Neskončna pot. jama

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x$$

$$E_n = \frac{\hbar^2\pi^2}{2ma^2} n^2$$

$$n = 1, 2, 3, \dots$$

2D harmonski oscilator

$$H = \frac{p^2}{2m} + \frac{1}{2} k_x x^2 + \frac{1}{2} k_y y^2 = H_x + H_y$$

če $k_x = k_y$ $\quad H = \frac{p^2}{2m} + \underbrace{\frac{1}{2} k r^2}_{V(r)}$

$$E_{nm} = \hbar\omega(n+m+1)$$

Centralni potencial $V(r)$

$$H = \frac{p^2}{2m} + V(r) \qquad [H, L_z] = 0$$

$$L_z = -i\hbar \frac{\partial}{\partial \varphi}$$

$$L_z \psi_m = \hbar m \psi_m \qquad \psi_m = \frac{1}{\sqrt{2\pi}} e^{im\varphi}$$

Vrtilna količina

$$\vec{\mu} = \gamma \vec{L} \qquad |lm\rangle = Y_{lm}(\varphi, \theta)$$

$$L^2 |lm\rangle = \hbar^2 l(l+1) |lm\rangle$$

$$L_z |lm\rangle = \hbar m |lm\rangle \qquad m = -l, \dots, l \quad l = 0, 1, \dots$$

$$L_\pm = L_x \pm i L_y$$

$$L_\pm |lm\rangle = \hbar \sqrt{l(l+1) - m(m\pm1)}\, |l\, m\pm1\rangle$$

$$L_x = \frac{1}{2}(L_+ + L_-) \qquad L_y = \frac{1}{2i}(L_+ - L_-)$$

Spin dveh delcev

$$\vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2}(S^2 - S_1^2 - S_2^2)$$

Klebsch-Gordon

Produktna baza $S_1^2, S_{z1}, S_2^2, S_{z2}$

Dobri skupni spin S^2, S_z, S_1^2, S_2^2

$S = |S_1 - S_2|, \dots, S_1 + S_2$

Y_{lm}

$$Y_{00} = \frac{1}{\sqrt{4\pi}}$$

$$Y_{10} = \sqrt{\frac{3}{4\pi}} \cos\theta$$

$$Y_{11} = -\sqrt{\frac{3}{8\pi}} \sin\theta\, e^{i\varphi}$$

$$Y_{20} = \sqrt{\frac{5}{4\pi}} \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_{21} = -\sqrt{\frac{15}{8\pi}} \sin\theta \cos\theta\, e^{i\varphi}$$

$$Y_{22} = \frac{1}{4}\sqrt{\frac{15}{2\pi}} \sin^2\theta\, e^{2i\varphi}$$

Harmonski oscilator

$$H = \frac{p^2}{2m} + \frac{1}{2} k x^2 \qquad \omega = \sqrt{\frac{k}{m}}$$

$$H = \hbar\omega(a^\dagger a + 1/2)$$

$$E_n = \hbar\omega(n + 1/2)$$

$$a = \frac{1}{\sqrt{2}}\left(\frac{x}{x_0} + i\frac{p}{p_0}\right)$$

$$x_0 = \sqrt{\frac{\hbar}{m\omega}} \qquad p_0 = \frac{\hbar}{x_0}$$

$$x = \frac{x_0}{\sqrt{2}}(a + a^\dagger) \qquad p = \frac{p_0}{\sqrt{2}i}(a - a^\dagger)$$

$$a|n\rangle = \sqrt{n}\, |n-1\rangle$$

$$a^\dagger |n\rangle = \sqrt{n+1}\, |n+1\rangle$$

$$a^\dagger a |n\rangle = n |n\rangle$$

$$[a, a^\dagger] = 1$$

$$\langle x, t\rangle = x_0\sqrt{2}\, \mathrm{Re}\langle a, t\rangle$$

$$\langle p, t\rangle = p_0\sqrt{2}\, \mathrm{Im}\langle a, t\rangle$$

Spin

e^-

$$s = 0, \frac{1}{2}, 1, \dots \qquad m_s = -s, s$$

$$S^2 |s m_s\rangle = \hbar^2 s(s+1) |s m_s\rangle$$

S_z, $S_\pm$ enako kot L_z, $L_\pm$

Spin - Paulijeve matrike

le za $s = \frac{1}{2}$, $m_s = -\frac{1}{2}, \frac{1}{2}$

baza $\alpha|\uparrow\rangle + \beta|\downarrow\rangle = \begin{pmatrix}\alpha \\ \beta\end{pmatrix}$

$$\vec{S} = (S_x, S_y, S_z) = \frac{\hbar}{2}\vec{\sigma} = \frac{\hbar}{2}(\sigma_x, \sigma_y, \sigma_z)$$

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Vodikov atom

$$H = \frac{p^2}{2m} - \frac{e^2}{4\pi\varepsilon_0 r} \qquad E_n = -\frac{R_y}{n^2}$$

13,6 eV

$$\psi_{nlm}(\vec{r}) = R_{nl}(r) Y_{lm}(\theta, \varphi)$$

$n = 1, 2, \dots \qquad l < n$

$$R_{10} = 2/r_b^{3/2}\, e^{-r/r_b}$$

$$R_{20} = 2/(2r_b)^{3/2} \left(1 - \frac{r}{2r_b}\right) e^{-r/2r_b}$$

$$R_{21} = \frac{r}{r_b} e^{-r/2r_b} / \sqrt{3}(2r_b)^{3/2}$$

$$R_{30} = \frac{1}{r_b^{3/2}} \frac{2}{81\sqrt{3}} \left(27 - 18\frac{r}{r_b} + 2\frac{r^2}{r_b^2}\right) e^{-r/3r_b}$$

$$R_{31} = \frac{1}{r_b^{3/2}} \frac{4}{81\sqrt{6}} \left(6 - \frac{r}{r_b}\right) \frac{r}{r_b} e^{-r/3r_b}$$

$$R_{32} = \frac{1}{r_b^{3/2}} \frac{4}{81\sqrt{30}} \frac{r^2}{r_b^2} e^{-r/3r_b}$$

$$\int_0^\infty u^t e^{-u}\, du = \Gamma(t-1) = t!$$

Operatorji

$$\hat{p} = -i\hbar\frac{\partial}{\partial x}, \quad -i\hbar\nabla$$

$$\hat{p}|\psi\rangle = -i\hbar\nabla|\psi\rangle = \hbar\vec{k}|\psi\rangle$$

$$|\psi\rangle = |k\rangle = e^{i\vec{k}\cdot\vec{r}}$$

$$\delta x^2 = \langle x^2\rangle - \langle x\rangle^2$$

$$L_z = x p_y - y p_x = -i\hbar\frac{\partial}{\partial\varphi}$$

Operator obrata časa

$$\langle T\psi_1 | T\psi_2\rangle = \langle\psi_1|\psi_2\rangle^*$$

Za spin $s = 1/2$ velja

$T = i\sigma_y K$ kompleksna konjugacija

Vezano stanje δ potenciala

$$H = \frac{p^2}{2m} - \lambda\delta(x)$$

Lastno st. $\psi_0(x) = \sqrt{\kappa}\, e^{-\kappa|x|}$

$$\kappa = \frac{m\lambda}{\hbar^2} \qquad E_0 = \frac{m\lambda^2}{2\hbar^2}$$

Nedegenerirana teorija motenj

$H_0 |n\rangle^0 = E_n^0 |n\rangle^0$ nemoten

$$H = H_0 + H'$$

$$E_n = E_n^0 + \langle n|H'|n\rangle^0 + \sum_{m\neq n} \frac{|\langle m|H'|n\rangle^0|^2}{E_n^0 - E_m^0}$$

$$|n\rangle = |n\rangle^0 + \sum_{m\neq n} \frac{|\langle m|H'|n\rangle^0|^2}{E_n^0 - E_m^0} |m\rangle^0$$

Degenerirana teorija motenj

Naj bo stanje Nx degenerirano

- perturbacijska matrika $\begin{bmatrix} \langle 1|H'|1\rangle & \dots & \langle 1|H'|N\rangle \\ \vdots & \ddots & \vdots \\ \langle N|H'|1\rangle & \dots & \langle N|H'|N\rangle \end{bmatrix}$
- diagonaliziraj mat., λ_j, $\vec{u}_j$

$$E_j = E^0 + \lambda_j \qquad |\psi_j\rangle = \sum_{k=1}^{N} u_{jk} |k\rangle$$

Časovno odvisna motnja

$$H(t) = H_0 + H'(t) \qquad H_0|n\rangle^0 = E_n^0 |n\rangle^0$$

$$|\psi, t\rangle = \sum_n c_n(t)\, e^{-i\frac{E_n}{\hbar}t} |n\rangle^0$$

$$c_n(t) = c_n(t_0) - \frac{i}{\hbar}\int_{t_0}^{t} \sum \langle n,t'|H'(t')|m,t'\rangle c_m(t')\, dt'$$

ponovno stavimo

$$c_n(t) = c_n(t_0) - \frac{i}{\hbar}\sum_m c_m(t_0) \int_{t_0}^{t} \langle n,t'|H'(t')|m,t'\rangle\, dt'$$

$$\langle n,t|H|m,t\rangle = \langle n| e^{i\frac{E_n}{\hbar}t} H e^{-i\frac{E_m}{\hbar}t} |m\rangle$$