**IB Physics**

Extended Essay

# What is the relationship between the number of coils & voltage and the exit velocity of a permanent magnet projectile in an electromagnetic linear accelerator?

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# Abstract

This essay aims to investigate the relationship between voltage (& no. of turns in coil) and the exit velocity of permanent magnet projectiles in a Magnetic Linear Accelerator (MLA or Coil Gun) projectile launching system. This is given the unconventionality of using permanent magnets as projectiles in Coil Guns, as soft iron materials are usually used. An MLA is simply a high energy coil (powered by a short high power pulse from a capacitor bank) through which a ferromagnetic projectile is sucked into and accelerated out of.

The first objective of this essay is to determine the relationship between the Voltage supplied to the capacitor bank and the resultant Projectile Exit Velocity. The second is to investigate the effects of changing the number of turns in the coil on the Exit Velocity of the Projectile.

Data was collected using a fully functional Magnetic Linear Accelerator (Coil Gun) powered by a capacitor bank of 3000$\mu$F which was varied from 100 to 300 Volts with 25 Volt increments. Projectile Speed was calculated using a single laser light gate.

Results were compared to the hypothesis that 'the Voltage supplied to the Capacitor Bank and the Projectile Exit Velocity are linearly related, not accounting for air resistance. Factoring in air resistance would result in a logarithmic-like relationship between Capacitor Bank and the Projectile Exit Velocity.' This hypothesis was established from equating the formulas for Energy stored in the capacitor bank and the kinetic energy of the projectile ($\frac{1}{2} \cdot C \cdot V^2 = \frac{1}{2} \cdot m \cdot v^2$), as well as taking into account the effects of factors such as air resistance and the suckback effect.

A linear relationship was found between Capacitor Bank Voltage and Projectile exit velocity with lower coil turns ($\therefore$ a less energetic coil). With higher coil turns ($\therefore$ a more energetic coil), this relationship was found to be logarithmic as air resistance and the suckback effect became more significant.

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# 1. Introductory Information

A Magnetic Linear Accelerator (MLA) is a projectile launching system that exploits the properties of induced magnetic fields. Said magnetic field is produced in a (relatively) high energy coil for a fraction of a second, pulling in and propelling out a magnetic projectile placed behind it.

The building of MLA's are undertaken in a number of households and academic institutions world wide for various purposes. However they have also attracted the interest of larger more prominent organisations such as NASA and the United States Navy as experimental launching systems. (Gardner, 2010) (Nowicki, 1999)

Information on building coil guns are an available resource, however very little is described about certain factors and parameters of them. Relationships between current and induced magnetic fields are well established, however within the context of MLA's, a non-vague relationship between more useful variables such as the projectile exit velocity and voltage supplied to the capacitor bank is far less prevalent (voltage is far more easily controlled in MLA structures). Also, all accounts of MLA's only use non-magnetised ferric projectiles (such as soft iron). There is little to no account of the usage of permanent magnets as projectiles as used in this investigation.

In order for the magnetic field produced to be strong enough to accelerate a projectile at significant speeds, a high amount of current is needed. A continuous supply of high enough current to provide a desired effect, however, cannot be practically applied in smaller scale coils builds with lower gauge wiring and lower current ratings. Also, the supply would have to be timed as the coil would tend to accelerate the projectile to the centre of it regardless of whether it is behind or in front of the coil, causing what is known as the suckback effect[1].

A suitable source to provide this current would be from discharging aluminium electrolytic capacitors. $E = \frac{1}{2}CV^2$ tells us that the energy stored in capacitors is proportional to voltage squared. High voltage capacitors are therefore used in order to store and transfer the most amount of energy and lose the least amount of energy in heat. These capacitors also discharge all their stored charge in a fraction of a second, and since $I = \frac{Q}{t}$, the quick discharge times would result in very high currents for a very short period of time. This would also mean that the coil would likely be unpowered before the projectile reaches its centre point, resulting in a reduction or even negation of the suck back effect.

This essay is an attempt to study the usage of permanent magnets as projectiles in MLA's and their possible outcomes. The first objective of this essay is to determine the relationship between the Voltage supplied to the capacitor bank ($V_{Capacitor\ Bank}$) and the resultant Projectile Exit Velocity ($v_{Projectile\ Exit\ Velocity}$). The second is to investigate the effects of changing the number of turns in the coil on $v_{\text{Projectile Exit Velocity}}$.

[1] Suckback effect: The result of pulse times being long enough such that the coil remains powered (i.e. current flowing through it) even after the projectile's centre of mass moves pasts the centre of the coil. As a result, a force is exerted on the magnetic projectile forcing it back towards the coils centre, or "sucking it in", effectively contributing to the deceleration of it.

# 2. Apparatus Description & Experimental Procedure

## 2.1 Magnetic Linear Accelerator Device

A specific support structure as well as electronic circuitry had to be designed and built in order to collect data, as shown in Figures 2A and 2B respectively.

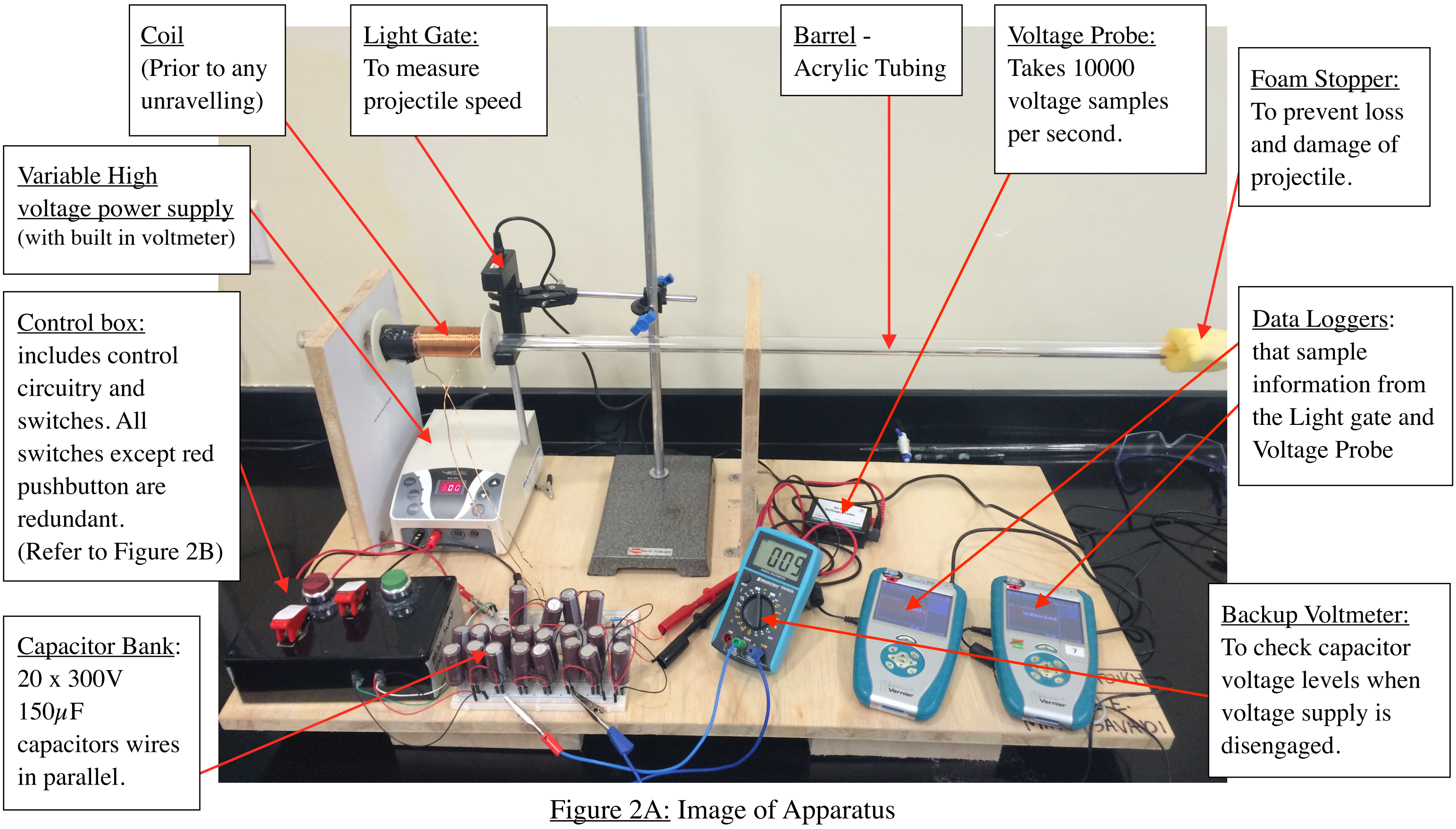


Figure 2A: Image of Apparatus

The circuit was designed to use a minimal amount of components in order to reduce resistances and discrepancies in performance of the circuit during data collection. A thyristor was used as a solid state trigger where in a switch controlled 9V source was attached to its gate. Thyristors have very high voltage and current ratings and are unlikely to be affected during the high energy discharging of the capacitors. They function similar to transistors except that their gate only closes when the current flowing from its anode to cathode ceases. A diode was placed in series with the capacitor bank so as to prevent reverse voltage damage to the high voltage power supply during discharge. The Voltage probe (refer to Figure 2B) is placed in parallel to the capacitor bank and takes samples of the potential difference across the capacitor bank over time, including during discharge. It then produces a voltage time graph after every trial which is then analysed to find the period of time in which the pulse (i.e. sudden relatively large change in voltage) took place. 450V 150$\mu$F capacitors were the most cost effective capacitors to buy. They had a voltage rating higher than that of the maximum power supplied, allowing a good safety margin. 20 of these capacitors were placed in parallel hence increasing their total capacitance. This allowed a greater amount of energy to be transferred to the coil. They weren't placed in series (where their voltage would been multiplied) because the collective internal resistance of the capacitor bank would have been relatively higher than if connected in parallel. A higher voltage power supply would also be required (which would be dangerous and expensive hence impractical to use given the scope of this investigation).

Q. What is the relationship between the number of coils & voltage and the exit velocity of a permanent magnet projectile in an electromagnetic linear accelerator?

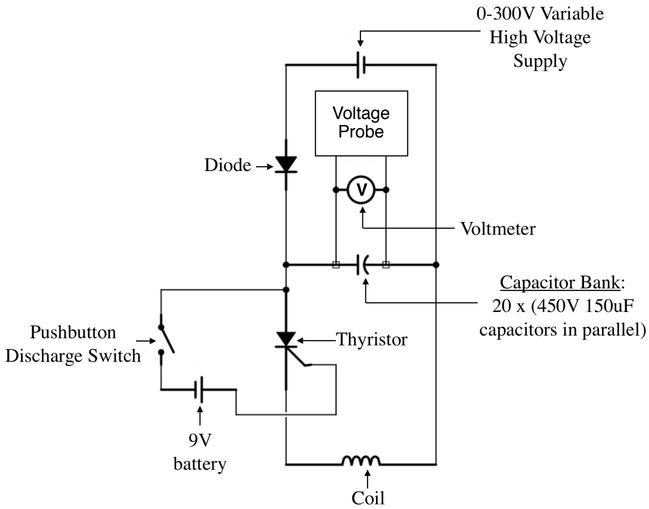


Figure 2B: Apparatus circuitry schematic

The placement characteristics of the projectile (shown in Figure 2E) and the tube can be illustrated by figure 2C and 2D below. Details on the parameters of each of the components used in the entire apparatus can be found in Table 1A in Appendix 1. Further information of specific parts used in the control circuit can be found in Table 2A of Appendix 1.

Figure 2E: Neodymium Iron Boron Magnetic Projectiles used. Image shows projectiles in a set of 6
(Source: http://www.apexmagnets.com/1-4-x-1-cylinders)

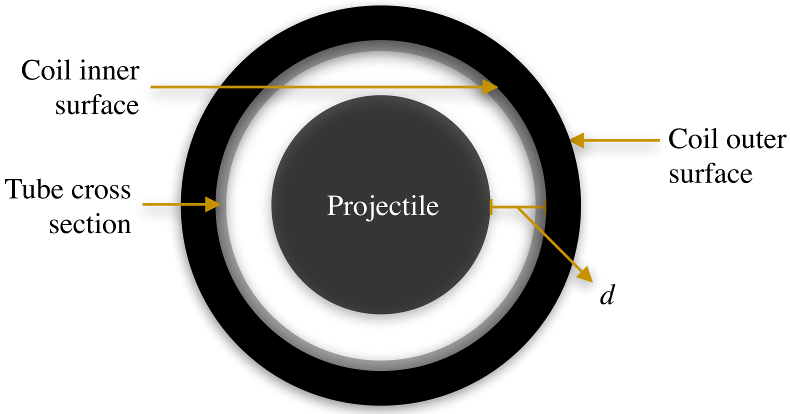


Figure 2C: Rear view into tube depicting projectile, tube and coil on a 2D plane

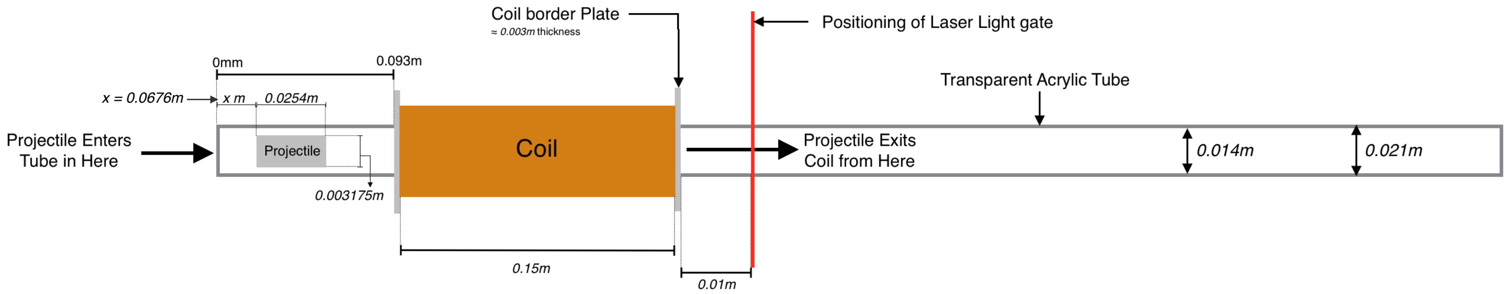


Figure 2D: Side view diagram of MLA.
*x = 0.0676m

## 2.2 Independent and Dependent Variables

**Independent Variables**

1. Voltage will be varied in between 100 and 300 Volts with 25 Volt increments.
2. The number of turns in the coil - will be varied from 700 to 100 turns with 100 turn decrements.

Data for the entire voltage range will be taken at every 100 coil turn increments. The resulting Voltage-velocity graph would therefore have 7 curves plotted representing the 7 sets of data for each number of turns, as shown in Figure 4A.

**Dependent Variables**

1. Projectile Exit Velocity
2. Pulse Duration

From these dependent variables, the average pulse current and efficiency of the system can be determined, which is given by the formula → $\dfrac{\text{projectile kinetic energy}}{\text{stored energy in capacitor bank}} = \dfrac{0.5 \cdot m \cdot v^2}{0.5 \cdot C \cdot V^2}$.

**Controlled Variables**

1. Total Circuit resistance
2. Projectile properties and dimensions (Structural Integrity)
3. Horizontal levelling of the barrel
4. Support Structure properties and dimensions (Structural Integrity)

## 2.3 Measuring Instruments

*note: positions of items are illustrated in figures 2B and 2D, unless otherwise specified.*

The idea in this experiment was to vary $V_{Capacitor\ Bank}$ - between 100 to 300V in 25V increments - and rapidly discharge the capacitor bank (or 'dump' the current stored in the capacitor bank) into the coil.

**Voltage**

The capacitor bank had to be charged to different voltages, and these could be accurately preset from the power supply which ranged from 0-300V with adjustable increments of 1V. A multimeter was attached in parallel to said capacitor bank to provide an indication for when the capacitors were fully charged to a prescribed voltage and therefore when it was appropriate to discharge them.

The capacitors however have a capacitance tolerance/variance of ±20% per capacitor (±30$\mu$F). For a capacitor bank rated nominally at 3000$\mu$F, its uncertainty would therefore be ±600$\mu$F.

**Velocity**

$v_{\text{Projectile Exit Velocity}}$ was measured with a light gate attached to a data logger. The length of the projectile was entered into the data logger which allowed automatic and reliable calculation of the speed at which the projectile passed through the light gate. Due to the casing of the light gate, the actual sensing laser was placed slightly ahead of the end of the solenoid - approximately 1.7cm. However, as long as this distance is kept constant, it shouldn't hinder data precision consistency. In order to keep it so, it was securely attached to a test tube stand which was secured to the wooden base of the apparatus.

Velocity readings were taken to 3 decimal places and hence had an uncertainty of ±0.0005m/s.

**Pulse time**

A Voltage probe (which was connected to a data logger) of high sample rates was connected in parallel to the capacitor bank. Its placements can be viewed in Figure 2B. The sensor was only rated up to 30V however. A simple potential divider circuit was installed allowing values to be read by said voltage sensor (refer to Appendix 4). The data logger operated at a maximum of 10000 samples per second. Pulse durations (in seconds) were therefore able to be calculated to 5 decimal places, therefore having an uncertainty of ±0.000005s. Graphs of recorded change in voltages against time for all trials were automatically plotted and compiled by computer software (Logger Pro), which was then analysed. The time at the last stable high voltage was subtracted from the corresponding to the closest lowest voltage reading, giving the pulse duration.

# 3. Hypothesis

## 3.1 General Hypothesis

Given the capacitor bank's capacitance, $C$, is always constant (allowing for ±20% error margin), the formula $Q = CV$ shows proportionality between Voltage, $V$, and charge stored, $Q$. This will directly affect the amount of current flowing through the coil which causes a magnetic field to form inside it which propels the projectile. It can therefore be said that changing $V_{Capacitor\ Bank}$ will affect $v_{\text{Projectile Exit Velocity}}$ because the magnetic field energy has to be transferred somewhere - in this case the projectile.

The energy stored in the capacitor bank, $U_{capacitors}$, is given by the equation $U_{capacitors} = \frac{1}{2}CV^2$, where C is capacitance and is constant. The kinetic energy of the projectile, KE, is given by the equation $KE = \frac{1}{2}mv^2$, where m is the mass of the projectile and is constant. Assuming 100% efficiency, $U_{capacitors}$ can be quested to KE, therefore giving $\frac{1}{2}CV^2 = \frac{1}{2}mv^2$ .Through this equivalence it can be deduced that $V_{Capacitor\ Bank} \propto v_{\text{Projectile Exit Velocity}}$. As a result, graphing $V_{Capacitor\ Bank}$ against $v_{\text{Projectile Exit Velocity}}$, a straight line should be obtained.

However once the projectile starts travelling at higher velocities due to higher voltage inputs and/or greater no. of turns in the coil (hence a more energetic coil), factors like air resistance, the suckback effect and heat loss due to resistance's of components will increase. This would likely influence the $V$ vs. $v$ curve such that as $V$ increases the slope of the curve will slowly flatten out, i.e. it will start out roughly linear then flatten out as $V$ progresses, tending to a logarithmical like shaped curve.

## 3.2 Prediction of Forces Exerted During Projectile Travel

In addition, the following describes predictions of the forces exerted on the magnetic projectile during its travel into and out of the powered coil in the MLA, which are in reference to the free body diagrams as shown in Figure 3.2A. These forces would heavily influence the Voltage-velocity characteristics of the Projectile and MLA, as shown in Figure 4A and Figure 4B.

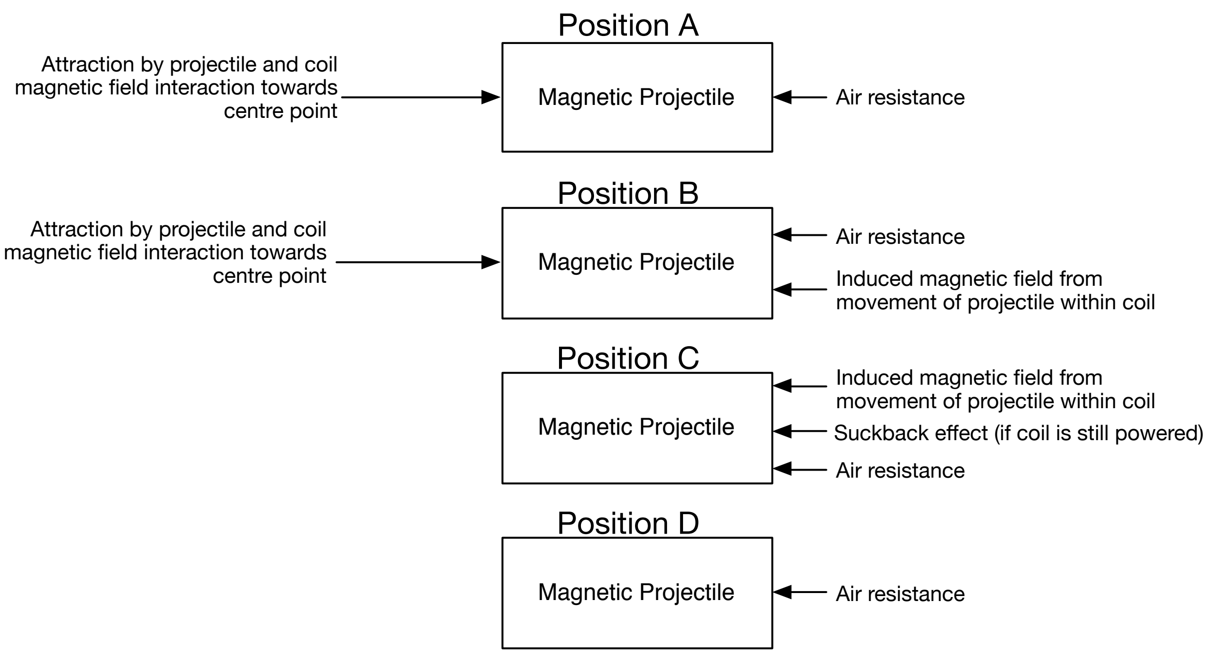


Figure 3.2A: Free body diagrams of projectile in different positions relative to Coil as described below.

<u>Position A</u> : **Before the projectile enters the coil**
The projectile's magnetic field acts to provide a stronger attractive force between it and the coil as it would be attracted to the already present magnetic field created by the coil. The result would be similar to that of two ordinary bar or cylindrical magnet being attracted to each other. The projectile is accelerated towards the centre of the coil (i.e. the centre point - where centre of mass of projectile meets centre of mass of coil) with only air resistance resisting its forward motion. There is net forwards acceleration on the projectile.

<u>Position B</u> : **When the projectile is inside coil (before reaching centre point[2])**
When a permanent magnet travels through a coil, it induces a current in the coil which in turn induces an opposing magnetic field that tries to repel said travelling permanent magnet, hence creating resistance to the direction of motion of the permanent magnet in the coil. This opposing induced magnetic field reduces the magnitude of the coil's initial field that existed before the magnetic projectile entered the coil. Therefore, once inside the coil, the force accelerating the permanent magnet projectile forwards decreases in magnitude. The net force on the projectile hence decreases, decreasing the average acceleration of the projectile (note: the decrease in field strength due to the opposing induced field is relatively far less than the initial field provided by the coil before the magnetic projectile entered it, hence net acceleration is still positive/ forwards).

<u>Position C</u> : **When the projectile crosses and surpasses the centre point[2] but is still in the coil.**
Because the centre of mass of the magnet is now past the centre point, if the coil is still powered it will try to accelerate the projectile towards its centre. This is known as the suckback effect which is likely to happen (especially as the coil becomes more energised) and hence would contribute to lower than potential speeds that the projectile could have travelled at if the coil were turned off right as the centre of mass of the projectile passed through the centre point. Due to the suckback effect, there will now be a backward acceleration on the projectile as there is no force actively propelling the projectile forward anymore. Air resistance contributes to this backwards acceleration (which was relevant to the projectile in all its positions relative to the coil). Also, the induced magnetic field from the moving projectile also acts in favour of the backward acceleration of the projectile, which like air resistance (but unlike the suckback effect) cannot be avoided. In this area relative to the coil, the projectile is slowing down at a faster rate compared to Position D. There is net backward acceleration on the projectile.

<u>Position D</u> : **When the projectile has exited the coil and passes the light gate to measure its speed.**
There is only air resistance acting upon the projectile which provides backward acceleration on it. The projectile is therefore still slowing down. Its backwards acceleration is lower than when it was in the area described in Position C as the induced magnetic field is not acting upon it anymore. However the distance between the end of the coil and the light gate is relatively very small at 39.4% of the length of the projectile. Hence the decelerating affects of air resistance in this position relative to the coil is likely to be insignificant.

### 3.2.1 Taking Air Resistance into account

Since air resistance is mentioned, it is useful to note the following.

The formula for drag force is $F_D = \frac{1}{2}\rho v^2 C_D A$ where $\rho$ is density of the fluid medium - (STP) air, $v$ is velocity of object, $C_D$ is drag coefficient and $A$ is the cross sectional area of the object. Here it can be seen that the drag force (*air resistance*) increases in proportion to $v^2$. Therefore the faster the projectile travels, the force exerted due to air resistance on the projectile would rise exponentially. Since higher energy coils would propel the projectile at higher speeds on average, it can be said that the more energetic the magnetic field of the coil, the lesser the rate of increase of $v_{\text{Projectile Exit Velocity}}$ as $V_{Capacitor\ Bank}$ increases, hence following a more logarithmic natured graph-plot.

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[2] <u>Centre Point:</u> The center of mass of the coil, which is the point where the centre of mass of the projectile is accelerated towards when the coil is energised.

Note that in the $F_D$ equation above, $C_D$ is a function of several further factors that will not be practical to measure given the scope of this investigation and is assumed a negligibly changing variable hence treated as a constant.

An attempt was made to derive a formula which would be able to map $V_{Capacitor\ Bank}$ against $v_{\text{Projectile Exit Velocity}}$. This can be found in Appendix 2. While being able to qualitatively exhibit a linear relationship as hypothesised (hence not actively taking into account air resistance and the suck back effect), the absolute values that the formula returns were substantially abnormal, suggesting errors in the derivation. Nonetheless, the process of the derivation still serves to only express assumptions and understandings of the more detailed dynamics of the MLA and this hypothesis in the form of a mathematical model, but should not be relied upon for specific results.

Hence in summary, the hypothesis to be tested states that the faster the projectile travels on average, the more logarithmic the relationship between $V_{Capacitor\ Bank}$ and $v_{\text{Projectile Exit Velocity}}$ becomes, as air resistance and the suckback effect become more significant. Likewise, the slower the projectile travels on average, the more linear the relationship between $V_{Capacitor\ Bank}$ and $v_{\text{Projectile Exit Velocity}}$ becomes, as air resistance and the suckback effect become less significant.

# 4. Results and Discussion

Figure 4A below illustrates a summary of all the data collected. Figure 4B also below shows the equations and $R^2$ values for the lines of regression as shown in Figure 4A. The different lines of regression represent the relationship between $V_{Capacitor\ Bank}$ against the speed of the projectile as it leaves the coil for every set of coil turn no.'s.

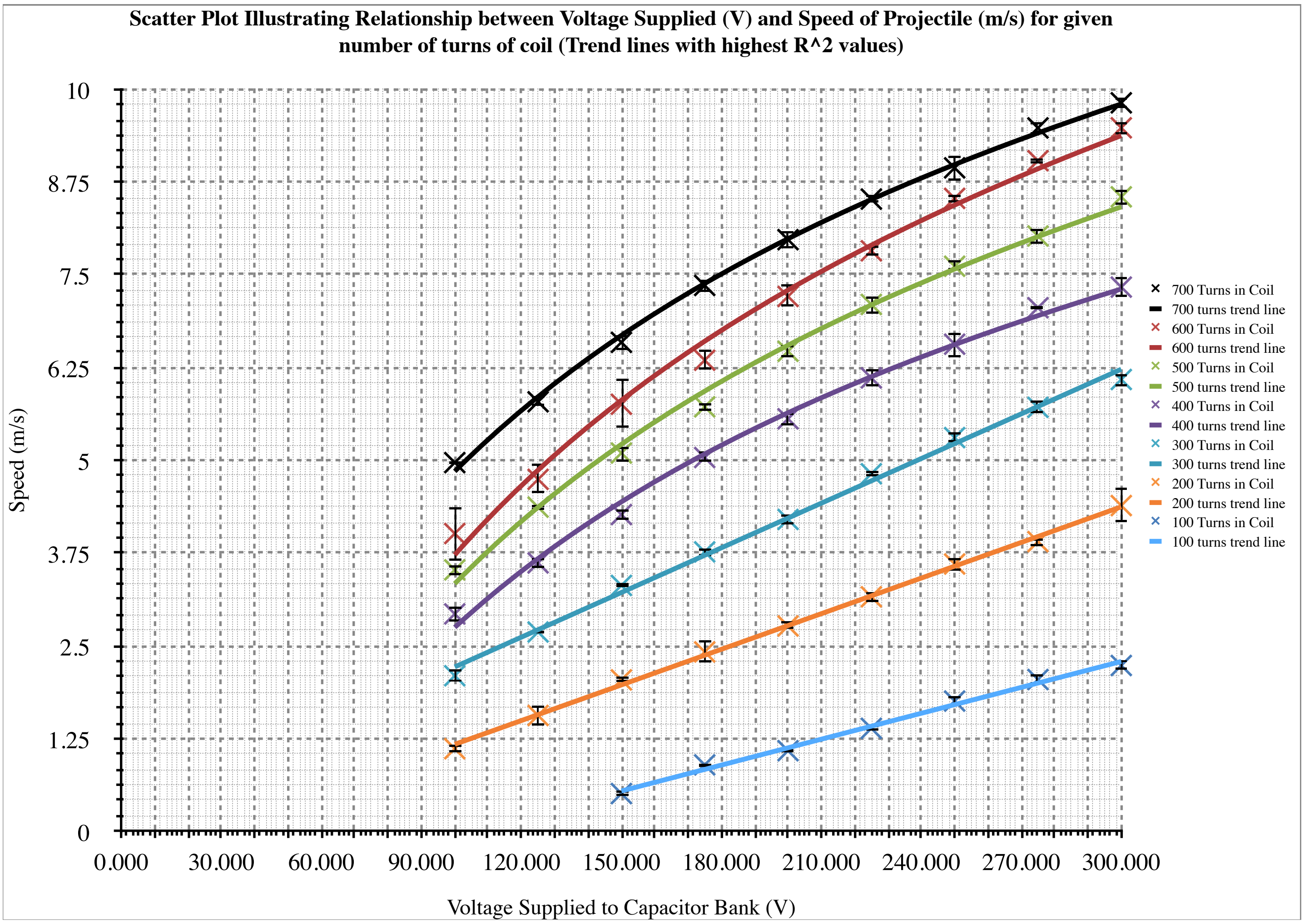


Figure 4A: Voltage-Velocity graph summarising all data collected.

| No. of turns in coil | Equation | $R^2$ |
|---|---|---|
| 700 | y = 4.5056*ln(x) - 15.8980 | 0.9985 |
| 600 | y = 5.1436*ln(x) - 19.9630 | 0.9928 |
| 500 | y = 4.6218*ln(x) - 17.9470 | 0.9952 |
| 400 | y = 4.1505*ln(x) - 16.3660 | 0.9958 |
| 300 | y = 0.0201*x - 0.2092 | 0.9959 |
| 200 | y = 0.0160*x - 0.4275 | 0.9983 |
| 100 | y = 0.0116*x - 1.2025 | 0.9938 |

Figure 4B: Table of Equations and $R^2$ values for regression lines shown in Figure 4A.

Each point on the graph (i.e. for a specific voltage and number of turns in the coil) is the result of the average of 6 repetitions/trials for a given voltage and number of turns of coil. Figure 4C below is an example of the table of data collected for these trials, specifically for 300V at 400 turns in the coil.

| No. of Trial | No. of Turns of Coil | Voltage charged Across Capacitor Bank (V) | No. of Capacitors in Parallel in Bank | Charge stored in capacitors (Q)(coulombs) | *Uncertainty (± x coulombs)* | Pulse Duration (s) | *Uncertainty (± x seconds)* | Average Pulse Current (A) | *Uncertainty (± x seconds)* | Projectile Velocity (m/s) | *Uncertainty (± x m/s)* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 400 | 300 | 20 | 0.900 | *0.180* | 0.03107 | *0.000005* | 28.96685 | *0.000005* | 7.206 | 0.0005 |
| 2 | 400 | 300 | 20 | 0.900 | *0.180* | 0.03070 | *0.000005* | 29.31596 | *0.000005* | 7.478 | 0.0005 |
| 3 | 400 | 300 | 20 | 0.900 | *0.180* | 0.02756 | *0.000005* | 32.65602 | *0.000005* | 7.379 | 0.0005 |
| 4 | 400 | 300 | 20 | 0.900 | *0.180* | 0.02786 | *0.000005* | 32.30438 | *0.000005* | 7.252 | 0.0005 |
| 5 | 400 | 300 | 20 | 0.900 | *0.180* | 0.02756 | *0.000005* | 32.65602 | *0.000005* | 7.280 | 0.0005 |
| 6 | 400 | 300 | 20 | 0.900 | *0.180* | 0.02787 | *0.000005* | 32.29279 | *0.000005* | 7.404 | 0.0005 |
| | | | | Average: | *0.180* | 0.02877 | *0.001755* | 31.36534 | *1.844587* | 7.333 | *0.1360* |
| | | | | Efficiency (ProjectileKE/CapStoredEnergy) = | | | *0.001215* | | | | |

Figure 4C: Data collection table for independent variables - 300V at 400 turns of coil.

The data as illustrated in Figure 4A and Figure 4B shows quite close agreement with the Hypothesis. From 700 to 400 turns in the coil, the regression line's with the highest $R^2$ values (i.e. that fit the plot most closely) were logarithmic. They increase at an ever decreasing rate. The higher the number of turns in the coil, the stronger the magnetic field generated by the coil would have become, which meant more energy was transferred to the kinetic energy (KE) of the projectile, hence causing the projectile to travel at higher speeds on average. The increase in the rate of decrease as Voltage increases for 700 to 400 coil turn data-sets could be due to the fact that the faster the projectile travelled the more air resistance it faced. In addition, the faster the projectile travelled the higher the likeliness that it would have passed the centre of the coil while the coil was still energised, hence increasing the chance that it would have faced the suckback effect. It can therefore be seen that from the '400 to 700 no. of turns in coils data sets', the hypothesised logarithmic relationship between $V_{Capacitor\ Bank}$ and $v_{\text{Projectile Exit Velocity}}$ follows, but the hypothesised linear relationship doesn't.

In contrast, for the '100 to 300 no. of turns in coil' data sets, it can be seen that the hypothesised linear relationship is followed (the regression lines with the highest $R^2$ values were linear). The lower the no. of turns in the coil, the weaker the magnetic field generated by the coil would have become. This means less energy was transferred to the KE of the projectile, resulting in projectile being propelled at lower speeds. With these slower speeds, air resistance would have become less and less significant. Hence, while travelling at a relatively low speed range, the changes in air resistance on the projectile would have become less significant and therefore have less influence the projectiles speed. Also, the suckback effect would have been less likely to occur as it is more likely that the slower travelling projectiles would not have passed the centre point of the coil while the coil it was still energised.

(Note: Raw data tables can be found in Appendix 3)

In terms of the attempted derivation in Appendix 3, the linear relationship between $V_{Capacitor\ Bank}$ and $v_{\text{Projectile Exit Velocity}}$ that it exhibits (hence not taking into account air resistance and suck back effect) is proven as shown in the Lower energy coils of 100 to 300 turns in Figure 4A and 4B. Despite this similarity the absolute values that the formula returns are still far from the values exhibited in Figure 4A and 4B by several orders of magnitude. The derivation in Appendix 3 is therefore sound in terms of its use and application of various formulae, however it also contains significant errors elsewhere (in mathematical values and various assumptions) as shown by the substantially large discrepancy between the data values and values returned by the derived formula.

# 5. Evaluation & Conclusion

## 5.1 Conclusion

This investigation has shown very close resemblances between its hypothesis and the collected data.

Inferring from the data the following can be concluded:

The strength/energy of the magnetic field produced by the coils [*given by* $B_0 = \mu_0 nI = \frac{\mu_m nCV}{2t_p} = \mu_0 n \frac{V}{R_C}$ *. See EQUATION 3 and '<u>Step 2</u>' in Appendix 3 for definitions of each variable*] is proportional to the number of turns in the coil, $n$. Hence coils with lower number of turns and voltage across it ($V_{Capacitor\ Bank}$) create weaker and less energetic fields, therefore transferring less KE to the projectile and propelling it slower on average. With slower projectiles, the air resistance and the suckback effect become <u>less</u> significant and a more linear relationship is hence followed between $V_{Capacitor\ Bank}$ and $v_{\text{Projectile Exit Velocity}}$.

On the other hand, coils with more turns and voltage across them ($V_{Capacitor\ Bank}$) create stronger and more energetic fields. As a result these coils propel the projectile at higher speeds on average, thus making the air resistance and the suckback effect <u>more</u> significant. As a result of this, a logarithmic relationship is followed between $V_{Capacitor\ Bank}$ and $v_{\text{Projectile Exit Velocity}}$ (the effect of air resistance and the likeliness and magnitude of the suckback effect increase as $v_{\text{Projectile Exit Velocity}}$ increases).

As a result, it can be seen that at a certain number of turns in the coil, the Voltage-velocity relationship of the projectile in the MLA changes from being more significantly linear to more significantly logarithmic, which in the case of this investigation is between 300 and 400 turns in the coil.

At higher $V_{Capacitor\ Bank}$, as well as with higher coil turn no.'s, the strength of the magnetic field created by the coil propels the projectile faster. However even while ignoring air resistance, a linear relationship cannot be held between $v_{\text{Projectile Exit Velocity}}$ and $V_{Capacitor\ Bank}$. This is because unlike air resistance, the suckback effect cannot be ignored as it will become more and more significant as the projectile travels faster hence making it more likely to cross the coil's centre point while the coil is still powered. This means that even while ignoring air resistance, the relationship between the energy of the coil (which is proportional to the $V_{Capacitor\ Bank}$ and Number of turns in the coil) and $v_{\text{Projectile Exit Velocity}}$ will look more and more logarithmic as the coil becomes more energised (albeit having a less dramatic logarithmic relationship than if air resistance were taken into account) . With lower powered coils, the suckback effect becomes less significant hence this logarithmic relationship is less visible.

It can therefore be concluded that over a broader domain of the strength/energy of the magnetic field of the coil a more logarithmic relationship between the two variables can be observed due to the effects of air resistance as well as the suckback effect. The broadness of this domain is determined by using low values of $V_{Capacitor\ Bank}$ <u>and/or</u> Number of turns in coil at the lower end of the domain, and higher values of $V_{Capacitor\ Bank}$ <u>and</u> Number of turns in coil at the higher end of the domain. The linear relationship that was hypothesised only holds if either the suckback effect and air resistance aren not significant or taken into account, which is illustrated in Figure 4A and Figure 4B when lower values of the no. of turns in coil are used where the projectile is propelled at lower speeds making air resistance and the suckback effect negligible (less significant).

## 5.2 Evaluation

This investigation heavily utilised electric power and control systems in its apparatus; the only moving part involved was the projectile itself. This meant that it was relatively easier to implement greater reliability measures in the apparatus during data collection. It was ensured that all components used were rated for short-pulse voltage and amperage levels far higher than the what the discharging capacitor bank could supply at its maximum power. This meant that all the parts could maintain consistency in their operations. This consistency is reflected by the small error bars visible in Figure 4A.

The physical characteristics of the projectile, which heavily influenced its drag and electromagnetic properties, were kept as constant as possible well. A foam stopper was placed at the end of the acrylic barrel so as to prevent the brittle Neodymium Iron Boron Projectile from chipping or cracking upon collision with foreign materials as it exited the barrel - instead it was safely caught in soft foam, ensuring the projectiles structural integrity after every trial. The projectile material - Neodymium Iron Boron - is also one of the worlds hardest materials to demagnetise, which meant travelling through the high powered coil at high velocities several times in a row meant minimal (if not any) demagnetisation and change in the projectiles magnetic properties.

However, the capacitors used in the capacitor bank had a ± 20% capacitance tolerance which meant that its capacitance could randomly vary (with accordance to the central limit theorem) within ± 20% bounds of its rated capacitance. This resulted in a significant drop in precision when measuring and comparing results, contributing to larger error bars. In addition, the capacitor banks and all other electronic components were secured to each other and the rest of the MLA circuit by means of a bread board and 22AWG jumper wires. While these could handle the short but very high powered pulses as discharged from the capacitor bank, their small form factor contributed to higher overall electrical resistance in the circuit which meant lower efficiencies could be obtained. To decrease the resistance and increase the efficiency of the circuit, solid copper bus bars could have been used to secure the capacitor bank together and thick gauge wire to connect all the electrical component terminals that had high voltage and current flowing through them. Thicker gauge wire could also have been used for the coil, reducing the coils resistance hence increasing the entire system's efficiency as well.

## 5.3 Areas for Further Investigation

It would be interesting to investigate a more detailed relationship between the energy of the coil (by varying the no. of coil turns and $V_{Capacitor\ Bank}$) and the magnitude of the suckback effect by measuring the resultant changes in the projectiles exit velocity.

It would also be interesting to collect data using a non-magnetised soft iron projectile instead of a strongly magnetised one, under the same conditions in this investigation. Even though it would lack the additional magnetic attractive forces before entering the coil, the soft iron projectile (unlike the magnetised one) wouldn't induce current as it travels through the coil, hence the coils initial magnetic field wouldn't be compromised. It would be interesting to compare the significance of these differences in terms of the exit velocity of the projectiles.

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# 7. Appendices

## Appendix 1

**Below → Table 1A:** Details and dimensions of parts used in apparatus

| Part | Parameter | Description |
|---|---|---|
| **Tube** | Material | Acrylic |
| | Inner Diameter | 0.014 m |
| | Outer Diameter | 0.021m |
| | Length | 1.21 m |
| **Coil** | Number of turn layers | 7 |
| | Number of turns per layer | 100 (≈ 200) |
| | Length of Coil | 0.15 m |
| | Inner Diameter of Coil | 0.021 m |
| | Outer Diameter of Coil (prior to any unravelling) | 0.028 m |
| | Wire external diameter (including enamel coating) | 1 mm = 0.001 m |
| | Wire conductor diameter | 19 AWG = 0.91186 mm |
| | Total length of wire (prior to any unravelling) | 64.3 m (≈53.6584 m) |
| | Coil inductance (prior to any unravelling) | 30.5 micro Henries (μH) |
| | Magnetic Flux Density (prior to unravelling and at 300V/ approximately 20A pulse) | 98000T |
| **Projectile** | Material | Neodymium Iron Boron |
| | Length | 0.0254 m (1 inch) |
| | Diameter | 0.003175 m (1/4 inch) |
| | Weight | 0.0061 kg |
| | Relative Permeability ( $\mu_m \div \mu_0$ ) | 1.05 |
| | Magnetic Permeability ( $\mu_m$ ) | 0.00000131946 |
| **Frame** | Material | Plywood |

**Below → Table 2A:** Component Part No.'s and Specifications

| Name | Part number/Model | Specifications |
|---|---|---|
| **Variable High Voltage Supply** | Major Science Mini Pro 300V | 0-300V range; 1V increments; 60W output power |
| **Data Logger** | Vernier LabQuest 1 | - |
| **Capacitors** | UNITED CHEMI-CON EKXG401ELL151MM40S | 400V; 150$\mu$F |
| **Thyristor** | LITTELFUSE S6025L | Peak Current = 300A<br>Peak Voltage = 600V |
| **Diode** | DIODES INC. 1N5404 | Peak Reverse Voltage = 400 V<br>Surge Current Rating = 200 A |
| **Coil Wire** | Rowan Cable Products Ltd<br>ECW1.0<br>(enamelled wire) | Conductor Material = Copper<br>Conductor Size = 19 AWG<br>Conductor CSA = 0.79mm<br>External diameter = 1mm |
| **Breadboard** | TWIN INDUSTRIES<br>TW-E40-1020 | - |
| **Jumper Cables** | MULTICOMP<br>MCBBJ65 | Conductor Size = 22 AWG<br>Conductor Material = solid core copper |

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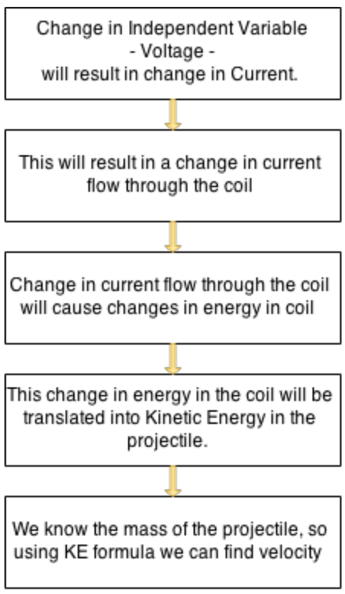
# Appendix 2 - (Detailed Mathematical Model/Derivation)

IMPORTANT: Please refer to the last paragraph in Section 3 (under subsection 3.1), to put the following derivation in context.

## MATHEMATICAL MODEL

The aim of derivation in this section is to get a function for Projectile velocity in terms of voltage.



In order to maintain a good level of coherence, the derivation for model 1 will take place in steps as can be seen in the following pages.

**Step 1**

Following the flow chart above, we can express the following equation:

$$U_{projectile} = \alpha \cdot (U_{filled} - U_{empty}) = KE \rightarrow \textbf{EQUATION 1}$$

where,

$U_{projectile}$ = Energy transferred to projectile

$U_{empty}$ = energy in coil while empty

$U_{filled}$ = energy in coil with projectile in it

$\alpha$ = efficiency constant which is <1. Accounts for loss of useful energy through dissipation due to resistances in circuitry.

**note: The* $(U_{filled} - U_{empty})$ *part of equation only accounts for space in coil occupied by projectile**

The equations for $U_{filled}$ and $U_{empty}$ are as follows:

$$U_{filled} = l\pi r^2 \cdot \frac{B^2}{2\mu_0} \rightarrow \textbf{EQUATION 2}$$

&

$$U_{empty} = l\pi r^2 \cdot \frac{B_0^{\ 2}}{2\mu_0} \rightarrow \textbf{EQUATION 3}$$

where,

$l$ = length of projectile

$r$ = radius of projectile

$B_0 = \mu_0 nI$ = magnetic field in empty coil

where,

$\mu_0$ = Magnetic Permeability of free space = $4\pi \times 10^{-7}$ T m/A

$n$ = Number of Turns per meter in coil

$I$ = Current through wire

$B = \frac{\mu_m}{\mu_0} B_0$ = magnetic field inside material

Substituting equation 2 and 3 into equation 1 we get the following expression of energy transferred to the projectile.:

$$U_{projectile} = \alpha(l\pi r^2 \cdot \frac{B^2}{2\mu_0} - l\pi r^2 \cdot \frac{B_0^{\ 2}}{2\mu_0}) = KE \rightarrow \textbf{EQUATION 4}$$

Equation 4(i) below expresses the energy in the periphery of the coil that is radiated away:

$$\text{Peripheral Energy Dissipated} = \alpha \cdot (L\pi R \cdot \frac{B_0^{\ 2}}{2\mu_0}) - \alpha \cdot (l\pi r^2 \cdot \frac{B^2}{2\mu_0} - l\pi r^2 \cdot \frac{B_0^{\ 2}}{2\mu_0}) \rightarrow \textbf{EQUATION 4(i)}$$

We can express the KE part of equation 1 with Velocity as the subject, as follows:

$$v = \sqrt{\frac{2U_{projectile}}{m}} \rightarrow \textbf{EQUATION 5}$$

where,
$v$ = Velocity of projectile
$m$ = mass of projectile

**Step 2**

In order to express equation 4 in more detail, the expressions for $B_0$ and $B$ can be expressed in terms of more useful variables and constants.

We know $Q = CV$ and $I = \frac{Q}{t_p}$, where $Q$ is charge in capacitors, $C$ is Capacitance, $V$ is voltage ($V_{Capacitor\ Bank}$), $I$ is current, and $t_p$ is the pulse time. From these two equations we can derive the equation $I = \frac{CV}{t_p}$. However during $t_p$, $I$ isn't constant. Using the description from figure 3.2.1A on page 8 we can closely approximate our expression for $I$ by dividing it by 2, giving us $I = \frac{CV}{2t_p}$. This can be substituted into our old formula for $B_0$ to give us a new expression for $B_0$ :-

$$B_0 \rightarrow B_0 = \frac{\mu_o nCV}{2t_p} \rightarrow \textbf{EQUATION 6}$$

We can now therefore substitute equation 6 into our expression for $B$ to get the following equation:

$$B = \frac{\mu_m nCV}{2t_p} \rightarrow \textbf{EQUATION 7}$$

We can now substitute equation 6 and 7 into equation 4, giving us equation 8, which can be simplified as follows:

$$U_{projectile} = \alpha\ [(l\pi r^2 \cdot \frac{(\frac{\mu_m nCV}{2t_p})^2}{2\mu_0}) - (l\pi r^2 \cdot \frac{(\frac{\mu_o nCV}{2t_p})^2}{2\mu_0})]$$

$$U_{projectile} = \alpha\ \{\ (\frac{(l\pi r^2)\cdot(nCV)^2}{8\cdot\mu_0\cdot(t_p)^2})\cdot[(\mu_m)^2 - (\mu_0)^2]\ \}$$

$$\rightarrow \textbf{EQUATION 8}$$

From now on, let us refer to the part of equation 8 excluding $\alpha$ as $Z$ for convenience, thereby allowing equation 8 to take the form of the following:

$$U_{projectile} = \alpha \cdot (Z)$$

**Step 3 - addressing the efficiency constant ($\alpha$) in more detail**

$\alpha$ can be expressed as the following

$$\alpha = 1 - \frac{\text{energy lost}}{\text{total initial energy}}$$

$\alpha$ accounts for the energy lost from the resistances in all components of the MLA circuit, including the coil. The energy dissipated from the coils resistance is transferred into useful magnetic flux and wasteful heat. It will also assumed that the resistances of the components will stay constant during and between trials. From these assumptions we can say that $\alpha$ will be constant and will therefore act as a proportionality constant between $KE_{projectile}$ and energy transferred to Projectile due to the coil's magnetic field.

We can thereby interpret equation 5 as the following:

$$v = \sqrt{\frac{2U_{projectile}}{m}} = \sqrt{\frac{2\alpha Z}{m}} = \sqrt{\frac{2\alpha}{m}}\sqrt{Z}$$

therefore,

$$v = \sqrt{\frac{2\alpha}{m}}\sqrt{Z} \rightarrow \text{EQUATION 9}$$

We can say that $\sqrt{\frac{2\alpha}{m}}$ (*where 'm' is the mass of the projectile*) is the proportionality constant between the projectiles exit velocity, $v$, and the square root of $Z$ - the energy transferred to to the projectile due to the magnetic field of solenoid.

For energy lost as heat due to resistance we can express it as $U_{Rlost} = \frac{V^2}{R_T}t_p$ where $t_p$ is the time of pulse (because $power \times time = total\ energy\ released$ and $\frac{V^2}{R_T}$ = power dissipated and $t_p$ = pulse time when said power was dissipated), and $R_T$ is the total resistance of the circuit including the coil[3]. Ideally, $U_{Rlost}$ would be expressed as $\int_0^t \frac{V^2}{R_T}dt$ because the current pulse isn't a digital high low step, but rather an analog like pulse curve which increases then recedes quickly over the pulse time, 0 to $t_p$, as shown in figure 3.2.1A below.

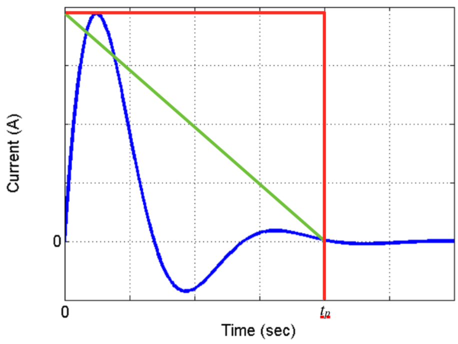



**Figure 3.2.1A:** Figure showing Pulse characteristics over time. Current *A* can be used to represent same pattern voltage *V* would experience.
(Su-Jeong Lee 2013)

[3] note: Values for $R_T$ would include the combined ESR of the capacitor bank and thyristor used in the circuit. Given the size of the coil being used, its resistance is considered negligible, hence shall not be included in the value for $R_T$.

Because of the lack of ability to record at high enough sample rates in order to get an accurate enough integral, we cannot treat the change in $\frac{V^2}{R_T}$ as a dampening sinusoidal curve as depicted in blue in figure 3.2.1A, but instead a digital step as shown by the red line. However to closer approximate our model we can assume a linear decrease from 0 to $t_p$ as shown by the green line in the figure. We therefore modify our expression for $U_{Rlost}$ as simply $\frac{1}{2}\frac{V^2}{R_T}t_p = \frac{V^2}{2R_T}t_p$.

The 'total supplied energy' in the expression for $\alpha$ is simply the energy stored by the capacitor bank which is given by $U_{capacitors} = \frac{1}{2}CV^2$. We can therefore more specifically address $\alpha$ as shown below by equation 10.

$$\alpha = 1 - \frac{\frac{V^2}{2R_T}t_p}{\frac{CV^2}{2}} = 1 - \frac{V^2 t_p}{2R_T}\frac{2}{CV^2} = 1 - \frac{t_p}{R_T} = \frac{R_T - t_p}{R_T}$$

therefore,

$$\alpha = \frac{R_T - t_p}{R_T} \rightarrow \text{EQUATION } \mathbf{10}$$

**Step 4 - Final step**

From steps 1 through 3, we can now derive an equation for exit velocity of a projectile given *n* (*number of turns in coil per meter*), *C* (*Capacitance of capacitor bank*), *V* (*Voltage*), $t_p$ (*pulse time*), *d* (*distance between projectile and inner circumference of coil*), and *m* (*mass of projectile*) variables, as shown in equation 11 below:

$$v = \sqrt{\frac{2(R_T - t_p)}{R_T m}} \text{ x } \sqrt{\left(\frac{(l\pi r^2)(nCV)^2}{8\mu_0 (t_p)^2}\right)\ ((\mu_m)^2 - (\mu_0)^2)}$$

$$\rightarrow \text{EQUATION } \mathbf{11}$$

This value of *v* however only reflects the speed of the projectile at the instant it leaves the centre point of the coil, assuming no suckback effect. However with our experimental set up, the speed is only recorded a small distance *x* frm the end of the coil. Due to air resistance the projectile would have fractionally slowed down by the time it reaches the light gate at the end of the coil[4], resulting in raw data which will likely deviate from the hypothesised values by a certain magnitude.

The formula for drag force is $F_D = \frac{1}{2}\rho v^2 C_D A$ where $\rho$ is density of the fluid medium i.e. air, *v* is velocity of object, $C_D$ is drag coefficient and *A* is the cross sectional area of the object (*assuming it is uniformly shaped along its length*). Here we can see that the drag force (*air resistance*) increases in proportion to $v^2$. Therefore, in our trials, the faster the projectile travels, the force exerted due to air resistance on the projectile will rise exponentially. From this we can say that the data we receive will be slightly lower in magnitude than the hypothesis *v(V)* equation - equation 11 - will predict.

[4] Here we are also assuming that the projectile hasn't lost enough forward momentum that it slows down and touches the barrels inner surface - causing even greater frictional resistances - by the time it reaches the light gate. We therefore do not account for this in the hypothesis.

However, because the distance between the coil centre point and the light gate is relatively small given the speed of the projectile, the effect of air resistance on its speed will be negligible as it would have covered this distance relatively quickly. $F_D$ will not be accounted for in the equation as $C_D$ is a function of several further factors that will not be practical to measure given the scope of this investigation. However we can approximately account for this by multiplying our value of $v$ (*velocity*) by ω, which is a number less than one which is not likely to be constant and will vary with the *KE* of the projectile as $v$ does. We can attempt to find a rough estimate of the function for ω(V) by plotting the differences between the hypothesised equation for $v$ (*equation 11*) and the actual processed data for $v$ for every V (*voltage*) interval, and then dividing these differences by the hypothesised values, followed by subtracting the resulting value from 1.

Due to the implementation of ω, we must slightly modify equation 11 to give equation 12 below. Here, ω is a function of voltage just like the rest of the equation, except that it always returns a variable value less than one.

$$v = \omega\sqrt{\frac{2(R_T - t_p)}{R_T m}} \text{ x} \sqrt{(\frac{(l\pi r^2)(nCV)^2}{8\mu_0 (t_p)^2})\ ((\mu_m)^2 - (\mu_0)^2)}$$

→ **EQUATION 12**

where,

- $v$ = Velocity of projectile
- $t_p$ = Pulse time period
- $m$ = mass of projectile = 0.0061 Kg
- $l$ = Length of projectile = 0.0254 m
- $r$ = Radius of projectile = 0.0015875 m
- $R_T$ = Total resistance of circuit = 860 Ohms
- $n$ = number of turns in coil = 700 turns without unwinding, *varies otherwise*
- $C$ = Total capacitance of capacitor bank = 3000 $\mu$F
- $V$ = Voltage that capacitor bank is charged up to = *varies*

We can see resemblance in equation 11 the hypothesis described in section 3.1. Ignoring ω, $v$ can be seen to be proportional to $V$. Also, the implementation of ω - where ω is a number less than one that decreases as $V$ (therefore also $v$) increases - represents part of the hypothesis which predicts a curve increasing at a decreasing rate due to air resistance.

If we now substitute in values specific to our apparatus into equation 11, as described in section 2 of this extended essay, we should get the following function.

$$v(V) = \frac{(2.03251 \cdot 10^{-8}) \cdot (\sqrt{860 - t_p})}{t_p} \cdot V$$

# Appendix 3 - (Raw Data Tables)

Note that the following tables only include (averaged) data as illustrated by each point in the graph of Figure 4A. They do not include data for each and every trial as illustrated in the table of Figure 4C.

**700 Turns in Coil**

| VOLTAGE (V) | Pulse Duration (seconds) | Uncertainty (± Seconds) | Avg. Pulse Current (Amps) | Uncertainty (±Amps) | Projectile Velocity (m/s) | Uncertainty (± m/s) |
|---|---|---|---|---|---|---|
| 300 | **0.02668** | 0.00276 | **33.87649** | 3.42286 | **9.81633** | 0.07050 |
| 275 | **0.03129** | 0.00237 | **26.43966** | 2.11044 | **9.47800** | 0.08100 |
| 250 | **0.02975** | 0.00192 | **25.26714** | 1.74386 | **8.93983** | 0.16450 |
| 225 | **0.02208** | 0.00160 | **30.63321** | 2.10265 | **8.51917** | 0.04400 |
| 200 | **0.01804** | 0.00106 | **33.30920** | 1.88922 | **7.97250** | 0.12050 |
| 175 | **0.01564** | 0.00080 | **33.60668** | 1.73846 | **7.35567** | 0.07450 |
| 150 | **0.01399** | 0.00142 | **32.33336** | 3.45203 | **6.58900** | 0.10450 |
| 125 | **0.01396** | 0.00078 | **26.91795** | 1.49698 | **5.78783** | 0.06450 |
| 100 | **0.01212** | 0.00157 | **24.97717** | 3.43768 | **4.96817** | 0.01700 |

**600 Turns in Coil**

| VOLTAGE (V) | Pulse Duration (seconds) | Uncertainty (± Seconds) | Avg. Pulse Current (Amps) | Uncertainty (±Amps) | Projectile Velocity (m/s) | Uncertainty (± m/s) |
|---|---|---|---|---|---|---|
| 300 | **0.02342** | 0.00179 | **38.52919** | 2.87278 | **9.47967** | 0.07600 |
| 275 | **0.02177** | 0.00453 | **38.62761** | 8.90842 | **9.03650** | 0.03550 |
| 250 | **0.01745** | 0.00257 | **43.32996** | 6.00935 | **8.52900** | 0.05300 |
| 225 | **0.01554** | 0.00151 | **43.66779** | 4.27143 | **7.82250** | 0.06400 |
| 200 | **0.01196** | 0.00172 | **50.66114** | 7.74597 | **7.20717** | 0.14800 |
| 175 | **0.01432** | 0.00067 | **36.71900** | 1.71429 | **6.34767** | 0.13850 |
| 150 | **0.01470** | 0.00253 | **31.16993** | 6.58507 | **5.75433** | 0.32700 |
| 125 | **0.01907** | 0.00473 | **20.21991** | 5.70907 | **4.74283** | 0.19600 |
| 100 | **0.02107** | 0.00176 | **14.28880** | 1.25460 | **4.01233** | 0.36000 |

**500 Turns in Coil**

| VOLTAGE (V) | Pulse Duration (seconds) | Uncertainty (± Seconds) | Avg. Pulse Current (Amps) | Uncertainty (±Amps) | Projectile Velocity (m/s) | Uncertainty (± m/s) |
|---|---|---|---|---|---|---|
| 300 | **0.03241** | 0.00219 | **27.85203** | 1.91338 | **8.54950** | 0.10300 |
| 275 | **0.02759** | 0.00071 | **29.90830** | 0.78820 | **8.02217** | 0.08650 |
| 250 | **0.02591** | 0.00096 | **28.97159** | 1.12202 | **7.61433** | 0.06300 |
| 225 | **0.02394** | 0.00026 | **28.19770** | 0.29949 | **7.09967** | 0.11400 |
| 200 | **0.02323** | 0.00018 | **25.83497** | 0.19484 | **6.46833** | 0.08050 |
| 175 | **0.02304** | 0.00013 | **22.78511** | 0.12357 | **5.72000** | 0.04300 |
| 150 | **0.02182** | 0.00018 | **20.62062** | 0.16564 | **5.09717** | 0.09800 |
| 125 | **0.02103** | 0.00010 | **17.82906** | 0.08487 | **4.36783** | 0.04400 |
| 100 | **0.01987** | 0.00017 | **15.10117** | 0.12828 | **3.52317** | 0.07150 |

Q. What is the relationship between the number of coils & voltage and the exit velocity of a permanent magnet projectile in an electromagnetic linear accelerator?

## 400 Turns in Coil

| VOLTAGE (V) | Pulse Duration (seconds) | Uncertainty (± Seconds) | Avg. Pulse Current (Amps) | Uncertainty (±Amps) | Projectile Velocity (m/s) | Uncertainty (± m/s) |
|---|---|---|---|---|---|---|
| 300 | **0.02877** | 0.00176 | **31.36534** | 1.84459 | **7.33317** | 0.13600 |
| 275 | **0.02606** | 0.00150 | **31.70929** | 1.93094 | **7.05500** | 0.01800 |
| 250 | **0.02452** | 0.00071 | **30.60344** | 0.89006 | **6.56383** | 0.16050 |
| 225 | **0.02286** | 0.00017 | **29.53034** | 0.21940 | **6.10917** | 0.11300 |
| 200 | **0.02238** | 0.00018 | **26.81634** | 0.20956 | **5.55833** | 0.08100 |
| 175 | **0.02149** | 0.00013 | **24.43415** | 0.14230 | **5.03683** | 0.06500 |
| 150 | **0.02024** | 0.00064 | **22.24247** | 0.67735 | **4.26818** | 0.06845 |
| 125 | **0.01888** | 0.00010 | **19.86255** | 0.11017 | **3.61633** | 0.05850 |
| 100 | **0.01780** | 0.00046 | **16.85871** | 0.42620 | **2.92933** | 0.09950 |

## 300 Turns in Coil

| VOLTAGE (V) | Pulse Duration (seconds) | Uncertainty (± Seconds) | Avg. Pulse Current (Amps) | Uncertainty (±Amps) | Projectile Velocity (m/s) | Uncertainty (± m/s) |
|---|---|---|---|---|---|---|
| 300 | **0.02729** | 0.00032 | **32.98174** | 0.38620 | **6.08817** | 0.07600 |
| 275 | **0.02537** | 0.00023 | **32.51590** | 0.30255 | **5.71317** | 0.07200 |
| 250 | **0.02404** | 0.00087 | **31.21021** | 1.12168 | **5.30517** | 0.06200 |
| 225 | **0.02190** | 0.00008 | **30.81972** | 0.10551 | **4.81783** | 0.04500 |
| 200 | **0.02058** | 0.00004 | **29.15933** | 0.06384 | **4.20267** | 0.05300 |
| 175 | **0.01959** | 0.00032 | **26.81016** | 0.43923 | **3.76533** | 0.03850 |
| 150 | **0.01859** | 0.00014 | **24.20722** | 0.18974 | **3.31300** | 0.00750 |
| 125 | **0.01736** | 0.00022 | **21.60059** | 0.26649 | **2.68583** | 0.02900 |
| 100 | **0.01626** | 0.00005 | **18.45593** | 0.05681 | **2.09617** | 0.06600 |

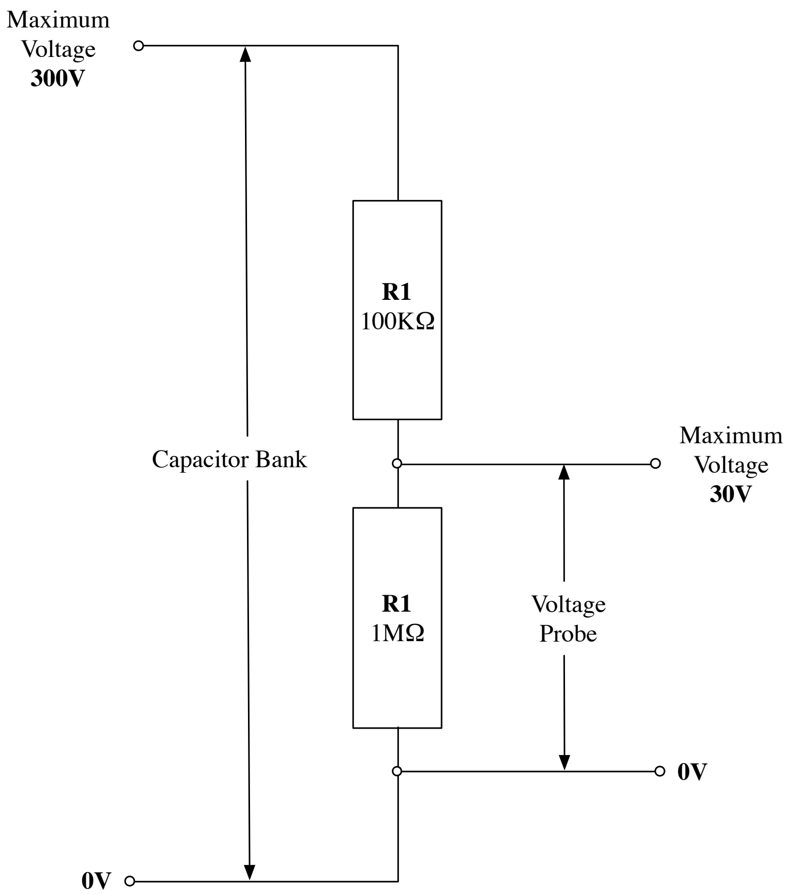
## 200 Turns in Coil

| VOLTAGE (V) | Pulse Duration (seconds) | Uncertainty (± Seconds) | Avg. Pulse Current (Amps) | Uncertainty (±Amps) | Projectile Velocity (m/s) | Uncertainty (± m/s) |
|---|---|---|---|---|---|---|
| 300 | **0.02543** | 0.00027 | **35.39082** | 0.36870 | **4.38910** | 0.23550 |
| 275 | **0.02372** | 0.00018 | **34.77918** | 0.25759 | **3.90350** | 0.05200 |
| 250 | **0.02183** | 0.00095 | **34.38681** | 1.56054 | **3.60183** | 0.08650 |
| 225 | **0.01971** | 0.00076 | **34.27092** | 1.29862 | **3.16000** | 0.06650 |
| 200 | **0.01813** | 0.00020 | **33.09892** | 0.36633 | **2.76667** | 0.04950 |
| 175 | **0.01745** | 0.00041 | **30.08930** | 0.70244 | **2.41750** | 0.13650 |
| 150 | **0.01619** | 0.00021 | **27.79965** | 0.36193 | **2.04017** | 0.04200 |
| 125 | **0.01497** | 0.00027 | **25.05197** | 0.44716 | **1.56083** | 0.13200 |
| 100 | **0.01390** | 0.00029 | **21.58149** | 0.43485 | **1.11267** | 0.04150 |

## 100 Turns in Coil

| VOLTAGE (V) | Pulse Duration (seconds) | Uncertainty (± Seconds) | Avg. Pulse Current (Amps) | Uncertainty (±Amps) | Projectile Velocity (m/s) | Uncertainty (± m/s) |
|---|---|---|---|---|---|---|
| 300 | **0.02259** | 0.00011 | **39.84993** | 0.19401 | **2.23417** | 0.05850 |
| 275 | **0.02053** | 0.00027 | **40.19924** | 0.53498 | **2.04533** | 0.06400 |
| 250 | **0.01936** | 0.00024 | **38.73614** | 0.47735 | **1.75483** | 0.07700 |
| 225 | **0.01675** | 0.00009 | **40.29431** | 0.21892 | **1.38750** | 0.04150 |
| 200 | **0.01557** | 0.00010 | **38.54877** | 0.24719 | **1.08600** | 0.02600 |
| 175 | **0.01511** | 0.00012 | **34.75011** | 0.28651 | **0.89515** | 0.01150 |
| 150 | **0.01348** | 0.00022 | **33.39528** | 0.52515 | **0.50850** | 0.02150 |
| 125 | **N.A.** | N.A. | **N.A.** | N.A. | ***Not Enough Power*** | N.A. |
| 100 | **N.A.** | N.A. | **N.A.** | N.A. | ***Not Enough Power*** | N.A. |

# Appendix 4 - (Potential Divider Circuit)




Above is a showing the layout of the potential divider circuit used to step down the voltage from the capacitor bank in order to not damage the Voltage probe. The voltage probe was then used to determine the pulse time of the capacitor discharges. Note that it is not shown in Figure 2B, but this circuit was placed in parallel between the capacitor bank and the regular Voltmeter.