(T) $M(A,Z) = Z m_p + (A-Z) m_n - W_v(A,Z)/c^2$

$m_p \approx m_n = 939 \frac{MeV}{c^2}$ $\qquad w_0 = 15{,}6\ MeV$ $\qquad w_2 = 0{,}7\ MeV$ $\qquad w_4 = 12\ MeV$

$m_n - m_p = 1{,}3\ MeV/c^2$ $\qquad w_1 = 17{,}2\ MeV$ $\qquad w_3 = 23{,}2\ MeV$

$$\boxed{W_v = w_0 A - w_1 A^{2/3} - w_2 \frac{Z^2}{A^{1/3}} - w_3 \frac{(A-2Z)^2}{A} - w_4 \frac{\delta^{ZN}}{A^{1/2}}}$$

Semiempirična masna formula

$\delta^{LL} = +1$

$\delta^{SL} = 0$

$\delta^{SS} = -1$

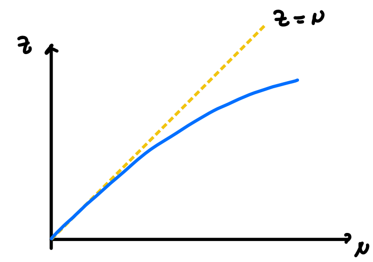
(1) Dolina stabilnosti

Za dani A določi izobar (isti A, različen Z) z največjo W_v (najbolj vezano jedro)

A .. liho $\qquad \frac{\partial W_v}{\partial Z} = -2w_2 \frac{Z}{A^{1/3}} - 2w_3 \frac{(A-2Z)}{A}(-2) = 0$

$$w_2 \frac{Z}{A^{1/3}} = w_3 \frac{A-2Z}{A}$$

$$Z = \frac{\frac{A}{2}}{1 + \underbrace{\frac{w_2}{4w_3}}_{0{,}08} A^{2/3}}$$



Velika jedra $\qquad A \gg 1$

$Z \approx \frac{1}{2\cdot 0{,}08} A^{1/3}$

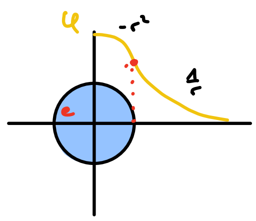
$W_v \approx w_0 A - w_3 A$ $\qquad w_0 = 15{,}7$

$\quad = A(w_0 - w_3) < 0$ $\qquad w_3 = 23{,}3$

Velika jedra niso stabilna

(2) $R = R_1 A^{1/3}$ $\qquad R_1 = 1{,}2\ fm$

Določi R_1 iz $w_2 = 0{,}71\ MeV$



$$\varphi(r>R) = \frac{e}{4\pi\varepsilon_0 r}$$

$$E(r<R) = \frac{e\, r^3}{R^3\, 4\pi\varepsilon_0 r^2} = \frac{e\, r}{4\pi R^3 \varepsilon_0}$$

$$\varphi(r<R) = -\frac{e r^2}{8\pi R^3 \varepsilon_0} + C$$

$$= \frac{e}{8\pi R\varepsilon_0}\left(3 - \frac{r^2}{R^2}\right)$$

elektrostatska energija $\qquad W_p = \frac{1}{2}\int \rho\varphi\, dV = \frac{1}{2}\int_0^R \frac{e}{8\pi R\varepsilon_0}\left(3 - \frac{r^2}{R^2}\right)\frac{e}{\frac{4}{3}\pi R^3} 4\pi r^2 dr$

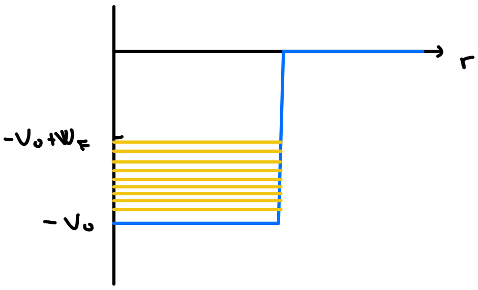
$$= \frac{1}{2}\frac{e}{8\pi R\varepsilon_0}\frac{e}{\frac{4}{3}\pi R^3} 4\pi \int_0^R \left(3 - \frac{r^2}{R^2}\right) r^2 dr$$

$$= \frac{3e^2}{16\pi R^4 \varepsilon_0}\left(\underbrace{R^3 - \frac{R^5}{5R^2}}_{\frac{4}{5}R^3}\right)$$

$$= \frac{3e^2}{20\pi R\varepsilon_0}$$

$w_2 \frac{Z^2}{A^{1/3}} = \frac{3e^2 Z^2}{20\pi\varepsilon_0 R_1 A^{1/3}}$ $\qquad R_1 = \frac{3e^2}{20\pi\varepsilon_0 w_2}$

③ Fermionski plin



celotna energija

$$E = \int_0^{W_F} \underbrace{\frac{dN}{dE}}_{g(E)} E\, dE$$

gostota stanj

$$N = \int \frac{dN}{dE}\, dE = \int g(E)\, dE$$

$$g(E) = V \underbrace{\frac{\sqrt{2}\, m_N^{3/2}}{\pi^2 \hbar^3}}_{C} \sqrt{E} = VC\sqrt{E}$$

$$N = VC \int_0^{W_F} \sqrt{E}\, dE = VC\, \frac{2}{3} E^{3/2} \Big|_0^{W_F} = \frac{2}{3} VC\, W_F^{3/2}$$

$$\Rightarrow \quad W_F = \left(\frac{3}{2}\frac{N}{C}\right)^{2/3}$$

$$E = VC \int_0^{W_F} E^{3/2}\, dE = VC\, \frac{2}{5} W_F^{5/2} = VC\, \frac{2}{5}\left(\frac{3}{2}\frac{N}{VC}\right)^{5/3} \propto \frac{N^{5/3}}{V^{2/3}}$$

proporcionalna konst. je b

$$V = \frac{4\pi R^3}{3} = \frac{4\pi R_1^3}{3} A$$

Taylorjev razvoj

$$E_{tot} = \frac{b}{V^{2/3}}\left(N^{5/3} + Z^{5/3}\right) = \frac{b'}{A^{2/3}}\left(N^{5/3} + Z^{5/3}\right)$$

$$= \frac{b'}{A^{2/3}}\left((A-Z)^{5/3} + Z^{5/3}\right)$$

$$\frac{dE_{tot}}{dZ} = \frac{b'}{A^{2/3}}\frac{5}{3}\left(-(A-Z)^{2/3} + Z^{2/3}\right) = 0$$

$$A - Z = Z \qquad A = 2Z$$

najnižja energija pri $A = 2Z$

$$\frac{d^2E_{tot}}{dZ^2}\Big|_{A=2Z} = \frac{5}{3}\frac{2}{3}\frac{b'}{A^{2/3}}\left((A-Z)^{-1/3} + Z^{-1/3}\right)$$

$$= \frac{10}{9}\frac{b'}{A^{2/3}}\left(\left(\frac{A}{2}\right)^{-1/3} + \left(\frac{A}{2}\right)^{-1/3}\right)$$

$$= \frac{10}{9}\, 2\sqrt[3]{2}\, b'\, \frac{1}{A} = \frac{\tilde{b}}{A}$$

Celotna Taylor

$$E_{tot} = E_{tot}\left(Z = \frac{A}{2}\right) + \overbrace{\frac{dE_{tot}}{dZ}}^{0}\left(Z - \frac{A}{2}\right) + \frac{d^2E_{tot}}{dZ^2}\left(Z - \frac{A}{2}\right)^2$$

$$= c_1 A + 0 + c_2\frac{(A-2Z)^2}{A}$$

④ Nevtronska zvezda

Minimalna masa stabilne zvezde, $A \gg 1$

Za homogeno kroglo velja $W_g = -\frac{3Gm^2}{5R}$

$$W = w_0 A - w_1 A + \underbrace{\frac{3Gm^2}{5R}}_{\text{gravitacijski člen}} =$$

$m \simeq A m_n \qquad R = R_1 A^{1/3}$

$$= \underbrace{(w_0 - w_1)}_{\sim -\text{MeV}} A + \underbrace{\frac{3Gm_n^2}{5R_1}}_{w_5 = 6\cdot 10^{-37}\,\text{MeV}} A^{5/3}$$

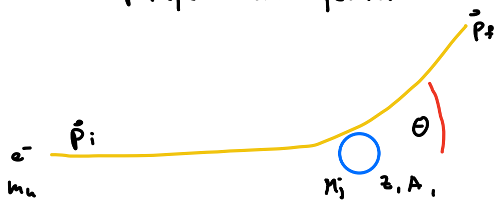
Iščemo $W = 0 \quad \Rightarrow \quad (w_1 - w_0)A = w_5 A^{5/3}$

$$A = \left(\frac{w_1 - w_0}{w_5}\right)^{3/2} = 5\cdot 10^{55}$$

$\Rightarrow \quad m = 0{,}04 \cdot m_{sonce}$

$\Rightarrow \quad R = 4\,\text{km}$

T EM sipanje na jedra



$\rho(\vec r) = Z e_0 \bar\rho(\vec r)$ $\qquad \int \bar\rho(\vec r)\, dV = 1$

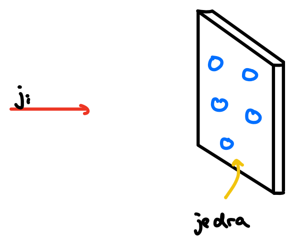
$M_j \gg m_e$

$|\vec p_i| = |\vec p_f| = p$ elastično sipanje

$$\boxed{\begin{aligned}\vec q &= \vec p_f - \vec p_i \\ q^2 &= \frac{4p^2}{\hbar^2}\sin^2\frac{\theta}{2}\end{aligned}}$$

$\hbar c = 197$ eV fm

Sipalni presek σ_{fi} verjetnost za sipanje na enoto površine



$j_i = \frac{\dot N_i}{S}$ $\qquad \dot N_f = \underbrace{\frac{N_t \sigma_{fi}}{S}}_{P_{fi} \text{ za tanko tarčo}} \dot N_i = N_t \sigma_{fi} j_i$

število delcev tarče

Fermijevo zlato pravilo (za izračun σ_{fi})

w_{fi} ... verjetnost na časovno enoto

$$\frac{1}{\tau} = \boxed{w_{fi} = \frac{2\pi}{\hbar} |V_{fi}|^2 \rho(E_f)} \qquad \sigma_{fi} = \frac{w_{fi}}{j_i}$$

$\rho(E_f) = \frac{dN}{dE_f}$ gostota končnih stanj

Gostota stanj v faznem prostoru $\quad dN = 2V\frac{d^3p}{h^3} = \frac{2V}{h^3} d\Omega\, p^2 dp = \dots$

$E = \frac{p^2}{2m}$ nerelativistično $\qquad E^2 = (mc^2)^2 + p^2c^2$ relativistično

$2E\,dE = 2c^2 p\,dp$

$$\dots = \frac{2V}{h^3} d\Omega\, p \frac{1}{c^2} E\,dE$$

$$\Rightarrow \frac{dN}{dE} = \frac{2V}{(hc)^3} pE\, d\Omega$$

$$d\sigma_{fi} = \frac{1}{j_i}\frac{2\pi}{\hbar}|V_{fi}|^2 \frac{2V}{(hc)^3} pE\, d\Omega$$

$$j_i = \frac{1}{V} v_i = \frac{1}{V}\frac{c^2 \gamma m v_i}{\gamma m c^2} = \frac{c^2}{V}\frac{p}{E}$$

iz predavanj $\quad V_{fi} = \frac{-Z e_0^2}{V\varepsilon_0 q^2} F(\vec q) \qquad F(\vec q) = \int d^3x\, \bar\rho(\vec x)\, e^{-i\vec q\cdot\vec x}$

oblikovna funkcija

$$\frac{d\sigma_{fi}}{d\Omega} = \frac{VE}{c^2 p}\frac{2\pi}{\hbar}\frac{Z^2 e_0^4}{V^2\varepsilon_0^2 q^4} F^2(\vec q) \frac{2V}{(hc)^3} pE = \frac{4\alpha^2 Z^2 (m_e c^2)^2}{(\hbar c)^2 q^4} |F(\vec q)|^2$$

$$\boxed{\frac{d\sigma_{fi}}{d\Omega} = \frac{\alpha^2 Z^2 (m_e c^2)^2 (\hbar c)^2}{4(pc)^4}\frac{|F(\vec q)|^2}{\sin^4\theta/2} \qquad F(\vec q) = \int d^3r\, \bar\rho(\vec r)\, e^{-i\vec q\cdot\vec r}}$$

$\hbar\vec q = \vec p_f - \vec p_i$
$\hbar^2 q^2 = 4p^2 \sin^2\frac{\theta}{2}$

gostota naboja v jedru

Za kotno simetrična jedra

$$F(\vec{q}) = \int 2\pi \; r^2 dr \; d\cos\theta \; \rho(r) \; e^{-iqr\cos\theta} =$$

$$= \int \frac{2\pi}{-iqr} \; r^2 dr \; \rho(r) \left(e^{-iqr} - e^{iqr}\right) =$$

$$= \int \rho(r) \; \frac{2\pi i}{q} \; r\,dr \; \sin(qr)\,(-2i) =$$

$$\boxed{F(\vec{q}) = \int_0^\infty \frac{4\pi}{q} \sin(qr) \; \rho(r) \; r\,dr}$$

5

$p_e = 10\,\frac{MeV}{c}$

$\sigma(\theta > \frac{\pi}{2}) = 2{,}3 \cdot 10^{-1}\,fm^2$

Sipanje nazaj na ${}^{90}_{38}Sr$

Iščemo $\sqrt{\langle r^2 \rangle}$ radij

Upoštevaj razvoj za $q\sqrt{\langle r^2\rangle} \ll 1$

$$F(q) = \int_0^\infty \frac{4\pi}{q}\left(qr - \frac{q^3r^3}{3!} + \dots\right) \rho(r)\, r\,dr =$$

$$= \underbrace{\int_0^\infty 4\pi r^2 \rho(r)\,dr}_{\rho dV = 1} - \int 4\pi r^2 dr \; \frac{q^2 r^2}{6} \; \rho(r)$$

$$= 1 - \frac{q^2}{6}\int r^2 \rho(r)\,dV = 1 - \frac{q^2}{6}\langle r^2\rangle$$

$$\frac{d\sigma}{d\Omega} = \frac{4\alpha^2 z^2 E^2}{(\hbar c)^2 q^4}\left(1 - \frac{q^2}{6}\langle r^2\rangle\right)^2$$

$$\approx \frac{4\alpha^2 z^2 E^2}{(\hbar c)^2 q^4}\left(1 - 2\frac{q^2}{6}\langle r^2\rangle + O(q^4)\right)$$

$$\sigma = \int d\Omega \underbrace{\frac{4\alpha^2 z^2 E^2}{(\hbar c)^2 q^4}}_{A/q^4}\left(1 - \frac{q^2}{3}\langle r^2\rangle\right) = \int \frac{A}{q^4}\left(1 - \frac{q^2}{3}\langle r^2\rangle\right) d\Omega =$$

$$= 2\pi A \int_{\frac{4p^2}{\hbar^2}}^{\frac{2p^2}{\hbar^2}} \left(\frac{1}{q^4} - \frac{\langle r^2\rangle}{3q^2}\right) dq^2 \left(-\frac{\hbar^2}{2p^2}\right)$$

$$= -\frac{\pi\hbar^2 A}{p^2}\left(-2\frac{1}{q^2} - \frac{\langle r^2\rangle}{3}\ln q^2\right)\Big|$$

$$= \frac{\pi\hbar^2 A}{p^2}\left(\frac{2\hbar^2}{2p^2} - \frac{2\hbar^2}{4p^2} + \frac{\langle r^2\rangle}{3}\ln\frac{1}{2}\right)$$

$$= \frac{\pi}{2}\frac{\hbar^4 A}{p^4} - \frac{\pi\hbar^2 A}{3p^2}\ln 2\,\langle r^2\rangle$$

$$\Rightarrow \quad \langle r^2\rangle = \frac{3p^2}{\pi\hbar^2 A}\left(\frac{\pi}{2}\frac{\hbar^4 A}{p^4} - \sigma\right)$$

$Z = 38 \qquad \alpha = \frac{1}{137} \qquad E = \sqrt{\underbrace{(m_e c^2)^2}_{\sim 0{,}5^2} + \underbrace{c^2 p^2}_{\sim 10^2}} \approx pc = 10\,MeV$

$\dots \sqrt{\langle r^2\rangle} = 5{,}1\,fm$

$q^2 = \frac{4p^2}{\hbar^2}\sin^2\frac{\theta}{2}$

$q^2 = \frac{2p^2}{\hbar^2}(1 - \cos\theta)$

$$\boxed{\begin{aligned} dq^2 &= -\frac{2p^2}{\hbar^2}\,d\cos\theta \\ d\cos\theta &= -\frac{\hbar^2}{2p^2}\,dq^2 \end{aligned}}$$

konstanta fine strukture

$\alpha = \frac{1}{137} = \frac{e^2}{4\pi\varepsilon_0\hbar c}$

$\hbar c = 200\,eVfm$

6 Enakomerno nabito jedro

Oceni radij jedra ^{58}Ni, $T_e = 450\,\mathrm{MeV}$. Prvi minimum σ pri $q = 1{,}1\,\mathrm{fm}^{-1}$

$$\frac{d\sigma}{d\Omega} = \frac{4\alpha^2 Z^2 E^2}{(\hbar c)^2 q^4} |F(q)|^2$$

$$F(q) = \frac{4\pi}{q}\int_0^\infty r\,dr \sin qr\ \bar{\rho}(r) \qquad \bar{\rho}(r) = \frac{1}{\frac{4}{3}\pi R^3}$$

$$= \frac{4\pi}{q^2}\frac{1}{\frac{4}{3}\pi R^3}\int_0^{qR} qr \sin qr \, drq \qquad u = rq$$

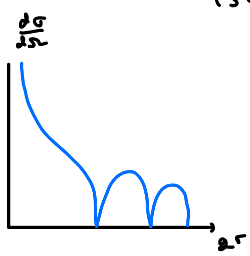
$$= \frac{3}{(qR)^3}\int_0^{qR} u \sin u \, du = \qquad u = u \quad \sin u\, du = dv$$
$$du = du \quad -\cos u = v$$

$$= \frac{3}{(qR)^3}\left(-u\cos u + \int \cos u \, du\right) =$$

$$= \frac{3}{(qR)^3}\left(-u\cos u + \sin u\right)\Big|_0^{qR} =$$

$$= \frac{3}{(qR)^3}\left(-qR\cos qR + \sin qR\right)$$

Iščemo minimum $\frac{d\sigma}{d\Omega} \sim |F|^2 \Leftrightarrow F = 0$

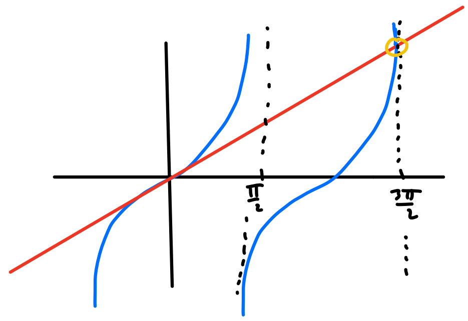


$$-qR\cos qR + \sin qR = 0$$

$$\tan qR = qR$$

$$qR \simeq \frac{3\pi}{2} = 4{,}7$$

$$\Rightarrow R = 4{,}1\,\mathrm{fm}$$



7 Oceni sproščeno energijo pri razpadu $^{A}_{Z}X \to {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha$

Sproščena energija $Q = m_x c^2 - m_y c^2 - m_\alpha c^2$

$$m_{He}^{atom} = 3728{,}4\ \frac{\mathrm{MeV}}{c^2}$$

$$M(A,Z) = Z m_p + (A-Z) m_n - W_b(A,Z)/c^2$$

$$Q = -W_b(A,Z) + W_b(A-4,Z-2) + \underbrace{W_b(4,2)}$$

$$W_b^\alpha = W_b(4,2) = -M_{He}c^2 + 2m_p c^2 + 2m_n c^2 + 2m_e c^2$$
$$= 28{,}4\,\mathrm{MeV}$$

$$Q = -\frac{\partial W_b(A,Z)}{\partial A}\,4 - \frac{\partial W_b(A,Z)}{\partial Z}\,2 + W_b^\alpha$$

$$W_b = w_0 A - w_1 A^{2/3} - w_2\frac{Z^2}{A^{1/3}} - w_3\frac{(A-2Z)^2}{A} + \dots$$

$$\frac{\partial W_b}{\partial A} = w_0 - \frac{2}{3}A^{-1/3} + w_2\frac{1}{3}\frac{Z^2}{A^{4/3}} - w_3\frac{2(A-2Z)A - (A-2Z)^2}{A^2}$$

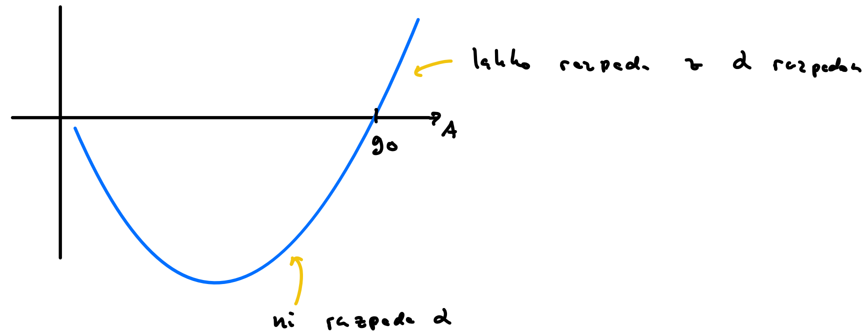
$$\frac{\partial W_b}{\partial Z} = -2w_2\frac{Z}{A^{1/3}} + w_3\frac{4(A-2Z)}{A}$$

$$-4\frac{\partial W_b}{\partial A} - 2\frac{\partial W_b}{\partial Z} = -4w_0 + \frac{8}{3}w_1 A^{-1/3} + w_2\frac{4Z}{A^{1/3}}\left(1 - \frac{Z}{3A}\right) - 4w_3\frac{(A-2Z)^2}{A^2} = \dots$$

$$z = \frac{A}{2}$$

$$\ldots = -4w_0 + \frac{8}{7} w_1 A^{-1/3} + w_2 \frac{5}{12} A^{2/3}$$

$Q = \ldots$



8 Po^{210}_{84} razpade prek α razpada

$$^{210}_{84}Po \rightarrow\ ^{206}_{82}Pb + \alpha$$

- $T_\alpha = ?$
- Relativistično / nerelativistično
- $T_{Pb} = ?$

$m^{atom}_{Po} = 209{,}983\ u$ $\qquad u = 931{,}494\ \frac{MeV}{c^2}$

$m^{jedro}_{Po} = 195554{,}862\ MeV/c^2$

$m^{jedro}_{Pb} = 191822{,}\ldots\ MeV/c^2$

$m^j_{Po} - m^j_{Pb} = 3733\ MeV/c^2$

$m_\alpha = 3727\ MeV/c^2$

$Q = 3733 - 3727 = 6\ MeV$

$$T_\alpha + T_{Pb} = Q \qquad -\vec{p}_\alpha = \vec{p}_{Pb} \Rightarrow |p_\alpha| = |p_{Pb}|$$

$$T_\alpha = \frac{p_\alpha^2}{2m} \qquad \text{nerelativistično}$$

$$\frac{p_\alpha^2}{2m_\alpha} + \frac{p_{Pb}^2}{2m_{Pb}} = Q$$

$$p_\alpha = \sqrt{\frac{2Q}{\frac{1}{m_\alpha} + \frac{1}{m_{Pb}}}} \approx \sqrt{2m_\alpha Q}$$

$$T_\alpha = \frac{2m_\alpha Q}{2m_\alpha} = Q$$

Nerelativistično ✓ ker $Q \ll m_\alpha c^2$

V splošnem $M \rightarrow m_1\ m_2$

$$p^\mu = p_1^\mu + p_2^\mu$$

$p^\mu = (Mc, \vec{0})$ v težiščnem sistemu

$p_1^\mu = (E/c, \vec{p}_1)$

$p \cdot p = M^2c^2$

$p_1 \cdot p_1 = m_1^2 c^2$

$$p - p_1 = p_2 \quad /^2$$

$$p^2 + p_1^2 - 2p\cdot p_1 = p_2^2$$

$$(M^2 + m_1^2)c^2 - 2ME_1 = m_2^2 c^2$$

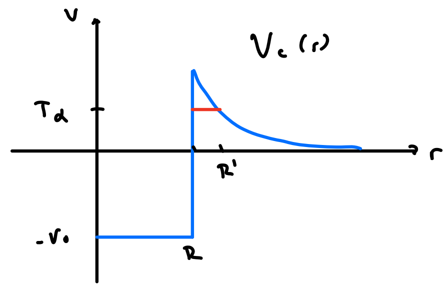
$$E_1 = \frac{(-m_2^2 + M^2 + m_1^2)c^2}{2M} \qquad \text{celotna energija}$$

$$E_2 = \frac{(M^2 + m_2^2 - m_1^2)c^2}{2M}$$

$$|\vec{p}_1| = \frac{1}{c}\sqrt{E^2 - m_1^2 c^4}$$

Mehanizem α razpada

Potencial za α



Iščemo R' $\qquad T_\alpha = V_c(R') \approx Q$ $\qquad V_c = \frac{2e_0\,(Z-2)e_0}{4\pi\varepsilon_0\, r} = \underbrace{\frac{e_0^2}{4\pi\varepsilon_0 \hbar c}}_{\alpha=\frac{1}{137}} \overbrace{\hbar c}^{200\ \text{MeV fm}} \frac{2(Z-2)}{r}$

$$R' = \alpha\hbar c\,\frac{2(Z-2)}{Q}$$

$$= \frac{1}{137}\,200\ \text{MeV fm}\,\frac{2\cdot 82}{6\ \text{MeV}} = 40\ \text{fm}$$

$$\vee$$

$$R = R_1 A^{1/3} = 1{,}2\ \text{fm}\cdot 206^{1/3} \doteq 10\ \text{fm}$$

9 α razpad $\quad {}^{243}_{93}Am \quad Q' = 5{,}275\ \text{MeV} \quad t'_{1/2} = ?$

$\qquad\qquad\quad {}^{241}_{93}Am \quad Q = 5{,}485\ \text{MeV} \quad t_{1/2} = 432$ let

transmisivnost $\quad T = e^{-2G}$

$$G = \frac{1}{\hbar}\int dx \sqrt{2m_\alpha (V(x) - T_\alpha)}$$

$$= \alpha Z_\alpha (Z - Z_\alpha)\sqrt{\frac{2m_\alpha c^2}{T_\alpha}}\left[\arccos\sqrt{\frac{R}{R'}} - \sqrt{\frac{R}{R'}\left(1-\frac{R}{R'}\right)}\right]$$

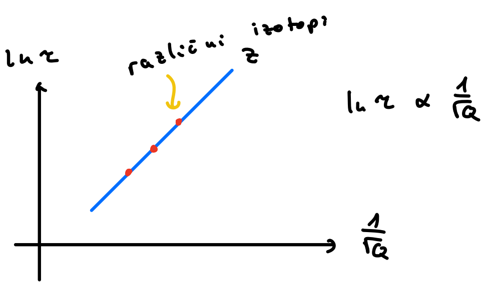
$R \ll R'$

$$\approx \frac{\alpha\pi}{\sqrt{2}}\sqrt{m_\alpha c^2}\, Z_\alpha \left(\frac{Z - Z_\alpha}{\sqrt{T_\alpha}} - \frac{4}{\pi}\sqrt{\frac{(Z-Z_\alpha)R}{Z_\alpha \alpha\hbar c}}\right)$$

$$\frac{1}{\tau} = D\,\frac{v}{2R}\,e^{-2G}$$

Geiser – Nuttalovo pravilo

$$\boxed{-\ln\tau = \ln D + \ln\frac{v}{2R} - 2G}$$



$\ln\tau \propto \frac{1}{\sqrt{Q}}$

$$-\ln\tau = \ln D + \ln\frac{v}{2R} - 2G$$
$$-\ln\tau' = \ln D' + \ln\frac{v'}{2R'} - 2G'$$

$$\ln\frac{\tau'}{\tau} = \ln\underbrace{\frac{D}{D'}}_{\approx 1} + \ln\underbrace{\frac{vR'}{v'R}}_{\approx 1} - 2(G - G')$$

$$\ln\frac{\tau'}{\tau} = -2(G - G')$$

$$= -2\,\frac{\alpha\pi}{\sqrt{2}}\sqrt{m_\alpha c^2}\, Z_\alpha\left[\overbrace{\frac{Z - Z_\alpha}{\sqrt{T_\alpha}} - \frac{Z' - Z_\alpha}{\sqrt{T'_\alpha}}}^{\gg} - \frac{4}{\pi\sqrt{Z_\alpha \alpha\hbar c}}\left(\overbrace{\sqrt{(Z-Z_\alpha)R} - \sqrt{(Z'-Z_\alpha)R'}}\right)\right]$$

R': radij ${}^{243}Am$

$Z \approx Z'$

$$= -\sqrt{2}\alpha\pi\sqrt{m_\alpha c^2}\, Z_\alpha (Z - Z_\alpha)\left[\underbrace{\frac{1}{\sqrt{T_\alpha}} - \frac{1}{\sqrt{T'_\alpha}}}_{\frac{Q'-Q}{2Q^{3/2}}}\right]$$

$$\frac{t'_{1/2}}{t_{1/2}} = \frac{\tau'}{\tau} = \exp\left[\alpha\sqrt{2m_\alpha c^2}\,\pi Z_\alpha (Z - Z_\alpha)\frac{Q' - Q}{2Q^{3/2}}\right] \quad\Rightarrow\quad t'_{1/2} = 8560\ \text{let}$$

(10) Površinska napetost ne zadržuje več Coulombskega potenciala. Jedro se deformira.

$$W_b = -w_1 A^{2/3} - w_2 \frac{Z^2}{A^{1/3}}$$

$a = R(1+\varepsilon)$

$$\frac{4\pi R^3}{3} = \frac{4\pi b^2 a}{3} \quad \Rightarrow \quad b = \frac{R}{\sqrt{1+\varepsilon}}$$

Gledamo spremembo površine jedra ⇒ sprememba W_b

Površina vrtenine $S = \int_{-a}^{a} 2\pi r(z)\sqrt{dz^2+dr^2} = \ldots = 4\pi b a \frac{1}{2}\left(\sqrt{1-e^2} + \frac{1}{e}\arcsin e\right)$ $\quad e = \sqrt{1-b^2/a^2}$

$\overset{\text{razvoj po } \varepsilon}{=} \ldots = 4\pi R^2\left(1+\frac{2}{5}\varepsilon^2\right)$

Ploskovna pot. en. elipse $\qquad g = Ze_0$

$$W_p = \frac{3g^2}{\underbrace{4\pi\varepsilon_0}_{A}} \int_0^\infty \frac{dg}{(b^2+g)\sqrt{a^2+g}} = \ldots = \frac{3g^2}{20\pi\varepsilon_0}\left(1-\frac{\varepsilon^2}{5}\right) = \frac{3Z^2\alpha\hbar c}{5}\left(1-\frac{\varepsilon^2}{5}\right)$$

$$\Rightarrow \quad W_b = -w_1\left(1+\frac{2}{5}\varepsilon^2\right)A^{2/3} - w_2\frac{Z^2}{A^{1/3}}\left(1-\frac{\varepsilon^2}{5}\right)$$

$\Delta W_b = -w_1 \frac{2}{5}\varepsilon^2 A^{2/3} + w_2 \frac{Z^2}{A^{1/3}}\frac{\varepsilon^2}{5}$ $\quad$ če $\Delta W_b > 0$ je jedro nestabilno

$\Rightarrow \quad \frac{Z^2}{2A}\frac{w_2}{w_1} > 1 \qquad \frac{Z^2}{A} > \frac{2w_1}{w_2} \qquad$ (Za $2Z = A$: $\quad A \geq 197$)

(T) Razpad $\beta^\pm$

- Razpad α, γ, fisije: N, Z, N_e se ohranja
- Pri β:

$n \to p\, e^-\, \bar\nu_e$ } šibka interakcija

B - barionsko število
L - leptonsko število

	p	n	${}^A_Z X$	e^-	e^+	ν_e	$\bar\nu_e$	$\bar p$
B	1	1	A	0	0	0	0	-1
L	0	0	0	1	-1	1	-1	0

β^-: ${}^A_Z X \to {}^A_{Z+1} X\ e^-\ \bar\nu_e \qquad \beta^+$: ${}^A_Z X \to {}^A_{Z-1} X\ e^+\ \nu_e$

Fermijev razpad

$$\frac{1}{\tau} = W_{fi} = \frac{2\pi}{\hbar}|V_{fi}|^2 \rho(E) \quad ; \quad V_{fi} = G_F \int \psi_f^* \psi_i \underbrace{\varphi_e^* \varphi_\nu^*}_{\propto e^{i\vec q\cdot\vec r/\hbar}} dV = \ldots \qquad \vec q = \vec p_e + \vec p_\nu$$

$$\varphi_e = \frac{1}{\sqrt V} e^{i\frac{\vec p_e\cdot\vec r}{\hbar}} \qquad \varphi_\nu = \frac{1}{\sqrt V} e^{i\frac{\vec p_\nu\cdot\vec r}{\hbar}}$$

$$\ldots = G_F\int \psi_f^*\psi_i \Big(\underset{l=0}{1} + \underset{l=1}{\frac{i\vec q\cdot\vec r}{\hbar}} + \underset{l=2}{\frac{(i\vec q\cdot\vec r)^2}{2\hbar^2}} + \underset{\ldots}{\ldots} \Big)$$

l ... tirna vrtilna količina teh dveh delcev, e in ν.

Fermi: $\vec s_e + \vec s_\nu = 0$ spinski singlet

$\vec j_2 = \vec j' + \vec l \qquad |j-j'| \leq l \leq j+j'$

Gamov - Tellerjev razpad

$$|\vec{S}_e + \vec{S}_\nu| = 1$$
$$|J - J'| - 1 \leq \ell \leq J + J' + 1$$

(11) $^{22}Na\ (3^+)$

$\hookrightarrow (j^P) = ($vrtilna količina parnost$)$

$^{22}Na\ (3^+) \rightarrow {}^{22}Ne\ (2^+)\ e^+\ \nu_e$ verjetnost 0,99944

$\rightarrow {}^{22}Ne\ (0^+)\ e^+\ \nu_e$ verjetnost 0,00056

Prehod $3^+ \rightarrow 2^+$

- Fermijev razpad

$1 \leq \ell \leq 5$ $\quad P = P'(-1)^\ell$ $\Rightarrow \ell = 2,4$ $\quad$ F2, F4 $\quad$ nižji dominanten

(začetna parnost, končna parnost)

- GT

$0 \leq \ell \leq 6$ $\quad$ GT 0 $\quad$ dominanten proces

Prehod $3^+ \rightarrow 0^+$

- Fermi

$3 \leq \ell \leq 3$ $\quad$ ni razpada

- GT

$2 \leq \ell \leq 4$ $\quad \ell = 2,4$ $\quad$ GT2 $\quad$ dominanten je nižji

$$\frac{\tau(3^+ \rightarrow 2^+)}{\tau(3^+ \rightarrow 0^+)} = \frac{|V_{fi}(3^+ \rightarrow 0^+)|^2}{|V_{fi}(3^+ \rightarrow 2^+)|^2} = \frac{|G_F \int \psi_f^* \psi_i \frac{1}{2}\left(\frac{i\vec{q}\cdot\vec{r}}{\hbar}\right)^2 dV|^2}{|G_F \int \psi_f^* \psi_i\ 1\, dV|^2}$$

$$\approx \left(\frac{1}{2}\left(\frac{qR}{\hbar}\right)^2\right)^2 \sim 10^{-6}$$

$$q \approx 1{,}5\,\text{MeV}$$

(12) Oceni življenjski čas, $\langle E_e \rangle$, E_{max} $\qquad X \rightarrow Y\ e^-\ \bar{\nu}_e$

$$\frac{d\Gamma}{dE}\Big|_{E_{max}} = max$$

aproksimacija $E_0 = E_e + E_\nu \gg m_e c^2$

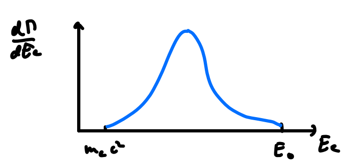
$= m_e c^2 + Q$ $\qquad m_\nu \approx 0$

sproščena energija

Fermijevo zlato pravilo

$$d\Gamma = \frac{2\pi}{\hbar} |V_{fi}|^2 \frac{d\,\varphi_f(E_f)}{dE_e} dE_e$$

$E_e + E_\nu$



$\varepsilon =$ 1 $\quad \varepsilon_0$ $\qquad \varepsilon = \frac{E_e}{m_e c^2}$

Število končnih stanj $\qquad dN = 2V \frac{d^3 p_e}{h^3}\, 2V \frac{d^3 p_\nu}{h^3}$

$d^3p_e = 4\pi p_e^2\, dp_e$ $\qquad$ $d^3p_\nu = 4\pi p_\nu^2\, dp_\nu$

$$dN = 4V^2 \frac{(4\pi)^2}{h^6}\, p_e^2 dp_e\, p_\nu^2 dp_\nu$$

$p_\nu = \frac{E_\nu}{c}$ ultrarelativistični

$V = 1$ naj bo

$$= 4\frac{(4\pi)^2}{h^6}\, \underline{\frac{p_e E_e dE_e}{c^2}}\, \frac{E_\nu^2}{c^3}\, dE_\nu$$

$E_e^2 = p_e^2c^2 + (m_ec^2)^2$

$2E_e dE_e = c^2\, 2p_e dp_e$

$$d\rho_f = \frac{dN}{dE}$$

$E = E_e + E_\nu$

zanima nas pri določenem E_e zato $dE = dE_\nu$

$$d\rho_f = \frac{dN}{dE_\nu} = \frac{4(4\pi)^2}{h^6c^5}\, p_e E_e dE_e E_\nu^2$$

Gostota stanj elektronov z energijo E_e na intervalu dE_e

$$\frac{d\Gamma}{dE_e} = \frac{2\pi}{\hbar}|V_{fi}|^2\, \frac{4(4\pi)^2}{h^6c^5}\, p_e E_e E_\nu^2$$

Nova spremenljivka $\varepsilon = \frac{E_e}{m_ec^2}$ $\qquad$ $c p_e = \sqrt{E_e^2 - (m_ec^2)^2}$

$$\frac{d\Gamma}{d\varepsilon} = \underbrace{4\frac{2\pi}{\hbar}|V_{fi}|^2 \frac{(4\pi)^2}{(hc)^6}(m_ec^2)^5}_{\Gamma_0}\; \underbrace{\varepsilon(\varepsilon_0-\varepsilon)^2\sqrt{\varepsilon^2-1}}_{f(\varepsilon)}$$

$$\Gamma = \int_1^{\varepsilon_0 \gg 1} \frac{d\Gamma}{d\varepsilon}\, d\varepsilon = \Gamma_0 \underbrace{\int_1^{\varepsilon_0} \varepsilon(\varepsilon_0-\varepsilon)^2\sqrt{\varepsilon^2-1}\, d\varepsilon}_{I(\varepsilon_0)}$$

$u = \frac{\varepsilon}{\varepsilon_0}$

$$I(\varepsilon_0) = \int_1^{\varepsilon_0} \varepsilon(\varepsilon_0-\varepsilon)^2\sqrt{\varepsilon^2-1}\, d\varepsilon =$$

$$= \varepsilon_0^5 \int_{1/\varepsilon_0}^1 u(1-u)^2\sqrt{u^2 - 1/\varepsilon_0^2}\, du$$

ε_0 velik, $\frac{1}{\varepsilon_0} \sim 0$

$$I^{(0)}(\varepsilon_0) \approx \varepsilon_0^5 \int_0^1 u^2(1-u)^2\, du = \frac{\varepsilon_0^5}{30}$$

$z = \frac{1}{\varepsilon_0}$ $\qquad$ $h(z) = \int_z^1 du\, u(1-u)^2\sqrt{u^2-z^2}$

$$\frac{dh}{dz} = -u(1-u)^2\sqrt{u^2-z^2}\Big|_{u=z} + \int_z^1 du\, \frac{-2u(1-u)^2 z}{2\sqrt{u^2-z^2}}$$

$$\frac{dh}{dz}\Big|_{z=0} = 0$$

$$\frac{d^2h}{dz^2}\Big|_{z=0} = \frac{d}{dz}\int_z^1 \frac{-u(1-u)^2 z}{\sqrt{u^2-z^2}} = \underbrace{\frac{u(1-u)^2 z}{\sqrt{u^2-z^2}}}_{0}\Big|_{\substack{u=z\\ z\to 0}} - \int_z^1 du\, \frac{u(1-u)^2\left(\sqrt{u^2-z^2} - \frac{1}{2}\frac{-2z^2}{\sqrt{u^2-z^2}}\right)}{u^2-z^2}$$

$$= -\int_0^1 \frac{u^2(1-u)^2}{u^2}\, du = -\frac{1}{3}$$

$\Rightarrow$ $I^{(1)}(\varepsilon_0) = 0$ $\qquad$ $I^{(2)}(\varepsilon) = -\frac{1}{6}\varepsilon_0^3$

$$\Gamma = \Gamma_0 \frac{\varepsilon_0^5}{30}\left(1 - \frac{5}{\varepsilon_0^2}\right)$$

$$\langle E_e\rangle = E_0\langle\varepsilon\rangle = E_0 \frac{\int_1^{\varepsilon} f(\varepsilon)\,\varepsilon\, d\varepsilon}{\int_1^{\varepsilon_0} f(\varepsilon)\, d\varepsilon} = \frac{E_0}{2}\left(1 + \frac{5}{2\varepsilon_0^2}\right)$$

$$\boxed{E_0 = E_e + E_\nu \qquad \varepsilon_0 = \frac{E_0}{m_ec^2} \qquad \langle E_e\rangle = \frac{E_0}{2}\left(1 + \frac{5}{2\varepsilon_0^2}\right)}$$

Vrh $f(\varepsilon)$

$$0 = \frac{d}{d\varepsilon}\ln(f) = \frac{d}{d\varepsilon}\ln\left(\varepsilon\,(\varepsilon_0-\varepsilon)^2\sqrt{\varepsilon^2-1}\right) = \frac{d}{d\varepsilon}\ln\varepsilon + \frac{d}{d\varepsilon}2\ln(\varepsilon_0-\varepsilon) + \frac{d}{d\varepsilon}\frac{1}{2}\ln \varepsilon^2-1$$

$$= \frac{1}{\varepsilon} - \frac{2}{\varepsilon_0-\varepsilon} + \frac{\varepsilon}{\varepsilon^2-1} = 0$$

$$\frac{(\varepsilon^2-1)(\varepsilon_0-\varepsilon)-2\varepsilon(\varepsilon^2-1)+\varepsilon^2(\varepsilon_0-\varepsilon)}{\varepsilon(\varepsilon_0-\varepsilon)(\varepsilon^2-1)} = 0$$

$$\varepsilon^2\varepsilon_0 - \varepsilon^3 - \varepsilon_0 + \varepsilon - 2\varepsilon^3 + 2\varepsilon + \varepsilon^2\varepsilon_0 - \varepsilon^3 = 0$$

$$-4\varepsilon^3 + 2\varepsilon_0\varepsilon^2 + \underbrace{3\varepsilon - \varepsilon_0}_{\text{majhno}} = 0$$

Približek $\varepsilon \approx \varepsilon_0$ velik

$\varepsilon = \frac{\varepsilon_0}{2} \longrightarrow$ prvi popravek $\varepsilon = \frac{\varepsilon_0}{2}(1+x)$

$$-4\frac{\varepsilon_0^3}{2^3}(1+3x) + 2\varepsilon_0\frac{\varepsilon_0^2}{2^2}(1+2x) + 3\frac{\varepsilon_0}{2}(1+x) - \varepsilon_0 = 0$$

$$-\frac{\varepsilon_0^3}{2}(1+3x-1-2x) + \varepsilon_0(3+x-1) = 0$$

$$x = 1/\varepsilon^2$$

13 $^{90}_{38}Sr$ $q = 0{,}54\ \frac{W}{g}$ specifična moč , β^- razpad

$$^{90}_{38}Sr \rightarrow {}^{90}_{39}Y\ e^-\ \bar\nu_e \qquad t_{1/2} = 28{,}8\,let$$

$$^{90}_{39}Y \rightarrow {}^{90}_{40}Zr\ e^-\ \bar\nu_e \qquad t'_{1/2} \ll t_{1/2}$$

večino energije se sprosti pri drugi reakciji

Ocena $\Delta m = m_Y - m_{Zr}$

$$q = \frac{P}{m} = \frac{P}{N\,m_Y}$$

$$P = \frac{dN}{dt}\langle E_e - m_e c^2\rangle$$

$$= \frac{dN}{dt}(\langle E_e\rangle - m_e c^2) \qquad A = \frac{dN}{dt} = \frac{N}{\tau} \quad \tau = \frac{t_{1/2}}{\ln 2}$$

$$= \frac{N\ln 2}{t_{1/2}}(\langle E_e\rangle - m_e c^2)$$

$$q = \frac{\ln 2}{m_Y t_{1/2}}(\langle E_e\rangle - m_e c^2)$$

90u

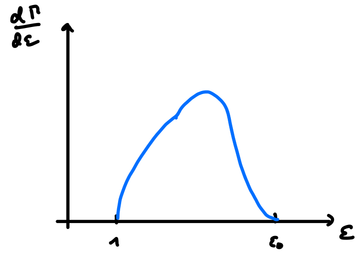
$$\langle E_e\rangle = m_e c^2\,\frac{\varepsilon_0}{2}\left(1+\frac{5}{2\varepsilon_0^2}\right) \qquad \varepsilon_0 = \frac{E_0}{m_e c^2}$$

$$q \rightarrow \langle E_e\rangle \rightarrow \varepsilon_0 \rightarrow \underset{2{,}0\,MeV}{E_0} = \Delta m\,c^2$$

14 $^3H \to {}^3He\ e^-\ \bar{\nu}_e$, $m_\nu > 0$ upoštevamo $m_\nu > 0$

Spekter pri $E_e \lesssim E_0$, kako se obnaša $\frac{d\Gamma}{d\varepsilon}$ pri $E_e \lesssim E_0$

Nariši $K(E_e) = \sqrt{\frac{1}{p_e E_e}\frac{d\Gamma}{dE_e}}$



Vemo: $\varepsilon = \frac{E_e}{m_e c^2}$ $\quad \varepsilon_0 = \frac{E_0}{m_e c^2}$ $\quad E_0 = E_e + E_\nu = Q + m_e c^2$

$$\frac{d\Gamma}{d\varepsilon} = A\,\varepsilon\sqrt{\varepsilon^2-1}\,(\varepsilon_0-\varepsilon)^2$$

$$dN = 4\frac{(4\pi)^2}{h^6}\underbrace{p_e^2\,dp_e}_{\frac{p_e}{c^2}E_e dE_e}\ \underbrace{p_\nu^2\,dp_\nu}_{\frac{p_\nu}{c^2}E_\nu dE_\nu}$$

$$p_\nu = \frac{1}{c}\sqrt{E_\nu^2 - m_\nu^2 c^4} \to \frac{1}{c}\sqrt{(\varepsilon_0-\varepsilon)^2 - \Big(\underbrace{\frac{m_\nu}{m_e}}_{\tilde{m}_\nu}\Big)^2}$$

$$dg = \frac{dN}{dE_\nu} = B\ p_e E_e\ p_\nu E_\nu\ dE_\nu \xrightarrow{E_e \to \varepsilon} \sqrt{\varepsilon^2-1}\,\varepsilon\,(\varepsilon_0-\varepsilon)\sqrt{(\varepsilon_0-\varepsilon)^2 - \tilde{m}_\nu^2}$$

Maks. vrednost energije (prej ε_0) sedaj

$$(\varepsilon_0-\varepsilon)^2 - \tilde{m}_\nu^2 = 0 \quad \rightsquigarrow \quad \varepsilon = \varepsilon_0 - \tilde{m}_\nu$$

$$K(\varepsilon) = \sqrt{\frac{1}{\varepsilon\sqrt{\varepsilon^2-1}}\frac{d\Gamma}{d\varepsilon}} \ \propto\ \sqrt{(\varepsilon_0-\varepsilon)\sqrt{(\varepsilon_0-\varepsilon)^2-\tilde{m}_\nu^2}} =$$

naj bo $\varepsilon_0 - \varepsilon = \tilde{m}_\nu + x$

$$= \sqrt{(\tilde{m}_\nu + x)\sqrt{(\tilde{m}_\nu+x)^2 + \tilde{m}_\nu^2}}$$

ⓐ $\tilde{m}_\nu = 0$ $\qquad K(\varepsilon) = x$

ⓑ $\tilde{m}_\nu \neq 0$

$$K(\varepsilon) \propto \sqrt{\tilde{m}_\nu}\,(2\tilde{m}_\nu x)^{1/4}$$

15 Razpad γ

$X^{**} \to X^*\ \gamma$ $\quad E1$

$X^{**} \to X\ \gamma$ $\quad M1$

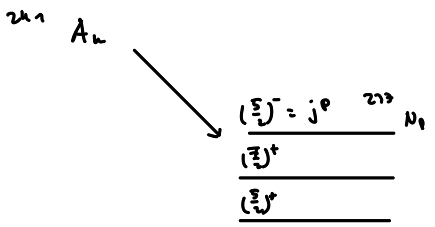
$$\Gamma_{E1} = \frac{E_\gamma^3}{3\pi\varepsilon_0\hbar^4 c^3}\left|\int d^3r\,\psi_f^*\,e\vec{r}\,\psi_i^*\right|^2$$

$$\Gamma_{M1} = \frac{\mu_0 E_\gamma'^3}{3\pi\hbar^4 c^3}\left|\int d^3r\,\psi_f^*\,\frac{e}{2m_N}(\vec{l}+g\vec{s})\,\psi_i\right|^2$$

$$\frac{\Gamma_{E1}}{\Gamma_{M1}} \approx \left(\frac{E_\gamma}{E_\gamma'}\right)^3 \frac{1}{\varepsilon_0\mu_0}\frac{e^2R^2}{\frac{e^2}{2m_N}\hbar^2} = \left(\frac{E_\gamma}{E_\gamma'}\right)^3\left(\frac{R\,2m_N c^2}{\hbar c}\right)^2 \sim 100\left(\frac{E_\gamma}{E_\gamma'}\right)^3$$

16

$^{241}Am \to \alpha\ ^{237}Np$



$\Delta m_1 = 26\ keV$ $\quad$ $\Delta m_2 = 59\ keV$

$$\frac{\Gamma\left((\frac{5}{2})^- \to (\frac{5}{2})^+\right)}{\Gamma\left((\frac{5}{2})^- \to (\frac{7}{2})^+\right)} = ? \approx \dots$$

$(\frac{5}{2})^- \to (\frac{5}{2})^+$ $\quad$ $\Delta J = 0$ $\quad$ $|J - J'| \le \ell \le J + J'$

$\Delta P = 1$ $\quad$ $0 \le \ell \le 5$

$\ell = 1$ je najnižji

Ali je M ali E?

E: $\Delta P = (-1)^\ell$

M: $\Delta P = -(-1)^\ell$

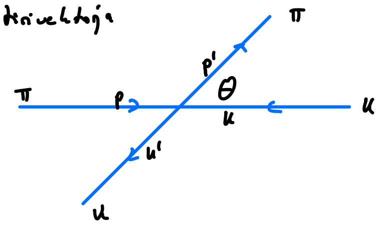
$\Rightarrow$ Ta prehod je E1

Tudi drugi prehod je E1

$$\Rightarrow \quad \dots \approx \left(\frac{E_f}{E_j}\right)^3 = \left(\frac{\Delta m_2}{\Delta m_1}\right)^3 = 11{,}7$$

17

$\pi^+(p)\ K^+(k) \to \pi^+(p')\ K^+(k')$ sipanje p, k štirivektorja

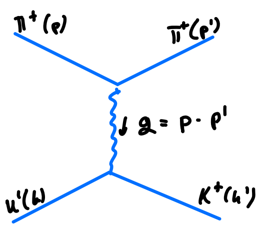


V težiščnem sistemu $\vec{p} + \vec{k} = 0$

$$d\sigma = \frac{|\mathcal{M}|^2}{F}\, dQ$$

fluks $\quad$ fazni prostor obeh delcev

Gre to μ



ν

$$T_{fi} = i \int j_\mu^{fi}(x)\, A^\mu\, d^4x$$

$$j_\mu^\pi = j_\mu^{fi} = e N^2 (p + p')_\mu\ e^{-i(p-p')x}$$

normalizacija

$$T_{fi} = -i \int j_\mu^\pi(x)\ \frac{-1}{q^2}\ j_\mu^k(x)\, d^4x$$

$$= (2\pi)^4 \delta^{(4)}(k + p - k' - p')\ \underbrace{(ie)(k+k')_\mu \frac{-i g^{\mu\nu}}{q^2} (ie)(p+p')_\nu}_{\mathcal{M}}$$

$$\mathcal{M} = ie\,(p+p')^\mu\ ie\,(k+k')^\nu\ \frac{-i g_{\mu\nu}}{q^2}$$

$$= \frac{ie^2}{q^2} (p+p')(k+k')$$

$$dQ = (2\pi)^4 \delta^{(4)}(p+k-p'-k') \frac{d^3p'}{(2\pi)^3\, 2E_{p'}} \cdot \frac{d^3k'}{(2\pi)^3\, 2E_{k'}}$$

$$2E_{p'} = 2\sqrt{m_\pi^2 + \vec{p}'^2}$$

$$F = |v_\pi - v_k|\, 2E_k\, 2E_p$$

V težiščnem sistemu

$$dQ = \frac{1}{4(2\pi)^2} \delta(E_p + E_k - E_{p'} - E_{k'}) \delta^{(3)}(\vec{p}' + \vec{k}') \frac{d^3p'}{\sqrt{m_\pi^2 + \vec{p}'^2}} \frac{d^3k'}{\sqrt{m_k^2 + \vec{k}'^2}} \qquad s = (p+k)^2$$

$$dQ = \frac{1}{4(2\pi)^2} \delta(\sqrt{s} - E_{p'} - E_{k'}) \frac{d^3p'}{\sqrt{m_\pi^2 + \vec{p}'^2}} \frac{1}{\sqrt{m_k^2 + \vec{p}'^2}}$$

$$dQ = \frac{1}{4(2\pi)^2} \delta\Big(\sqrt{s} - E_{p'} - \underbrace{\sqrt{m_k^2 + E_{p'}^2 - m_\pi^2}}_{E_{k'}}\Big) \frac{|p'|\, E_{p'}\, dE_{p'}\, d\Omega}{E_{p'}\, E_{k'}}$$

Ničle

$$* \quad \sqrt{s} - E_{p'} - \sqrt{m_k^2 + E_{p'}^2 - m_\pi^2} = 0$$

$$s - 2\sqrt{s} E_{p'} + E_{p'}^2 = m_k^2 - m_\pi^2 + E_{p'}^2 \Rightarrow E_{p'} = \frac{s + m_\pi^2 - m_k^2}{2\sqrt{s}}$$

Odvod

$$\frac{d}{dE_{p'}} * = -1 - \frac{2E_{p'}}{2E_{k'}} = -\frac{E_{k'} + E_{p'}}{E_{k'}}$$

$$dQ = \frac{1}{4(2\pi)^2} \frac{E_{k'}}{\sqrt{s}} \frac{|p_f|}{E_{k'}} d\Omega \quad \text{fazni prostor dvodelčnega sistema}$$

$$dQ = \frac{1}{4(2\pi)^2} \frac{p_f}{\sqrt{s}} d\Omega \quad \text{dvodelčni fazni prostor v težiščnem sistemu}$$

$$F = |v_\pi - v_k|\, 2E_k\, 2E_p$$

$$p^2 = p\cdot p = (E_p, \vec{p})(E_p, \vec{p}) = E_p^2 - \vec{p}^2 = \vec{p}^2 + m_\pi^2 - \vec{p}^2 = m_\pi^2$$

$$= 4\sqrt{(pk)^2 - m_\pi^2 m_k^2} =$$

$$= 4\sqrt{\tfrac{1}{2}((p+k)^2 - p^2 - k^2) - m_\pi^2 m_k^2} =$$

$$= 4\sqrt{\tfrac{1}{4}\left((s - m_\pi^2 - m_k^2)^2 - 4m_\pi^2 m_k^2\right)} =$$

$$= 2\sqrt{(s + m_\pi^2 + m_k^2)^2 - 4(m_\pi^2 m_k^2 + s m_\pi^2 + s m_k^2)} = 4\sqrt{s}\, |\vec{p}_i|$$

$$\left[\begin{aligned} s &= (p+k)^2 \\ t &= (p-p')^2 \\ u &= (p-k')^2 = (p'-k)^2 \\ q^2 &= t \end{aligned}\right. \qquad s+t+u = \sum_i m_i \qquad p'k' = p\cdot k = \frac{s - m_\pi^2 - m_k^2}{2} \qquad p\cdot k' = \frac{-u + m_\pi^2 + m_k^2}{2} = p'k$$

$$\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2 s} \frac{p_f}{p_i}$$

$$\mathcal{M} = \frac{ie^2}{q^2}(p+p')(k+k') = \frac{ie^2}{t}(pk + pk' + p'k + p'k') =$$

$$= \frac{ie^2}{t}(s - m_\pi^2 - m_k^2 - u) \approx \frac{ie^2}{t}(s-u) = \ldots$$

$$\left[\begin{aligned} p^\mu &= \left(\tfrac{\sqrt{s}}{2}, \tfrac{\sqrt{s}}{2}\hat{e}_z\right) \\ k^\mu &= \left(\tfrac{\sqrt{s}}{2}, -\tfrac{\sqrt{s}}{2}\hat{p}\right) \\ p'^\mu &= \left(\tfrac{\sqrt{s}}{2}, \tfrac{\sqrt{s}}{2}\hat{p}\right) \\ k'^\mu &= \left(\tfrac{\sqrt{s}}{2}, -\tfrac{\sqrt{s}}{2}\hat{e}_z\right) \end{aligned}\right.$$

Ker $s \gg m_k^2 \approx (500\,\mathrm{MeV})^2$

$$u = (p-k')^2 = -2p\cdot k' = -2\frac{s}{4}(1 + \underbrace{\hat{p}\,\hat{e}_z}_{\cos\theta}) = -\frac{s}{2}(1+\cos\theta)$$

$$\Rightarrow t = -u - s = -\frac{s}{2}(1-\cos\theta)$$

$$\ldots = ie^2 \frac{\frac{s}{2}(2+1+1\cos\theta)}{-\frac{s}{2}(1-\cos\theta)} = -ie^2 \frac{3+\cos\theta}{1-\cos\theta}$$

$$\Rightarrow \frac{d\sigma}{d\Omega} = \frac{e^4}{64\pi^2 s} \left(\frac{3+\cos\theta}{1-\cos\theta}\right)^2$$

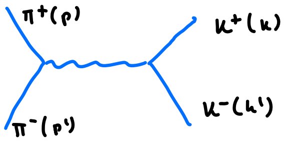
$\pi^+ K^+ \to \pi^+ K^+$

$s = (p+p')^2$
$t = (k-p)^2$
$u = (p+p')^2$

$|M|^2 = e^4 \left(\frac{s'-u'}{t'}\right)^2 = e^4 \left(\frac{u-t}{s}\right)^2$

Zanima nas te reakcija

$\pi^+ \pi^- \to K^+ K^-$



$\pi^+(p)\ \pi^-(p') \to K^+(k)\ K^-(k')$

$\pi^+(p)\ K^+(-k') \to K^+(k)\ \pi^+(-p')$

$s' = (p-k')^2$
$t' = (p+p')^2$
$u' = (p-k)^2$

rešitev za to že poznamo

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{p_f}{p_i} |M|^2 = \frac{\alpha^2}{4s} \cos^2\theta$$

18 Dokaži da močne sile ne loči med nukleonoma

$n \leftrightarrow p$

$^{31}_{15}P \leftrightarrow {}^{31}_{16}S$

odštejemo elektrostatski člen

$$\tilde{W}_b(P) = W_b^{exp}(P) + w_c \frac{Z^2}{A^{1/3}}$$

$$\frac{\tilde{W}_b(P)}{A} = 8{,}48\ \text{MeV}$$

$$\frac{\tilde{W}_b(P) - \tilde{W}_b(S)}{A} = -0{,}027\ \text{MeV} \quad \text{majhno}$$

19 Nukleon $N = \binom{p}{n}$

$N \to UN$

$SU(2) = \{ M \in \mathbb{C}^{2\times 2} ;\ M^+M = I ,\ \det M = 1 \}$

$N^+N = |p|^2 + |n|^2$
$(UN)^+(UN) = N^+U^+UN = N^+N$

Dokaži, da je $SU(2)$ grupa

① $g_1 \circ g_2 \in G$

$M_1, M_2 \in SU(2) \qquad M_1 M_2$

det = 1 ✓

$(M_1M_2)^+(M_1M_2) = M_2^+M_1^+M_1M_2 = M_2^+M_2 = I \in SU(2)$ ✓

② $e \circ g = g \circ e = g \qquad e = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \in G$

③ $g^{-1} \circ g = g \circ g^{-1}$ $\qquad M^{-1} = M^{\dagger}$

Vzemimo hermitsko matriko H, $H^{\dagger} = H$

$$M = e^{iH}$$

$$H = \alpha_0 I + \alpha_1 \frac{\sigma_1}{2} + \alpha_2 \frac{\sigma_2}{2} + \alpha_3 \frac{\sigma_3}{2}$$

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\det e^{iH} \overset{\alpha_i \ll 1}{=} \det(1 + iH) = \det(I + i(\alpha_0 I + \alpha_1 \tfrac{\sigma_1}{2} + \alpha_2 \tfrac{\sigma_2}{2} + \alpha_3 \tfrac{\sigma_3}{2}))$$

$$= \det \begin{bmatrix} 1 + i\alpha_3/2 + i\alpha_0 & i\frac{\alpha_1}{2} + \frac{\alpha_2}{2} \\ i\frac{\alpha_1}{2} - \frac{\alpha_2}{2} & 1 + i\alpha_0 - \alpha_3/2 \end{bmatrix}$$

$$= (1 + i\alpha_0 + i\tfrac{\alpha_3}{2})(1 + i\alpha_0 - i\tfrac{\alpha_3}{2}) - (i\tfrac{\alpha_1}{2} + \tfrac{\alpha_2}{2})(i\tfrac{\alpha_1}{2} - \tfrac{\alpha_2}{2})$$

samo lin. člani

$$= 1 + i\alpha_0 - i\tfrac{\alpha_3}{2} + i\alpha_0 + i\tfrac{\alpha_3}{2} = 1 + 2i\alpha_0 = 1 \quad \Rightarrow \quad \alpha_0 = 0$$

$$M = e^{iH} = e^{i\alpha_i \underbrace{\frac{\sigma_i}{2}}_{\text{generator}}}$$

Izospin $\qquad I_i = \frac{\sigma_i}{2} \qquad [I_i, I_j] = i\varepsilon_{ijk} I_k$

$|p\rangle = |\tfrac{1}{2}\ \tfrac{1}{2}\rangle_I \quad \ldots |I\ I_3\rangle$

$|n\rangle = |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle_I$

$\hat{I}^2 |I\ I_3\rangle = I(I+1)|I\ I_3\rangle$

$\hat{I}_3 |I\ I_3\rangle = I_3 |I\ I_3\rangle$

$I_{\pm} = I_1 \pm i I_2 \qquad I_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$I_{\pm}|I\ I_3\rangle = \sqrt{I(I+1) - I_3(I_3 \pm 1)}\ |I\ I_3 \pm 1\rangle$

$|p\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |n\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad I_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

⑳ $[H, I_i] = 0 \Rightarrow m_p = m_n$

$m_p = \langle p|H|p\rangle = \langle n|I_- H I_+|n\rangle =$ $\qquad |p\rangle = I_+|n\rangle$

$= \langle n|I_- I_+ H|n\rangle = \ldots$ $\qquad \langle p| = \langle n|I_-$

$$(I_1 - iI_2)(I_1 + iI_2) = I_- I_+ = I_1^2 + I_2^2 - iI_2 I_1 + iI_1 I_2 = I^2 - I_3^2 + i[I_1, I_2] =$$

$$= I^2 - I_3^2 - I_3$$

$$\ldots = \langle n|H(I^2 - I_3^2 - I_3)|n\rangle = \langle n|H(\tfrac{1}{2}\tfrac{3}{2} - (-\tfrac{1}{2})^2 + \tfrac{1}{2})|n\rangle = \langle n|H|n\rangle = m_n$$

㉑ Imamo NN (pp, np, pn, nn)

$I = 1$ triplet $\qquad |1\,1\rangle = |p\rangle|p\rangle$

$|1\,0\rangle = \frac{1}{\sqrt{2}}(|p\rangle|n\rangle + |n\rangle|p\rangle)$

$|1\,{-1}\rangle = |n\rangle|n\rangle$

$I = 0$ singlet $\qquad |0\,0\rangle = \frac{1}{\sqrt{2}}(|p\rangle|n\rangle - |n\rangle|p\rangle)$

(22) ^2H devteron, $J=1$

Določi možne I, S, L

$I=1$ triplet $\frac{1}{\sqrt{2}}(|p\rangle|n\rangle + |n\rangle|p\rangle)$ ni možen, ker drugače bi opazili tudi druge

$I=0$ singlet $\frac{1}{\sqrt{2}}(|p\rangle|n\rangle - |n\rangle|p\rangle)$

$\Rightarrow I=0$

$|\psi\rangle = |\psi\rangle_r |\chi\rangle_s |\chi\rangle_I$ Za vse fermione je $|\psi\rangle$ antisim

ψ more biti antisim. na zamenjavo 1. in 2. delca, zaradi Paulijevega izključitvenega načela (dva delca ne moreta biti na istem delu)

Simetrijske lastnosti pri zamenjavi $1\leftrightarrow 2$

$|\chi\rangle_I \to (-1)^{I+1}|\chi\rangle_I$ $\Rightarrow I=0 \Rightarrow$ antisim

$|\chi\rangle_s \to (-1)^{S+1}|\chi\rangle_s$

$|\psi\rangle_r = \psi(\vec r) \to (-1)^{\ell}|\psi\rangle_r$

$|\psi\rangle \to (-1)^{I+1}(-1)^{S+1}(-1)^{\ell}|\psi\rangle = (-1)^{I+S+\ell}|\psi\rangle$

$\Rightarrow I+S+\ell =$ liho $\quad I=0$

izkaže se da je dominantno

S	ℓ	J
0	0	0
1	0	1
0	1	0
1	1	0,1,2
1	2	1,2,3

(23) $\{u,d\}$, fermioni, $s=\frac{1}{2}$, $Q_u=\frac{2}{3}e_0$, $Q_d=-\frac{1}{3}e_0$, $u=|\frac{1}{2}\ \frac{1}{2}\rangle_I$, $d=|\frac{1}{2}\ -\frac{1}{2}\rangle_I$ ($I\ I_3$)

$u=I_+d$, $d=I_-u$

Poišči vsa možna stanja treh kvarkov u,d; $2\otimes2\otimes2$ (barion)

- $2\otimes2$
  - $I=1$: $|I\ I_3\rangle$: $|11\rangle$ uu; $|10\rangle$ $1/\sqrt{2}\,(ud+du)$; $|1\,-1\rangle$ dd
  - $I=0$: $|00\rangle$ $1/\sqrt{2}\,(ud-du)$

- $2\otimes2\otimes2$ ($2^n=8$ stanj), $4\oplus2\oplus2$

$I=\frac{3}{2},\frac{1}{2}$

$|11\rangle u = uuu = |\frac{3}{2}\ \frac{3}{2}\rangle$

$|\frac{3}{2}\ \frac{1}{2}\rangle = \frac{1}{\sqrt{3}}(duu+udu+uud)$

$|\frac{3}{2}\ -\frac{1}{2}\rangle = \frac{1}{\sqrt{3}}(ddu+dud+udd)$

$|\frac{3}{2}\ -\frac{3}{2}\rangle = ddd$

I_-, kvartet

$|\frac{1}{2}\ \frac{1}{2}\rangle = \alpha|10\rangle u + \beta|11\rangle d = \alpha\frac{1}{\sqrt{2}}(ud+du)u + \beta\, uud = \frac{1}{\sqrt{6}}(udu+duu) - \sqrt{\frac{2}{3}}\,uud$

$|\frac{1}{2}\ -\frac{1}{2}\rangle =$ DN

Določimo α,β

$\langle\frac{3}{2}\ \frac{1}{2}|\frac{1}{2}\ \frac{1}{2}\rangle = 0 = \alpha\frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}} + \alpha\frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}} + \beta\frac{1}{\sqrt{3}} \Rightarrow \frac{2}{\sqrt{2}}\alpha = -\beta \quad \beta=-\sqrt{2}\alpha$

Normalizacija $\left(\alpha\frac{1}{\sqrt{2}}\right)^2\cdot 2 + (\sqrt{2}\alpha)^2 = 1$

$\alpha = \frac{1}{\sqrt{3}}$ $\quad \beta = -\sqrt{\frac{2}{3}}$

$|00\rangle u = |\frac{1}{2}\ +\frac{1}{2}\rangle = \frac{1}{\sqrt{2}}(ud-du)u$

$|00\rangle d = |\frac{1}{2}\ -\frac{1}{2}\rangle = \frac{1}{\sqrt{2}}(ud-du)d$

} antisim na prva dva delca

(24) Δ^{++} (uuu) $J_z = \frac{3}{2}$

$J = \frac{3}{2}$

$I = \frac{3}{2}$

$S = \frac{3}{2}$ $L = 0$

$$|\Delta^{++}\rangle = \underbrace{|uuu\rangle |\uparrow\uparrow\uparrow\rangle_s}_{\text{simetrično prostor}} \quad \frac{1}{\sqrt{6}}\underbrace{(RGB - GRB + \dots)}_{\text{antisimetričen del}}$$

Določi ostala stanja v izospinskem kvartetu

$$|\Delta^+\rangle = \frac{1}{\sqrt{3}}(uud + udu + duu)\,|\uparrow\uparrow\uparrow\rangle$$

$$|\Delta^0\rangle = \frac{1}{\sqrt{3}}(udd + dud + ddu)\,|\uparrow\uparrow\uparrow\rangle \qquad \text{barva je enaka}$$

$$|\Delta^-\rangle = ddd\,|\uparrow\uparrow\uparrow\rangle$$

Določi magnetni moment μ stanja $|\Delta^+, J_z = \frac{3}{2}\rangle$

$$\hat{\mu}_z = \frac{e\hbar}{2m} g\, \hat{s}_z$$

$$\hat{\mu}_z^{a} = \frac{e_a \hbar}{m_a}\hat{s}_z \qquad \hat{\mu}_z = \hat{\mu}_z^{1} + \hat{\mu}_z^{2} + \hat{\mu}_z^{3}$$

Iščemo $\langle\hat{\mu}_z\rangle = \frac{1}{\sqrt{3}}(uud + \dots)\langle\uparrow\uparrow\uparrow|\,\hat{\mu}_z\,\frac{1}{\sqrt{3}}(uud + udu + duu)|\uparrow\uparrow\uparrow\rangle$

$$\hat{\mu}_u = \frac{\frac{2}{3}e_0\hbar}{m_u}\frac{1}{2} = \frac{e_0\hbar}{3m_u} \qquad \hat{\mu}_d = -\frac{e_0\hbar}{6m_d}$$

$$\hat{\mu}_z\,|uud\rangle|\uparrow\uparrow\uparrow\rangle = (2\mu_u + \mu_d)\,|uud\rangle|\uparrow\uparrow\uparrow\rangle$$

$$\langle\hat{\mu}_z\rangle = \frac{1}{3}(2\mu_u + \mu_d)\cdot 3 = 2\mu_u + \mu_d$$

(25) $m_\Delta = 1232$ MeV

$$p\pi^+ \to \Delta^{++} \longrightarrow p\pi^+ \qquad \frac{\sigma(p\pi^+ \to \Delta^{++} \to p\pi^+)}{\sigma(p\pi^- \to \Delta^0 \to p\pi^-,\ n\pi^0)} = \frac{|M_{3/2}|^4}{\frac{1}{9}|M_{3/2}|^4 + \frac{2}{9}|M_{3/2}|^4} = 3$$

- $p = |\frac{1}{2}\ \frac{1}{2}\rangle$

$\pi^+ = |1\ 1\rangle$

$\Delta^{++} = |\frac{3}{2}\ \frac{3}{2}\rangle$

Clebsch-Gordan

$$|\tfrac{1}{2}\ \tfrac{1}{2}\rangle|1\ 1\rangle = 1\ |\tfrac{3}{2}\ \tfrac{3}{2}\rangle$$

$$\langle\Delta^{++}|H|p\pi^+\rangle = \langle\tfrac{3}{2}\tfrac{3}{2}|H|\tfrac{3}{2}\tfrac{3}{2}\rangle$$

$$\langle p\pi^+|H|\Delta^{++}\rangle = \underbrace{\langle\tfrac{3}{2}\tfrac{3}{2}|H|\tfrac{3}{2}\tfrac{3}{2}\rangle}_{M_{3/2}}$$

$$\sigma(p\pi^+ \to \Delta^{++} \to p\pi^+) \propto |M_{3/2}|^4$$

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \quad \pi = \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} \quad \Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^+ \\ \Delta^0 \\ \Delta^- \end{pmatrix}$$

dublet $I = \frac{1}{2}$ triplet $I = 1$ kvartet $I = \frac{3}{2}$

- $\Delta^0 = |\frac{3}{2}, -\frac{1}{2}\rangle$

$$p\pi^- = |\tfrac{1}{2}\ \tfrac{1}{2}\rangle|1\ -1\rangle = \frac{1}{\sqrt{3}}|\tfrac{3}{2}\ -\tfrac{1}{2}\rangle - \frac{2}{\sqrt{3}}|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$$

$\frac{1}{2} + 1$

$-\frac{1}{2} + 1$

Izospin se mora ohraniti

$$\langle\Delta^0|H|p\pi^-\rangle = \langle\tfrac{3}{2}\ -\tfrac{1}{2}|H|\frac{1}{\sqrt{3}}|\tfrac{3}{2}\ -\tfrac{1}{2}\rangle - \cancel{\frac{2}{\sqrt{3}}|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle}$$

$$= \frac{1}{\sqrt{3}}\underbrace{\langle\tfrac{3}{2}\ -\tfrac{1}{2}|H|\tfrac{3}{2}\ -\tfrac{1}{2}\rangle}_{M_{3/2}}$$

$$\sigma(p\pi^- \to \Delta^0 \to p\pi^-) \propto \frac{1}{9}|M_{3/2}|^4$$

$$n\pi^0 = |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle|1\ 0\rangle = \sqrt{\tfrac{2}{3}}|\tfrac{3}{2}\ -\tfrac{1}{2}\rangle + \frac{1}{\sqrt{3}}\cancel{|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle}$$

$$\langle\Delta^0|H|n\pi^0\rangle = \sqrt{\tfrac{2}{3}}\,M_{3/2}$$

$$\sigma(p\pi^- \to \Delta^0 \to n\pi^0) \propto \left(\frac{1}{\sqrt{3}}M_{3/2}\cdot\sqrt{\tfrac{2}{3}}M_{3/2}\right)^2 = \frac{2}{9}M_{3/2}^4$$

26 Pri sipanju dveh nukleonov lahko nastane devteron D, pri čemer končnem stanju nastane še pion. (Zakaj reakcija brez izsevanega piona ne more potekati? Primerjaj mase nukleonov in devterona!) Z uporabo izospinske simetrije določi razmerje sipalnih presekov

$$\sigma_{pn\to D\pi^0} : \sigma_{pp\to D\pi^+}.$$

$$NN \to D\pi$$

$D = pn \qquad I = 0$

$$\frac{\sigma_{pn\to D\pi^0}}{\sigma_{nn\to D\pi^-}} = ?$$

1/2 1/2	1	0
	0	0
1/2 −1/2	$\frac{1}{2}$	$\frac{1}{2}$

$$pn = |\tfrac{1}{2}\,\tfrac{1}{2}\rangle|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{2}}|10\rangle + \tfrac{1}{\sqrt{2}}|00\rangle \qquad D\pi^0 = |10\rangle$$

$$\langle pn|H|D\pi^0\rangle = \tfrac{1}{\sqrt{2}}\langle 10|H|10\rangle = \tfrac{1}{\sqrt{2}}M_1$$

$$nn = |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle = |1\,-1\rangle \qquad D\pi^- = |1\,-1\rangle$$

$$\langle nn|H|D\pi^-\rangle = 1\, M_1$$

$$\frac{\sigma_{pn\to D\pi^0}}{\sigma_{nn\to D\pi^-}} = \frac{1}{2}$$

T Valovne funkcije barionov 222

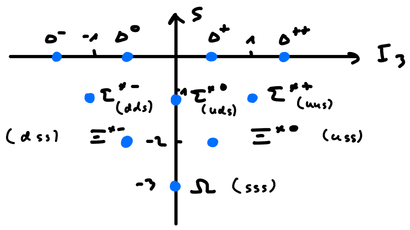
$\binom{u}{d}$ izospinski dublet $\qquad 4\times 0$ $\qquad 4 \oplus 2 \oplus 2$ $\quad I=\frac{3}{2}\;\; I=\frac{1}{2}\;\; I=\frac{1}{2}$

$\begin{pmatrix}u\\d\\s\end{pmatrix}$ SU(3) triplet $\qquad 10 \oplus 8 \oplus 8 \oplus 1$

dekuplet

Dekuplet 10 $\qquad J = \frac{3}{2}$

Čudnost : S $\qquad S|s\rangle = -|s\rangle$



← en s kvark

27 Zapiši val. funkcijo $|\Sigma^{*+}, J_z = \frac{1}{2}\rangle = ?$ $\qquad \Sigma^{*+} = uus$

P(kvark s spin ↑) = ?

$$|\Sigma^{*+}\rangle = |11\rangle_I\, |\tfrac{3}{2}\,\tfrac{1}{2}\rangle_s\, |\psi\rangle_{barvna} \qquad |\psi\rangle_{barvna} = \tfrac{1}{\sqrt{6}}\varepsilon_{ijk}|ijk\rangle$$

Mora biti simetrična

$$|11\rangle_I = \tfrac{1}{\sqrt{3}}(uus + usu + suu)$$

$$|\tfrac{3}{2}\,\tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)$$

$$|\Sigma^{*+}\rangle = \tfrac{1}{3}(uus + usu + suu)(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)$$

$$= \tfrac{1}{3}\big(|u\uparrow u\uparrow s\downarrow\rangle + |u\uparrow u\downarrow s\uparrow\rangle + \dots\big)$$

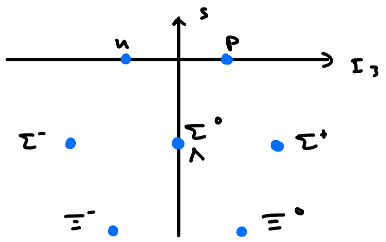
9 členov

$$P(\text{kvark } s \text{ spin } \uparrow) = \frac{2\cdot 3}{9} = \frac{2}{3}$$

$$|\Sigma^{*0}\, J_z=\tfrac{1}{2}\rangle = I^-|\Sigma^{*+}\rangle = |\tfrac{3}{2}\,\tfrac{1}{2}\rangle\, I^-|11\rangle_s$$

$$= |\tfrac{3}{2}\,\tfrac{1}{2}\rangle \tfrac{1}{\sqrt{6}}(dus + uds + dsu + usd + sdu + sud)$$

(T) Oktet 8 $J = \frac{1}{2}$



$$|p,\uparrow\rangle = \frac{1}{\sqrt{18}}\big[uud\,(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) + udu\,(2\uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow) + duu\,(2\downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow)\big]$$

(28) Zapiši valovno funkcijo bariona $\Sigma^0(\uparrow)$ (okteta $(\frac{1}{2})^+$) s spinsko komponento $\uparrow$. Lahko začneš z usrezno valovno funkcijo protona, kjer najprej nadomestiš $d \to s$, da dobiš v.f. Σ^+. Nato deluješ z izospinskim operatorjem nižanja I^-. Kakšna je verjetnost, da ima v tem stanju kvark s spin $\uparrow$? Kakšna je verjetnost, da ima kvark u spin $\downarrow$? Izrazi magnetni moment μ_{Σ^0} z magnetnimi momenti μ_u, μ_d, μ_s.

$\Sigma^0\ (uds)$

$$p \xrightarrow{d \to s} \Sigma^+ \xrightarrow{I_-} \Sigma^0$$

$$|\Sigma^+,\uparrow\rangle = \frac{1}{\sqrt{18}}\big[uus\,(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) + usu\,(2\uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow) + suu\,(2\downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow)\big]$$

$$|\Sigma^0,\uparrow\rangle = \frac{1}{\sqrt{36}}\big[(dus + uds)(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) + \dots \big]$$

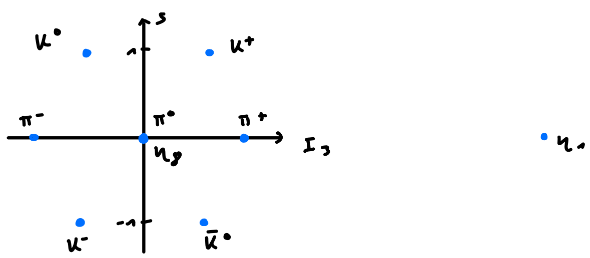
poseben singlet

$$|\Lambda^0,\uparrow\rangle = \frac{1}{\sqrt{12}}\big[s\underbrace{(ud-du)}_{I=0}\ \uparrow\underbrace{(\uparrow\downarrow-\downarrow\uparrow)}_{S=\frac{1}{2}} + (usd - dsu)(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow) + (ud-du)s(\uparrow\downarrow-\downarrow\uparrow)\uparrow\big]$$

(T) Mezoni $q\bar{q}$

$3 \otimes \bar{3} = 8 \oplus 1$

Pseudo skalar
Oktet $J=0$ $P=-1$
Singlet



$$|\pi^+\rangle = |u\bar{d}\rangle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\ \frac{1}{\sqrt{3}}(|R\bar{R}\rangle + |G\bar{G}\rangle + |B\bar{B}\rangle)$$

I_-

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}(|d\bar{d}\rangle - |u\bar{u}\rangle)\ *$$

$$|K^+\rangle = |u\bar{s}\rangle$$

$$|\eta_1\rangle = \frac{1}{\sqrt{3}}(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$$

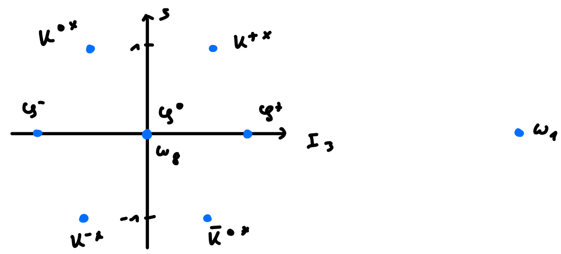
$|\eta_8\rangle$ mora biti ortogonalen na $|\eta_1\rangle$ in $|\pi^0\rangle$

$$|\eta_8\rangle = A(|d\bar{d}\rangle + |u\bar{u}\rangle + \alpha|s\bar{s}\rangle) = \frac{1}{\sqrt{6}}(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle)$$

$$\langle\pi^0|\eta_8\rangle = 0 \qquad \langle\eta_1|\eta_8\rangle = \frac{A}{\sqrt{3}}(1+1+\alpha) = 0 \qquad \alpha = -2$$

Vektorski oktet

$J = 1 \qquad P = -1$



Razlikujejo se v spinu od pseudoskalarnih

$|\uparrow\uparrow\rangle$

$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$

$|\downarrow\downarrow\rangle$

29 Ko stanje z vrtilno količino j in projekcijo m na z-os, $|j\,m\rangle$, zavrtimo okrog y-osi za kot θ, dobimo rezultat z delovanjem operatorja končne rotacije

$$e^{-iJ_2\theta}\,|j\,m\rangle = \sum_{m'} d^j_{m'm}(\theta)\,|j\,m'\rangle\,.$$

Unitarna matrika $d^j(\theta)$ je dimenzije $(2j+1)\times(2j+1)$ in jo imenujemo Wignerjeva rotacijska d. Določi matriko $d^{1/2}(\theta)$, tako, da eksponent matrike razviješ v vrsto! Določi tudi matriko za rotacijo stanja s spinom $j=1$, $d^1_{m'm}$!

$$e^{-iJ_2\theta}|jm\rangle = \sum_{m'} d^j_{m'm}(\theta)\,|jm'\rangle \qquad \Big|\cdot\langle jm|$$

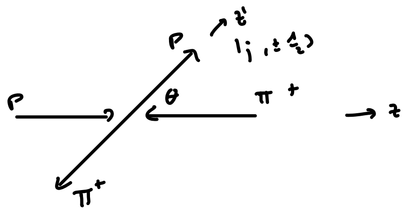
$$\langle jm|\,e^{-iJ_2\theta}\,|jm\rangle = d^j_{mm}(\theta)$$

$$e^{-i\theta J_2} \underset{j=\frac{1}{2}}{=} e^{-i\theta\frac{\sigma_2}{2}} \overset{\text{Taylor}}{=} \qquad j=s=\tfrac{1}{2} \qquad J_i = \frac{\sigma_i}{2}$$

$$= 1 - i\theta\frac{\sigma_2}{2} + \frac{(-i\theta)^2}{2!}\frac{\sigma_2^2}{2^2} + \frac{(-i\theta)^3}{3!}\left(\frac{\sigma_2}{2}\right)^3 + \dots \qquad \sigma_i^2 = \sigma_i^4 = \sigma_i^6 = I$$

$$= \left[1 - \frac{1}{2!}\left(\frac{\theta}{2}\right)^2 + \frac{1}{4!}\left(\frac{\theta}{2}\right)^4\right] - i\sigma_2\left(\frac{\theta}{2} + i\frac{(-i\theta)^3}{3!}\frac{1}{2^3} + \dots\right)$$

$$= \cos\frac{\theta}{2} - i\sigma_2\sin\frac{\theta}{2} = \begin{bmatrix}\cos\frac{\theta}{2} & \sin\frac{\theta}{2}\\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{bmatrix}_{m'm} = d^j_{m'm}(\theta)$$



$p = |\tfrac{1}{2}\ \tfrac{1}{2}\rangle$

$J_z = |j\ \tfrac{1}{2}\rangle$

$$\frac{d\sigma}{d\Omega} \propto |M|^2 \qquad M = \langle j, \pm\tfrac{1}{2}|H|j\ \tfrac{1}{2}\rangle$$

$$M_{\pm 1/2} = \sum_{m'} \langle j, \pm\tfrac{1}{2}|H|j m'\rangle\, d^j_{\frac{1}{2}m'}(-\theta)$$

ohranitev gil. kol.

$$= \underbrace{\langle j, \pm\tfrac{1}{2}|H|j, \pm\tfrac{1}{2}\rangle}_{M_j}\, d^j_{\frac{1}{2}\pm\frac{1}{2}}(-\theta)$$

$$|M_{\pm\frac{1}{2}}|^2 = |M_j|^2\,|d^j_{\frac{1}{2}\pm\frac{1}{2}}(-\theta)|^2$$

$$|M|^2 = |M_j|^2\left(|d^j_{\frac{1}{2}\frac{1}{2}}|^2 + |d^j_{\frac{1}{2}-\frac{1}{2}}|^2\right) = \dots$$

$j=\frac{1}{2}$: $\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} = 1$ $\qquad$ $j=\frac{3}{2}$: $\left(\frac{3\cos\theta}{2}\sin\frac{\theta}{2}\right)^2 + \left(\frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}\right)^2$

$$\dots = |M_j|^2\,\frac{1}{4}(1+3\cos^2\theta)$$

30 Preoblikuj Diracovo enačbo s Hamiltonianom $H = \boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta m$ v kovariantno obliko $(i\gamma^\mu\partial_\mu - m)\psi = 0$. Kakšna je zveza med matrikami $(\beta,\boldsymbol{\alpha})$ in matrikami γ^μ? Pokaži, da je $(\gamma^0)^\dagger = \gamma^0$ in $(\gamma^i)\dagger = -\gamma^i$.

$$\underline{(\vec\alpha \underbrace{\vec p}_{-i\nabla} + \beta m)\psi = i\frac{\partial\psi}{\partial t}}$$

$$(i\vec\alpha\nabla - \beta m + i\frac{\partial}{\partial t})\psi = 0 \qquad /\cdot\gamma^0$$

$$(i\underbrace{\gamma^0\vec\alpha}_{\vec\gamma}\nabla - \beta m + i\gamma^0\frac{\partial}{\partial t})\psi = 0 \qquad \underline{\gamma^0 = \beta} \quad \underline{(\gamma^0)^2 = 1}$$

$$\underline{(i\gamma^\mu\partial_\mu - m)\psi = 0}$$

$$\vec\gamma = \gamma^0\vec\alpha \qquad \vec\alpha = \gamma^0\vec\gamma$$

če $\mu \neq \nu \Rightarrow \gamma^\mu\gamma^\nu = \gamma^\nu\gamma^\mu$

$$H = \vec\alpha\vec p + \beta m = \gamma^0\vec\gamma\cdot\vec p + \gamma^0 m = \gamma^0(\vec\gamma\cdot\vec p + m)$$

$$\underline{\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}}$$

$$H^\dagger = \vec\gamma\cdot\vec p\,\gamma^{0\dagger} + m\gamma^{0\dagger} = H \quad\Rightarrow\quad \gamma^0 = \gamma^{0\dagger}$$

$$\underline{(\gamma^i)^\dagger = -\gamma^i}$$

$$(\gamma^i)^2 = -1$$
$$(\gamma^0)^2 = 1$$

31 $\gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0$ dokaži

$\mu = 0$ $\quad \gamma^0 = \gamma^0\gamma^0\gamma^0 = \gamma^0$

$\mu = i$ $\quad \gamma^{i\dagger} = \gamma^0\gamma^i\gamma^0 = -\gamma^i$

Pauli - Dirac

$$\gamma^0 = \begin{pmatrix} I_{2\times2} & 0 \\ 0 & -I_{2\times2}\end{pmatrix} \qquad \vec\gamma = \begin{pmatrix} 0 & \vec\sigma \\ -\vec\sigma & 0\end{pmatrix}$$

Weyl $\quad \gamma^{0\prime} = \begin{pmatrix} 0 & I \\ I & 0\end{pmatrix} \qquad \vec\gamma' = \vec\gamma$

32 γ^μ zadoščajo $\quad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$

$\gamma^{\mu\prime} = U^\dagger\gamma^\mu U \qquad U^{-1} = U^\dagger$

$$\{\gamma'^\mu, \gamma'^\nu\} = \{U^\dagger\gamma^\mu U, U^\dagger\gamma^\nu U\} = U^\dagger\gamma^\mu \underbrace{UU^\dagger}_{1}\gamma^\nu U + U^\dagger\gamma^\nu\underbrace{UU^\dagger}_{1}\gamma^\mu U = U^\dagger 2g^{\mu\nu} U = 2g^{\mu\nu}$$

33 $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = -\frac{i}{4!}\varepsilon^{\mu\nu\alpha\beta}\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta$

Pokaži: $\quad \gamma^5$ hermitska, $\quad (\gamma^5)^2 = 1$, $\quad [\gamma^5, \gamma^i] = 0$

$$(\gamma^5)^\dagger = -i\gamma^{3\dagger}\gamma^{2\dagger}\gamma^{1\dagger}\gamma^{0\dagger} = i\gamma^3\gamma^2\gamma^1\gamma^0 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \gamma^5$$

$$(\gamma^5)^2 = \ldots = 1$$

$$[\gamma^5, \gamma^i] = -i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^i + i\gamma^i\gamma^0\gamma^1\gamma^2\gamma^3$$

$i = 0$

$i = 1 \quad \ldots$

34

$$\mathrm{Tr}\,\gamma^\mu = 0$$
$$\mathrm{Tr}\,\gamma^5 = 0$$

35

2.38, 2.39

Izpelji: $(i\gamma^\mu \partial_\mu - m)\psi = 0$

Pokaži, da j^μ $\quad j^\mu = e\bar{\psi}\gamma^\mu\psi$ $\qquad \bar{\psi} = \psi^\dagger\gamma^0$

$$\partial_\mu j^\mu = 0$$

$$(i\gamma^\mu\partial_\mu - m)\psi = 0 \quad /^\dagger$$

$$\psi^\dagger(-i\gamma^0\gamma^\mu\gamma^0\partial_\mu - m) = 0$$

$$-i\partial_\mu\psi^\dagger\gamma^0\gamma^\mu\gamma^0 - m\psi^\dagger = 0 \quad |\gamma^0$$

$$-i\partial_\mu\bar{\psi}\gamma^\mu\gamma^0\gamma^0 - m\underbrace{\psi^\dagger\gamma^0}_{\bar{\psi}} = 0$$

$$\Rightarrow \quad \bar{\psi}(i\partial_\mu\gamma^\mu + m) = 0$$

$$\partial_\mu j^\mu = e\bar{\psi}\gamma^\mu\partial_\mu\psi + e(\partial_\mu\bar{\psi})\gamma^\mu\psi$$

Diracova en.

$$= e\bar{\psi}\gamma^\mu(-im\psi) + e\,im\bar{\psi}\gamma^\mu\psi = 0$$

vektor (4 komp)

$$j^0 = e\bar{\psi}\gamma^0\psi = e\psi^\dagger(\gamma^0)^2\psi = e(|\psi_1|^2 + |\psi_2|^2 + |\psi_3|^2 + |\psi_4|^2)$$

36

2.7.10

$$\psi_{\vec{p}}^{(s)}(x) = N e^{-ip\cdot x}\, u^{(s)}(\vec{p})$$

$$(i\gamma^\mu\partial_\mu - m)\psi_{\vec{p}}^{(s)}(x) = 0$$

$$(\underbrace{p^\mu\gamma_\mu}_{\not{p}} - m)\,u^{(s)}(\vec{p}) = 0$$

$$\not{p} = \underbrace{E\gamma_0}_{E\begin{bmatrix} I & 0 \\ 0 & -I\end{bmatrix}} - \vec{\gamma}\cdot\vec{p} = \begin{bmatrix} E & -\vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -E \end{bmatrix}$$

$$\begin{bmatrix} E-m & -\vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -E-m \end{bmatrix}_{4\times4}\begin{pmatrix} u_A \\ u_B \end{pmatrix}_{4\times1} = 0 \quad \Rightarrow \quad \begin{aligned} &(E-m)u_A - \vec{\sigma}\cdot\vec{p}\,u_B = 0 \\ &\vec{\sigma}\cdot\vec{p}\,u_A - (E+m)u_B = 0 \end{aligned} \Rightarrow u_B = \frac{\vec{\sigma}\cdot\vec{p}}{E+m}u_A$$

$$\left((E-m) - \frac{(\vec{\sigma}\cdot\vec{p})^2}{E+m}\right)u_A = 0$$

$$(\vec{\sigma}\cdot\vec{p})^2 = p^i p^j \underbrace{\sigma^i\sigma^j}_{\delta^{ij} + i\varepsilon^{ijk}\sigma^k} = p^2$$

$$\frac{E^2 - m^2 - p^2}{E+m}\,u_A = 0$$

$$\Rightarrow \quad E^2 = m^2 + p^2$$

$$\Rightarrow u^{(s)}(\vec{p}) = N \begin{pmatrix} \chi^{(s)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(s)} \end{pmatrix} \qquad \chi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \uparrow \quad \chi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \downarrow$$

za antidelca $v^{(s)}(\vec{p}) = N \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(-s)} \\ \chi^{(-s)} \end{pmatrix}$ $\quad \chi^{(-1)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \uparrow \quad \chi^{(-2)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \downarrow$

$$N = \sqrt{E+m} \qquad \int \rho\, dV = \int \psi^\dagger \psi\, dV = 2E$$

(37) 2.7.12

e^-
$\vec{p} = p\hat{k}$
$s_z = -1/2$

$$\Rightarrow u^{(2)}(\vec{p}) = \sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ \frac{p\sigma_3}{E+m}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix} = \sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ \frac{p}{E+m}\begin{pmatrix} 0 \\ -1 \end{pmatrix} \end{pmatrix}$$

$$\underset{\substack{E\to m \\ \text{nerel.}}}{=} \sqrt{2m}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -\frac{v}{2} \end{pmatrix}$$

e^+
$\vec{p} = p\hat{k}$
$s_z = -1/2$

$$v^{(2)}(p\hat{k}) = \sqrt{E+m}\begin{pmatrix} \frac{p}{E+m}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ 1 \\ 0 \end{pmatrix}$$

(38) $u^{(r)}(\vec{p})^\dagger\, u^{(s)}(\vec{p}) = \delta^{rs}\, 2E$

$v^{(r)}(\vec{p})^\dagger\, v^{(s)}(\vec{p}) = 2E\,\delta^{rs}$

(39) 2.7.16

$$\overline{u^{(s)}(\vec{p})}\, u^{(s)}(\vec{p}) = u^{(s)}(0)^\dagger u^{(s)}(0) \overset{p=0}{=} 2E = 2m$$

Lorentzova invarianta
lahko izberemo $\vec{p}=0$

$$\overline{u^{(s)}(0)} = \left(N\begin{pmatrix} \chi^{(s)} \\ 0 \end{pmatrix}\right)^\dagger \gamma^0$$
$$= N\begin{pmatrix} \chi^{(s)\dagger} & 0 \end{pmatrix}\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$
$$= N\begin{pmatrix} \chi^{(s)\dagger} & 0 \end{pmatrix}$$

$$\overline{v^{(s)}(\vec{p})}\, v^{(s)}(\vec{p}) \underset{\vec{p}=0}{=} -v^{(s)}(0)\, v^{(s)}(0) = -2m$$

(40) 2.7.14

Sučnost $h = \vec{s}\cdot\frac{\vec{p}}{|\vec{p}|}$ dobro kvantno število ker $[H,h]=0$

lastni vrednosti $h = \pm\frac{1}{2}$

$$\vec{s} = \frac{1}{2}\begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$$

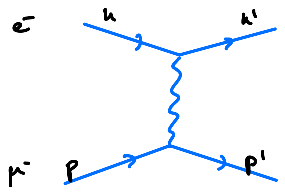
$$[H,h] = [\gamma^0(\vec{\gamma}\cdot\vec{p} - m), \vec{s}\cdot\vec{p}] \overset{DN}{=} 0$$

(41)

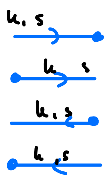
$$\sum_s u^{(s)}(\vec{p})\,\overline{u^{(s)}(\vec{p})} = \not{p} + m$$

$$\sum_s v^{(s)}(\vec{p})\,\overline{v^{(s)}(\vec{p})} = \not{p} - m$$

(42) 2. 3. 18 $\quad e^-(k)\ \mu^-(p) \to e^-(k')\ \mu^-(p')$



Feynmanova pravila

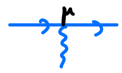


$u^{(s)}(k)$

$\overline{u^{(s)}(k)}$

$v^{(s)}(k)$ antidelec

$\overline{v^{(s)}(k)}$

Vozlišče  $-ieQ\gamma^\mu$ $\qquad e = \sqrt{4\pi\alpha}$

Q naboj delca v os. enotah $\frac{e}{e_0}$ ki gre vzdolž puščice

Propagator fotona $\frac{-ig^{\mu\nu}}{q^2}$ $\qquad q = k - k'$

(43) 2.3.14

$$-i\mathcal{M} = \overline{u^{(s)}(k')}\,(-ie\overset{-1}{Q}\gamma^\mu)\,u^{(s)}(k)\,\frac{-ig^{\mu\nu}}{q^2}\,\overline{u^{(r)}(p')}\,(-ie\overset{-1}{Q}\gamma^\nu)\,u^{(r)}(p)$$

$$= \frac{ie^2}{q^2}\,\bar{u}^{s'}(k')\gamma^\mu u^s(k)\,\bar{u}^{r'}(p')\gamma_\mu u^r(p) = \dots \qquad \gamma_\mu = g^{\mu\nu}\gamma^\nu$$

$$u^s(k) = \sqrt{E+m}\begin{pmatrix}\chi^s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^s\end{pmatrix} \overset{\substack{\text{nerel.}\\ E\to m}}{=} \sqrt{2m}\begin{pmatrix}\chi^s \\ 0\end{pmatrix}$$

$$\gamma^0 = \begin{pmatrix} I & \\ & -I\end{pmatrix}$$

$$\dots = \frac{ie^2}{q^2}\left(\sqrt{2m_e}\right)^2\left(\sqrt{2m_\mu}\right)^2 (\chi^{s'\dagger}\ 0)\,\gamma^0\gamma^\mu\begin{pmatrix}\chi^s\\0\end{pmatrix}(\chi^{r'\dagger}\ 0)\,\gamma^0\gamma_\mu\begin{pmatrix}\chi^r\\0\end{pmatrix} = \dots$$

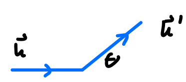
Seštevamo po vseh μ, prispevek γ^0, ostali so izvendiagonalni in so enaki 0

$$\dots = \frac{ie^2}{q^2}\,2m_e\,2m_\mu\,\underbrace{\chi^{(s')\dagger}\chi^{(s)}}_{\delta^{ss'}}\,\underbrace{\chi^{(r')\dagger}\chi^{(r)}}_{\delta^{rr'}} = \frac{4ie^2 m_e m_\mu}{q^2}\,\delta^{ss'}\delta^{rr'}$$

$$\overline{|\mathcal{M}|^2} = \frac{1}{4}\sum_{ss'rr'}|\mathcal{M}^{ss'rr'}|^2 = \frac{1}{4}\sum\frac{16e^4}{q^4}m_e^2m_\mu^2\,\delta^{ss'}\delta^{rr'} = \frac{1}{4}\,4\,\frac{16e^4}{q^4}m_e^2m_\mu^2$$

$\frac{1}{4}$: število vhodnih stanj

$\delta^{ss'}$: 11, 11, 22, 22 $\quad \delta^{rr'}$: 11, 22, 11, 22

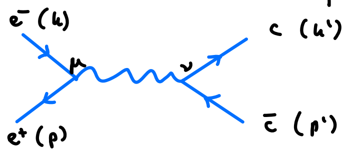


$$q^2 = (k-k')^2 = 2m_e^2 - 2k\cdot k' = -8m_eT_e\sin^2\frac{\theta}{2}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\,\overline{|\mathcal{M}|^2}\,\frac{p_f}{p_i}$$

44

2. 7. 21 Anihilacija



$Q_c = \frac{2}{3}$

$q^2 = s = (k+p)^2$

$$\mathcal{M} = \bar v(p)\, ie\gamma^\mu\, u(k)\, \frac{-i g_{\mu\nu}}{q^2}\, \bar u(k')\, \left(-ie^2 \tfrac{2}{3}\gamma^\nu\right) v(p')$$

$$= \frac{ie^2}{q^2}\frac{2}{3}\, \bar v(p)\gamma^\mu u(k)\, \bar u(k')\gamma_\mu v(p')$$

$$\left[\ \left(\bar v(p)\gamma^\mu u(k)\right)^\dagger = u(k)^\dagger \gamma^0\gamma^\mu\gamma^0\gamma^0 v(p) = \bar u(k)\gamma^\mu v(p)\right.$$

vektor, matrika, vektor

$$\overline{|\mathcal{M}|^2} = \frac{1}{4}\sum_{\text{spini}}\left(\frac{2}{3}\right)^2 \frac{e^4}{q^4}\, \bar v_A(p)\gamma^\mu_{AD} u_B(k)\, \bar u_C(k)\gamma^\nu_{CD} v_D(p)\, \bar u_E(k')\gamma_{\mu EF} v(p')_F\, \bar v_G(p')\gamma_{\nu GH} u_H(k')$$ index notacija

$$= \frac{e^4 (2/3)^2}{4s^2}\sum_{\text{spini}} v_F(p)\bar v_A(p)\gamma^\mu_{AD}\, u_D(k)\, u_H(k')\bar u_C(k')\gamma_{\mu CD}\, v_D(p')\, \bar u_E(k)\gamma^\nu_{EF}\, \bar v_G(p')\gamma^\nu_{GH}$$

$$= \frac{e^4 (2/3)^2}{4s^2}\operatorname{Tr}\Big(\underbrace{\sum_{\text{spini}}}\ \underbrace{v(p)\bar v(p)}_{\not p - m_e}\gamma^\mu \underbrace{u(k)\bar u(k)}_{\not k + m_e}\gamma^\nu\Big)\operatorname{Tr}\Big(\underbrace{\sum_{\text{spini}}}\ \underbrace{u(k')\bar u(k')}_{\not k' + m_q}\gamma_\mu \underbrace{v(p')\bar v(p')}_{\not p' - m_q}\gamma_\nu\Big)$$

$$= \frac{e^4 (2/3)^2}{4s^2}\operatorname{Tr}\left((\not p - m_e)\gamma^\mu(\not k + m_e)\gamma^\nu\right)\operatorname{Tr}\left((\not k' + m_q)\gamma_\mu(\not p' - m_q)\gamma_\nu\right) = \dots$$

Lastnosti Tr γ matrik

$$\operatorname{tr}\gamma^\mu = 0$$
$$\operatorname{tr}\gamma^\mu\gamma^\nu = \frac{1}{2}\operatorname{tr}(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu) = \frac{1}{2}\operatorname{tr}\{\gamma^\mu,\gamma^\nu\} = \frac{1}{2}\, 2g^{\mu\nu}\operatorname{Tr} I = 4g^{\mu\nu}$$
$$\operatorname{tr}\gamma^\mu\gamma^\nu\gamma^\rho = \operatorname{tr}\gamma^\mu\gamma^\nu\gamma^\rho(\gamma^5)^2 = -\operatorname{tr}(\gamma^5)^2\gamma^\mu\gamma^\nu\gamma^\rho = -\operatorname{tr}\gamma^\mu\gamma^\nu\gamma^\rho = 0$$ za liho število γ
$$\operatorname{tr}\gamma^\mu\gamma^\nu\gamma^5 = \operatorname{tr}(\gamma^\mu\gamma^\nu i\gamma^0\gamma^1\gamma^2\gamma^3) = 0$$
$$\operatorname{tr}\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta =$$
$$\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta = (2g^{\mu\nu} - \gamma^\nu\gamma^\mu)\gamma^\alpha\gamma^\beta = 2g^{\mu\nu}\gamma^\alpha\gamma^\beta - \gamma^\nu(2g^{\mu\alpha} - \gamma^\alpha\gamma^\mu)\gamma^\beta =$$
$$= 2g^{\mu\nu}\gamma^\alpha\gamma^\beta - 2g^{\mu\alpha}\gamma^\nu\gamma^\beta + \gamma^\nu\gamma^\alpha(2g^{\mu\beta} - \gamma^\beta\gamma^\mu) =$$
$$= 2g^{\mu\nu}\gamma^\alpha\gamma^\beta - 2g^{\mu\alpha}\gamma^\nu\gamma^\beta + 2g^{\mu\beta}\gamma^\nu\gamma^\alpha \underbrace{- \gamma^\nu\gamma^\alpha\gamma^\beta\gamma^\mu}_{\operatorname{tr}\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta} \quad /\operatorname{tr}$$
$$2\operatorname{tr}\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta = 2\left(g^{\mu\nu}\operatorname{tr}\gamma^\alpha\gamma^\beta - g^{\mu\alpha}\operatorname{tr}\gamma^\nu\gamma^\beta + g^{\mu\beta}\operatorname{tr}\gamma^\nu\gamma^\alpha\right)$$
$$\operatorname{tr}\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta = 4\left(\underset{12}{g^{\mu\nu}g^{\alpha\beta}} - \underset{13}{g^{\mu\alpha}g^{\nu\beta}} + \underset{14}{g^{\mu\beta}g^{\nu\alpha}}\right)$$

$$\operatorname{Tr}\left((\not p - m_e)\gamma^\mu(\not k + m_e)\gamma^\nu\right) = -m_e^2\operatorname{Tr}\gamma^\mu\gamma^\nu + \operatorname{Tr}\not p\gamma^\mu\not k\gamma^\nu + 0$$
$$= -m_e^2\, 4g^{\mu\nu} + p_\alpha k_\beta \operatorname{Tr}(\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu)$$
$$= -m_e^2\, 4g^{\mu\nu} + p_\alpha k_\beta\, 4\left(g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\beta}g^{\mu\nu} + g^{\alpha\nu}g^{\mu\beta}\right)$$
$$= 4\left(-m_e^2 g^{\mu\nu} + p^\mu k^\nu - (p\cdot k)g^{\mu\nu} + p^\nu k^\mu\right)$$

$$\overline{|\mathcal{M}|^2} = \frac{e^4 (2/3)^2}{4s^2}\operatorname{Tr}\left((\not p - m_e)\gamma^\mu(\not k + m_e)\gamma^\nu\right)\operatorname{Tr}\left((\not k' + m_q)\gamma_\mu(\not p' - m_q)\gamma_\nu\right) =$$

$m_e, \approx 0$

$$= \frac{e^4 (2/3)^2}{4s^2}\, 4\left(p^\mu k^\nu - (p\cdot k)g^{\mu\nu} + p^\nu k^\mu\right) 4\left(p'_\mu k'_\nu - (m_q^2 + p'\cdot k')g_{\mu\nu} + p'_\nu k'_\mu\right)$$

$$= \frac{4e^4(2/3)^2}{s^2}\Big(p\cdot p'\; k\cdot k' - (m_q^2 + p'\cdot k')\; p\cdot k + p\cdot p'\; k\cdot k'$$

$g^{\mu\nu} g_{\mu\nu} = 4$

$$- p\cdot k\; p'\cdot k' + 4\,(p\cdot k)\,(m_q^2 + p'\cdot k') - p\cdot k\; p'\cdot k'$$

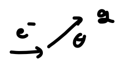
$$k\cdot p'\; p\cdot k' - (m_q^2 + p'\cdot k')\; p\cdot k + k\cdot p'\; p\cdot k' \Big)$$

$$= \frac{4e^4(2/3)^2}{s^2}\left(2\, p\cdot p'\; k\cdot k' + 2\, m_q^2\; p\cdot k + 2\, k\cdot p'\; p\cdot k' \right)$$

$$\boxed{\overline{|\mathcal{M}|^2} = \frac{8e^4Q^2}{s^2}\left(p\cdot p'\; k\cdot k' + m_q^2\; p\cdot k + k\cdot p'\; p\cdot k' \right)}$$

$m_q = 0 \qquad s = (k+p)^2 \qquad t = (k-k')^2 \qquad u = (k-p')^2$

$$\Rightarrow \overline{|\mathcal{M}|^2} = \frac{8e^4Q^2}{s^2}(u^2+t^2) = 16\pi^2\alpha^2 Q_c^2 (1+\cos^2\theta)$$

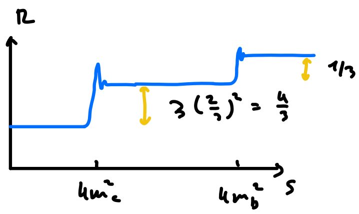
če 

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\overline{|\mathcal{M}|^2}\; \underbrace{3}_{\text{barve}}$$

$$\sigma_{e^+e^-\to q\bar q} = 3Q_q^2\frac{4\pi\alpha^2}{3s}$$

$$\sigma_{e^+e^-\to \mu^+\mu^-} = \frac{4\pi\alpha^2}{3s}$$

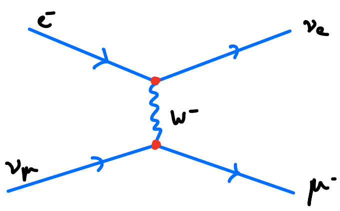
$$R(s) = \frac{\sum_q \sigma_{e^+e^-\to q\bar q}}{\sigma_{e^+e^-\to \mu^+\mu^-}}$$



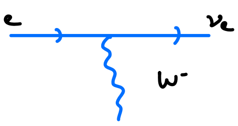
45 Šibka interakcija

$W^\pm \qquad m_W = 80{,}4$ GeV

$Z^0 \qquad m_Z = 91$ GeV



$e^-(k)\,\nu_\mu(p) \to \nu_e(k')\,\mu^-(p')$



Ohranja se leptonsko št.

levoročni projektor

$$\frac{-ig}{\sqrt{2}}\gamma^\mu P_L$$

$$P_L = \frac{1-\gamma_5}{2} \qquad P_R = \frac{1+\gamma_5}{2}$$

$P_L u = u_L$

$P_L^2 = P_L \qquad$ enako za R

$P_L P_R = P_R P_L = 0$

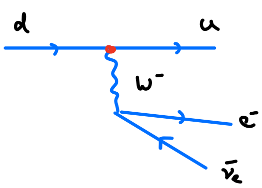
$u = u_L + u_R$

$$\mathcal{M} = \bar u(k')\left(\frac{-ig}{\sqrt{2}}\gamma^\mu P_L\right)u(k)\cdot \frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_W^2}\right)}{q^2 - m_W^2}\cdot \bar u(p')\left(\frac{-ig}{\sqrt{2}}\gamma^\nu P_L\right)u(p)$$

Projektor za

$$\frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_W^2}\right)}{q^2 - m_W^2}$$

Primer razpada β



$$\frac{-ig}{\sqrt{2}}\gamma^\mu P_L\; V_{ud}$$

Kabibo - Kobajaši

$$V = \begin{bmatrix} V_{ud} & & V_{us} \\ & \ddots & \\ V_{td} & & V_{ts} \end{bmatrix}$$

Efektivna teorija šibke sile $\qquad g^2 \ll m_W^2$

$$\mathcal{M} = \underbrace{-\frac{ig^2}{2}\,\frac{1}{m_W^2}}_{-i\frac{4G_F}{\sqrt{2}}}\;\bar{u}(k')\,\gamma^\mu P_L\, u(k)\;\bar{u}(p')\,\gamma_\nu P_L\, u(p)$$

$$G_F = 1{,}17\cdot 10^{-5}\ \mathrm{GeV}^{-2} = \frac{g^2}{4\sqrt{2}\,m_W^2}$$