1. kolokvij

Absolutna in relativna napaka

$u = f(x_1, \dots, x_n)$

$\Delta u = \sum_i \frac{\partial f}{\partial x_i}\Big|_{x_i} \Delta x_i$ abs. napaka

$\frac{\Delta u}{u} = \frac{1}{f} \sum_i \Delta x_i \frac{\partial f}{\partial x_i}\Big|_{x_i}$ rel. napaka

Optimalno združevanje

Meritvi: $(\bar{z}_1, \sigma_1^2), (\bar{z}_2, \sigma_2^2)$

$\bar{z}_1 = x + r_1 \quad \bar{z}_2 = x + r_2$ šum

Opt. združitev $(\hat{x}, \hat{\sigma}^2)$

Varianca σ^2

Kovarianca $\sigma_{12} = \langle r_1 r_2 \rangle$

$r_2 = \alpha r_1 + w \quad \langle w^2 \rangle = \sigma_w^2 \quad \langle w r_1 \rangle = 0$

$\rho_{12} = \alpha \frac{\sigma_1}{\sigma_2}$

$\sigma_{12} = \rho_{12} \sigma_1 \sigma_2$

$$\hat{x} = \bar{z}_1 + \frac{\sigma_1^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} (\bar{z}_2 - \bar{z}_1)$$

$$\hat{\sigma}^2 = (1 - \rho_{12}^2)\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} - \frac{2\rho_{12}}{\sigma_1 \sigma_2}\right)^{-1}$$

Varianca povprečja

$\bar{z} = \frac{1}{N} \sum_i z_i \quad z_i \sim N(x, \sigma_i^2) \quad \sigma_{ij} \neq 0$

$$\sigma_m^2 = \frac{1}{N^2}\Big(\sum_i \sigma_i^2 + 2 \sum_{i<j} \sigma_{ij}\Big)$$

Kalmanov filter, neodvisna konst. količina

Opt. ocena $(\hat{x}_n, \hat{\sigma}_n^2)$,

meritev $(z_{n+1}, \sigma_{n+1}^2)$

$$\hat{x}_{n+1} = \hat{x}_n + \frac{\hat{\sigma}_n^2}{\hat{\sigma}_n^2 + \sigma_{n+1}^2}(z_{n+1} - \hat{x}_n)$$

$$\hat{\sigma}_{n+1}^2 = \left(\frac{1}{\hat{\sigma}_n^2} + \frac{1}{\sigma_{n+1}^2}\right)^{-1}$$

Če združujemo M meritev

$$\hat{x} = \sum_i \frac{z_i}{\sigma_i^2} \Big(\sum_i \sigma_i^{-2}\Big)^{-1}$$

$$\hat{\sigma}^{-2} = \sum_i \sigma_i^{-2}$$

Gaussova porazdelitev

$$\frac{dP}{dz} = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

$\mathrm{erf}(x) = \int_{-\infty}^{x} \frac{dP}{dz} dz = F(x)$

$F(-x) = 1 - F(x) \qquad F = F\left(\frac{x-\mu}{\sigma}\right)$

Merjenje skalarne količine

$x_{n+1} = \Phi_n x_n + c_n + \Gamma_n w_n$

Poznamo $(\hat{x}_n, \hat{\sigma}_n^2)$, $(z_{n+1}, \sigma_{n+1}^2)$

Napoved $\bar{x}_{n+1}, \bar{\sigma}_{n+1}^2$

Naj bo $\hat{\sigma}_n^2 = P_n$, $\bar{\sigma}_{n+1}^2 = M_{n+1}$

$\langle w_n^2 \rangle = Q_n, \quad \sigma_{n+1}^2 = R_{n+1}$

- $\bar{x}_{n+1} = \Phi_n x_n + c_n$

  $M_{n+1} = \Phi_n^2 P_n + \Gamma_n^2 Q_n$

- $\hat{x}_{n+1} = \bar{x}_{n+1} + \frac{M_{n+1}}{M_{n+1} + R_{n+1}}(z_{n+1} - \bar{x}_{n+1})$

  $P_{n+1} = M_{n+1} - \frac{M_{n+1}^2}{M_{n+1} + R_{n+1}}$

Vektorska količina

- $\bar{\vec{x}}_{n+1} = \Phi_n \hat{\vec{x}}_n + \vec{c}_n$

  $M_{n+1} = \Phi_n P_n \Phi_n^T + \Gamma_n Q_n \Gamma_n^T$

- $\hat{\vec{x}}_{n+1} = \bar{\vec{x}}_{n+1} + K_{n+1}(\vec{z}_{n+1} - H \bar{\vec{x}}_{n+1})$

  $P_{n+1} = M_{n+1} - M_{n+1} H^T (H M_{n+1} H^T + R_{n+1})^{-1} H M_{n+1}$

  $K_{n+1} = P_{n+1} H^T R_{n+1}^{-1}$

  $\vec{z} = H\vec{x} + \vec{r}$

Vektorska količina - zvezna slika

$\dot{\vec{x}} = A\vec{x} + \vec{c} + \Gamma \vec{w}$

$\dot{\hat{\vec{x}}} = A\hat{\vec{x}} + \vec{c} + K(\vec{z} - H\hat{\vec{x}})$

$K = P H^T R^{-1}$

$\dot{P} = AP + PA^T + \Gamma Q \Gamma^T - P H^T R^{-1} H P$

$PA^T = (AP)^T$

Senzorji

$x(t) \quad z(t)$

$x(s) = H(s)\, z(s)$

$z(0^-) = x(0^-) = 0$

- 1. red

  $\tau \dot{x}(t) + x(t) = z(t)$

  $H(s) = \frac{1}{1 + \tau s}$

- 2. red

  $\ddot{x} + 2\zeta\omega_0 \dot{x} + \omega_0^2 x = \omega_0^2 z$

  $\zeta_{opt} = 1/\sqrt{2}$

  $$H(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

Širjenje napak

$u = f(x, y)$

Poznamo $\bar{x}, \sigma_x^2 \quad \bar{y}, \sigma_y^2$, iščemo $\bar{u}, \sigma_u^2$

$\bar{u} = f(\bar{x}, \bar{y})$

$$\sigma_u^2 = \left(\frac{df}{dx}\right)^2 \sigma_x^2 + \left(\frac{df}{dy}\right)^2 \sigma_y^2 + 2\frac{df}{dx}\frac{df}{dy}\sigma_{xy}\Big|_{\bar{x}, \bar{y}}$$

	$f(t) = \mathscr{L}^{-1}(F(s))$	$F(s) = \mathscr{L}(f(t))$
1.	e^{at}	$\frac{1}{s-a}$
2.	1	$\frac{1}{s}$
3.	t^n	$\frac{1}{s^{n+1}} n!$
4.	$\delta(t)$	1
5.	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
6.	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
7.	koračna f. $\Theta(t-T)$	$\frac{1}{s} e^{-Ts}$
8.	$f(t-T)\Theta(t-T)$	$F(s)\, e^{-Ts}$
9.	$f(t)\, e^{at}$	$F(s-a)$
10.	$\frac{d^n}{dt^n} f(t)$	$s^n F(s) - f(0)$
11.	$-t f(t)$	$\frac{d}{ds} F(s)$

2. kolokvij

Senzorji

$x(t)$ $z(t)$

$x(s) = H(s)\, z(s)$

$z(0^-) = x(0^-) = 0$

- 1. red

$\tau \dot{x}(t) + x(t) = z(t)$

$H(s) = \frac{1}{1+\tau s}$

$z(t) = \delta(t) \quad x(t) = \frac{1}{\tau} e^{-t/\tau}$

$z(t) = kt \quad x(t) = k(t - \tau(1 - e^{-t/\tau}))$

- 2. red

$\ddot{x} + 2\zeta\omega_0 \dot{x} + \omega_0^2 x = \omega_0^2 z + (2\zeta\omega_0 \dot{z})$

$\zeta_{opt} = 1/\sqrt{2}$

$H(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$

$s^2 + 2\zeta\omega_0 s + \omega_0^2 = (s+\zeta\omega_0)^2 + \omega_0^2(1-\zeta^2)$

$z(t) = \delta(t) \quad x(t) = \frac{\omega_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_0 t} \sin(\omega_0^2(1-\zeta^2)t)$

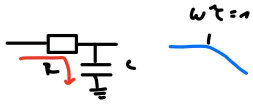
Periodičen signal

$z(t) = z_0 e^{i\omega t}, \quad H = \frac{U_{iz}}{U_{vh}}$

$Z_C = \frac{1}{i\omega C} \qquad Z_L = i\omega L$

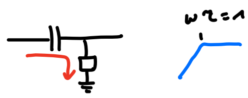
- LPF $\quad H = \frac{Z_C}{R + Z_C} = \frac{1}{1 + i\omega\tau}$

$\tau = RC$



- HPF $\quad H = \frac{R}{Z_C + R} = \frac{i\omega\tau}{1 + i\omega\tau}$

$\tau = RC$

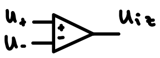


	$f(t) = \mathcal{L}^{-1}(F(s))$	$F(s) = \mathcal{L}(f(t))$
1.	e^{at}	$\frac{1}{s-a}$
2.	1	$\frac{1}{s}$
3.	t^n	$\frac{1}{s^{n+1}} n!$
4.	$\delta(t)$	1
5.	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
6.	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
7.	koračna f. $\Theta(t-T)$	$\frac{1}{s} e^{-Ts}$
8.	$f(t-T)\Theta(t-T)$	$F(s) e^{-Ts}$
9.	$f(t) e^{at}$	$F(s-a)$
10.	$\frac{d^n}{dt^n} f(t)$	$s^n F(s) - f(0$
11.	$-t f(t)$	$\frac{d}{ds} F(s)$

$t \sim \frac{1}{s^2}$

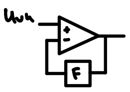
$\sin \omega t \; e^{at} = \frac{\omega}{(s-a)^2 + \omega^2}$

Aktivna vezja

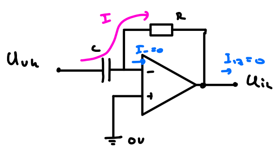


$U_{iz} = A_0 (U_+ - U_-)$

$I_+, I_- = 0$

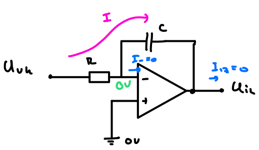


$H = \frac{1}{F}$



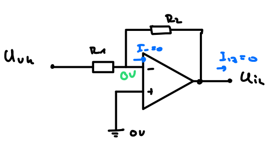
Diferenciator

$H = -i\omega\tau$



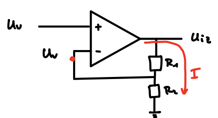
Integrator

$H = -\frac{1}{i\omega\tau}$



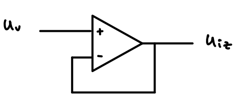
Invertirajoči ojač

$H = -\frac{R_2}{R_1}$



Neinvertirajoči ojač

$H = 1 + R_2/R_1$



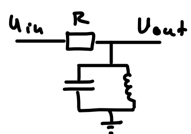
Napetostni sledilnik

$H = 1$

Resonančna vezja (filtri 2. reda)

Pasovno prepustni filter

$H = \frac{s\omega_c}{s^2 + s\omega_c + \omega_0^2}$



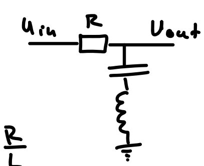
$\omega_c = \frac{1}{RC} \qquad \omega_0^2 = \frac{1}{LC}$

$\Delta\omega_{-3dB} : \quad -3dB = 10 \log\left(\frac{|H|^2}{|H_{max}|^2}\right)$

$Q = \frac{\omega_0}{\Delta\omega_{-3dB}} \qquad \Delta\omega = 2|\mathrm{Re}\, s_1| \; (= \omega_c)$

Pasovni neprepustni filter

$H = \frac{s^2 + \omega_0^2}{s^2 + s\omega_c + \omega_0^2}$



$\omega_0^2 = \frac{1}{LC} \qquad \omega_c = \frac{R}{L}$

Bodejev diagram: $20 \log |H|$ in δ odv. od ω

Poli $\quad s^2 + \omega_c s + \omega_0^2 = 0$

$s_{1,2} = -\frac{\omega_c}{2} \pm i\omega_0$

Statistika

$a = \frac{1}{N} \sum z_i \qquad s^2 = \frac{1}{N-1} \sum (z_i - \bar{z})^2$

$s^2 = \frac{1}{N} \sum (z_i - a)^2 \qquad \sigma^2 = \langle s^2 \rangle$

$a = \langle z_i \rangle$

$\chi^2 = \sum \frac{(z_i - \bar{z})^2}{\sigma^2} \sim \chi^2(N-1)$

$\sigma_z^2 = (N-1) \frac{s^2}{\chi^2} \qquad P(\chi^2 > \chi_1^2) = \frac{\alpha}{2}$

$P(\chi^2 > \chi_2^2) = 1 - \frac{\alpha}{2}$

$T = \frac{\bar{z} - a}{s} \sqrt{N} \sim S(N-1)$

$a = \bar{z} \pm T_1 \frac{s}{\sqrt{N}} \qquad P(|T| > T_1) = \alpha$

Primerjava dveh vzorcev

$a_x, a_y \quad , \quad \sigma_x = \sigma_y = \sigma$

$T = \frac{(\bar{x} - \bar{y})\sqrt{N_x + N_y - 2}}{\sqrt{\frac{1}{N_x} + \frac{1}{N_y}} \sqrt{s_x^2(N_x - 1) + s_y^2(N_y - 1)}} \sim S(N_x + N_y - 2)$

σ_x, σ_y

$F = \frac{s_1^2 / \sigma_1^2}{s_2^2 / \sigma_2^2} \sim F(N_1 - 1, N_2 - 1)$

$F_1 = F_{\alpha/2}(k_1, k_2) \qquad F_2 = F_{1-\frac{\alpha}{2}} = (F_{\frac{\alpha}{2}}(k_2, k_1))^{-1}$