TRIG IDENTITIES

Pythagorean Identities

- $\sin^2(x) + \cos^2(x) = 1$
- $\tan^2(x) + 1 = \sec^2(x)$
- $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

Even-Odd identities

- $\sin(-x) = -\sin(x)$
- $\cos(-x) = \cos(x)$
- $\tan(-x) = -\tan(x)$

Double angle formulas

- $\sin(2x) = 2\cdot\sin(x)\cdot\cos(x)$
- $\cos(2x) = \cos^2(x) - \sin^2(x)$
- $\cos(2x) = 1 - 2\sin^2(x)$
- $\cos(2x) = 2\cdot\cos^2(x) - 1$
- $\tan(2x) = \dfrac{2\tan(x)}{1-\tan^2(x)}$

Half-Angle Formulas

- Literally two of the cos(2x) double angle formulas re-arranged
- $\tan\left(\frac{x}{2}\right) = \dfrac{(1-\cos(x))}{\sin(x)}$

Product to Sum formulas

- $\sin(x)\cdot\sin(y) = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$
- $\cos(x)\cdot\cos(y) = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$
- $\sin(x)\cdot\cos(y) = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$
- $\cos(x)\cdot\sin(y) = \frac{1}{2}\left[\sin(x+y) - \sin(x-y)\right]$

Sum to Product formulas

- $\sin(x) \pm \sin(y) = 2\cdot\sin\left(\frac{x\pm y}{2}\right)\cdot\cos\left(\frac{x\mp y}{2}\right)$
- $\cos(x) + \cos(y) = 2\cdot\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)$
- $\cos(x) - \cos(y) = -2\cdot\sin\left(\frac{x+y}{2}\right)\cdot\sin\left(\frac{x-y}{2}\right)$

Sum and difference formulas

- $\sin(x\pm y) = \sin(x)\cdot\cos(y) \pm \cos(x)\cdot\sin(y)$
- $\cos(x\pm y) = \cos(x)\cdot\cos(y) \mp \sin(x)\cdot\sin(y)$
- $\tan(x\pm y) = \dfrac{\tan(x)\pm\tan(y)}{1\mp\tan(x)\cdot\tan(y)}$

Unit Circle Angles Trigonometry

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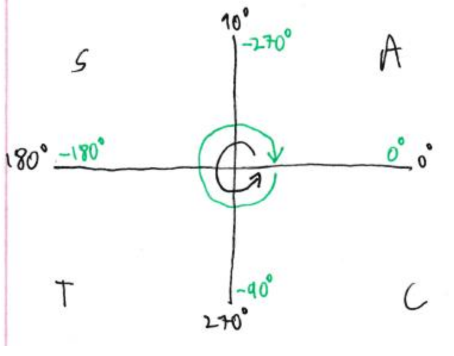
$\theta°$	0	30	45	60	90
	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
$\sin(\theta°)$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta°)$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan(\theta°)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Easy way to remember

1. Write from → 0, 1, 2, 3, 4
2. Square root all → $\sqrt{0}, \sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4}$
3. Divide all by two → $\frac{\sqrt{0}}{2}, \frac{\sqrt{1}}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, \frac{\sqrt{4}}{2}$
4. Simplify → $0, \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}, 1$
5. Tadaa, these are your sin(x) values for $x = 0, 30, 45, 60, 90$ respectively.
6. cos(x) is the same as sin(x) but in reverse
7. tan(x) is just $\frac{\sin(x)}{\cos(x)}$

- 

The C.A.S.T circle

- C → cos(x) ; A → All ; S → sin(x) ; T → tan(x)
- if 'x' is in any of the respective quadrants of trig operations, it will give a positive value; if not, a negative value
- eg. cos(60) would be positive, but cos(120) would be negative

Derivative's & Integrals

(1) $\frac{d}{dx} x^n = nx^{n-1}$

(2) $\frac{d}{dx} e^x = e^x$

(3) $\frac{d}{dx} \ln(x) = \frac{1}{x}$

(4) $\frac{d}{dx} n^x = n^x \ln x$

(5) $\frac{d}{dx} \sin(x) = \cos(x)$

(6) $\frac{d}{dx} \cos(x) = -\sin(x)$

(7) $\frac{d}{dx} \tan(x) = \sec^2(x)$

(8) $\frac{d}{dx} \cot(x) = -\mathrm{cosec}^2(x)$

(9) $\frac{d}{dx} \sec(x) = \sec(x) \cdot \tan(x)$

(10) $\frac{d}{dx} \mathrm{cosec}(x) = -\mathrm{cosec}(x) \cdot \cot(x)$

(11) $\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}$

(12) $\frac{d}{dx} \arccos(x) = -\frac{1}{\sqrt{1-x^2}}$

(13) $\frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$

(14) $\frac{d}{dx} \mathrm{arccot}(x) = -\frac{1}{1+x^2}$

(15) $\frac{d}{dx} \mathrm{arcsec}(x) = \frac{1}{x\sqrt{x^2-1}}$

(16) $\frac{d}{dx} \mathrm{arccosec}(x) = -\frac{1}{x\sqrt{x^2-1}}$

(1) $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$

(2) $\int e^x \, dx = e^x + C$

(3) $\int \frac{1}{x} \, dx = \ln(x) + C$

(4) $\int n^x \, dx = \frac{n^x}{\ln(n)} + C$

(5) $\int \cos(x) \, dx = \sin(x) + C$

(6) $\int \sin(x) \, dx = -\cos(x) + C$

(7) $\int \sec^2(x) \, dx = \tan(x) + C$

(8) $\int \mathrm{cosec}^2(x) dx = -\cot(x) + C$

(9) $\int \tan(x) \cdot \sec(x) = \sec(x) + C$

(10) $\int \cot(x) \cdot \mathrm{cosec}(x) = -\mathrm{cosec}(x) + C$

(11) $\int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin(x) + C$

(12) $\int -\frac{1}{\sqrt{1-x^2}} \, dx = \arccos(x) + C$

(13) $\int \frac{1}{1+x^2} \, dx = \arctan(x) + C$

(14) $\int -\frac{1}{1+x^2} \, dx = \mathrm{arccot}(x) + C$

(15) $\int \frac{1}{x\sqrt{x^2-1}} \, dx = \mathrm{arcsec}(x) + C$

(16) $\int -\frac{1}{x\sqrt{x^2-1}} \, dx = \mathrm{arccosec}(x) + C$

$$\boxed{\begin{aligned} \int \tan(x) \, dx &= \ln|\sec(x)| + C \\ \int \sec(x) \, dx &= \ln|\sec(x) + \tan(x)| + C \end{aligned}}$$