega stanja

$N = 2V_F\,\frac{L^3}{(2\pi)^3}$ $\qquad /: V = L^3$

$$\frac{N}{L^3} = n_e = 2\,\frac{4\pi k_F^3}{3}\,\frac{1}{(2\pi)^3} = \frac{k_F^3}{3\pi^2}$$

$$k_F = \sqrt[3]{3\pi^2 n_e}$$

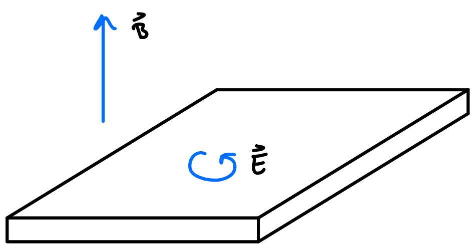
$$v_F = \frac{k_F\hbar c}{m_e c^2}\,c = \frac{\hbar c}{mc^2}\,c\,(3\pi^2 n_e)^{1/3}$$

$$v_{F,Cu} = \frac{200\ \text{eV nm}}{0,5\ \text{MeV}}\;3\cdot10^8\,\frac{m}{s}\left(3\cdot\pi^2\cdot 8,5\cdot10^{28}\,\frac{1}{m^3}\right)^{1/3} = 1,6\cdot10^6\,\frac{m}{s}$$

$v_{F,Na} = \dots = 1,0\cdot10^6\,\frac{m}{s}$ $\qquad$ Tipično $\sim 0,01\,c$

$l_e = \tau v_F$ $\qquad l_{eCu} = 0,4$ nm $\qquad l_{eNa} = 0,3$ nm $\qquad$ Tipično $\sim$nm

4.11



$\vec E = E_0\, e^{i\omega t}(1, i, 0)$

$\vec B = B_0(0,0,1)$

Kolikšna prevodnost σ

Max od σ ?

2.N.Z. $\qquad \frac{\partial\langle\vec p\rangle}{\partial t} = -e\vec E - e\langle\vec v\rangle\times\vec B - \frac{\langle\vec p\rangle}{\tau}$ $\quad$ ①

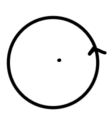
$$\vec j = -en\langle\vec v\rangle = \sigma\vec E$$

Če je $B = 0$ $\qquad \sigma = \frac{ne^2\tau}{m}$

① $\cdot\,(-n_e e)$

$$-mn_e e\,\frac{d\langle\vec v\rangle}{dt} - ne\,\frac{m_e}{\tau}\langle\vec v\rangle - ne^2\vec E - n_e e^2\,\vec v\times\vec B = 0$$

$$\frac{d\vec j}{dt} + \frac{\vec j}{\tau} - \frac{ne^2}{m}\vec E + \frac{e}{m}\,\vec j\times\vec B = 0$$

$\odot\,B$  $\qquad evB = m\frac{v^2}{r}$ $\qquad \omega_c = \frac{eB}{m}$ Ciklotronska frekvenca

$$\frac{d\vec j}{dt} + \frac{\vec j}{\tau} - \frac{e^2 n}{m}\vec E + \underbrace{\frac{eB_0}{m}}_{\omega_c}\vec j \times \hat e_z = 0 \qquad /\cdot\tau$$

$$\tau\frac{d\vec j}{d(t)} + \vec j - \underbrace{\frac{e^2 n \tau}{m}}_{\sigma_0}\vec E + \omega_c\tau\, j \times \hat e_z = 0$$

$E = E_0\,(1, i, 0)\, e^{-i\omega t}$

Nastavek $\quad j = j_0\, e^{-i\omega t}$

$$-\omega i\tau\, j_0 e^{-i\omega t} + j_0 e^{-i\omega t} - \sigma_0 E_0(1, i, 0)\, e^{-i\omega t} + \omega_c\tau\, \vec j_0 e^{-i\omega t} \times \hat e_z = 0$$

$$j_0 \times \hat e_z = \begin{vmatrix} x & y & z \\ j_x & j_y & j_z \\ 0 & 0 & 1 \end{vmatrix} = (j_y, -j_x, 0)$$

$$-i\omega\tau\, j^x + j^x + \omega_c\tau\, j^y = \sigma_0$$
$$-i\omega\tau\, j^y + j^y - \omega_c\tau\, j^x = i\sigma_0$$

$$(1 - i\omega\tau)\, j^x + \omega_c\tau\, j^y = \sigma^0$$
$$(1 - i\omega\tau)\, j^y - \omega_c\tau\, j^x = i\sigma^0 \qquad /\cdot i$$
$$(1 - i\omega\tau)\, i\, j^y - i\omega_c\tau\, j^y = -\sigma^0 \qquad \Big]+$$

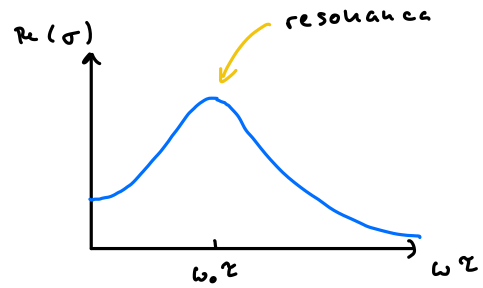
$$(1 - i(\omega + \omega_c)\tau)\, j^x + (i + (\omega + \omega_c)\tau)\, j^y = 0$$
$$(1 - i(\omega + \omega_c)\tau)(j^x + i j^y) = 0$$

$$j^x + i j^y = 0 \qquad j^x = -i j^y$$
$$1 - i(\omega + \omega_c)\tau = 0$$

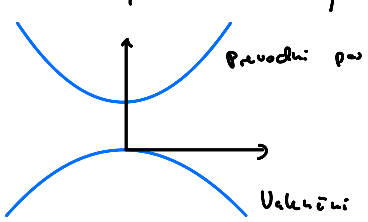
$$(1 - i\omega\tau + i\omega_c\tau)\, j^x = \sigma^0$$

$$\sigma(\omega) = \frac{\sigma_0}{1 - i(\omega - \omega_c)\tau}$$

$$\sigma(\omega) = \frac{\sigma_0(1 + i(\omega - \omega_c)\tau)}{1 + (\omega - \omega_c)^2\tau^2}$$



Polprevodniki, kadar je $E_g \sim k_B T$ pri sobni tempr. $k_B T = \frac{1}{40}$ eV

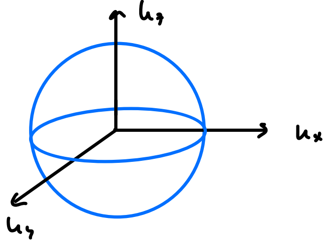


Si: $E_g = 1{,}1$ eV

Ge: $E_g = 0{,}7$ eV

E_{Fermi} za Germanij ; $m_e = 0{,}041\, m_e$ $\qquad T = 0K$, $T = 1000K$

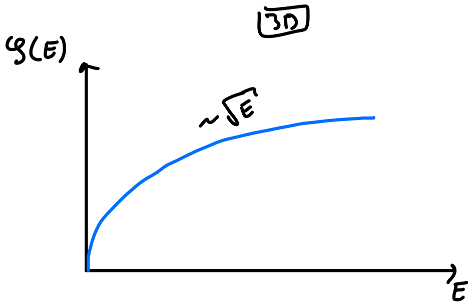
$m_v = 0{,}28\, m_e$



Število vseh stanj $N = 2\frac{V_{Fermi}}{V_0}$

$= \frac{k_F^3}{3\pi^2}$

$E_{Fermi} = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$ $\qquad n = \frac{N}{L^3}$

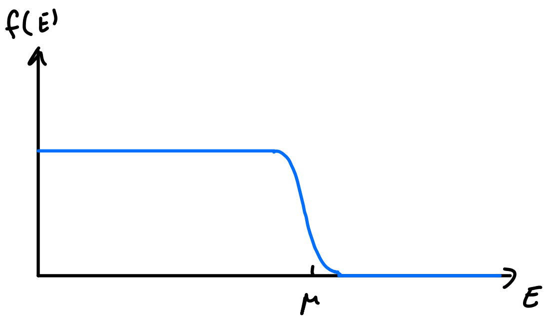


Gostota stanj (št. stanj na energijo)

$$\frac{N}{V} = n = \frac{1}{3\pi^2}\left(\frac{2mE}{\hbar^2}\right)^{3/2}$$

$$g(E) = \frac{dN}{dE} = \underbrace{\frac{1}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\frac{3}{2}}_{A_p V} E^{1/2}$$

Statistična fizika za fermione



Fermi-Dirac

$$f(E) = \frac{1}{1+e^{\beta(E-\mu)}}$$

$\beta = \frac{1}{k_B T}$ $\qquad \mu$ ... kemijski potencial

Prevodni pas

$$N_e^c = \int_0^\infty f(E)\, g(E)\, dE = \int_0^\infty \frac{1}{1+e^{\beta(E-\mu)}} g(E)\, dE$$

$$n_e^c = \frac{N_e^c}{V} = \int_0^\infty \frac{1}{1+e^{\beta(E+\mu)}} g(E)\, dE = \dots \qquad g(E) = \frac{g(E)}{V}$$

$$\dots = \int_{E_g}^\infty \frac{1}{1+e^{\beta(E+\mu)}} g(E)\, dE = \int_{E_g}^\infty e^{-\beta(E-\mu)} A_v \sqrt{E-E_g}\, dE = A_p \int_0^\infty e^{-(x+E_g-\mu)\beta}\sqrt{x}\, dx =$$

ker v reži ni stanj

$T \to 0$, $\beta \to \infty$ $\qquad x = E - E_g$

$$= A_p e^{-(E_g-\mu)\beta}\int_0^\infty e^{-x\beta}\sqrt{x}\, dx = A_p e^{-(E_g-\mu)\beta}\int_0^\infty e^{-t}\beta^{-3/2}\sqrt{t}\, dt =$$

$$= A_p e^{-(E_g-\mu)\beta}\beta^{-3/2}\Gamma\left(\frac{3}{2}\right) = A_p e^{-(E_g-\mu)\beta}\beta^{-3/2}\frac{\sqrt{\pi}}{2}$$

Valenčni pas

$$N_v = N - N_e^v$$

$$n_v = n - n_e^v = \int_{-\infty}^0 g_v(E)\left(1-f(E)\right) dE = \dots$$

$$1 - f(E) = 1 - \frac{1}{e^{\beta(E-\mu)}+1} = \frac{e^{\beta(E-\mu)}}{e^{\beta(E-\mu)}+1} = \frac{1}{1+e^{-\beta(E-\mu)}}$$

$$g_v(E) = A_v\sqrt{-E}$$

$$\ldots = A_v \int_{-\infty}^{0} \sqrt{-E}\, \frac{1}{1+e^{-\beta(E-\mu)}}\, dE \overset{\varepsilon=-E}{=} A_v \int_0^{\infty} \sqrt{\varepsilon}\, \frac{1}{1+e^{\beta(\varepsilon+\mu)}}\, d\varepsilon \underset{\substack{T\to 0\\ \beta\to\infty}}{\approx} A_v \int_0^{\infty} \sqrt{\varepsilon}\, e^{-\beta(\varepsilon+\mu)}\, d\varepsilon =$$

$$= A_v e^{-\beta\mu} \beta^{-3/2} \int_0^{\infty} \sqrt{u}\, e^{-u}\, du = \frac{\sqrt{\pi}}{2} A_v e^{-\beta\mu} \beta^{-3/2}$$

$$n^p = n^v$$

$$A_p\, e^{-(E_g-\mu)\beta} \beta^{-3/2} \frac{\sqrt{\pi}}{2} = \frac{\sqrt{\pi}}{2} A_v e^{-\beta\mu} \beta^{-3/2}$$

$$A_p\, e^{-(E_g-\mu)\beta} = A_v e^{-\beta\mu}$$

$$e^{2\mu\beta} = \frac{A_v}{A_p} e^{E_g\beta} = \left(\frac{m_v}{m_p}\right)^{3/2} e^{E_g\beta}$$

$$2\mu\beta = \frac{3}{2} \ln \frac{m_v}{m_p} + E_g\beta$$

$$\mu = \frac{3}{4\beta} \ln \frac{m_v}{m_p} + \frac{E_g}{2}$$

Poudarki

ⓐ $T\to 0$ $\quad \mu = \frac{1}{2} E_g$

ⓑ $m_e = m_v$ $\quad \mu = \frac{1}{2} E_g$

ⓒ linearna odvisnost od temperature

ⓓ $n_{ep} \cdot n_{ev} = n_0^2\, e^{-\beta E_g}$ $\qquad n_0 = \frac{1}{2\pi^2} \left(\frac{2 m_v m_e}{\hbar}\right)^{3/4} \frac{\sqrt{\pi}}{2} \beta^{-3/2}$

za čist polprevodnik

$E_g = 0{,}72\ \mathrm{eV}$

$m_e = 0{,}04\, m_0$

$m_v = 0{,}28\, m_0$

pri $T=0$ $\qquad \mu(T=0) = 0{,}36\ \mathrm{eV}$

zelo majhni premiki

pri $T = 1000\,\mathrm{K}$ $\qquad \mu = \frac{1}{2} E_g + \frac{3}{4} k_B T \ln \frac{m_v}{m_e} = 0{,}24\ \mathrm{eV}$

V/29 $\quad \Delta T$ okoli $T = 20\,\mathrm{K}$ $\qquad \frac{\Delta R}{R} = 0{,}1\,\%$ $\qquad E_g = 0{,}72\,\mathrm{eV}$

$$j = \sigma E \qquad \sigma = \frac{n e^2 \tau}{m}$$

$$\sigma = e^2 \left(\frac{n_{ep} \tau_e}{m_e} + \frac{n_v \tau_v}{m_v} \right)$$

$$n_v = n_{ep} = n_0(T)\, e^{-\frac{E_g}{2}\beta} \qquad n_0(T) \propto (k_B T)^{3/2}$$

$$\sigma = e^2 n_0(T)\, e^{-\frac{E_g \beta}{2}} \left(\frac{\tau_e}{m_e} + \frac{\tau_v}{m_v} \right)$$

$$R = \rho \frac{l}{S} \qquad \sigma = \frac{1}{\rho}$$

$$\left|\frac{dR}{R}\right| = \left|\frac{d\rho}{\rho}\right| \qquad d\sigma = -\frac{1}{\rho^2} d\rho \Rightarrow \frac{d\sigma}{\sigma} = -\frac{d\rho}{\rho}$$

$$\ln\sigma = \ln\left(e^2 n_0(T)\left(\frac{\tau_e}{m_e}+\frac{\tau_v}{m_v}\right)\right) - \frac{E_g\beta}{2} \qquad n_0 \propto (k_B T)^{3/2}$$

$$\ln\sigma = C + \frac{3}{2}\ln T - \frac{E_g}{2k_BT} \qquad \Big/ \text{ odvaj.}$$

$$\frac{1}{\sigma}\frac{d\sigma}{dT} = \frac{3}{2}\frac{1}{T} + \frac{E_g}{k_BT^2}$$

$$\left|\frac{d\sigma}{\sigma}\right| = \left|\frac{dR}{R}\right| = \frac{3}{2}\frac{dT}{T} + \frac{E_g}{2k_B}\frac{dT}{T^2}$$

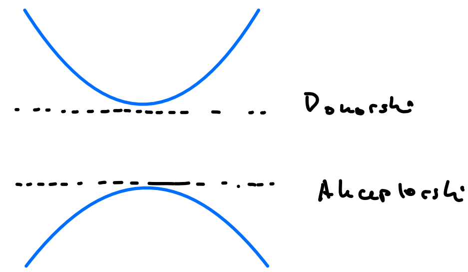
$$\frac{dT}{T} = \left|\frac{dR}{R}\right| \frac{1}{\frac{3}{2}+\frac{E_g}{2k_BT}}$$

Dopirani polprevodniki

Polprevodnik tipa n

Si (4 val. e^-)

As (5 val. e^-)



$$\Delta E_d = \frac{e^4 m_e^*}{32\pi^2(\varepsilon\varepsilon_0)^2\hbar^2} \qquad \Delta E_d \sim 0{,}05\,eV$$

vezavna energija donorskega e^-

$$n_{ep}\, n_v = n_0^2\, e^{-\frac{E_g}{k_BT}}$$

$$n_e = n_v + n_d(1-f(\mu)) =$$

$$= n_v + \frac{n_d}{e^{-\beta(E_d-\mu)}+1}$$

Sobna T $\quad n_v \sim 0 \quad n_{ep} = \frac{n_d}{e^{-\beta(E_d-\mu)}\cancel{+1}} = n_d\, e^{\beta(E_d-\mu)}$

zanemarimo

V/70

$n_{As} = 10^{21}\,m^{-3} \qquad T = 300K \qquad \beta_e = 0{,}38\,\frac{m^2}{Vs} \qquad \beta_v = 0{,}18\,\frac{m^2}{Vs} \qquad \sigma_{Ge} = ?$

$$n_{ep}\cdot n_v = n_0^2\, e^{-E_g\beta}$$

Čisti polprevodnik

$$n_{ep} = n_v = n_0\, e^{-\frac{E_g}{2k_BT}} = 1{,}66\cdot 10^{19}\,\frac{1}{m^3} \qquad n_0 = \frac{1}{2\pi^2}\left(\frac{2\sqrt{m_e m_v}}{\hbar^2}\right)^{3/2}\frac{\sqrt{\pi}}{2}(k_BT)^{3/2} = 9\cdot 10^{23}\,\frac{1}{m^3}$$

n-tip

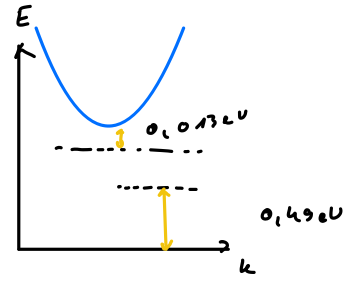
$$n_{ep} = n_d(1-f(\mu)) + n_v$$

$n_d \ll n_0$

$n_v \sim 0 \quad T = 300K \qquad e^{\beta(E_g-\mu)} \gg 1$

$$n_d \approx n_c = n_0\, e^{-\beta(E_g-\mu)} \left(\frac{m_e}{m_0}\right)^{3/4}$$

$$\mu = E_g + k_B T\left(\ln\frac{n_d}{n_0} + \frac{3}{4}\ln\frac{m_e}{m_0}\right) = 0{,}49\ \mathrm{eV}$$



$$f(E_g-E_d) = \frac{1}{\frac{1}{2}e^{(E_g-E_d-\mu)\beta}+1} = 2e^{-\beta(E_g-E_d-\mu)}$$

samo en e^- lahko tvori par

$$E_g - E_d - \mu = 0{,}16\ \mathrm{eV}$$

$$\sigma = (n_{ep}\,\beta_e + n_v\,\beta_v)\,e$$

$$n_{ep}\cdot n_v = n_0^2\, e^{-\beta E_g} = n_v(n_v+n_d)$$

$$n_v^2 + n_v n_d - \underbrace{n_0^2\, e^{-\beta E_g}}_{n_e^{\circ}} = 0$$

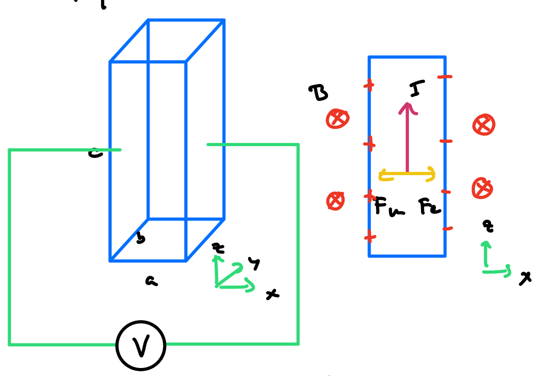
$$\sqrt{1+x} = 1+\frac{x}{2}$$

$$n_v = -\frac{n_d}{2} \pm \frac{n_d}{2}\sqrt{1+4\underbrace{\left(\frac{n_e^{\circ}}{n_d}\right)^2}_{\ll 1}} = -\frac{n_d}{2}\left(1 \mp \left(1+2\left(\frac{n_e^{\circ}}{n_d}\right)^2\right)\right) =$$

$$n_v \approx 2\frac{n_e^{\circ 2}}{n_d} \approx 1\cdot 10^{5}\ \frac{1}{m^3} \ll n_{ep} \approx n_d$$

$$\Rightarrow \quad \sigma \doteq n_{ep}\,\beta_e\, e = 10^{21}\ \frac{1}{m^3}\cdot 0{,}38\ \frac{m^2}{Vs}\cdot 1{,}6\cdot 10^{-19}\ As = 61\ \frac{1}{\Omega m}$$

(T) Hallov pojav



$$\vec F_m = \vec F_e$$

$$eE_H = e\langle v\rangle B \qquad j = en\langle v\rangle$$

$$E_H = \frac{jB}{en}$$

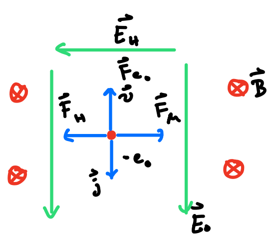
$$j = \frac{I}{S} = \frac{I}{ab} \qquad E_H a = U_H$$

$$U_H = \frac{IB}{ben}$$

(V/50)

$B = 0{,}3\ T$
$\varphi = 5{,}7°$
n-tip
(siloje se e^-)
$\sigma = 100\ \frac{1}{\Omega m}$

$\beta = ?$
$n_{ep} = ?$



$$\tan\varphi = \frac{e\langle v\rangle B}{eE_0} = \beta B$$

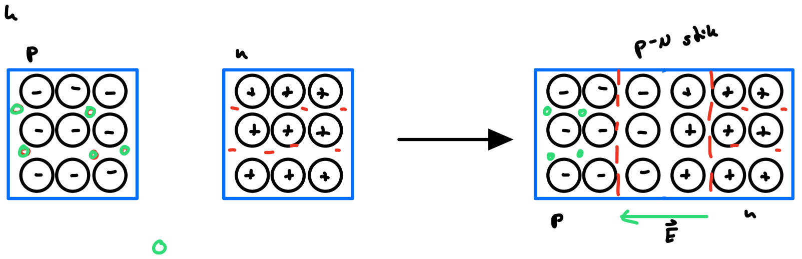
$$\langle v\rangle = \beta E_0$$

$$\beta = \frac{\tan\varphi}{B} = \ldots = 0{,}33\ \frac{m^2}{Vs}$$

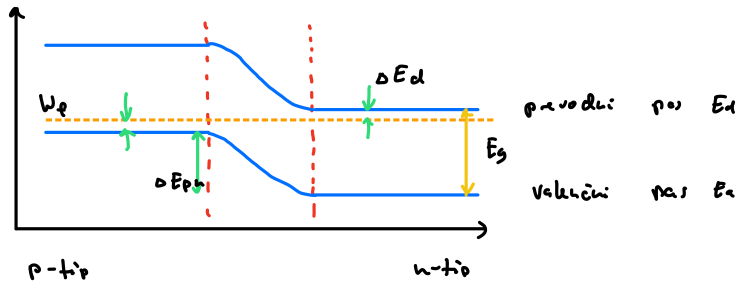
$$\sigma = e n_e \beta \qquad j = \sigma E_0 = en\langle v\rangle \qquad n_e = \frac{\sigma}{e\beta} = \ldots =$$

$$\langle v\rangle = \beta E_0$$

T P-N stik



- e^- in vrzeli se rekombinirajo
- izpraznjen sloj je brez nosilcev naboja



- v ravnovesju, ko na stiku p-n ni zun. napetosti je Fermijeva en. na obeh straneh enaka.
- Približek

$$E_{Fn} \simeq E_d \qquad \Delta E_{pn} = E_{Fn} - E_{Fp}$$
$$E_{Fp} \simeq E_a \qquad \simeq E_d - E_a$$

V/43 Dioda

$T = 300\,K$

p-tip: $\rho_p = 0{,}01\ \Omega m$ specifična upornost $\sigma = \frac{1}{\rho}$

n-tip: $\rho_n = 0{,}001\ \Omega m$

$\Delta E_{pn} = ?$

$E_g = 0{,}67\,eV$

$m_e = 0{,}56\, m_0$

$m_v = 0{,}35\, m_0$

$\beta_e = 0{,}39\ m^2/Vs$

$\beta_v = 0{,}19\ m^2/Vs$

$\varepsilon = 15{,}8$

$$j = e n \langle v \rangle = e n \beta E = \sigma E = \frac{E}{\rho} \qquad n = \frac{1}{e \rho \beta}$$

$$n_{ep} = \frac{1}{e \beta_e \rho_n} \qquad n_v = \frac{1}{e \beta_v \rho_p}$$

$$n_d \simeq n_{ep} = 1{,}6 \cdot 10^{22} \frac{1}{m^3} \qquad n_a \simeq n_v =$$

Pregled količin

$$\boxed{\begin{aligned}
n_{ep} &= A_p \frac{\sqrt{\pi}}{2} (k_B T)^{3/2} e^{-\beta(E_g - \mu)} \qquad \beta = \frac{1}{k_B T} \\
n_v &= A_v \frac{\sqrt{\pi}}{2} (k_B T)^{3/2} e^{-\beta \mu} \\
n_{ep} n_v &= n_0^2 e^{-\beta E_g} \\
n_0 &= \sqrt{A_v A_p} \frac{\sqrt{\pi}}{2} (k_B T)^{3/2} = \frac{1}{2\pi^2} \left(\frac{2}{\hbar^2} \sqrt{m_e m_v}\right)^{3/2} \frac{\sqrt{\pi}}{2} (k_B T)^{3/2} \\
&\text{Gostota } e^- \text{ v čistem polprev.} \quad n_e^0 = n_0 e^{-\frac{E_g}{2 k_B T}}
\end{aligned}}$$

$\ldots = 7{,}4 \cdot 10^{24}\ m^{-3}$

$\ldots = 1{,}1 \cdot 10^{19}\ m^{-3}$

n-tip $\quad k_B T = 25\ meV \gg \Delta E_d = E_g - E_d$

$$\Delta E_d \sim 10^{-3}\,eV$$

$$n_{ep} = n_v + (1 - f(E_g - \Delta E_d))\, n_d \simeq n_d$$

$$\boxed{\begin{aligned} n_e &= \left(\frac{m_e}{m_v}\right)^{3/4} n_0\, e^{-(E_g - E_{Fn})\beta} \\ n_v &= \left(\frac{m_v}{m_e}\right)^{3/4} n_0\, e^{-E_{Fp}\beta} \end{aligned}} \qquad \begin{aligned} &\simeq n_d \\ &\simeq n_a \end{aligned}$$

$$\dots \quad E_{Fn} = E_g + k_B T \left(\ln \frac{n_d}{n_0} + \frac{3}{4} \ln \frac{m_v}{m_e} \right) = 0{,}51 \text{ eV}$$

p-tip $\quad k_B T \gg E_a$

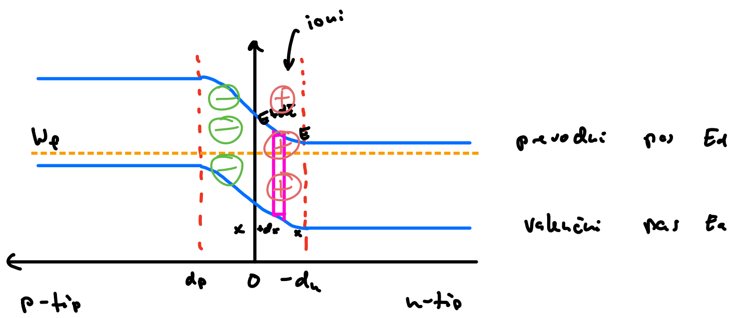
$n_v \simeq n_a$

$$\dots \quad E_{Fp} = -k_B T \left(\ln \frac{n_a}{n_0} + \frac{3}{4} \ln \frac{m_e}{m_v} \right) = 0{,}18 \text{ eV}$$

$$\Delta E_{pn} \simeq E_{Fn} - E_{Fp} = 0{,}33 \text{ eV}$$

Ⓣ Širina zaporne plasti (p-n stik)

$$d = d_p + d_n = ?$$



Gaussov zakon

$$\varepsilon\varepsilon_0 \oint \vec{E} \cdot d\vec{S} = e$$

$$\varepsilon\varepsilon_0 \left((E + dE) S - E S \right) = n_d e_0 \, \overset{dx\,S}{dV}$$

$$\frac{dE_n}{dx} = \frac{n_d e_0}{\varepsilon\varepsilon_0}$$

$$E_n(x) = n_d \frac{e_0}{\varepsilon\varepsilon_0} x + C$$

Robni pogoj $\quad E_n(x = -d_n) = 0 \quad \Rightarrow \quad C = \frac{n_d e_0}{\varepsilon\varepsilon_0} d_n$

$$\boxed{E_n(x) = \frac{e_0 n_d}{\varepsilon\varepsilon_0} (x + d_n)}$$ El. polje na n strani

P - stran

$$\varepsilon\varepsilon_0 \left((E + dE) S - E S \right) = -e_0 n_a S\, dx$$

Robni pogoj $\quad E_p(x = d_p) = 0$

$$\boxed{E_p(x) = \frac{e_0 n_a}{\varepsilon\varepsilon_0} (d_p - x)}$$

Pogoj zveznosti

$$E_p(x=0) = E_n(x=0)$$

$$\boxed{n_d d_n = n_a d_p}$$ ohranitev naboja

$$\Delta E_{pn} = e_0 \int_{-d_n}^{d_p} \vec{E}\cdot d\vec{s} = e_0 \left(\int_{-d_n}^{0} E_n(x)\,dx + \int_{0}^{d_p} E_p(x)\,dx \right) = \frac{e_0^2}{\varepsilon\varepsilon_0}\left(n_d \int_{-d_n}^{0} (x+d_n)\,dx + n_a \int_{0}^{d_p} (d_p - x)\,dx \right) =$$

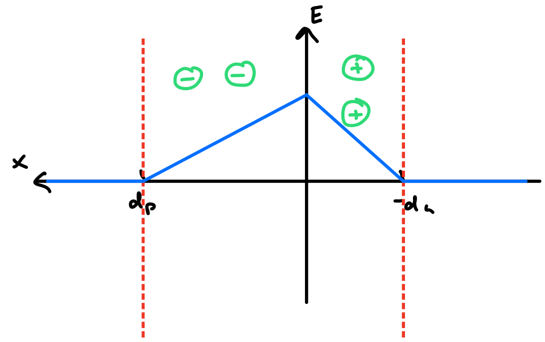
$$= \frac{e_0^2}{\varepsilon\varepsilon_0}\left(n_d \left(-\frac{d_n^2}{2} + d_n^2\right) + n_a \left(d_p^2 - \frac{d_p^2}{2}\right)\right) = \frac{e_0^2}{2\varepsilon\varepsilon_0}\left(n_d d_n^2 + n_a d_p^2 \right) =$$

$$= \frac{e_0^2}{2\varepsilon\varepsilon_0} n_d \left(1 + \frac{n_d}{n_a}\right) d_n^2$$

$d_n = \ldots = 79\,nm$

$d_p = d_n \frac{n_d}{n_a} = 381\,nm$

$d = d_n + d_p = 460\,nm$



ker $n_d > n_a$

$\Rightarrow d_p > d_n$

$d_p \ll d_n \Rightarrow n_d d_n = n_a d_p \Rightarrow n_d \ll n_a$

$$\Delta E_{pn} = e_0 V_0 = \frac{e_0^2}{2\varepsilon\varepsilon_0} n_d \left(1 + \frac{n_d}{n_a}\right) d_n^2 \simeq \frac{e_0^2}{2\varepsilon\varepsilon_0} n_d d_n^2$$

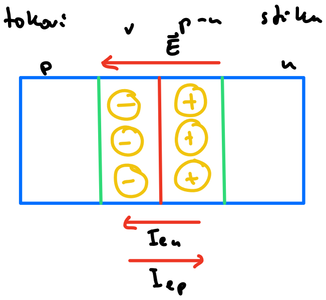
skok el. potenciala

$$\Rightarrow d_n = \sqrt{\frac{2\varepsilon\varepsilon_0 V_0}{e_0 n_d}} \approx d \qquad d_n = \sqrt{\frac{2\varepsilon\varepsilon_0 (V_0 - V)}{e_0 n_d}}$$

Prevodna smer $V < 0$ d_n se poveča

Zaporna smer $V > 0$ d_n se zmanjšuje

T El. tokovi v p-n stiku



$I = I_e + I_v$

$I_e = I_{en} - I_{ep}$

$I_{en}, I_{ep}, I_{vn}, I_{vp} > 0$

Brez zun. el. napetosti

ker smo v ravnovesju

$I_e = I_{en} - I_{ep} = 0$

$I_{en} = A n_0 e^{-\beta(\Delta E_{pn} + E_s - E_{Fn})}$

$I_{ep} = A n_0 e^{-\beta(E_s - E_{Fp})}$

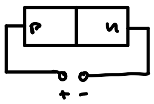
$I_v = I_{vp} - I_{vn} = 0$

$I_{vp} = A' n_0 e^{-\beta(\Delta E_{pn} + E_{Fp})}$

$I_{vn} = A' n_0 e^{-\beta E_{Fn}}$

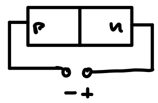
Zunanja el. napetost

$V > 0$



tok v prevodni smeri

$V < 0$



tok v zaporni smeri

$V \neq 0$ $\quad \Delta E_{pn} \to \Delta E_{pn} - e_0 V$

apostrofi zaradi zun. el. pot.

$I_e' = I_{en}' - I_{ep}' = I \ldots = I_{en} e^{\beta e_0 V} - I_{ep} = I_{en}\left(e^{\beta e_0 V} - 1\right)$

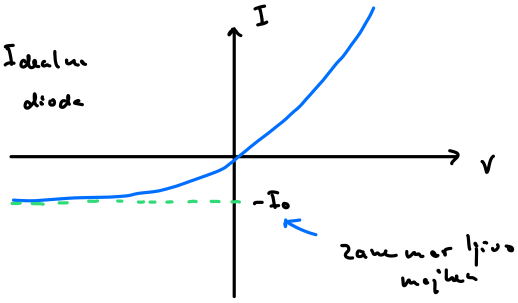
$I_{en}' = I_{en} e^{\beta e_0 V}$ $\quad I_{ep}' = I_{ep}$ $\quad I_{ep} = I_{en}$

$I_v' = I_{vp}' - I_{vn}' = I_{vp} e^{\beta e_0 V} - I_{vn} = I_{vn}\left(e^{\beta e_0 V} - 1\right)$

$I_{vp}' = I_{vp} e^{\beta e_0 V}$ $\quad I_{vn} = I_{vn}'$ $\quad I_{vp} = I_{vn}$

$$I' = I_e' + I_v' = I_0\left(e^{\beta e_0 V} - 1\right)$$

Idealna dioda



N/18 p-n stik iz kocke z robom $a = 1\,mm$

$\sigma_n = 500\ 1/\Omega m$

$\sigma_p = 200\ 1/\Omega m$

$T = 300\,K$

$I = 10\,mA$

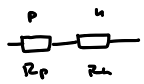
$I_0 = 1\,\mu A$

$V_0 = ?$

$$\boxed{I = I_0\left(e^{\beta e_0 V} - 1\right)}$$

$$V = \frac{k_B T}{e_0} \ln \frac{I}{I_0} = \ldots = 0{,}24\,V$$

Neidealna dioda



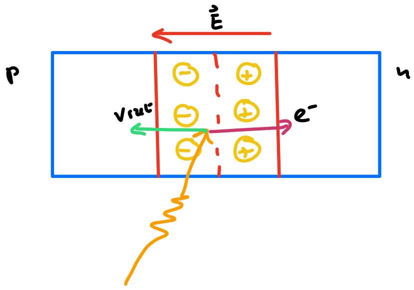
$L = a \qquad S = a^2$

$$R = R_p + R_n = \frac{1}{\sigma_p}\frac{L}{S} + \frac{1}{\sigma_n}\frac{L}{S} = \frac{1}{a}\left(\frac{1}{\sigma_p} + \frac{1}{\sigma_n}\right) = 7\,\Omega$$

$$V_0 = V + RI = 1\mu A + 1mA \cdot 7\Omega$$

T Foto dioda

- p-n stik uporabimo za detekcijo svetlobe
- energija fotona mora biti večja od $E_g \Rightarrow e^-$ preide iz val. v prevodni pas
  - skače nazaj že.
  - dobimo tok I_f (v zaporni smeri)



$$I = I_0(T)\left(e^{\beta e_0 V} - 1\right) - I_f$$

$$I_f = 2 e_0 \frac{\eta P_s}{h\nu}$$

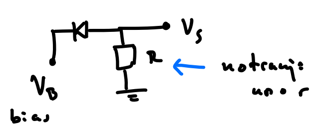
η .. izkoristek detekcije
P_s .. moč svetila

- želimo si: linearen odziv
- zelo tanke ene od plasti
- čim nižja temperatura
- $V < 0$ čim bolj negativen V

Fotodioda z notranjim uporom

$$I = I_0\left(e^{\beta e_0 V} - 1\right) - I_f$$

$$I = I_0\left(e^{-\beta e_0 |V_B - IR|} - 1\right) - I_f$$



$$V = -(V_B - V_s) = -|V_B - IR|$$

Nizke temp. $\quad \beta e_0 |V_B - IR| \gg 1$

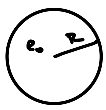
Jedra

Z – vrstno št.
A – masno št.
N = A − Z

$^{A}_{Z}X_{N}$

Naivni model – imamo samo elektrostatsko energijo

V/2 Količina elektrostatične energije
- naboj e_0
- radij R
- $\frac{e_0}{V}$ = konst



$$\int \vec{D}\, d\vec{S} = e$$
$$\vec{D} = \varepsilon_0 \vec{E}$$

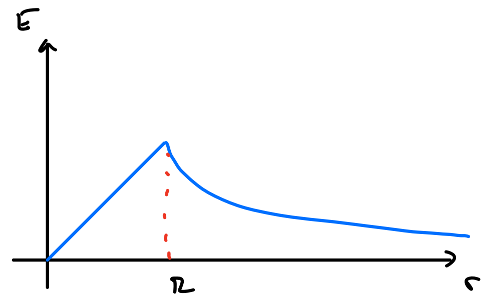
1

a El. polje znotraj jedra

$$\frac{e_0}{V} = \frac{e_0}{\frac{4\pi R^3}{3}} = e(r)$$

$$e(r) = \int \vec{D}\, d\vec{S} = \varepsilon_0 E\, 4\pi r^2 = e_0 \frac{r^3}{R^3}$$

$$\Rightarrow \quad E(r) = \frac{e_0}{4\pi\varepsilon_0}\frac{r}{R^3}$$



b El. polje zunaj jedra

$$\int \vec{D}\, d\vec{S} = e_0$$

$$\varepsilon_0 E\, 4\pi r^2 = e_0$$

$$E = \frac{e_0}{4\pi r^2}$$

2 Potencial $\vec{E} = -\nabla\varphi$
- umeritev : $\varphi(r\to\infty) = 0$
- sferične koordinate : $\vec{\nabla} = \frac{\partial}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\hat{e}_\phi$

$$E(r) = -\frac{\partial}{\partial r}\varphi(r)$$ potencial

$$\varphi(r) = -\int E(r)\, dr$$

a Potencial znotraj jedra

$$E(r) = \frac{e_0}{4\pi\varepsilon_0}\frac{r}{R^3}$$

$$\varphi_z(r) = -\frac{e_0}{4\pi\varepsilon_0 R^3}\int r\, dr = -\frac{e_0}{4\pi\varepsilon_0}\frac{r^2}{2R^3} + C$$

b Potencial zunaj jedra

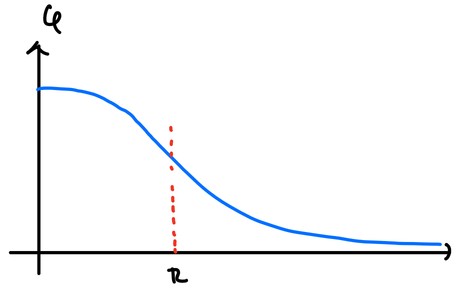
$$\varphi_z(r) = -\frac{e_0}{4\pi\varepsilon_0}\int\frac{dr}{r^2} = \frac{e_0}{4\pi\varepsilon_0}\frac{1}{r} + D$$

Rešimo posoji:

$\varphi_>(r\to\infty)=0 \quad\Rightarrow\quad D=0$

$\varphi_>(r=R) = \varphi_<(r=R) \qquad -\frac{e_0}{4\pi\varepsilon_0}\frac{1}{2R} + C = \frac{e_0}{4\pi\varepsilon_0}\frac{1}{R}$

$C = \frac{3}{2}\frac{e_0}{4\pi\varepsilon_0 R}$



$$\varphi_<(r) = \frac{1}{2}\frac{e_0}{4\pi\varepsilon_0 R}\left(3-\left(\frac{r}{R}\right)^2\right)$$

$$\varphi_>(r) = \frac{e_0}{4\pi\varepsilon_0}\frac{1}{r}$$

$$\varphi(0) = \frac{3}{2}\frac{e_0}{4\pi\varepsilon_0 R}$$

3 Elektrostatska energija

$$E_{el} = \frac{1}{2}\int \varphi_<(r)\, de \qquad e(r) = e_0\frac{r^3}{R^3} \qquad de = e_0\frac{3r^2dr}{R^3}$$

$$E_{el} = \frac{1}{2}\int_0^R \frac{e_0}{8\pi\varepsilon_0 R}\left(3-\left(\frac{r}{R}\right)^2\right)\frac{3e_0}{R^3}r^2dr$$

$$= \frac{3}{2}\frac{e_0^2}{8\pi\varepsilon_0 R^4}\int_0^R\left(3r^2 - \frac{r^4}{R^2}\right)dr =$$

$$= \frac{3}{2}\frac{e_0^2}{8\pi\varepsilon_0 R^4}\,\frac{4}{5}R^3 = \frac{3}{5}\frac{e_0^2}{4\pi\varepsilon_0 R}$$

Podatki: $R = 2\,fm$

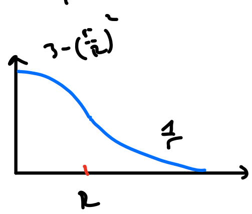
$\hbar c = 197\,MeV\,fm$

$\frac{e^2}{4\pi\varepsilon_0\hbar c} = \alpha = \frac{1}{137}$

$$E_{el} = \frac{3}{5}\alpha\frac{1}{R}\hbar c = \frac{3}{5}\frac{1}{137}\frac{197\,MeV\,fm}{2\,fm} = 0{,}4\,MeV$$

Komentar
- tipične energije so MeV
- homogena porazdelitev naboja $\frac{e_0}{V} = konst$ (dodatna sila ≈ močna sila)
- sistem ni stabilen

Efektivni el potencial

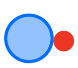


VI(3) α delec pri obstreljevanju $^{56}_{26}Fe$

prodre do centra atoma

$\alpha = [^4_2He]^{2+} \qquad N=2 \quad Z=2 \quad A=4$

Fe $\qquad N=30 \quad Z=26 \quad A=56$

(a)  $R = r_{Fe} + r_\alpha \qquad E_1 = e_\alpha\varphi_{Fe}(r_{Fe}+r_\alpha) = \frac{Z_\alpha e_0\, Z_{Fe}e_0}{4\pi\varepsilon_0(r_{Fe}+r_\alpha)}$

(b)  $E_2 = e_\alpha\varphi_{Fe}(0) = \frac{Z_\alpha e_0\, Z_{Fe}e_0}{4\pi\varepsilon_0 r_{Fe}}\,\frac{9}{2}$

Radij jedra $\boxed{r_j = r_1\sqrt[3]{A_j} \qquad r_1 \sim 1{,}25\,fm}$

$E_1 = 11{,}04\,MeV \qquad E_2 = 23\,MeV \qquad$ Kin. en. α delca je $\sim 10\,MeV$

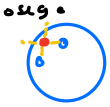
T Semiempirična masna formula

- močna sila
- masa jeder je manjša kot masa vseh nukleonov

$m(A,Z)c^2 = Zm_p c^2 + N m_n c^2 + E_v(A,Z)$

$$\boxed{E_v(A,Z) = -w_0 A + w_1 A^{2/3} + w_2 \frac{Z^2}{A^{1/3}} + w_3 \frac{(A-2Z)^2}{A} + w_4 \frac{\delta_{ZN}}{A^{3/4}}}$$

0) Jedrska sila kratkega dosega (privlačna sila) $w_0 = 15{,}6\ \mathrm{MeV}$

1) Površinska energija  $S \propto r^2 \propto A^{2/3}$ $w_1 = 17{,}3\ \mathrm{MeV}$

2) Elektrostatska energija  (protoni se odbijajo) $w_2 \frac{Z^2}{r} = w_2 \frac{Z^2}{A^{1/3}}$ $w_2 = 0{,}7\ \mathrm{MeV}$

3) Mešalna energija (št p^+ in n^0 skoraj enako $w_3 A \frac{(N-Z)^2}{N+Z} = w_3 \frac{(A-2Z)^2}{A}$ $w_3 = 23{,}7\ \mathrm{MeV}$

4) Paritvena energija $\delta_{ZN} = \begin{cases} 1 & (Z,N)\ (\text{sod},\text{sod}) \\ 0 & (Z,N)\ (\text{sod},\text{lih})\ (\text{lih},\text{sod}) \\ -1 & (Z,N)\ (\text{lih},\text{lih}) \end{cases}$ $w_4 = 33{,}5\ \mathrm{MeV}$

VI/6 a) $n + {}^{235}_{92}U \rightarrow {}^{141}_{55}Cs + {}^{95}_{37}Rb$

- Koliko energije se sprosti?

$Q = E_k - E_z$ posoj: $T = 0$

Začetek: $E_z = m_n c^2 + m_U c^2$ $m_i c^2 = m_p Z_i c^2 + N_i m_n c^2 + E_v(A_i, Z_i)$

Konec: $E_k = m_{Cs} c^2 + m_{Rb} c^2$

${}^{235}_{92}U$ $Z = 92$ $A = 235$ $N = 143$

n $Z = 0$ $A = 1$ $N = 1$

${}^{141}_{55}Cs$ $Z = 55$ $A = 141$ $N = 86$

${}^{95}_{37}Rb$ $Z = 37$ $A = 95$ $N = 58$

Začetek $A_z = 236$ $Z_z = 92$

Konec $A_k = 236$ $Z_k = 92$

$\Rightarrow Q = E_k - E_z = E_{vk} - E_{vz} = E_v(Cs) + E_v(Rb) - E_v(U) - \underset{=0}{E_v(n)} = -179{,}7\ \mathrm{MeV}$

sproščena energija

b) Obravnavaj reakcijo v težiščnem sistemu, kjer imata n in U nasprotno enake gib. kol.

Prostor Minkowskega

$a_\mu = (a_0, a_1, a_2, a_3)$

$(a \cdot a) = (a_\mu)^2 = g_{\mu\nu} a^\mu a^\nu = a_0^2 - a_1^2 - a_2^2 - a_3^2$

$$g_{\mu\nu} = \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

$p_\mu c = (E, \vec{p}c)$

$E = mc^2 + T = \sqrt{(pc)^2 + (mc^2)^2}$

mirovna energija

$p_{np} = (E_n, 0, 0, p_z c)$ nevtron $\qquad p_z c = 0{,}5\ \mathrm{GeV}$

$p_u = (E_u, 0, 0, -p_z c)$ uran

$p_{np} c + p_{up} c = (E_n + E_u, 0, 0, 0)$

	$mc^2\,[\mathrm{GeV}]$
U	218,94
n	0,940
Cs	131,27
Rb	88,43

Energija po trku

težiščna energija $\rightarrow E^{*2} = (E_n + E_u)^2 - |\vec{0}|^2$

$E^* = E_n + E_u = \sqrt{(m_n c^2)^2 + (p_z c)^2} + \sqrt{(m_u c^2)^2 + (p_z c)^2} = 1{,}064\ \mathrm{GeV} + 218{,}94\ \mathrm{GeV}$

$= 220{,}0\ \mathrm{GeV}$

$m_{Cs} c^2 + m_{Rb} c^2 = 219{,}7\ \mathrm{GeV}$ dovolj energije da reakcija poteče.

$p_z c = p_{Cs} c = -p_{Rb} c$

$E_n + E_u = E^* = E_{Rb} + E_{Cs} = (T_{Rb} + m_{Rb} c^2) + (T_{Cs} + m_{Cs} c^2)$

$T = T_{Rb} + T_{Cs} = E^* - m_{Rb} c^2 - m_{Cs} c^2 = 300\ \mathrm{MeV}$

$T = \sqrt{(mc^2)^2 + (pc)^2} - mc^2 = mc^2 \left(\sqrt{1 + \left(\frac{pc}{mc^2}\right)^2} - 1 \right)$

ravno j

$\simeq mc^2 \left(1 + \frac{1}{2}\left(\frac{pc}{mc^2}\right)^2 - 1\right) = \frac{1}{2}\frac{(pc)^2}{mc^2} = \frac{p^2}{2m}$

$T = T_{Rb} + T_{Cs} = \frac{1}{2}\frac{(p_{Rb} c)^2}{m_{Rb} c^2} + \frac{1}{2}\frac{(p_{Rb} c)^2}{m_{Cs} c^2} = \frac{(p_K c)^2}{2}\left(\frac{1}{m_{Rb} c^2} + \frac{1}{m_{Cs} c^2}\right)$

$p_K c = 5{,}63\ \mathrm{GeV}$

$T_{Rb} = 175\ \mathrm{MeV} \qquad T_{Cs} = 121\ \mathrm{MeV}$

VI/8 Poišči najmanjšo energijo fotona, da iz jedra ^{56}Fe izbije proton

ⓐ $\gamma + {}^{56}_{26}Fe \longrightarrow {}^{1}_{1}p + {}^{55}_{25}Mn \qquad E_\gamma^{min} = ?$

ⓑ $\gamma + {}^{56}_{26}Fe \longrightarrow {}^{1}_{0}n + {}^{55}_{26}Fe \qquad E_\gamma^{min} = ?$

Fe ———— γ $\Rightarrow$ $\cdot\cdot$ Tež. sist.

$p_1 = p_2 = 0$

$E_\gamma = p_\gamma c = -p_{Fe} c$

$E_z = E_\gamma + \sqrt{(m_{Fe} c^2)^2 + E_\gamma^2} = E^* = E_u = m_p c^2 + m_{Mn} c^2$

$E_\gamma + m_{Fe} c^2 = m_p c^2 + m_{Mn} c^2$) $mc^2 = Z m_p c^2 + N m_n c^2 + E_v$

ⓐ $E_\gamma + E_v^{56Fe} = E_v^{p} + E_v^{Mn}$

ⓑ $E_\gamma + E_v^{56Fe} = E_v^{55Fe}$

$E_v^{56Fe} \overset{SEMF}{=} \dots = -491,67 \text{ MeV}$

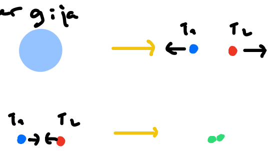
$E_v^{Mn} = -482,17 \text{ MeV}$

$E_v^{55Fe} = -479,56 \text{ MeV}$

ⓐ $E_\gamma = 9,5 \text{ MeV}$

ⓑ $E_\gamma = 12,1 \text{ MeV}$

ⓒ odrivna energija



Odriv. en. $= T_{56Fe}$

Zanimamo: $\frac{T_{Fe}}{E_\gamma} = \frac{(p_\gamma c)^2}{2 m_{Fe} c^2 (p_\gamma c)} = \frac{E_\gamma}{2 m_{Fe} c^2} \sim 10^{-4}$

5.8 $A = 2Z$, razpad α. Katera jedra so obstojna na α razpad. $|\delta Z| \ll Z$

$$^A_Z X \longrightarrow {}^{A-4}_{Z-2} Y + {}^4_2 He$$

Kriterij za stabilnost elementa X: $E_v(Z,A) \le E_v(Z-2, A-4) + E_v(2,4)$

ker $A = 2Z$

$$E_v(Z,A) = -w_0 A + w_1 A^{2/3} + w_2 \frac{Z^2}{A^{1/3}} + w_3 \cancel{\frac{(A-2Z)^2}{A}} + w_4 \underbrace{\frac{\delta_{pp}}{A^{3/4}}}$$

Gre proti 0 za velik A.

$\frac{d f(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$E_v(Z,A) - E_v(Z-2, A-4) - E_v(2,4) \le 0$$

$$\frac{\partial E_v}{\partial Z} \cdot 2 + \frac{\partial E_v}{\partial A} \cdot 4 - E_v(2,4) \le 0$$

$$2 w_2 \frac{2Z}{A^{1/3}}\Big|_{A=2Z} \quad -w_0 + \frac{2}{3}\frac{w_1}{A^{1/3}} - \frac{1}{3} w_2 Z^2 \frac{1}{A^{4/3}}\Big|_{A=2Z}$$

$$2(2^{2/3} w_2 Z^{2/3}) + 4\left(-w_0 + \frac{2^{2/3}}{3} w_1 Z^{-1/3} - \frac{2^{-4/3}}{3} w_2 Z^{2/3}\right) - E_v(2,4) \le 0$$

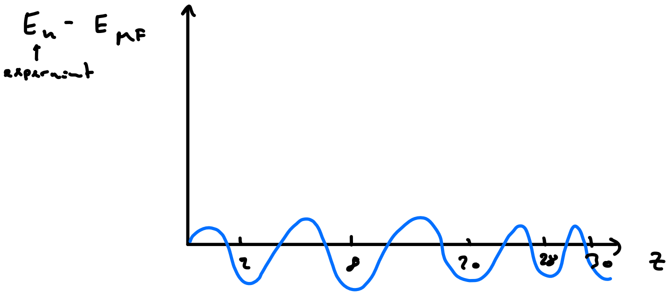
$$\dots \frac{5 \cdot 2^{2/3}}{3} w_2 Z + \frac{2^{5/3}}{3} w_1 - \left(2^{4/3}(w_0 + w_2) - 2^{-7/12} w_4\right) Z^{1/3} \le 0$$

$$\le \overset{lim}{\Rightarrow} =$$

$$Z_{min} = \frac{3}{w_2 5 \cdot 2^{2/3}} \left(\left(2^{4/3}(w_0 + w_2) - 2^{-7/12} w_4\right) Z^{1/3} - \frac{2^{5/3}}{3} w_1 \right)$$

Rešimo z numerično iteracijo $Z_{min} = 44,22 \Rightarrow Z_{min} = 44$

T Lupinski model



magična števila

- krogelno sim. potencial ⇒ nukloni v enodelčnih stanjih
- lupine
- n, p^+ seštejemo ločeno (Pauli)

1. Harmonski oscilator $V(r) = \frac{1}{2} m\omega^2 r^2$ 3D — Glede na št. degeneracij pri določeni energiji
2. Realističen Saxon-Woods potencial + sklopitev spin-tir

1
a) zapiši energije
b) napovej magična števila
c) iz spektra $^{19}_{9}F$ ... $E_5 - E_1$ poznan izračunaj ω

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V(r) \qquad V(r) = V_0 + \frac{1}{2} m\omega^2 r^2$$

$$H\psi(r) = E_n \psi(r) \qquad \psi(r) = R_{n_x}(x) R_{n_y}(y) R_{n_z}(z)$$

$$x: \quad -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} R_{n_x}(x) + V_0 + \frac{1}{2} m\omega^2 x^2 R_{n_x}(x) = E_n R_{n_x}(x)$$

Vemo rešitev $V_0 + E_{n_x} = \hbar\omega\left(n_x + \frac{1}{2}\right)$

$$V_0 + E_n = \hbar\omega\left(n_x + n_y + n_z + \frac{3}{2}\right) = \hbar\omega\left(n + \frac{1}{2}\right) \qquad n = n_x + n_y + n_z + 1$$

$\frac{E+V_0}{\hbar\omega}$	n_x n_y n_z	Degeneracija	Kumulativna vsota
$\frac{9}{2}$	$(3,0,0)\cdot 3$ $(1,1,1)$ $(2,1,0)\cdot 6$	$10\cdot 2 = 20$	40
$\frac{7}{2}$	$(1,1,0)\cdot 3$ $(2,0,0)\cdot 3$	$6\cdot 2 = 12$	20
$\frac{5}{2}$	1 0 0, 0 1 0, 0 0 1	$3\cdot 2 = 6$	8
$\frac{3}{2}$	0 0 0	$1\cdot 2 = 2$	2

zaradi spina

$$\Delta E = E_{n_5} - E_{n_1} = \frac{7}{2}\hbar\omega - \frac{3}{2}\hbar\omega = 2\hbar\omega$$

Komentar: sferične koordinate

$$\psi(r,\theta,\varphi) = R_{nl}(r)\, Y_{lm}(\theta,\varphi)$$

$$-\frac{\hbar^2}{2m}\nabla^2 = -\frac{\hbar^2}{2m}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{\hat{l}^2}{2mr^2}\right)$$

$$\hat{l}^2 Y_{lm} = \hbar^2 l(l+1) Y_{lm}$$

$$\Rightarrow E = \hbar\omega\left(2n_r + l + \frac{3}{2}\right) = \hbar\omega\left(n + \frac{1}{2}\right)$$

Pogoji: $n_r \geq 0$

$$\ell = \begin{cases} 0, 2, 4 & n \text{ sod} \\ 1, 3, 5 & n \text{ lih} \end{cases}$$

$$-\ell \leq m \leq \ell$$

Degeneracija pri nivoju n

$$2 \sum_{\ell=0}^{n} (2\ell+1) = n(n+1)$$

$n = 2n_r + \ell + 1$

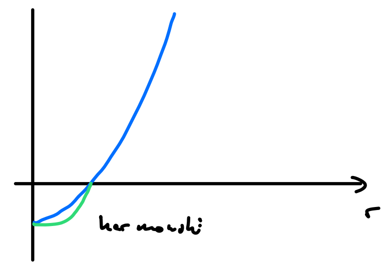
n	(n_r, ℓ)	spektroskopski o.	deg. $n(n+1)$
4	$(1,1)$ $(0,3)$	2p 1f	20
3	$(1,0)$ $(0,2)$	2s 1d	12
2	$(0,1)$	1p	6
1	$(0,0)$	1s	2

2 Saxon-Woods in spin-tir sklopitev

a p^+, n konfiguracija

b magična števila (posebej močno vezana stanja)

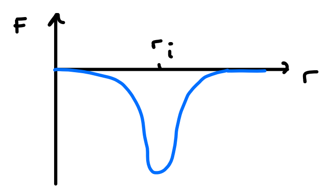
c napovej magnetni moment



$$V(r) = -\frac{V_0}{e^{(r-r_j)/s}+1} \qquad r_j = r_0 \sqrt[3]{A_j}$$

$$s \approx 0{,}55 \text{ fm} \qquad V_0 \approx 50 \text{ MeV}$$

$$F = -\frac{dV}{dr} = -\frac{V_0}{s} \frac{e^{(r-r_j)/s}}{\left(e^{(r-r_j)/s}+1\right)^2}$$



Numerično reševanje: večji $\ell \to$ nižje energije napram LHO.

$$E \propto \frac{\ell(\ell+1)}{r^2}$$

Sklopitev spin-tir

$$\hat{H} = -\eta \, \hat{\vec{\ell}} \, \hat{\vec{s}} = -\eta \frac{\vec{j}^2 - \vec{\ell}^2 - \vec{s}^2}{2} \qquad \eta = 20 \, A^{-2/3} \, \frac{\text{MeV}}{\hbar^2}$$

$$\vec{j} = \vec{\ell} + \vec{s}$$

$$\vec{j}^2 = \vec{\ell}^2 + \vec{s}^2 + 2\,\vec{\ell}\cdot\vec{s}$$

$$\hat{\ell}^2 \, Y_{\ell m} = \hbar^2 \ell(\ell+1) \, Y_{\ell m}$$

$$E = -\frac{\eta}{2} \hbar^2 \left(j(j+1) - \ell(\ell+1) - s(s+1) \right)$$

$j, m_j \qquad j = |\ell + s|, \ldots, |\ell - s|$

$\qquad\qquad m_j = -j, \ldots, j$

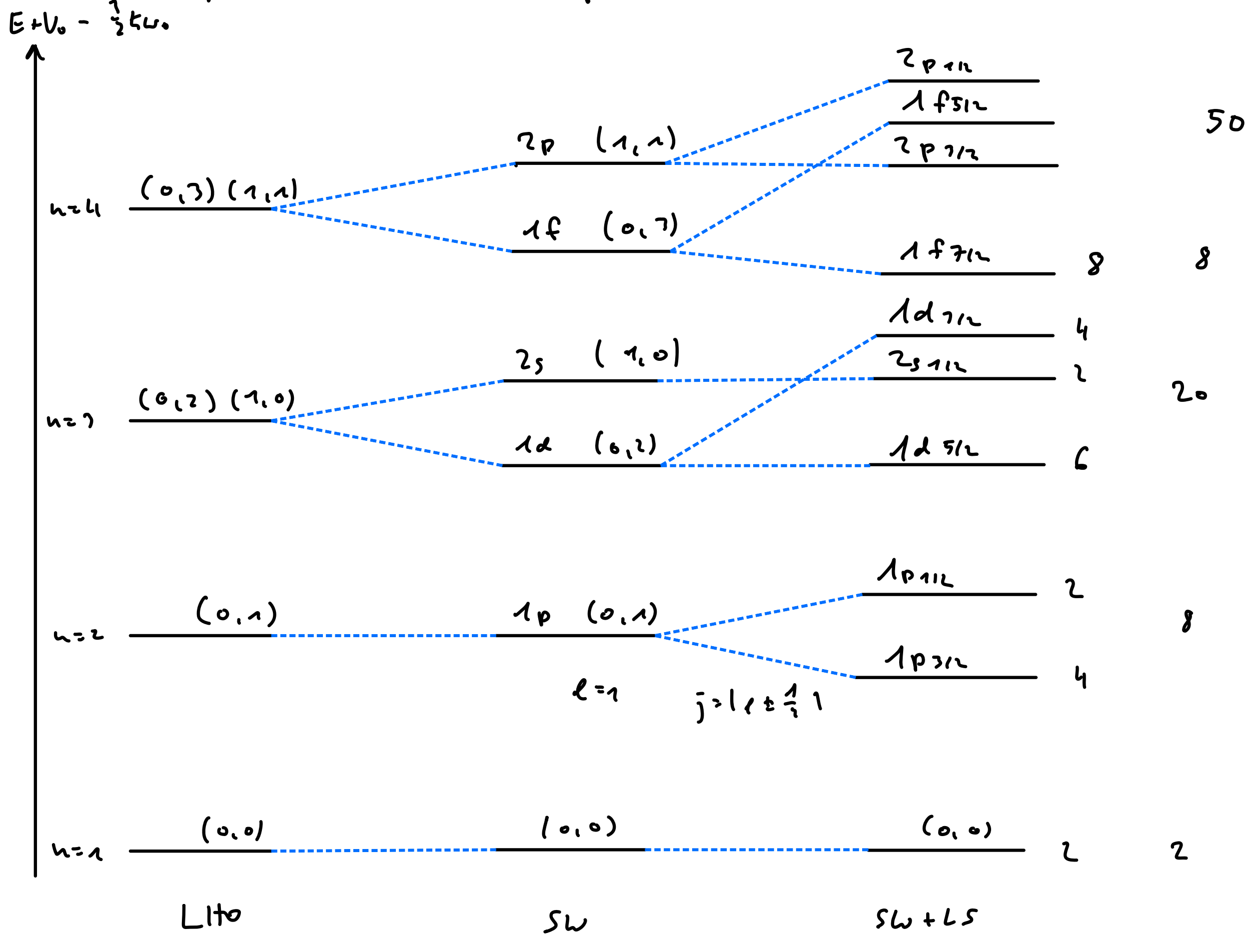
Spin $1/2$ $\qquad j = |l \pm \frac{1}{2}|$

$j = l + \frac{1}{2} \qquad E = -\frac{4}{2}\hbar^2 \left((l+\frac{1}{2})(l+\frac{3}{2}) - l(l+1) - \frac{1}{2}\frac{3}{2} \right)$

$\qquad = \frac{4}{2}\hbar^2 (-l)$

$j = l - \frac{1}{2} \qquad E = \ldots = \frac{4}{2}\hbar^2 (l+1)$

Višji j ima nižjo energijo.



Primeri

$^{15}_{7}N \qquad$ 7p $\qquad p \quad (1s_{1/2})^2 \, (1p_{3/2})^4 \, (1p_{1/2})^1$

$\qquad\qquad$ 8n $\qquad\quad (1s_{1/2})^2 \, (1p_{3/2})^4 \, (1p_{1/2})^2$

Osn. vrt. moment $\qquad j = \frac{1}{2} \qquad l = 1$

Parnost $\qquad \underline{(-1)^l} \qquad p \to l = 1$

$^{16}_{8}O \qquad$ 8p 8n $\qquad$ 2x magično $\qquad p^+ \quad (1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2$

$\qquad\qquad n \quad (1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2$

Ni: ne sparjenih nukleonov zato $J = 0 \quad L = 0$

$^{33}_{16}S \qquad$ DN

$^{7}_{3}Li \qquad p^+ \quad (1s_{1/2})^2 \, (1p_{3/2})^1$

$\qquad\quad n^- \quad (1s_{1/2})^2 \, (1p_{3/2})^1$

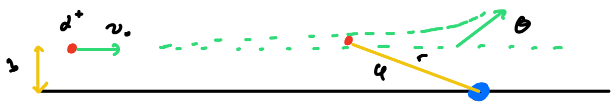
$L_p = 1 \qquad L_n = 1 \qquad J_p = \frac{3}{2} \qquad J_n = \frac{3}{2}$

$|J_p - J_n|, \dots, |J_p + J_n|$

$0, 1, 2, 3$

Parnost $(-1)^{L_p + L_n} = +1$

Rutherfordovo sipanje



Sipanje α^+ na Au foliji

1. Ohranitev vrtilne količine $\frac{d\vec{\Gamma}}{dt} = 0$ (ni navora)
2. $|\vec{v}| = $ konst
3. Porazdelitev po kotih

(1)

$$r = (-r\cos\varphi, r\sin\varphi)$$
$$\vec{\Gamma} = m\vec{r} \times \vec{v}$$
$$v = \dot{\vec{r}} = (-\dot{r}\cos\varphi + r\sin\varphi\,\dot{\varphi},\ \dot{r}\sin\varphi + r\cos\varphi\,\dot{\varphi})$$
$$(\vec{r}\times\vec{v})_z = r_x v_y - r_y v_x = -r\cos\varphi(\dot{r}\sin\varphi + r\cos\varphi\,\dot{\varphi}) - r\sin\varphi(-\dot{r}\cos\varphi + r\sin\varphi\,\dot{\varphi}) = -r^2\dot{\varphi}$$
$$\Gamma_z = -mr^2\dot{\varphi} \qquad |\vec{\Gamma}| = mr^2\dot{\varphi}$$

Začetek $|\Gamma(t \to -\infty)| = m v_0 b = m r^2 \dot{\varphi} \Rightarrow \dot{\varphi} = \frac{v_0 b}{r^2}$

(2)

$$F_y = m\frac{dv_y}{dt} = \frac{z_1 z_2 e_0^2}{4\pi\varepsilon_0 r^2}\sin\varphi$$
$$m\frac{dv_y}{dt} = m\frac{dv_y}{d\varphi}\frac{d\varphi}{dt} = \frac{z_1 z_2 e_0^2}{4\pi\varepsilon_0 r^2}\sin\varphi$$
$$\frac{dv_y}{dt} = \frac{z_1 z_2 e_0^2}{4\pi\varepsilon_0 r^2}\frac{r^2}{v_0 b m}\sin\varphi$$
$$v_y = \frac{z_1 z_2 e_0^2}{4\pi\varepsilon_0 v_0 b m}\int_0^{\varphi}\sin\varphi\,d\varphi$$
$$v_y = -\frac{z_1 z_2 e_0^2}{4\pi\varepsilon_0 v_0 b m}(\cos\varphi - 1)$$

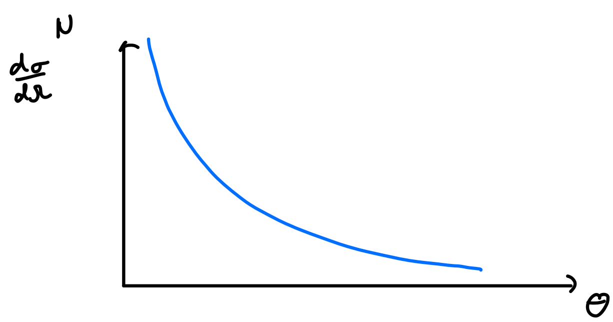
$$\varphi(t \to \infty) = \pi - \theta$$

$t \to \infty$ $\quad v_y(\theta) = \frac{z_1 z_2 e_0^2}{4\pi\varepsilon_0 v_0 b m}(1 + \cos\theta) = v_0 \sin\theta$

$$\Rightarrow b = \frac{z_1 z_2 e_0^2}{4\pi\varepsilon_0 v_0^2 m}\frac{1+\cos\theta}{\sin\theta} = c\,\frac{\cos\frac{\theta}{2}}{\sin\theta/2} = c\cot\frac{\theta}{2}$$

(3) Sipalni presek $\frac{d\sigma}{d\Omega}d\Omega = \frac{\#\ \text{delcev sipanih v kot } \Omega \text{ na enoto časa}}{\text{vhodna intenziteta}}$

$$dS = 2\pi b\,db$$
$$\frac{db}{d\theta} = -\frac{1}{2}c\frac{1}{\sin^2\theta/2}$$
$$d\Omega = 2\pi\sin\theta\,d\theta$$



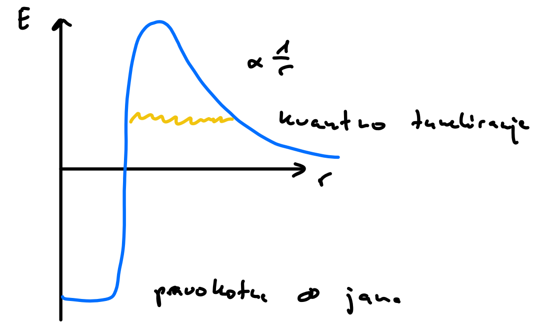
Št. delcev se ohranja

$$I\,2\pi b\,db = I\frac{d\sigma}{d\Omega}d\Omega = I\frac{d\sigma}{d\Omega}2\pi\sin\theta\,d\theta$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\frac{db}{d\theta} = -\frac{1}{2}c^2\frac{\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2}\,\sin\theta\,\sin\frac{\theta}{2}} = -\frac{1}{4}c^2\frac{1}{\sin^4\theta/2}$$

Razpad α

$${}^{A}_{Z}X \longrightarrow {}^{4}_{2}\alpha + {}^{A-4}_{Z-2}Y$$



Geiger Nuttallova enačba

$$\ln \tau = \frac{a'}{\sqrt{T}} - b' \qquad a' = \frac{e_0^2 (Z-2)(2m_r)^{1/2}}{2\varepsilon_0 \hbar}$$

T kin. en. α

$$\frac{1}{m_r} = \frac{1}{m_\alpha} + \frac{1}{m_Y} \qquad R = r_\alpha + r_Y$$

$$b' = \frac{4e_0}{\hbar}\left(\frac{(Z-2)m_r R}{\pi\varepsilon_0}\right)^{1/2} + \ln\sqrt{\frac{T}{2m_r R^2}}$$

- kratkoživi izotopi izsevajo bolj energetske α delce

3.7.2 $\quad {}^{210}_{84}Po \longrightarrow {}^{206}_{82}Pb + \alpha$

$m_\alpha = 4{,}0026u$

$m_{Po} = (210 - 0{,}0171)u$

$m_{Pb} = (206 - 0{,}0256)u$

$\tau = ?$

$$Q = (m_\alpha + m_{Pb} - m_{Po})c^2 = (4 + 0{,}0026 + 206 - 0{,}0256 - 210 + 0{,}0171)uc^2$$

$$u = 931{,}494 \frac{MeV}{c^2}$$

$Q = -5{,}5\,MeV$ energija se sprosti / razpoložljiva energija

Težiščni sistem

$v_{Po} = 0 \qquad |\vec{p}_{Pb}| = |\vec{p}_\alpha| = p \qquad T_\alpha = \frac{p^2}{2m} \qquad T_{Pb} = \frac{p^2}{2m_{Pb}}\frac{m_\alpha}{m_\alpha} = T_\alpha \frac{m_\alpha}{m_{Pb}}$

Ohranitev energije $\quad m_{Po}c^2 = m_{Pb}c^2 + m_\alpha c^2 + T_{Pb} + T_\alpha$

$$(m_{Po} - m_{Pb} - m_\alpha)c^2 = -Q = T_{Pb} + T_\alpha = \left(\frac{m_\alpha}{m_{Pb}} + 1\right)T_\alpha$$

$$T_\alpha = \frac{-Q}{1 + \frac{m_\alpha}{m_{Pb}}} = 5{,}4\,MeV$$

$$\frac{1}{m_r} = \frac{1}{m_\alpha} + \cancel{\frac{1}{m_{Pb}}}^{\approx 0} \Rightarrow m_r \simeq m_\alpha$$

$$R = r_\alpha + r_{Po} = r_1\left(A_\alpha^{1/3} + A_{Pb}^{1/3}\right)$$

$$a' = \frac{e_0^2 (Z-2)\sqrt{2m_r c^2}}{2\varepsilon_0 \hbar c}\frac{2\pi}{2\pi} = \qquad \boxed{\alpha\hbar c = \frac{e_0^2}{4\pi\varepsilon_0}}$$

$$a' = 2\pi\alpha(\overset{84}{Z} - 2)\sqrt{2m_r c^2} = 325\sqrt{MeV}$$

$$b' = \frac{4e_0}{\hbar}\left(\frac{(Z-2)m_r R}{\pi\varepsilon_0}\right)^{1/2} + \ln\sqrt{\frac{T}{2m_r R^2}}$$

$$= \frac{8}{\sqrt{\hbar c}}\left((Z-2)m_r c^2 \alpha R\right)^{1/2} + \ln\sqrt{\frac{Tc^2}{2m_r c^2 R^2}} = b_0 + b_1 = 77{,}7 + 48{,}3$$

$$\ln\tau = \frac{a'}{\sqrt{T}} - b_0 - b_1 = \frac{a'}{T} - b - \ln\sqrt{\frac{T}{2m_r R^2}}$$

$$\ln\tau\sqrt{\frac{T}{2m_r R^2}} = \frac{a'}{T} - b_0$$

$$\tau = \sqrt{\frac{2m_r c^2 R^2}{Tc^2}}\, e^{\frac{a'}{\sqrt{T}} - b_0} = 1{,}52 \cdot 10^6\,s = 17{,}6\text{ dni}$$

$t_{1/2} = \tau \ln 2 = 12{,}2\text{ dni}$

15 Beta razpad

$${}^{A}_{Z}X_N \rightarrow {}^{A}_{Z+1}Y_{N-1} + e^- + \bar{\nu}_e$$

$$n \rightarrow p + e^- + \bar{\nu}_e$$

$m_n = 939{,}56$ MeV/c^2
$m_p = 938{,}27$ MeV/c^2
$m_e = 0{,}51$ MeV/c^2
$m_\nu \approx 0$

$$Q = (m_p + m_e - m_n)c^2 = -0{,}78 \text{ MeV}$$

a) ν_e, p sta pri miru

relativistično

$$|Q| = T_e = E_e - m_e c^2 = \gamma m_e c^2 - m_e c^2 = (\gamma - 1) m_e c^2$$

$$\gamma = \frac{|Q|}{m_e c^2} + 1 = 2{,}53 \qquad \beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0{,}92 \qquad v = 0{,}92c$$

b) p pri miru

težiščni sistem

$p_n c = (m_n c^2, \vec{0})$
$p_p c = (m_p c^2, \vec{0})$
$p_e c = (E_e, \vec{p}c) = \left(\sqrt{(m_e c^2)^2 + (pc)^2}, \vec{p}c\right)$
$p_\nu c = (E_\nu, -\vec{p}c) = (pc, -\vec{p}c)$

ohranitev energije

$$m_n c^2 = m_p c^2 + \sqrt{(m_e c^2)^2 + (pc)^2} + pc$$

$$((m_n - m_p)c^2 - pc)^2 = (m_e c^2)^2 + (pc)^2$$

$$(m_n - m_p)^2 c^4 - 2pc(m_n - m_p)c^2 + (pc)^2 = (m_e c^2)^2 + (pc)^2$$

$$pc = \frac{1}{2(m_n - m_p)c^2}\left(((m_n - m_p)c^2)^2 - (m_e c^2)^2\right) = 0{,}54 \text{ MeV}$$

$$E = \gamma m c^2 \qquad p = \gamma m v \qquad pc = \gamma m v c$$

$$\frac{pc}{E} = \frac{v}{c} = \beta$$

$$\beta = \frac{pc}{E_e} = \frac{pc}{\sqrt{(m_e c^2)^2 + (pc)^2}} = 0{,}73$$

razpad

β $\quad T \sim 15$ MeV
α $\quad T \sim 5$ MeV

T Sevanje gama

$$X^* \longrightarrow X + \gamma$$

Električno dipolno sevanje

Verjetnost za prehod na časovno enoto $\quad \dfrac{1}{\tau} = \dfrac{\omega_{12}^3 \, p_{e_{12}}^2}{3\pi \varepsilon_0 c^3 \hbar} \qquad \Delta E_{12} = \hbar \omega_{12}$

$$p_{e_{12}} = \langle 1 | \hat{p}_e | 2 \rangle = \int \underbrace{\psi_1^*}_{\text{končno}} e \hat{r} \underbrace{\psi_2}_{\text{začetno}} \, d^3r$$

Izbirna pravila

$\Delta L = \pm 1$ $\quad L, S$
$\Delta M_j = \pm 1, 0$

ali:

$\Delta J = \pm 1, 0$ $\quad J, M_j$
$\Delta M_j = \pm 1, 0$

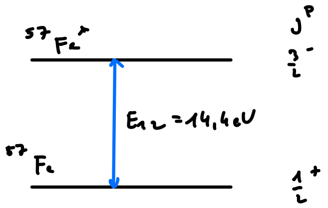
Parnost $\quad P \cdot P' = (-1)$

$P_\gamma = -1 \qquad \Delta M_j = 0, \pm 1$

Parnost pove kako se val. funk. spremeni pri zrcaljenju prostora

$$\vec{r} \xrightarrow{P=-1} -\vec{r}$$
$$\vec{l} = \vec{r} \times \vec{p} \xrightarrow{P=1} (-\vec{r}) \times (-\vec{p}) = \vec{l}$$

3.3.6 $\quad {}^{57}Fe(2) \to {}^{57}Fe(1) + \gamma \qquad \tau = 10^{-7}\,s$



$\delta E = ?$
$p_{e12} = ?$

a) $\quad \delta E\, \delta t \geq \hbar$

$$\delta E \approx \frac{\hbar}{\tau} = 6,58 \cdot 10^{-9}\,eV$$

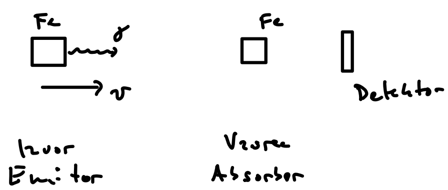
$$\frac{\delta E}{E_{12}} = 4,6 \cdot 10^{-13}$$

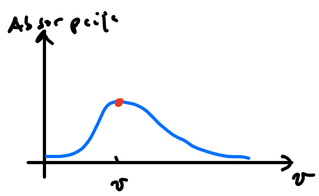
b) $$\boxed{\frac{1}{\tau} = \frac{\omega_{12}^3 p_{e12}}{3\pi\varepsilon_0 c^3 \hbar}}$$

$$p_{e12} = e_0 \sqrt{\frac{3\pi \varepsilon_0 c^3 \hbar\, 4c}{e_0^2\, \tau\, \omega_{12}^3\, 4c}} \qquad \boxed{\frac{e_0^2}{4\pi\varepsilon_0 \hbar c} = \alpha}$$

$$\frac{p_{e12}}{e_0} = \sqrt{\frac{3}{4}\, 137 \left(\frac{\hbar c}{E_{12}}\right)^3 \frac{1}{c}\, \frac{1}{\tau}} = 0,1\,fm$$

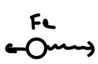
c)





Prost atom ^{57}Fe, pri kateri hitrosti je absorpcija največja

Emitor



$p_{Fe} = p_\gamma$

$T_{Fe} + E_{\gamma E} = E_{12}$

$E_{\gamma E} = E_{12} - \frac{p_\gamma^2}{2m_{Fe}} \quad \Rightarrow \quad E_\gamma = E_{12} - \frac{E_{12}^2}{2m_{Fe}c^2}$

$E_{12} \simeq E_\gamma$

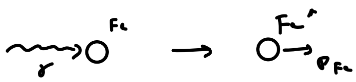
Ker $\quad \frac{T_{Fe}}{E_{12}} \ll 1$

zakaj?

$T_{Fe} \simeq \frac{E_{12}^2}{2m_{Fe}c^2} = 1,8 \cdot 10^{-9}\,keV$

$$E_{\gamma E} = E_{12}\left(1 - \frac{E_{12}}{2m_{Fe}c^2}\right)$$

Absorber



$p_\gamma = p_{Fe} \qquad E_{12} + T_{Fe} = E_{\gamma A}$

$$E_{\gamma A} = E_{12} + \frac{p_\gamma^2}{2m_{Fe}} = E_{12} + \frac{E_\gamma^2}{2m_{Fe}c^2} = E_{12}\left(1+\frac{E_{12}}{2m_{Fe}c^2}\right)$$

Dopplerjev efekt

- izvor mir., spr. prem.

$$\nu' = \nu\left(1 \pm \frac{v}{c}\right)$$

približevanje oddaljevanje

- izvor premik, spr. mir

$$\nu' = \nu / \left(1 \mp \frac{v}{c}\right)$$ za $\frac{v}{c} \ll 1$ taylor $\nu' = \nu\left(1 \pm \frac{v}{c}\right)$

$$E'_{\gamma E} = E_{\gamma E}\left(1+\frac{v}{c}\right) = E_{\gamma A}$$

$$v = c\left(\frac{E_{\gamma A}}{E_{\gamma E}} - 1\right)$$

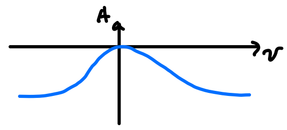
$$v = c\left(\frac{1+\frac{E_{12}}{2mc^2}}{1-\frac{E_{12}}{2mc^2}} - 1\right)$$

$$1+\frac{v}{c} \approx 1 + \frac{E_{12}}{mc^2}$$

$$v = c\,\frac{E_{12}}{m_{Fe}c^2} = c\,\frac{14{,}4\cdot 10^{-3}\,MeV}{57\cdot 931\,MeV} = 81\,\frac{m}{s}$$

(d) $T_{Fe} = 0$

$v = ?$ $v_{1/2} = ?$



$E_{\gamma E} = E_{12}$

$E_{\gamma A} = E_{12}$ $\qquad E_{12} \pm \frac{\delta E}{2}$

$$v = 0$$

$$E'_{\gamma E} = \left(E_{12} - \frac{\delta E}{2}\right)\left(1+\frac{v}{c}\right) = E_{\gamma A} = E_{12}$$

$$v_{1/2} = c\left(\frac{E_{12}}{E_{12}-\frac{\delta E}{2}} - 1\right) = c\left(\frac{1}{1-\frac{\delta E}{2E_{12}}} - 1\right) \approx c\left(1+\frac{\delta E}{2E_{12}} - 1\right)$$

$$v_{1/2} = c\,\frac{\delta E}{E_{12}}\,\frac{1}{2} = 6{,}5\cdot 10^{-5}\,\frac{m}{s}$$

(T) Dinamika jedrskih reakcij

$$N(t) = N_0\, e^{-t/\tau} = N_0\, 2^{-t/t_{1/2}}$$

$$-\frac{dN}{dt} = \lambda N \qquad \lambda = \frac{1}{\tau}$$

Aktivnost $A = -\frac{dN}{dt}$ $[Bq = s^{-1}]$

$$A(t) = \lambda N(t) = \frac{N(t)}{\tau}$$

2 razpadna kanala $\quad X \overset{\tau_1}{\nearrow} \underset{\tau_2}{\searrow} \qquad \frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2} \qquad t_{1/2} = \tau \ln 2$

(34) $R_0 = R(t=0) = \frac{A(^{133}I)}{A(^{131}I)} = 2{,}14$

(a) $R(t) = 0{,}1$ $\quad t = ?$

(b) $\frac{N(^{133}I)}{N(^{131}I)}(t' = 1 \text{ teden}) = ?$

$t_1 = t_{1/2}^{133I} = 20{,}8$ ur $\qquad t_2 = t_{1/2}^{131I} = 8$ dni

(a) $R(t) = \frac{A_1}{A_2} = \frac{\tau_2 N_{10}}{\tau_1 N_{20}} e^{-t/\tau_1 + t/\tau_2}$

$$t = \frac{1}{\frac{1}{\tau_2} - \frac{1}{\tau_1}} \ln\left(\frac{R(t)}{R_0}\right) = \frac{1}{\ln 2\,(t_2^{-1} - t_1^{-1})} \ln\left(\frac{R(t)}{R_0}\right) = 103\,\text{ur} = 4{,}3\,\text{dni}$$

(b) $\frac{N_1}{N_2}(t') = \frac{\tau_1 A_1}{\tau_2 A_2} = \frac{\tau_1}{\tau_2} R(t') = \frac{t_1}{t_2} R_0\, e^{-t\ln 2\left(\frac{1}{t_1} - \frac{1}{t_2}\right)} =$

(33)

$$^{212}_{83}Bi \xrightarrow[\tau_1]{33{,}7\%} {}^{208}_{81}Tl + \alpha \xrightarrow{\tau_3} {}^{208}_{82}Pb + e^- + \bar{\nu}_e$$

$$^{212}_{83}Bi \xrightarrow[\tau_2]{66{,}3\%} {}^{212}_{84}Po + e^- + \bar{\nu}_e$$

$\frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2}$

$\tau \ln 2 = t_{1/2} = 60{,}5\,\text{min}$ $\quad \tau = 87{,}28\,\text{min}$

$t_{1/2,3} = 3{,}1\,\text{min}$ $\quad \tau_3 = 4{,}47\,\text{min}$

$\frac{m_{Tl}}{m_{Bi}}(t = 10\,\text{min}) = ?$

$\frac{\tau_1}{\tau_2} = \frac{\lambda_2}{\lambda_1} = \frac{66{,}3\%}{33{,}7\%} = 1{,}97$ $\qquad \tau_1 = 1{,}97\,\tau_2$

$\frac{1}{\tau} = \frac{1}{\tau_1}\, 2{,}97$ $\qquad \tau_1 = 2{,}97\,\tau = 259\,\text{min}$

Bizmut: $N_{Bi} = N_{Bi}^0 e^{-t/\tau}$

Talij: $\frac{dN_{Tl}}{dt} = \frac{N_{Bi}}{\tau_1} - \frac{N_{Tl}}{\tau_3} = \frac{N_{Bi}^0 e^{-t/\tau}}{\tau_1} - \frac{N_{Tl}}{\tau_3}$

nastavek $\quad N_{Tl} = A e^{-t/\tau} + B e^{-t/\tau_3} = A\left(e^{-t/\tau} - e^{-t/\tau_3}\right)$

začetni pogoj $\quad N_{Tl} = 0 \qquad A = -B$

$$-A\frac{1}{\tau} e^{-t/\tau} + A\frac{1}{\tau_3} e^{-t/\tau_3} = \frac{N_{Bi}}{\tau_1} e^{-t/\tau} - \frac{A}{\tau_3} e^{-t/\tau} + \frac{A}{\tau_3} e^{-t/\tau_3}$$

$$-\frac{A}{\tau} = \frac{N_{Bi}}{\tau_1} - \frac{A}{\tau_3} \qquad A = N_{Bi}\frac{1}{\tau_1}\left(\frac{1}{\tau_3} - \frac{1}{\tau}\right)^{-1} = \frac{N_{Bi}^0}{\tau_1}\frac{\tau\tau_3}{\tau - \tau_3}$$

$\frac{m}{M} = \frac{N}{N_A}$

$$\frac{m_{Bi}}{m_{Tl}} = \frac{N_{Bi} M_{Bi}}{N_{Tl} M_{Tl}} = \frac{M_{Bi}}{M_{Tl}} \frac{1}{\tau_1} \frac{\tau\tau_3}{\tau - \tau_3}\left(1 - e^{-t\left(\frac{1}{\tau_3} - \frac{1}{\tau}\right)}\right) = 0{,}016$$

(35) Fisija

$$^{235}_{92}U \longrightarrow 2\,{}^{A/2}_{Z/2}X + Q$$

$Q = ?$ sproščena energija

$m_{TNT} = 5 \cdot 10^4$ ton

$q_s = 4{,}18\ \frac{GJ}{ton}$

$$Q = \left(2m\left(\frac{A}{2}, \frac{Z}{2}\right) - m(A, Z)\right)c^2$$

$$Q = 2E_v\left(\frac{A}{2}, \frac{Z}{2}\right) - E_v(A, Z)$$

masa nevtronov in protonov se odšteje

Semiempirična

$$Q = w_0\overbrace{\left(-2\frac{A}{2} + A\right)}^{=0} + w_1\left(2\left(\frac{A}{2}\right)^{2/3} - A^{2/3}\right) + w_2\left(2\frac{(Z/2)^2}{(A/2)^{1/3}} - \frac{Z^2}{A^{1/3}}\right) +$$

$$+ w_3\left(2\frac{\left(\frac{A}{2} - Z\right)^2}{A/2} - \frac{(A - 2Z)^2}{A}\right) + w_4\left(\frac{2\,\delta_{Z/2,N/2}}{(A/2)^{3/4}} - \frac{\delta_{Z,N}}{A^{3/4}}\right)$$

$$\frac{\delta_{Z_1 N_1}^{=-1}}{(A_1)^{3/4}} + \frac{\delta_{Z_2 N_2}^{=0}}{(A_2)^{3/4}}$$

$A = 235 \qquad Z = 92 \qquad N = 143 \xrightarrow{\text{po razpadu}} \frac{Z}{2} = 46 \qquad \frac{N}{2} = \begin{cases} 71 & \text{LS jedro} \quad \delta = 0 \\ 72 & \text{SS jedro} \quad \delta = 1 \end{cases}$

liho sodo jedro $\delta = 0$

$$Q = -184\ \text{MeV} - \frac{w_4}{(A/2)^{1/4}} = -185\ \text{MeV}$$

$$Q_{TNT} = m\,q = 4{,}18\ \frac{\text{GJ}}{\text{ton}}\ 5 \cdot 10^4\ \text{ton} = 2{,}09 \cdot 10^{14}\ \text{J} = 1{,}31 \cdot 10^{27}\ \text{MeV}$$

$$N = \frac{Q_{TNT}}{|Q|} = 7{,}06 \cdot 10^{24}$$

$$m_U = \frac{N M}{N_A} = 2{,}76\ \text{kg}$$

36 5.20 C-N zlivanje jeder

$$^{12}C + p \to {}^{13}N + \gamma$$
$$^{13}N \to {}^{13}C + e^+ + \nu_e \qquad E_{\nu_1} = 0{,}72\ \text{MeV}$$
$$^{13}C + p \to {}^{14}N + \gamma$$
$$^{14}N + p \to {}^{15}O + \gamma$$
$$^{15}O \to {}^{15}N + e^+ + \nu_e \qquad E_{\nu_2} =$$
$$^{15}N + p \to {}^{12}C + {}^{4}He$$

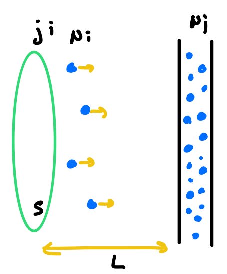
$$4p \to 3\gamma + 2e^+ + 2\nu_e + {}^4_2He \quad / +2e^- \qquad e^+ + e^- \to 2\gamma$$
$$4p + 2e^- \to 7\gamma + 2\nu_e + {}^4_2He$$

$$E_\gamma = \Delta E = 4 m_p c^2 + 2 m_e c^2 - \left[(m_{He} - 2m_e) c^2 + E_{\nu_1} + E_{\nu_2} \right]$$

$$= (4 \cdot 1{,}0073 - 4{,}0026)\, u c^2 + 4 m_e c^2 - E_{\nu_1} - E_{\nu_2} = 25{,}1\ \text{MeV}$$

T Sipanje

N_i ... št. vpadnih delcev
N_j ... št. delcev v tarči
N_r ... št. interakcij v času
S ... presek vpadnega curka



$$\boxed{N_r = N_i N_j \frac{\sigma}{S}}$$

σN_j = celotni eff. presek sipanja ≠ geometrijski presek

$$j_i = n_i v = \frac{N_i v}{V} = \frac{N_i}{SL} \frac{L}{t} = \frac{N_i}{St}$$

$$\sigma = \frac{S N_r}{N_i N_j} = \frac{N_r}{j_i t N_j}$$

Diferencialni sipalni presek

$$\frac{d\sigma}{d\Omega} = \frac{1}{j_i t N_j} \frac{dN_r}{d\Omega} \overset{N_j = 1}{=} \frac{1}{j_i t} \frac{dN_r}{d\Omega}$$

$$j_i = \frac{N_i}{St} = \frac{N_r}{St} \Rightarrow j_i\, dS = \frac{dN_r}{t} \Rightarrow j_i = \frac{dN_r}{dS} \frac{1}{t}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{j_i t} \frac{dN_r}{d\Omega} = \frac{1}{\frac{dN_r}{dS} \frac{1}{t} t} \frac{dN_r}{d\Omega} = \frac{dS}{d\Omega} = \frac{2\pi b\, db}{d\varphi \sin\theta\, d\theta}$$

integriramo po φ / osna simetrija

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|}$$

Coulombska interakcija $\quad b(\theta) = \frac{e_1 e_2}{8\pi\varepsilon_0 T}\cot\left(\frac{\theta}{2}\right)$

T kin. en.

$$\frac{d\sigma}{d\Omega} = \left(\frac{e_1 e_2}{16\pi\varepsilon_0 T}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}$$

37 $E_\alpha = 5{,}7\,MeV$

$$\frac{P_1}{P_2} = \frac{P(45° < \theta < 90°)}{P(90° < \theta < 180°)} = ? \qquad P \propto \sigma$$

$$\frac{d\sigma}{d\Omega} = \underbrace{\left(\frac{e_1 e_2}{16\pi\varepsilon_0 T}\right)^2}_{A} \frac{1}{\sin^4\frac{\theta}{2}} = A\frac{1}{\sin^4\frac{\theta}{2}} \qquad \sin^2\frac{\theta}{2} = \frac{1}{2}(1-\cos\theta)$$

$$\frac{\sigma}{A} = \int \frac{1}{\sin^4\frac{\theta}{2}}\,d\Omega = \int_0^{2\pi}\int_{\theta_1}^{\theta_2} \frac{1}{\sin^4\theta/2}\,d\varphi\,\sin\theta\,d\theta = -2\pi\int_{\cos\theta_1}^{\cos\theta_2}\frac{d(\cos\theta)}{\sin^4\frac{\theta}{2}} = -2\pi\int_{\cos\theta_1}^{\cos\theta_2}\frac{4\,du}{(1-u)^2} = 8\pi\int_{1-\cos\theta_1}^{1-\cos\theta_2}\frac{dt}{t^2} =$$

$$\boxed{\frac{\sigma}{A} = 8\pi\left(\frac{1}{1-\cos\theta_1} - \frac{1}{1-\cos\theta_2}\right)}$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{\frac{1}{1-\cos 45°} - \frac{1}{1-\cos 90°}}{\frac{1}{1-\cos 90°} - \frac{1}{1-\cos 150°}} = 5{,}2$$

38 α delci na zlati foliji ${}^{197}_{79}Au$ (pod vplivom Coulombske interakcije), elastično sipanje $(m_\alpha \ll m_{Au})$

$T = 7\,MeV$

$\rho_{Au} = 19{,}3\,g/cm^3$

Prepusti se $99{,}97\%$ delcev

$d = ?$

$$\frac{d\sigma}{d\Omega} = \left(\frac{e_1 e_2}{16\pi\varepsilon_0 T}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}$$

$$\sigma(\theta_1 < \theta < \theta_2) = \left(\frac{e_1 e_2}{16\pi\varepsilon_0 T}\right)^2 8\pi\left(\frac{1}{1-\cos\theta_1} - \frac{1}{1-\cos\theta_2}\right)$$

Prepuščeni delci $\quad 0° < \theta < 90°$

Odsik $\quad 90° < \theta < 180°$

$$\sigma\left(\frac{\pi}{2} < \theta < \pi\right) = 8\pi\left(\frac{e_1 e_2}{16\pi\varepsilon_0 T}\right)^2\left(\frac{1}{1-0} - \frac{1}{1-(-1)}\right) = 4\pi\left(\frac{e_1 e_2}{16\pi\varepsilon_0 T}\right)^2 = 4\pi\left(\frac{z_{Au} z_{He}}{4T}\overset{\alpha\hbar c}{\frac{e_0^2}{4\pi\varepsilon_0}}\right)^2 =$$

$$= 4\pi\left(\frac{79\cdot 2\cdot\frac{1}{137}\,197{,}3\,MeV\,fm}{4\cdot 7\,MeV}\right)^2 = 4{,}5\cdot 10^7\,fm^2 = 45\,b \quad [barn]$$

$1\,b = 10^{-28}\,m^2$

verjetnost odboja

$$P = 1 - 0{,}9997 = \frac{N_r}{N_i} \qquad \frac{P}{N_j} = \frac{\sigma}{S} = \frac{N_r}{N_i N_j}$$

$$P = \frac{\sigma}{S}N_j = \frac{\sigma}{S}n_j V = \frac{\sigma}{S}n_j S d = \sigma n_j d = \boxed{\sigma\frac{\rho N_A}{M_{Au}}d = P}$$

$$d = \frac{P\,M_{Au}}{\rho\,N_A} = 1{,}1\,\mu m$$

39 **a** $E_{e^-} = 104\,GeV = E_{e^+}$ težiščna energija $E_T = 208\,GeV \qquad e^+e^- \to X \qquad m_X < \frac{E_T}{c^2}$

$E_{e^+} = ?$ če e^- miruje da dobimo isto težiščno en. $E_T = 208\,GeV$

- $p_{e^+}c = (E_{e^+}, pc)$
  $p_{e^-}c = (E_{e^-}, -pc)$

$$\overset{0}{p^2c} + E_T^2 = (p_{e^+}c + p_{e^-}c)^2 = (E_{e^+} + E_{e^-}, 0)^2$$

kvadrat četvorca

$$E_T = E_{e^+} + E_{e^-} = 2e^+$$

- $e^+ \to e^-$ $\quad p_{e^+}c = (E_+, pc) \quad p_{e^-}c = (m_e c^2, 0) \qquad p_{e^+}c + p_{e^-}c = (E_+ + m_e c^2, pc)$

$\hookrightarrow\ ()^2 \quad (pc)^2 = E_+^2 - m^2c^4$

$$E_T^2 = \left((p_{e^+} + p_{e^-}) c \right)^2 = (E_+ + m_e c^2)^2 - (pc)^2 = E_+^2 + 2mc^2 E_+ + m^2c^4 \sim E_+^2 + m^2 c^4$$

$$E_+ = \frac{E_T^2 - 2(mc^2)^2}{2mc^2} = 4{,}2 \cdot 10^3\,\text{GeV}$$

T Standardni model

Delci snovi		Delci prenašalci interakcij	
Fermioni	spin = 1/2	Bozoni	spin = 1
Leptoni	Kvarki		
- nimajo barve - ne interagirajo z močno - nevtralni nevtrini interagirajo le s šibko interakcijo			

Leptoni

Leptonsko št. po posameznih številih: L_e, L_μ, L_τ; celotno leptonsko število: $L = L_e + L_\mu + L_\tau$

Generacija	Ime	Naboj	L_e	L_μ	L_τ	L	m
1.	ν_e	0	1	0	0	1	0
	e^-	-1	1	0	0	1	0,511 MeV
2.	ν_μ	0	0	1	0	1	0
	μ^-	-1	0	1	0	1	105,7 MeV
3.	ν_τ	0	0	0	1	1	0
	τ	-1	0	0	1	1	1777 MeV

$m_\nu \simeq 1\,\text{eV}$

Pri antidelcih se obrne naboj in vsa kvantna števila, spin ostane enak

Pri vseh interakcijah se ohranja leptonsko število

Kvarki $\Rightarrow B = \frac{1}{3}$ barionsko št.

I_3: 3. komp. izospina; C: charm; S: strange; T: top; B': bottomness

Generacija	q	Ime	Naboj	I_3	C	S	T	B'	m
1.	u	up (gor)	2/3	1/2	0	0	0	0	1 MeV - 10 MeV
	d	down (dol)	-1/3	-1/2	0	0	0	0	
2.	c	charm (čarobni)	2/3	0	1	0	0	0	≃ 1,3 GeV
	s	strange (čuden)	-1/3	0	0	-1	0	0	≃ 100 MeV
3.	t	top (vrh)	2/3	0	0	0	1	0	≃ 173 GeV
	b	bottom (dol / beauty)	-1/3	0	0	0	0	-1	≃ 4,2 GeV

Kvarki 3 barvni naboji: R B G
antikvarki 3 antibarvni naboji: $\bar{R}$ $\bar{B}$ $\bar{G}$

Kvarki tvorijo barvno nevtralna vezana stanja

Hadroni

Mezoni
- $q\bar{q}$
- bozoni

π^+ $u\bar{d}$

π^- $d\bar{u}$

π^0 $\left(\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})\right)$

K^+ $u\bar{s}$

K^- $s\bar{u}$

Barioni
- qqq
- fermioni

p^+ uud

n udd

Σ^+ uus

Δ^+ uud (spin drugačen kot pri p^+)

Ω^- sss

Ξ^- dss

Δ^{++} uuu

Izberi kakšne ni isto

40

$\pi^+(u\bar{d})$ $S_\pi = 0$ $m_\pi c^2$

$\rho^+(u\bar{d})$ $S_\rho = 1$ $m_\rho c^2$

$K^+(u\bar{s})$ $S_K = 0$ $m_K c^2$

a) Določi m_u, m_s, a

$$M_{mu} c^2 = m_1 c^2 + m_2 c^2 + a \frac{\langle \vec{S}_1 \cdot \vec{S}_2 \rangle}{m_1 m_2 c^4}$$

$$\langle \vec{S}_1 \cdot \vec{S}_2 \rangle = ?$$

$$\vec{S} = \vec{S}_1 + \vec{S}_2 \Rightarrow |\vec{S}|^2 = |\vec{S}_1|^2 + |\vec{S}_2|^2 + 2\, \vec{S}_1 \cdot \vec{S}_2$$

$$\langle \vec{S}_1 \cdot \vec{S}_2 \rangle = \frac{1}{2}\left(\langle \vec{S}^2 \rangle - \langle \vec{S}_1^2 \rangle - \langle \vec{S}_2^2 \rangle \right)$$

$$= \frac{\hbar^2}{2}\left(s(s+1) - \underbrace{s_1}_{1/2}(s_1+1) - \underbrace{s_2}_{1/2}(s_2+1) \right)$$

$$= \frac{\hbar^2}{2}\left(s(s+1) - \frac{3}{2} \right)$$

$c = \hbar = 1$

$$= \frac{1}{2}\left(s(s+1) - \frac{3}{2} \right)$$

(1) $m_\pi = m_u + m_d + a \frac{1}{m_u m_d} \frac{1}{2}\left(0(0+1) - \frac{3}{2} \right) = 2m_u - \frac{3a}{4m_u^2}$

(2) $m_\rho = m_u + m_d + a \frac{1}{m_d m_u} \frac{1}{2}\left(1(1+1) - \frac{3}{2} \right) = 2m_u + \frac{a}{4m_u^2}$

(1) + 3 (2) $\Rightarrow m_\pi + 3m_\rho = 2m_u + 6m_u = 8m_u$

$$\Rightarrow m_u \approx m_d = \frac{m_\pi + 3m_\rho}{8} = 308{,}3\,\text{MeV}$$

(2) − (1) $a = m_u^2 (m_\rho - m_\pi) = 6{,}05 \cdot 10^7\,\text{MeV}$

(3) $m_K = m_u + m_s + \frac{a}{m_u m_s} \frac{1}{2}\left(-\frac{3}{2} \right)$ $| \cdot 4m_u m_s$

$$4 m_u m_s^2 + 4(m_u - m_K) m_u \cdot m_s - 3a = 0$$

$$m_s = 487\,\text{MeV}$$

(b) $m(J/\psi) - m(\chi) =$

$$= 2m_c + \frac{a}{m_c^2}\left(1\cdot 2 - \frac{3}{2}\right) - \left(2m_c + \frac{a}{m_c^2}\left(0 - \frac{3}{2}\right)\right)$$

$$= \frac{a}{m_c^2} = 36\ \mathrm{MeV}$$

(T) Interakcije

Elektromagnetna interakcija → nosilec foton γ $m=0$, $S=1$

Šibka interakcija → nosilci W^+ W^- Z $m_W = 80$ GeV, $m_Z = 91$ GeV

nabiti nevtralen

šibki vektorski bozoni

Močna interakcija → nosilci 8 gluonov $m=0$

gluoni imajo barvni naboj

Higgsov bozon h spin $=0$

$m_h = 125$ GeV

(T) Feynmanovi diagrami

———	fermion
∿∿∿	vektorski bozon ($W^\pm$, Z^0, γ)
꩜꩜꩜	gluoni
- - - - -	higgsov bozon

→ t

→ delec

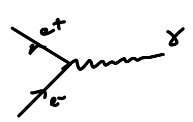
← antidelec

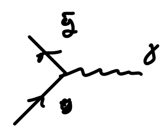
Elektromagnetna int.

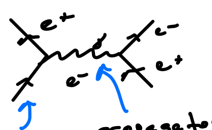
- kvant. elektrodinamika (QED)

γ, e^-, e^+

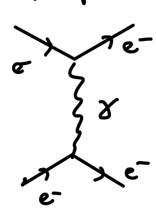
Osnovna vozlišča



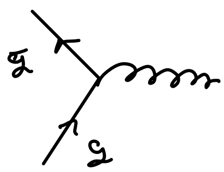


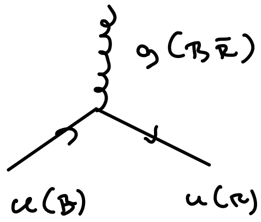


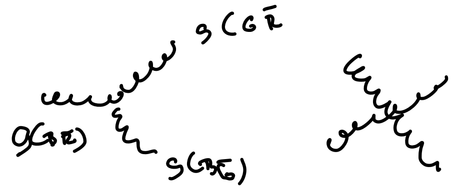
Sipanje



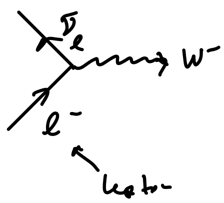
Močna interakcija
- kvantna kromodinamika

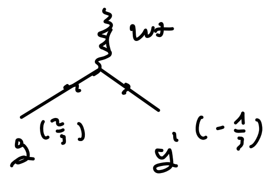




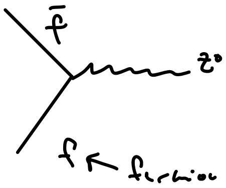


Šibka interakcija
- nabita interakcija $(W^\pm)$

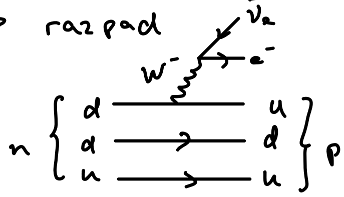




- nevtralna interakcija (Z^0)



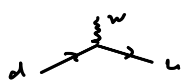
β razpad



Ⓣ Ohranitveni zakoni

Za vse interakcije
- $p^\mu \Rightarrow \vec{p}$ in E ter vrtilna količina
- naboj → EM: električni naboj
  ↳ močna: barvni naboj
- barionsko število B
- leptonsko število L in L_e, L_μ, L_τ

Za EM, močno in nevtralno šibko interakcijo
- okus se ohrani
  ↳ pri nabiti šibki se ta ne ohrani PAZI



Za šibko interakcijo
- krši se parnost (P) in nabojna parnost (C)

(T) Razpadna širina, razvejitveno razmerje, sipalni presek

Fermijevo zlato pravilo — invariantna amplituda / matrični element

- za razpadno širino $\Gamma_{i\to f} = \frac{2\pi}{\hbar}|\mathcal{M}(i\to f)|^2 \rho_f \propto |\mathcal{M}|^2$

  ρ_f - gostota končnih st. v energ. prostoru

  $\mathcal{M}(i\to f) = \langle f | H' | i \rangle \qquad H = H_0 + H'$

  $|f\rangle, |i\rangle$ lastni stanji od H_0

  Γ - verjetnost za prehod na časovno enoto $\qquad \tau_{i\to f} = \frac{\hbar}{\Gamma_{i\to f}}$

  Skupna razpadna širina $\Gamma_i^{tot} = \sum_f \Gamma_{i\to f}$

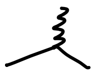
  Razvejitveno razmerje $Br(i\to f) = \frac{\Gamma_{i\to f}}{\Gamma_i^{tot}}$

- za diferencialni sip. presek

  $\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{|\mathcal{M}(i\to f)|^2}{E_{cm}^2}\frac{|\vec{p}_f|}{|\vec{p}_i|} \propto |\mathcal{M}|^2$ Za 2 delca ki se sipljeta v 2 delca

  E_{cm} - težiščna energija

(T) Feynmanova pravila


verteks ima podatek o sklopitvi

Primer EM


sklopitvena konst.

Močna


Šibka


$$|V_{CKM}| = \begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0{,}97428 & 0{,}2253 & 0{,}00347 \\ 0{,}2252 & 0{,}97345 & 0{,}0410 \\ 0{,}00862 & 0{,}0403 & 0{,}999152 \end{bmatrix}$$

$\mathcal{M} \propto$ produkt vseh verteksov

Γ in $\sigma \propto |\mathcal{M}|^2 \propto |$produkt vseh verteksov$|^2$

Zgornje kvarki gredo vedno v spodnje in na obratno

(u1) Ali je proces možen

(a) $p\, e^- \to n\, \bar{\nu}_e$ — naboj se ohrani, $m_p + m_e < m_n + m_\nu$

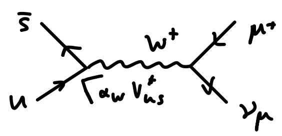
L_e	0	+1	0	−1
L	0	+1	0	−1

$\Delta L_e = -2 \neq 0$ proces ni možen

(b) $K^+ \to \mu^+ \nu_\mu$ — naboj ✓ L ✓ B ✓ kinematično možno ker $m_{K^+} > m_{\mu^+}$

$(u\bar{s})$ — Okus se spremeni → šibka interakcija

S	1	→	0
I_3	1/2	→	0



(c) $\pi^0 \to \mu^+ \; e^- \; \bar{\nu}_e$

Naboj	0	1	−1	0	✓
B	0		0		✓

$m_\pi < m_\mu + m_e + m_\nu$ ✗

(d) $\Lambda^0 \to K^+ \; K^-$

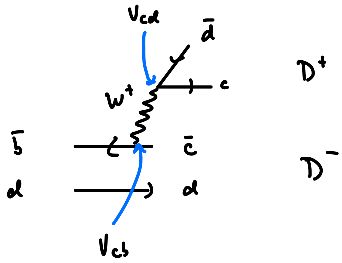
(uds) $\quad (u\bar{s}) \quad (s\bar{u})$

B $\quad 1 \quad 0 + 0$ ✗

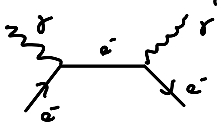
(e) $B^0 \to D^- \; D^+$

$(d\bar{b}) \quad (d\bar{c}) \quad (c\bar{d})$

Našita šibka interakcija

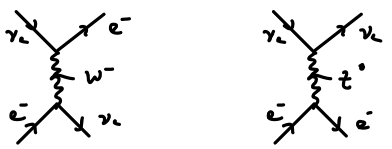


(f) Comptonovo sipanje



EM

(g) $\nu_e \; e^- \to \nu_e \; e^-$



(h) $\Delta^0 \to p \; \pi^-$

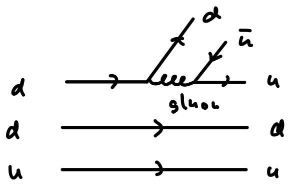
$(udd) \quad (uud) \quad (d\bar{u})$

1,2 GeV > ~1 GeV 140 MeV ✓

B ✓

Q ✓

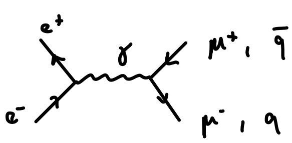
L ✓



(42)

3 barve

$$\frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{3\sum_{q=u,d,s,c,b} \sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} \doteq \frac{3\sum_q |\mathcal{M}(e^+e^- \to q\bar{q})|^2}{|\mathcal{M}(e^+e^- \to \mu^+\mu^-)|^2} =$$

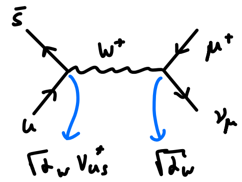


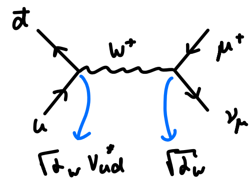
$$= \frac{3\left(2\left(\frac{2}{3}e\cdot e\right)^2 + 3\left(\frac{1}{3}e\cdot e\right)^2\right)}{(e\cdot e)^2} = \frac{11}{3}$$

$$\frac{e^2}{4\pi\varepsilon_0 \hbar c} = \alpha$$

43

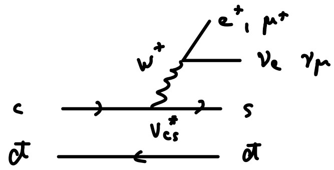
(a) $$R = \frac{|\mathcal{M}(\underset{\bar{s}u}{K^+} \to \mu^+ \nu_\mu)|^2}{|\mathcal{M}(\underset{u\bar{d}}{\pi^+} \to \mu^+ \nu_\mu)|^2}$$



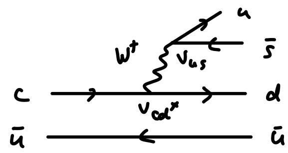


$$R = \frac{\alpha_w \alpha_w |V_{us}^*|^2}{\alpha_w \alpha_w |V_{ud}^*|^2} = \left(\frac{0{,}23}{0{,}97}\right)^2 = 0{,}06$$

(b) $$R = \frac{|\mathcal{M}(D^+ \to \bar{K}^0 e^+ \nu_e)|^2}{|\mathcal{M}(\underset{c\bar{d}}{D^+} \to \underset{s\bar{d}}{\bar{K}^0} \mu^+ \nu_\mu)|^2} = 1$$



(c) $$\frac{|\mathcal{M}(\overset{c\bar{u}}{D^0} \to \overset{d\bar{u}}{\pi^-} \; \overset{u\bar{s}}{K^+})|^2}{|\mathcal{M}(\underset{c\bar{u}}{D^0} \to \underset{u\bar{d}}{\pi^+} \; \underset{s\bar{u}}{K^-})|^2} =$$



$$= \frac{|V_{us} \; V_{cd}^*|^2}{|V_{ud} \; V_{cs}^*|^2} = \frac{|V_{us}|^2 |V_{cd}|^2}{|V_{ud}|^2 |V_{cs}|^2} = 0{,}003$$