# STATISTIKA

Vaje

3.10.24

# Vaje 1

1.

a) Študentje 1. letnika UP.
b) 1 študent
c)
d)
e) 8 sivo ⟹
f)

Stat. spremen.
- Opisne (prikaz s stolp: fortni.)
- Številske (-ll- histogr.)

1. Opisne:
- Imenske
  - Ime
  - Priimek
  - Spol
  - Kraj st. bival.
  - Štipendist (✓/x)
  - Poštna številka
- Urejenostne
  - Izobrazba očeta/mater

2. Številske
- Intervalske (ni abs. ničle)
  - Let. rojstva
- Razmernostna
  - Starost
  - Oddaljenost st. biv. od univerze
  - Št. otrok v družini

2.

a) $f_i^{\circ} = \frac{f_i}{\sum_{i=1} f_i}$ $f_i^{\%} = \frac{f_i}{\sum_{i=1} f_i} \cdot 100\%$

$f_Z^{\%} = \frac{57}{100} \cdot 100\% = 57\%$
$f_M^{\%} = \frac{43}{100} \cdot 100\% = 43\%$

b) $f_{rj}^{\%} = 17\%$
$f_Z^{\%} = 48\%$
$f_{rd}^{\%} = 24\%$
$f_b^{\%} = 11\%$

c) $f_Z^{rj\%} = 82{,}4\%$
$f_M^{rj\%} = 17{,}6\%$

$f_Z^{\check{c}\%} = 41{,}7\%$ $f_Z^{b\%} = 72{,}7\%$
$f_M^{\check{c}\%} = 58{,}3\%$ $f_M^{b\%} = 27{,}3\%$

$f_Z^{rd\%} = 62{,}5\%$
$f_M^{rd\%} = 37{,}5\%$

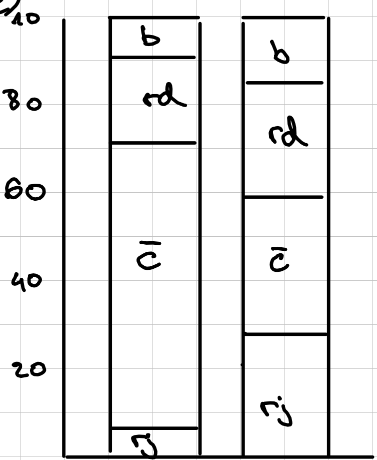
d) $f_{rj}^{M\%} = 7{,}0\%$ $\quad f_{rd}^{M\%} = 20{,}9\%$

$f_{\bar{c}}^{M\%} = 65{,}1\%$ $\quad f_{b}^{M\%} = 7{,}0\%$

$f_{rj}^{\bar{z}\%} = 24{,}6$ $\quad f_{rd}^{\bar{z}\%} = 26{,}3$

$f_{\bar{c}}^{\bar{z}\%} = 35{,}1$ $\quad f_{b}^{\bar{z}\%} = 14{,}0$

e)



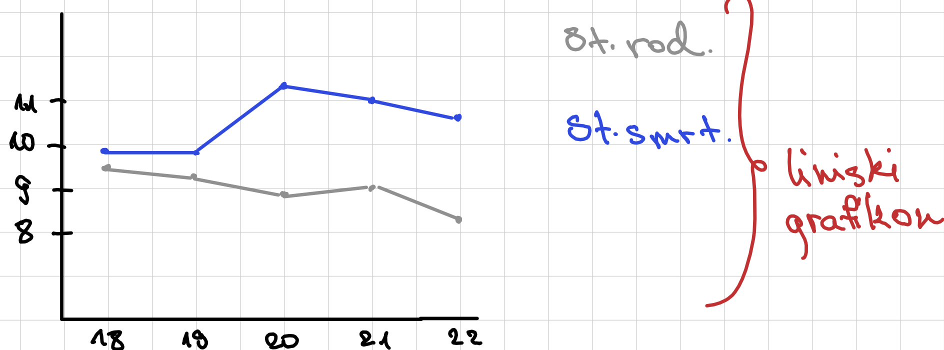

3. DN

4. KOEFICIENTI

Stopnja rodnosti $\frac{\text{št. roj.}}{\text{št. preb}} \times 1000$

| | a) št. rod. | b) št. smrt. |
|---|---|---|
| 2018 | 9,43 | 9,86 |
| 2019 | 9,23 | 9,83 |
| 2020 | 8,89 | 11,37 |
| 2021 | 9,00 | 11,03 |
| 2022 | 8,36 | 10,67 |

c)




5.

Rodilo se je 485600 otrok, umrlo pa je 649490 ljudi.

6.

INDEXI:

Index z osnovo $x_0$: $I_{i \setminus 0} = 100 \cdot \frac{x_i}{x_0}$, $i = 1 \ldots k$

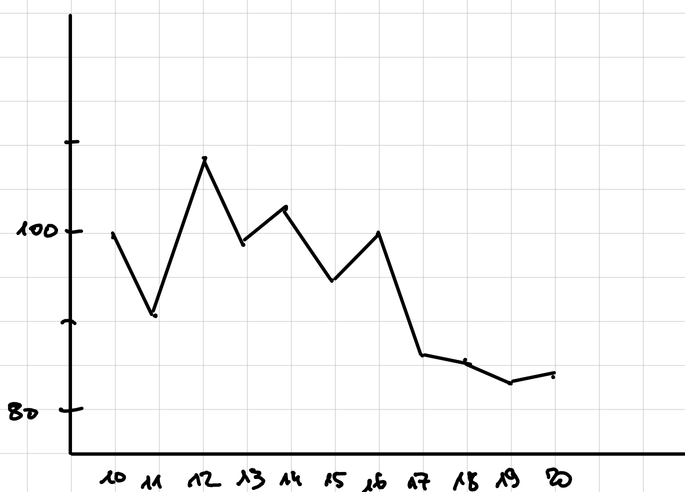
$x_i = x_1 \ldots x_k$

Verižni index: $I_{t \setminus t-1} = 100 \cdot \frac{x_t}{x_{t-1}}$ $t = 2, \ldots, T$ (T dolžina stat. vrste)

Stopnja rasti: $S_{t \setminus t-1} = I_{t \setminus t-1} - 100\% = 100 \cdot \frac{x_t - x_{t-1}}{x_{t-1}}$

i)

| | $I_{t\backslash 2010}$ | $I_{t\backslash t-1}$ | $S_{t\backslash t-1}$ |
|---|---|---|---|
| 2010 | 100,0 | / | / |
| 2011 | 92,8 | 92,8 | -7,2 |
| 2012 | 108,1 | 116,5 | 16,5 |
| 2013 | 98,9 | 91,5 | -8,5 |
| 2014 | 103,1 | 104,2 | 4,5 |
| 2015 | 94,9 | 92,0 | -8 |
| 2016 | 99,8 | 101,9 | 1,9 |
| 2017 | 86,5 | 89,5 | -10,5 |
| 2018 | 86,0 | 99,5 | -0,5 |
| 2019 | 83,0 | 96,5 | -3,5 |
| 2020 | 84,4 | 101,6 | 1,6 |



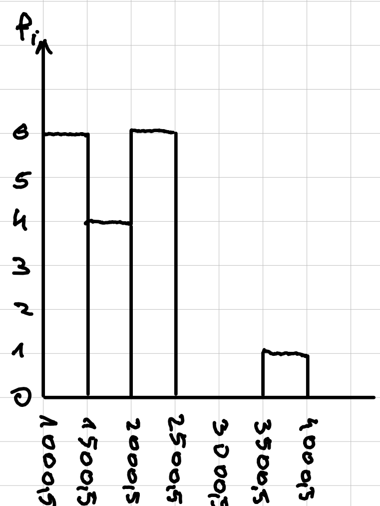

ii) DN

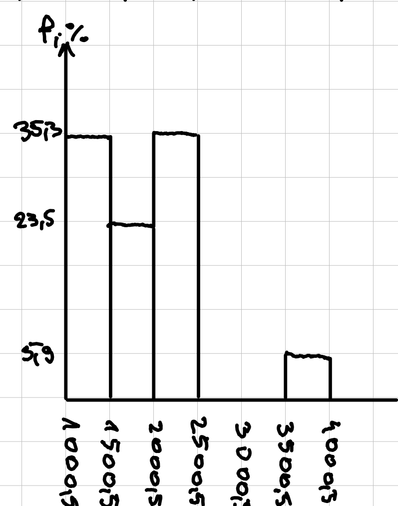
10.10.24

# Vaje 2

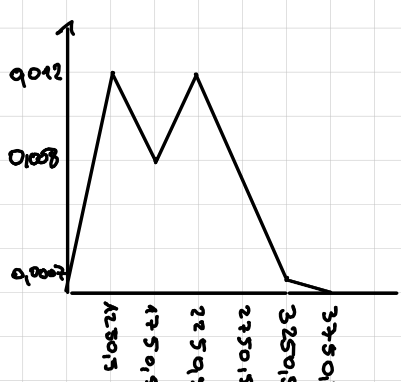
## 2.

| i | Razred | $f_i$ | $x_{i,min}$ | $x_{i,max}$ | $x_i$ | $d_i$ | $f_i\%$ |
|---|---|---|---|---|---|---|---|
| 1 | 1001-1500 | 6 | 1000,5 | 1500,5 | 1250,5 | 500 | 35,3 |
| 2 | 1501-2000 | 4 | 1500,5 | 2000,5 | 1750,5 | 500 | 23,5 |
| 3 | 2001-2500 | 6 | 2000,5 | 2500,5 | 2250,5 | 500 | 35,3 |
| 4 | 2501-3000 | 0 | 2500,5 | 3000,5 | 2750,5 | 500 | 0 |
| 5 | 3001-3500 | 0 | 3000,5 | 3500,5 | 3250,5 | 500 | 0 |
| 6 | 3501-4000 | 1 | 3500,5 | 4000,5 | 3750,5 | 500 | 5,9 |

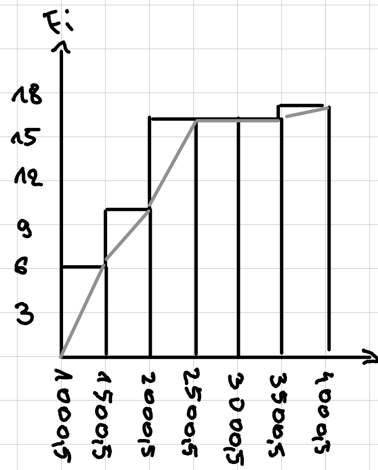


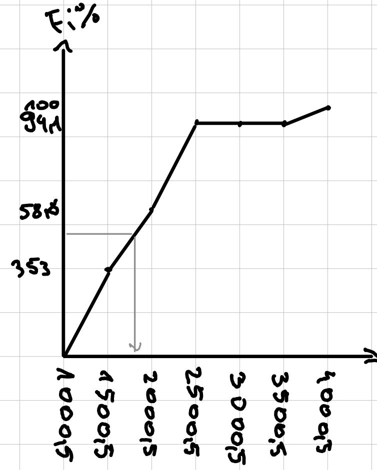




| i | Nova porazdelitev | nova $d_i$ | $g_i$ |
|---|---|---|---|
| 1 | 6 | 500 | 0,012 |
| 2 | 4 | 500 | 0,008 |
| 3 | 6 | 500 | 0,012 |
| 4 | | | |
| 5 | 1 | 1500 | 0,0007 |
| 6 | | | |



| i | $F_i$ | $F_i\%$ |
|---|---|---|
| 1 | 6 | 35,3 |
| 2 | 10 | 58,8 |
| 3 | 16 | 94,1 |
| 4 | 16 | 94,1 |
| 5 | 16 | 94,1 |
| 6 | 17 | 100 |




3.

Ranžirna vrsta

| | 16,2 | 16,8 | 17,1 | 17,8 | 17,9 | 18,2 | 18,5 | 19,0 | 19,1 | 19,5 | 19,9 | 20,0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| R: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| r: | 0,036 | | | | | | | | 0,607 | | | |

| | 20,3 | 20,3 |
|---|---|---|
| R: | 13 | 14 |

$\underbrace{13 \quad 14}_{13,5}$

$R(Q_{0,25}) = 0,25 \cdot 14 + 0,5 = 4$

$R(Q_{0,50}) = 0,5 \cdot 14 + 0,5 = 7,5$

$Q_{0,50} = 7$

$R(a) < R(Q) < R(b)$

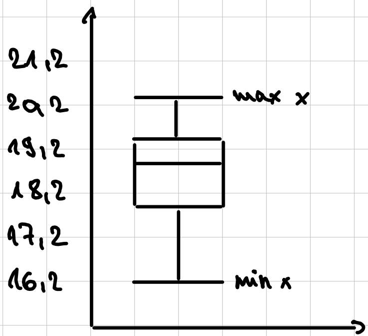
$Q = a + (R(Q) - R(a))(b-a)$

$R(Q_{0,30}) = 0,3 \cdot 14 + 0,5 = 4,7$

$Q_{0,30} = 17,8 + (4,7 - 4)(17,8 - 17,9)$

$= 17,8 + 0,7 \cdot 0,1$

$= 17,87$




5.

a) 22°C. b) Boxplot za temp. v juniju simetrična.

c) 2 10 in 400. d) Približno 2. e) Temp. M/A najmanj.
Small mero..J največ.

24. 10. 24

4.

kvantil: $Q_\gamma$

$F(Q_\gamma) = n \cdot Q_\gamma + 0.5$

$Q_p = x_{0,min} + \frac{F(Q_p) - F(x_{0,min})}{f_0} \cdot d_0$

Če kvantil $Q_p$ leži v razredu s sp. mejo $x_{0,min}$, zg. mejo $x_{0,max}$, frekvenco $f_0$ in širino $d_0$

| $x_{i,min}$ | $x_{i,max}$ | $f_i$ | $F_i$ | $f_i\%$ |
|---|---|---|---|---|
| 0 | 9,5 | 17 | 17 | |
| 9,5 | 19,5 | 35 | 52 | |
| 19,5 | 29,5 | 42 | 84 | |
| 29,5 | 39,5 | 56 | 150 | |
| 39,5 | 49,5 | 72 | 222 | |
| 49,5 | 59,5 | 113 | 335 | |
| 59,5 | 69,5 | 85 | 420 | |
| 69,5 | 79,5 | 46 | 466 | |
| 79,5 | 89,5 | 24 | 490 | |
| 89,5 | 99,5 | 10 | 500 | |

$$F(Q_{0,25}) = 500 \cdot 0{,}25 + 0{,}5 = 125{,}5$$

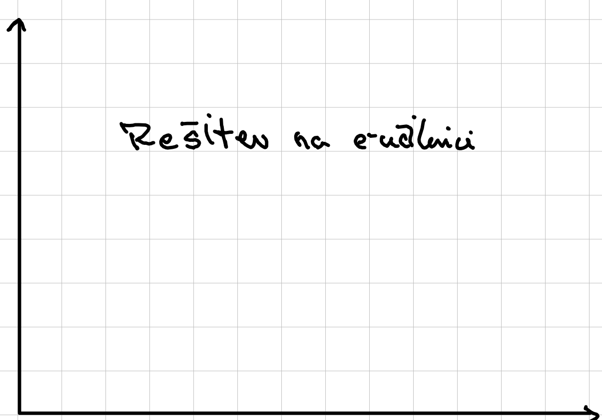
$$Q_{0,25} = 29{,}5 + \frac{125{,}5 - 84}{56} \cdot 10 = 35{,}1$$

$$F(Q_{0,5}) = 500 \cdot 0{,}5 + 0{,}5 = 250{,}5$$

$$Q_{0,50} = 49{,}5 + \frac{250{,}5 - 222}{113} \cdot 10 = 52{,}0$$

$$F(Q_{0,75}) = 500 \cdot 0{,}75 + 0{,}5 = 375{,}5$$

$$Q_{0,75} = 59{,}5 + \frac{375{,}5 - 335}{85} \cdot 10 = 64{,}3$$



# Vaje 3

1.

a) Mere centralne tendence:

Povprečje (aritmetična sredina):

vzorčno povprečje: $\bar{x} = \frac{x_1 + \dots + x_n}{n}$

$$\bar{x} = \frac{178,4 + 165,5 + 159,4 + \dots + 167,4}{12} = 170,7$$

populacijsko povprečje: $\mu = \frac{x_1 + \dots + x_N}{N}$

Mediana:

$$Me = \frac{170 + 171,8}{2} = 170,9$$

Modus: Ne obstaja

b) Mere variabilnosti

Variacijski razmik

$$VR = x_{max} - x_{min} = 179,6 - 159,4 = 20,2$$

IQR:

$$IQR = Q_{0,75} - Q_{0,25} \qquad Q = a + (R(Q) - R(a))(b - a)$$

$$R(Q_{0,75}) = 12 \cdot 0,75 + 0,5 = 9,5$$
$$Q_{0,75} = 175,3 + (9,5 - 9)(175,6 - 175,3) = 175,45$$

$$R(Q_{0,25}) = 12 \cdot 0,25 + 0,5 = 3,5$$
$$Q_{0,25} = \frac{165,5 + 167,4}{2} =$$

Varianca:

Pop. varianca:

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$$

Pop. standardni odklon:

$$\sigma = \sqrt{\sigma^2}$$

Vzorčna varianca:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Vzorčni standardni odklon:

$$s = \sqrt{s^2}$$

(V R-ju:
- sd(x) - vzorčni st. odklon
- var(x) - vzorčna varianca)

$$s^2 = \frac{1}{12-1}\left((178{,}4-170{,}7)^2 + \ldots + (167{,}4-170{,}7)^2\right)$$

$$= \frac{1}{11}(7{,}7^2 + 5{,}2^2 + 11{,}3^2 + 1{,}9^2 + 4{,}6^2 + 0{,}7^2 + 2{,}5^2 + 4{,}9^2 + 7{,}4^2 + 1{,}1^2 + 8{,}9^2 + 3{,}3^2)$$

„Poročajte ustrezne mere variabilnosti."

Če je sim. porazdelitev: AS in st. odklon in varianco

Če je asim. porazdelitev: Me in IQR

2.

a) $VR = 99 - 12 = 87$

$IQR = Q_3 - Q_1 = 74 - 31 = 43$

$Q_3 = 70 \cdot 0{,}75 + 0{,}5 = 53$ $\quad$ $Q_1 = 70 \cdot 0{,}25 + 0{,}5 \approx 31$

b)

| $x_{i,min}$ | $x_{i,max}$ | $f_i$ | $F_i$ | $x_i$ |
|---|---|---|---|---|
| 9,5 | 19,5 | 7 | 7 | 14,5 |
| 19,5 | 29,5 | 4 | 11 | 24,5 |
| 29,5 | 39,5 | 14 | 25 | 34,5 |
| 39,5 | 49,5 | 11 | 36 | 44,5 |
| 49,5 | 59,5 | 8 | 44 | 54,5 |
| 59,5 | 69,5 | 3 | 47 | 64,5 |
| 69,5 | 79,5 | 7 | 54 | 74,5 |
| 79,5 | 89,5 | 6 | 60 | 84,5 |
| 89,5 | 99,5 | 10 | 70 | 94,5 |

c)

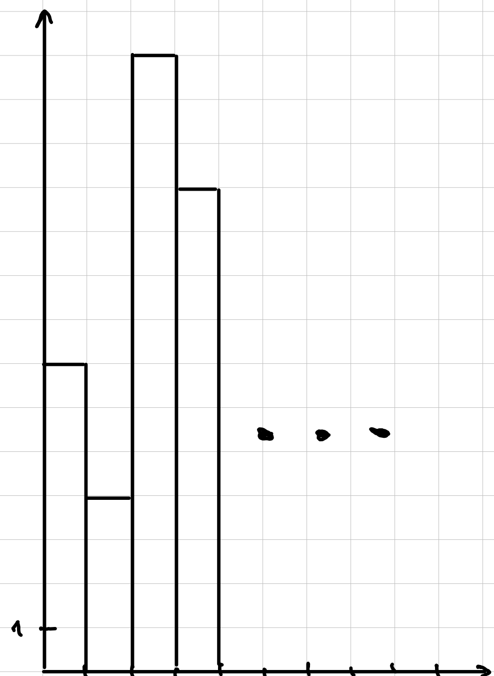
$$\mu = \frac{f_1 x_1 + \ldots + f_k x_k}{n}$$ $k$ = št. razredov, $x_i$ = sred. razreda

$$\mu = \frac{7 \cdot 14{,}5 + \ldots + 10 \cdot 94{,}5}{70} = 53{,}9$$

Tehtana varianca:

$$\sigma^2 = \frac{1}{70} \cdot \left(7 \cdot (14{,}5 - 53{,}9)^2 + \ldots + 10 \cdot (94{,}5 - 53{,}9)^2\right) = DN$$

e)




3. (ustno odgovarjanje - napisane le opombe)

d) desno asim. zato $Me < \bar{x}$

Vaje 4 7.11.'24

Slučajne spremenljivke

Diskretne sluč. spr. zavzemajo diskretne vrednosti, npr. v mn. $\mathbb{N}$, $\mathbb{Z}$, $\{1, 2, \ldots, n\}, \ldots$

1.

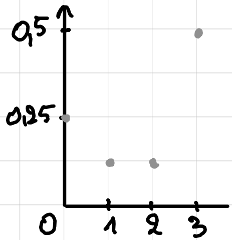
$P(X=0) = 0{,}25$ Možne vrednosti: 0, 1, 2, 3

$P(X=1) = 0{,}125$ Verjetnost: $0 \leq p < 1$ $\sum_{i=1}^{k} p_i = 1$

$P(X=2) = 0{,}125$

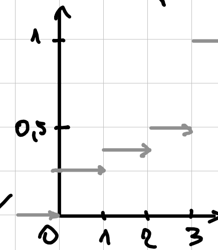
$P(X=3) = 0{,}5$

Graf fje verjetnosti




$p(x) = P(X=x)$

Graf porazdelitvene fje




$F(x) = P(X \leq x)$

**Lastnosti porazdelitvene fje:**
- Pri diskr. sluč. spr. je stopničaste oblike
- $\lim_{x \to -\infty} F(x) = 0$, $\lim_{x \to \infty} F(x) = 1$
- Nepadajoča

$P(X \leq 0) = 0{,}25$
$P(X \leq 1) = P(X=0) + P(X=1)$
$= 0{,}25 + 0{,}125$
$= 0{,}375$
$P(X \leq 2) = 0{,}5$
$P(X \leq 3) = 1$

2.

$X$ = št. padlih šestic v 5 metih igralne kocke

Vrednosti $X$: 0, 1, 2, 3, 4, 5

a) $P(X=5) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \left(\frac{1}{6}\right)^5 = 0{,}00013$

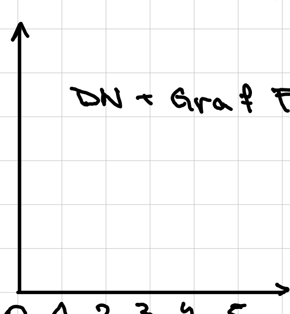
b) $P(X \geq 4) = P(X=4) + P(X=5) = \binom{5}{4} \cdot \left(\frac{1}{6}\right)^4 \cdot \frac{5}{6} + \left(\frac{1}{6}\right)^5 = 0{,}0033$

c) $X \sim B(n,p)$ (št. uspehov v $n$ poskusih) $n$ = št. poskusov
$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$ $p$ = verjetnost uspeha

$X \sim B(5, \frac{1}{6})$

$P(X=0) = 0{,}40188$
$P(X=1) = 0{,}40188$
$P(X=2) = 0{,}16075$
$P(X=3) = 0{,}03215$
$P(X=4) = 0{,}0033$
$P(X=5) = 0{,}00013$

DN: Graf $F(X)$



d) $E(X)=\sum_{i=1}^{\infty} k\cdot P(X=k)$, $Var(X)=E(X^2)+E^2(X)=E((X-E(X))^2)$

$$\boxed{\begin{aligned}&\text{Primer iz 1. naloge:}\\ &E(X)=0\cdot 0{,}25+1\cdot 0{,}125+2\cdot 0{,}125+3\cdot 0{,}5\\ &\quad=0+0{,}125+0{,}250+1{,}500\\ &\quad=1{,}875\end{aligned}}$$

$X\sim B(n,p) \Rightarrow E(X)=n\cdot p$, $Var(X)=n\cdot p\cdot(1-p)$

$E(X)=5\cdot\frac{1}{6}=\frac{5}{6}=0{,}8\overline{3}$, $Var(X)=69\overline{4}$

3.

$X=$ št. živih miši, $X\sim B(10,0.2)$

a) $P(X=1)=0{,}268$

b) $P(X=0)=0{,}107$

Nasprotje od vsaj 1 jenob ena

c) $P(X\geq 1)=P(X=1)+\ldots+P(X=10)=\underbrace{1-P(X=0)}=0{,}893$

d) $E(X)=n\cdot p=10\cdot 0{,}2=2$

4.

$X \sim Poisson(\lambda)$; $P(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$ $\forall k \in 0,1,\dots$; $E(X)=\lambda$, $Var(X)=\lambda$

Poisson($\lambda$) je št. sluč. dogodkov v fiks. čas. intervalu

$X$ = št. mutacij celic v 1h

$\lambda = 0,6$

$P(X=3) = \frac{0,6^3 e^{-0,6}}{3!} = \frac{0,216 \cdot 0,5488}{6} = 0,0198$

$P(X=10) = \frac{0,6^{10} e^{-0,6}}{10!} =$

$E(X) = 0,6$

5.

$E(A) = n \cdot p = 5 \Rightarrow n = 0,5, p = 10$

$E(B) = n \cdot p = 50 \Rightarrow n = 0,5, p = 100$

$E(C) = n \cdot p = 1 \Rightarrow n = 0,1, p = 10$

$E(D) = n \cdot p = 10 \Rightarrow n = 0,1, p = 100$

6.

$X \sim Geom(p)$, $P(X=k) = (1-p)^{k-1} p$ $\forall k \in 1,2,\dots$, $E(X) = \frac{1}{p}$, $Var(X) = \frac{1-p}{p^2}$

$Geom(\frac{1}{4})$

$P(X=5) = (1-\frac{1}{4})^{5-1} \cdot \frac{1}{4} = \frac{81}{64} \cdot \frac{1}{4} = 0,079$

$E(X) = \frac{1}{p} = \frac{1}{\frac{1}{4}} = \frac{4}{1} = 4$

$Var(X) = \frac{1-p}{p^2} = \frac{1-\frac{1}{4}}{(\frac{1}{4})^2} = \frac{\frac{3}{4}}{(\frac{1}{4})^2} = 12$

7.

a) Poissonova b) Binomska c) Geometrična

# Vaje 5 

14.11.24

Zvezne sluč. spr.

$P(a \leq X \leq b) = P(a < X < b) = \int_a^b \underbrace{f_X(x)}_{\text{gostota}}\,dx \qquad \int_{-\infty}^{\infty} f(x)\,dx = 1$

$P(X = a) = 0 \quad \forall a$

Porazdelitvena f-ja

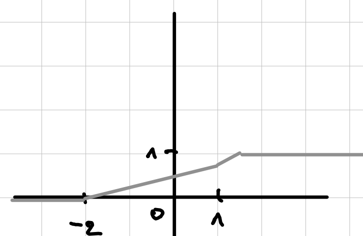
$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f_X(t)\,dt \qquad F_X'(x) = f_X(x)$

- $F(x)$ je nepadajoča
- $\lim_{x\to-\infty} F(x) = 0, \ \lim_{x\to\infty} F(x) = 1$

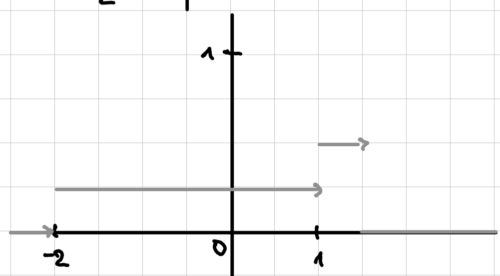
$P(a \leq X \leq b) = F(b) - F(a)$

1.

$$F(x) = \begin{cases} 0, & \text{če } x < -2 \\ \frac{1}{4}x + \frac{1}{2}, & \text{če } -2 \leq x < 1 \\ \frac{1}{2}x + \frac{1}{4}, & \text{če } 1 \leq x < 1{,}5 \\ 1, & \text{če } x \geq 1{,}5 \end{cases}$$



$$f(x) = F'(x) = \begin{cases} 0, & \text{če } x < -2 \\ \frac{1}{4}, & \text{če } -2 \leq x < 1 \\ \frac{1}{2}, & \text{če } 1 \leq x < 1{,}5 \\ 0, & \text{če } x \geq 1{,}5 \end{cases}$$



2.

$$f(x) = \begin{cases} cx^2, & 0 < x < 2 \\ 0, & \text{sicer} \end{cases}$$

a) $c = ?$

$\int_0^2 cx^2\,dx = 1 \qquad \int_0^2 cx^2\,dx = c \cdot \frac{x^3}{3}\Big|_0^2 = c \cdot \frac{2^3}{3} - c \cdot \frac{0^3}{3}$

$= c \cdot \frac{8}{3} \Rightarrow c = \frac{3}{}$

b)

$$F(x) = \int_{-\infty}^{x} f(t)\,dt \Rightarrow F(x) = \int_0^x \frac{3}{8} t^2\,dt$$

$$= \frac{3}{8} \frac{t^3}{3} \Big|_0^x = \frac{x^3}{8}$$

c)

$$P(0 \leq X \leq 1) = F(1) - F(0) = \frac{1}{8}$$

$$P(1 \leq X \leq 2) = F(2) - F(1) = \frac{8}{8} - \frac{1}{8} = \frac{7}{8}$$

$$Var(X) = E(X^2) - E^2(X), \quad E(X) = \int_{-\infty}^{\infty} x f(x)\,dx,$$

$$E(g(x)) = \int_{-\infty}^{\infty} g(x) f(x)\,dx$$

$$E(X^2) = \int_0^2 x^2 \cdot \frac{3}{8} x^2\,dx = \frac{3}{8} \frac{x^5}{5} \Big|_0^2 = \frac{3}{8} \cdot \frac{2^5}{5} = \frac{12}{5}$$

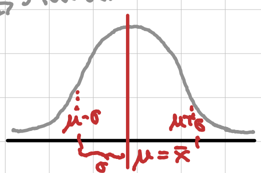
$$E^2(X) = \left(\int_0^2 x \cdot \frac{3}{8} x^2\,dx\right)^2 = \left(\frac{3}{8} \cdot \frac{x^4}{4}\Big|_0^2\right)^2 = \left(\frac{3}{8} \cdot \frac{16}{4}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$Var(X) = \frac{12}{5} - \frac{9}{4} = \frac{48-45}{20} = \frac{3}{20}$$

3. Normalna porazdelitev

$X \sim N(\mu, \sigma^2) = N(65, 25) \Rightarrow \mu = 65, \ \sigma = 5$ $\Rightarrow$ stand. odklon

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$




Standardna normalna porazdelitev

$Z \sim X(0,1)$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Če $X \sim N(\mu, \sigma^2) \Rightarrow \frac{X-\mu}{\sigma} \sim N(0,1)$

a) $\geq 70$

$P(X \geq 70) = P\left(\frac{X-65}{5} \geq 1\right) = P(Z \geq 1) = 1 - 0{,}8413 = 0{,}1587$

b) $\leq 70$

$P(X \leq 70) = P\left(\frac{X-65}{5} \leq 1\right) = P(Z \leq 1) = 0{,}8413$

$= Z$

c) $\leq 120$

$P(X \leq 120) = P\left(\frac{X-65}{5} \leq 11\right) = 1$

d) $70 < X < 120$

$P(70 \leq X \leq 120) = P(X \leq 120) - P(X \leq 70) = 1 - 0{,}8413 = 0{,}1587$

1. kvartil: $P(X \leq x) = 0{,}25$

$$P\left(\frac{X-65}{5} \leq \frac{x-65}{5}\right) = 0{,}25$$

$$P\left(Z \leq \frac{x-65}{5}\right) = 0{,}25$$

$$\frac{x-65}{5} = -0{,}67$$

$$x = -0{,}67 \cdot 5 + 65 = 61{,}65$$

4. $X \sim N(\mu, \sigma^2)$; $\mu = 13$, $\sigma = 3$

a) $< 12$

$P(X < 12) = P\left(\frac{X-13}{3} < -\frac{1}{3}\right) = 0{,}3707$

b) $10 \leq X \leq 15$

$P(10 \leq X \leq 15) = P(X \leq 15) - P(X \leq 10) = P\left(\frac{X-13}{3} \leq \frac{2}{3}\right) - P\left(\frac{X-13}{3} \leq -1\right)$

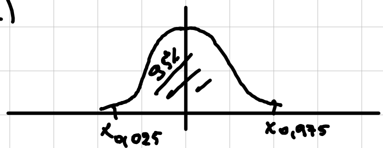
$= 0{,}7486 - 0{,}1587 = 0{,}5899$

c) $\geq 15$

$P(X \geq 15) = 1 - P(X < 15) = 1 - 0{,}7486 = 0{,}2514$

5. $X \sim N(500, 2500)$

a)




$P(X \le x) = 0{,}975$

$P\left(\frac{X-500}{50} \le \frac{x-500}{50}\right) = 0{,}975$

$\frac{x-500}{50} = 1{,}96$

$x = 1{,}96 \cdot 50 + 500 = 598$

$P\left(\frac{X-500}{50} \le \frac{x-500}{50}\right) = 0{,}025$

$\frac{x-500}{50} = -1{,}96$

$x = -1{,}96 \cdot 50 + 500 = 402$

$x \in [402, 598]$

21.11.24

# Vaje 6

1. (5. od zadnjič)

b) $P(X \le x) = 0{,}995$

$P\left(\frac{X-500}{50} \le \frac{x-500}{50}\right) = 0{,}995$

$\frac{x-500}{50} = 2{,}575 \Rightarrow 500 - 2{,}57 \cdot 50$

$x_2 = 500 + 2{,}57 \cdot 50$

c) $P(X \le x) = 0{,}05$

$P\left(\frac{X-500}{50} \le \frac{x-500}{50}\right) = 0{,}05$

$\frac{x-500}{50} = -1{,}645$

$x = 500 - 1{,}65 \cdot 50$

d) $P(X \ge x) = 0{,}5$

e) $P(X = x) = 0$

f) $P(X \ge 450) = 1 - P(X < 450) = 1 - P\left(\frac{X-500}{50} \le -1\right) = 1 - 0{,}1587 = 0{,}84$

2.

$K = L \cdot 0{,}45359237$

$L \sim N(500, 50^2)$

$K \sim ?$

$K \sim N(226{,}8; 22{,}7^2)$

$K = c \cdot L \Rightarrow K \sim N(?, ?)$

$E(K) = E(cL) = c \cdot E(L)$

$Var(K) = Var(cL) = E((cL)^2) - E^2(cL)$

$= E(c^2 L^2) + (cE(L))^2$

$= c^2 E(L^2) + c^2 E(L)^2$

$= c^2 (E(L^2) + E(L)^2)$

3.

$X \sim \underbrace{Exp(\lambda)}_{\text{zvezna sluč. spr.}}$, $f_X(x) = \lambda e^{-\lambda x}, x > 0$

$F_X(x) = 1 - e^{-\lambda x}; x \geq 0$

$E(X) = \frac{1}{\lambda}$

$Var(X) = \frac{1}{\lambda^2}$

$F(x) = P(X \leq x)$

$X$ = čas do 1. mutacije

$X \sim Exp(\lambda)$

$\Downarrow$

$X \sim Exp(\frac{1}{5})$

$E(X) = 5 \Rightarrow 5 = \frac{1}{\lambda}$

$5\lambda = 1$

$\lambda = \frac{1}{5}$

a) $P(X < 3) = F(3)$

$P(X < 3) = 1 - e^{-\frac{1}{5} 3}$

$= 0{,}45$

b) $P(X > 6) = 1 - P(X \leq 6) = 1 - F(6)$

$= 1 - (1 - e^{-\frac{1}{5} 6}) = 0{,}30$

$sd(X) = \sqrt{Var(X)} = \sqrt{\frac{1}{(\frac{1}{5})^2}} = \sqrt{25} = 5$

$F(x) = \int_{-\infty}^{x} f_X(t)\,dt = \int_0^x \lambda e^{-\lambda t}\,dt = \left| \begin{matrix} u = -\lambda t \\ du = -\lambda dt \\ dt = \frac{du}{-\lambda} \end{matrix} \right| = \lambda \int_0^{-\lambda x} e^u \frac{du}{-\lambda} = -\int_0^{-\lambda x} e^u\,du$

$= \int_{-\lambda x}^{0} e^u\,du = e^u \Big|_{-\lambda x}^{0} = 1 - e^{-\lambda x}, x \geq 0$

5.

$X$ = masa zavitka masla

$X \sim N(250, 100)$

a) $P(X \geq 242) = 1 - P(X < 242) = 1 - P\left(\frac{X-250}{10} < \frac{242-250}{10}\right)$

$= 1 - P(Z < \frac{8}{10}) = 1 - 0{,}21 = 0{,}79$

b) $n = 25$ $\quad \bar{X}$ = vzorčno povprečje

$\bar{X} = \frac{x_1 + \dots + x_n}{n} \qquad \bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$

$P(248 < \bar{X} < 252{,}5) = P(\bar{X} < 252{,}5) - P(\bar{X} \leq 248) =$

$= P\left(\frac{\bar{X}-250}{\frac{10}{\sqrt{25}}} < \frac{252{,}5-250}{\frac{10}{\sqrt{25}}}\right) - P\left(\frac{\bar{X}-250}{\frac{10}{\sqrt{25}}} \leq \frac{248-250}{\frac{10}{\sqrt{25}}}\right) =$

$= 0{,}8844 - 0{,}1587 = 0{,}7357$

6.

$X$ = vrednost beljakovin v mleku, $X \sim N(\mu, \sigma^2)$

$\bar{x} = 3{,}15\%$ $\quad sd(X) = 0{,}30\%$ $\qquad X \sim N(3{\cdot}15, 0{\cdot}30^2)$

$n = 4$ $\qquad X \sim N(3{\cdot}15, 0{\cdot}08)$

$P(\bar{X} < 3) = P\left(\frac{\bar{X}-3{,}15}{\frac{0{,}3}{\sqrt{4}}} < \frac{3-3{,}15}{\frac{0{,}3}{\sqrt{4}}}\right) = P(Z < -1) = 0{,}1587$

$P(3 \leq \bar{X} \leq 3{,}2) = P(\bar{X} \leq 3{,}2) - 0{,}1587 = P(Z < \frac{0{,}05}{0{,}15}) - 0{,}1587$

$= P(Z < 0{,}33) - 0{,}1587 = 0{,}6293 - 0{,}1587$

$= 0{,}4706$

$P(\bar{X} > 3{,}2) = 1 - P(\bar{X} \leq 3{,}2) = 1 - 0{,}6293 = 0{,}3707$

5. b) Velikost vzorca, da bo 80% vzorčnih $\bar{x}$ med 247,5 in 252,5?

$$P(247{,}5 < \bar{X} < 252{,}5) = P\left(\frac{\bar{X}-250}{\frac{10}{\sqrt{n}}} < \frac{252{,}5-250}{\frac{10}{\sqrt{n}}}\right) - P\left(\frac{\bar{X}-250}{\frac{10}{\sqrt{n}}} < \frac{247{,}5-250}{\frac{10}{\sqrt{n}}}\right)$$

$$= \underbrace{2 \cdot P\left(\frac{\bar{X}-250}{\frac{10}{\sqrt{n}}} < \frac{252{,}5-250}{\frac{10}{\sqrt{n}}}\right) - 1 = 0{,}80}$$

$$2 \cdot P\left(\frac{\bar{X}-250}{\frac{10}{\sqrt{n}}} < \frac{252{,}5-250}{\frac{10}{\sqrt{n}}}\right) = 0{,}90 + 1 = 1{,}90$$

$$P\left(\frac{\bar{X}-250}{\frac{10}{\sqrt{n}}} < \frac{252{,}5-250}{\frac{10}{\sqrt{n}}}\right) = 0{,}95$$

$$P\left(Z < \frac{2{,}5\sqrt{n}}{10}\right) = 0{,}95 \qquad \frac{2{,}5\sqrt{n}}{10} = 1{,}65$$

$$\sqrt{n} = \frac{16{,}5}{2{,}5} = 6{,}6$$

$$n = 43{,}56$$

Vzorec mora biti velik 44.

4.

$P(X=-1) = \frac{1}{4}$, $P(X=0) = \frac{1}{4}$, $P(X=1) = \frac{1}{2}$, $Y = X^2 - 1$

a) $E(X) = -1 \cdot \frac{1}{4} + 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} = \frac{1}{4}$

$Var(X) = E(X^2) - E(X)^2 = 0 \cdot \frac{1}{4} + 1 \cdot \frac{3}{4} - \frac{1}{16} = \frac{3}{4} - \frac{1}{16} = \frac{11}{16}$

$X$:

| $k$ | $P(X=k)$ |
|---|---|
| $-1$ | $\frac{1}{4}$ |
| $0$ | $\frac{1}{4}$ |
| $1$ | $\frac{1}{2}$ |

$X^2$:

| $k$ | $P(X=k)$ |
|---|---|
| $0$ | $\frac{1}{4}$ |
| $1$ | $\frac{3}{4}$ |

# Vaje 7

Preizkušanje domnev o $\mu$ = pop. povprečje

1. $\mu = 1600$, $\sigma = 120$, $n = 100$, $\bar{X} = 1570$, $\alpha = 0{,}05$

a) $H_0: \mu = 1600$ $(\mu = \mu^*)$ — ničelna domneva/hipoteza

$H_1: \mu \neq 1600$ $(\mu \neq \mu^*)$ — alt. domneva/hipoteza

Ko je $\sigma$ poznan, uporabimo Z-test.

Testna stat.: $Z = \dfrac{\bar{X} - \mu^*}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$

$$Z = \frac{1570 - 1600}{\frac{120}{\sqrt{100}}} = \frac{-30}{12} = -2{,}5$$

| Alt. domneva | $H_0$ zavrnemo, če |
|---|---|
| $H_1: \mu \neq \mu^*$ | $\lvert Z \rvert > z_{1-\frac{\alpha}{2}}$ |
| $H_1^+: \mu > \mu^*$ | $Z > z_{1-\alpha}$ |
| $H_1^-: \mu < \mu^*$ | $Z < -z_{1-\alpha}$ |

$H_0$ zavrnemo, če

$\lvert Z \rvert > z_{1-0{,}025}$

$\lvert -2{,}5 \rvert > z_{0{,}975}$

$\lvert -2{,}5 \rvert > 1{,}96$ ✓

Naši podatki kažejo, da lahko zavrnemo $H_0$ in potrdimo $H_1$

b) $H_0: \mu = 1600$ $H_1: \mu < 1600$

$H_0$ zavrnemo, če: $Z < -z_{0{,}95}$

$-2{,}5 < 1{,}65$ ✓

Naši podatki kažejo, da je pričakovana doba manjša

c) $H_0: \mu = 1600$ $H_1: \mu > 1600$

$-2,5 > 1,65$ //

Življenska doba ni večja od pričakovane.

d) Stopnja zaupanja $\beta$: $\beta \cdot 100\%$ IZ, $0 < \beta < 1$

$(\mu_{min}, \mu_{max}) = (\bar{x} - \frac{\sigma}{\sqrt{n}} z_{\frac{1+\beta}{2}}, \bar{x} + \frac{\sigma}{\sqrt{n}} z_{\frac{1+\beta}{2}})$

$\beta = 0,95$:

$(\mu_{min}, \mu_{max}) = (1570 - 12 \cdot 1,96, 1570 + 12 \cdot 1,96)$

$= (1546'5, 1593'5)$

e) širina IZ 100h

$\frac{\sigma}{\sqrt{n}} \cdot z_{\frac{1+\beta}{2}} = 50$

$\frac{120}{\sqrt{n}} \cdot 1,96 = 50$

$\sqrt{n} = 4,704 \Rightarrow n = 22,13 \Rightarrow$ uporabimo 23

2. $\alpha = 0,05$, $\sigma = 0,02$, $n = 10$, $\bar{x} = 9,987$

$H_0: \mu = 10$ $H_1: \mu \neq 10$

$z = \frac{9,987 - 10}{\frac{0,02}{\sqrt{10}}} = \frac{0,013 \sqrt{10}}{0,02} = -2,06$ lahko zavrnemo $H_0$

3. $n = 10$, $\alpha = 0,1$, $\mu^* = 46,25$, $\mu = 46,483$, $\sigma$ ni znan

$H_0: \mu = 46,25$ $H_1^+: \mu > 46,25$

t-test

$$T = \frac{\bar{x} - \mu^*}{\frac{s}{\sqrt{n}}} \sim t(df)$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{9}\left((46{,}2 - 46{,}493)^2 + \ldots + (46{,}5 - 46{,}493)^2\right)$$
$$= 0{,}077$$
$$s = 0{,}28$$

$$T = \frac{46{,}493 - 46{,}25}{\frac{0{,}28}{\sqrt{10}}} = 2{,}74$$

| Alt. domneva | $H_0$ zavrnemo, če |
|---|---|
| $H_1: \mu \neq \mu^*$ | $\lvert T\rvert > t_{1-\frac{\alpha}{2}}(df = n-1)$ |
| $H_1^+: \mu > \mu^*$ | $T > t_{1-\alpha}(df = n-1)$ |
| $H_1^-: \mu < \mu^*$ | $T < -t_{1-\alpha}(df = n-1)$ |

$T > t_{1-0{,}1}(9)$ (V tabeli iščemo vrednost $\alpha$ in df)

$2{,}74 > t_{0{,}9}(9)$

$2{,}74 > 1{,}383$ Potrdimo alt. hipotezo.

4. $\alpha = 0{,}05$, $n = 20$, $\bar{x} = 30{,}6$

$H_0: \mu = 35$ $H_1: \mu < 35$

$$s^2 = \frac{1}{24}\cdot\sum_{i=1}^{n}\left(f_i(x_i - \bar{x})^2\right) = 46{,}9$$
$$s = 6{,}8$$

$$T = \frac{30{,}6 - 35}{\frac{6{,}8}{\sqrt{25}}} = -3{,}24$$

$H_0$ zavrnemo, če:

$T < -t_{1-0{,}05}(24)$

$-3{,}24 < -1{,}711$

Vaje 8 5.12.24

1.

Odvisna vzorca. Ali novo zdravilo učinkuje?

a) $H_0: \mu_{ZD} = \mu_{PL}$ $H_1: \mu_{ZD} < \mu_{PL}$

$H_0: \mu_{ZD} - \mu_{PL} = 0$ $H_1: \mu_{ZD} - \mu_{PL} < 0$

Zapišemo lahko tudi: $X = \text{zdravilo} - \text{placebo}$

$H_0: \mu_X = 0$ $H_1: \mu_X < 0$

$n = 10, \mu^* = 0 \Rightarrow$ t-test

$$T = \frac{\bar{X} - \mu^*}{\frac{S_X}{\sqrt{n}}} =$$

$$\bar{X} = \frac{2 - 35 + \ldots + 4}{10} = -11{,}2$$

$$S_X = \sqrt{\frac{1}{9}\left((2+11{,}2)^2 + (-4+11{,}2)^2 + \ldots + (4+11{,}2)^2\right)} = 14{,}34$$

$$T = \frac{-11{,}2 - 0}{\frac{14{,}34}{\sqrt{10}}} = \frac{-11{,}2}{4{,}53} = -2{,}47$$

$H_0$ zavrnemo, če: $T < -t_{1-\alpha}(df = n-1)$ (če $\alpha$ ni podana izberemo $\alpha = 0{,}05$)

$T < -t_{1-0{,}05}(9) = -1{,}833$

$-2{,}47 < -1{,}833$ ✓

Naši podatki kažejo da zdravilo učinkuje

2. DN

3. $\underbrace{95\%}_{\beta = 0{,}95}$

a) 95% interval zaupanja za pop. stand. odklon

$$\frac{n-1}{\sigma^2} S^2 \sim \chi^2(df = n-1)$$

$\left(\frac{n-1}{b} s^2, \frac{n-1}{a} s^2\right) \Rightarrow$ za varianco

$\left(\sqrt{\frac{n-1}{b}} s, \sqrt{\frac{n-1}{a}} s\right) \Rightarrow$ za st. odklon

$$a = \chi^2_{\frac{(1+\beta)}{2}} (df)$$

$$b = \chi^2_{\frac{1-\beta}{2}} (df)$$

$$\bar{x} = \frac{160 + 165 + \ldots + 170}{15} = 166{,}47$$

$$s = \sqrt{\frac{1}{14} \left((160 - 166{,}47)^2 + \ldots + (170 - 166{,}47)^2\right)} = 6{,}41$$

$$a = \chi^2_{\frac{1+0{,}95}{2}} (14) = \chi_{0{,}975}(14) = 5{,}63$$

$$b = \chi_{\frac{1-0{,}95}{2}} (14) = \chi_{0{,}025} (14) = 26{,}12$$

95% IZ za $\sigma$: $\left(\sqrt{\frac{15-1}{26{,}12}} \cdot 6{,}41, \sqrt{\frac{15-1}{5{,}63}} \cdot 6{,}41\right) = (4{,}6; 10{,}2)$

b)

$H_0: \sigma = 7{,}2$ $\qquad$ $H_1: \sigma \neq 7{,}2$

Testna stat.: $\chi^2 = \frac{(n-1) s^2}{(\sigma^*)^2}$ $\qquad$ $\chi^2 = \frac{14 \cdot 6{,}41^2}{7{,}2^2} = 11{,}1$

| $H_1$ | $H_0$ zavrnemo, če |
|---|---|
| $H_1^+: \sigma > \sigma^*$ | $\chi^2 > \chi^2_\alpha (df = n-1)$ |
| $H_1^-: \sigma < \sigma^*$ | $\chi^2 < \chi^2_{\alpha-1} (df = n-1)$ |
| $H_1: \sigma \neq \sigma^*$ | $\chi^2 < \chi_{1-\frac{\alpha}{2}}(df)$ ali $\chi^2 > \chi^2_{\frac{\alpha}{2}} (df)$ |

$H_0$ zavrnemo, če

$$\chi^2 < \chi^2_{1-\frac{0,05}{2}}(14) = \chi^2_{0,975}(14) = 5,63$$
$$11,1 \not< 5,63$$

$$\chi^2 > \chi^2_{\frac{0,05}{2}}(14) = \chi^2_{0,025}(14) = 26,12 \not< 11,1$$

Naši podatki niso stat. značilni.

4.

Ali je delež modrookih študentov UP FAMNIT različen od povp. v Sloveniji?

$H_0: p = 0,17$ $\quad\quad$ $H_1: p \neq 0,17$

Z-test → ker imamo delež

$$Z = \frac{x - n \cdot p^*}{\sqrt{n \cdot p^*(1-p^*)}}$$ ($x$ = št. osebkov iz vzorca z izbrano last.)

$$Z = \frac{49 - 165 \cdot 0,17}{\sqrt{165 \cdot 0,17 \cdot 0,83}} = 4,34$$

$H_0$ zavrnemo, če

$$|Z| > z_{1-\frac{\alpha}{2}} = z_{0,975} = 1,96$$

$$|4,34| > 1,96 \checkmark$$

$H_0$ lahko zavrnemo, ker so naši pod. stat. značilni. Zato vemo da se podatki ne skladajo

5.

$H_0: p = 0{,}85 \qquad H_1^-: p < 0{,}85$

$$z = \frac{41 - 50 \cdot 0{,}85}{\sqrt{50 \cdot 0{,}85 \cdot 0{,}15}} = -0{,}594$$

$H_0$ zavrnemo, če

$z < -z_{1-\alpha} = -z_{0,95} = -1{,}65$

$-0{,}594 < 1{,}65$ $\not\!\!/$

Naši podatki niso stat. značilni, zato ne moremo trditi da semenarna laže.

6.

$n = 15$

c) $H_0$ zavrnemo, če $|T| > t_{1-\frac{\alpha}{2}}(df = n-1) = t_{1-0{,}025}(14) = 2{,}145$

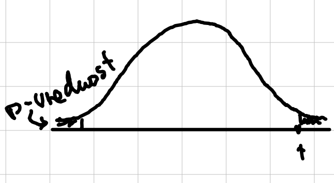
$H_0$ ne moremo zavrniti, če je $-2{,}145 < T < 2{,}145$

$H_0$ sprejmemo (nikoli)

Vaje 9 12.12.24

p-vrednost = verjetnost, da ob veljavni $H_0$ dobimo rezultat, ki smo ga dobili, ali bolj ekstremen rezultat

$p = P(|T| > t \mid H_0 \text{ velja})$, $t$ = vrednost, ki smo jo dobili




$p < \alpha \Rightarrow H_0$ zavrnemo
podatki stat. značilni

$p \geq \alpha \Rightarrow H_0$ obdržimo
podatki niso stat. značilni

1.

Stat. test: t-test za neodvisna vzorca

$H_0: \mu_S = \mu_{NS}$ $H_1: \mu_S \neq \mu_{NS}$

$$\frac{\max(s_S^2, s_{NS}^2)}{\min(s_S^2, s_{NS}^2)} < 2$$

$1{,}07 < 2$

$$T = \frac{(\bar{x}_{NS} - \bar{x}_S) - (\mu_{NS} - \mu_S)}{s\sqrt{\frac{1}{n_{NS}} + \frac{1}{n_S}}}$$

$$s = \sqrt{\frac{s_{NS}^2 + s_S^2}{2}}$$

$$T \sim t(df = n_{NS} + n_S - 2)$$

$$T = \frac{(15-14) - 0}{5{,}8 \cdot \sqrt{\frac{1}{28} + \frac{1}{10}}} \approx 0{,}47$$

$T \sim t(df = 36)$

$H_0$ zavrnemo, če: $|T| > t_{1-\frac{\alpha}{2}}(36)$

$|0{,}47| > t_{1-0{,}025}(36$

$0{,}47 > 2{,}028$ //

$H_0$ obdržimo, podatki niso stat. značilni

2.

$H_0: \mu_B = \mu_A$ $H_1: \mu_B < \mu_A$

t-test za neodvisna vzorca

$$\frac{\max(s_A^2, s_B^2)}{\min(s_A^2, s_B^2)} = \frac{162{,}1}{79{,}6} = 2{,}04 > 2$$

$$T = \frac{(\bar{x}_B - \bar{x}_A) - (\mu_B - \mu_A)}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}} \sim t(df = \min(n_A - 1, n_B - 1)$$

$$T = \frac{(16{,}1 - 12{,}3) - 0}{\sqrt{\frac{162{,}1}{90} + \frac{79{,}6}{80}}} = 2{,}3$$

$H_0$ zavrnemo, če: $T > t_{1-\alpha}$ $(df = \min(n_A - 1, n_B - 1))$

$T > t_{1-0,05}$ $(df = 89)$

$2,3 > 1,664$

$H_0$ zavrnemo, naši podatki so stat. značilni.
Naši podatki kažejo, da je povp. količina žvepla v smrekovih iglicah na lokaciji B večja kot povp. količina žvepla v smrekovih iglicah na lokaciji A.

3. Primerjava Bernulijevih verjetnosti (Primerjava deležev)

$H_0: p_A = p_B$

$H_1^+: p_A > p_B$

$p_A - p_B > 0$

$\hat{p}_A = \frac{x_A}{n_A} = \frac{87}{150} = 0,58$

$\hat{p}_B = \frac{x_B}{n_B} = \frac{33}{100} = 0,33$

$\hat{p} = \frac{x_A + x_B}{n_A + n_B} = \frac{120}{250} = 0,48$

$$Z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}}$$

$$Z = \frac{0,58 - 0,33}{\sqrt{0,48(1-0,48)\left(\frac{1}{150} + \frac{1}{100}\right)}} = 3,88$$

$H_0$ zavrnemo, če: $Z > z_{1-\alpha}$

$3,88 > z_{1-0,05} = z_{0,95}$

$3,88 > 1,645$

$H_0$ zavrnemo in potrdimo $H_1$, ker so pod. stat. značilni.
Naši podatki kažejo, da je kaljivost sorte A boljša od kaljivosti sorte B.

4.

$H_0: p_A = p_B$

$H_1: p_A \neq p_B$

$$Z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}}$$

$\hat{p}_A = 0,08$, $\hat{p}_B = 0,117$

$\hat{p} = 0,097$

$$Z = \frac{0,08 - 0,117}{\sqrt{0,097 \cdot 0,903\left(\frac{1}{180} + \frac{1}{200}\right)}} = -1,203$$

# Vaje 10

Linearna regresija

- Linearna odvisnost spr. Y od spr X


razsevni grafikon

v populaciji:

sluč. vplivi

$$Y = \alpha + \beta X + \varepsilon$$

z vzorcem ocenimo regresijsko premico:

$$\hat{y} = a + bX$$

Metoda najmanjših kvadratov

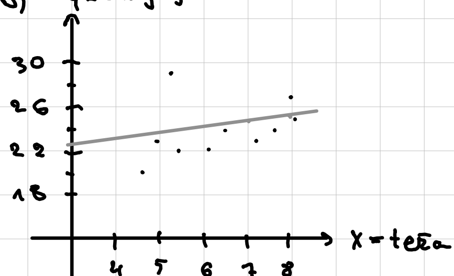
$$\min \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \min \sum_{i=1}^{n} (y_i - a - bx_i)^2 = \min S(a,b)$$

$$\frac{\partial S}{\partial a} = 0, \quad \frac{\partial S}{\partial b} = 0 \qquad b = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}, \quad a = \bar{y} - b\bar{x}$$

1.

a) lin. regresijo

b) $y$ = št. jajčec




c)

$b = 0{,}72$

$a = 19{,}24$

$\hat{y} = 19{,}24 + 0{,}72 \cdot X$

d)

$\hat{y} = 19{,}24 + 0{,}72 \cdot 7 = 24{,}28$

e)

Ne smemo izračunati, ker lahko računamo le na območju [min(X), min(Y)]

f) Koeficient determinacije

5,38 29 23,11

$$r^2 = \frac{\sum_{i=1}^{n} \hat{y}_i - n\bar{y}_i^2}{\sum_{i=1}^{n} y_i - n\bar{y}_i^2}, \quad 0 \le r^2 \le 1$$

24,54

23,65

22,66

$$r^2 = \frac{5704{,}75 - 10 \cdot 23{,}8^2}{5773 - 10 \cdot 23{,}8^2}$$

25,07

24,84

$$r^2 = \frac{\sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$

23,14

25,01

22,78

g) $H_0: \beta = 0 \qquad H_1: \beta \neq 0$

23,88

$$t = \frac{b - \beta_0}{s(b)} \qquad s(a) = \sqrt{s^2 \left( \frac{1}{n} + \frac{\bar{x}^2}{\sum_{i=1}^{n} (x - \bar{x})^2} \right)}$$

$$s = \sqrt{\frac{1}{n-2} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2}$$

$$s(b) = \sqrt{\frac{s^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2}}$$

$$s^2 = \frac{1}{8} \cdot \left( (29 - 23{,}11)^2 + \ldots + (24 - 23{,}88)^2 \right) = 6{,}61$$

$$s(b) = \sqrt{\frac{6{,}61}{428{,}74 - 10 \cdot 6{,}43^2}} = 0{,}86$$

$H_0$ zavrnemo, če $|T| > t_{1-\frac{\alpha}{2}}(df = 8)$

$(1{,}1) > 2{,}306$ // ne moremo zavrniti

h)

$[b - t_{0,025}(df=8) \cdot s(b),\ b + t_{0,025}(df=8) \cdot s(b)]$

$[0{,}72 - 2{,}306 \cdot 6{,}61,\ 0{,}72 + 2{,}306 \cdot 6{,}61] = [-0{,}3 ; 2{,}2]$

2.

1. tabela = vzorec 3
2. tabela = vzorec 2
3. tabela = vzorec 4
4. tabela = vzorec 1