Derivation of weak form

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = f(x,t) \qquad (x,t) \in (0,1) \times (0,1)$$

$$u(x,0) = \sin(\pi x)$$
$$u(0,t) = u(1,t) = 0$$

$$f(x,t) = (\pi^2 - 1) e^{-t} \sin(\pi x)$$
$$u(x,t) = e^{-t} \sin(\pi x)$$

$$\int_0^1 \frac{\partial u}{\partial t} V(x)\,dx - \int_0^1 \frac{\partial^2 u}{\partial x^2} V(x)\,dx = \int_0^1 f(x,t)$$

$$u = V(x) \qquad du = V'(x)\,dx \qquad V = \frac{\partial u}{\partial x}$$
$$V = \frac{du}{dx} \qquad dv = \frac{d^2 u}{dx^2}\,dx$$

$$\int_0^1 \frac{\partial u}{\partial t} V(x)\,dx - \left[V(x) \frac{du}{dx} \right]_0^0 + \int_0^1 \frac{\partial u}{\partial x} \frac{\partial v}{\partial x} = \int_0^1 f(x,t) V(x)\,dx$$

$$V(x) = \phi_i(x)$$

$$\int_0^1 \frac{\partial u}{\partial t} \phi_i(x)\,dx + \int_0^1 \frac{\partial u}{\partial x} \frac{\partial \phi_i}{\partial x}\,dx - \int_0^1 f(x,t)\,\phi_i(x)\,dx = 0$$