Neomejena domena

$$\mathcal{L} u(\vec r) = f(\vec r)$$

Razvoj f po δ funkciji $\Longrightarrow$ $f(\vec r) = \int_V \delta(\vec r - \vec r_0) f(\vec r_0)\, d^3 r_0$

Tudi def δ funkcije

$$\mathcal{L} G(\vec r, \vec r_0) = \delta(\vec r - \vec r_0)$$

Rešitev $\quad u(\vec r) = \int_V G(\vec r, \vec r_0) f(\vec r_0)\, d^3 r_0$

Laplace $\quad \mathcal{L} = \nabla^2$

2D $\quad G = \frac{1}{2\pi} \ln |\vec r - \vec r_0|$

3D $\quad G = -\frac{1}{4\pi |\vec r - \vec r_0|}$

Difuzijska $\quad \mathcal{L} = \frac{\partial}{\partial t} - D\nabla^2$

ND $\quad G = \Theta(t)\left(\frac{1}{4\pi D t}\right)^{\frac{N}{2}} e^{-|\vec r - \vec r_0|^2/4Dt}$

Helmholtz $\quad \mathcal{L} = \nabla^2 - k^2$

2D
$$G = \begin{cases} -\frac{i}{4} H_0^{(1)}(k|\vec r - \vec r_0|) & \text{vpadni valovi} \\ \frac{i}{4} H_0^{(2)}(k|\vec r - \vec r_0|) & \text{valovi navzven} \\ \frac{1}{4} Y_0(k|\vec r - \vec r_0|) & \text{stoječi valovi} \end{cases}$$

3D
$$G = \begin{cases} \frac{1}{4\pi|\vec r - \vec r_0|} e^{\pm i k |\vec r - \vec r_0|} & \text{potujoči valovi} \\ \frac{1}{4\pi|\vec r - \vec r_0|} \cos(k|\vec r - \vec r_0|) & \text{stoječi valovi} \end{cases}$$

Omejena domena

$G = G_\infty + g$

dobimo z zrcaljenjem

Greenova formula

$$\int_V \left(u(\vec r)\, \mathcal{L}_r G(\vec r, \vec r_0) - G(\vec r, \vec r_0)\, \mathcal{L}_r u(\vec r)\right) d^3 r = \int_{\partial V} \left(u(\vec r_b) \frac{\partial G(\vec r, \vec r_b)}{\partial n_b} - G(\vec r, \vec r_b) \frac{\partial u(\vec r_b)}{\partial n_b}\right) d^2 r$$

Laplace

Dirichlet $\quad u|_{\partial D} = u(\vec r_b) = h(\vec r_b)$ border $\qquad$ Na robu $G = 0$ ne sledi na pogoje

$$u(\vec r) = \int_D G(\vec r, \vec r_0) f(\vec r_0) d^3 r_0 + \int_{\partial D} h(\vec r_b) \frac{\partial G(\vec r, \vec r_b)}{\partial n_b} d\vec S_b$$

Neumann $\quad \frac{\partial u}{\partial n}\Big|_{\partial D} = h(\vec r) \qquad \mathcal{L} G_N = \delta(\vec r - \vec r_0) - \frac{1}{V}$

$$u(\vec r) = \int_D G(\vec r, \vec r_0) f(\vec r_0) d^3 r_0 - \int_{\partial D} h(\vec r_b) G_N(\vec r, \vec r_b)\, dS_b + \text{konst.}$$

Difuzijska

Dirichlet

$u(\vec r, 0) = g(\vec r)$

$$u(\vec r, t) = \int_D \int_0^\infty f(\vec r_0, \tau) G(\vec r, \vec r_0, t - \tau) d^3 r_0\, d\tau - D \int_{\partial D} \int_0^\infty h(\vec r_b, \tau) \frac{\partial G(\vec r, \vec r_b, t - \tau)}{\partial \vec r_b} d\vec S_b\, d\tau + \int_D g(\vec r) G(\vec r, \vec r_0, t) d^3 r_0$$

Ali je G vedno na robu 0