Edge Detection Methods (First order Derivative filters)

① Roberts operator (cross-gradient) :-

$$\begin{bmatrix} -1 & 0 \\ 0 & +1 \end{bmatrix} \qquad \begin{bmatrix} 0 & -1 \\ +1 & 0 \end{bmatrix}$$

② Sobel operator

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \rightarrow$$

∂x

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

∂y

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

detects pixels close to Center pixel; but not on diagonals.

③ Prewitt operator :-

$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \rightarrow$$

detects Horizontal & Vertical

Problems with Roberts Cross operator :-

→ 2*2 masks are not easy to implement.
→ No. of calculations are more.
→ No. of neighboring pixels considered in one go are less.

And to solve these problems, we make the following changes:-
→ Change in Size of the ~~pixels~~ mask.
→ Change in number of ~~pix~~ neighbouring pixels considered.

Problem :- Apply Roberts, Sobel & Prewitt operators on the pixel (1, 1) in the following given image :-

50	50	100	100
50	50	100	100
50	50	100	100
50	50	100	100

Input image (4*4)

Sol :- Given :- pixel (1, 1)

	0	1	2	3
0	50	50	100	100
1	50	50	100	100
2	50	50	100	100
3	50	50	100	100

pixel (1, 1)

3 * 3 from center pixel

50	50	100
50	50	100
50	50	100

3*3

① **Robert's operator :-**

Mask's for Robert operator is :- $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (or) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

take this mask :-

origin ← $\begin{bmatrix} -1 & 0 \\ 0 & +1 \end{bmatrix}$ mask.

Here origin is not mentioned so, take '-1' pixel value as origin & multiply with pixel (1, 1) i.e; 50 :-

50	50	100
50	50	100
50	50	100

original Image

$= 50*(-1) + 100*1$

$= -50 + 100 = \underline{50}$

② **Sobel operator :-**

mask for Sobel operator is :- $\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$ origin

Here, take center pixel value '0' as origin & multiply with (3*3) original pixel elements :-

50	50	100
50	50	100
50	50	100

Original Image

$= 50(-1) + 50(-2) + 100(-1) + 50(1) + 50(2) + 100(1) =$

$= -50 - 100 - 100 + 50 + 100 = 0$

(3) Prewitt operator :-

50	50	100	100
50	50	100	100
50	50	100	100
50	50	100	100

Input Image

$\times$

-1	-1	-1
0	0	0
1	1	1

Mask for prewitt operator

$= 50(-1) + 50(-1) + 100(-1) + 50(1) + 50(1) + 100(1)$

$= -50 - 50 - 100 + 50 + 50 + 100$

$= 0$.

Example problem on Harris Corner Detection :-

1. Input Image :-

0	0	1	4	9
1	0	5	7	11
1	4	9	12	16
3	8	11	14	16
8	10	15	16	20

5 * 5

Differentiation Kernel :-

-1	0	1

d/dx

-1
0
1

d/dy

Sol :- Step 1 :- x-axis :- | -1 | 0 | 1 | d/dx

0	0	1	4	9
1	0	5	7	11
1	4	9	12	16
3	8	11	14	16
8	10	15	16	20

I/P Image

$\Rightarrow$

Ωx

X	X	X	X	X
X	4	7	6	X
X	8	8	7	X
X	8	6	5	X
X	X	X	X	X

O/P Image

$= -1(1) + 0*0 + 1*5 = -1 + 5 = 4$

$= -1(0) + 0*5 + 1*7 = 0 + 0 + 7 = 7$

$= -1(5) + 0*7 + 1*11 = -5 + 0 + 11 = 6$

$= -1(1) + 0*4 + 1*9 = -1 + 0 + 9 = 8$

$= -1(4) + 0*9 + 1*12 = -4 + 0 + 12 = 8$

$= -1(9) + 0*12 + 1*16 = -9 + 0 + 16 = 7$

$= -1(3) + 0(8) + 1(11) = -3 + 0 + 11 = 8$

$= -1(8) + 0(11) + 1(14) = -8 + 0 + 14 = 6$

$= -1(11) + 0(14) + 1(16) = -11 + 0 + 16 = 5$

Step 2 :- y-axis :- $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ d/dy

0	0	1	4	9
1	0	5	7	11
1	4	9	12	16
3	8	11	14	16
8	10	15	16	20

Input Image

x	x	x	x	x
x	4	8	6	x
x	8	6	6	x
x	8	7	4	x
x	x	x	x	x

output Image

$= -1(0) + 0(0) + 1(4) = 0 + 0 + 4 = 4$

$= -1(0) + 0(4) + 1(8) = 0 + 0 + 8 = 8$

$= -1(4) + 0(8) + 1(10) = -4 + 0 + 10 = 6$

$= -1(1) + 0(5) + 1(9) = -1 + 0 + 9 = 8$

$= -1(5) + 0(9) + 1(11) = -5 + 0 + 11 = 6$

$= -1(9) + 0(11) + 1(15) = -9 + 0 + 15 = 6$

$= -1(4) + 0(7) + 1(12) = -4 + 0 + 12 = 8$

$= -1(7) + 0(12) + 1(14) = -7 + 0 + 14 = 7$

$= -1(12) + 0(14) + 1(16) = -12 + 0 + 16 = 4$

Step 3 :-

Ix

x	x	x	x	x
x	4	7	6	x
x	8	8	7	x
x	8	6	5	x
x	x	x	x	x

Iy

x	x	x	x	x
x	4	8	6	x
x	8	6	6	x
x	8	7	4	x
x	x	x	x	x

$Ix = 4^2 + 7^2 + 6^2 + 8^2 + 8^2 + 7^2 + 8^2 + 6^2 + 5^2$

$Ix = 403$

$Iy = 4^2 + 8^2 + 6^2 + 8^2 + 6^2 + 6^2 + 8^2 + 7^2 + 4^2$

$Iy = 381$

$Ix * Iy = 4*4 + 7*8 + 6*6 + 8*8 + 8*6 + 7*6 + 8*8 + 6*7 + 5*4$

$= 16 + 56 + 36 + 64 + 48 + 42 + 64 + 42 + 20$

$= 388.$

$\therefore$ Harris corner detector, $H = \begin{bmatrix} 403 & 388 \\ 388 & 381 \end{bmatrix}$

Step 4 :-

$$H = \begin{bmatrix} 403 & 388 \\ 388 & 381 \end{bmatrix}$$

Corner detector :- $\boxed{C = \det(H) - K \cdot \text{trace}(H)^2}$ (K: constant)

$$\det(H) = (403 * 381 - 388 * 388)$$
$$= 153543 - 150544$$
$$= 2999$$

$K \cdot \text{trace}(H)^2 =$

'K' is constant $= 0.04$

$\text{trace}(H)^2 = (403 + 381)^2$

$= (784)^2$

$K \cdot \text{trace}(H)^2 = 0.04 * (784)^2$

$\therefore C = 2999 - 0.04 * (784)^2$

$= 2999 - 0.04 * 614656$

$= 2999 - 24586.24$

$= -21587.24 \longrightarrow$ edge.

Note :- If 'c' is large value : it is corner.

If 'c' is negative :- it is edge.

If $|c|$ is small :- it is flat.