$\vec{p} = m\vec{v}$ $\quad W_k = \frac{mv^2}{2} = \frac{p^2}{2m}$

Polarne koordinate

$\vec{F} = m\left(\vec{e}_r\,(\ddot{r} - \dot{\varphi}^2 r) + \vec{e}_\varphi\,(r\ddot{\varphi} + 2\dot{r}\dot{\varphi})\right)$

$W_k = \frac{1}{2} m\,(\dot{r}^2 + r^2\dot{\varphi}^2)$

$\vec{v} = \dot{r}\,\vec{e}_r + \dot{\varphi}\,\vec{e}_\varphi$

Sferne

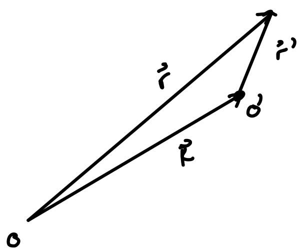
$W_k = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\varphi}^2\right)$

$\vec{l} = \vec{\Gamma} = \vec{r}\times\vec{p} = \vec{r}\times m\vec{v}$

$\vec{M} = \vec{r}\times\vec{F} = \dot{\vec{l}}$

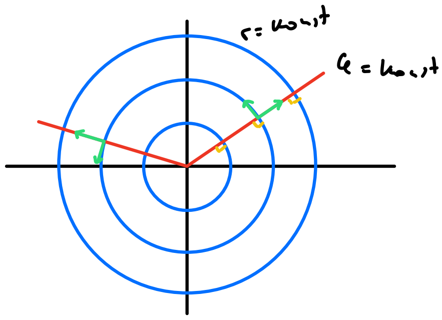
$F = -\nabla U$

Sistemske sila $\quad F + F_s = m\frac{d^2}{dt^2}r'_{rel}$ ← lokalni postopek



$\vec{F}_s = -m\underbrace{\frac{d^2\vec{R}}{dt^2}}_{?\ \text{zun. sile}} \underbrace{- 2m\vec{\omega}'\times\vec{v}'_{rel}}_{\text{Coriolis}} \underbrace{- m\vec{\omega}'\times(\vec{\omega}'\times\vec{r}')}_{\text{Centri fuga}} - m\,\dot{\vec{\omega}}'\times\vec{r}'$

Polarni koord. sistem



$\vec{r} = x\,\hat{i} + y\,\hat{k} = r\cos\varphi\,\hat{i} + r\sin\,\hat{j}$

$x = r\cos\varphi \qquad \vec{e}_r = \frac{\partial\vec{r}}{\partial r} = \cos\varphi\,\hat{i} + \sin\varphi\,\hat{j}$

$y = r\sin\varphi \qquad \vec{e}_\varphi = \frac{\partial\vec{r}}{\partial\varphi} = -r\sin\varphi\,\hat{i} + r\cos\varphi\,\hat{j}$

Komentar $\quad \vec{r} = \cancel{r\,\vec{e}_r + \varphi\,\vec{e}_\varphi} = r\,\vec{e}_r$

$|\vec{e}_r|^2 = \cos^2\varphi\,|\hat{i}|^2 + \sin^2\varphi\,|\hat{j}|^2 = 1$

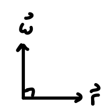
$|\vec{e}_\varphi|^2 = r^2\sin^2\varphi\,|\hat{i}|^2 + r^2\cos^2\varphi\,|\hat{j}|^2 = r^2$

$s:\quad m\ddot{\vec{r}} = \sum\vec{F}$

$s':\quad m(\ddot{\vec{R}} + \ddot{\vec{r}}') = \sum\vec{F}$

$m\vec{a}'_{rel} = \sum\vec{F} - m\Big(\underbrace{\underbrace{2\vec{\omega}\times\vec{v}'_{rel}}_{\text{coriolisova sila}} + \underbrace{\vec{\omega}\times(\vec{\omega}\times\vec{r}') + \vec{\omega}\times(\vec{\omega}\times\vec{R})}_{\text{centrifugalna sila}}}_{\text{sistemske sile}}\Big)$

običajno 0



$\underbrace{-m\,\vec{\omega}\times\vec{\omega}\times\vec{r}}_{+m\omega^2\vec{r}}$

$\vec{\omega}\times\vec{v}'_{rel} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & \omega\\ \dot{x}' & \dot{y}' & 0\end{vmatrix} = (-\omega\dot{y}',\ \omega\dot{x}',\ 0)$

Lagrangev formalizem

$L = T - V$

$\frac{d}{dt}\frac{\partial L}{\partial\dot{q}_i} = \frac{\partial L}{\partial q_i}\ (+Q_i)$

$Q_i = \sum_j \vec{F}_j\cdot\frac{\partial\vec{r}_j}{\partial q_i}$ ← tiste sile ki niso v V

$3N - K$ ← vezi

N delcev

Nihanje $\quad \omega_0 = \frac{V}{T}$

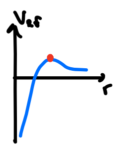
Orbite

$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$

$V_{eff} = \frac{p_\varphi^2}{2\mu r^2} + V(r) \qquad p_\varphi = \frac{\partial L}{\partial\varphi} = mr^2\dot{\varphi}$

$p_\varphi = m\,b\,v_\infty$

$V_{ef} \to \frac{dV_{ef}}{dr}\Big|_{r_0} = 0 \to V_{ef}(r_0)$



$\Rightarrow \frac{1}{2}mv_0^2 = V_{ef}(r_0) \Rightarrow b$

$\Rightarrow \sigma_{TOT} = \pi b^2$

$H = T + V = \frac{m}{2}(\dot{r}^2 + r^2\dot{\varphi}^2) + V(r) = \frac{m\dot{r}^2}{2} + V_{eff}$

Sipalni kot $\quad \sigma(\Omega) = \frac{d\sigma_{TOT}}{d\Omega} \qquad \sigma(\Omega) = \left|\frac{b\,db}{\sin\theta\,d\theta}\right|$

Rutherfordovo sipanje

$V(r) = -\frac{\alpha}{r} \qquad \alpha = \frac{e_1 e_2}{4\pi\varepsilon_0}$

$\alpha > 0 \quad r(\varphi) = \frac{p}{1+\varepsilon\cos\varphi}$

$\alpha < 0 \quad r(\varphi) = \frac{p}{-1+\varepsilon\cos\varphi}$

$p = \frac{p_\varphi^2}{m\alpha} \qquad \varepsilon = \sqrt{1 + \frac{2 p_\varphi^2 E}{m\alpha^2}}$

$b = \frac{\alpha}{2E}\cot\frac{\theta}{2}$

$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha}{4E}\right)^2\frac{1}{\sin^4\theta/2}$

Taylorjeve vrste

$(1\pm x)^m = 1 \pm m x + \frac{m(m-1)}{2!}x^2$

$(1\pm x)^{-m} = 1 \mp m x + \frac{m(m+1)}{2!}x^2$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$

$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5$

$\mathrm{sh}\, x = x + \frac{x^3}{3!} + \frac{x^5}{5!}$

$\mathrm{ch}\, x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!}$

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3}$

Odvodi:

$x^{n\prime} = n x^{n-1}$

$e^{x\prime} = e^x$

$\ln x' = \frac{1}{x}$

$a^{x\prime} = a^x \ln a$

$\log_a x' = \frac{1}{x \ln a}$

$\tan x' = \frac{1}{\cos^2 x}$

$\cot x' = \frac{-1}{\sin^2 x}$

$\arcsin x' = \frac{1}{\sqrt{1-x^2}}$

$\arccos x' = \frac{-1}{\sqrt{1-x^2}}$

$\arctan x' = \frac{1}{1+x^2}$

$\mathrm{arccot}\, x' = \frac{-1}{1+x^2}$

Integrali

$\int x^n = \frac{x^{n+1}}{n+1}$

$\int e^x = e^x$

$\int a^x = \frac{a^x}{\ln a}$

$\int \frac{1}{x} = \ln|x|$

$\int \frac{1}{\cos^2 x}dx = \tan x$

$\int \frac{1}{\sin^2 x}dx = -\cot x$

$\int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\frac{x}{a}$

$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\arctan\frac{x}{a}$

$\int \frac{1}{a^2 \mp x^2}dx = \frac{1}{2a}\ln\left|\frac{x\pm a}{x\mp a}\right|$

$\int \sqrt{a^2 \pm x^2}\,dx = \frac{x}{2}\sqrt{a^2\pm x^2} \pm \frac{a^2}{2}\ln\left|\sqrt{a^2\pm x^2}+x\right|$

$\int \ln x = x\ln x - x$

Hiperbolične funkcije

$\sinh x = \frac{e^x - e^{-x}}{2}$

$\cosh x = \frac{e^x + e^{-x}}{2}$

$\tanh x = \frac{\sinh x}{\cosh x}$

$\sinh' x = \cosh x$

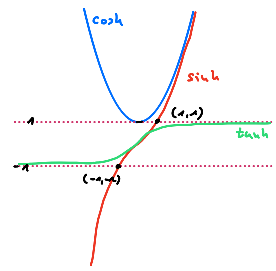
$\cosh' x = \sinh x$

$\tanh' x = 1 - \tanh^2 x$

$\mathrm{ch}^2 x - \mathrm{sh}^2 = 1$

$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$

$\cos x = \frac{e^{ix} + e^{-ix}}{2i}$



Trigonometrija

$\sin(\pi - x) = \sin x$ $\quad \sin(\frac{\pi}{2} \pm x) = \cos x$

$\cos(\pi - x) = -\cos x$ $\quad \cos(\frac{\pi}{2} \pm x) = \mp \sin x$

$\tan(\pi - x) = -\tan x$ $\quad \tan(\frac{\pi}{2} \pm x) = \mp \cot x$

$\cot(\pi - x) = -\cot x$

$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

Γ funkcija

$\Gamma(s) = \int_0^\infty x^{s-1} e^{-x} dx$

$\Gamma(n) = (n-1)!$

$\Gamma(n+1) = n\Gamma(n)$

$\Gamma(\frac{1}{2}) = \sqrt{\pi}$

$\Gamma(x+1) = \left(\frac{x}{e}\right)^x \sqrt{2\pi x}$

$\Gamma(s)\Gamma(1-s) = \frac{\pi}{\sin(\pi s)} = B(s, 1-s) \qquad 0 < s < 1$

B funkcija

$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$

$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1} dx$

$B(a,b) = \int_0^\infty x^{a-1}/(1+x)^{a+b} dx$

$B(a,b) = 2\int_0^{\pi/2} \cos^{2a-1}x \sin^{2b-1}x \, dx$

Fizikalne enačbe

$W_r = \frac{J\omega^2}{2}$

$\vec{M} = \vec{r} \times \vec{F}$

$\vec{\Gamma} = \vec{\ell} = \vec{r} \times \vec{p} = J \cdot \vec{\omega}$

Steiner $\quad \vec{\Gamma} = \vec{\Gamma}^* + m\vec{r} \times \vec{v}$

$J = mr^2$

$J = \frac{1}{2}mr^2$

$J = \frac{1}{12}ml^2$ palica

$J = \frac{1}{3}ml^2$

$J = \frac{2}{5}mr^2$ krogla

$J = \frac{2}{3}mr^2$ sfera

Toga telesa

$\dot{\vec{L}} = \sum \vec{M} \qquad \vec{L} = \underline{J}\,\vec{\omega} \qquad \vec{M} = \vec{r} \times \vec{F}$

$(J)_{ij} = \int dm\,(r^2\delta_{ij} - r_i r_j)$

Pri vrtavki $\quad \vec{M} = 0 \quad \vec{L} = \text{konst}$

$\dot{\vec{r}} = \vec{\omega} \times \vec{r} \qquad J\alpha = J\ddot{\varphi} = M$

Eulerjeve enačbe (v lastnem sistemu)

$M_x = \dot{L}_x = J_x \dot{\omega}_x + \omega_y \omega_z (J_z - J_y)$

$M_y = \dot{L}_y = J_y \dot{\omega}_y + \omega_x \omega_z (J_x - J_z)$

$M_z = \dot{L}_z = J_z \dot{\omega}_z + \omega_x \omega_y (J_y - J_x)$

Kovanec / frizbi $\quad J_z = \frac{1}{2}mr^2 \quad J_x = J_y = \frac{1}{4}mr^2$

Mala nihanja

Posplošene koord. $\quad q_1, \dots, q_n \qquad \underline{q} = \underline{q}_0 + \underline{\eta}$

$V = V_0 + \frac{1}{2}\underline{\eta}^T \underline{\underline{V}}\,\underline{\eta} \qquad (\underline{\underline{V}})_{ij} = \frac{\partial^2 V}{\partial q_i \partial q_j}\Big|_{q=q_0} \qquad T = \frac{1}{2}\dot{\underline{\eta}}^T \underline{\underline{T}}\,\dot{\underline{\eta}} \qquad L = \frac{1}{2}\dot{\underline{\eta}}^T \underline{\underline{T}}\,\dot{\underline{\eta}} - \frac{1}{2}\underline{\eta}^T \underline{\underline{V}}\,\underline{\eta}$

E-L enačba $\quad \underline{\underline{T}}\,\ddot{\underline{\eta}} + \underline{\underline{V}}\,\underline{\eta} = 0 \qquad \underline{\eta} = \underline{a}\,\overset{\text{amplituda}}{\alpha}\, e^{-i\omega t} \qquad (\underline{\underline{V}} - \omega^2 \underline{\underline{T}})\,\underline{a} = 0 \Rightarrow$ lastne vrednosti

Splošna rešitev $\quad \underline{\eta}(t) = \sum_n \underline{a}_n \begin{cases} \alpha_n e^{-i\omega_n t} & \omega_n^2 > 0 \\ b_n + c_n t & \omega_n^2 = 0 \end{cases}$ $\quad$ če so a_n normirani

Ortogonalnost $\quad \underline{a}_n^T \underline{\underline{T}}\,\underline{a}_\ell = \delta_{n\ell} \underline{\underline{T}}' \qquad \underline{\underline{A}}^T \underline{\underline{T}}\,\underline{\underline{A}} = \underline{\underline{T}}'(= I)$ diagonalna $\qquad \underline{\underline{A}}^T \underline{\underline{V}}\,\underline{\underline{A}} = \underline{\underline{V}}' \left(= \begin{pmatrix} \omega_1^2 & & \\ & \ddots & \\ & & \omega_n^2 \end{pmatrix}\right)$

$\underline{\underline{A}} = (\underline{a}_1, \dots, \underline{a}_n)$

Linearni člen $= 0$ v minimumu, razvijemo do kvadratnega člena.

Razvoj po energijah $\quad H_n = \frac{1}{2}\dot{\alpha}_n^2 T_n' + \frac{1}{2}\alpha_n^2 V_n' \qquad T' = \underline{\underline{A}}^T \underline{\underline{T}}\,\underline{\underline{A}} \qquad V' = \underline{\underline{A}}^T \underline{\underline{V}}\,\underline{\underline{A}}$