Metoda karakteristike

PDE 1. reda

$$A(x,y,z)\frac{\partial z}{\partial x}+B(x,y,z)\frac{\partial z}{\partial y}=C(x,y,z) \qquad z=z(x,y)$$

$$A = \frac{\partial x}{\partial s}, \quad B = \frac{\partial y}{\partial s}, \quad C = \frac{\partial z}{\partial s}$$

Na karakteristiki $z(x(s),y(s))=\text{konst.}=$
$=z(x_0,y_0)=z_0$

PDE 2. reda

$A=A(x,y),\ldots \qquad Af_{xx}+Bf_{xy}+Cf_{yy}+Df_x+Ef_y+Ff=G$

Karakteristike $\frac{dy}{dx}=\frac{1}{2A}\left(B\pm\sqrt{B^2-4AC}\right)$

$y=\ldots+C_1$

C_1

>0	Hiperbolične	$f_{yy}-f_{xx}=0$	val. en.
$=0$	Parabolične	$f_y-f_{xx}=0$	dif. en.
<0	Eliptične	$f_{xx}+f_{yy}=0$	poisson. en.

Valovna enačba

majhni odmiki $\quad u_{tt}=c^2u_{xx}+f \qquad dy\approx 0 \qquad dx\approx ds$

$$\mu x_{tt}=\frac{d}{ds}\left(F(s)\frac{dx}{ds}\right)+f^x$$
$$\mu y_{tt}=\frac{d}{ds}\left(F(s)\frac{dy}{ds}\right)+f^y \qquad ds^2=dx^2+dy^2$$

μ – dolžinska gostota; f^y – gostota zun. sil

$$f^y=\frac{dF_y^{zun}}{ds}=\ldots$$

Energija nihanja $E=\frac{1}{2}\mu\int_0^s y_t^2\,dx+\frac{1}{2}F\int_0^s y_x^2\,dx$

d'Alembertova rešitev (za neskončno struno)

$$u(x,t)=\frac{1}{2}\big(u(x-ct,0)+u(x+ct,0)\big)+\frac{1}{2c}\int_{x-ct}^{x+ct}u_t(x,0)\,dx$$

Masa $^{(m)}$ na struni R.P. $u_1(0,t)=u_2(0,t)$, $\quad m u_{tt}=F\left(\frac{\partial u}{\partial x}\Big|_{0^+}-\frac{\partial u}{\partial x}\Big|_{0^-}\right) \qquad \frac{F_y}{F}=\frac{dy}{dx}$

Val. en. za opna

$$z_{tt}-c^2\nabla^2z=0 \qquad c^2=\frac{\gamma^2}{\varrho} \qquad z_{tt}=\frac{P}{\varrho h}+\underbrace{\frac{\gamma}{\varrho h}}_{c^2}\nabla^2 z$$

$\frac{dP}{dS}$

Vemo $F_y=\gamma^2\frac{dz}{dx} \qquad F_s=-F_0\frac{\partial u}{\partial x} \qquad$ R.P. $u_{1x}=u_{2x} \qquad u_1=u_2\,\big|_{x=0}$

$$\boxed{u(x,t)=\sum_n T_nX_n=\sum(A_n\cos\omega_nt+B_n\sin\omega_nt)(C_n\cos k_nx+D_n\sin k_nx)}$$

Besselova DE
$$z^2u_{zz}+zu_z+(z^2-n^2)u=0$$
$$u(z)=A\,J_n(z)+B\,Y_n(z)$$

Legendrova DE
$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+l(l+1)y=0$$
$$y(x)=P_n(x)$$