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Unit 1

Method 1

Q
H 5

⟹ • Graph: A set of vertices connected by edges

• Connected graph: A graph where there is a path between every pair of vertices

• Degree of vertex: The number of edges incident to the vertex

• Circuit: A closed path with no repeated edges or vertices except the starting and ending vertex.

• tree: A connected graph with no circuits

• Path tree: A tree where intermidiate vertices have 2 and end vertices have degree 1

• Star ~~graph~~ tree: A tree with one central vertex of degree n-1 and all other vertices (leaves) of degree 1.

Q
H 6

⟹ • Family trees: Representing ancestry and descendants

• File systems: Organizing files and folders hierarchically.

• Decision Trees: Modeling decisions and their possible Consequences.

QH7 ⟹

let G_1 be tree with vertices 5

G_1

And the Given tree G is

→ The degree sequences of both Graphs are given below

$G_1 = (1,1,2,2,2)$

$G = (1,1,2,2,2)$

We know that

$\sum d(v)_{t_5} = 8$

for G_1 $\sum d(v) = 8$

for G $\sum d(v) = 8$.

$\therefore$ G_1 & G are isomorphic Graph/tree

Method 2.

QH5 ⟹

- Distance : Number of edges in shortest path between two vertices.
- Eccentricity : Maximum distance from vertex to any other vertex
- Radius : Minimum eccentricity among all vertices.
- Center : Vertex or vertices with eccentricity equal to radius.

QH6 ⟹

let a tree be with 3 vertices (path tree)

t $\begin{bmatrix} E(1) = 2 \\ E(2) = 1 \\ E(3) = 2 \end{bmatrix}$

We know that

Diameter of t = 2

Radius of t = 1 [2 is the center of t]

Hence Diameter = 2 × Radius

Hence Proved

Q H7

Tree with more leaves push some vertices farther from "the farthest leaf", raising those vertices' eccentricity; however, the center(s) remains the vertices of minimum eccentricity.

~~Total degree sum is fixed at 2(n-1)~~

Q H8

(i) $d(a,b) = 1$ $\quad$ $d(j,k) = 1$

$d(a,c) = 2$

$d(a,d) = 3$

$d(a,e) = 4$

$d(a,f) = 5$ . . .

(ii)

$E(a) = 6$	$E(f) = 5$	$E(k) = 6$
$E(b) = 5$	$E(g) = 6$	$E(l) = 6$
$E(c) = 4$	$E(h) = 6$	$E(m) = 6$
$E(d) = 3$	$E(i) = 5$	$E(n) = 6$
$E(e) = 4$	$E(j) = 5$	

(iii) Radius $= 3$

(iv) Diameter $= 6$

(v) Center $= d$

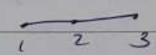
H9

Minimum $= 2$. Every tree with $n \geq 2$

$\because$ Sum of degree is $2(n-1)$;

Distribute this over n positive degrees leaves two 1's.

Ex: A path tree with 3 v.

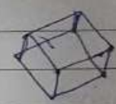


Method 3

H5

Given graphs

G_1



G_2

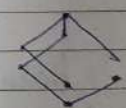
for G_1

1)

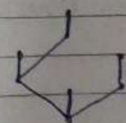
2)

for G_2

1)



2)



H6

$\Longrightarrow$ In $G = P_n$, then G is connected and has no circuits.

$\longrightarrow$ The P_n contains n vertices with $(n-1)$ edges

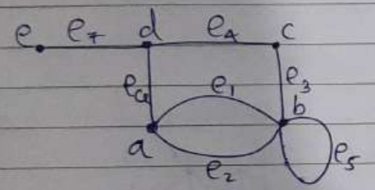
$\longrightarrow$ It is minimally connected.

$\therefore$ P_n is a ~~tree~~ spanning tree ~~own~~ on its own

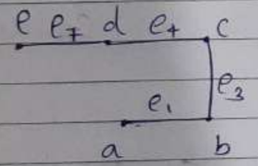
H7

$\Longrightarrow$ Given

G.



spanning tree from G =



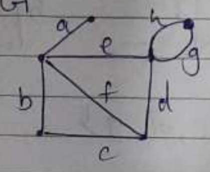
branch $\Rightarrow \{e_1, e_3, e_4, e_7\}$

chord $\Rightarrow \{e_2, e_5, e_6\}$

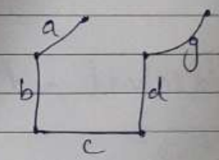
Method 4

H3

$\Longrightarrow$ Given G



Spanning tree from G.



branch = $\{a, b, c, d, g\}$

chord = $\{e, f, h\}$

Fundamental circuits :-

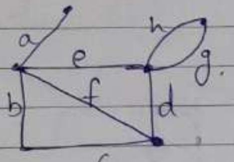
1. $\{e, b, c, d\}$

2. $\{f, b, c\}$

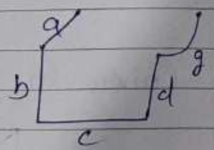
3. $\{h, g\}$

H4

⟹ Given G



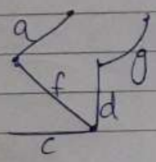
tree

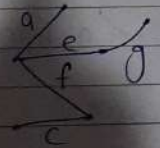


branch = {a, b, c, d, g}

chord = {e, f, h}

1) add 1 chord and remove 1 branch.





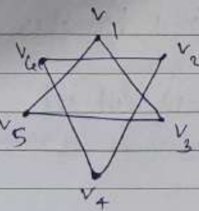
① ②

Unit 2

Method 1

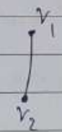
H5

⟹ for G_1,



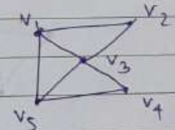
cut set ⇒ $\{(v_1, v_5), (v_1, v_3)\}$

for G_2



cut-set = $\{(v_1, v_2)\}$

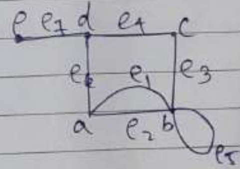
for G_3



cut-set = $\{(v_1, v_2), (v_1, v_3), (v_1, v_5)\}$

H6

⟹ Given G



a) $\{e_4, e_1, e_2\}$ = cut-set

b) $\{e_1, e_6, e_4\}$ = not cut-set

c) $\{e_1, e_2\}$ = not cut-set

H7

⟹ A tree contains $(n-1)$ edges.

∴ A tree have $(n-1)$ branches

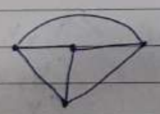
∴ A tree has $(n-1)$ cut-set

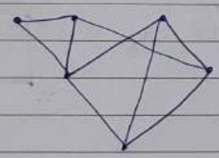
Method 2

H3

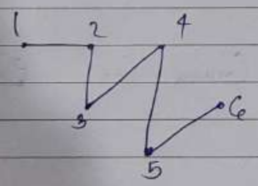
⟹ Given

G_1 G_2





Spanning tree

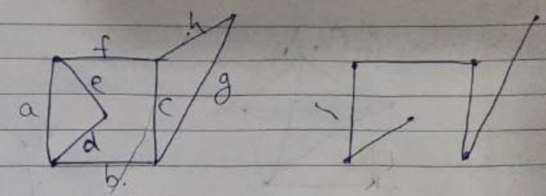


fundamental cut-set ⟹ {(1,3), (1,2), (1,4)} {(1,2), (1,3)}

No. of f.C.S ⟹ 3 [no. of branches] 5 [No. of branches]

H4

⟹ Given G.



Given spt = {d, a, f, c, g}

f.c.s ⟹ {h, g}.

There are 4 other fundamental cutset

{h, c, b}, {f, b}, {e, d}, {a, e, b}

H5

⟹ Given that

The graph is made by removing an edge from minimally connected graph

And given its Rank = 9.

We know that

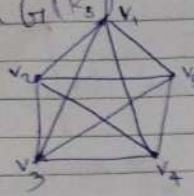
Rank = n - K [Here K is component]

9 = n - 2 [it's a disconnected graph]

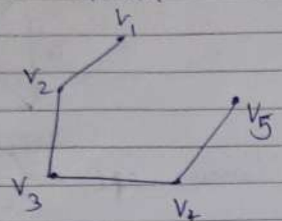
$\boxed{\therefore n = 11}$

H6

⟹ Given $G(K_5)$



Sp. tree



* fundamental Circuits.

⟶ $\{v_1, v_2, v_3\}$, $\{v_1, v_2, v_4\}$, $\{v_1, v_2, v_5\}$, $\{v_1, v_3, v_4\}$,

$\{v_1, v_3, v_5\}$, $\{v_1, v_4, v_5\}$

where.

$\{v_2, v_3, v_4, v_5\}$ ∴ $\{v_2 v_3, v_2 v_4, v_2 v_5, v_3 v_4, v_3 v_5, v_4 v_5 \ldots\}$

* fundamental cutset

$\{(v_1 v_5), (v_1 v_4), (v_1 v_3), (v_1 v_2)\}$,

$\{(v_1 v_5), (v_1 v_4), (v_1 v_3), (v_2 v_5), (v_2 v_4) (v_2 v_3)\}$

$\{(v_1 v_5) (v_1 v_4) (v_2 v_5) (v_3 v_5) (v_2 v_4) (v_3 v_4)\}$

$\{(v_1 v_5) (v_2 v_5) (v_3 v_5) (v_4 v_5)\}$

H7

⟹ ~~A tree on n vertices has exactly n-1 cut-set~~

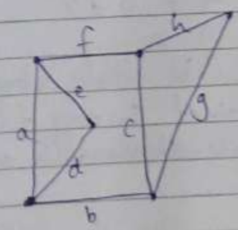
Let S be a cut-set of G. By definition, G-S is disconnected. The Complement of S are the edges remaining after deleting S. Since G-S is disconnected, no spanning tree can be contained entirely in $E(G) \setminus S$. Therefore the Complement of cut-set Cannot contain a Spanning tree.

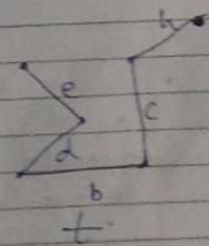
⟶ Let T be a spanning tree of G. The complement $E(G) \setminus T$ are the chords. By the notes, every cut-set in a connected graph must Contain at least one Branch of every spanning tree of G; Since $E(G) \setminus T$ contains no branch of T, it cannot Contain a cut-set. Therefore the complement of a spanning tree does not contain a cut-set.

Method 3

H3

⟹ Given G.





Here, Branch = $\{e, d, b, c, h\}$

chords = $\{a, f, g\}$

→ With respect to a spanning tree T, a chord C_i that determines a fundamental circuit C occurs in every fundamental cut-set associated with the branches in that circuit and in no other fundamental cut-set.

for example.

for chord a

which creates a fundamental circuit

$\Rightarrow \langle a, e, d \rangle$

∴ chord will be present in the fundamental cut-set of all branches of that circuits.

for fundamental cut-sets.

$\Rightarrow \langle b, a, e \rangle$

$\Rightarrow \langle f, a, d \rangle$

- Dually, a branch b_i that determines a fundamental cut-set S is contained in every fundamental circuit associated with the chords in S and in no other fundamental circuit.

for example.

for branch e

fundamental cut-set = $\{a, e, f\}$.

∴ fundamental circuits containing e ⇒

$\langle a, e, d \rangle$, $\langle f, e, d, b, c \rangle$

Method 4

H5

⟹ Given: Tree T and a vertex v with $d(v) > 1$

To prove: v is a cut-vertex.

→ A tree has no cycles and is minimally connected.

If $d(v) \geq 2$, then there are at least two distinct neighbors u & w of v.

→ In a tree, the unique path between u to w must contain v

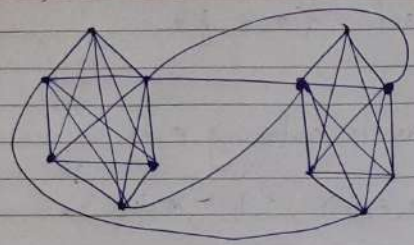
→ By removing vertex v, you are removing the distinct path between u & w

which makes them a two different Components.

∴ The graph becomes disconnected.

∴ The vertex v is a cut-vertex.

H6 $\Longrightarrow$



H7 $\Longrightarrow$

Edge Connectivity :→ ($\lambda(G)$): The ~~minimum~~ minimum number of edges whose removal disconnects G.

Example :→

In a tree $\lambda = 1$ because trees are minimally connected.

Vertex Connectivity :→ the minimum no. of vertices whose removal disconnects G.

for Example

K_{1n} [Here $\kappa = 1$ Because all n vertices are only connected to 1 virtex]

H8 $\Longrightarrow$ The vertex Connectivity of K_{1n} = 1

The Edge Connectivity of K_{1n} = 1.

H9 $\Longrightarrow$ A cut-edge is an edge whose removal increases the no. of connected components of the graph.

→ Yes, In a tree, every edge is a bridge. Thus every tree with $n \geq 2$ has at least one cut-edge

H10 $\Longrightarrow$ Given, degree 2 Regular Graph with $n \geq 3$

The vertex & Edge connectivity of this graph is 2

unit 3

H5 ⟹ Given: place a minimum no. of queens on an 8X8 board so every square is controlled.

→ The domination no. of for queens is 5.

let Horizontal scalling be (a, b, c, d, e, f, g, h)

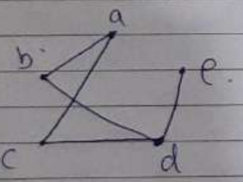
& Vertical Scalling be (1, 2, 3, 4, 5, 6, 7, 8)

Then one of the Solution is

⇒ (a1, d1, g2, b6, f8)

H6 ⟹ Given: $V = \{a, b, c, d, e\}$

$E = \{(a,b), (a,c), (b,d), (c,d), (d,e)\}$.



$d_1 = \{a, d\}$

$d_2 = \{b, d\}$

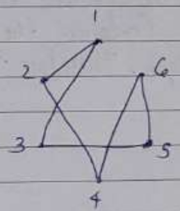
H7 ⟹ Given: $V = \{1, 2, 3, 4, 5, 6\}$

$E = \{(1,2), (1,3), (2,4), (3,5), (4,6), (5,6)\}$

1) $\{2, 5, 6\}$ → yes, This set can cover all the vertex.

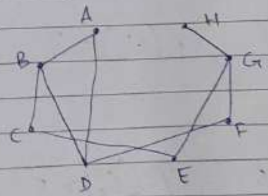
2) $\{1, 3, 4\}$ → yes, ″

3) $\{2, 5\}$ → yes. ″



H8 ⟹ Given: $V = \{A, B, C, D, E, F, G, H\}$

$E = \{(AB), (AD), (B,C), (BD), (CE), (DF), (EG), (FG),$
$(GH)\}$



for the Given Graph

→ Minimum Number of Cameras = 2.

{B, G}.

H9

⟹ Given $S_n = K_{1,n-1}$

→ The center vertex has degree (n-1), so its closed neighborhood is the entire vertex set. Thus the singleton set contains the center dominates the whole graph. Therefore $\gamma(S_n) = 1$

Method 2

H7

⟹ Find $\gamma(P_n) = ?$,

$\gamma(C_n) = ?$, for $6 \leq n \leq 9$

for $n = 6$.

$\gamma(P_6) = 3$, $\gamma(C_n) = 3$.

for $n = 9$

$\gamma(P_9) = 5$, $\gamma(C_9) = 5$

$\therefore \gamma(P_n) = \gamma(C_n)$ [for $6 \leq n \leq 9$] [3, 4, 4, 5]

H8

⟹ 1) K_6: $\gamma(K_6) = 1$ & $\gamma_t(K_6) = 2$

2) P_6: $\gamma(P_6) = 2$ & $\gamma_t(P_6) = 3$.

3) C_6: $\gamma(C_6) = 2$ & $\gamma_t(C_6) = 3$

4) $K_{4,3}$: $\gamma(K_{4,3}) = 3$ & $\gamma_t(K_{4,3}) = 4$.

H9

⟹ Given: $\gamma(G) = 2 \times \gamma_t(G)$

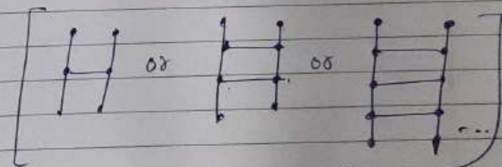
⟹ K_6

H10

⟹ Given $\gamma(G) = \gamma_t(G) = k$; $k \geq 2$.

for Graph such as ladder graph.

like



k is ≥ 2. & $\gamma(G) = \gamma_t(G)$

H2 → Given G

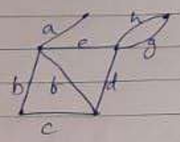
1) The domination number of G = 3.

2) The total domination number of G = 4.

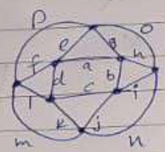
Method 3

H4 → Given:

G_1,

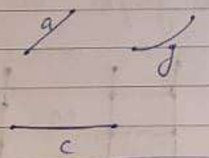


G_2



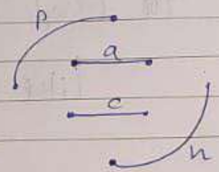
Maximum matching of G_1

⇒



$\boxed{\therefore \text{Matching no. of } G_1 = 3}$

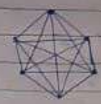
Maximum matching of G_2



$\boxed{\therefore \text{Matching no of } G_2 = 4}$

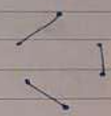
H5 → Given $G = K_6$

K_6 =



for maximum matching pairs = $\frac{n}{2} = \frac{6}{2} = 3$

$\boxed{\therefore \nu(K_6) = 3}$ →



H6 → Given $G = C_6$

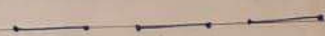
C_6 =

$\boxed{\nu(C_6) = 3}$ →

H7 → Given $G = P_6$

P_6 =

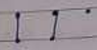
$\boxed{\nu(P_6) = 3}$ →



H8

⟹ Given $G = K_{2,3}$

$K_{2,3} \rightarrow$ 

$\boxed{\nu(K_{2,3}) = 2} \rightarrow$ 

H9

⟹ Given $G = S_n$

$S_n =$ 

$\boxed{\nu(S_n) = 1} \rightarrow$ Because there is only one non-pendent vertex & which has $n-1$ degree

H10

⟹ Prove = maximum is always maximal but maximal is not necessarily maximum.

We know that,

Maximal matching :→ maximum no. of edges with no adjecent edges.

Maximum Matching :→ It is a maximal matching which contains the highest no. of edges.

∵ Maximum is sub-set of Maximal

∴ Hence every Maximum is Maximal but not converse is not true.

Method 4

H5

⟹ Given = Connected graph of order 4 that has no perfect Maching

let Graph be $K_{1,3}$

$K_{13} =$ 

In above graph, which has order 4 not have perfect matching.

H6

⟹

let G be K_{BG} [B = set of Boys, G = set of Girls]

By Hall's theorem,

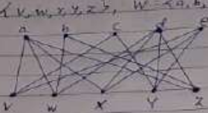
if for every subset $X \subseteq B$

$|N(X)| \geq |X|$, $\therefore \boxed{|N(B)| \geq |B|}$

H7

Given $U = \{v, w, x, y, z\}$, $W = \{a, b, c, d, e\}$

G_1 =

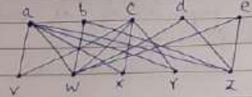


for G_1, for every $X \subseteq U$

$|N(X)| \geq |X|$

$\therefore$ U can be matched to W for G_1

G_2 =



for G_2, let X be $= \{b, d, e\}$

$\therefore N(X) = \{w, z\}$

Here, $|X| > |N(X)|$

→ it doesn't follows the Hall's theorem

$|N(X)| \geq |X|$

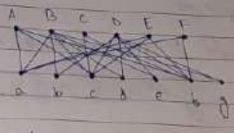
$\therefore$ U can not be Matched in G_2

H8 Given

Applicants $= \{A, B, C, D, E, F\}$

positions $= \{a, b, c, d, e, f, g\}$

G_1 =



or

A = a, c, f

B = a, b, c, d, e, g

C = c, f

D = b, c, d, e, f, g

E = a, c, f

F = a, f

for G_1,

$X \subseteq$ Applicants,

$|N(X)| \not\geq |X|$

$\therefore$ There, a matching doesn't exist

for $X = \{A, C, E, F\}$

$N(X) = \{a, c, f\}$ $\therefore |N(X)| < |X|$

H9 $\Longrightarrow$ 1) for every nonempty $X \subseteq X$-partite set.

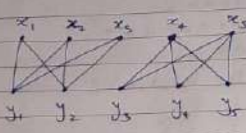
Compute neighbourhood $N(X)$. If any X has $|N(X)| < |X|$, Hall fails and no perfect matching exists. Otherwise perfect matching exists.

for example $\Rightarrow$

let $X = \{x_1, x_2, x_3, x_4, x_5\}$,

$Y = \{y_1, y_2, y_3, y_4, y_5\}$

And



Here, for $X' = \{x_1, x_2, x_3\}$

$N(X') = \{y_1, y_2\}$

so, $|N(X')| = 2 < 3 = |X'|$.

Thus Halls fails.

$\therefore$ No perfect Matching exists.

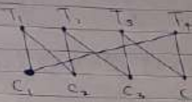
H10 $\Longrightarrow$ Given $T = \{T_1, T_2, T_3, T_4\}$, $C = \{C_1, C_2, C_3, C_4\}$

$T_1 : C_1, C_2$

$T_2 : C_2, C_3$

$T_3 : C_3, C_4$

$T_4 : C_1, C_4$



$\left[\begin{array}{c} \text{By, Hall's theorem} \\ |N(X)| \leq |X| \end{array} \right]$

for $\forall X \subseteq T$ $\because$

$|N(X)| \leq |X|$ or $|N(T)| \leq |T|$

$\therefore$ There is a perfect matching for T & C.

Ex $\Rightarrow$ $T_1 : C_1$

$T_2 : C_2$

$T_3 : C_3$

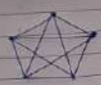
$T_4 : C_4$

H11 $\Longrightarrow$ Same as $\boxed{H8}$.

Unit 4

Method 1

H5 $\Longrightarrow$ Given K_5, $d(v_i) = 4$,



We Know $L = D - A$

for K_n: $D = (n-1)I$, $A = J - I$ [where J is the all-ones matrix]

$$\therefore L(K_5) = \begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & -1 & 4 \end{bmatrix}$$

H7 $\Longrightarrow$ Given $G = C_5$, $d(v_i) = 2$.

By $L = D - A$

$D = [2, 2, 2, 2, 2]$

$$\therefore L = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}$$

H8 $\Longrightarrow$ Given $G = K_{4,4}$, $d(v_i) = 4$

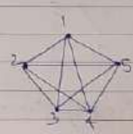
$D = \mathrm{diag}(4,4,4,4,4,4,4,4)$ or $4\,\mathrm{diag}(I_8)$.

$A = \begin{bmatrix} O_{4\times4} & J_{4\times4} \\ J_{4\times4} & O_{4\times4} \end{bmatrix}$ (where J is all 1's matrix)

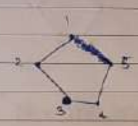
$\therefore L(K_{4\times4}) = D - A = \begin{bmatrix} 4I_{4\times4} & -J_{4\times4} \\ -J_{4\times4} & 4I_{4\times4} \end{bmatrix}$

Method 2

H5 $\Longrightarrow$ Given $G = K_5$



$\longrightarrow$



$n = 5$, $e = 10$, $c = e - n + 1 = 6$, $B = 4$.

f - cut-sets = {(13, 14, 15), (13, 14, 15, 24, 25), (14, 15, 24, 25, 35), (15, 25, 35)}

[chords only]

$$\therefore C_f(K_5) = \begin{array}{cccccc|cccc} 13 & 14 & 15 & 24 & 25 & 35 & 12 & 23 & 34 & 45 \\ \end{array}$$
$$\left[\begin{array}{cccccc|cccc} 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

H6

$\Longrightarrow$ Given $G = S_5$, $n = 5$ [S_5 is tree]

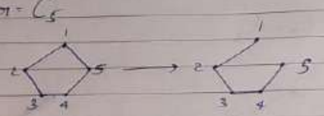
$B = 4$,

$C = 0$

$$\therefore C_f(S_5) = I_5 \text{ or } \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

H7

$\Longrightarrow$ Given $G = C_5$



$n = 5$, $B = 4$

$C = 1$

f - cut-sets = {(15, 12), (15, 23), (15, 34), (15, 45)}

$$\therefore C_f(C_5) = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

H8

$\Longrightarrow$ Given G -

f - cut sets = {(e, f), (e, h, d), (c, i, d), (c, g, b), (a, b)}

$$\therefore C_f(G) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccc|ccccc} e & c & d & b & f & h & i & g & a \end{array} \\ \left[\begin{array}{cccc|ccccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \end{array}$$

Method 3

H4

$\Longrightarrow$ Given, $A = \begin{bmatrix} -5 & 4 & 34 \\ 0 & 0 & 4 \\ 0 & 0 & 4 \end{bmatrix}$.

for triangular Matrices, eigenvalues are diagonal entries.

$\therefore \lambda \in \{-5, 0, 4\}$.

$|A| = \lambda_1 \times \lambda_2 \times \lambda_3 = -5 \times 0 \times 4 = 0$.

$\therefore A$ is not invertible

H5 $\Longrightarrow$ Given $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$\because$ characteristic polynomial is

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3$$

$S_1 = 0+0+0 = 0$ [tras of A]

$S_2 = (0-1)+(0-1)+(0-1) = -3$

[sum of principal 2x2 minors]

$S_3 = |A| = 2$

$\therefore \lambda^3 - 3\lambda - 2 = 0$

$(\lambda+1)(\lambda+1)(\lambda-2) = 0.$

$$\boxed{\therefore \begin{cases} \lambda_1 = -1 \\ \lambda_2 = -1 \\ \lambda_3 = 2 \end{cases}}$$

H6 $\Longrightarrow$ Given $G = K_4$

We know that

$L = nI - J$

$L = 4I - J$

$\therefore L = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$

& we know that

for $K_n \Rightarrow \lambda \in (0^1, n^{n-1})$

$$\boxed{\therefore \lambda \in \langle 0, 4, 4, 4 \rangle}$$

for $L = \begin{bmatrix} 3-\lambda & -1 & -1 & -1 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ -1 & -1 & -1 & 3-\lambda \end{bmatrix}$

let $a = 3-\lambda$ & $d = 4-\lambda$

after Row operations

$L = \begin{bmatrix} a & -1 & -1 & -1 \\ -d & d & 0 & 0 \\ -d & 0 & d & 0 \\ -d & 0 & 0 & d \end{bmatrix}$

By Solving eq

we get $-\lambda(4-\lambda)^3 = 0$.

$\therefore \lambda \in (0, 4, 4, 4)$

H7

$\Longrightarrow$ for A_{P_4}

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$(A-\lambda I) = \begin{bmatrix} -\lambda & 1 & 0 & 0 \\ 1 & -\lambda & 1 & 0 \\ 0 & 1 & -\lambda & 1 \\ 0 & 0 & 1 & -\lambda \end{bmatrix}$$

$|A-\lambda I| = \lambda^4 - 3\lambda^2 + 1 = 0$.

let $x = \lambda^2$; then

$$x^2 - 3x + 1 = 0$$

$$x = \frac{3 \pm \sqrt{5}}{2}$$

$$\therefore \lambda = \pm\sqrt{\frac{3 \pm \sqrt{5}}{2}}$$

$\therefore \lambda = \pm 1.6180, \pm 0.6180.$

for L_{P_4}

$$(L-\lambda I) = \begin{bmatrix} 1-\lambda & -1 & 0 & 0 \\ -1 & 2-\lambda & -1 & 0 \\ 0 & -1 & 2-\lambda & -1 \\ 0 & 0 & -1 & 1-\lambda \end{bmatrix}$$

after solving $|L-\lambda I|$

$$= \lambda^4 - 6\lambda^3 + 10\lambda^2 - 4\lambda = 0$$

$$\lambda(\lambda^3 - 6\lambda^2 + 10\lambda - 4) = 0$$

$$\lambda(\lambda-2)(\lambda^2 - 4\lambda + 2) = 0$$

$$\boxed{\therefore \lambda = 0, 2, 2 \pm \sqrt{2}}$$

for S_4

A_{S_4}

$$(A-\lambda I) = \begin{bmatrix} -\lambda & 1 & 1 & 1 \\ 1 & -\lambda & 0 & 0 \\ 1 & 0 & -\lambda & 0 \\ 1 & 0 & 0 & -\lambda \end{bmatrix}$$

$|A-\lambda I| = \lambda^4 - 3\lambda^2 = \lambda^2(\lambda^2 - 3) = 0$.

$$\boxed{\therefore \lambda = 0, 0, \pm\sqrt{3}}$$

for L_s

$$(L - \lambda I) = \begin{bmatrix} 3-\lambda & -1 & -1 & -1 \\ -1 & 1-\lambda & 0 & 0 \\ -1 & 0 & 1-\lambda & 0 \\ -1 & 0 & 0 & 1-\lambda \end{bmatrix}$$

$$|L - \lambda I| = \lambda(\lambda-4)(\lambda-1)^2 = 0$$

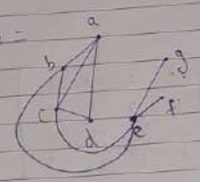
$$\boxed{\therefore \lambda = 0, 4, 1, 1}$$

Method 4

H2

$\Longrightarrow$ Given.

$G =$



Here.

→ The graph contains two dense triads $\{a, b, c, d\}$ & $\{e, f, g\}$ with bridging be, ce.

→ Spectral bisection separates these as:

$$S = \{a, b, c, d\}, \quad T = \{e, f, g\}.$$