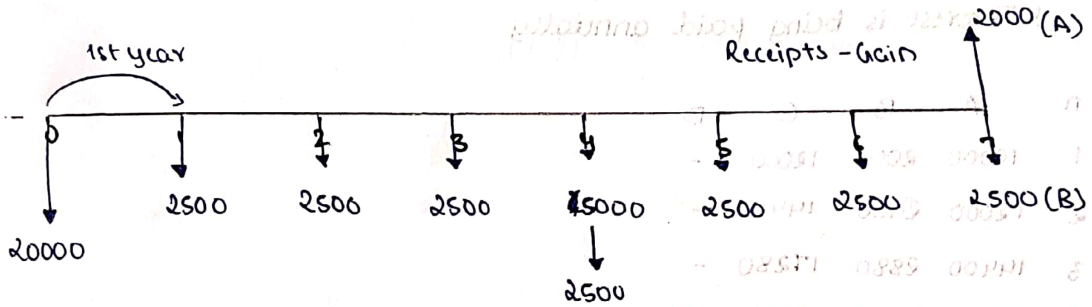
CASH FLOW DIAGRAM

A company is considering buying a machine that will cost them ₹20000. After 7-years its salvage value will be ₹2000. An overhaul costing ₹5000 will be needed in Year 4. O&M costs will be ₹2500 per year. Draw the cash flow diagram.



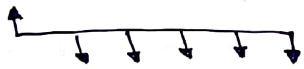
Disbursements
↓
Cash outflow

Net receipt: (A-B)
Net disbursements: (A-B)

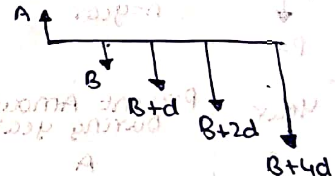
→ Types of CFD

① Single CFD

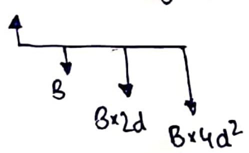
② Uniform (Annual CFD)



③ Linear Gradient CFD



④ Geometric gradient:



⑤ Irregular CFD

→ Simple Interest

$I = PNi$

I → Simple interest

P → Principle amount

N → Time period for borrowed amount

i → Interest rate.

→ Compound Interest

Q] Amount borrowed = ₹10000; time period = 4 years; annual interest rate = 20%

1] Interest paid annually:

B = interest
C = total amount
D = amount that you pay
A = principle

n	A	B	C	D
1	10000	2000	12000	2000
2	10000	2000	12000	2000
3	10000	2000	12000	2000
4	10000	2000	12000	12000
				18000

* Interest is being paid annually

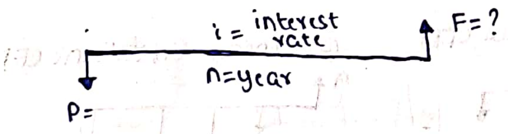
2]

n	A	B	C	D
1	10000	2000	12000	-
2	12000	2400	14400	-
3	14400	2880	17280	-
4	17280	3456	20736	20736

* Interest is allowed to be compounded, not paid annually.

[TYPE 1]

→ Single payment compound interest Factor



Year	Present Amount During year	Interest owed for the year	Total compounded amount for year
n	A	B	C
1	P	Pi	$P + Pi = P(1+i)$
2	$P(1+i)$	$P(1+i).i$	$P(1+i)^2$
3	$P(1+i)^2$	$P(1+i)^2.i$	$P(1+i)^3$
⋮	⋮	⋮	⋮
n	$P(1+i)^{n-1}$	$P(1+i)^{n-1} \times i$	$P(1+i)^n = F$

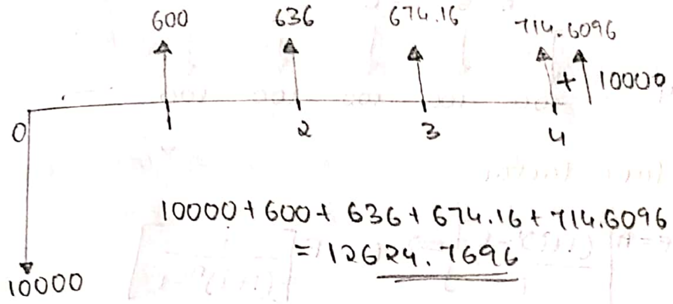
$F = P(1+i)^n$ - Factor

$F = P\left(\frac{F}{P}, i, n\right)$ - Single payment compound interest factor

$a = b^n$

$\frac{F}{P} = (1+i)^n$ $\quad \sqrt[n]{\frac{F}{P}} = (1+i)$

5) If ₹10000 is invested at 6% compounded interest in a project annually at the beginning of year 1, what will be the compounded amount of a single payment at the end of year 4. [Use compounding interest table]



10000 × 1 × 0.06
= 600
10600 × 0.06
= 636
11236 × 0.06 = 674.16
11910.16 × 0.06 =

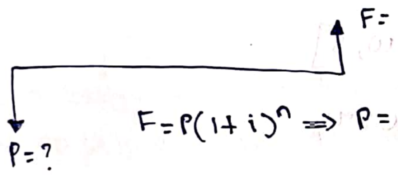
10000 + 600 + 636 + 674.16 + 714.6096
= 12624.7696

In table, $F = P\left(\frac{F}{P}, i, n\right)$

$= 10000 \times 1.262 = 12620$ $\left[\frac{F}{P} = 1.262 \text{ given } n = 4 \text{ \& } i = 6\%\right]$

→ Single payment present worth Factor

We don't know present amount but we have the value of F.



$F = P(1+i)^n \Rightarrow P = \frac{F}{(1+i)^n}$

From table, $\boxed{P = F\left(\frac{P}{F}, i, n\right)}$

How much must be invested now at 6% compounded annually so that ₹5,00,000 can be received after 5 years?

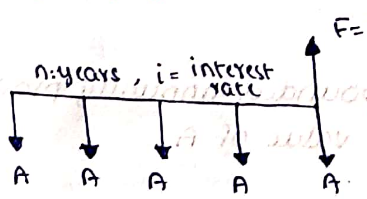
$P = 500000\left[\frac{P}{F}, 5, 6\right]$

$= 500000 \times 0.7473 = 373650$

[TYPE 2]

Equal payment series:

1) Equal payment compound amount factor



$F = A(1) + A(1+i)^1 + A(1+i)^2 + A(1+i)^3 \ldots A(1+i)^{n-1}$ — (1)

Multiply $(1+i)$ on both sides

$F(1+i) = A(1+i) + A(1+i)^2 + \ldots A(1+i)^n$ — (2)

Subtract (2) − (1)

$F(1+i-1) = A[(1+i)^n - 1]$

$F = \left[\frac{A(1+i)^n - 1}{i}\right] \Rightarrow \boxed{F = A\left[\frac{F}{A}, i, n\right]}$

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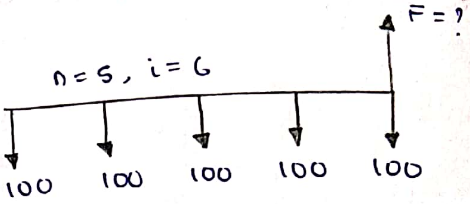
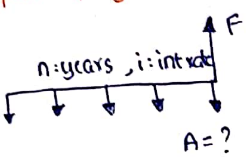
Determine the future amount of ₹100 payment deposited at the end of each of the next 5 years and earning 6% per annum.

$n = 5$ years $\quad A = 100 \quad i = 6\%$



$$F = 100\left[\frac{F}{A}, 6, 5\right] = 100 \times 5.637$$
$$= 563.7 \approx 564$$

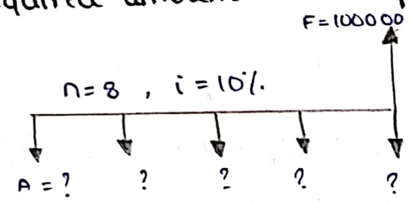
② Equal payment series: Sinking fund factor



$$F = A\left[\frac{(1+i)^n - 1}{i}\right] \Rightarrow A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

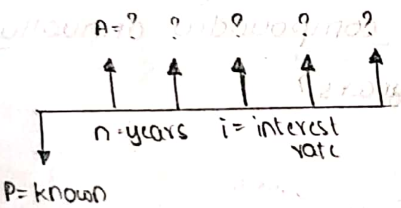
$$\boxed{A = F\left[\frac{A}{F}, i, n\right]}$$

It is desired to accumulate 1 lakh rupees by making a series of 8 equal annual payments at 10% interest compounded annually. What is required amount of each payment?



$$A = 100000\left[\frac{A}{F}, 10, 8\right]$$
$$= 100000 \times 0.0874$$
$$= 8740$$

③ Equal payment series - Capital Recovery Factor



$$A = F\left[\frac{i}{(1+i)^n - 1}\right] \quad \text{→ From sinking fund factor} - ①$$

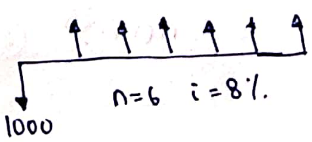
$$F = P(1+i)^n \quad \text{→ From single payment compound amount factor} - ②$$

Substituting value of F from ② in ①..

$$A = P(1+i)^n\left[\frac{i}{(1+i)^n - 1}\right] \Rightarrow A = \frac{P[(1+i)^n \cdot i]}{[(1+i)^n - 1]}$$

$$\boxed{A = P\left[\frac{A}{P}, i, n\right]}$$

₹1000 has been invested now at 8% interest compounded annually provide for 6 equal future year increments. Determine the value of A.



$$A = 1000\left[\frac{A}{P}, 8, 6\right]$$
$$= 1000 \times 0.2163$$
$$= 216.3 \approx 216$$

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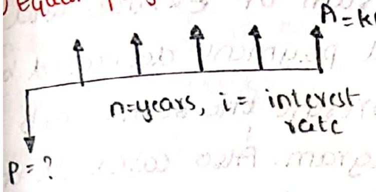
Equal payment series: Present Worth Factor



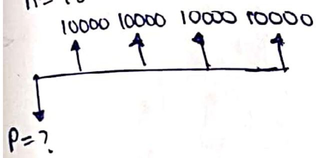
$$A = P\left[\frac{A}{P}, i, n\right]$$

$$\Rightarrow P = A\left[\frac{(1+i)^n - 1}{i.(1+i)^n}\right]$$

$$\Rightarrow \boxed{P = A\left[\frac{P}{A}, i, n\right]}$$

Determine the present worth of a series of 15 equal annual payments of ₹10000 at an interest rate of 12% compounded annually.

$n = 15 \quad A = 10000 \quad i = 2\%$



$$P = 10000\left[\frac{P}{A}, 2, 15\right]$$

$$= 10000 \times 6.811$$

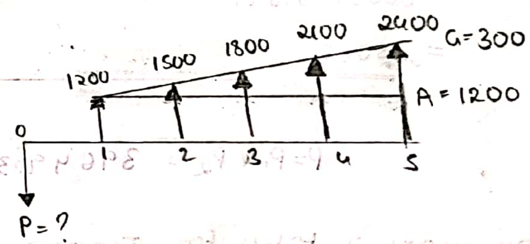
$$= \underline{68110}$$

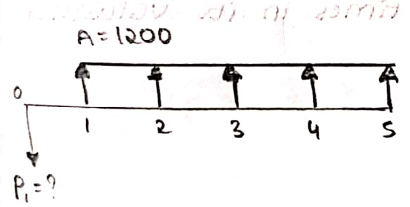
[TYPE 3]

Linear Graded | Arithmetic Series :

A person buys a car. He wishes to set aside enough money in his bank account to pay the maintenance on the car for the first 05 years. The anticipated maintenance cost of the car are as follows: The rate of interest is 5%.

Year	1	2	3	4	5
Maintenance Cost	1200	1500	1800	2100	2400





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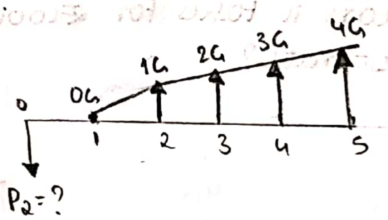
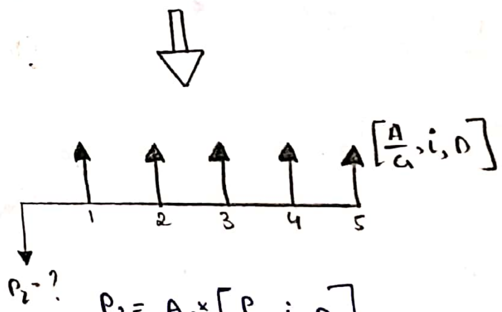


$$P_1 = A\left[\frac{P}{A}, i, n\right]$$

$$P_1 = 1200\left[\frac{P}{A}, 5, 5\right]$$

$$P_1 = 1200 \times 4.329$$

$$\boxed{P_1 = \underline{5194.8}}$$



$$P_2 = A_2 \times \left[\frac{P}{A}, i, n\right]$$

$$P_2 = G\left[\frac{A}{G}, i, n\right] \times \left[\frac{P}{A}, i, n\right]$$

$$= 300 \times \left[\frac{A}{G}, 5, 5\right] \times 4.329$$

$$= 300 \times 1.902 \times 4.329$$

$$\boxed{P_2 = 2470.13}$$

$$P = P_1 + P_2$$

$$= 5194.8 + 2470.13$$

$$= \underline{7664.93}$$

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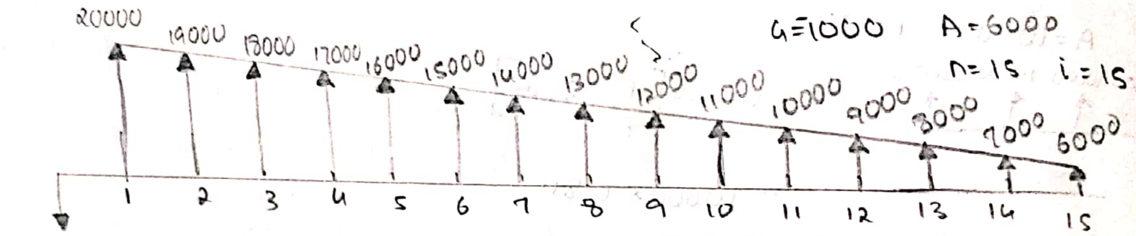
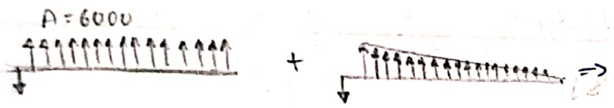
A person is paying in the first year end a sum of ₹20000 and the next 14 years i.e upto 15 years the year end payment decreased by ₹1000. What equivalent annual payment will represent the same cash flow diagram as that of the gradient series diagram. Also calculate the value of principle amount. Consider the rate of interest 15% compounded annually.

A = ₹20000 19000 18000 17000 16000





$P_1 = A\left[\frac{P}{A}, i, n\right]$

$= 6000\left[\frac{P}{A}, 15, 15\right]$

$= 6000 \times 5.847$

$= \underline{35082}$

+

$P_2 = G \times \left[\frac{A}{G}, i, n\right] \times \left[\frac{P}{A}, i, n\right]$

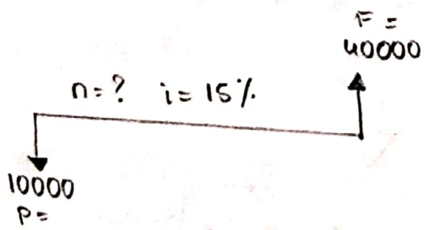
$= -1000\left[\frac{A}{G}, 15, 15\right] \times 5.847$

$= 4.565 \times 1000 \times 5.847$

$= \underline{-26691.55}$

$P = P_1 + P_2 = 8390.45 \approx \underline{8390}$

How many years it takes for ₹10000 to be 4 times in its valuation at 15% interest rate?



$\frac{F}{P} = (1+i)^n$

$\frac{40000}{10000} = (1+0.15)^n$

$4 = (1.15)^n$

$\log 4 = n \log 1.15$

$n = \frac{\log 4}{\log 1.15} = 9.91 \approx \underline{10 \text{ years}}$

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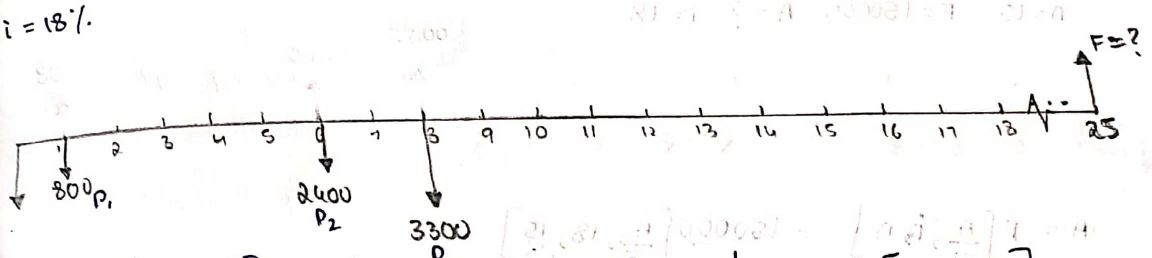
How much money will be accumulated in 25 years if ₹800 is deposited in first year from now, ₹2400 is deposited in 6 years from now, and ₹3300 is deposited in 8 years from now. All at an interest rate of 18% per year.

$n = 25$ years
$i = 18\%$



$F_1 = P_1\left[\frac{F}{P_1}, i, n\right]$

$= 800\left[\frac{F}{P_1}, 24, 18\right]$

$= 800 \times 53.109$

$= 42487.2$

$F_2 = P_2\left[\frac{F}{P_2}, i, n\right]$

$= 2400\left[\frac{F}{P_2}, 18, 19\right]$

$= 2400 \times 23.214$

$= 55713.6$

$F_3 = P_3\left[\frac{F}{P_3}, i, n\right]$

$= 3300\left[\frac{F}{P_3}, 18, 17\right]$

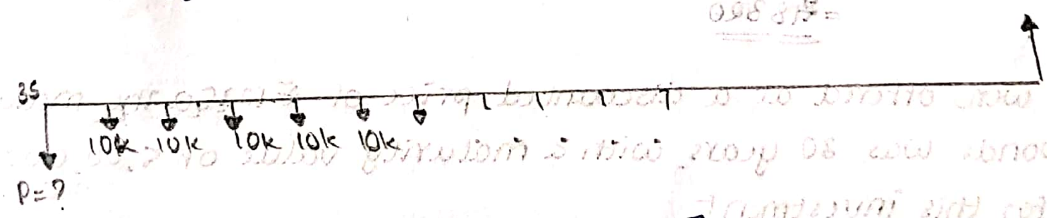
$= 3300 \times 16.672$

$= 55017.6$

$F = F_1 + F_2 + F_3 = 42487.2 + 55713.6 + 55017.6 = ₹1{,}53{,}218.4$

A 35 year old person is planning to invest an equal sum of ₹1000 at the end of every year for the next 25 years from the end of the next year. The bank gives 20% interest rate compounded annually what amount will he get when he will become 60 years.

$n = 25 \quad i = 20 \quad A = 10000 \quad F = ?$



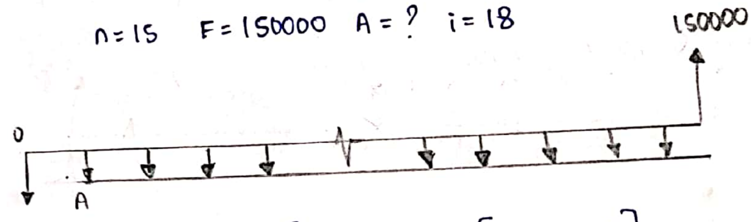
$A = F\left[\frac{A}{F}, i, n\right] \Rightarrow F = A\left[\frac{F}{A}, i, n\right]$

$F = 10000\left[\frac{F}{A}, 20, 25\right]$

$F = 10000 \times 471.981 = ₹471981$

A company has to represent its present facility after 15 years at a c of ₹1,50,000. It plans to deposit an equal amount at the end of every year. What equal amount the company should deposit at the end of every year for the next 15 years at an interest rate of 18% per year to meet the cost?

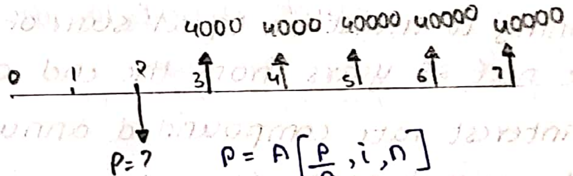
$n = 15 \quad F = 150000 \quad A = ? \quad i = 18$



$$A = F\left[\frac{A}{F}, i, n\right] = 150000\left[\frac{A}{F}, 18, 15\right]$$

$$= 150000 \times 0.0164 = ₹2460$$

Determine the amount P that you should deposit into an account 2 years from now, in order to be able to withdraw ₹4000 per year for 5 years starting 3 years from now at an interest rate of 15% per year.



$n = 5$ years

$i = 15\%$

$P = ? \quad A = 4000$

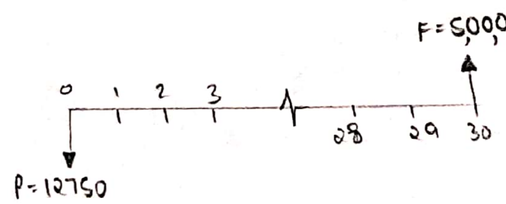
$$P = A\left[\frac{P}{A}, i, n\right]$$

$$= 4000\left[\frac{P}{A}, 15, 5\right]$$

$$= 4000 \times 3.352$$

$$= ₹13408$$

A bank bonds were offered at a discounted price of ₹12750. The maturity period for the bonds was 30 years, with a maturity value of 5,00,000. Determine ROI for this investment



$$F = P(1+i)^n$$

$$500000 = 12750(1+i)^{30}$$

$$(1+i)^{30} = 39.2157$$

$$30\log(1+i) = \log(39.2157)$$

$$\log(1+i) = 0.05311$$

$$1+i = 1.13008$$

$$i = 1.13008 - 1$$

$$= 0.13008 = 13\%$$

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Nominal and Effective Interest Rates:

$P = 10000 \quad i = 10\%$ per annum $\quad n = 1$

$F = P(1+i)^n$

$= 10000(1+0.1)^1$

$= 11000$

(Compounded Annually) ⇒ compounded semi annually

$$F = 10000\left[1 + \frac{10}{100 \times 2}\right]^{1 \times 2}$$

$$= 10000[1.05]^2 = 11025$$

Interest Rate → Divide

Time → Multiply

E = Effective interest rate

r = Nominal rate of interest

m = Reciprocal of compounded period for Time Interval

$$E = \left[1 + \frac{r}{m}\right]^{l \times m} - 1$$

l = Time interval (converted in terms of years)

Nominal Rate of Interest is 9% (m) compounded monthly with time interval of 1 month. Findout E?

$$E = \left[1 + \frac{0.09}{12}\right]^{1/\cancel{12} \times \cancel{12}} - 1$$

$= 0.0075 = 0.75\%$

$r = 9\%$

$l = 0.083$ [1 month of 12 months]

$m = 1/1/12 = 12$ → Compounded annually

Nominal rate of 12% compounded monthly with time interval of 1 year.

$$E = \left[1 + \frac{0.12}{12}\right]^{12} - 1 = 0.1268$$

$= 12.68\%$

$r = 12\%$

$l = 1$

$m = 1/1/12 = 12$ → Compounded annually

Nominal rate of 18% compounded weekly with time interval of 1 year.

$$E = \left[1 + \frac{0.18}{52}\right]^{1 \times 52} - 1$$

$= 0.1968$

$= 19.68\%$

$r = 0.18$

$l = 1$

$m = 1/1/52 = 52$

Nominal rate of 14% compounded quaterly with a time interval of 6 month

$$E = \left[1 + \frac{0.14}{4}\right]^{1/\cancel{2} \times \cancel{4}^2} - 1$$

$= (1.035)^2 - 1$

$= 0.0712 \Rightarrow 7.12\%$

$r = 0.14$

$l = \frac{6}{12} = 1/2$

$m = 1/3/12 = 4$

NR is 10% compounded weekly with TI of 6 months

$$\mathcal{E} = \left[1 + \frac{0.1}{52}\right]^{1/2 \times 52} - 1$$

$r = 0.1\%$

$l = \frac{6}{12} = 1/12$

$m = 52$

$= 1.050 - 1$

$= 0.05 = 5\%$

NR is 13% compounded monthly with TI of 2 years

$$\mathcal{E} = \left[1 + \frac{0.13}{12}\right]^{2 \times 12} - 1$$

$r = 0.13$

$l = 2$

$m = 1/1/12 = 12$

$= 29.51\%$

NR is 9% compounded semi annually TI of 36 months

$$\mathcal{E} = \left[1 + \frac{0.09}{2}\right]^{3 \times 2} - 1$$

$r = 0.09\%$

$l = 3$

$m = 1/6/12 = 2$

$= 0.302 = 30.2\%$

→ Comparison of Interest Rates

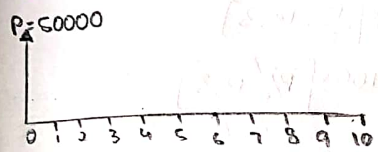
S.No	Compounded frequency	No. of periods per year	Effective interest rate per period(%)	Annual IR (%)
1	Annually	1	12	12
2	Semi-annually	2	6	12.36
3	Quaterly	4	3	12.55
4	Monthly	12	1	12.68
5	Weekly	52	0.23	12.73
6	Daily	365	0.032	12.747

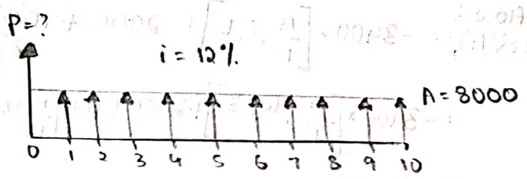
Economic Evaluation of Alternatives

Option 1: Take £50,000 now

Option 2: Take £8,000 per year for next 10 years at 12% interest rate. [The amount will be deposited at the end of every year]

Case 1: Present Worth





Option 1

$P = 50000$

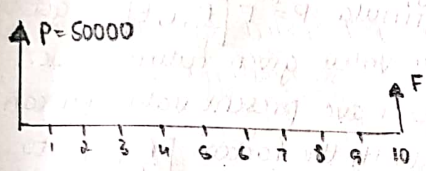
Option 2

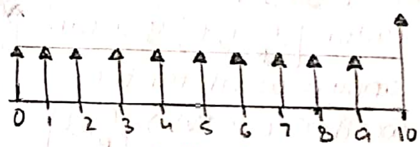
$$P = A\left[\frac{P}{A}, i, n\right]$$

$$= 8000 \times 5.650 = \underline{45200}$$

Option 1 is better

Case 2: Future Worth





Option 1

$$F = P(1+i)^n$$

$$= P\left[\frac{F}{P}, i, n\right] = 50000 \times 3.106$$

$$= \underline{155300}$$

Option 2

$$F = A\left[\frac{F}{A}, i, n\right]$$

$$= 8000 \times 17.549$$

$$= \underline{140392}$$

Option 1 is better

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3 methods of economic evaluation of alternatives

① Present Worth Method:

$$PW(i) = \sum_{t=0}^{n} F_t (1+i)^t \Rightarrow PW(i) = \sum_{t=0}^{n} F\left[\frac{P}{F}, i, t\right]$$

② Future Worth Method:

$$FW(i) = PW(i) \times \left[\frac{F}{P}, i, t\right]$$

③ Equivalent uniform annual worth method [EUAW | EUAC]

$$EUAC(i) = PW(i) \times \left[\frac{A}{P}, i, t\right]$$

Q)

	Machine A	Machine B	Interest Rate
Initial Cost (First cost)	£3400	£6500	12%
Service Life	03 years	06 years	
Salvage Value	£100	£500	
Net cash flow after taxes	£2000/year	£1800/year	

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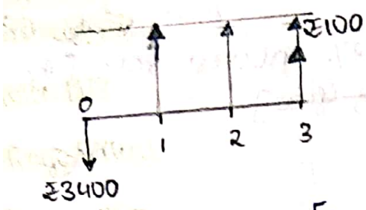
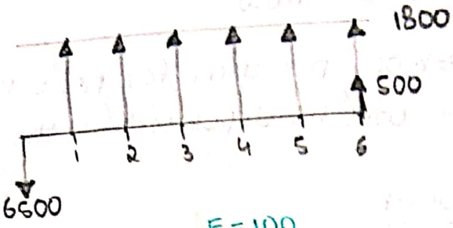
EUAW(i)

Machine A



Machine B



EUAW(A):- $-3400 \times \left[\frac{A}{P}, i, t\right] + 2000 + 100\left[\frac{P}{F}, 12, 3\right] \times \left[\frac{A}{P}, 12, 3\right]$

$F = 100$

$P = F\left[\frac{P}{F}, 12, 3\right]$

$= -3400 \times \left[\frac{A}{P}, 12, 3\right] + 2000 + 100\left[\frac{P}{F}, 12, 3\right] \times \left[\frac{A}{P}, 12, 3\right]$ $= 100 \times \left[P/F, 12, 3\right]$

$= \underline{\underline{614.21}}$

This is already the value that you yield every year given in question

EUAW(B):- $\underbrace{-6500 \times \left[\frac{A}{P}, 12, 6\right]} + 1800 + \underbrace{500\left[\frac{P}{F}, 12, 6\right] \times \left[\frac{A}{P}, 12, 6\right]}$

$= \underline{\underline{280.80}}$

When multiplying present worth with a factor $\left[\frac{A}{P}, i, t\right]$ you can get the annual worth for the present worth. EUAW = PW(i) × $\left[\frac{A}{P}, i, t\right]$

The initial part $500\left[\frac{P}{F}, 12, 6\right]$ is the original formula $P = F\left[\frac{P}{F}, i, t\right]$ to get the principle value given future value. Now that you have present value you co[uld] multiply it with the factor $\left[\frac{A}{P}, i, t\right]$ to get annual worth :- EUAW = PW(i) × $\left[\frac{A}{P}\right.$

→ EUAW of A is greater than EUAW(B) hence Machine 'A' is selected.

II] Present Worth Method

$PW(A) = -3400 + 2000 \times \left[\frac{P}{A}, 12, 3\right] + 100\left[\frac{P}{F}, 12, 3\right]$

$= -3400 + 2000(2.402) + 100(0.7118)$

$=$

$PW(B) = -6500 + 1800 \times \left[\frac{P}{A}, 12, 6\right] + 500\left[\frac{P}{F}, 12, 6\right]$

$= -6500 + 1800(4.111) + 500(0.5066)$

$=$

III] Future worth method

$FW(A) = -3400\left[\frac{F}{P}, 12, 3\right] + 2000\left[\frac{F}{A}, 12, 3\right] + 100$

$= -3400($

$FW(B) = -6500\left[\frac{F}{P}, 12, 6\right] + 1800\left[\frac{F}{A}, 12, 6\right] + 500$

$=$

As a manager of your firm, you would like to buy a printer for your subordinates. In this regard you receive two proposals:-

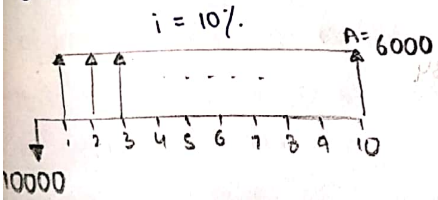
Proposal 1:- Printer A costs you ₹10000 with a useful life of 10 years. It will give an annual saving of ₹6000

Proposal 2:- Printer B costs you ₹25,000. It has associated with a cost saving of ₹20000 in the first year, ₹18000 in second year, ₹16000 in third year and so forth for 7 years.

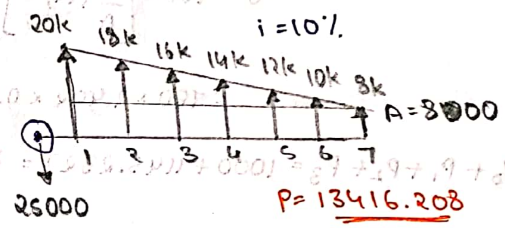
Which proposal would you like to opt for at MARR of 10% using AW Method

EU:

Machine A



Machine B



P= 13416.208

$EUAW(A) = -10000\left[\frac{A}{P}, 10, 10\right] + 6000$

$= -10000\,(0.1627) + 6000$

$= \underline{4373}$

$P_1 = 8000\left[\frac{P}{A}, 10, 7\right]$

$= 8000(4.868) = 38944$

$P_2 = G\left[\frac{A}{G}, i, n\right] \times \left[\frac{P}{A}, i, n\right]$

$= -2000\left[\frac{A}{G}, 10, 7\right] \times [4.868] = 25527.792$

$EUAW(B) = 13416.208\left[\frac{A}{P}, 10, 7\right]$

$= 13416.208 \times 0.2054$

$= \underline{2755.6891}.$

Since EUAW(A) > EUAW(B) hence Machine A is better than Machine B.

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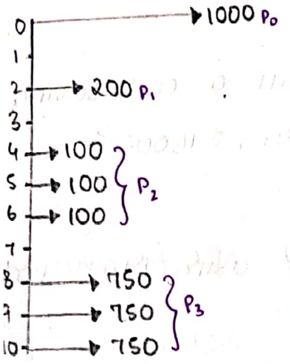
Principles of Equivalence

① Receipts or Disbursements can be directly added or subtracted only if they occur at same point in time.



Case 1: Equivalence of all transactions at $t=0$.

$$P_1 = 200 \times \left[\frac{P}{F}, 12, 2\right] = 200 \times 0.7972 = 159.44$$

$$P_2 = 100 \times \left[\frac{P}{A}, 12, 3\right] \times \left[\frac{P}{F}, 12, 3\right]$$

$$= 100 \times 2.402 \times 0.7118 = 170.97436$$

$$P_3 = 750 \times \left[\frac{P}{A}, 12, 3\right] \times \left[\frac{P}{F}, 12, 7\right]$$

$$= 750 \times 2.402 \times 0.4523 = 814.8184$$

$$P = P_0 + P_1 + P_2 + P_3 = 1000 + 1145.2327 = 2145.2327$$

Case 2: Equivalence of all transactions at $t=10$

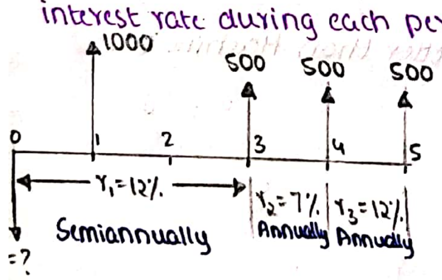
$$F_0 = P_0 \times \left[\frac{F}{P}, 12, 10\right] = 1000 \times 3.106 = 3106$$

$$F_1 = P_1 \times \left[\frac{F}{P}, 12, 8\right] = 200 \times 2.476 = 495.2$$

$$F_2 = P_2 \times \left[\frac{F}{P}, 12, 3\right] \times \left[\frac{F}{P}, 12, 7\right] = 100 \times 2.402 \times 2.211 = 531.0822$$

$$F_3 = P_3 \times \left[\frac{P}{A}, 12, 3\right] \times \left[\frac{F}{P}, 12, 3\right] = 750 \times 2.402 \times 1.405 = 2531.1075$$

② When cashflows are converted to their equivalences from one period to another interest rate during each period must be taken into consideration.



Case 1: Equivalence of transactions at $t=0$.

$$P_1 = F\left[\frac{P}{F}, 12, 1\right]$$

$$P_1 = 500 \times 0.8929 = 446.45$$

$$P_2 = 946.45 \times 0.9346 = 884.55$$

[500 + 446.45 = 946.45]

$$P_3 = 1384.55\left[\frac{P}{F}, 12, 4\right] = 1384.55 \times 0.6355 = 879.8$$

[500+884.55 = 1384.55]

$$P_4 = 1879.88 \times \left[\frac{P}{F}, 12, 2\right]$$

$$= 1879.88 \times 0.7972 = 1498.64$$

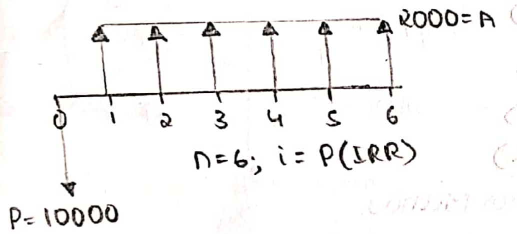
Case 2: Equivalence of transactions at t=1

Internal Rate Of Return Method (IRR)

- For any investment, IRR is that rate of interest at which the equivalent value of receipts is equal to the equivalent value of disbursements.
- It is the rate of interest for which present worth of any investment is equal to 'zero'.
- For any investment with proposal life of 'n' years:

$$P(W)(i) = 0 = \sum_{t=0}^{n} F_t (1+i^*)^{-t}$$

Q] What is the IRR for the investment of ₹10000 for which ₹2000 is the return at the end of every year for 6 years?



$$P = A \times \left[\frac{P}{A}, i, n\right]$$

$$10000 = 2000\left[\frac{P}{A}, i^*, 6\right]$$

$$\boxed{5 = \left[\frac{P}{A}, i^*, 6\right]}$$ → Compound amount factor

$\left[\frac{P}{A}, 5\%, 6\right] = 5.076$

$\left[\frac{P}{A}, 6\%, 6\right] = 4.917$ Using linear interpolation method

$$x^* = x_1 + (x_2 - x_1)\left(\frac{y^* - y_1}{y_2 - y_1}\right)$$

$x_1 = 5\% = 0.05$; $y_1 = 5.076$; $x_2 = 6\% = 0.06$; $y_2 = 4.917$; $y^* = 5$

$$x^* = 0.05 + (0.06 - 0.05)\left(\frac{5 - 5.076}{4.917 - 5.076}\right)$$

$$= 0.05 + 0.004779$$

$$x^* = 0.05477 = \underline{5.48\%}$$

$i^* = \underline{5.48\%}$

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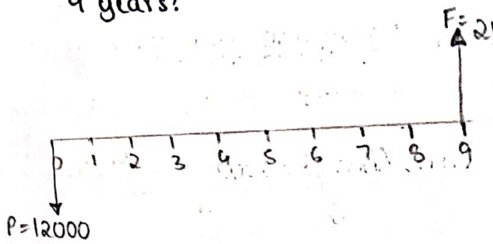
Q] What will be the interest rate that will convert ₹12000 into ₹21000 in 9 years?



$F = P\left[\frac{F}{P}, i^*, n\right]$

$21000 = 12000\left[\frac{F}{P}, i^*, 9\right]$

$1.75 = \left[\frac{F}{P}, i^*, 9\right]$

$\left[\frac{F}{P}, 6\%, 9\right] = 1.689$

$\left[\frac{F}{P}, 7\%, 9\right] = 1.838$ $\Rightarrow$ $x^* = x_1 + (x_2 - x_1)\left[\frac{y^* - y_1}{y_2 - y_1}\right]$

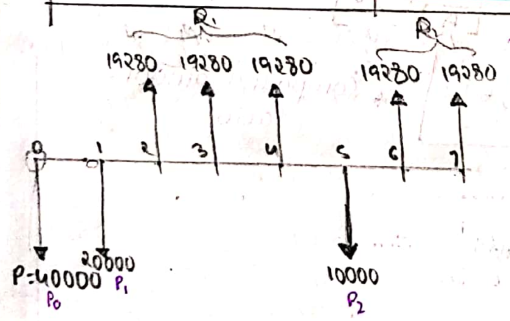
$= 0.06 + (0.07 - 0.06)\left[\frac{1.75 - 1.689}{1.838 - 1.689}\right]$

$= 0.06 + 0.00409$

$= 0.06409 = \underline{6.409\%}$

Q]

End of the year	Receipts (or) Disbursements
0	40000 (D)
1	20000 (D)
2	19280 (R)
3	19280 (R)
4	19280 (R)
5	10000 (D)
6	19280 (R)
7	19280 (R)



→ Trial and Error Method

Case 1: At $t=0$

$P_1 = 20000\left[\frac{P}{F}, i^*, 1\right]$

$P_2 = 10000\left[\frac{P}{F}, i^*, 5\right]$

$P = P_0 + P_1 + P_2 = 40000 + 20000\left[\frac{P}{F}, i^*, 1\right] + 10000\left[\frac{P}{F}, i^*, 5\right]$

$= 40000 + 20000\,[0.9091] + 10000\,[0.6209] = \underline{\underline{64391}}$

$R_1 = 19280\left[\frac{P}{A}, i^*, 3\right] \times \left[\frac{P}{F}, i^*, 1\right]$

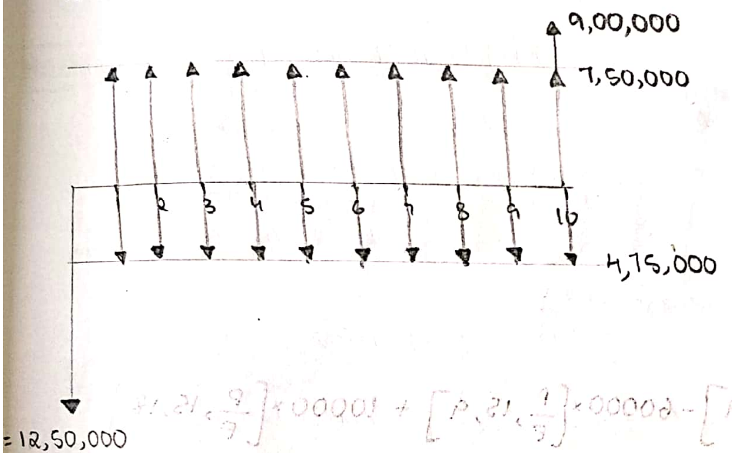
$R_2 = 19280\left[\frac{P}{A}, i^*, 2\right] \times \left[\frac{P}{F}, i^*, 5\right]$

$R = [19280 \times (2.487) \times (0.9091)] + [19280 \times (1.736) \times (0.6209)]$

$= 43590.763 + 20781.5726 = \underline{\underline{64372.333}}$

The above was done by trying all values of i^* until value of receipts is equal to the value of disbursements.

Q] As a manager of a company, you are planning to launch a new product. this regard, you are setting up a facility where the land cost is ₹3,00,000 building cost is ₹6,00,000, the machine tools cost is ₹2,50,000 and ₹1,00,0 is needed as a working capital. It is expected that the product's sales will of ₹7,50,000 per year for 10 years at which the land can be sold for ₹4,00 the building for ₹3,50,000 the Machine for ₹50,000 and all of the worki capital is recovered. The annual maintenance cost is estimated as ₹4,75,00 If the MRR is 25%., determine the feasibility of investment.



$$PW(A) = -12,50,000 + 2,75,000 \times \left[\frac{P}{A}, 25, 10\right] + 9,00,000 \left[\frac{P}{F}, 25, 10\right]$$

$$= -12,50,000 + [2,75,000 \times 3.571] + [9,00,000 \times 0.1074]$$

$$= \underline{-1,71,315}$$

The investment is not feasible.

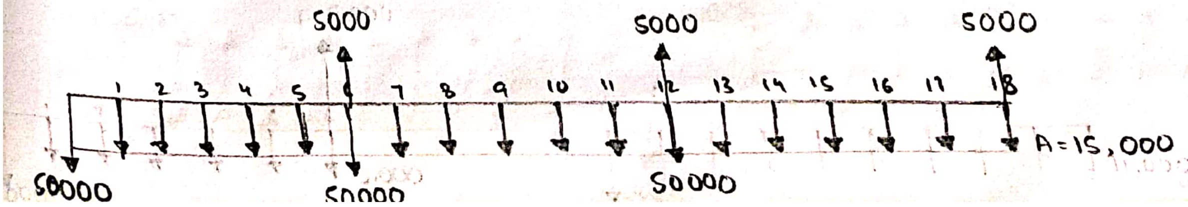
Q] As a manager of a company you are planning to launch a new pro In this regard, you are setting up a facility for which you received tw proposals:

	Machine A	Machine B
Initial cost	50,000	60,000
Salvage value	5,000	10,000
Annual operating cost	15,000	11,000
Life (in years)	6 18	9 18

We need atleast 2 common factor One is interest which is 15% ar the other can b life which is dor calculating LCM of th two machines.

If MARR is 15%. which machine will you prefer?

Machine A



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$$PW(A) = -50000 + 5000\left[\frac{P}{F}, 15, 6\right] - 50000\left[\frac{P}{F}, 15, 6\right] + 5000\left[\frac{P}{F}, 15, 12\right]$$

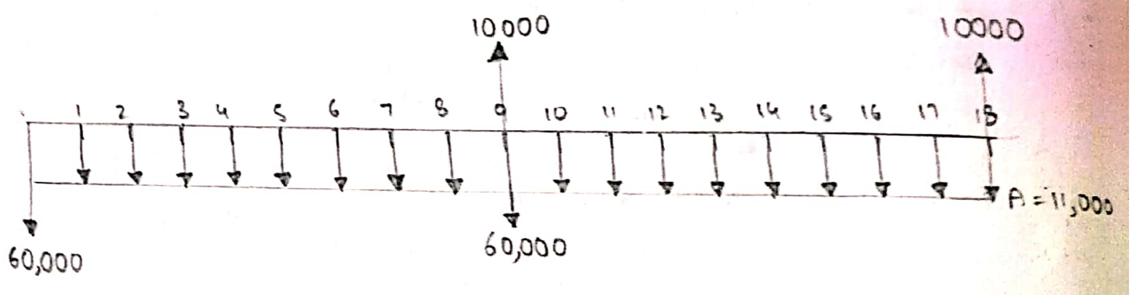
$$-50000\left[\frac{P}{F}, 15, 12\right] + 5000\left[\frac{P}{F}, 15, 18\right] - 15{,}000\left[\frac{P}{A}, 15, 18\right]$$

$$= -50000 + [5000 \times 0.4323] - [50000 \times 0.4323] + [5000 \times 0.1869]$$

$$[-50000 \times 0.1869] + [5000 \times 0.0808] - [15000 \times 6.128]$$

$$= -50000 + 2161.5 - 21615 + 934.5 - 9345 + 404 - 91920 = \underline{\underline{-169380}}$$

<u>Machine B</u>



$$PW(B) = -60000 + 10000 \times \left[\frac{P}{F}, 15, 9\right] - 60000 \times \left[\frac{P}{F}, 15, 9\right] + 10000 \times \left[\frac{P}{F}, 15, 18\right]$$

$$-11000 \times \left[\frac{P}{A}, 15, 18\right]$$

$$= -60000 + [10000 \times 0.2843] - [60000 \times 0.2843] + [10000 \times 0.0808]$$

$$-[11000 \times 6.128]$$

$$= -60000 + 2843 - 17058 + 808 - 67408 = \underline{\underline{-140815}}$$

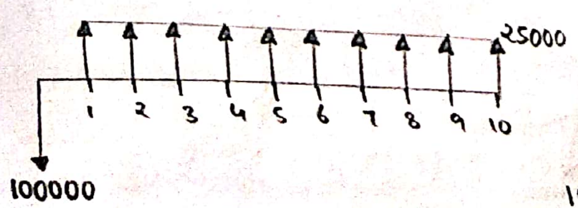
Q8] As a manager of a firm, you plan to buy a machine for a specific product. In this regard you have a few proposals as follows:-

	Machine 1	Machine 2	Machine 3
Initial Cost	1,00,000	1,50,000	2,00,000
Annual saving	25,000	34,000	46,000

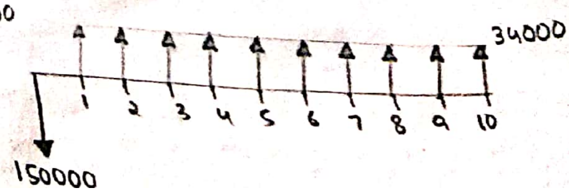
All these machines have a life of 10 years with zero salvage value. If MARR is 15%., find out the best option.

Incremental IRR Method (Mutually available/exclusive options)

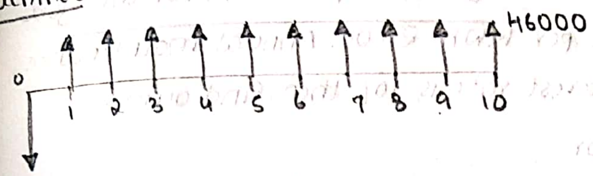
<u>Machine 1</u>



<u>Machine 2</u>



Machine 3



Machine 1: $P = 100000$ $A = 25000$ $n = 10$

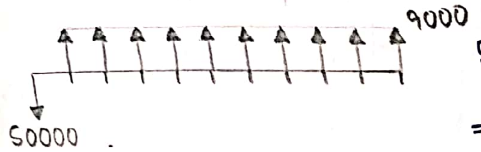
$$100000 = 25000\left[\frac{P}{A}, i^*, 10\right] \Rightarrow 4 = \left[\frac{P}{A}, i^*, 10\right]$$

** From the table $\underline{\underline{i^* = 21.4\%}}$ $\Rightarrow \left[\frac{P}{A}, 21.4, 10\right] = 4$

Machine 2: Incremental Cost:- 50000

Incremental Saving:- 9000

Incremental is calculated w.r.t Machine 1 (150000 - 100000) and (34000 - 25000)



$$50000 = 9000\left[\frac{P}{A}, i^*, 10\right]$$

$$\Rightarrow 5.555 = \left[\frac{P}{A}, i^*, 10\right]$$

$\left[\frac{P}{A}, 15, 10\right] = 5.019$

$\left[\frac{P}{A}, 12, 10\right] = 5.650$

$$x^* = x_1 + (x_2 - x_1)\left[\frac{y - y_1}{y_2 - y_1}\right]$$

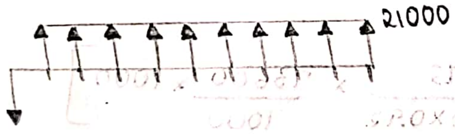
$$x^* = 12 + (15 - 12)\left[\frac{5.5555 - 5.650}{5.019 - 5.650}\right] = 12 + 0.449 = \underline{\underline{12.449}}$$

12.449 < MARR hence discarded.

Machine 3: Incremental Cost:- 100000

Incremental Saving:- 21000

Incremental is again calculated w.r.t Machine 1 since Machine 2 is discarded.



$$100000 = 21000\left[\frac{P}{A}, i^*, 10\right]$$

$$4.7619 = \left[\frac{P}{A}, i^*, 10\right]$$

$\left[\frac{P}{A}, 18, 10\right] = 4.494$

$\left[\frac{P}{A}, 15, 10\right] = 5.019$

$$x^* = x_1 + (x_2 - x_1)\left[\frac{y_0^* - y_1}{y_2 - y_1}\right]$$

$$= 15 + (18 - 15)\left[\frac{4.7619 - 5.019}{4.494 - 5.019}\right] = 15 + 1.4691 = \underline{\underline{16.4691}}$$

Machine 3 is the minimum interest rate greater than MARR.

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8] As a manager of your firm, you have to buy a 100Hp motor. For this you have two proposals, Also the power cost per KWH ₹0.06. Annual maintenance and taxes are 16% of original cost. If interest rate is 10%. then find out:-

a) Equivalent annual cost for each motor.

b) Which motor will you recomment using Annual Worth Method

(Assume 1HP = 736W)

	Motor A	Motor B
First Cost	₹4500	₹4000
'η' ½ load	85%	83%
'η' ¾ load	92%	89%
'η' full load	89%	88%

Hours of use per year

½ load	800	800
¾ load	1000	1000
Full load	600	600
Salvage Value	0	0
Life	12 years	12 years

Machine A

8a] Annual Cost = $\left[\frac{\text{load}}{\eta} \times \text{no. of hours} \times \text{power}\right] \times \text{unit cost}$

$$= \left[\left[\frac{50}{100 \times 0.85} \times \frac{736 \times 100}{1000} \times 800\right] + \left[\frac{75}{100 \times 0.92} \times \frac{73600}{1000} \times 1000\right]\right.$$

$$\left. + \left[\frac{100}{100 \times 0.89} \times \frac{73600}{1000} \times 600\right]\right] \times 0.06 =$$

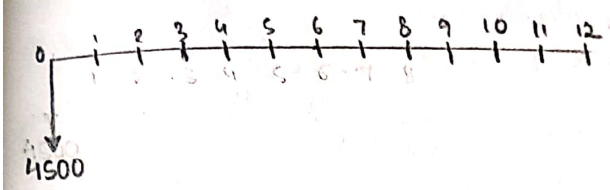
$$= [34635.294 + 60000 + 49617.97] \times 0.06 = \underline{\underline{8655.19}}$$

Machine B

$$\text{Annual Cost} = \left[\left[\frac{50}{100 \times 0.83} \times \frac{73600}{1000} \times 800\right] + \left[\frac{75}{100 \times 0.89} \times \frac{73600}{1000} \times 1000\right]\right.$$

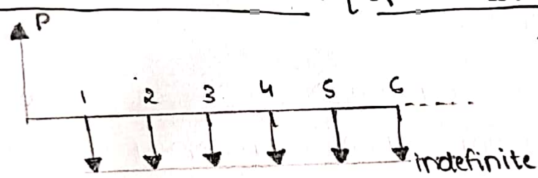
$$\left. + \left[\frac{100}{100 \times 0.88} \times \frac{73600}{1000} \times 600\right]\right] \times 0.06$$

$$= [35469.88 + 62022.47 + 50181.81] \times 0.06 = \underline{\underline{8860.45}}$$



taxes = 1.6% × 4500 = 72

Capitalized Cost Method [Special Case in Present Worth]



$A = P \cdot i$

$P = \frac{A}{i}$

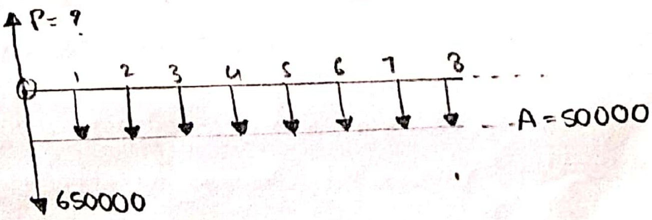
A → indefinite
perpetuity

Q] You have two projects as follows:-

	Project A	Project B
Initial Cost	₹ 6,50,000	₹ 2,30,000
Annual Maintenance Cost	₹ 5,000	₹ 13,500
Intermittent Maintenance (every 15 years)	₹ —	₹ 90000

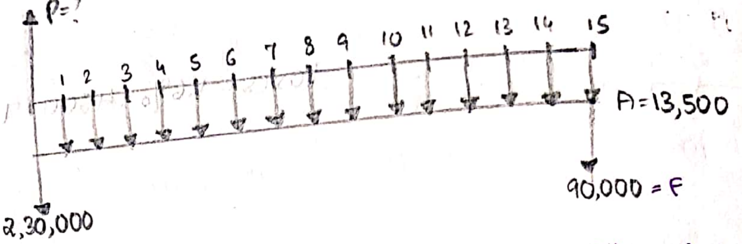
Assume both projects have infinite lives. Compare the present worth of these projects using an interest rate of 8%, which project will you prefer?

Project A:-



$PW_A = 6,50,000 + \frac{5000}{0.08} = ₹ 7,12,500$

Project B:-



$$PW_B = 2,30,000 + \frac{13500}{0.08} + \frac{90000\left[\frac{F}{A},8,15\right]}{0.08}$$

$$= ₹4,40,150$$

F is a future amount so we have to get $A = F \times \left[\frac{A}{F}, i, n\right]$

$$A = 90000\left[\frac{A}{F}, 8\%, 15\right].$$

We prefer project B since investment should be as low as possible.

3] An automobile owner is trying to decide between buying new radial tyres or having the worn out tyres recapped. Radial tyres for the car will cost \$85 each and will last 60,000 kilometers. The old tyres can be recapped for \$35 each but will last for 30,000 kilometers. Since it is a second hand car it registers only 10,000 kilometers per year. If radial tyres are purchased, gasoline mileage will increase by 10%. If the cost of the gasoline is \$0.42 per litre and the car gets 10 kilometers per litre, which tyres should be purchased if interest rate is 12% per year. Use present worth method. Assume salvage value of tyres as zero.

Replacement Analysis :

(1) Defender (2) Challenger

Reasons for replacement

→ Physical Damage (affects performance, sustainability, maintenance cost, production cost)

→ Obsolescence

- Functional
- Economics

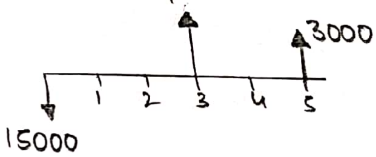
→ Inadequecy

Eg : Life of asset = 10 years

Study | Comparison period for replacement = 3 years

Implied salvage value ⇒ P = 15000 ; F = 3000 ; n = 5 years ; i = 12%

F^* → on replacing, salvage value



$$CRR = (P-F)\left[\frac{A}{P}, i, n\right] + Fi$$

Entire life of the asset :

$$CRR = (15000-3000)\left[\frac{A}{P}, 12, 5\right] + 3000 \times 0.12 \rightarrow (1)$$

$$CRR = 15000\left[\frac{A}{P}, 12, 3\right] - F^*\left[\frac{A}{F}, 12, 3\right] \rightarrow (2)$$

Equate (1) and (2)

$$12000\left[\frac{A}{P}, 12, 5\right] + 3000 \times 0.12 = 15000\left[\frac{A}{P}, 12, 3\right] - F^*\left[\frac{A}{F}, 12, 3\right]$$

$$(12000 \times 0.2774) + 360 = (15000 \times 0.4163) - (F^* \times 0.2963)$$

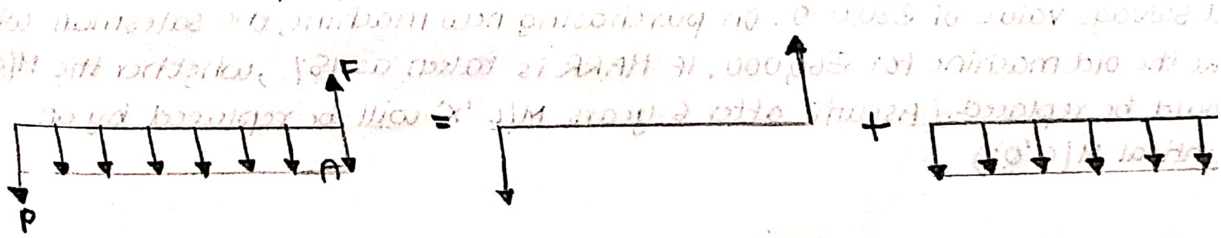
$$3688.8 = 6244.5 - 0.2963F^*$$

$$0.2963F^* = 2555.7$$

$$\underline{\underline{F^* = 8625.38}}$$

<u>Capital Recovery with Return</u>

CFD from lenders perspective



→ Equal annual cashflow over its life that is equivalent to the capital costs of investments

→ The capital cost of investment includes the initial outlay and the final salvage value.

→ Hence two transactions are included here - initial cost and salvage value of the asset.

$$CRR = (P) - (F)$$

$$= P \times \left[\frac{A}{P}, i, n\right] - F \times \left[\frac{A}{F}, i, n\right]$$

↓

Converting further using relations from equal payment series

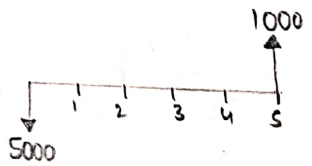
$$\left[\frac{A}{F}, i, n\right] = \left[\frac{A}{P}, i, n\right] - i$$

$$= P \times \left[\frac{A}{P}, i, n\right] - F\left[\left[\frac{A}{P}, i, n\right] - i\right]$$

$$= P \times \left[\frac{A}{P}, i, n\right] - F\left[\frac{A}{P}, i, n\right] + Fi$$

$$\boxed{CRR = (P - F)\left[\frac{A}{P}, i, n\right] + Fi}$$

Q] Asset First Cost = 5000, $n = 5$ years, $F = 1000$, $i = 10\%$.



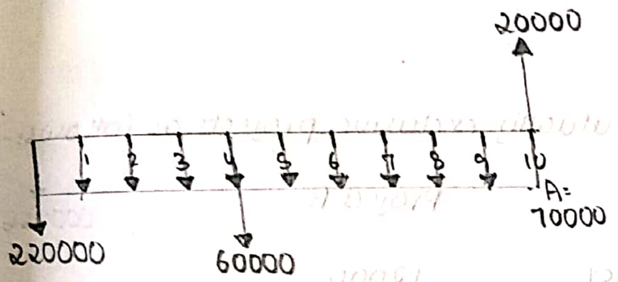
$$CRR = 5000\left[\frac{A}{P}, 10, 5\right] - 1000\left[\frac{A}{F}, 10, 5\right]$$

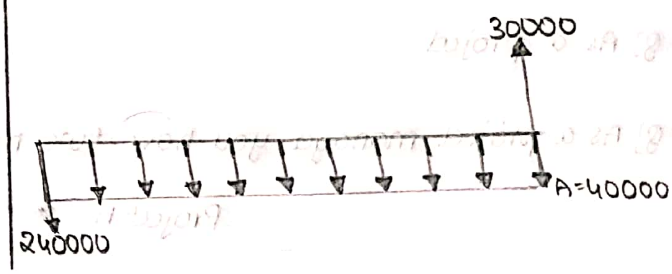
$$\Rightarrow CRR = (5000 - 1000)\underbrace{\left[\frac{A}{P}, 10, 5\right]}_{0.2635} + 1000(0.1)$$

$$= \underline{\underline{1155.2}}$$

Q] Machine X was purchased by a company 4 years ago for ₹2,20,000 with an estimated life of 10 years and salvage value of ₹20,000. Operating cost on the M/c is ₹70,000 per year. Presently a salesman has offered a new Machine 'y' for ₹2,40,000 with an estimated life of 10 years, operating cost ₹40,000/year and salvage value of ₹30,000. On purchasing new machine, the salesman will take the old machine for ₹60,000. If MARR is taken as 15%, whether the M/c should be replaced. (Assume after 6 years, M/c 'x' will be replaced by an identical M/c 'Q')

Machine 'X'	Machine 'y'
I.C = 2,20,000	I.C = 2,40,000
O.C = 70000/year	O.C = 40000/year
S.V = 20000	S.V = 30000
n = 10 years	n = 10 years
Study period = 6 years	Study period = 6 years
Present valuation = 60000	



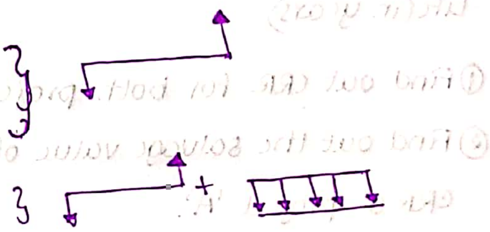


CRR for Machine X:

$$CRR = (60000 - 20000) \times \left[\frac{A}{P}, 15\%, 6\right] + 20000(0.15)$$

$$= (40000 \times 0.2642) + 3000 = \underline{13568}$$

$$EAC(x) = CRR + O.C = 13568 + 70000 = \underline{83568}$$

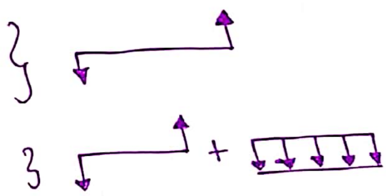


CRR for Machine Y:

$$CRR = (2{,}40{,}000 - 30000)\left[\frac{A}{P}, 15\%, 10\right] + 30000(0.15)$$

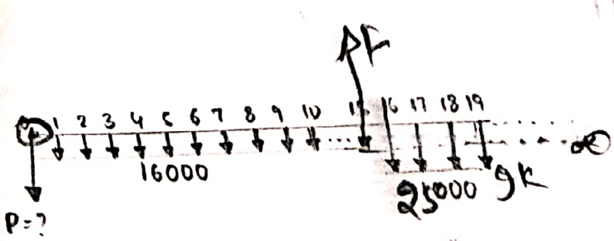
$$= (210000 \times 0.1993) + 4500 = 46353$$

$$EAC(Y) = CRR + O.C = 46353 + 40000 = \underline{\underline{86353}}$$



EAC(x) < EAC(Y) hence Machine X will be considered

9] BMC is planning to build and maintain a park in Govindapura, Yelahank. If the annual interest is 8% and annual maintenance cost is 16000/year. fo first 15 years and increasing ₹25000/year later on, then how much BMC should invest initially on this project?



Q] As a project

Q] As a project manager you have two mutually exclusive projects as follows

	Project A		Project B
First Cost	10000	$i = 15\%$	12000
Salvage Value	1500		3500
Life (in years)	5		5

① Find out CRR for both projects

② Find out the salvage value of project 'B' when its CRR is equal to the CRR of project 'A'.