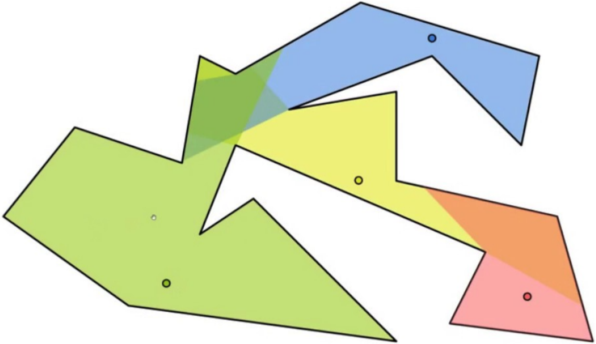
Maximizing the Guarded Boundary of a Dynamic Art Gallery

Mihir Patel

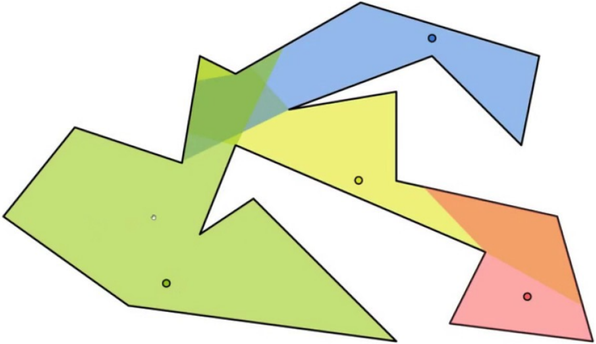
April 16, 2025

Art Gallery



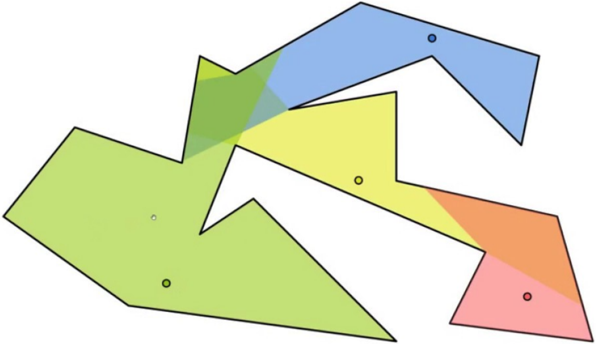
Problem: Given a polygon P, find the minimum number of guards needed to guard the whole polygon.

Flipping what we optimize



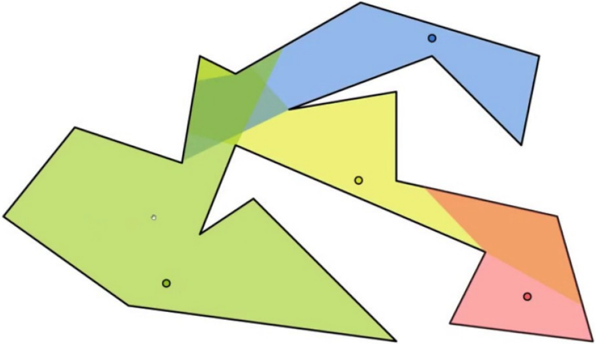
Problem: Given a polygon P and $k \in \mathbb{N}$, find the maximum area that can be guarded by k guards.

Flipping what we optimize



Problem: Given a polygon P and $k \in \mathbb{N}$, find the maximum area that can be guarded by k vertex guards.

Flipping what we optimize



Problem: Given a polygon P and $k \in \mathbb{N}$, find the maximum boundary length that can be guarded by k vertex guards.

A weighted case

Some paintings are more valuable than others, and with limited guards they should be of greater concern.

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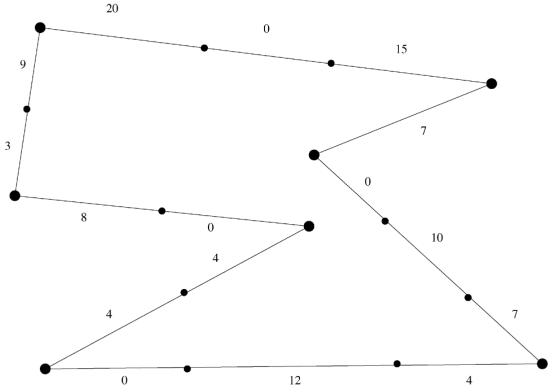

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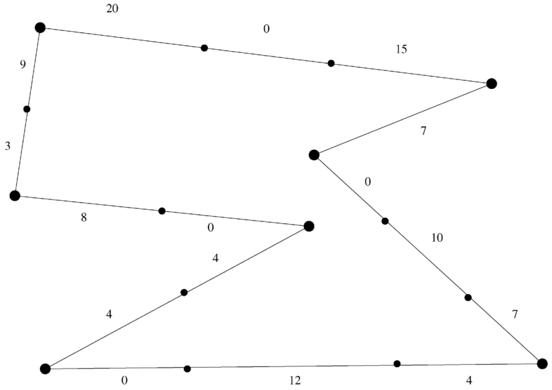

Problem: Given a polygon P and $k \in \mathbb{N}$, find the maximum value that can be guarded by k vertex guards.

Existing Results

Fragoudakis et. al (2005,2006,2007)

Maximum Length Vertex Guard

Input:	A simple polygon P and positive integer $k \in \mathbb{N}$.
Problem:	Find a set of vertices $S \subseteq V_P$ of size at most k such that $L(S)$ is maximized.

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Maximum Value Vertex Guard

Input:	A simple polygon P and positive integer $k \in \mathbb{N}$.
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MAXIMUM LENGTH VERTEX GUARD

Input: A simple polygon P and positive integer $k \in \mathbb{N}$.
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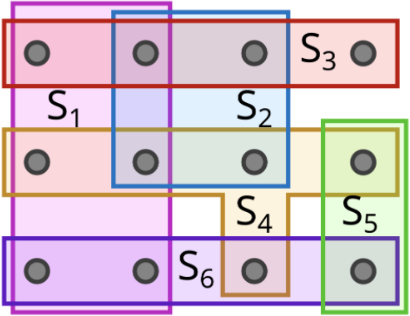
MAXIMUM VALUE VERTEX GUARD

Input: A simple polygon P and positive integer $k \in \mathbb{N}$.
Problem: Find a set of vertices $S \subseteq V_P$ of size at most k such that $W(S)$ is maximized.

Both problems are APX-complete and permit $(1 - 1/e)$-approximations.

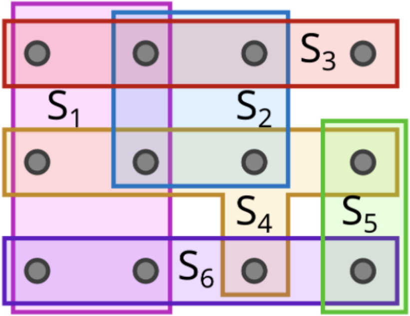
Set Cover



SET COVER

Input:	A universe U of n elements, m subsets of U.
Problem:	What is the minimum number of subsets whose union covers all of U?

Max Coverage



Max Coverage

Input: A universe U of n elements, m subsets of U, $k \in \mathbb{N}$.

Problem: What is the maximum number of elements in U covered by the union of k subsets?

Monotonicity and Submodularity

- **Monotonicity:**
  For any sets $S \subseteq T$, we have:

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Theorem

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Example: Max Coverage

Proposed Contributions

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- Hardness reduction from Set Cover, not 3SAT variation.
- Difficult to break past monotone/submodular $\rightarrow$ hardness of approximation results?

A dynamic version

Paintings may move around, new paintings may arrive. You need to find an optimal camera placement that does not require extensive reinstallment.

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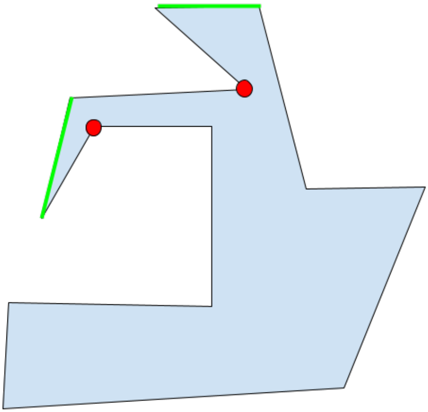

Thank You!

Questions?