Binarni zapis

$0\,11\,01100_2 \Rightarrow$ pozitivno

$1\,\underbrace{1101100}_{}{}_2 \Rightarrow$ negativno

$0 \to 1 \quad 1 \to 0$

$$\begin{array}{r} 0010011 \\ +\quad 1 \\ \hline 0010100 \end{array}$$

BCD - Binary coded decimal

123_{10}

$0001 \quad 0010 \quad 0011$ BCD

Fourierova analiza

$x(t) \to F(x(t)) = X(\omega) \Rightarrow$

Energije v odv. od frekvence

koeficienti

c_k periodični signali

$c(i\omega)$ neperiodični signali

Periodično:

$$F(x(t)) = c_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-i\omega t k}\, dt$$

perioda

$$x(t) = \sum_k c_k e^{i\omega k t}$$

Neperiodično:

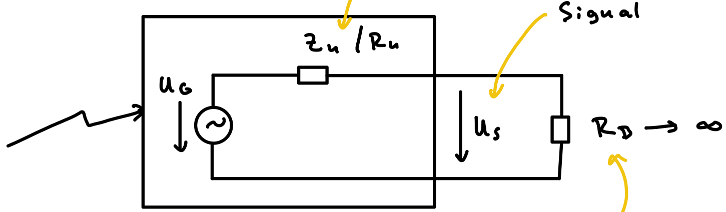
$$F(x(t)) = c(i\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-i\omega t}\, dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} c(i\omega)\, e^{i\omega t}\, d\omega$$

Analogni signali

Notranja impedanca / upornost

Signal

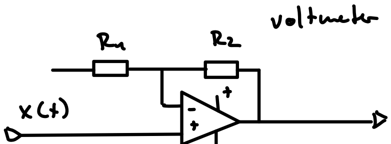


Breme (voltmeter)

$$U_S = U_G \frac{R_B}{R_B + R_n}$$

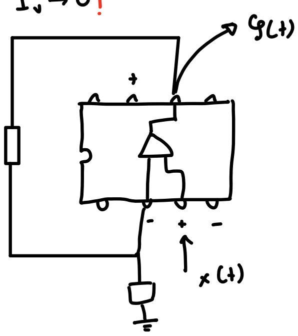
Ne želimo, da bi kateri koli voltmeter vplival na signal

Ojačanje



$$y(t) = x(t)\left(1 + \frac{R_2}{R_1}\right)$$

$I_v \to 0$!

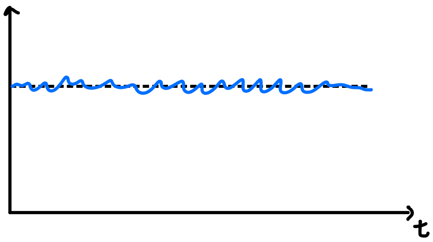


Operacijski ojačevalnik

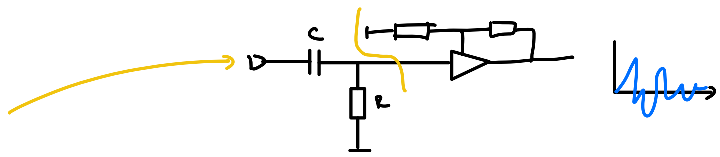
Če R_1 ni $R_2 = 0 \to y(t) = x(t)$

Bufferski ojačevalnik

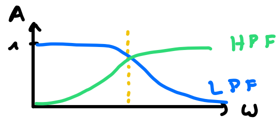
Filtriranje

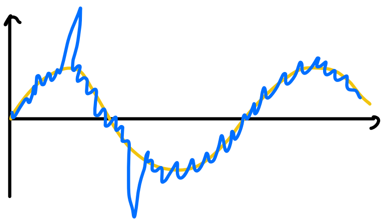


HPF (high pass filter)

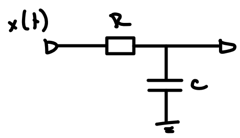


Kondenzator prepušča pri visokih frekvencah

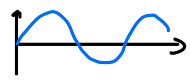




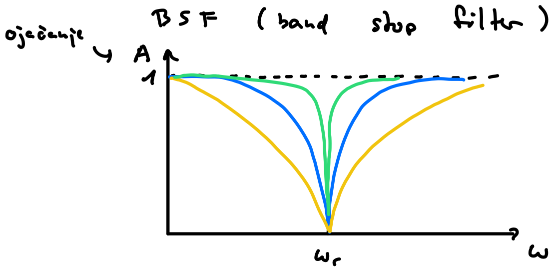
LPF (low pass filter



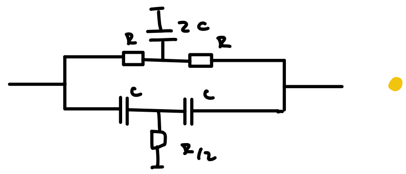
⇒ ojačevalnik



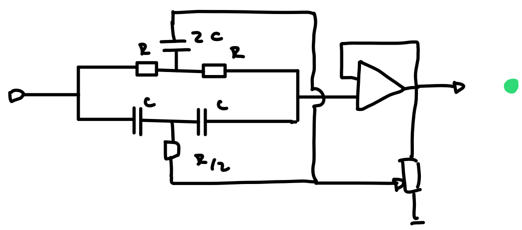
BSF (band stop filter)



izloči eno frekvenco

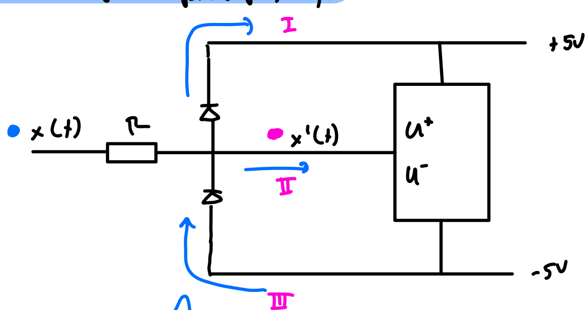


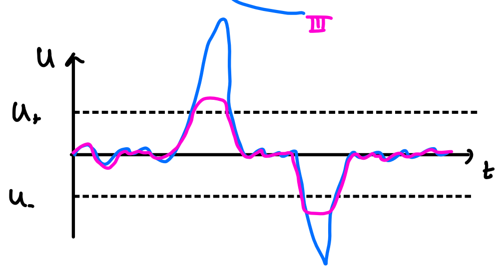
$\omega_r = \frac{1}{RC}$



Bolj uspešna karakteristika

Zaščita pred prenapetostjo

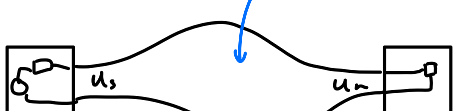




Tok teče za
I $u > u_+$
II $u_- < u < u_+$
III $u < u_-$

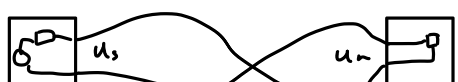
Oklaplanje

Inducira se lahko napetost

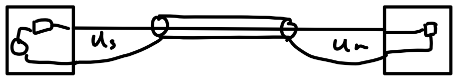


$U_s \neq U_m$

Prepletemo žico



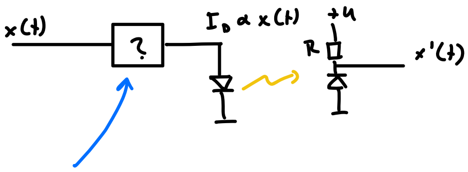
Inducirani napetosti se odštejeta

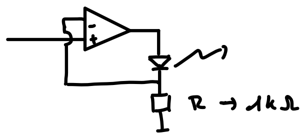


Koaksialni kabel

Izniči vpliv zunanjega izmeničnega E

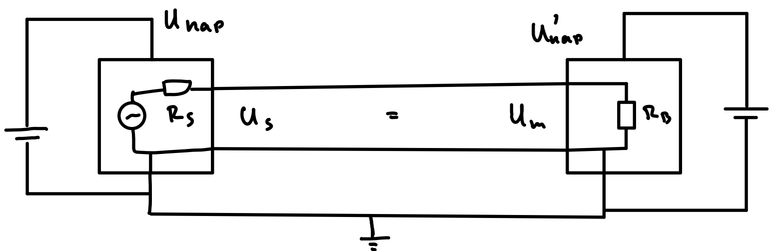
Optične ločitve signalov





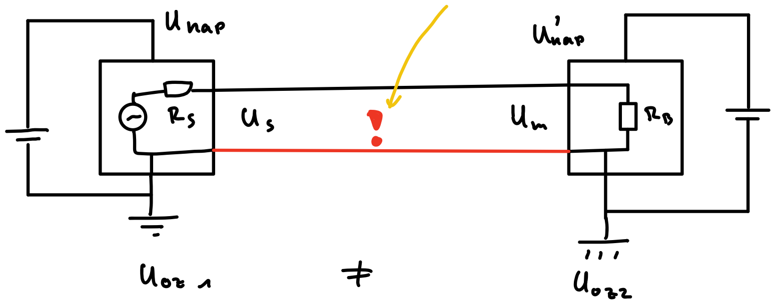
Tokovni generator

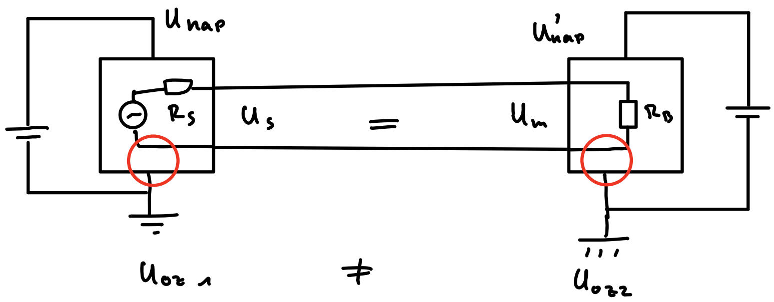
Priključitev senzorjev



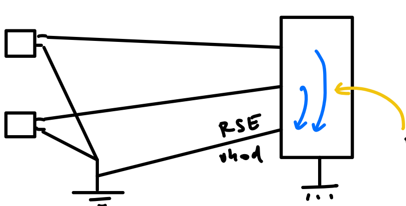
Če sta senzorja blizu

Ne smemo, ker bi tekel tok zaradi $U_{oz1} \neq U_{oz2}$



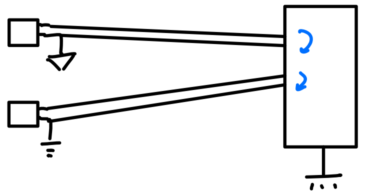


Več senzorjev



Če sta merilnik in senzor daleč

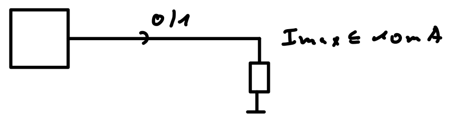
merimo skok na tujo zemljo



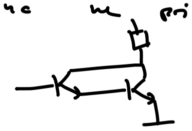
Če sta merilnih in senzor daleč
↳ merilniki daleč med sabo

Diferencialni način

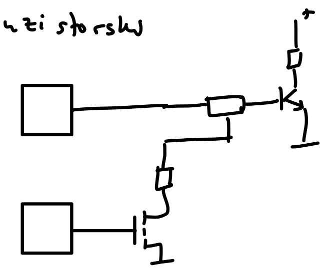
decibeli : razmerje $w\,[dB] = 20 \log \frac{|x|}{|y|}$



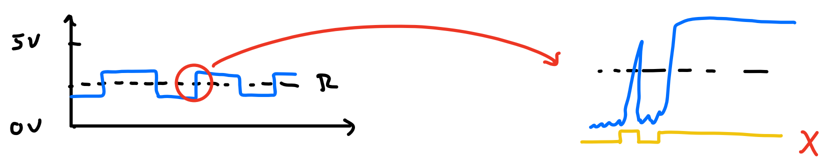
Bremena ne priključimo direktno na izhod, uporabimo tokovni generator



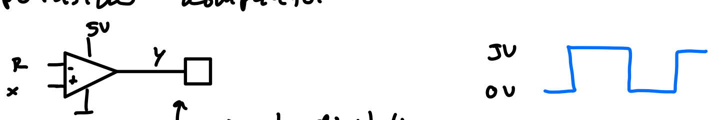
Tranzistorsko



Signali na vhodu v vezje

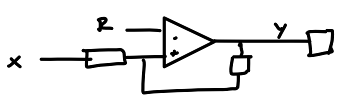


uporabimo komparator

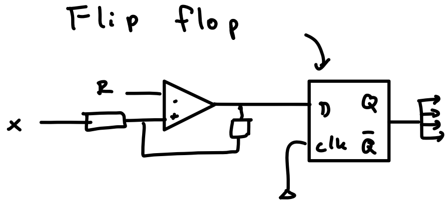


↑ signal digitaliziramo

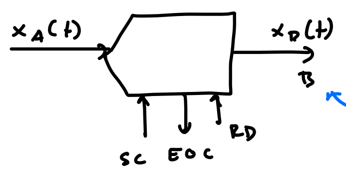
Komparator s histerezo



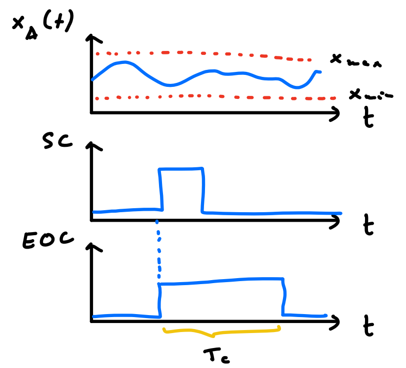
Flip flop

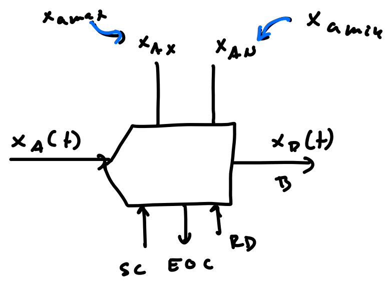


Analogno digitalni pretvornik ADC



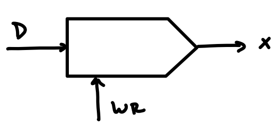
št bitov na outputu

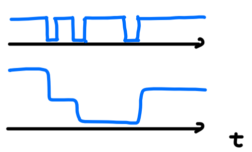




$$x_D = \frac{2^B - 1}{x_{Amax} - x_{Amin}} \left(x_A - x_{Amin} \right)$$

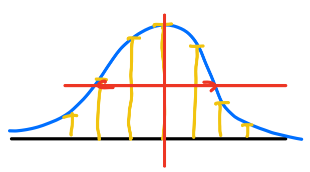
Digitalno analogni pretvornik DAC





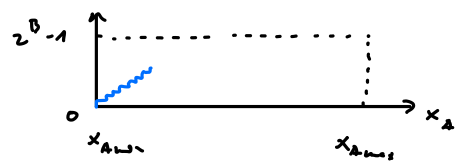
Lastnosti ADC in DAC

- Ločljivost (resolucija, natančnost)

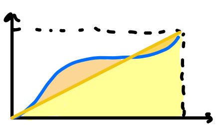


SNR (signal to noise ratio) $= 20 \log \frac{S+N}{N}$ [dB]

- Nelinearnost: diferencialna : integralna, THD, SFDR

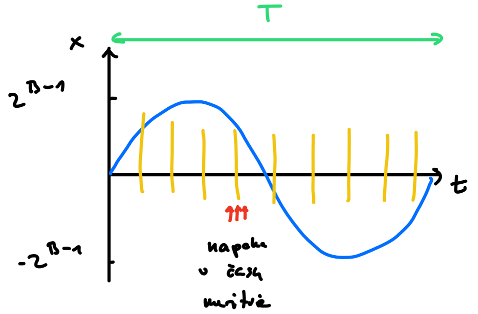


Integralna nelinearnost =



THD, SFDR

Napaka zaradi časa zajemanja



$$x(t) = 2^{B-1} \sin(\omega t)$$
$$x'(t) = 2^{B-1} \omega \cos \omega t$$
$$\max x' = 2^{B-1} \omega$$

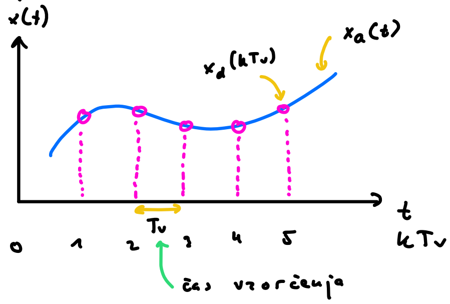
$$x' \Delta \le B$$

največja napaka

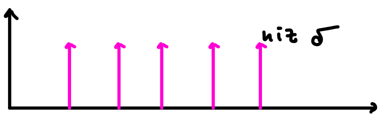
$$2^{B-1} 2\pi \frac{1}{T} \Delta \le B$$

$$\boxed{\Delta = \frac{T}{\pi 2^B}}$$

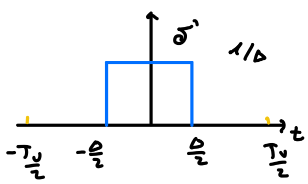
Vzorčenje



$$x_a(t) \longrightarrow x_d(kT_v) = x_a(t) \sum_k \delta(t-kT_v)$$



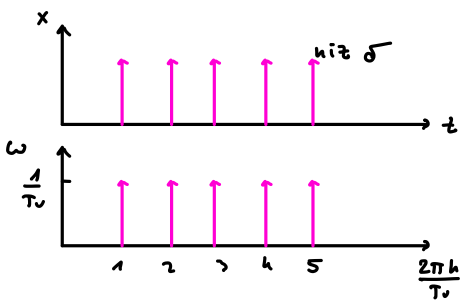
$$= \sum_k \delta(kT_v)$$
$$\delta(t-kT_v)$$



$$F(\text{niz } \delta') = \frac{1}{T_v} \int_{-\frac{T_v}{2}}^{T_v/2} \delta' e^{-ik\omega t}\, dt = \frac{1}{T_v \Delta} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} e^{-\frac{ik2\pi}{T_v} t}\, dt \qquad \omega = \frac{2\pi}{T_v}$$

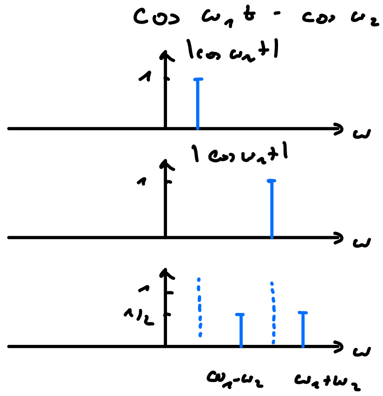
$$= \frac{i}{T_v k \omega \Delta} e^{-ik\omega t} \Big|_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} = \frac{i}{T_v k \omega \Delta} \left(\cos k\omega t - i \sin k\omega t - \cos k\omega t - i \sin k\omega t \right)$$

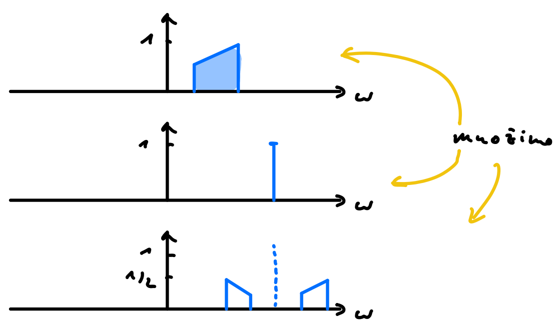
$$= \frac{2 \sin k\omega \Delta/2}{T_v \Delta k \omega} = \frac{\sin k\omega \Delta/2}{T_v k\omega \Delta/2}$$



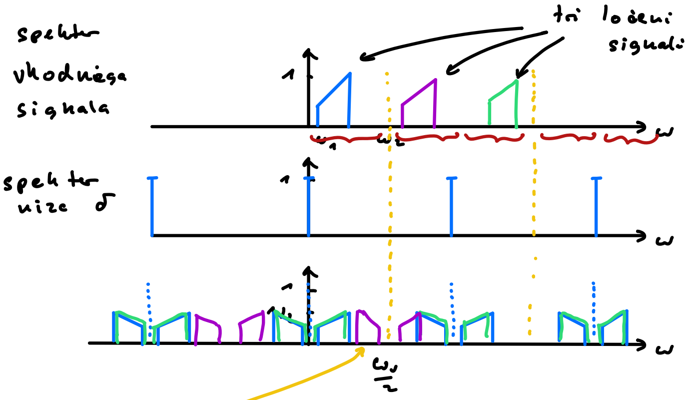
$$F(\text{niz } \delta) = \lim_{\Delta \to 0} F(\delta') = \frac{1}{T_v} \quad \Rightarrow \quad \text{niz } \delta = \frac{1}{T_v} \sum e^{ik\omega t}$$

$$\cos \omega_1 t \cdot \cos \omega_2 t = \frac{1}{2} \left(\cos(\omega_1 + \omega_2) t + \cos(\omega_1 - \omega_2) t \right)$$





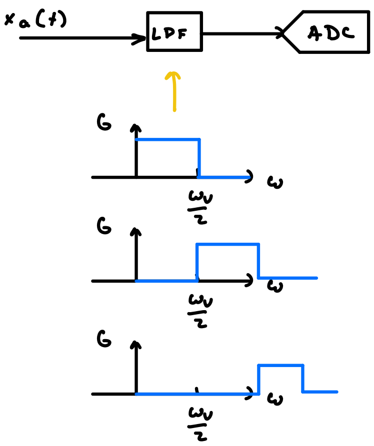
Nyquist:
- frekvenca vhodnega signala je lahko največ polovica frekvence vzorčenja



under sampling

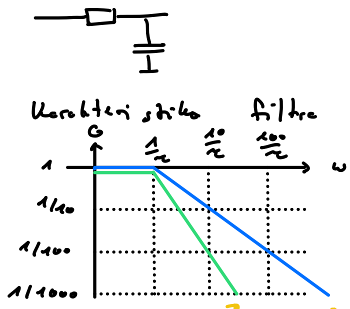
signal mora biti znotraj ——

Spektralni obrat



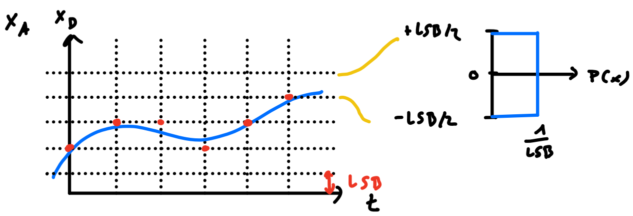
spektralni obrat

undersampling



2 RC filtra

ni dovolj dober filter



$$SNR = 20 \log \frac{U_{SEF}}{U_{NEF}}$$

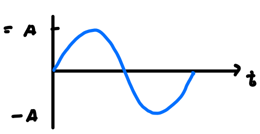
signal, efektivna

šum, efektivna

$$= 10 \log \frac{U_{SEF}^2}{U_{NEF}^2} = \dots$$

a) $U_{SEF}^2 = \frac{1}{2\pi} \int_0^{2\pi} x^2 \, dt = \frac{1}{2\pi} \int_0^{2\pi} A^2 \sin^2 \varphi \, d\varphi = \frac{A^2}{2}$

$LSB \cdot 2^{B-1} = A$



b) $U_{NEF}^2 = \sigma^2 = \int_{-LSB/2}^{LSB/2} (x - \underbrace{\langle x \rangle}_{=0})^2 \underbrace{P(x)}_{=\frac{1}{LSB}} \, dx =$

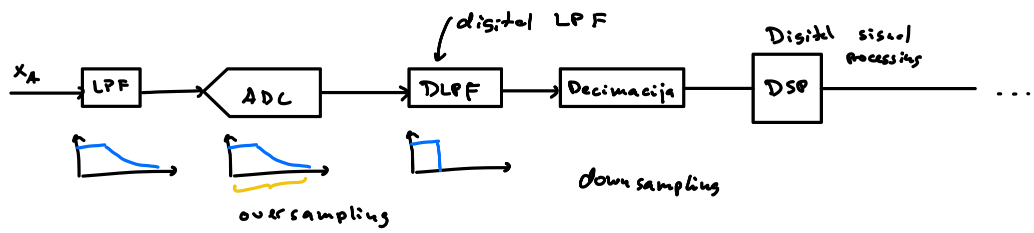
$$= \frac{1}{LSB} \frac{x^3}{3} \Big| = \frac{1}{LSB} \frac{2}{3} \frac{LSB^3}{2^3} = \frac{LSB^2}{12}$$

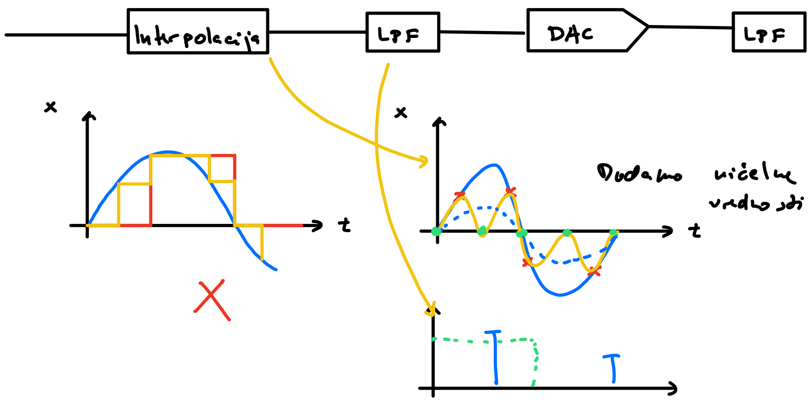
$$SNR = \dots = 10 \log \frac{A^2/2}{LSB^2/12} = 10 \log \frac{6A^2}{LSB^2}$$

$$= 10 \log \frac{6\, LSB^2\, 2^{(B-1)\cdot 2}}{LSB^2} = 10 \log \frac{3}{2} 2^{2B} =$$

$$SNR = 10 \log \frac{3}{2} + 10 \cdot 2B \log 2$$

B	SNR
8	50dB
16	98dB
24	146dB





Spekter digitalnega signala

$$x_a(t) \to x_d(kT_v)$$

$$x_d = x_a \sum_k \delta(t-kT_v) = x_a \sum_k \frac{1}{T_v} e^{ik\omega_v t}$$

$$F(x_d(kT_v)) = \int_{-\infty}^{\infty} x_d(kT_v)\, e^{-i\omega t}\, dt = \int_{-\infty}^{\infty} x_a\Big(\sum_k \frac{1}{T_v} e^{ik\omega_v t}\Big) e^{-i\omega t}\, dt =$$

$$= \sum_k \frac{1}{T_v} \int_{-\infty}^{\infty} x_a(t)\, e^{-i(\omega-k\omega_v)t}\, dt = \dots$$

$$F(x_a) = \int_{-\infty}^{\infty} x(t)\, e^{-i\omega t}\, dt$$

$$\dots = \sum_k \frac{1}{T_v} \int_{-\infty}^{\infty} x_a\, e^{-i\omega' t}\, dt = \frac{1}{T_v} \sum_k F(x_a, \omega - k\omega_v)$$

$\omega' = \omega - k\omega_v$

Linearni sistem

Značilnosti:

- vzročnost $\quad x(t) \to \boxed{} \to y(t) \quad y(t) = f(x(t)) \quad y(t) = 0$ zgodovina $\big|_{x=0}$
- homogenost $\quad x(t) \to y(t) \Rightarrow A\, x(t) \to A\, y(t)$
- aditivnost $\quad x_1(t) \to y_1(t) \quad x_2(t) \to y_2(t) \Rightarrow x_1(t) + x_2(t) \to y_1(t) + y_2(t)$
- časovna neodvisnost $\quad x(t) \to y(t) = x(t-T) \to y(t-T)$

Lin. sis. lahko zapišemo kot DE

$$\sum_m A_m x^{(m)} = \sum_n B_n y^{(n)}$$

$$\Downarrow$$

prenosna funkcija $T = \frac{y}{x}$

Laplaceova transformacija $\quad \mathcal{L}(x(t)) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$

$$T(s) = \frac{\mathcal{L}(y(t))}{\mathcal{L}(x(t))}$$

Zgled

$$\mathcal{L}(\delta(t)) = \lim_{\Delta\to 0}\int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} e^{-st}\,dt = \lim -\frac{1}{s}\frac{1}{\Delta}e^{-st}\Big|_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} =$$

$$\delta = \begin{cases} \frac{1}{\Delta} & -\frac{\Delta}{2} < x < \frac{\Delta}{2} \\ 0 & \text{sicer} \end{cases}$$

$$= \lim_{\Delta\to 0} -\frac{1}{\Delta s}\left(\cancel{1} - \frac{st}{1!} + \cancel{\frac{s^2t^2}{2!}} - \frac{s^3t^3}{3!} + \dots\right)\Big|_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}}$$

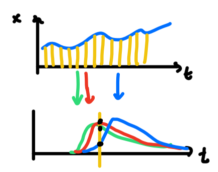
$$= \lim_{\Delta\to 0} -\frac{1}{\Delta s}\left(-\frac{s\Delta}{1!} - \frac{s^3\Delta^3}{4\cdot 3!} - \dots\right) = 1$$

$T(s) = \mathcal{L}(y(t))$ če $x(t) = \delta(t)$

Če nas zanima T sistema vzbujamo z $x(t) = \delta(t)$ izmerimo $y(t) \Rightarrow \mathcal{L}(y) \Rightarrow T$

$h(t)$

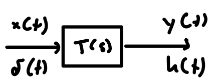
značilni / impulzni odziv

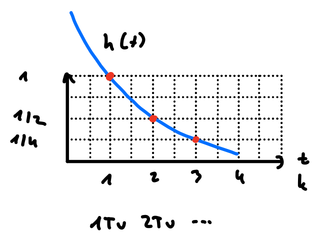
- 

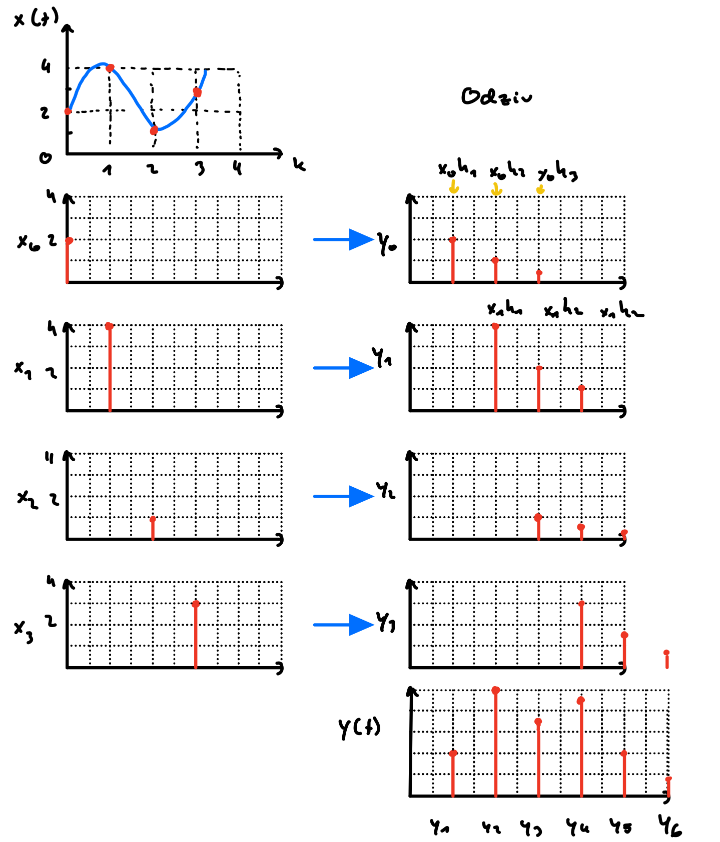
signal ≈ niz delta funkcij

signal → diskretizacija → Σ značilnih odziv na vsak δ

Zgled



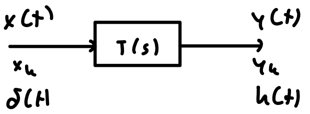


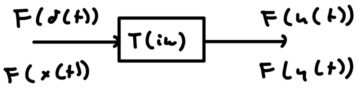


$$y_3 = x_0 h_3 + x_1 h_2 + x_2 h_1$$

$$\boxed{y_k = \sum_{m=1}^{M} x_{k-m} h_m}$$

Konvolucijska formula

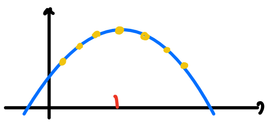


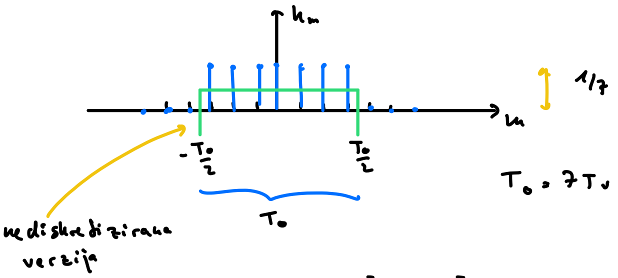


$$\Rightarrow \quad T(i\omega) = \frac{F(y(t))}{F(x(t))} = \frac{F(h(t))}{\underset{=1}{F(\delta(t))}} =$$

$$\Rightarrow \quad T(i\omega) = F(h(t))$$

- Drsno povprečenje





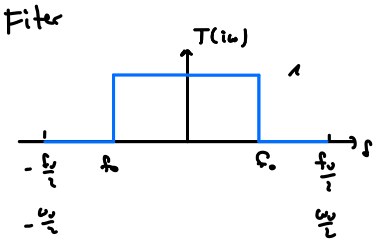
$T_0 = 7T_v$

$$h(t) = \begin{cases} 1/T_0 \ ; & -\frac{T_0}{2} < t < \frac{T_0}{2} \\ 0 \ ; & \text{sicer} \end{cases}$$

$$F(h(t)) = T(i\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-i\omega t}\, dt = \int_{-T_0/2}^{T_0/2} \frac{1}{T_0}\, e^{-i\omega t}\, dt = \frac{1}{T_0}\frac{1}{-i\omega}\left(-2i \sin\frac{T_0}{2}\omega\right)$$

$$T(i\omega) = \frac{\sin\frac{T_0\omega}{2}}{\frac{T_0\omega}{2}}$$ Slab filter

- Filter



$$h(t) = F^{-1}(T(i\omega))$$

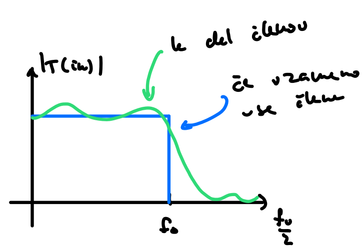
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} T(i\omega)\, e^{i\omega t}\, d\omega = \frac{1}{2\pi}\int_{-f_0}^{f_0} e^{i\omega t}\, d\omega = \frac{1}{2\pi i t}\left(e^{i\omega_0 t} + e^{-i\omega_0 t}\right)$$

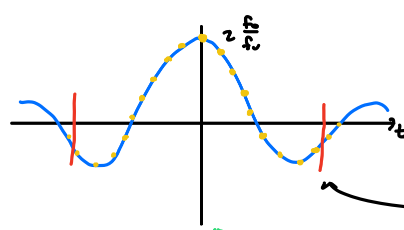
$$= \frac{1}{2\pi i t}\left(2i \sin \omega_0 t\right) = \frac{1}{\pi t}\sin\omega_0 t = \frac{\sin 2\pi f_0 t}{2\pi f_0 t}\, 2f_0$$

Diskretizacija $\quad t = m T_v$

$$\underset{\text{analog}}{T_A(i\omega)} = \underset{\text{digital}}{T_D(i\omega)} \cdot T_v \qquad T_v = \frac{1}{f_v}$$

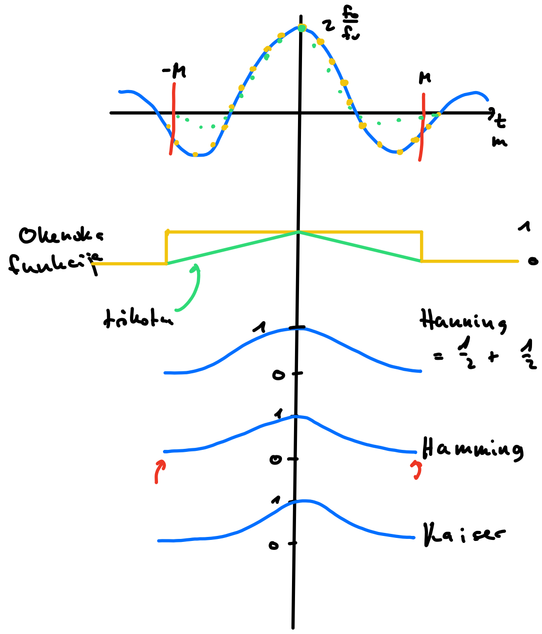
$$h_m = \frac{\sin(2\pi f_0 T_v m)}{2\pi f_0 m T_v}\, 2f_0 T_v = \frac{\sin 2\pi m \frac{f_0}{f_v}}{2\pi m \frac{f_0}{f_v}}\, 2\frac{f_0}{f_v}$$





Filter bo zakasnil signal k šele tu računamo za vrednost tu

Kako izbojsjeti signal

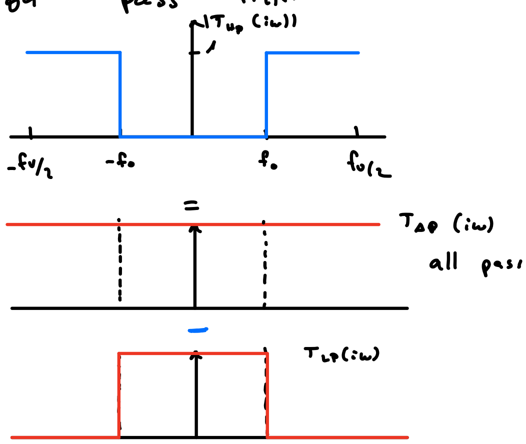


Hanning $= \frac{1}{2} + \frac{1}{2}\cos\left(\pi \frac{m}{M}\right)$

Hamming $= 0{,}56 + 0{,}44 \cos\left(\pi \frac{m}{M}\right)$

Kaiser fit Gaussove funkcije

High pass filter



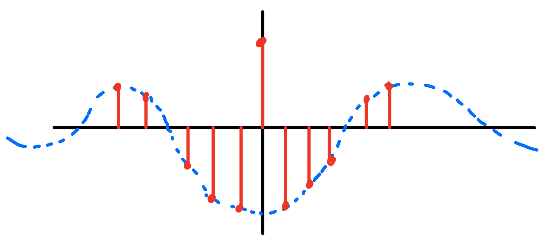
$$h(t) = F^{-1}(T_{HP}(i\omega))$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} T_{HP}(i\omega)\, e^{i\omega t}\, d\omega$$

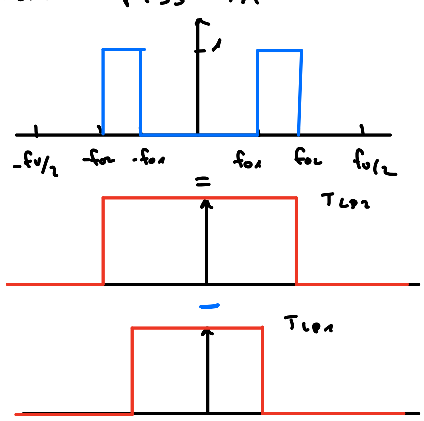
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} (T_{AP}(i\omega) - T_{LP}(i\omega))\, e^{i\omega t}\, d\omega$$

$$= \underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} 1\, e^{i\omega t}\, d\omega}_{h_{AP\,m} = \begin{cases} 1 & m = 0 \\ 0 & m \neq 0 \end{cases}} - 2\frac{f_0}{f_v}\, \frac{\sin 2\pi m \frac{f_0}{f_v}}{2\pi m\, f_0/f_v}$$

$$\boxed{\begin{array}{ll} \text{za } m = 0 & \rightarrow h_0 = 1 - 2\frac{f_0}{f_v} \\ \text{za } m \neq 0 & \rightarrow h_m = -2\frac{f_0}{f_v}\, \dfrac{\sin 2\pi m \frac{f_0}{f_v}}{2\pi m\, f_0/f_v} \end{array}}$$

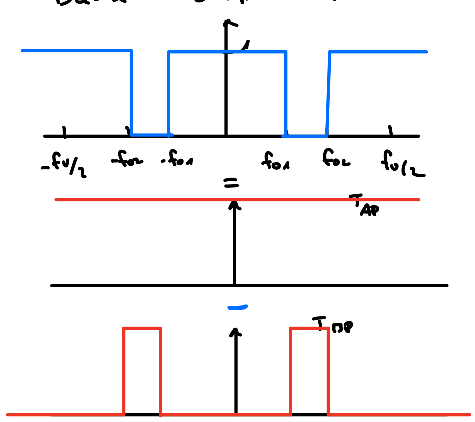


Band pass filter



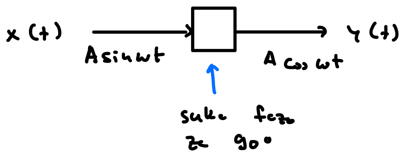
$$h_{BP\,m} = 2\frac{f_{02}}{f_v}\, \frac{\sin 2\pi m \frac{f_{02}}{f_v}}{2\pi m \frac{f_{02}}{f_v}} - 2\frac{f_{01}}{f_v}\, \frac{\sin 2\pi m \frac{f_{01}}{f_v}}{2\pi m \frac{f_{01}}{f_v}}$$

Band stop filter

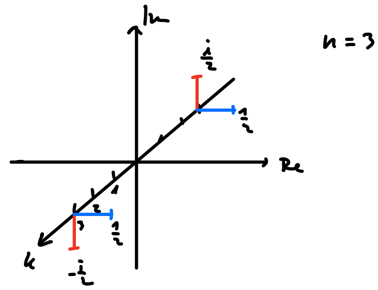


$$h_{BS\,m} = \begin{cases} m = 0 & h_0 = 1 - 2\frac{f_{02}}{f_v} + \frac{f_{01}}{f_v} \\ m \neq 0 & h_m = -h_{BP\,m} \end{cases}$$

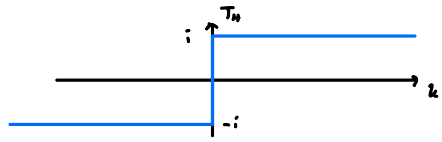
Hilbertov transform



$$T_H(i\omega) = \frac{F(y(t))}{F(x(t))} = \frac{Y(i\omega)}{X(i\omega)} = \frac{\frac{1}{2}(\delta(n-k) + \delta(n+k))}{\frac{i}{2}(-\delta(n-k) + \delta(n+k))}$$

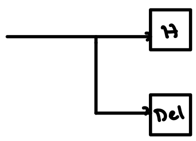


$$T_H(i\omega) = \begin{cases} n = k; & i \\ n = -k; & -i \end{cases}$$



Diskretna verzija

$$h(m) = \frac{(-1)^m - 1}{\pi m}$$

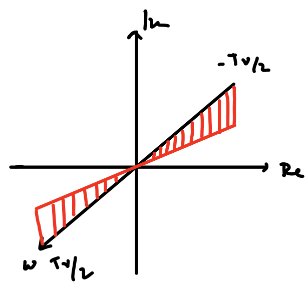


zakasni za $M T_r$
ker H ne računamo
v real-time

Uporaba: Lahko dobimo amplitudo signala
o vseh točkah n k v vrhovih.

Odvod

$\sin wt \to w \cos wt$
→ zasuk za 90°
→ prilagodimo amplitudo



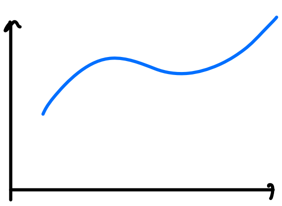
$T(i\omega) = i\omega\, d$

FIR filtriranje

$$y_n = \sum_{m=0}^{M} x_{n-m} h_m \quad \text{upoštevamo le zgodovino}$$

$$y_n = \sum_{m=-M}^{M} x_{n-M-m} h_m \quad \text{upoštevamo zgodovino in prihodnost}$$

IIR - uporabimo prejšnje rezultate za izračun novih

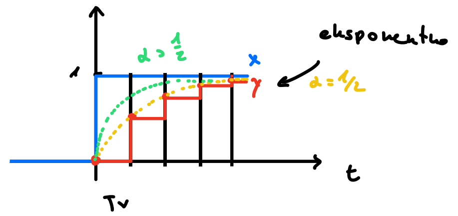


$$y_n = a_0 x_n + a_1 x_{n-1} + a_2 x_{n-2} + \dots$$

$$- (b_1 y_{n-1} + b_2 y_{n-2} + \dots)$$

$$= \sum_{m=0}^{M} a_m x_{n-m} - \sum_{n=0}^{N} b_n y_{n-n}$$

$$y_k = y_{k-1} + (x_k - y_{k-1})\,\alpha \qquad 0 \le \alpha \le 1$$

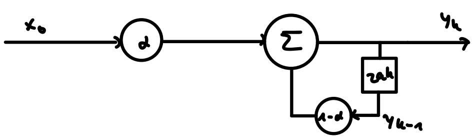


za $\alpha = \frac{1}{2}$ $\qquad y_k = \frac{1}{2} y_{k-1} + \frac{1}{2} x_k$

Filter z eksponentnim pozabljanjem

$$y_N = \alpha^N y_0 \quad \Rightarrow \quad N = -\frac{1}{\ln \alpha} \quad \text{časovna konstanta}$$

α	$1-\alpha$	$1-\alpha$	$\tau_s = N$
0,5	0,5	$1-\frac{1}{2}$	1,44 $\approx$ 2
0,125	0,875	$1-\frac{1}{8}$	7,48 $\approx$ 8
$\frac{1}{64}$	0,98...	$1-\frac{1}{64}$	63,5 $\approx$ 64
$\frac{1}{1024}$		$1-\frac{1}{1024}$	1023,5 $\approx$ 1024

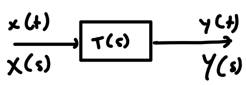


$$y_k = \sum_{n=0}^{N} a_n x_{k-n} - \sum_{m=0}^{N} b_m y_{k-m}$$

Katerikoli filter lahko zapišemo s to formulo, le določiti moramo koeficiente.

$$T(s) = \frac{Y(s)}{X(s)} \qquad X(s) = \mathcal{L}(x(t)) = \int_{-\infty}^{\infty} x(t) e^{-st}\, dt$$

$$s = \beta + i\omega$$



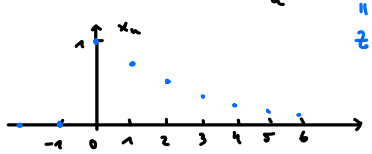
Podobno želimo narediti za diskretne signale

$$x_D = x_A \sum_k \delta(t - kT_v) = \sum_k x_A(t = kT_v)\,\delta(t - kT_v) = \sum_k x_k\, \delta(t - kT_v)$$

$$\mathcal{L}(x_D) = \int_{-\infty}^{\infty} \sum_k x_k\, \delta(t - kT_v)\, e^{-st} dt = \sum_k x_k \int_{-\infty}^{\infty} \delta(t - kT_v)\, e^{-st} dt =$$

$$= \sum_k x_k\, \underbrace{e^{-skT_v}}_{= z} = \sum_k x_k z^{-k} = X(z)$$

Zgled:



$$x_k = \begin{cases} 0 & k < 0 \\ c^k & k \ge 0 \end{cases}$$

$$X(z) = \sum_{k=0}^{\infty} c^k z^{-k} = \sum_{k=0}^{\infty} \left(\frac{c}{z}\right)^k = \frac{1}{1 - \frac{c}{z}}$$

x_k	$\delta(k)$	$\cos k\alpha$	$\sin k\alpha$	$u(k)$	$e^{-\alpha k}$	α^k
$X(z)$	1	$\frac{z(z - \cos\alpha)}{z^2 + 2z\cos\alpha + 1}$	$\frac{z \sin\alpha}{z^2 - 2z\cos\alpha + 1}$	$\frac{z}{z-1}$	$\frac{z}{z - e^{-\alpha}}$	$\frac{z}{z-\alpha}$
konvergenca	povsod	$\lvert z\rvert > 1$	$\lvert z\rvert > 1$	$\lvert z\rvert > 1$	$\lvert z\rvert > e^{-\alpha}$	$\lvert z\rvert > \alpha$
$X(s)$	1	$\frac{s}{s^2+\alpha^2}$	$\frac{\alpha}{s^2+\alpha^2}$	$\frac{1}{s}$	$\frac{1}{\alpha+s}$	

Lastnosti Laplaceove transformacije

$X(z) \ldots X(z)\, z^{-m}$ kasnitev osnovne vrste

$$X(z)\, z^{-m} = \sum_k x_k z^{-k} z^{-m} = x_0 z^{-(0+m)} + x_1 z^{-(1+m)} + \ldots$$

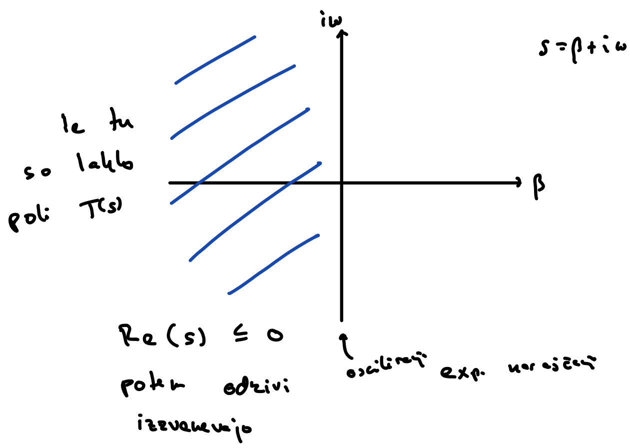
$$T(s) \to H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{n=0}^{N} \frac{a_n}{z^n}}{\sum_{m=0}^{M} \frac{b_m}{z^m}} \qquad z = e^{sT_v}$$

$$\sum_m b_m Y(z)\, z^{-m} = \sum_n a_n X(z)\, z^{-n}$$
$$\sum_m b_m \sum_k y_k z^{-k} z^{-m} = \sum_n a_n \sum_k x_k z^{-k} z^{-n}$$
$$\sum_k z^{-k} \sum_m b_m y_k z^{-m} = \sum_k z^{-k} \sum_n a_n x_k z^{-n}$$

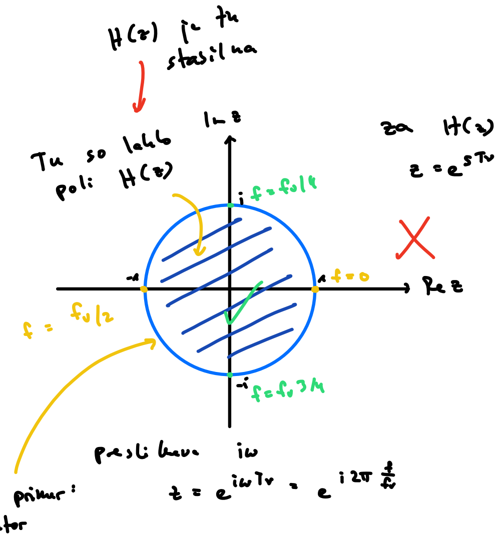
$$\Rightarrow \sum_{m=0}^{M} b_m \underbrace{y_k z^{-m}}_{y_{k-m}} = \sum_{n=0}^{N} a_n \underbrace{x_k z^{-n}}_{x_{k-n}}$$

$$b_0 y_{k-0} + b_1 y_{k-1} + \ldots = a_0 x_{k-0} + a_1 x_{k-1}$$

$$y_k = \frac{1}{b_0}\left(\sum_{n=0}^{N} a_n x_{k-n} - \sum_{m=1}^{M} b_m y_{k-m} \right)$$

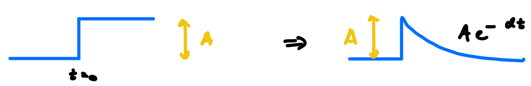


za $T(s)$



meja $\Rightarrow i\omega$

Primer: želim predelati



$Au(t)$ $\Downarrow$ iz tabele $\frac{Az}{z-1}$ $\qquad \Downarrow \frac{Az}{z-e^{-\alpha}}$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{Az}{z-e^{-\alpha}}}{\frac{Az}{z-1}} = \frac{z-1}{z-e^{-\alpha}} \quad /\cdot z^{-1}$$
$$= \frac{1-z^{-1}}{1-z^{-1}e^{-\alpha}}$$

Ničla $z = 1$

Pol $z = e^{-\alpha} < 1$ da bo stabilna

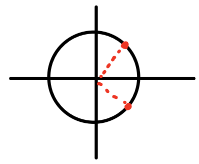
$$\Rightarrow Y(z) - Y(z) z^{-1} e^{-\alpha} = X(z) - X(z) z^{-1}$$

High-pass filter

odstrani konstantno komponento $\quad y_k = x_k - x_{k-1} + y_{k-1} e^{-\alpha} \quad$ Diferenčna enačba

Primer: oscilator

$X(z) = 1$ $\qquad$ $Y(z) = \dfrac{z(z-\cos\alpha)}{z^2 - 2z\cos\alpha + 1}$



$$H(z) = \frac{Y(z)}{X(z)} = \frac{z(z-\cos\alpha)}{z^2-2z\cos\alpha+1} \quad \Big| \cdot z^{-2}$$

$$= \frac{1-\cos\alpha\, z^{-1}}{1-2z^{-1}\cos\alpha + z^{-2}}$$

Poli: $z^2 - 2z\cos\alpha + 1 = 0$

$$z = \frac{2\cos\alpha \pm \sqrt{4\cos^2\alpha - 4}}{2} = \cos\alpha \pm i\sin\alpha$$

$\Rightarrow \quad y_k = x_k - x_{k-1}\cos\alpha + 2y_{k-1}\cos\alpha - y_{k-2}$ $\qquad$ Pola na krožnici $\Rightarrow$ oscilator.

x_k	y_k
0	
0	
0	
1	
0	
0	
0	
0	
0	

$T(s)\big|_{s\to i\omega} \to T(i\omega)$: $\quad |T(i\omega)|$ = ojačanje

$\varphi = \arctan\dfrac{\operatorname{Im} T(i\omega)}{\operatorname{Re} T(i\omega)}$ $\quad$ fazni zasuk

$H(z)\big|_{z\to e^{sT_v}}\Big|_{s\to i\omega} \to H(i\omega)$

$z \to e^{i\omega T_v}$

Zgled: $\quad y_k = y_{k-1} + (x_k - y_{k-1})\alpha$ $\qquad$ eksponentno pozabljanje $\qquad 0<\alpha<1$

$= y_{k-1}(1-\alpha) + x_k\alpha$

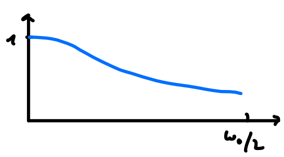
$y_k + y_{k-1}(\alpha-1) = \alpha x_k$

$Y(z) + Y(z)z^{-1}(\alpha-1) = \alpha X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\alpha}{1+(\alpha-1)z^{-1}}$$

$$H(i\omega) = H(z)\Big|_{z=e^{i\omega T_v}} = \frac{\alpha}{1+(\alpha-1)e^{-i\omega T_v}}$$

$$|H(i\omega)| = \frac{\alpha}{\sqrt{\ldots}}$$

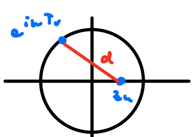


Vaje po inverznem $|H(i\omega)| \to H(i\omega)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_n a_n z^{-n}}{\sum_m b_m z^{-m}} = \frac{\prod_n (z-z_n)}{\prod_m (z-z_m)}$$

$$|H(z)| = \frac{\prod_n |z-z_n|}{\prod_m |z-z_m|}$$

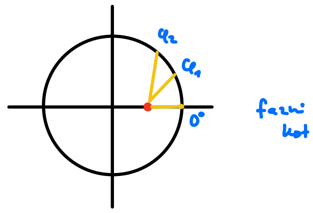
$$|H(i\omega)| = \frac{\prod |e^{i\omega T_v} - z_n|}{\prod_m |e^{i\omega T_v} - z_m|} \leftarrow d$$



$$= \frac{\text{produkt razdalj od točke do vseh ničel}}{\text{produkt razdalj od točke do vseh pola}}$$

$$H(z) = \frac{\prod_n |z - z_n| e^{i\varphi_n}}{\prod_m |z - z_m| e^{i\varphi_m}}$$

fazni kot $= \sum_n \varphi_n - \sum_m \varphi_m$



$$T(s) = \frac{1}{1+s}$$

za LPF pri ω_n $\quad s \to \frac{s}{\omega_n}$

za HPF pri ω_n $\quad s \to \frac{\omega_n}{s}$

za BPF pri ω_1, ω_2 $\quad s \to \frac{s^2 + \omega_1 \omega_2}{(\omega_2 - \omega_1) s}$

za BSF pri ω_1, ω_2 $\quad s \to \frac{(\omega_2 - \omega_1) s}{s^2 + \omega_1 \omega_2}$

$T(s) \to H(z)$ $\qquad$ analogno $\to$ digitalno

$$z = e^{sT_v} \Rightarrow s = \frac{\ln z}{T_v}$$

$H(z) = T\left(\frac{\ln z}{T_v}\right)$ $\qquad$ kaj je $\ln z$, čudno

$$z = e^{sT_v} = \underbrace{1 + sT_v + \frac{(sT_v)^2}{2!}} + \frac{(sT_v)^3}{3!} + \ldots$$

$$z = e^{\frac{sT_v}{2}} e^{\frac{sT_v}{2}} = \frac{e^{\frac{sT_v}{2}}}{e^{-\frac{sT_v}{2}}} = \frac{1 + \frac{sT_v}{2} + \frac{1}{2!}\left(\frac{sT_v}{2}\right)^2 + \frac{1}{3!}\left(\frac{sT_v}{2}\right)^3 + \ldots}{1 - \frac{sT_v}{2} + \frac{1}{2!}\left(\frac{sT_v}{2}\right)^2 - \frac{1}{3!}\left(\frac{sT_v}{2}\right)^3 + \ldots}$$

$$\left(1 + \frac{sT_v}{2}\right) : \left(1 - \frac{sT_v}{2}\right) = \underbrace{1 + sT_v + \frac{(sT_v)^2}{2}} + \frac{(sT_v)^3}{4} + \ldots$$

je dosti aproximacija

$$\begin{array}{l}
-1 + \frac{sT_v}{2} \\
\hline
sT_v \\
-sT_v + \frac{1}{2}(sT_v)^2 \\
\hline
\frac{1}{2}(sT_v)^2 \\
-\frac{1}{2}(sT_v)^2 + \frac{1}{4}(sT_v)^3 \\
\hline
\frac{1}{4}(sT_v)^3
\end{array}$$

zamenjava $\qquad z = \frac{1 + \frac{sT_v}{2}}{1 - \frac{sT_v}{2}} \to s = \frac{2}{T_v} \frac{z-1}{z+1}$

bilinearna transformacija

$$|z|_{s=i\omega} = \frac{\left|1 + \frac{i\omega T_v}{2}\right|}{\left|1 - \frac{i\omega T_v}{2}\right|} = 1$$

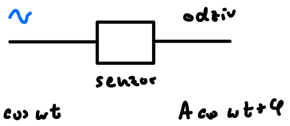
Karakteristika

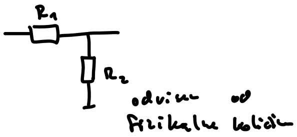
$$\underbrace{T(s)}\Big|_{s=i\omega_A} = H(z)\Big|_{z=e^{sT_v}=e^{i\omega_D T_v}} : \underbrace{T(s)}\Big|_{s=\frac{2}{T_v}\frac{1-z^{-1}}{1+z^{-1}}}\Big|_{z=e^{i\omega_D T_v}}$$

$$\Rightarrow \quad i\omega_A = \frac{2}{T_v} \frac{1 - e^{-i\omega_D T_v}}{1 + e^{-i\omega_D T_v}} = \frac{2i}{T_v} \tan \frac{\omega_D T_v}{2}$$

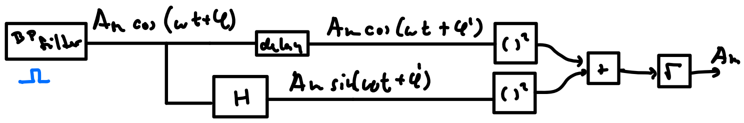
$$\boxed{\omega_A = \frac{2}{T_v} \tan \omega_D \frac{T_v}{2}}$$

Merjenje





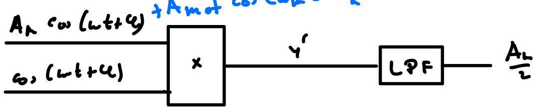
Signal, moduliran $A_n \cos \omega t + \varphi$



$$f_v \approx \frac{\omega}{2\pi}\cdot\varphi$$

Lock-in: na razpolago je informacija o:

- ω
- φ

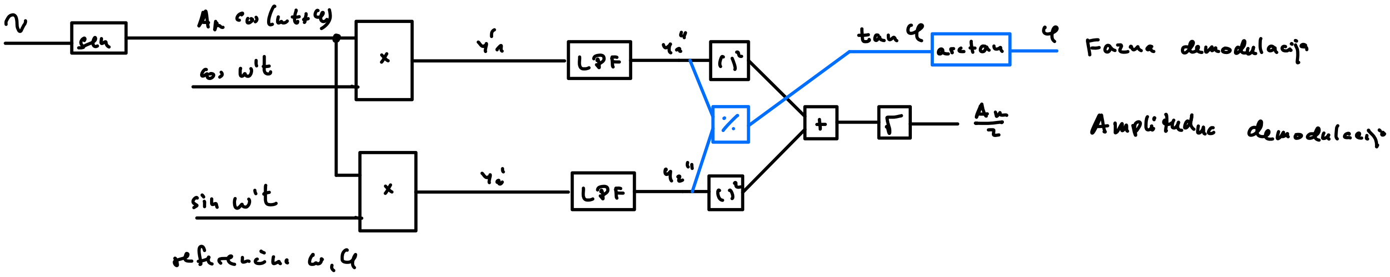


$$y' = A_n \cos(\omega t+\varphi) \cos(\omega t+\varphi')$$
$$= A_n \frac{1}{2}\left(\cos(2\omega t + \overset{\varphi+\varphi'}{2\varphi}) + \cos \overset{\varphi-\varphi'}{0}\right) =$$
$$= \frac{A_n}{2}\left(\cos(2\omega t + 2\varphi) + 1\right)$$

z motnjo

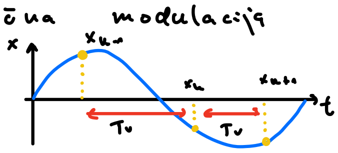
$$y' = \cos(\omega t+\varphi)\left(\cos(\omega t+\varphi)A_n + A_m \cos(\omega_m t+\varphi_m)\right)$$
$$= A_n \frac{1}{2}\left(\cos(2\omega t + 2\varphi) + \cos 0\right) + A_m \frac{1}{2}\left(\cos[(\omega_m+\omega)t + \varphi+\varphi_m] + \cos((\omega_m-\omega)t + \varphi_m - \varphi)\right)$$
$$= \frac{A_n}{2}\left(\cos(2\omega t + 2\varphi) + 1\right) \quad + \quad —\text{"}—$$

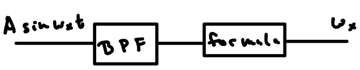
Kaj če ne poznamo frekvence vzbujanja ω in φ?



$$y_1' = A_n \cos(\omega t+\varphi)\cos\omega' t = \frac{A_n}{2}\left(\cos(\overset{\omega+\omega'}{\widetilde{2\omega}} t + \varphi) + \cos \overset{(\omega-\omega')t+\varphi}{\tilde{\varphi}}\right) \rightarrow y_1'' = LPF(y_1') = \frac{A_n}{2}\cos\tilde{\varphi}$$
$$y_2' = A_n \cos(\omega t+\varphi)\sin\omega' t = \frac{A_n}{2}\left(\sin \underbrace{2\omega}_{\omega+\omega'} t + \varphi - \sin \underbrace{\varphi}_{(\omega-\omega')t+\varphi}\right) \rightarrow y_2'' = LPF(y_2') = \frac{A_n}{2}\sin\tilde{\varphi}$$

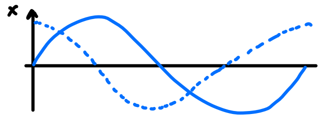
Frekvenčna modulacija



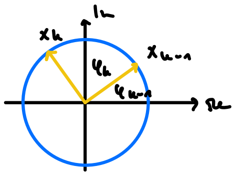


$$x = A \sin \omega_x t$$
$$x_n = A \sin(\omega_x t + \varphi)$$
$$x_{n-1} = A \sin(\omega_x (t - T_v) + \varphi) = A\left(\sin(\omega_x t+\varphi)\cos\omega_x T_v - \cos(\omega_x t+\varphi)\sin\omega_x T_v\right)$$
$$x_{n+1} = A \sin(\omega_x (t + T_v) + \varphi) = A\left(\sin(\omega_x t+\varphi)\cos\omega_x T_v + \cos(\omega_x t+\varphi)\sin\omega_x T_v\right)$$
$$x_{n-1} + x_{n+1} = 2A\sin(\omega_x t+\varphi)\cos\omega_x T_v = 2x_n\cos\omega_x T_v$$
$$\omega_x = \frac{1}{T_v}\arccos\frac{x_{n-1}+x_{n+1}}{2x_n}$$

Pogoj: sinusni signal, $x_n \neq 0$, vsi izmerki v isti periodi; signal ne sme imeti offseta.



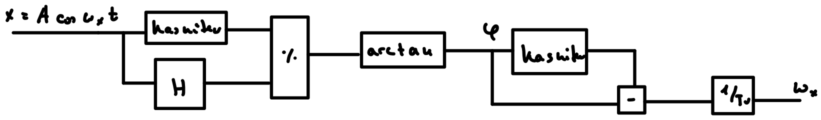
$$x = A\cos\omega_x t \rightarrow \mathrm{Re} \qquad \Rightarrow \qquad x = Ae^{i\omega_x t}$$
$$x' = A\sin\omega_x t \rightarrow \mathrm{Im}$$



$$x_{k-1} = A\cos(\omega_x(t-T_v)) + iA\sin(\omega_x(t-T_v)) \qquad \varphi_{k-1} = \arctan\frac{A\sin\omega_x(t-T_v)}{A\cos\omega_x(t-T_v)}$$
$$x_k = A\cos(\omega_x t) + iA\sin(\omega_x t) \qquad \varphi_k = \arctan\frac{A}{A}\frac{\sin\omega_x t}{\cos\omega_x t}$$

$$-\Delta\varphi = -(\varphi_k - \varphi_{k-1}) = \arctan\frac{\sin\omega_x(t-T_v)}{\cos\omega_x(t-T_v)} - \arctan\frac{\sin\omega_x t}{\cos\omega_x t} = \omega_x t - \omega_x T_v - \omega_x t$$

$$\omega_x = \frac{\varphi_k - \varphi_{k-1}}{T_v}$$



Pogoj: lepa harmonska oblika

Kaj če je signal pravokotne oblike?