Mihir Bendre
COE 352

Project 2 : Hand-Calculations

$$u_t - u_{xx} = f(x,t)$$

If $v(x)$ is a function which disappears at boundary (as per Dirichlet), we can do the following:

$$v(x)\left(u_t - u_{xx}\right) = v(x)\, f(x,t)$$

since we're given domain $(0,1)$:

$$\int_0^1 v(x)\left(u_t - u_{xx}\right) dx = \int_0^1 v(x)\, f(x,t)\, dx$$

Doing integration by parts, we get : $\displaystyle\int_0^1 v(x)\, u_{xx}\, dx = \underbrace{v(x)\, u(x)\Big|_0^1}_{=\varnothing \text{ by Dirichlet conditions.}} - \int_0^1 v'(x)\, u_x\, dx$

$\therefore$ Our equation simplifies to :

$$\boxed{\int_0^1 v(x)\, u_t\, dx + \int_0^1 v'(x)\, u_x\, dx = \int_0^1 v(x)\, f(x,t)\, dx}$$