Theoretical time complexity

Insertion sort

$T(0) = 0$

$T(n) = T(n-1) + n$
$T(n) = T(n-2) + (n-1) + n$

$\vdots$

$T(n) = T(0) + 1 + 2 + \cdots + n$
$T(n) = \frac{n(n+1)}{2}$

$\therefore \text{Constant} = \frac{1}{2}$

To find $O(g(n))$,

$T(n) \leq cg(n);\ n > n_0$

$\therefore \text{Time complexity} = O(n^2)$

Bubble sort

$T(0) = 0$

$T(n) = T(n-1) + n$
$T(n) = T(n-2) + (n-1) + n$

$\vdots$

$T(n) = T(0) + 1 + 2 + \cdots + n$
$T(n) = \frac{n(n+1)}{2}$

$\therefore \text{Constant} = \frac{1}{2}$

To find $O(g(n))$,

$T(n) \leq cg(n);\ n > n_0$

$\therefore \text{Time complexity} = O(n^2)$