Smoothing Spacial filters

- Linear filter
  - Mean/Box filter
  - Weighted average filter
  - Gaussian filter
- Non-Linear filter
  - Median filter
  - Min filter
  - Max filter

(1) **Box/Mean filter** :- all coefficients are equal

Eg:-

$$\frac{1}{9} * \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \rightarrow \text{Mask/filter}$$

3*3

$$= (1+1+1+1+1+1+1+1+1) * 1/9 = 9/9 = \underline{1}.$$

(2) **Weighted Average filter** :- gives more / less weight to pixels near (away from) the output location.

Eg:-

$$\frac{1}{16} * \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array} \longrightarrow \text{Mask/filter}$$

$$= (1+2+1+2+4+2+1+2+1)/16 = 16/16 = \underline{1}.$$

(3) **Gaussian filter**:- The weights are samples of 2D Gaussian function:

$$G_{\sigma}(x,y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}$$ (2D Gaussian function)

Eg:-

$$\frac{1}{16} * \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array} \rightarrow \text{Mask/filter}$$

→ used to blur edges & reduce contrast

→ Similar to median filter but is faster.

Example Problem :-

① Consider the ~~pixel~~ image below & calculate the output of the pixel (2,2) if Smoothing is done using (3*3) neighbourhood using all the filters below :-

(a) Box / Mean filter
(b) Weighted average filter
(c) Median filter
(d) Min filter
(e) Max filter

1	8	8	0	7
4	7	9	5	7
5	4	(6)	8	6
4	2	0	1	5
0	1	0	2	0

3*3

I/P Image 5*5

$\frac{1}{9} * \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}_{3*3}$

Mask/filter

(a) **Box filter :-**

$= \frac{1}{9}\left[7+9+5+4+6+8+2+0+1\right]$

$= \frac{1}{9}\left[42\right] = 4.66 \approx 5$. $\longrightarrow$ o/p =

1	8	8	0	7
4	7	9	5	7
5	4	(5)	8	6
4	2	0	1	5
0	1	0	2	0

3*3

5*5

(b) **Weighted average filter :-**

$f(x,y) =$

1	8	8	0	7
4	7	9	5	7
5	4	(6)	8	6
4	2	0	1	5
0	1	0	2	0

3*3

5*5

I/p Image

$* \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}_{3*3}$

Mask/filter

$= \frac{1}{16}\left[7*1+9*2+5*1+4*2+6*4+8*2+2*1+0*2+1*1\right]$

$= \frac{1}{16}\left[81\right] = 5.0625 \approx 5$ o/p =

1	8	8	0	7
4	7	9	5	7
5	4	(5)	8	6
4	2	0	1	5
0	1	0	2	0

5*5

(c) Median filter :-

1	8	8	0	7
4	7	9	5	7
5	4	6	8	6
4	2	0	1	5
0	1	0	2	0

3*3

5*5

* Arrange $\begin{bmatrix} 7 & 9 & 5 \\ 4 & 6 & 8 \\ 2 & 0 & 1 \end{bmatrix}$ elements in ascending order :-

0 1 2 4 5 6 7 8 9

↓

Median = 5

(d) Min filter & Max filter :-

From $\begin{bmatrix} 7 & 9 & 5 \\ 4 & 6 & 8 \\ 2 & 0 & 1 \end{bmatrix}$ (3*3), identify minimum & maximum elements.

Min = 0

Max = 9.