Time value of money

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In today's class we will learn

- ✓Interest and Interest rate
- ✓2 viewpoints on interest
- ✓Meaning of Time Value of Money
- ✓Simple and compound interest
- ✓Cash Flow Diagrams (CFD)

INTEREST AND INTEREST RATE

Interest:

is the rental amount charged by the financial institutions for the use of money.

Interest rate:

is the rate at which interest is paid by a borrower for the use of money that they borrow from a lender.

is the rate of gain received from an investment.

Generally, Interest rates are stated on a per-year basis.

2 aspects of interest

1. Money received as a result of investing.

   GAIN or PROFIT

2. Money paid out as a result of borrowing funds.

   COST

1. Lender's viewpoint
2. Borrower's viewpoint

The interest rate
LENDER'S VIEWPOINT

- May exchange money for personal satisfaction (by purchasing consumer/producer goods)
- May hoard the money and wait for opportunity to use
- May lend the money on condition that may or may not charge interest from borrower.

Factors influencing the interest rate
LENDER'S VIEWPOINT

Factors deciding interest rate for lender

- probability of not repaying loan by borrower
- expense in investigating the borrower, loan agreement etc
- adequate return (comparing among all alternatives)
- Inflationary effects

The interest rate
BORROWER'S VIEWPOINT

- Has to follow the terms and conditions set by the lender
- Should be ready for consequences if fails to repay the loan
- Should check the interest rate on which (s)he is borrowing
- Has to fulfill the desires at the present time rather than at a future time

Factors influencing the interest rate

Borrower's viewpoint

The lender limits alternatives open to the borrower for the use of funds.

a) If the borrower uses funds for personal use

Interest rate < Privilege of having satisfaction immediately

b) If the borrower uses funds to finance business operations

Interest rate< expected gain

Time Value of Money

Because money can earn at certain interest rate, an amount in hand now is not equivalent to the same amount at some point of time in future.

- Money has a time value because it can earn more money over time (earning power).
- Money has a time value because its purchasing power changes over time (inflation).
- Time value of money is measured in terms of interest rate.

Time Value of Money

It Grows!

Time Value of Money

- Earning and purchasing power of money will not be the same in the future.
- There will be the effect of inflation or deflation on money.
- Effect of inflation
  - Basically, it is beneficial for the borrower
  - Reduced value of money during this time. Hence, the borrower will pay less amount on interest.
- Effect of deflation
  - Conversely, there will be an extra burden on the borrower.
- While deciding the interest rate, one must take care of inflation and deflation. This is why the time value of money is important.

The relationship between _____ and _____ is the concept of Time Value of Money (TVM)

The relationship between time and interest rate is the concept of Time Value of Money (TVM)

Cash Flow Diagram

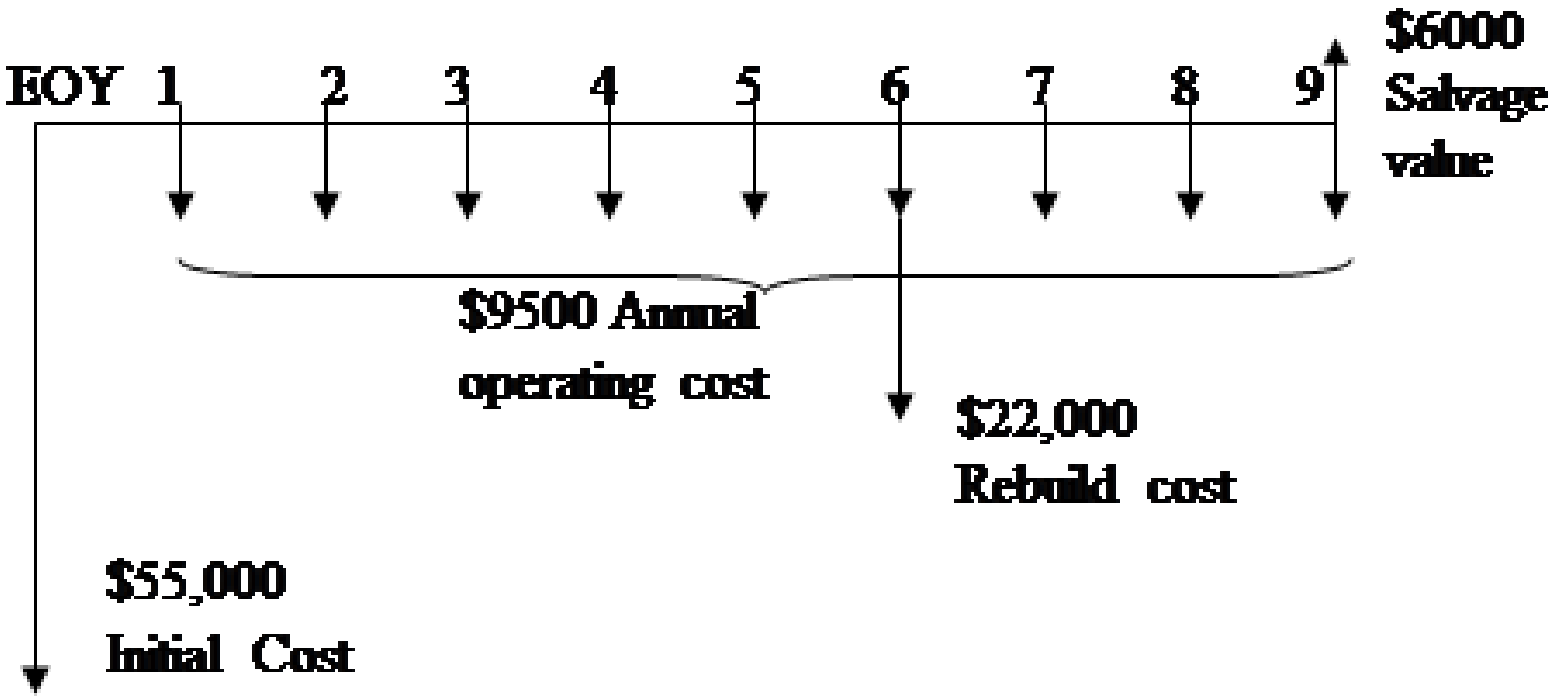
- To represent real-life problems, in engineering, we use models (Physical, mathematical, computer generated)
- In finance, the model used is known as a cash flow diagram (CFD).
- CFD allows us to visualize (in a particular economic problem):
  - The size of payments
  - The timing of payments

Cash Flow Diagram

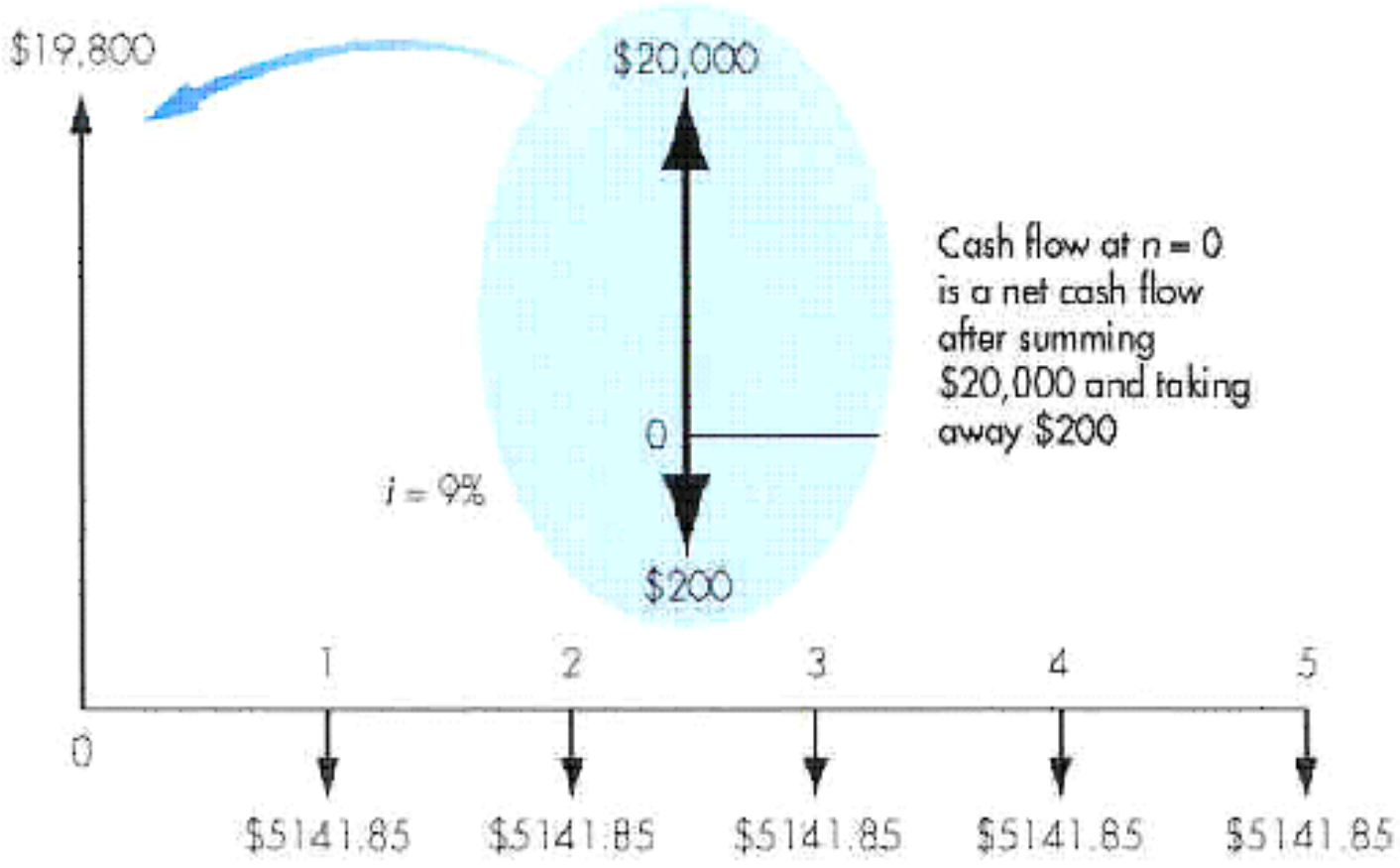
- Total time to model (represent it on a straight horizontal line)
- The money that comes out of our pocket (Cost) or the money we have to pay is drawn as down arrows (Disbursement – cash outflows).
- The money that comes into our pocket (Profit/Gain) or the money we receive is drawn as up arrows (Receipts- cash inflows).

A company is considering buying a machine that will cost them $20,000. After 7-years its salvage value will be $2,000. An overhaul costing $5,000 will be needed in Year 4. O&M costs will be $2,500 per year. Draw the cash flow diagram associated with the purchase of this machine.

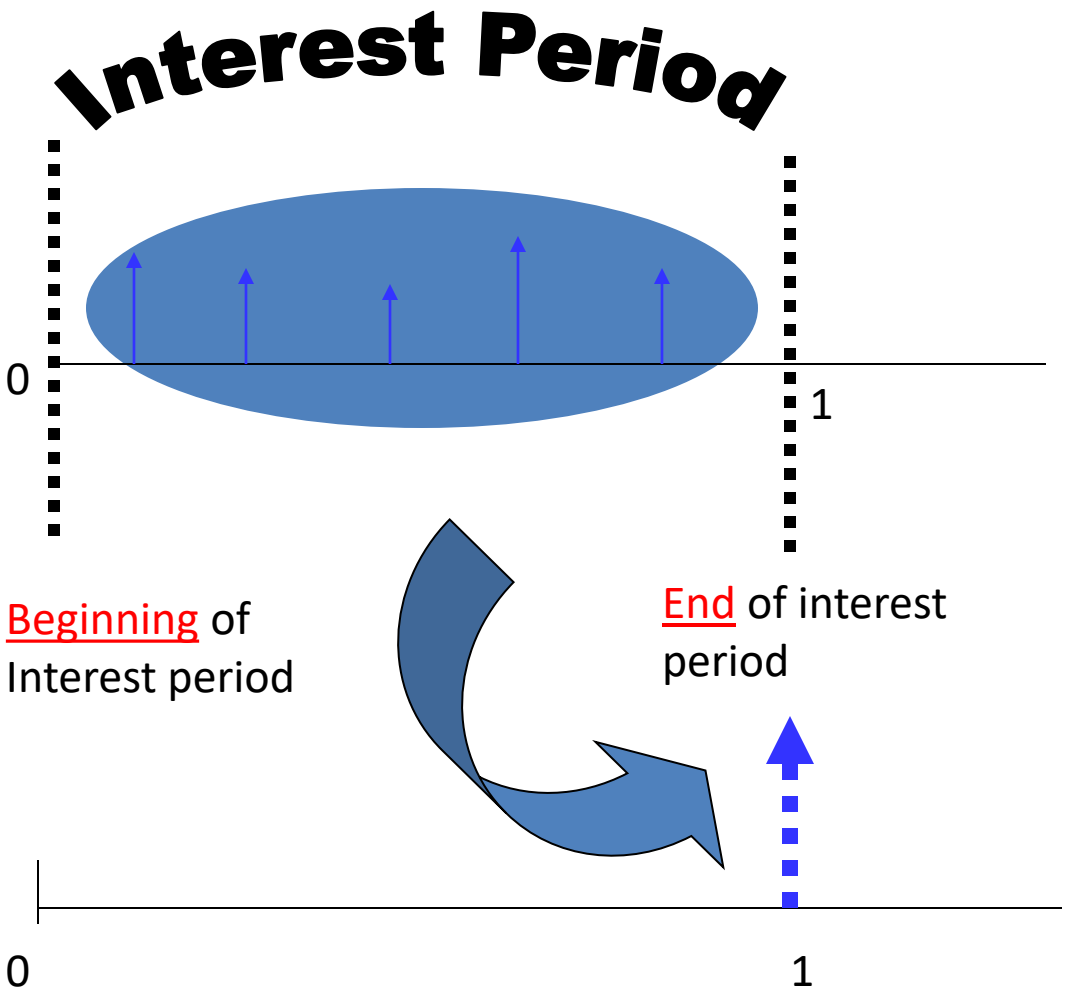
Cash Flow diagram (CFD)



Cash Flow Diagram (CFD)



End-of-Period Convention



Simple Interest

- *Interest owed upon repayment of a loan is proportional to the length of time the principal sum has been borrowed.*
- *Mathematically,*

$$I = P.n.i$$

- *I = Interest earned*
- *P = Principal amount*
- *i* = Interest rate
- *n* = Number of interest periods
- A simple interest loan may be made for any period of time. Interest and principal become due only at the end of the time period.
- It means that the borrower will be paying the whole amount (Interest earned + principal amount) only at the end of the borrowing time. He will not be paying it in the middle of the borrowed time period.

Simple Interest

- *P* = Principal amount
- *i* = Interest rate
- *n* = Number of interes periods
- Example:
  - *P* = $1,000
  - *i* = 8%
  - *n* = 3 years

End of Year	Beginning Balance	Interest earned	Ending Balance
0			$1,000
1	$1,000	$80	$1,080
2	$1,080	$80	$1,160
3	$1,160	$80	$1,240

Simple Interest Formula

$$F = P + (iP)n$$

where

P = Principal amount

i = simple interest rate

N = number of interest periods

F = total amount accumulated at the end of period N

$$F = \$1{,}000 + (0.08)(\$1{,}000)(3) = \$1{,}240$$

Compound Interest

- *When a loan is made for several interest periods, interest is calculated and payable at the end of each period of interest.*
- Basically, there may be a condition from the lender that we should pay the interest at the end of every year. Otherwise, the lender may say that this interest amount may be compounded or it will be added to the principal amount that is owed at the beginning of the year again.
- It means that the interest amount which is generated every year will go on increasing. This is known as the compounding of interest.

Compound Interest

- *Two cases:*
- A lender may tell the borrower to pay the interest every year.
- A lender may tell the borrower not to pay the interest every year, but the interest of every year will be added to the amount owed every year.
- If we have a certain amount with a certain interest rate at the end of 1st year, we will have certain interest generated. This interest is added to the principal amount. And this principal amount along with the interest generated during 1st year becomes the principal amount at the beginning of the second year.
- In this way, the compounding of interest is generated.

Compound Interest

- *So far, we have discussed about two types of compounding:*
- The interest is paid annually.
- Interest is not paid annually; interest is allowed to be compounded.

Example

Amount borrowed = ₹10000; time period = 4 years; annual interest rate = 20%

Year	Amount owed at beginning of year	Interest to be paid of year	Amount owed at the end of year	Amount paid by the borrower at the end of year
n	A	B	C	D

Compound Interest

- *P* = Principal amount
- *i* = Interest rate
- *n* = Number of interes periods
- Example:
  - *P* = $1,000
  - *i* = 8%
  - *n* = 3 years

End of Year	Beginning Balance	Interest earned	Ending Balance
0			$1,000
1	$1,000	$80	$1,080
2	$1,080	$86.40	$1,166.40
3	$1,166.40	$93.31	$1,259.71

Compound Interest Formula

$$n = 0: P$$

$$n = 1: F_1 = P(1 + i)$$

$$n = 2: F_2 = F_1(1 + i)$$

$$= P(1 + i)^2$$

$$\vdots$$

$$n = N: \boxed{F = P(1 + i)^n}$$

Example:

In 1626, Peter Minuit of DWI Co., paid $24 to purchase Manhattan Island in NY from the Indians. In retrospect, if Minuit had invested the $ 24 in a savings account that earned 8% interest, how much it would be worth in 2010?

Calculate both SI and CI.

Draw CFDs

- If $1,500 is invested now, $1,800 two years from now, and $2,000 four years from now at an interest rate of 6% compounded annually, what will be the total amount in 15 years?

Draw CFDs

- A student is buying an automobile that costs $12,000. She will pay $2000 as down payment, borrow rest of the money from a bank, and pay the loan off in four beginning of year annual instalments, starting from year 3. At 15% interest rate, what is the installment amount ? Prepare a cash flow diagram to represent this situation.

Draw CFDs

a) Suppose you make an annual contribution of $3000 to your savings account at the end of each year for 10 years. If the account earns 7% interest annually, how much can be withdrawn at the end of 10 years?

b) Suppose the ten deposits were made at the beginning of each period, what is the balance at the end of period 10?